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DIGITAL COMPUTER SIMULATION OF THE PERFORMANCE OF SMALL SOLID PROPELLANT ROCKET MOTORS

by

JAMES M. DE LEO

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DIGITAL COMPUTER SIMULATION OF THE PERFORMANCE OF SMALL SOLID PROPELLANT ROCKET MOTORS

OMS Code 5010.11.844.00 DA Project 5897-01-006

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ABSTRACT

A mathematical model of a small solid propellant rocket has been programmed for use with a Univac Solid State 90 Digital Computer by means of Fortran I and Fortran II. In addition to calculating the pressure-time and thrust-time transients, the program computes time-averaged pressure, thrust coefficient, thrust, flow rate, and burning rate, as well as total impulse and specific impulse.

The report includes the derivation of equations, the method of solution, the program in Fortran I and Fortran II, an example solution compared with experiment, and a recommended procedure for use of the computer program in the design and development of a small solid propellant rocket motor.
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INTRODUCTION

Basic rocket equations and empirically derived propellant constants and motor dimensions may be used to predict the performance of small solid propellant rocket motors. Programming these equations on a digital computer reduces the solution time for such performance estimates to a matter of minutes. This report is concerned with the preparation of a suitable digital computer program for this purpose.

Specifically, this analysis is intended to predict the pressure and thrust vs time transients in rocket motors that are large enough to assume equilibrium between the gas generation rate and the gas discharge rate, and small enough so that temperature gradients in the propellant grains may be considered negligible.

The objective of this report is to present a digital computer program which will serve as a satisfactory tool in the design and development of certain small solid propellant rocket motors.

ASSUMPTIONS

The following assumptions have been made in the construction of the mathematical model of the solid propellant rocket motor.

1. Initially (time = 0), the igniter system has pressurized the chamber to a pressure, $P_0$.

2. All propellant surfaces are ignited isochronically, and burning is directed normal to all exposed propellant surfaces.

3. Propellant geometry can be described by a form function, and propellant is completely burned upon disappearance of the web.

4. The space-averaged temperature of the combustion products remains constant at the isobaric (i.e., constant volume, adiabatic) flame temperature, $T$.

5. The gas generation rate and the gas discharge rate are equal.

6. Temperature gradients in the propellant grain are negligible.

*See REFERENCES.*
EQUATIONS

The equations predicting the pressure vs time relationship are combined in Appendix A into the following:

\[ P = P_0 + \frac{12.0 \left\{ \left[ -\theta z^2 + z(1 + \theta) \right] c_p - \frac{C_d A_t}{B} \int_0^w \frac{P}{p_{n_{av}}} dz \right\}}{\rho v_0 + \left[ -\theta z^2 + z(1 + \theta) \right] c_p} \rho v_p T_k \]

(1)

The thrust value corresponding to a given pressure value is defined by:

\[ F = \varphi \delta C_f A_t P \] (Ref 2)

(2)

in which:

\[ C_f = \left\{ \frac{2y^2}{\gamma - 1} \left( \frac{2}{\gamma + 1} \right)^{\gamma + 1} \left[ 1 - \left( \frac{P_e}{P} \right)^\gamma \right] \right\}^{1/2} \left( \frac{P_e - P_a}{P} \right) \cdot \epsilon \] (Ref 3)

(3)

and

\[ \epsilon = \frac{\left( \frac{\gamma - 1}{2} \right)^{1/2} \left( \frac{2}{\gamma + 1} \right)}{\left( \frac{P_e}{P} \right)^{1/\gamma} \left[ 1 - \left( \frac{P_e}{P} \right)^{\gamma + 1} \right]^{1/2}} \] (Ref 3)

(4)

Equations 1 to 4 are used to predict the pressure and thrust vs time curves. In addition to predicting these relationships, it is desirable to compute additional parameters that characterize a specific solid propellant rocket motor. Additional parameters computed in this program are:

A. Time-Averaged Pressure

\[ \bar{P} = \frac{1}{t_c} \int_0^{t_c} P \, dt \] (5)

*Unless otherwise stated, the word "pressure" will refer to head pressure.
B. Time-Averaged Thrust Coefficient

\[ \bar{C}_f = C_f^C - \frac{c^v}{P} \]  

C. Time-Averaged Thrust

\[ \bar{F} = \phi \delta \bar{C}_f \bar{P} A_t \]  

D. Time-Averaged Gas Flow Rate Through the Nozzle

\[ c_{ex} = C_d A_t \bar{P} \]  

E. Specific Impulse

\[ I_{sp} = \frac{\bar{F}}{c_{ex}} \]  

F. Total Impulse

\[ I_{tot} = \bar{F} t_c \]  

G. Time-Averaged Burning Rate

\[ \bar{r} = B \bar{P}^n \]  

These additional parameters are computed after the computation of the pressure and thrust vs time relationships.

**METHOD OF SOLUTION**

Equation 1 is an implicit expression of pressure, \( P \), as a function of the fraction of web burned, \( z \). All other values in this equation are considered constants. Since \( z \) ranges from 0 to 1, it is conveniently used as the independent variable. For a given value of \( z \) and an assumed value of \( P \), the right hand side of Equation 1 may be calculated. If the result of this calculation is in close agreement with the assumed value of \( P \), then the assumed value is adequate. If there is poor agreement, then another estimate for \( P \) is made and the calculation is repeated. This process continues until good agreement between the assumed value and the calculated value is obtained. If systematic estimates are made, the number of iterations are reduced. The specific details of this iteration as used in the computer program are found in Appendix B.
For each value of $P$ computed in this fashion, a corresponding value of $F$ is calculated by means of Equation 2. Before Equation 2 is solved, the parametric Equations 3 and 4 must be solved for $C_f$. Details of the $C_f$ calculation by use of the Newton-Rapthson Method are found in Appendix C.

An explanation of the $\phi \delta$ quantity is in Appendix D.

COMPUTER PROGRAM

The method of solution described above and in Appendices B and C is the basis for the computer program which was written in Fortran I and Fortran II for use with a Univac Solid State 90 Computer. These programs appear in Appendix E of this report.

To test this computer model, parameters describing the XM15 Catapult, an experimental solid propellant rocket motor designed to eject the nose capsule of a fighter aircraft, were used as input data with the program. These parameters, together with the computer results compared to experimental results, are shown in Appendix F.

CONCLUSIONS

1. The method presented is useful in predicting the performance of small solid propellant rocket motors as used in Propellant Actuated Devices (PAD).

2. Fortran has proved effective in accomplishing a digital computer simulation of small solid propellant rocket motors.

RECOMMENDED PROCEDURE

It is recommended that the computer program presented here be incorporated in the performance estimation and early design and development phases of small propellant rocket motors as used in PAD. The procedure is as follows:

1. Select initial design parameters consistent with envelope requirements.
2. Perform exploratory computer run to evaluate initial design parameters.

3. Adjust parameters and make computer tests until agreement between the computer output and the design requirements is obtained.

4. Build and test and experimental model based on the parameters selected in 3 above.

5. Adjust propellant parameters to match the experimental performance. This is the calibrated analytical model.

6. Use calibrated analytical model to establish optimum design parameters.

FUTURE WORK

Future work will be directed toward the preparation of programs for the automated design of more complex PAD systems.
APPENDIX A
DERIVATION OF EQUATION 1

Each time the propellant web regresses by an increment \( dz = \frac{dw}{w} \),
the weight of charge burnt \( (c_b) \), in pounds, is given by

\[
c_b = z(1 + \theta f) \ c_p
\]  \hspace{1cm} (A-1)

where \( z \) is the total fraction of web burnt (dimensionless);
\( f \) is the total fraction of web remaining, i.e., \( 1 - z \)
(dimensionless);
\( \theta \) is the form factor (dimensionless);
\( c_p \) is the total charge weight at the start (lb).

The free volume \( (V) \), in cu in., is the sum of the initial free
volume \( (V_o) \), in cu in., and the volume left by the burned propellant.

\[
V = V_o + \frac{c_b}{\rho}
\]  \hspace{1cm} (A-2)

where \( \rho \) is the gravimetric density of the propellant \((lb/in.^3)\).

The rate of web regression \( (r) \) in inches per second is given by

\[
r = B P_{av} n
\]  \hspace{1cm} (A-3)

where \( B \) is the regression rate coefficient \((in./sec)(in.^2/lb)^n\);
\( n \) is the regression rate exponent (dimensionless);
\( P_{av} \) is the pressure \((lb/in.^2)\) averaged between the pressures cor-
responding to \( z \) and \( z + dz \), i.e., \( P_{av} = \frac{P(z) + P(z+dz)}{2} \).

The time \( (dt) \), in seconds, required for the web to burn a
fraction \( dz \) is given by

\[
dt = \frac{w dz}{r}
\]  \hspace{1cm} (A-4)
where \( w \) is the total web (in.).

The weight of gas \( (cex) \), in pounds, that has been expelled up to time \( t \) is given by

\[
cex = C_d A_t \int_0^t P \, dt
\]

where \( C_d \) is the discharge coefficient \((\text{sec}^{-1})\);

\( A_t \) is the throat area of the nozzle \((\text{in.}^2)\);

\( P \) is the pressure at time \( t \) \((\text{lb/in.}^2)\).

The pressure at time \( t \) is given by

\[
P = P_0 + 12.0 \left( c_b - cex \right) \frac{F_p T_k}{V}
\]

where \( (c_b - cex) \) is the weight of gas in the chamber \((\text{lb})\);

\( F_p \) is the impetus of the propellant \((\text{ft-lb/lb})\);

\( T_k \) is the ratio of the isobaric flame temperature \((^\circ \text{K})\) of the propellant to the isochoric flame temperature \((^\circ \text{K})\) of the propellant.

Combining Equations A-1 through A-6,

\[
P = P_0 + \frac{12.0 \left[ z(1 + \theta f) c_p - C_d A_t \int_0^t P \, dt \right] F_p T_k}{V_0 + z(1 + \theta f) c_p}
\]

(A-7)

Since \( f = 1 - z \),

\[
P = P_0 + \frac{12.0 \left[ -\theta z^2 + z(1 + \theta) \right] c_p - C_d A_t \int_0^t P \, dt \} F_p T_k}{V_0 + \left[ -\theta z^2 + z(1 + \theta) \right] c_p}
\]

(A-8)

Combining Equations A-3 and A-4,

\[
dt = \frac{w \, dx}{BP_n \, \text{av}}
\]

(A-9)
Integrating,
\[ \int_{0}^{t} dt = \int_{z(0)}^{z(t)} \frac{w}{B \rho_{av}} \frac{dz}{w} \]

(A-10)

Since \( z(0) = 0 \) and \( z(t) = z \), and since \( w \) and \( B \) are constants,
\[ t = \frac{w}{B} \int_{0}^{z} \frac{dz}{\rho_{av}} \]

(A-11)

Substituting Equations A-9 and A-11 into the integral expression in the numerator of Equation A-8, and multiplying the numerator and denominator by \( \rho \),
\[
P = P_{o} + \frac{12.0 \left\{ \left[ -\theta z^2 + z(1 + \theta) \right] c_{p} - \frac{C_{d} A_{t} w}{B} \int_{0}^{\rho_{av}} \frac{dz}{\rho_{av}} + \frac{P}{\rho_{av}} \int_{0}^{\rho_{av}} dz \right\} \rho F_{p} T_{k}}{\rho V_{o} + \left[ -\theta z^2 + z(1 + \theta) \right] c_{p}}
\]

(A-12)

Equation A-12 is Equation 1 in the text.
APPENDIX B

ITERATIVE PROCEDURE FOR SOLVING EQUATION 1

The procedure for calculating $P$ in Equation 1 is as follows:

1. At $t = 0$, $P = P_0$ and $z = 0$.
2. $z = z + dz$.
5. Estimate a value of $P$, making sure the estimated value is less than the actual value.
8. Compute $c_{ex}$ using Equation A-5.
10. If the difference between the value of $P$ estimated in step 5 and the value of $P$ calculated in step 9 is large, increment the estimated value of $P$ by some $\Delta P > 0$ and go back to step 6. If this difference is small, the value of $P$ computed in step 9 is adequate as the pressure, $P(z + dz)$, at the time $t + dt$, where $t$ is the time corresponding to the pressure $P(z)$ and $dt$ is the value last computed in step 7. Proceed to step 11.
11. If $z < 1$, go back to step 2; otherwise, set $z = 1$ and proceed.
12. Since $z = 1$, the propellant is completely consumed and gas is no longer being generated. Hence, the gas in the chamber will be expelled and the pressure will rapidly decrease. The independent variable is now time, $t$. So, $t = t + dt$, where $dt$ is designated.
13. Set $c_b = c_p$.
15. Estimate a value for $P$, making sure the estimated value is less than the actual value.


18. Compare the pressures, as was done in step 10; increment the estimated pressure and go back to step 16. When good agreement is obtained, proceed to step 19.

19. Increment $t$ by $dt$, and go back to step 15. Continue in this manner until the pressures become relatively low; then stop.

As indicated above and in the schematic flow chart (Figure B-1), the procedure is to assume the web has regressed by a fraction $dz$; estimate what the resulting pressure would be, and compare the calculated pressure with the estimated pressure. This comparison is made by checking the truth of the following inequality:

$$|P - P_{e}| < \text{EPS}.$$ 

If the inequality is false, a new estimated pressure is obtained by incrementing the present estimated pressure by EPS. This process is continued until the inequality holds true.

The question that naturally arises is: "What values of $dz$ and EPS are required to produce a satisfactory simulation?" Apparently, smaller values of $dz$ and EPS are closer to the limit than larger values. However, smaller values of $dz$ and EPS mean not only more iterations in finding the pressure corresponding to a given value of $dz$, but also more values of $dz$ to evaluate.

To determine the values of $dz$ and EPS that could be used to produce a good simulation and yet not require unrealistic computer time, a hypothetical rocket motor was assumed and pressure vs time curves were obtained for two cases: Case I, $dz = 0.01, \text{EPS} = 25\text{ psi}$; Case II, $dz = 0.001, \text{EPS} = 2.5\text{ psi}$. Figure B-2 illustrates the significance of using these sets of variables for EPS and $dz$. For Case II, ten equally-spaced points, whose ordinates are correct within $\pm 2.5\text{ psi}$, are determined on the interval $z = 0$ to $z = 0.01$. In Case I, only one point whose ordinate is correct within $\pm 25\text{ psi}$ is determined.

Figure B-3 shows pressure-time curves produced by the computer in Case I and Case II. It can be seen that the curves are remarkably close. Since Case II requires a considerable amount of computer time, the small over-all precision obtained is hardly warranted. Thus, the values of $dz = 0.01$ and $\text{EPS} = 25\text{ psi}$ may be considered sufficient for good simulation.

The computation of the pressure and thrust vs time curves is terminated when the values computed for the thrust coefficient, $C_f$, become negative.
Figure B-1. Schematic Flow Chart
Figure B-2. Illustration of dz and EPS choices
Figure B-3. Pressure vs Time for

dz = 0.001, EPS = 2.5 psi

dz = 0.01, EPS = 25 psi
APPENDIX C

USE OF NEWTON-RAPHTSON METHOD IN SOLVING FOR $C_f$

The values of $\epsilon$, $\gamma$, $p_a$, and $p_e/P$ in the pair of parametric equations (Equations 3 and 4) are considered constant. Hence, the thrust coefficient, $C_f$, varies with the pressure, $P$. To reduce these equations into a form easier to work with, the following substitutions are made:

\[
G_1 = \left(\frac{\gamma - 1}{2}\right)^{1/2} \left(\frac{2}{\gamma + 1}\right)^{(\gamma + 1)/2}
\]

\[
G_2 = \frac{1}{\gamma}
\]

\[
G_3 = \frac{\gamma - 1}{\gamma}
\]

\[
x = \frac{p_e}{p}
\]

Equation 4 then becomes

\[
\epsilon = \frac{G_1}{G_2^{1/2} (1 - x G_3)^{1/2}}
\]

which may be rearranged into the following form

\[
x^{2G_2} - x^{2G_2 + G_3} = \left(\frac{G_1}{\epsilon}\right)^2
\]

or

\[
x^m - x^n = \ell
\]

in which

\[
\ell = \left(\frac{G_1}{\epsilon}\right)^2 = \frac{1}{\epsilon^2} \left(\frac{\gamma - 1}{2}\right) \left(\frac{2}{\gamma + 1}\right)^{(\gamma + 1)/2}
\]

\[
m = 2G_2 = \frac{2}{\gamma}
\]

\[
n = 2G_2 + G_3 = \frac{2}{\gamma} + \frac{\gamma - 1}{\gamma} = \frac{1 + \gamma}{\gamma}
\]
In the physical situation, \( n \) is always greater than \( m \), and \( x \) is always less than 1. We are interested in determining the value of \( x \) that makes Equation C-7 hold true, or, we are looking for the roots of \( f(x) \) in Equation C-10.

\[
f(x) = x^m - x^n - \ell \tag{C-10}
\]

As a typical example, consider the case when \( \gamma = 1.2 \) and \( \epsilon = 4 \). Using Equations C-8, C-9, and C-10, respectively, it is determined that

\[
\ell = 0.0021905869
\]
\[
m = 1.6666667
\]
\[
n = 1.8333333
\]

Figure C-1 is a plot of \( y = x^m \) and \( y = x^n \), in which \( m \) and \( n \) have the above values. Figure C-2 shows a plot of \( y = x^m - x^n \), a plot of \( y = \ell \), and, also, a plot of the difference of these graphs, viz, \( f(x) = x^m - x^n - \ell \). It is evident from this figure that \( f(x) \) has two roots. The smaller root is desired since it is physically meaningful.

To use the Newton-Raphson (Ref 4) method, we must have an analytical expression for the derivative of \( f(x) \).

\[
f'(x) = m x^{m-1} - n x^{n-1} \tag{C-11}
\]

Making sure that the initial guess for \( x \) is far to the left of the point where \( f(x) \) crosses the abscissa, compute the value for \( f(x) \) at \( x = x_0 \), determine the value of \( f(x)/f''(x) \), add this to \( x_0 \) and repeat the process using this new value for \( x \) as dictated by Equation C-12.

\[
x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)} \tag{C-12}
\]

The iteration ceases when \( f(x) \) is zero or very small.

Table C-I shows the result of this procedure using the values of \( \gamma \) and \( \epsilon \) cited above and using an initial guess of 0.001 for \( x \). Only four iterations were required in determining the desired root.

Having employed the Newton-Raphson method in solving for \( x \), and recalling that \( x \) is really \( p_0/P \), the value for \( C_f \) may be calculated directly by means of Equation 3.
Figure C-2. Plot of $y = x^2 - 1.8333$ and $y = 0.002191$.

$y = x^{1.6667} - 1.8333$.
<table>
<thead>
<tr>
<th>$x_n$</th>
<th>$f(x_n)$</th>
<th>$f'(x_n)$</th>
<th>$f(x_n)/f'(x_n)$</th>
<th>$x_{n+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0010000000</td>
<td>-0.0021837492</td>
<td>0.018691532</td>
<td>-0.20091255</td>
<td>0.20191255</td>
</tr>
<tr>
<td>0.20191255</td>
<td>0.14074955</td>
<td>0.090326570</td>
<td>0.15582298</td>
<td>0.046089570</td>
</tr>
<tr>
<td>0.046089570</td>
<td>0.00018654790</td>
<td>0.073131940</td>
<td>0.002508403</td>
<td>0.043538730</td>
</tr>
<tr>
<td>0.043538730</td>
<td>0.0000018187</td>
<td>0.071691210</td>
<td>0.0000253685</td>
<td>0.043513361</td>
</tr>
<tr>
<td>0.043513361</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The dimensionless quantity \( \phi \delta \) (ref 2) in Equation 2 is a combined efficiency factor and nozzle expansion factor. The value \( \phi \) is an empirically derived constant and is usually about 0.96 for good nozzle designs. The value \( \delta \) is express as

\[
\delta = 0.5 \left( 1 + \cos \alpha \right)
\]

where \( \alpha \) is the half angle of the expansion cone.

Figure D-1 is a plot of the combined factor \( \phi \delta \) as a function of \( \alpha \) for values of \( \phi \) equal to 0.94, 0.96, and 1.0.
Figure D-1. Combined Energy Loss Correction, $\phi$, and Divergence Angle Correction, $\delta$, vs Divergence Angle, $\alpha$
APPENDIX E

THE FORTRAN PROGRAMS

Rocket Motor Performance Studies - Fortran I

```
TRACe
ROCKET MOTOR PERFORMANCE
STUDIES
READ P
READ CP
READ FP
READ DEN
READ VOLUME
READ CD
READ EPS
PRINT P
PRINT CP
PRINT FP
PRINT DEN
PRINT VOLUME
PRINT CD
PRINT EPS
DIF (1/2+1)
EXP(G1/G)
G2=G2/G1/G
AM2=AM1/6
AN1=AN2/6
C4=(2*U+G)/(U+1)*U+1)
C5=AN1/42
1 PXA AN=1-(X+AN)-AL
IF(AVPS>(1.0E-7)G2+3)
3 PXA AN=1-(AN+1))
HEX/FXK
AN=H
2 DELCPS=SNF(X)
IF(UE>3.5)
5U DELRU+U-UE
21G1=EXP(UE)
G0 TO 52
51 21GEXP(FUE)
54 21G(x)+1.0-Z1)
Z2S=RT(I/21)
Z3=EXK
C3K=(Z2+Z3)
C4=CN5+PA
T3=U
G3=U0
PX3K=0.0
Z2I
61 FXA=U-2
C3=211.0+0.0(FORM+PR1)+CP
UE+VU+62/DEN
P3SP
64 PGPA=PR1
PAVX(I+1/2)+U
RATE=60*(PAV*BN)
DTND/MATE
PRMTK+(PAV*UT)
C3=CDATK
GAS(1)=C3
7U PR1(U+1)K=FO
IF(AVSP-(1.0E-7)X)
53 21G
54 21G
END
```
ROCKET MOTOR PERFORMANCE
STUDIES

READ 20, P
READ 23, CP, WEB, FORM
READ 20, FPE, DEK, B, BMI, G, TX
READ 25, VOL, AT, E, DELT
READ 26, CD, PA
READ 27, EPS, GUESS, XO, DZ, TI, AID
PRINT 100, P
PRINT 101, CP, WEB, FORM
PRINT 102, FPE, DEK, B, BMI, G, TX
PRINT 103, VOL, AT, E, DELT
PRINT 104, CD, PA
PRINT 105, EPS, GUESS, XO, DZ, TI, AID

DIN02*WEB
E18(1.0+1.012,0)
EXP0(1.0+1.010)
E28(1.0+1.010)*EXP0
ALB(1.0+1.010)
AHE2.0/8
AHI1.0+G1/8
CSN2.00001(6.101)+G2
CSSN(1.01/6)
PRINT 1115
ZKX
1 F=EX(S*AN)=X(AN)+AL
IF (1-ABS (FX)) = 1.0*1.0 2.0*1.0
3 FPE(AM=XAM(AM-AN.1))
1 -AM(AN(AM-AN.1))
INTER=FPEX
ZKX=H
GO TO 1
2 DELCSEQMN(X)
51 ZINEP(DEL)
52 ZINS4.1(0-121)
Z# SQRT(121)
Z2MEX
CFM222X2
CFM222PA
TUX=80
PGEQ.0
PRTX=0
2ZTQ
61 PDEP-EPS
PAVE(PBP+PSI+2.0)
RATEX=PAVE(BN)
DINORATE
PATXPATX+PAYVOT
CERACOAPX0
70 PRIN0+GASSMFPX+WIN+PA
IF (ABS (P-PG 1-EPS)) 63.66.62
63 PATXFPATX+PAYVOT
CFSQPPATQF(P)
IF(CF17*67.66
64 FORCSEPAPP+DEL
TQ+DT
65 SMTPK=QAS

PRINT 106, ZT+T+P0
1IFZ-1.015+66.66
65 ZN+OZ
PGRF=GUESS
GO TO 61
66 ZN+OZ
PGRF=AID(GUESS)
RATE0,0
DINTI
CENG
VWVOL+(CBS/DEI)
VPAP
PDEP
P21
PAVMP+PSI+PSI+2.0
PXTMPX+PAYVOT
CERACOAPX0
BASECB-EX
PRIN0+GASSMFPX+WIN+PA
IF (ABS (P-PG 1-EPS)) 63.66.62
67 PAVERFSKFT
PEBFAPAEVAX
PRINT 107, PAVG, PEX, X
CFAV=CFN+CFN/PAVG
PRINT 108, CFAV
FAVMP+PAEPAVAT+DEL
PRINT 109, PAVG
VWVOL+CB/DEI
PRINT 110, ROOT
SPIMAVG/DOT
PRINT 111, SPI
TIME=FAVAG
PRINT 112, TIME
RAVGB=PAVG=BN
PRINT 113, RAVG
12 FORMAT (E13.8)
23 FORMAT (E13.8)
24 FORMAT (6E13.8)
25 FORMAT (4E13.8)
26 FORMAT (2E13.8)
27 FORMAT (6E13.8)
100 FORMAT (6HPRSS + E14.8 + 2)
101 FORMAT (6HCP + E14.8 + 2X 6HWEB + E14.8 + 2X 6HE)
102 FORMAT (6HCP + E14.8 + 2X 6HWEB + E14.8 + 2X 6HE)
1 = E14.8 + 2X 6HE + E14.8 + 2X 6HE
6HTK + E14.8 + 2X 6HE + E14.8 + 2X 6HE
6HTK + E14.8 + 2X 6HE + E14.8 + 2X 6HE
6HTK + E14.8 + 2X 6HE + E14.8 + 2X 6HE
3 = E14.8 + 2X 6HE + E14.8 + 2X 6HE
114 FORMAT (7HPRACQ OF WEB-)
1 THRUST + /
100 FORMAT (E14.8 + E14.8 + 4)
107 FORMAT (3HMPAV + E14.8 + 3X 3HMPX)
100 FORMAT (3HMPAV + E14.8 + 3X 3HMPX)
129 FORMAT (4HMPAV + E14.8 + 3)
110 FORMAT (5HMPAV + E14.8 + 3)
111 FORMAT (3HMPAV + E14.8 + 3)
112 FORMAT (4HMPAV + E14.8 + 3)
113 FORMAT (4HMPAV + E14.8 + 3)
END

Rocket Motor Performance Studies - Fortran II
APPENDIX F
EXAMPLE - COMPARISON OF COMPUTER AND EXPERIMENTAL RESULTS

The parameters used to simulate the operation of the XM15 Rocket Catapult are presented in Table F-I.

TABLE F-I. Parameters Describing the XM15 Rocket Motor

\[
\begin{align*}
P_0 & = 250 \text{ lb/in.}^2 \\
\epsilon & = 4.662 \\
\omega & = 0.88 \text{ in.} \\
\theta & = .18 \\
A_t & = 19.24 \text{ in.}^2 \\
\epsilon & = 4.662 \\
\varphi & = .95 \\
C_D & = .00691 \\
F & = 297,700 \text{ ft lb/lb} \\
\rho & = 0.0576 \text{ lb/in.}^3 \\
\beta & = 0.086 \text{ (in./sec)(in.}^2/\text{lb)}^n \\
\gamma & = 1.23 \\
\nu & = .42 \\
\phi & = .95 \\
C_l & = .815 \\
\nu & = 2120 \text{ in.}^3 \\
\gamma & = 1.23 \\
T_k & = .815 \\
\nu & = 2120 \text{ in.}^3 \\
\end{align*}
\]

Figure F-1 shows a comparison between the calculated pressure vs time curve and a pressure vs time curve obtained from a test firing of the XM15 rocket catapult. Figure F-2 is a plot of the corresponding calculated thrust vs time curve.

Table F-II is a listing of additional parameters computed at the end of the program.

TABLE F-II. Additional Computed Parameters

\[
\begin{align*}
\frac{P_{ex}}{P} & = .03303 \\
P_{ex} & = 38.38 \text{ lb/in.}^2 \\
\tilde{F} & = 1101.4 \text{ lb/in.}^2 \\
C_l & = 1.561 \\
\tilde{F} & = 3,142 \text{ lb} \\
\end{align*}
\]

\[
\begin{align*}
\dot{w} & = 146.4 \text{ lb/sec} \\
I_{sp} & = 214.6 \text{ sec} \\
I_{tot} & = 17,590 \text{ lb-sec} \\
\tilde{F} & = 1.629 \text{ in./sec}
\end{align*}
\]
Figure F-1. Comparison between Computer Pressure vs Time Curve and Experimental Pressure vs Time Curve obtained from XM15 Test Firing
Figure F-2. Computer Thrust vs Time Curve, simulating XM15 Test Firing
REFERENCES


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The report includes the derivation of equations, the method of solution, the program in Fortran I and Fortran II, an example solution compared with experiment, and a recommended procedure for use of the computer program in the design and development of a small solid propellant rocket motor.
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