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Geophysical Sciences Laboratory Report 63-12

A PROPOSED SPECTRAL FORM FOR FULLY DEVELOPED WIND SEAS BASED ON THE SIMILARITY THEORY OF S. A. KITAIGORODSKII

by

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Technical Report Prepared for
U. S. Naval Oceanographic Office
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Abstract

The data for the spectra of fully developed seas obtained by Moskowitz (1963) for wind speeds from 20 to 40 knots as measured by anemometers on two weather ships are used to test the similarity hypothesis and the idea that, when plotted in a certain dimensionless way, the power spectra for all fully developed seas should be of the same shape as proposed by Kitaigorodskii (1961). Over the important range of frequencies that define the total variance of the spectrum within a few percent, the transformed plots yield a non-dimensional spectral form that is nearly the same over this entire range of wind speeds within the present accuracies of the data. However, since slight variations of the wind speed have large effects on the location of this non-dimensional spectral form, inaccuracies in the determination of the wind speed at sea allow for some latitude in the final choice of the form of the spectrum. Also since the winds used to obtain the non-dimensional form were measured at a height greater than ten meters, the problem of relating the spectral form to a standard anemometer height arises. The variability introduced by this factor needs to be considered.

The results, when errors in the wind speed, the sampling variability of the data, and the anemometer heights are considered, suggest a spectral form that is a compromise between the various proposed spectra and that has features similar to many of them. A spectral form is recommended for tentative application to the problem of wave forecasting by spectral techniques.

Given improved wind speed measurements at several elevations for longer averaging times, longer wave records analyzed so as to fit the procedures more readily, and better wave data, the form of the spectra of fully developed wind seas and seas limited by either fetch or duration can be determined to even greater precision than that obtained here.

Introduction

Although dimensional analysis is a useful tool in many problems, Neumann and Pierson (1957) have in the paper just cited and in other papers cast a skeptical eye on its application to wave theory. A brief note in preparation by Pierson will comment on this point in connection with the very high frequency part of the spectrum. Some recent work by Kitaigorodskii provides such a thorough discussion of the application of the theory of similarity to the problem of determining the spectra of wind waves and such explicit conclusions that it was essential to test his proposals by means of the data obtained by Moskowitz (1963). Also, the upper bound provided by Phillips has apparently been verified over a certain range of frequencies by a number of individual spectral estimates (Phillips, 1963; Longuet-Higgins, Cartwright, and Smith, 1963) and provides a part of the theory needed to obtain a possible spectral form.

As the content of this paper develops, it will be seen that such procedures provide a very powerful tool, when combined with the statistical tests applied by Moskowitz (1963), for the precise determination of the spectra of fully developed wind seas over an important range of frequencies. It will also be seen that present data provide a spectral form that can serve for many practical applications.

The work of Kitaigorodskii

Kitaigorodskii (1961) makes the assumption that for the dominant part of the power spectrum of a wind sea -- that part that determines the total variance within a few percent--the spectrum can be a function of only four variables as in equation (1).

$$S(f) = F(f, g, U_{+}, X) \tag{1}$$

In equation (1), f is the frequency, g is gravity, U_+ is the friction velocity, and X is the fetch. A transformation can readily be made to duration if it, instead of fetch, is the limiting factor for a partially developed sea.

The Charnock-Ellison friction velocity (Ellison, 1956) might conceivably be the definitive parameter in this problem, but recent work by Schmitz (1962) raises a question even here. Since in any case, U_+ is not measured at sea, we shall use the wind (U) as obtained by an anemometer and reported by the weather ships that took the wave observations instead of introducing an assumption as to the form of the drag coefficient to determine U_+. It must be kept in mind that this may introduce other factors for future consideration. In particular, the anemometers were not ten meters above the surface which complicates the interpretation of the results.

With this change, equation (1) becomes equation (2). It will be assumed that the properties deduced in terms of U_+ will also hold for U.

$$S(f) = F(f, g, U, X) \qquad (2)$$

A dimensionless spectrum as a function of a dimensionless frequency and a dimensionless fetch is then obtained according to the following equations.

$$\bar{S}(\bar{f}) = \bar{F}_1(\bar{f}, \bar{X}) \qquad (3)$$

$$\bar{S}(f) = \frac{S(f)g^3}{U^5} \qquad (4)$$

$$\bar{f} = Uf/g \qquad (5)$$

$$\bar{X} = gx/U^2 \qquad (6)$$

Since the data of Moskowitz are for fully developed seas the dependence on $\bar{X}$ vanishes and

$$\bar{S}(f) = \bar{F}_2(\bar{f}) \tag{7}$$

Kitaigorodskii has deduced a number of properties of the spectrum from this analysis. The wave height must follow a U^2 law, for example. The law that $\frac{f_{max} U}{g}$ = const must also hold. If U_2 is greater than U_1 for fully developed seas, then

$$S(f, g, U_2) \geq S(f, g, U_1) \text{ for all } f . \tag{8}$$

The only presently proposed spectral forms that agree with this analysis are the spectra proposed by Roll and Fischer (1956), the asymptotic form proposed by Phillips (1958), and some of the functions given by Bretschneider (1963).

Application to new data

It is a simple matter to test these considerations by means of the data provided by Moskowitz. One way is to divide the tabulated values by the fifth power of the wind velocity and plot the results against the product of f and U. Such a plot has the rather odd units of $(FT)^2$, $(KNOT)^{-5}$ for the vertical scale,and $(KNOT)(SEC)^{-1}$ for the horizontal scale, but this can easily be corrected by re-labeling the scale after the appropriate powers of g are introduced and a consistent system of units is used. Figure 1 shows the results of this calculation for the range of frequencies where the data are believed reliable. The original units are shown on the outside vertical and bottom horizontal scales and the dimensionless values are shown on the inside vertical and upper horizontal scales.

With the usual scatter in such data, the curves have been transformed

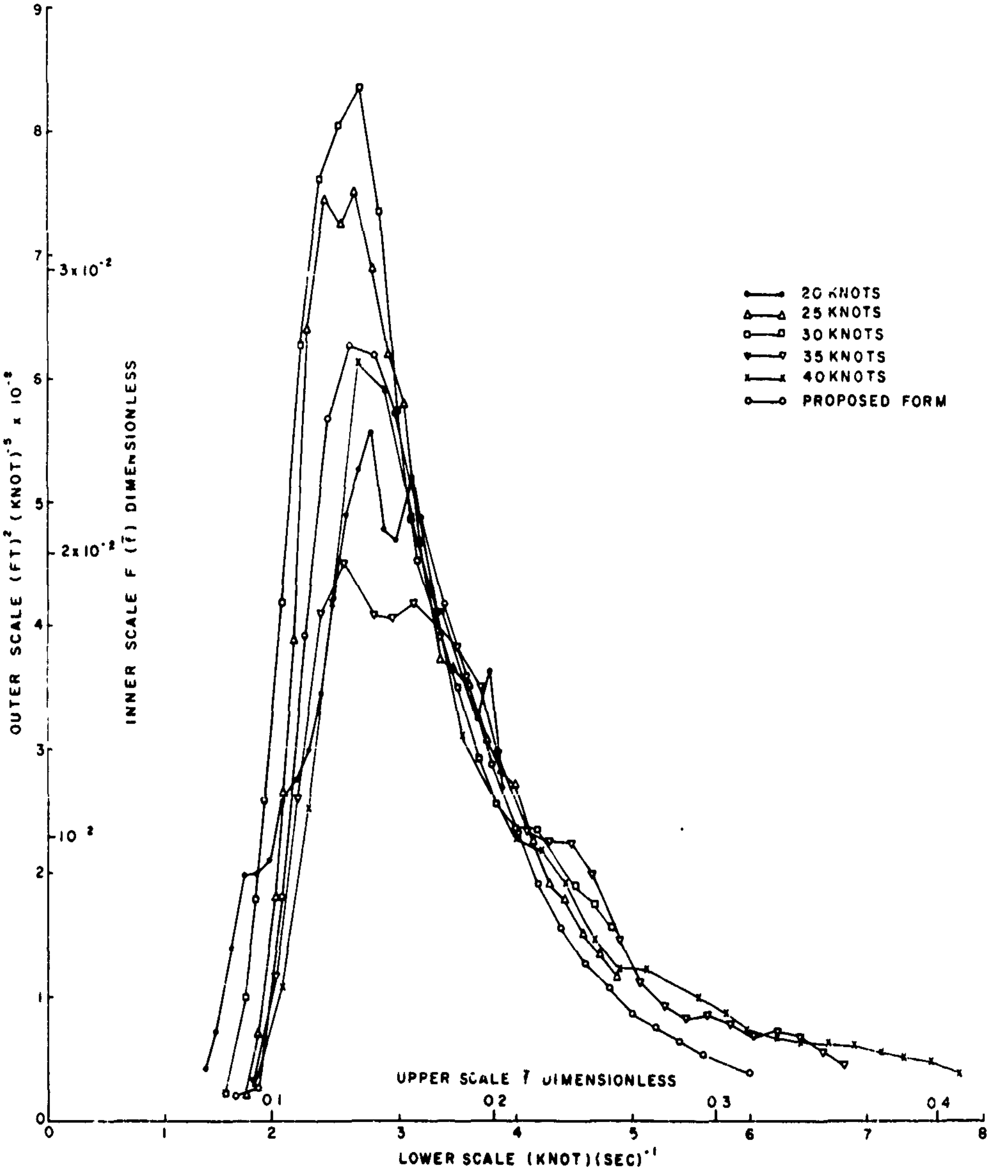

FIG I TRANSFORMED PLOTS OF THE SPECTRA FOR THE FIVE NOMINAL WIND SPEEDS to curves that are quite similar, one to the other. The curve for 20 knots comes quite close to the curve for 40 knots even though the area under the 40 knot curve was about sixteen times the area under the 20 knot curve. For values corresponding to the original units from 3.5 onward, the agreement is more or less within the sampling variability of the data. The forward faces of the different curves do not fall exactly one on the other, but again they are not too different. The peaks, however, scatter over a range from 4.5×10^{-8} to 8.3×10^{-8} and the scatter in the peak values is in excess of that which should occur given that the spectra are truly from observations with exactly the given wind speeds and that the sampling variability of the averages is properly determined from the product of the number of spectra used in the average and 20 degrees of freedom for each spectrum. The results might well be compared with some of the curves given by Wiegel (1963) and with the graphs of Kitaigorodskii and Strekalov (1962).

The reasons for this discrepancy are probably that the winds are not precisely enough determined, that a certain amount of blurring of the spectral values due to variations in the wind about the values tabulated has occurred. It proved very difficult to select the data for the subsets that comprise each of the average spectra as stronger winds prior to the observation and swell eliminated many spectra for the low winds and wind shifts and rapid variations in wind speed eliminated many spectra for the high winds. No effects of air sea temperature difference could be detected, but they may have been present.

Location of peak and adjustment for wind speed variability

In the basic data, the peaks of the various spectra were at the

Table I. Frequency at which the peak occurred in the spectrum for different wind speeds.

Wind speed U (knots, nominal)	Frequency at peak (f)	Product
20	$\frac{25}{180}$	$\frac{500}{180}$
25	$\frac{19}{180}$ (secondary peak)	$\frac{475}{180}$
30	$\frac{16}{180}$	$\frac{480}{180}$
40	$\frac{12}{180}$	$\frac{480}{180}$
	[Average]	$\frac{484}{180}$

values shown in Table I. The value chosen for the 25 knot wind is the secondary of two peaks, and no value has been used for the 35 knot wind as this averaged spectrum seems to be distorted in shape compared to the other four. With these qualifications, the product of the observed wind speed and the frequency at the peak is sensibly constant, varying from a maximum of 500/180 (2.72) to a minimum of 475/180 (2.64) with an average of 484/180 (2.69). The resolution of these spectra is not great enough to refine this calculation and so a value of 480/180 will be used. On the lower scale of figure 1 this corresponds to a value 2.67. This in turn corresponds to a dimensionless frequency, $\bar{f}_{max} = 0.140$.

At this frequency, or near it, the values of $S(FT)^2/U^5(KNOTS)^5$ are shown in Table II, along with their upper and lower confidence intervals as determined by 20 degrees of freedom per sample times the sample size. The confidence intervals do not overlap.

Table II. The values of S $(FT)^2/U^5$ (KNOTS)^5 near the peaks of the different averaged spectra, their 90% confidence intervals, and the derived wind speed correction.

Nominal wind	Average reported winds	S/U^5 $\times 10^{-8}$	Upper $\times 10^{-8}$	Lower $\times 10^{-8}$	Factor	New Wind
20	20	5.55	6.43	4.77	.974	19.5
25	25.3	7.43	8.92	6.16	1.033	25.8
30	29.9	8.36	9.69	7.20	1.057	31.7
35	35.3	4.14	4.96	3.44	.959	33.6
40	40.1	6.11	7.03	5.20	.9955	39.8
Mean		6.32				

The mean of these five maxima in the units shown on the outside scale of figure 1 is 6.32×10^{-8}. If the ratio of the value tabulated for a given wind to the mean is taken and if the fifth root is computed, the value tabulated as a factor is obtained. Thus the fifth root of 5.55/6.32 is 0.974. If the nominal wind speed of 20 knots is multiplied by this factor the result is 19.5 as indicated in the column "New wind". Similar values for the other winds are shown. Certainly with the present accuracy of observed winds at sea, with the variable effects of stability, and in view of the fact that the winds reported for the spectra that comprise the individual subsets were not exactly at the values shown for the nominal wind, one certainly could not argue with the assumption that the new wind value as tabulated is as representative of the wind for which the average is taken as the nominal wind value. This procedure quite evidently is highly sensitive to the correct value of the wind speed as the fifth root of the ratios involved lead to differences of at most 6 percent and usually 3 or 4 percent.

A longer averaging time for the wind observation and perhaps measurements at two different heights could improve the data in this connection. It would have been possible to define a slightly different friction velocity for each wind speed and obtain this same result. Before this could be accurately done, however, it would be necessary to reduce the sampling variability of the spectra and improve the wind speed measurements.

The dimensionless spectra for the corrected winds

If these new wind values are then used instead of the nominal values to transform the spectra to a non-dimensional form, the result is shown in figure 2. These curves all come quite close together compared to the usual variability that occurs. Our procedure has suppressed a large amount of this variability.

At this point, we might remark that the results of Kitaigorodskii suggest that wave data should be taken in a special way. In order for plots such as this one to be more easily comparable, the length of the record should be proportional to the wind speed. If, for example, a wave record is taken when the wind is 20 knots, and if this record is 20 minutes long, then when the winds are 40 knots the record ought to be 40 minutes long. In figures 1 and 2, there are twice as many values on the transformed spectrum for a 20 knot nominal wind over a given interval of $\overline{f}$ as there are for a 40 knot nominal wind. With the same Nyquist frequency, a record for a wind of 40 knots that is twice the length of a record for a wind of 20 knots can be analyzed at twice the resolution of the 20 knot record to provide a transformed spectrum with the same resolution as the 20 knot spectrum. A second important consequence, if the theory is correct, is that one carefully made series of observations offshore

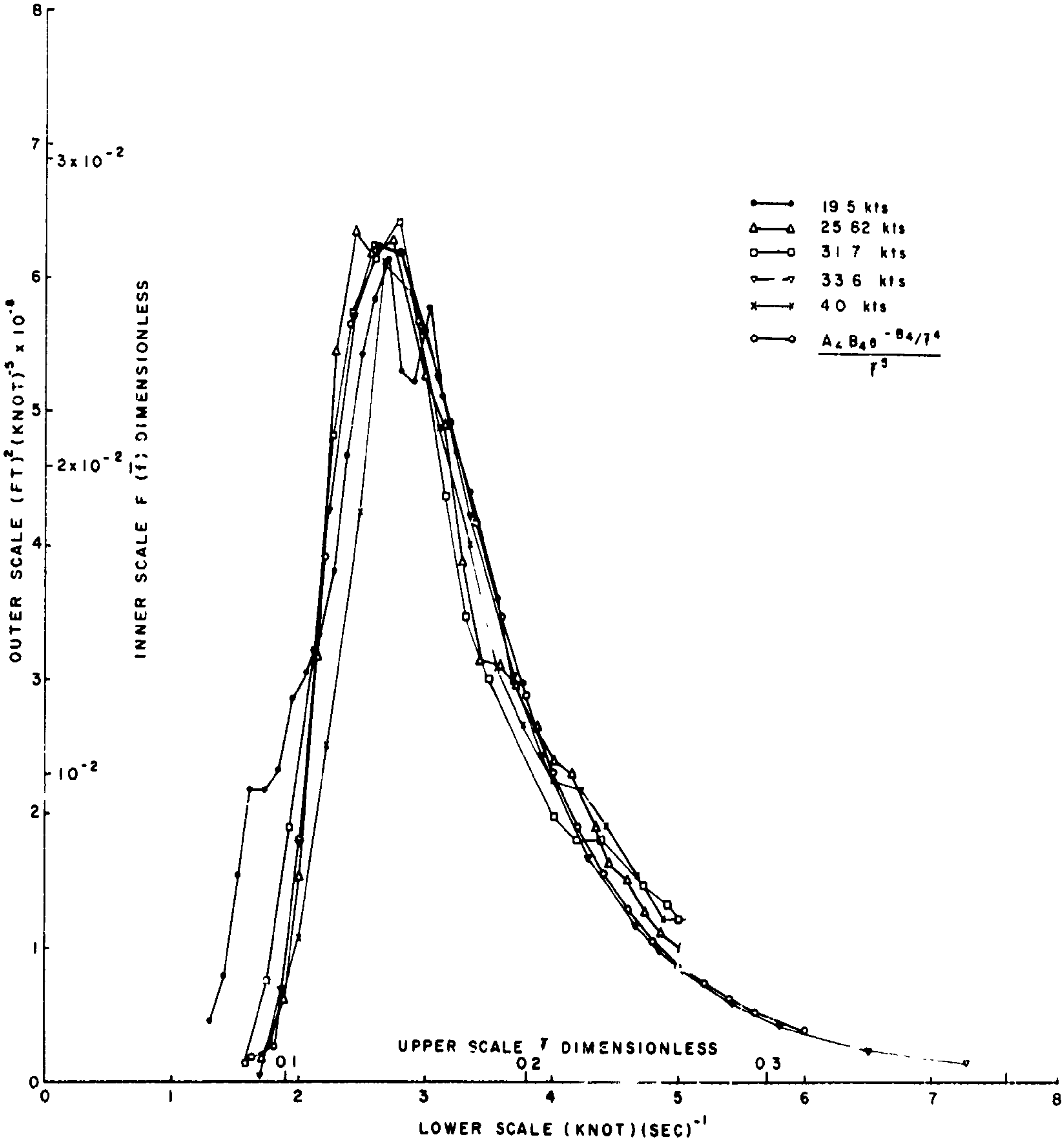

FIG. 2 TRANSFORMED PLOTS OF THE SPECTRA FOR THE NEW WINDS

from a given land mass for 20 knot winds or so could settle the whole problem of the effects of fetch and duration.

Moskowitz calculated each of his average spectra at triple the original resolution. Nothing significant appeared to result except for the 40 knot spectrum. In this spectrum, the forward face of the spectrum was shifted forward, with a compensating loss after the peak. If this higher resolution spectrum is plotted instead of the one shown in figure 2, it comes much closer to the spectra for 25 and 30 knots. This point will be illustrated in another way by a figure to be given later. It might also be remarked here that the behavior of the curve for 20 knots in figure 1 (and for 19.5 knots in figure 2) leads us to believe that there was a slight contribution left over at low frequencies from previously higher winds.

Curve fitting

There are many possible ways to proceed given figure 2. One way would be to correct the curve for 40 knots with the higher resolution data and omit the low frequency part of the curve for 20 knots. Then the five curves could be averaged across lines of constant value for the vertical scale so as to locate a central value for $\overline{f}$. A smoothed curve drawn through these points could then be read at a sufficiently high resolution for $\overline{f}$ so that upon transforming this smooth curve back to a spectrum with appropriate dimensions a spectral form for any wind speed would be obtained. (The difference between a wind speed of 40 knots and 39.82 knots did not seem to warrant recalculating the curve for 40 knots.) This would be an objective way to determine $S_{\zeta}(\overline{f})$. However, the calibration of the shipborne wave recorder that was used leads to some questions concerning the high frequency behavior of these curves and it was thought more appropriate to try to find an analytical form for $S_2(\bar{f})$. This was done by assuming certain analytical forms with undetermined dimensionless parameters and forcing the analytical form to have a maximum of 6.32×10^{-8} at the value $f^* = 2.67$ in terms of the outer scale for the vertical and the bottom scale for the horizontal in figure 2.

It is evident from the work of Kitaigorodskii that the spectrum must be of a form such that $\bar{f}^5$ occurs in the denominator. Since the spectra are nested, there must be some other function of the dimensionless frequency $\bar{f}$ involved. The form suggested by Roll and Fischer (1956) with a choice left open as to the constants in both the exponent and as the coefficient is one possibility, and this is shown by equation (9). Also, Bretschneider (1963) has suggested spectra of the form shown by (11). Another possible candidate would be (10). The exponential term can be thought of as a high-pass filter acting on the limiting form propsed by Phillips (1958), and the question is which of these possible forms would give the best fit to the curves presented in figure 2.

$$S(\bar{f}) = \frac{AB_2 e^{-B_2/\bar{f}^2}}{\bar{f}^5} \tag{9}$$

$$S(\bar{f}) = \frac{AB_3 e^{-B_3/\bar{f}^3}}{\bar{f}^5} \tag{10}$$

$$S(\bar{f}) = \frac{AB_4 e^{-B_4/\bar{f}^4}}{\bar{f}^5} \tag{11}$$

As fitted to pass through the peak of the set of curves given in figure 2, these three possible forms are graphed in figure 3. The form shown by equation (11) is the steepest and the other forms open out as

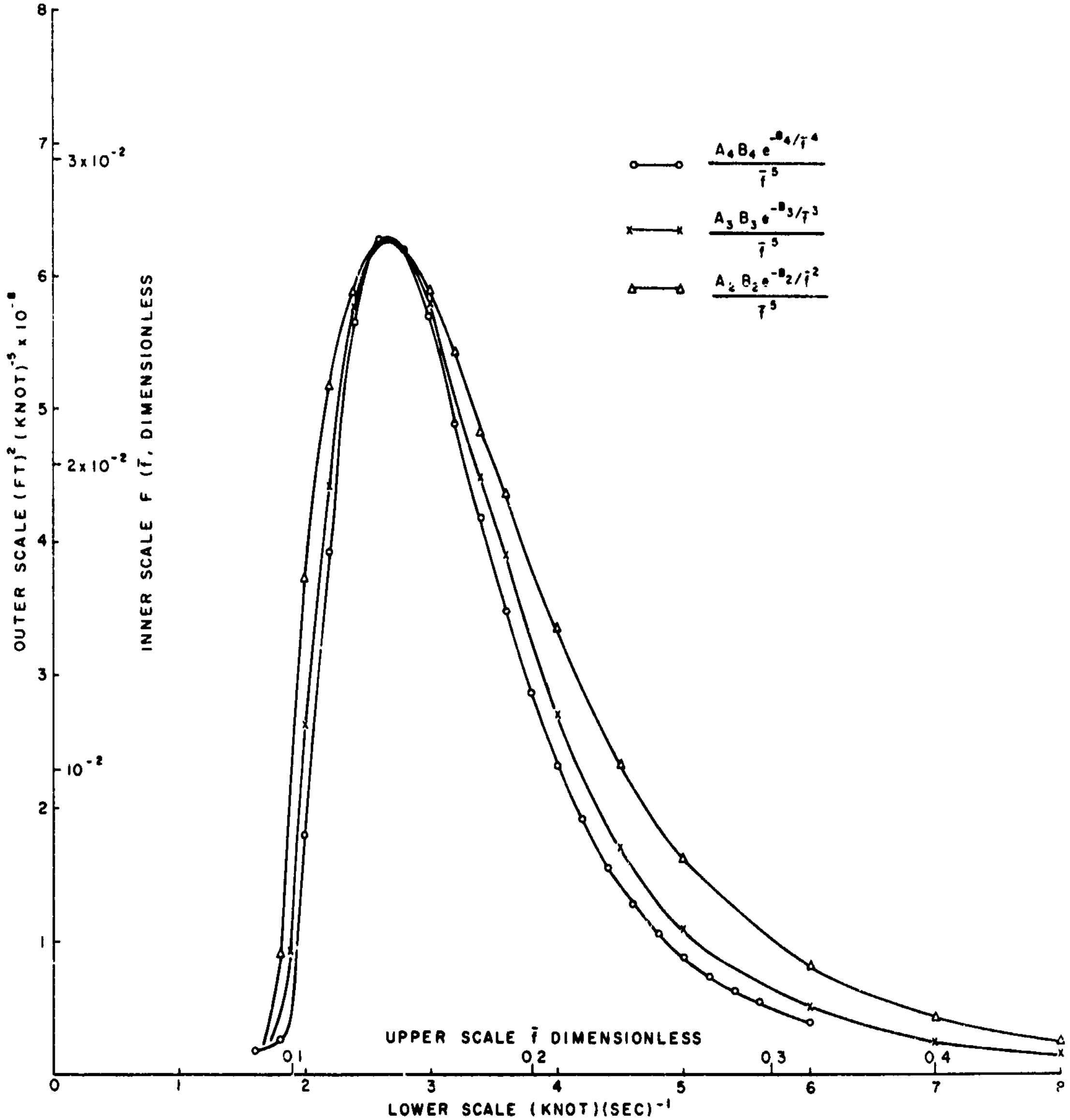

FIG. 3 GRAPHS OF THE POSSIBLE SPECTRAL FORMS

the exponent of $\bar{f}$ in the denominator of the exponential term is decreased. Quite evidently, since these three forms are so very close together the final proof as to which one is superior cannot now be given. However, for the purpose of forecasting the properties of the larger waves in a wind sea it would appear that the form given by eq. (11) is slightly better. This form is shown by open circles on both figures 1 and 2. It can be seen that it comes very close to the two curves for the 25 (25.82) and the 30 (31.7) knot winds. Moreover, when the 40 knot curve is corrected for the effect of resolution, it comes close to this curve on the forward face of the spectrum. The steepness of the forward face cannot be determined more sharply than this due to wind speed variations within the sample. It is also not too bad a fit on the high frequency side of the maximum out to a value of approximately 4 on the lower horizontal scale. Since the curve for 33.6 (35) knots does not seem to fit in too well with all the other curves and since it is believed that the curve for 19.5 knots has left over contributions from higher winds at low frequencies, this form seems slightly better than the other two.

For values on the lower horizontal scale greater than 5 the curves for 40 knots and 33.6 knots seem to rise so that they would agree better with the form given by eq. (9). However again it is suspected that this may be due to the calibration of the shipborne recorder. By increasing the noise level for the spectra in the computations described by Moskowitz, Pierson, and Mehr (1962, 1963) this part of the curve could easily be pulled down whereas the changes near the peak would be relatively unimportant.

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The proposed spectral form in various units

For these reasons, then, it would seem that a form given by equation (11) could serve the purposes of wave forecasting until more data of this nature analyzed by these techniques and better wind speed observations reduce the scatter of the results to point where a non-dimensional form can be determined with greater precision. In terms of the outer vertical scale and the bottom horizontal scale, the function that fits the peak and has the form indicated by equation (11) is given by equation (12). For such curve-fitting and preliminary calculations the proper non-dimensional forms are really not needed, as in fact, they complicate the analysis by making it necessary to calculate all sorts of additional factors every time the data are processed. If as in (12), values on the lower horizontal scale are substituted into the function for $\bar{f}^*$ (which is not really non-dimensional) the result will be the values on the outer vertical scale.

$$F_2^*(\bar{f}^*) = \frac{4.72\ 10^{-7}(63.21)e^{-63.21/(\bar{f}^*)^4}}{\bar{f}^{*5}} \qquad (12)$$

The analytical spectral form in proper scientific units

With equation (12), it is now possible to apply the various conversion factors that are necessary and obtain an analytical spectral form. This spectral form is given by eq. (13) in terms of $\omega(2\pi f)$, ω_o, and two dimensionless parameters, α and β. In this equation $\alpha = 7.79 \times 10^{-3}$, $\beta = 0.74$, and $\omega_o = g/U$ where U is the wind speed reported by the weather ships. The units are $cm^2 sec$ for $S(\omega)$, radians per second for ω, cm/sec^2 for g, and cm/sec for the wind speed.

$$S(\omega)\,d\omega = \frac{\alpha g^2}{\omega^5} e^{-\beta(\omega_0/\omega)^4} d\omega \tag{13}$$

The spectrum is defined only from zero to infinity, and the area under the spectrum is defined to be equal to the variance of the wave record. With improved data, the actual location of the peak may shift and the value of α has considerable sampling variability along with its basic lack of precision due to inaccuracies in the wind speed.

Comparison with original data

Figure 4 shows the graphs of the original spectra obtained by Moskowitz with their confidence limits and plots of this proposed spectral form as given by equation (13) both for the original nominal wind speed and for the modified wind speed. For a wind of 19.5 knots the proposed spectral form fits the data obtained by Moskowitz over a substantial range of frequencies with the exception of low frequencies, which, as mentioned before, may still have contributions left over from higher winds. For a wind speed of 25.82 knots the fit is again quite good over a substantial range of frequencies. For 31.7 knots the fit is not quite as good. The proposed form decreases a little too rapidly at frequencies of 10/180 and 11/180 to agree with the data. The curve is a little too high just past the peak and too low for high frequencies. The adjustment from 30 to 31.7 knots does, however, improve the fit and illustrates how sensitive these curves are to small changes in the wind speed. The curve for 35 (or 33.6) knots does not agree with the data at all. The curve for 40 knots does fairly well. The circles correspond to the values that would be used instead of the spectral values as originally obtained if three times the resolution had been used for these points.

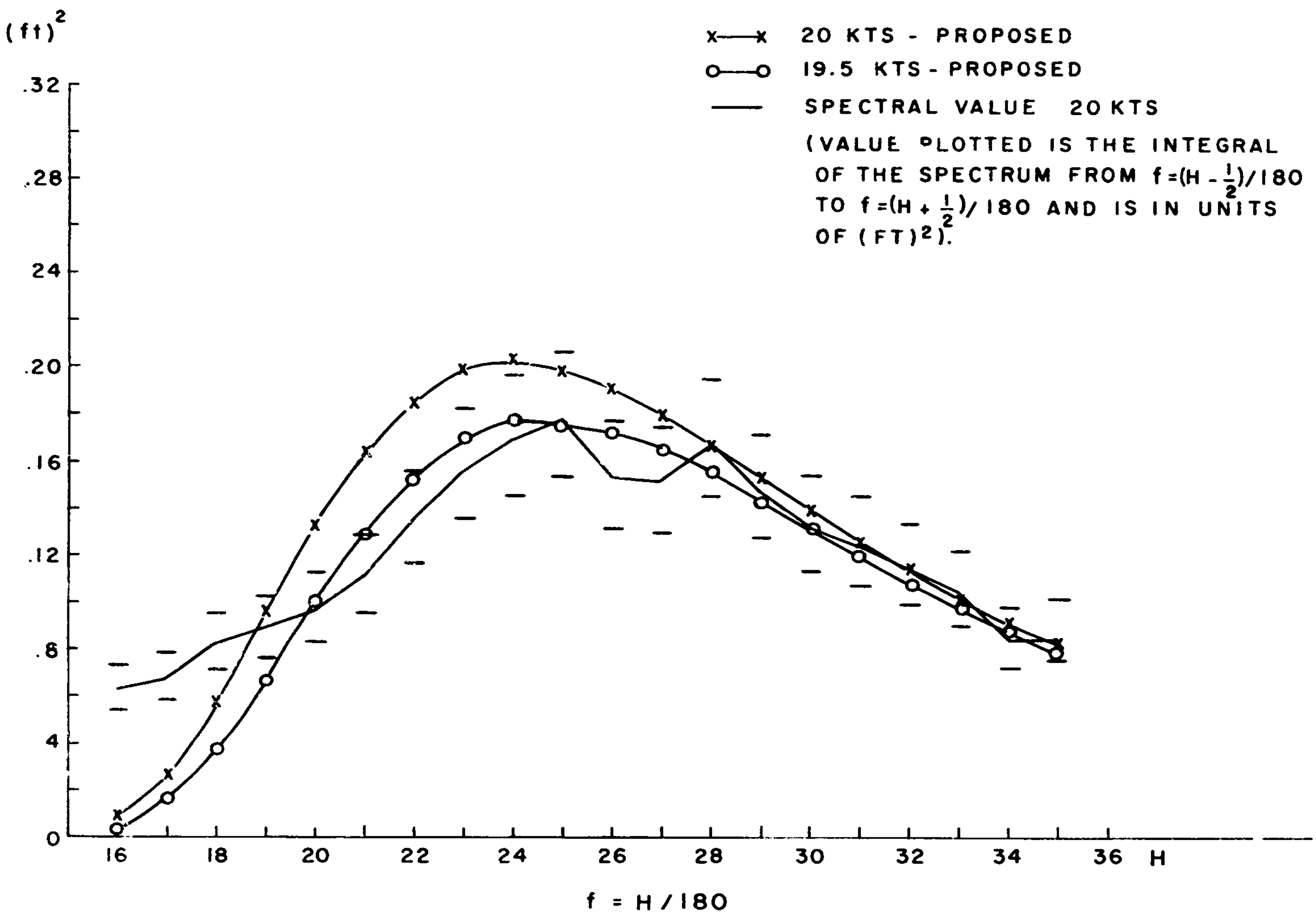

FIG. 4A COMPARISON OF PROPOSED SPECTRUM WITH ORIGINAL DATA FOR 20 KNOT WINDS

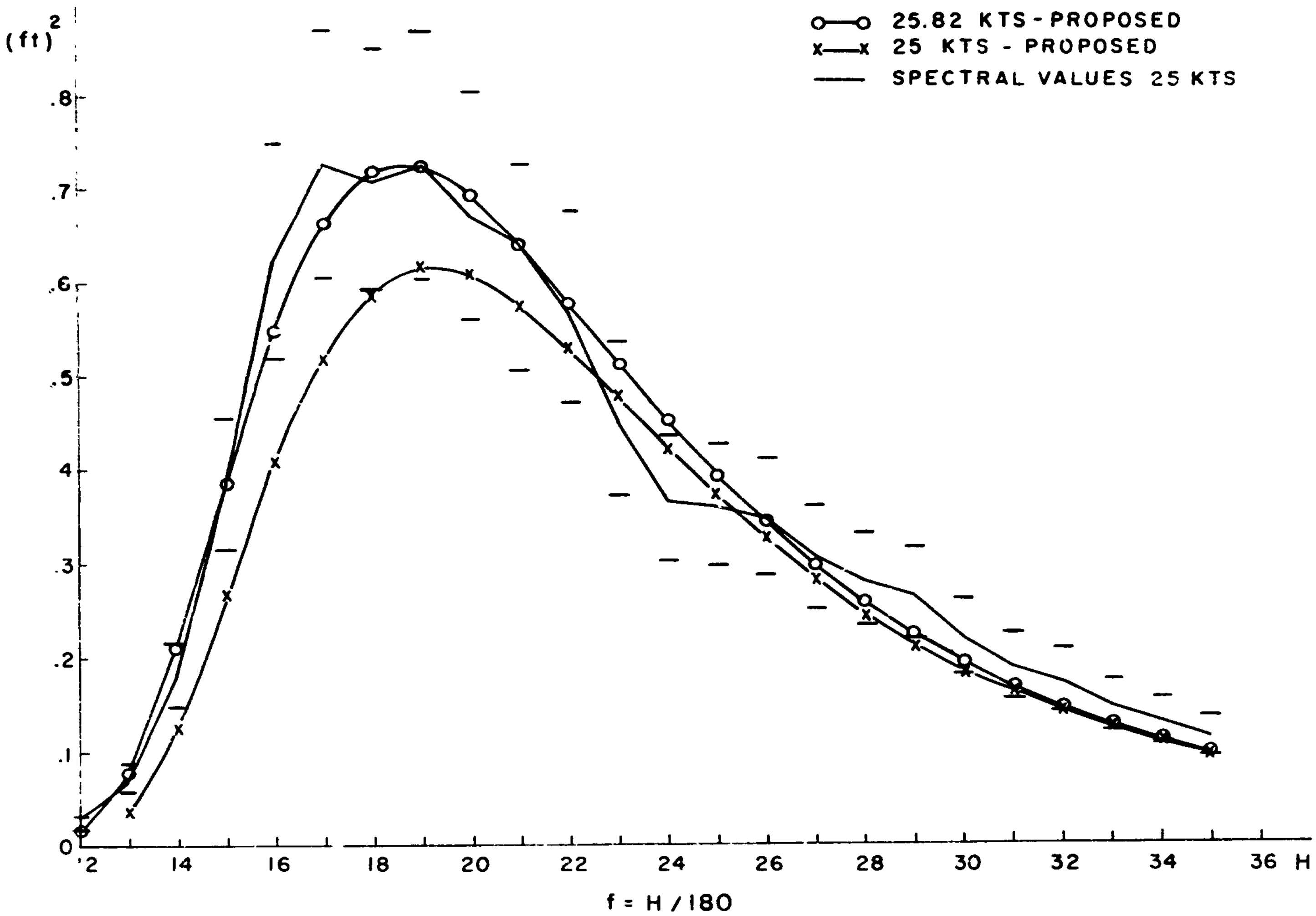

FIG. 4B COMPARISON OF PROPOSED SPECTRUM WITH ORIGINAL DATA FOR 25 KNOT WINDS

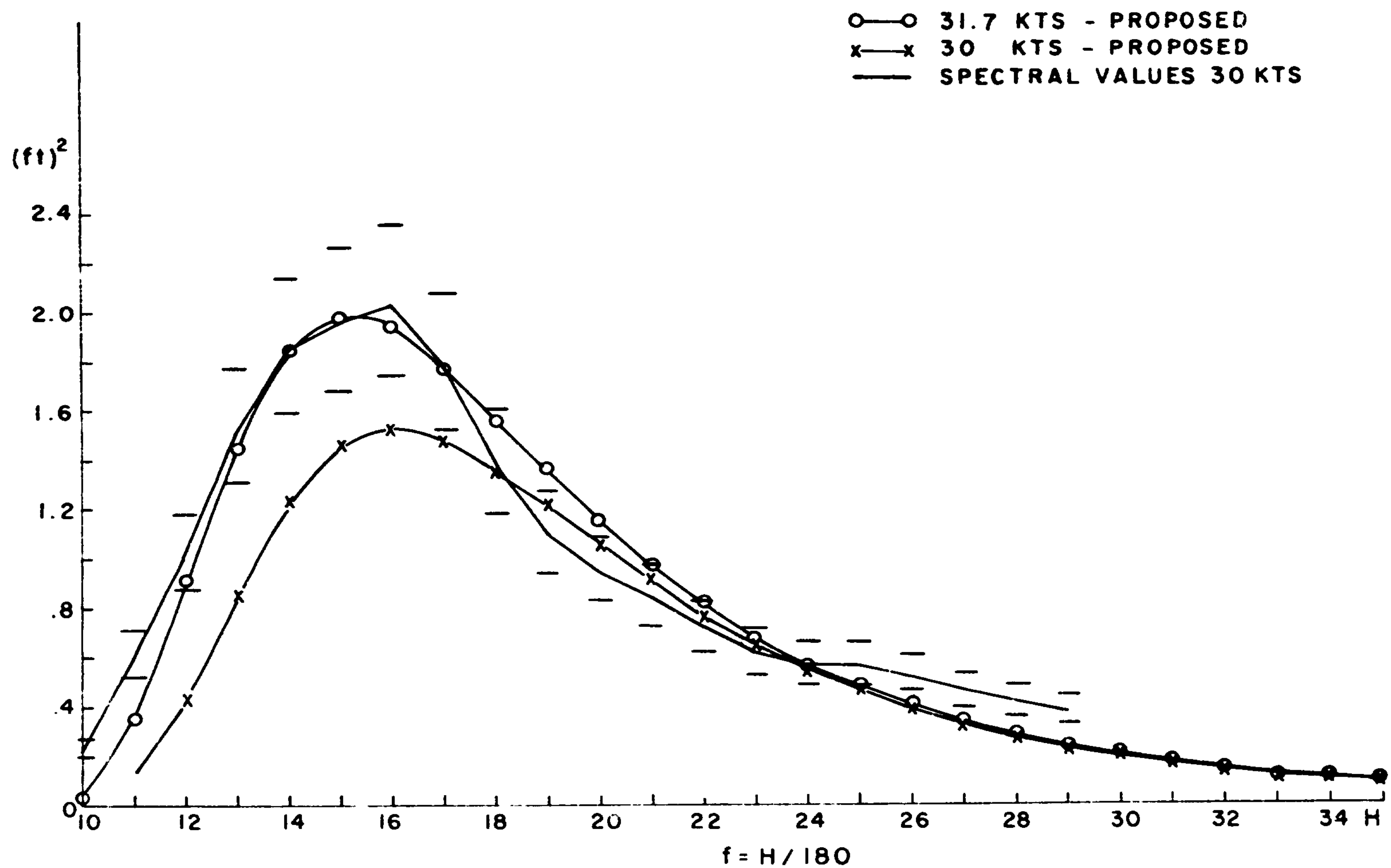

FIG 4C COMPARISON OF PROPOSED SPECTRUM WITH ORIGINAL DATA FOR 30 KNOT WINDS

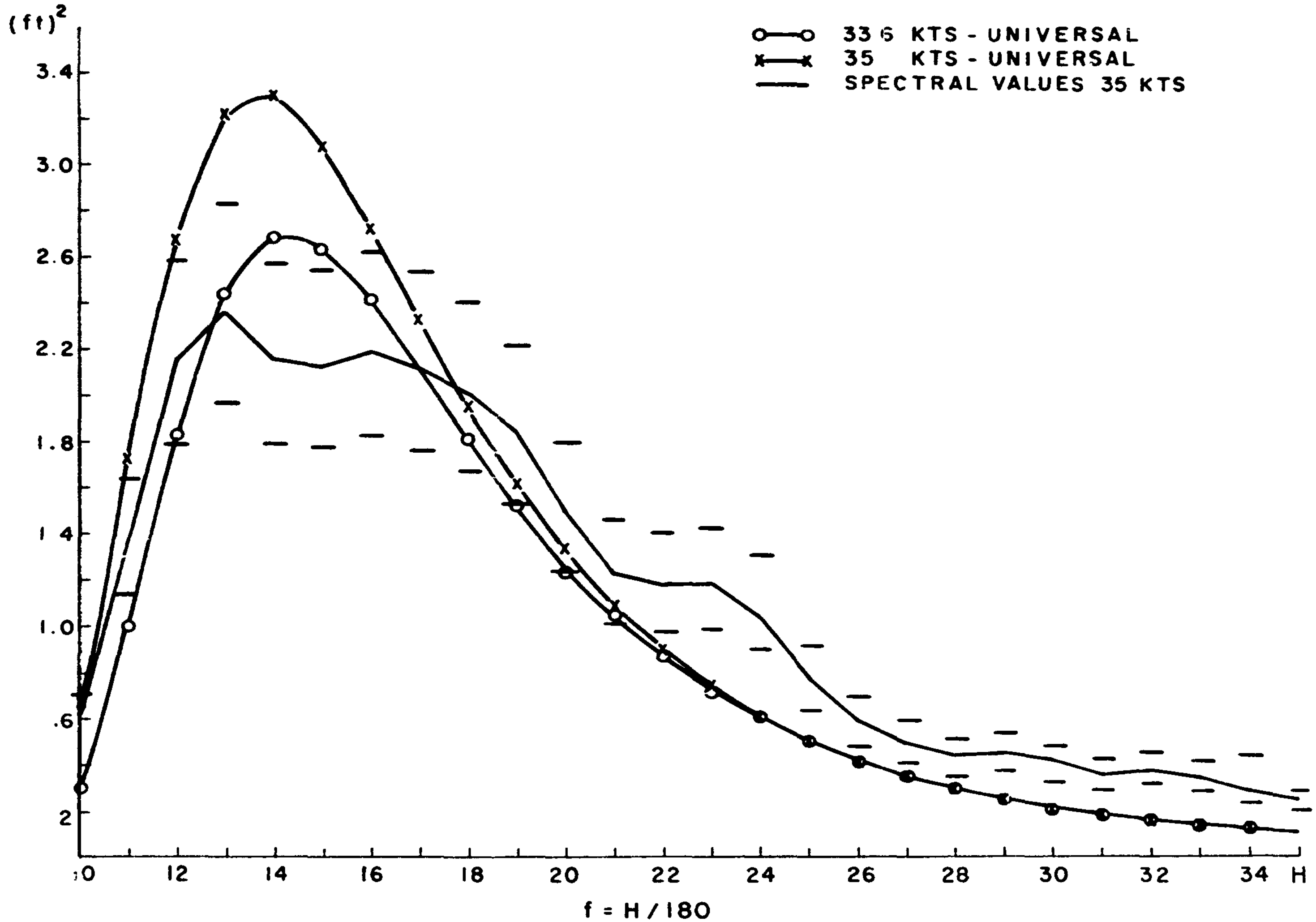

FIG 4D COMPARISON OF PROPOSED SPECTRUM WITH ORIGINAL DATA FOR 35 KNOT WINDS

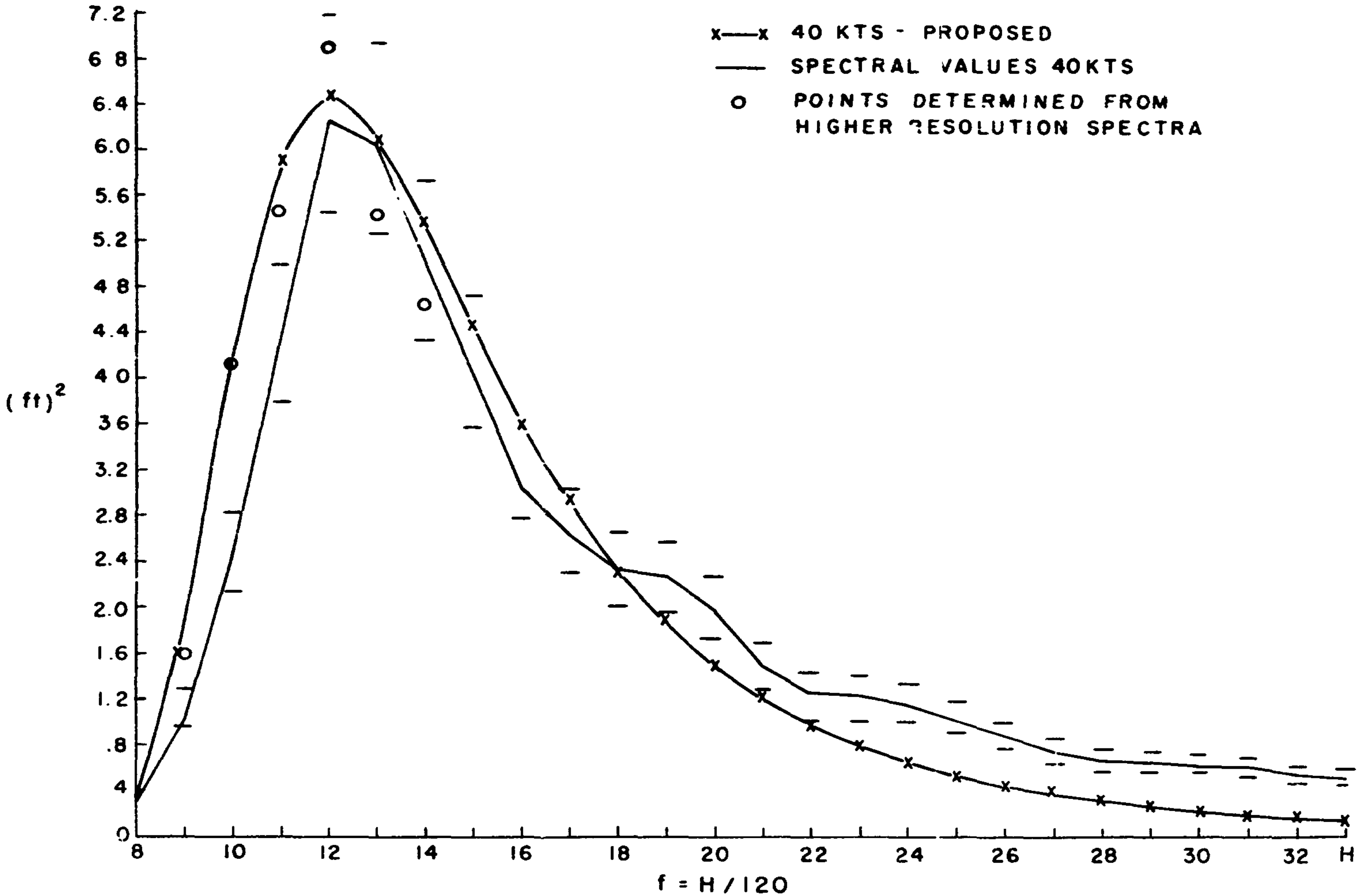

FIG 4E COMPARISON OF PROPOSED SPECTRUM WITH ORIGINAL DATA FOR 40 KNOT WINDS

Here the forward face of the 40 knot spectrum agrees very well with the data and the fit is good all the way out to a frequency of 0.10. From there on, the observed values are higher than the theoretical values.

The agreement between the analytical form and the averaged spectra is quite good. It shows that a family of spectra can be obtained theoretically that agree well with observed spectra and that this family can depend on the one parameter observed by a ship, namely the wind speed at some elevation above the sea surface.

The effect of anemometer heights

The anemometers on the *Weather Explorer* and *Weather Reporter* were located as far as possible above the disturbing influence of the ship at an elevation of about 19.5 meters above the sea surface. The anemometer masts were located about three quarters of the way aft. The effect of the ship in disturbing the wind is not known, but indications are that it is not too great.

The wind speeds used by Moskowitz and in the equations just given are for this height. Results on wave spectra by other authors are in general for wind speeds measured at lower heights, and the variation of wind speed with height needs to be considered so as to interpret these results completely.

The analysis of this problem requires a hypothesis concerning the nature of the stress on the sea surface and the variation of the drag coefficient as a function of wind speed. This subject is treated in a paper by Pierson (1963). The spectral form given by equation (13) will describe the spectrum of a fully developed wind sea for a wind measured at 19.5 meters. This spectral form also yields wave properties in remarkable agreement with the results of other investigators if the variation of the wind with height is considered.

Conclusions

Within the present limitations of the data, the spectra of fully developed wind seas for winds measured at 19.5 meters is given very nearly by equation (13). The proposals of Kitaigorodskii (1961) when systematically applied to more data should provide continuously refined and increasingly more accurate estimates of wind sea spectra.

Acknowledgments

We wish to thank the National Institute of Oceanography for providing the data, and Mr. D. E. Cartwright and Dr. J. Darbyshire for their helpful correspondence. The assistance of Mr. S. Press and Mr. T. Inoue in the analysis of the data is appreciated.

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