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FINAL RESEARCH REPORT

on

The Foundations of Scattering Theory

and

Applications to Physical Problems

(7) NA

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Office of Scientific Research

by the

Department of Physics and Astronomy
University of Maryland
College Park, Maryland

I. INTRODUCTION

This is the final report on the research on "The Foundations of Scattering Theory and Applications to Physical Problems" carried out under contract AF 49(638)-24.

Section II contains a review of work done since the last report. In Section III a complete list of publications and technical reports supported in whole or in part by this contract, since its inception in November, 1956, is given. Also included is a list of students who have received degrees wholly or partially supported by this contract. Concluding remarks are made in Section IV.

II. REVIEW OF RECENT WORK

The work done since the inception of this contract (on November 1, 1956) to the present has been reported in quarterly status reports, technical reports, a year-end summary, and published articles. In addition, a general review of the work done up to May, 1961, was given in the renewal proposal submitted at that time. We give below a brief resumé of the work done in the period May, 1961, to the end of the contract period. A list of all publications related to the contract program is given in Section III and references to this list will be indicated by superscripts.

Following is an outline of the work done during the final period. This is followed by a series of abstracts giving further detail.

A. Foundations of Scattering Theory.

1. Normalization Condition for the Bethe-Salpeter Wave Function and a Formal Solution to the Bethe-Salpeter Equation.
2. Relativistic Invariance and the Square-root Klein-Gordon Equation.
3. Rotational Invariance and the S-matrix in Non-relativistic Quantum Mechanics.
4. Asymptotic Behavior of the S-matrix for High Angular Momentum.
5. Regularity of the T-matrix in the Case of Dirac Potential Scattering.
6. Heisenberg Fields which Vanish on Domains of Momentum Space.
7. Completeness Identity in Field Theory.

B. Applications to Physical Problems.

1. A Field Theoretical Calculation of the One-pion Exchange and Two-pion Exchange Contributions to the Phase Shifts with Higher Angular Momenta for Nucleon-Nucleon Scattering.

2. Dispersion Theory Methods for Pion-deuteron Scattering.
3. Scattering of the K-meson from the Deuteron.
4. Calculations of the Energies of the Higher Resonances in Pion-nucleon Scattering.
5. A model of the Pionic Decays of the Hyperons.

C. In addition, lecture notes on the following topics appeared:

1. Scattering Theory.
2. Quantum Electrodynamics.
3. Multipole Expansions in Elementary Particle Physics.

. . . .

A. Foundations of Scattering Theory.

1. Normalization Condition for the Bethe-Salpeter Wave Function and a Formal Solution to the Bethe-Salpeter Equation.

By the use of an inhomogeneous Bethe-Salpeter equation, a normalization condition for the Bethe-Salpeter wave function is obtained. This condition requires the normalization integral to be positive. A formal solution is obtained in the ladder approximation, and convergence of the normalization integral is proved by the use of this solution. This solution is also used to prove a dispersion relation for the vertex function of the compound particle, and to give an approximate solution. The positiveness of the normalization integral is proved in the non-relativistic limit. The bound state of nucleon and anti-nucleon is studied in the ladder-chain approximation, and it is found that the normalization condition gives finite wave function in spite of divergency of the normalization integral.

This work, done by Dr. I. Sato, has appeared as PDTR #262, and has been submitted for publication.

2. <u>Relativistic Invariance and the Square-root Klein-Gordon Equation</u>.

It is stressed that the usual operator invariance requirements and corresponding commutation condition encountered in the study of the invariance of, for example, relativistic wave equations are only sufficient conditions for invariance and are by no means necessary. More general conditions are given and illustrated with the so-called square-root Klein-Gordon equation. A new proof is thereby given of the Lorentz invariance of this equation . The methods developed are extended to cover the presence of external fields and it is proved that the usual gauge invariant modification of the relativistic Hamiltonian of a spinless particle which takes into account the presence of an external electromagnetic field leads in the quantum mechanical case, to an equation which does <u>not</u> admit the proper Lorentz group. This theorem and its generalization is discussed in connection with a statement of Dirac, to the effect that the square-root equation cannot be extended to include interaction without losing Lorentz invariance.

This work, by Dr. J. Sucher, appeared as PDTR #246, and has been published in the Journal of Mathematical Physics.

3. <u>Rotational Invariance and the S-matrix in Non-relativistic Quantum Mechanics</u>.

The connection between an observable invariance property of the scattering-matrix and the interaction, V, generating the scattering is studied for a particular case: rotational invariance in non-relativistic "potential" scattering. It is shown that a determination that the scattering cross section depends only on the angle between the incident and outgoing beams by no means implies that V is invariant under rotation. This is true even if coherent incident beams are used to probe the target. Some aspects of these results and their relevance for the construction of physical theories are briefly discussed.

This work, by R. Fong and J. Sucher, has appeared as PDTR #269, and has been submitted for publication in the Physical Review.

4. Asymptotic Behavior of the S-matrix for High Angular Momentum.

The behavior of the partial wave transition matrix is discussed for large values of the angular momentum. For physical values of the angular momentum, it is shown that the N-channel T-matrix vanishes in the high angular momentum limit. The validity of the optical model is discussed. In the Gelfand-Levitan formalism, it is shown that the two Jost functions coincide as the angular momentum goes to infinity along the real axis. For the Yukawa-type potentials, it is shown that the transition matrix reduces to its Born term as the real part of the complex angular momentum variable goes to infinity.

This work, by Y. S. Kim and A. Jaffe, has appeared as PDTR #267, and has been accepted for publication in the Physical Review.

5. Regularity of the T-matrix in the Case of Dirac Potential Scattering.

The functional-theoretic methods of Hunziker have been applied to the case of Dirac potential scattering to prove analyticity of the T-matrix in energy and momentum transfer. The conditions on the potential are somewhat weaker than those used by Khuri and Treiman and the domains of analyticity contain the known ones as special cases.

This work, by A. Pagnamenta, has appeared as PDTR #274, and has been submitted for publication in the Journal of Mathematical Physics.

6. Heisenberg Fields which Vanish on Domains of Momentum Space.

If a local Heisenberg field vanishes, or, where appropriate, has an infinite zero, on one of the momentum space domains: A, $p^2 = -a^2$; B, $0 \leq p^2 \leq m^2$ and $p = 0$; or C', $p^2 > M^2$, then the field is a generalized free field. Counter examples show that this conclusion cannot be drawn if the field vanishes on the momentum space domains: D, $0 \leq M_1^2 < p^2 < M_2^2$, $p \neq 0$; or E, $p = 0$. It follows that if two fields in the same Borchers class are equal on one of the domains A, B, or C' then the fields differ at most by a generalized free field in their Borchers class.

This work, by O. W. Greenberg, has appeared as PDTR #251, and has been published in the Journal of Mathematical Physics.

7. Completeness Identity in Field Theory.

We discuss briefly three notions which are related to the completeness identity for Heisenberg fields: first, Gram-Schmidt orthogonalization of the states $|0\rangle$, $A^*(k)|0\rangle$, $A^*(k_1)A^*(k_2)|0\rangle$, etc. to construct an "ideal" or "improper" complete orthogonal basis for the Hilbert space generated by polynomials in the smeared (momentum space) Heisenberg field A(k) acting on the (assumed cyclic) unique vacuum state $|0\rangle$; secondly, the use of this orthogonal basis to construct an expression for the unit operator $\mathbf{1}$, which we call the completeness identity; and finally, the use of alternative completeness identities to test the mutual consistency of different completeness assumptions; for example, the compatibility of a given kind of space-time completeness with asymptotic completeness.

This work, by O. W. Greenberg in collaboration with H. J. Schnitzer and E. C. G. Sudarshan, has been published in Il Nuovo Cimento.

B. **Applications to Physical Problems.**

1. Nucleon-Nucleon Scattering.

The one-pion-exchange and two-pion-exchange parts of the S-matrix for nucleon-nucleon scattering are calculated field-theoretically. The rescattering of virtual pions by nucleons and the pion-pion interaction between virtual pions are taken into account. The S-matrix is then decomposed into the partial-wave amplitudes, and the phase shifts are calculated. Numerical evaluations are carried out for the 310-Mev proton-proton scattering, and the results are compared with the phase shifts obtained by analyzing the experimental data. It is found that, without contribution of the pion-pion interaction, the results are far from agreement with experiment because of too strong attraction arising from the contributions of the two-pion-exchange part, but the contribution of the pion-pion resonance in the I = J = 1 state improves the results considerably by largely canceling the attraction. It is, however, also found that definite discrepancies still remain between the theory and the experiments, and this suggests that some unknown effects must play important roles in determining the nuclear force in the region of the internucleon distance around the Compton wave length of the pion.

This work, by Dr. I. Sato, has appeared as PDTR #238 and has been published in the Physical Review.

2. Dispersion Theory Methods for Pion-Deuteron Scattering.

The problem of low energy pion-deuteron elastic scattering is undertaken using the methods of dispersion theory. Complex singularities do not occur if the momentum transfer is used as the variable in a dispersion relation with the energy taken to be fixed and real. Anomalous discontinuities are developed by analytic continuation in the deuteron mass. Subsequent analytic continuation in the energy provides the representation of the physical amplitude.

A sample calculation is made ignoring spin. The contributions of pion-nucleon scattering are estimated by assuming the dominance of the T=3/2 resonance in the energy channel and of the ABC resonance in the momentum transfer channel. The angular distribution is computed at 85 Mev and some agreement with experimental data is obtained.

This work by J.J. Brehm and J. Sucher has been submitted for publication in Annals of Physics.

3. Scattering of the K-meson from the Deuteron.

The reliability of calculations of low-energy meson-deuteron elastic scattering in which the nucleons are treated adiabatically is studied. A simple model is considered in which nucleon recoil and binding effects are taken into account and which permits numerical evaluation of the double scattering terms in the multiple scattering expansion of the scattering operator. It is found that the effects in question are quite significant numerically for K^--d scattering up to 200 Mev/c K^- lab. momentum. A crude attempt to distinguish the Dalitz solutions for the $K^- - \mathcal{N}$ scattering lengths is made; the results are in agreement with recent work of Chand and Dalitz.

This work constitutes the thesis of Dr. A. K. Bhatia (PDTR #265), and in abbreviated form, will shortly be submitted to Il Nuovo Cimento for publication.

4. Calculation of the Energies of the Higher Resonances in Pion-Nucleon Scattering.

The model of Cook and Lee for the coupled processes

$\pi + N \rightarrow \pi + N, \quad \pi + N \rightarrow \pi + \pi + N$ and

$\pi + \pi + N \rightarrow \pi + \pi + N$ is used to

calculate the positions of the $D_{3/2}$ and $F_{5/2}$ resonances. The three-body state is considered as a nucleon plus a two pion system at the T=1, J=1 pion-pion resonance energy. For the left hand cuts, only the one pion exchange contribution to the production amplitude is retained. The branch cut in the helicity amplitudes for this diagram is found by requiring that the exchanged pion be on the mass shell. The one pole approximation is made and resonance energies in fair agreement with the observed energies are found. No T=3/2, $F_{5/2}$ resonance is predicted at any energy, while for T=3/2, $D_{3/2}$ state, the result is ambiguous.

The neglect of the T=3/2, $P_{3/2}$ final state pion-nucleon interaction in the production amplitude is considered and it is shown that, although this interaction is essential for understanding the production cross section at the resonances, it has negligible effect on the location of the resonance energies.

This work, which constitutes Dr. V. Teplitz's thesis, has appeared as PDTR #270, and is being prepared for publication.

5. A Model of the Pionic Decays of Σ-Hyperons.

Σ-hyperons are assumed to decay via the $\Lambda N\pi$-vertex. Calculations are made on both even and odd (Σ, Λ) relative parity. The results of even parity cannot fit the experimental data. For the odd parity we get $\alpha^0 \sim \alpha_\Lambda$, $\alpha^\pm \sim 0$ for $G_{\Sigma\Sigma\pi} \sim 2G_{NN\pi}$ and $\alpha^0 \sim -\alpha_\Lambda$, $\alpha^\pm \sim 0$ for $2G_{\Sigma\Sigma\pi} \sim G_{NN\pi}$.

This work by S. Y. Shieh has appeared as PDTR #252.

III. LIST OF PUBLICATIONS UNDER CONTRACT AF 49(638)-24

1. "Causality and the Dispersion Relation: Logical Foundations", by John S. Toll; Phys. Rev. 104, 1760 (1956), Physics Department Technical Report No. 27.

2. "Dispersion Relations for the S-matrix", by John S. Toll, David Y. Wong, and James M. Knight; Bulletin of the American Physical Society (BAPS) Series II, 2, 13 (1957).

3. "Causality and the Dispersion Relation: S-matrix for the Maxwell Field", by David Y. Wong and John S. Toll, Annals of Physics 1, 91 (1957).

4. "Dispersion Relations for Non-relativistic Particles", by David Y. Wong; Phys. Rev. 107, 302 (1957); PDTR 62, ASTIA 115 073, AFOSR TN 57-35.

5. "Bound State Contribution to Dispersion Relations for the Scattering Matrix", by James M. Knight and John S. Toll; BAPS Series II, 2, 177 (1957).

6. "Dispersion Relations for Non-relativistic Particles", by David Y. Wong; BAPS, Series II, 2, 177 (1957).

7. " A Survey of Literature Pertaining to the Analytical Properties, Physical Realizability, and the Kronig-Kramers Relations of the System Functions in Electric Network Theory", by Alfred Chi-Tai Wu; PDTR 57.

8. " Application of Dispersion Relation and Unitarity Condition to Non-coherent Forward Scattering of Light by Light", by David Y. Wong; PDTR 81, ASTIA AD 132,487, AFOSR TN 57 351A.

9. "Remarks on the Implication of 'Causality Condition in Non-interaction Region' and the Unitarity of the S-matrix", by David Y. Wong; PDTR 82, ASTIA AD 132 424, AFOSR TN 57-35[illegible].

10. "A Possible Proof of Dispersion Relations", by H. J. Bremermann, R. Oehme, and J. G. Taylor, PDTR #84, ASTIA AD 136, 458, AFOSR TN 57-467. (This paper was delivered by J. G. Taylor before the International Conference on Mathematical Problems of Quantum Field Theory at Lille in June, 1957.)

11. "Uniqueness of Solutions to Dispersion Relations and an Iterative Scheme", by David Y. Wong; PDTR #85, ASTIA AD 136 459, AFOSR TN 57-468.

12. "Causality and the Dispersion Relation: S-matrix for Klein-Gordon and Dirac Fields", by James M. Knight and John S. Toll; PDTR #87 (64), ASTIA AD 136 536, AFOSR TN 57-552; Annals of Physics 3, 49 (1958).

13. "Connection of Phase Shift and Potential for the Dirac Equation", by Francisco Prats and John S. Toll; BAPS, Series II 3, 36 (1958).

14. "Note on the Radiative Corrections to the Scattering of Mesons in an External Electromagnetic Field", by Anand K. Bhatia and Joseph Sucher; PDTR #103, ASTIA 158 251, AFOSR TN 58-446; Prog. Theoret. Phys. 20, 397 (1958).

15. "Disperson Relations and Schwartz' Distributions", by John G. Taylor; PDTR #109, AFOSR TN 58-630, ASTIA 162-160.

16. "Note on the Formulation of the Causality Requirement", by John G. Taylor and John S. Toll; PDTR 110, AFOSR TN 58-631, ASTIA 162-161.

17. "The Formation of Discontinuities in Classical Non-linear Electrodynamics", by Morton Lutsky; PDTR #111, AFOSR TN 58-633, ASTIA 162 163; Phys. Rev. 113, 1649 (1959).

18. "The Construction of the Dirac Equation Central Potential from Phase Shifts and Bound States", by Francisco Prats; PDTR #112, AFOSR TN 58-632, ASTIA 162-162; Phys. Rev. 113, 363 (1959)

19. "Lectures on Dispersion Relations in Quantum Field Theory and Related Topics", by J. G. Taylor; PDTR #115, Vol. 1= AFOSR TN 58-816A, ASTIA 202-639,
Vol. 2= AFOSR TN 58-616B, ASTIA 202-640,
Vol. 3= AFOSR TN 58-816C, ASTIA 202-641.

20. "The V-A Four-fermion Interaction and the Intermediate Charged Vector Meson", by S. Oneda and J. Pati; PDTR #126, AFOSR TN 59-46, ASTIA 209-417; Phys. Rev. Lett. 2, 125 (1959).

21. "Integral Equation for Production Processes", by J. Sucher and T. B. Day; PDTR #130, AFOSR TN 59 153, ASTIA 211-113; Il Nuovo Cimento 13, 1111 (1959)

22. "K^-, K^0 Relative Parity from the K^--D Charge Exchange Reaction", by T. B. Day, G. A. Snow and J. Sucher; PDTR #132, AFOSR TN 59-435; ASTIA AD 214-797; Il Nuovo Cimento 13, 614 (1959).

23. Λ^0 and Σ^0 Production from the (Σ^-,D) System", by T. B. Day, G. A. Snow, and J. Sucher; PDTR #137, AFOSR TN 59-606, ASTIA AD 216-272; Phys. Rev. Lett. 2, 468 (1959).

24. "On the Suppression of P-state Capture in (K^-,p) Atoms", by T. B. Day, G. A. Snow and J. Sucher; PDTR #137, AFOSR TN 59-606, ASTIA AD 217-373; Phys. Rev. Lett. 3, 61 (1959)

25. "An Asymptotic Causality Requirement for Systems with Constrained Inputs", by J. Sucher; PDTR 141, AFOSR TN 59-773; Nuclear Physics 14 263 (1959).

26. "An Attempt at Universal Four-Fermian Interaction", by J. C. Pati, and S. Oneda; PDTR #139, AFOSR TN 59-630; Il Nuovo Cimento 16, 365 (1960).

27. "K^--Nucleon Scattering Lengths and the K^--d Scattering Reactions", by T. B. Day, G. A. Snow and J. Sucher; PDTR #142, AFOSR TN 59-772; Il Nuovo Cimento 10, 637 (1959).

28. "Collisional Auger Process for Mu-Mesic Atoms", by T. B. Day and J. Sucher; PDTR 143, AFOSR TN 59-771.

29. "K^--Nucleon Scattering Lengths and the K^--d Inelastic Cross Section", by T. B. Day and J. Sucher; PDTR 146, AFOSR TN 59-819.

30. "On the Leptonic Decay Modes of K-meson", by K. Chadan and S. Oneda; PDTR 151, AFOSR TN 59-877; Phys. Rev. Lett. 3, 292 (1959).

31. "Hyperfragment Formation from the K^- Deuteron Interaction in Flight", by T. B. Day, G. A. Snow and J. Sucher; PDTR 154, AFOSR TN 59-1072.

32. "High Orbital S-state Capture of π-mesons by Protons", by T. B. Day, G. A. Snow, and J. Sucher; PDTR 159, AFOSR TN 59-1295; Phys. Rev. 118, 864 (1960).

33. "$|\Delta I| = \frac{1}{2}$ -Rule and the Weak Four-fermion Interaction", by S. Oneda and J. C. Pati, PDTR #160, AFOSR TN 60-12; Phys. Rev. 119, 482 (1960).

34. "On the Decay Interaction of Strange Particles", by B. Sakita and S. Oneda; PDTR #165, AFOSR TN 60-304; Nuclear Physics 16, 72 (1960).

35. "Determination of K^+-n P-Wave Phase Shifts from K^+-d Reactions", by T. B. Day, L. S. Rodberg, G. A. Snow and J. Sucher; PDTR #166, AFOSR TN 60-306; Il Nuovo Cimento 16, [illegible] (1960).

36. K^--Deuteron Scattering and the K^--Nucleon Scattering Lengths", by T. B. Day, G. A. Snow and J. Sucher, PDTR #167, AFOSR TN 60-354; Phys. Rev. 119, 1100 (1960).

37. "Final State Interactions and $|\Delta I| = \frac{1}{2}$ Rule", by K. Chadan and S. Oneda; PDTR #168, AFOSR TN 60-303; Phys. Rev. 119, 1126 (1960).

38. "Multiple Scattering Corrections in K^--d Reactions", by T. B. Day, G. A. Snow and J. Sucher, PDTR #169.

39. "On the Leptonic and Non-leptonic Decay Modes of K-meson", by J. C. Pati and S. Oneda; PDTR #17[illegible], AFOSR TN 60-455; Nuclear Physics 18, 2 (1960).

40. "Absorption Mechanisms of Negative K-mesons and Pions in Liquid Hydrogen", by T. B. Day; PDTR #17[illegible], AFOSR TN 60-301.

41. "Capture of K^--Mesons from High S-orbitals in Helium", by T. B. Day and G. A. Snow; PDTR #181, AFOSR TN 60-761; Phys. Rev. Lett. 5, 113 (1960).

42. "Regularization and Renormalization through Finite-part Integrals", by E. R. Caianiello, A. Campolattaro (Institute of Theoretical Physics, University of Naples), and B. Preziosi (Institute of Theoretical Physics, University of Naples); PDTR #182, AFOSR TN 60-762.

43. "K^--Meson Capture by Helium", T. B. Day; PDTR #183, AFOSR TN 60-819.

44. "On the Analytic Properties of the 4-Point Function in Perturbation Theory", by Alfred Chi-Tai Wu; PDTR #186, AFOSR TN 60-804.

45. "Lie Equations for a Lee Model", by E. R. Caianiello and S. Okuho (Institute of Theoretical Physics, University of Naples); PDTR 185a, AFOSR TN 60-898.

46. "Strict Localization in Quantum Field Theory", by James M. Knight; PDTR #187, AFOSR TN 60-818.

47. "(Σ^{0}, Λ^{0}) Relative Parity and the Dalitz Decay of the Σ^{0} Hyperon", by J. Sucher and G. A. Snow, PDTR #188, AFOSR TN 60-8[illegible]; Il Nuovo Cimento 18, 195 ([illegible]).

48. "Relativistic Quantum Mechanics and Quantum Field Theory", Vol. 1, Lectures by J. Sucher, PDTR 192, AFOSR TN 60-948.

49. "A New Reduction Technique in Quantum Field Theory", by John S. Toll; PDTR #194, AFOSR TN 60-1089.

50. "Strange Particle Decays and the Nature of Weak Interactions", by J. C. Pati; Vol. I TN 60-1051a, Vol. II – TN 60-1051b; PDTR #193.

51. "Analyticity in the Coupling Constant and Bound States in Potential Theory", by B. Bosco and J. Sucher, PDTR #199, AFOSR TN 60-1434; Il Nuovo Cimento Series XIX 1183 (1961).

52. "The Possibility of a Spin Wave Magnetic Moment Detector", by T. B. Day and J. Sucher, PDTR 201, Journal of Applied Physics 32, 1788 (1961).

53. "Cusp Phenomena in the Region of Two Neighboring Thresholds", by J. Sucher, G. A. Snow and T. B. Day, PDTR #[illegible], AFOSR TN 60-1[illegible]; Phys. Rev. 122, 1645 (1961).

54. "A Note on Rearrangement Collisions", by T. B. Day, L. S. Rodberg, G. A. Snow and J. Sucher; PDTR #[illegible], Phys. Rev. [illegible], 10[illegible] (1961).

55. "Two-point Functions and Generalized Free Fields", A. L. Licht and J. S. Toll; PDTR #210, Il Nuovo Cimento, Series X, Vol. 21, pp. 346-52.

56. "Lectures on the Use of Perturbation Methods in Dispersion Theory", by Richard J. Eden, PDTR #2[illegible].

57. "A Field Theoretical Calculation of the One-pion Exchange and Two-pion Exchange Contributions to the Phase-Shifts with Higher Angular Momenta for Nucleon-Nucleon Scattering", by Iwao Sato, PDTR #238, Phys. Rev. 127, 1352 (1962).

58. "Introductory Notes on Scattering Theory", by O. W. Greenberg; PDTR #243.

59. "Relativistic Invariance and the Square-root Klein-Gordon Equation", by J. Sucher; PDTR #246 (Journal of Mathematical Physics, January 1963 issue).

60. "Relativistic Quantum Field Theory II", by J. Sucher; PDTR #249.

61. "Multipole Expansions in Elementary Particle Physics", by R. Stora, PDTR #250.

62. "Heisenberg Fields which Vanish on Domains of Momentum Space", by O. W. Greenberg; PDTR #251, Journal of Mathematical Physics 3, 859 (1962).

63. "A Model of the Pionic Decays of Hyperons", by Shien, S. W., PDTR #252.

64. "Normalization Condition for the Bethe-Salpeter Wave Function and a Formal Solution to the Bethe-Salpeter Equation", by Iwao Sato; PDTR #262 (has been submitted for publication).

65. "Scattering of the K^- Meson from the Deuteron", by A. K. Bhatia; PDTR 265. (will be submitted to Il Nuovo Cimento for publication)

66. "Some Asymptotic Behaviors of the S-matrix for High Angular Momentum", by A. M. Jaffe and Y. S. Kim; PDTR 267. (accepted for publication in Phys. Rev.)

67. "Rotational Invariance S-matrix in Non-relativistic Quantum Mechanics", by R. Fong and J. Sucher; PDTR #269 (accepted for publication in Physical Review).

68. "Calculations of the Energies of the Higher Resonances in Pion-nucleon Scattering", by V. Teplitz; PDTR 270 (is being prepared for publication).

69. "Regularity of the T-matrix in the Case of Dirac Potential Scattering", by A. Pagnamenta; PDTR 274(has been submitted for publication to the Journal of Mathematical Physics).

Other publications which, because of their nature, did not have TN numbers but were nevertheless related to the work of this contract are:

1. "Test of Global Symmetry in Pion-baryon Interactions by K^-+ P-Reactions", by Jogesh C. Pati, Physical Review, Vol.123, 2, 705-710 (1961).

2. "Asymmetry Parameter of Λ Decay and the Intermediate Boson of Weak Interactions", by S. Oneda, J. C. Pati, and B. Sakita: Phys. Rev. Lett. 6, 24 (1961).

3. MEMORIAL VOLUME TO WOLFGANG PAULI, by G. Källén and John S. Toll; Helvetica Physica Acta 33, 753-772 (1960); Article on "Integral Representations for the Vacuum Expectation Value of Three Scalar Local Fields".

4. QUANTUM FIELD THEORY, edited by Caianiello: Chapter "Analyticity Domain of Vacuum Expectation Values", published by Academic Press, 1960.

5. "On the Commutator of a Free Klein-Gordon Field Operator with an 'Arbitrary' Heisenberg Field Operator", abstract of a paper presented at the New York meetings of the American Physical Society (February, 1961) and written by Z. Fried, A. L. Licht and J. S. Toll: Bulletin of the American Physical Society.

6. "Completeness Identity in Quantum Field Theory", by O. W. Greenberg, H. J. Schnitzer and E. C. G. Sudarshan, Il Nuovo Cimento X, 25, 461 (1962).

7. "Dispersion Theory Methods for Pion-Deuteron Scattering", by J.J.Brehm, and J. Sucher (submitted for publication in Annals of Physics).

Students who have received degrees wholly or partially supported by this contract, since its inception are:

	D. Wong	Ph. D., 1957
	F. Prats	Ph. D., 1958
	M. Lutzky	M. S., 1958
	J. Knight	Ph. D., 1960
	A. C. Wu	Ph. D., 1960
	J. C. Pati	Ph. D., 1960
	A. K. Bhatia	Ph. D., 1962
	V. Teplitz	Ph. D., 1962
	R. Fong	Ph. D., 1963 (expected)
	S. Aks	Ph. D., 1963 (expected)
	A. Licht	Ph. D., 1963 (expected)
Also	J. J. Brehm	Ph. D., 1962

Although Dr. Brehm was not supported by this contract, his thesis work resulted in joint publication with his advisor, Dr. Sucher

IV. CONCLUDING REMARKS

We believe that the support given this research has been more than justified by the quantity and quality of the work done, as well as by the number and caliber of the Ph. D.'s in physics it has produced.

During the 5-year period of the contract work around 75 technical reports were produced, most of which led to publications in leading physics journals. Work of this contract has been the subject of invited and contributed papers at meetings of the American Physical Society, at the International High Energy Physics Conferences, etc. Also as another measure of the timeliness and frontier nature of some of the work done, there have been 5 Physical Review Letter publications during this period 1959-1962 by members of this group.

Nine students have obtained the Ph. D. degree partially supported by the contract. Many of them are now distinguishing themselves by their post-doctoral work. The three students who gained their degree this year are currently, Research Associate at the Lawrence Radiation Laboratory (V. L. Teplitz); post-doctoral N.S.F. Fellow at Princeton University (J. J. Brehm) and Assistant Professor at Wesleyan University (K. K. Bhatta). Among earlier outstanding students are D. Y. Wong, now Assistant Professor at the University of California at La Jolla; and J. C. Pati, Richard C. Tolman fellow at Cal. Tech. for two years and currently at the Institute for Advanced Study; A. C. Wu, currently Assistant Professor at the University of Michigan; F. Prats, permanent staff member of the National Bureau of Standards, and J. Knight, currently on military duty at Ft. Monmouth Signal Laboratories. Three more students who have been assistants on this contract will probably gain the Ph. D. in 1963.

We have also been particularly fortunate in the caliber of the work of the Research Associates we have been able to attract to this program. As an example we quote here the comments of the referee on a paper on Nucleon-Nucleon Scattering submitted to the Physical Review by Dr. I. Sato in 1961:

"This is an important as well as a monumental paper. The latter adjective can be used in several ways, one of them being that the paper is a monument to the skill and perseverance of one man's calculational ability. The method used is novel in as much as it combines conventional field theory and dispersion relations in a useful way, and calculates the phase shifts directly instead of the potential - a welcome innovation after the many years of potential theoretical morass . . ."

In conclusion, we would like to express our appreciation to the Air Force Office of Scientific Research and Development Command for the support it has given this work. We hope to continue the production of significant research on the basis of the grant-form rather than the contract-form of support.

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J. Sucher
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JS:csw