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August 1963

P-2759

(5) 737 700

(4) NA

(7-9) NA

(13) NA

(15-17) NA

(18-19) NA

(20) U

(21) NA

(6) **THE NON-CYCLIC VARIATION DURING QUIET DAYS,**

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(11) ~~August 1963~~, (12) 17p.

(14) Rept. no. P-2759

THE NON-CYCLIC VARIATION DURING QUIET DAYS

Albert T. Price*

Consultant, The RAND Corporation, Santa Monica, California

and

Professor of Mathematics
The University, Exeter, England

SUMMARY

New methods of studying Sq during the IGY have revealed a clear pattern of the non-cyclic variation field which was not discernible from earlier studies. The field is mainly zonal about the geomagnetic axis, though there are significant departures from axial symmetry. The zonal part corresponds closely to a uniform field plus a dipole field along the geomagnetic axis. The relative values of these two parts accord well with the assumption that the internal part arises from currents induced in the earth by Dst variations. The relationship between the non-cyclic variations and hourly values of equatorial Dst is discussed.

1. Introduction

In analyses of the Sq field, it is necessary to determine and remove the non-cyclic variation, which is regarded as an after effect of magnetic storms and not part of the true Sq variation.

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This paper was prepared for submission to the Journal of Geophysical Research.

Most Sq studies have been based on the international quiet day variations, and consequently the total non-cyclic change (N) for the day has been estimated from differences in the mean values of the magnetic elements for two intervals of time of an hour or more, centered near the beginning and the end of the Greenwich day. This has meant that, for stations differing in longitude from Greenwich by more than 90°, the estimate of N has depended on values of the elements occurring during daylight hours, and may therefore have been affected by the day-to-day variability of Sq as well as by the irregular fluctuations of the field that are more likely to occur during daylight hours. Apart from the well known fact that the change in the H-component is generally positive at all stations except those in high latitudes, Price and Wilkins (1963) were not able to find any definitely systematic pattern of the N-field, when N was obtained in this way, and concluded that it was not a satisfactory way of determining the true non-cyclic variation.

In a study of the Sq-field during the IGY, being made by D. J. Stone and myself, we have used "local quiet days" instead of the international quiet days, defining a local quiet day for any station as that 24-hour period extending from 0h to 24h local time which has the greatest overlap with an international quiet (Greenwich) day. We have correspondingly taken the H, D and Z components of the mean N for the five quiet days of each month to be the mean values for those days of the algebraic increases of the mean hourly values (m.h.v.) of H, D and Z, from near the first local midnight to a corresponding time near the second. More precisely, when the given m.h.v. data are centered on the hours of local meridian standard time (L.M.T.), the differences of the m.h.v. centered at successive midnights L.M.T. are used. When the given data are centered on the half-hours, the differences of the m.h.v. centered half an hour before midnight L.M.T. are used. Precautions have of course to be taken to see that the local days used do not contain any appreciable disturbance, which may happen, for example, if an international quiet day is followed by a day which becomes disturbed during the morning. While our main purpose is to try to improve the accuracy of our study of the Sq-field, the data also afford an opportunity of studying the N-field itself. This will be done in detail by D. J. Stone and myself. The present note calls attention to some preliminary results.

2. The field pattern of the non-cyclic variation

When the H, D, Z components of N obtained in this way are converted to X, Y, Z components, a fairly clear pattern of the N-field emerges. There are differences in the pattern from month to month but general features remain discernible. A good test of the reliability and significance of the horizontal components X, Y (or H, D) thus obtained, is to see whether their distribution over the earth can be derived from a scalar potential. I have roughly checked that this is so for the mean N field derived from the 60 quiet days of 1958. More accurate applications of this test for each separate month will, it is hoped, give numerical estimates of the reliability of the values obtained for X and Y.

The general pattern of the horizontal field corresponding to the values obtained for X and Y can be conveniently exhibited by a representative current system flowing in a shell immediately above the earth's surface, which would give rise to these values. This current system will not, of course, be the actual physical source of the field. The real external current source is probably at a considerable distance from the earth's surface and there is also a contribution to the field from currents within the earth. Nevertheless it conveniently exhibits the main features of the field. Current vectors, showing the strength and direction of the representative current system are obtained by rotating the vectors (X, Y) at each station clockwise through 90°. These are shown in Fig. 1 for some of the stations operating during 1958. Vectors for many of the European stations are omitted, but they correspond closely with those shown. The lines of geomagnetic latitudes for 0°, 30° and 60° are also shown, and it will be noted that the representative currents closely follow these lines in direction, especially in Europe and Asia. This shows that the field contains an important axial component parallel to the geomagnetic axis.

The non-cyclic variation is generally attributed mainly to the decay of current flowing westwards in an ionized belt surrounding the earth at a distance of several earth radii. This "ring current" is produced or intensified during magnetic storms, and its subsequent decay produces a gradual rise in the observed value of H. Calculations by Akasofu, Cain and Chapman (1961) indicate that the field of the ring current would be nearly uniform at the earth's surface, with its direction parallel at all times to the earth's dipole axis. The resultant surface field would be modified by currents induced in the earth, but it is likely that the field of these would be mainly a dipole field with its axis along the same direction.

The present preliminary study of N tends to confirm the above explanation of its origin, particularly with respect to the close relationship with the geomagnetic axis. A feature of Fig. 1 which requires further examination, however, is the considerable variation in the magnitude of the representative current vectors on moving around the geomagnetic equator. The current intensity is apparently more than twice as great between geographic longitudes 60°E and 150°E as it is between 60°W and 180°W. Correspondingly there should be some divergence of current away from the equator in one region and a convergence in another. There is some evidence of this in the figure. It appears therefore that the N-field, as here obtained, though largely controlled by the direction of the dipole axis, is not entirely symmetrical about this axis, as is the theoretical field of the ring current.

This lack of symmetry is also shown by plotting the contours of the H component of N on a geomagnetic grid as in Fig. 2. Note that the H component of N (say δH) is not the same as $|\delta\mathbf{H}|$, i.e., $\sqrt{(X^2 + Y^2)}$; in fact $|\delta\mathbf{H}|^2 = (\delta H)^2 + (H\delta D)^2$. But here $H\delta D$ is usually small compared with δH, so that the magnitudes of the vectors, $\sqrt{(X^2 + Y^2)}$, in Fig. 1 approximately correspond to the H components of N in Fig. 2. The Z-component of N is similarly shown on a geomagnetic grid in Fig. 3. The general zonal character of the field is again in evidence, though again there are significant departures from axial symmetry.

Departures of the field from axial symmetry could be caused by (a) some non-symmetrical features of the ring current, (b) ionospheric currents distinct from the ring current, (c) currents induced in the Earth, if it behaves as a non-uniform conductor. It seems very unlikely that (a) could explain the observed departures unless the ring current flows much nearer the Earth than is generally supposed. Some of the features of the Z-component distribution and to a lesser extent that of H could probably be explained by (c), but the major features of the H and Z distributions seem to require an explanation of the kind indicated in (b). The ionospheric currents involved may be associated with fluctuations in the strength of the ring current, or may be the direct effect of solar particle radiation. In this connection, the study by Sugiura (1963) of the hourly values of the equatorial Dst for the IGY period is of great interest. He finds a remarkably close correlation between his Dst values (averaged over three-hourly intervals) and Bartels three-hourly *ap* indices for the whole of the IGY period. This is all the more remarkable in view of the fact that the Dst values are deduced from hourly values of H for stations between 9° and 34° geomagnetic latitude, whereas the *ap* indices are derived from three-hourly ranges for stations between 48° and 63° geomagnetic latitude. This tends to suggest that Sugiura's Dst values, and therefore also our values of N, contain significant contributions from ionospheric sources of geomagnetic disturbance, in addition to the main source, namely the fluctuations in the strength of the ring current.

3. The quiet day non-cyclic variation and hourly Dst values

The relationship between Sugiura's hourly Dst values and the non-cyclic variation on quiet days is worth closer examination. The Sq variation for each month is obtained by subtracting from the m.h.v. (averaged over the five quiet days) a linear function N(t) chosen so as to remove the total non-cyclic change N. Now the non-cyclic variation is attributed to Dst, and Sugiura's tables give hourly values of Dst for all days, including quiet days. At first sight, it might therefore seem possible to get a more accurate (non-linear) estimate of N(t) by using these Dst values, and hence by its removal get a more accurate determination of Sq. However, on closer examination this is found not to be so, because the derivation of the Dst hourly values involves removing the average Sq(t) for the 8 stations used by Sugiura from the average m.h.v. for these stations. But Sq(t) is itself obtained for any station and any month from the m.h.v. on the assumption that the mean N(t) for the five quiet days of the month is a linear function. It follows that the mean derived values of Dst for the five days should represent a linear function, though of course this will not be true for the individual quiet days. Suppose the m.h.v. at any particular station, at time t measured from some chosen epoch, is written in the form

$$h(t) = M(t) + Sq(t) + L(t) + D(t) \ , \qquad (1)$$

where M(t) represents the contribution from the main field and its secular variation, L(t) is the lunar diurnal variation, D(t) the disturbance fluctuation. The variation Sq(t) for any month is defined as the periodic function

$$Sq(t) = \{h(t)\}_Q - N(t) \ , \qquad (2)$$

where the suffix Q denotes the mean values of the corresponding m.h.v.'s for the 5 quiet days of the month, and N(t) is the <u>linear</u> function which will make Sq(t) periodic.

Taking the average of (1) over the eight stations used by Sugiura gives

$$\overline{h(t)} = \overline{M(t)} + \overline{Sq(t)} + \overline{L(t)} + \overline{D(t)} \qquad (3)$$

In this expression $\overline{M(t)}$ is found to be practically constant, M say, $\overline{L(t)}$ is negligible, D(t) is identified with Dst for the for the mean latitude of the stations, and from (2)

$$\overline{Sq(t)} = \{\overline{h(t)}\}_Q - \overline{N(t)} \qquad (4)$$

It follows that

$$\begin{aligned} Dst &= \overline{h(t)} - M - \overline{Sq(t)} \\ &= \overline{h(t)} - M - \{\overline{h(t)}\}_Q + \overline{N(t)} \qquad (5) \end{aligned}$$

This is true for each day of the month. Hence, taking the average over the five quiet days,

$$\{Dst\}_Q = - M + \overline{N(t)} \quad , \tag{6}$$

showing that $(Dst)_Q$ is a linear function of t.

Actually the hourly values of Dst given in Sugiura's tables for the IGY period do not quite satisfy this condition, e.g., for the quiet days of January 1958 the average Dst deviates by amounts of up to 3γ from the linear variation, which totals 10γ for the day. The reason for this is that Sugiura did not in fact remove precisely the average $\overline{Sq(t)}$ as defined in (4) because this function of t changes somewhat for each successive month and, therefore, subtracting it from h(t) would introduce corresponding discontinuous changes in the derived Dst values. He therefore used instead a smoothed version of $\overline{Sq(t)}$, say $\Sigma(t)$, got by approximating to the 18 successive monthly functions $\overline{Sq(t)}$ with a double Fourier series in the time T from Greenwich midnight for each day, and the time M measured in tenths of a month from the start of the IGY. Thus Sugiura's Dst values correspond to

$$Dst = h(t) - M - \Sigma(t) \tag{7}$$

and therefore

$$\{Dst\}_Q = \{h(t)\}_Q - M - \{\Sigma(t)\}_Q \tag{8}$$

Hence, from (4) and (8)

$$\{Dst\}_Q = \overline{N(t)} - M + \overline{Sq(t)} - \{\Sigma(t)\}_Q \tag{9}$$

This shows that the fluctuations of the mean of Sugiura's Dst values, about the linear change $\overline{N(t)} - M$ throughout the average quiet day for a given month, are due to the difference between

$\overline{Sq(t)}$ for the month and the smoothed version $\{\Sigma(t)\}_Q$ of $\overline{Sq(t)}$. It is doubtful how much physical significance should be attached to this, though the fluctuations of Dst on individual quiet days certainly have significance. The result (9) rather suggests that the fluctuations of $\{Dst\}_Q$ from a linear law may be indicative of the order of magnitude of the uncertainties in the Dst values due to the seasonal variation of Sq. If this is so, these uncertainties are notably small and relatively unimportant.

Probably a greater uncertainty in the Dst values arises from the day-to-day variability of Sq. It might be possible to refine Sugiura's values to allow for this by assigning to each day an amplitude factor for Sq as was done by Vestine (1947) in his classic investigation of the main field, and then removing the corresponding modified $\Sigma(t)$ from $\overline{h(t)}$.

Insofar as these studies relate to the average non-cyclic variation N(t) for the five quiet days of each month, we conclude from them that Sugiura's table of hourly Dst values do not afford a method of obtaining an improved estimate of N(t).

4. The zonal part of the N-field

Figures 1, 2 and 3 indicate that a major part of the N-field is zonal. This part has been roughly separated from the remainder by obtaining the mean values of H and Z over zones of width 10° of latitude, using either geomagnetic or geographic latitudes. The results are shown in Fig. 4. This figure also shows the total H and Z values which would correspond to a uniform field parallel to the axis (geomagnetic or geographic) together with an internal dipole field along the corresponding axis. It will be seen that the fit for the H component is good for the geomagnetic axis case, but less good when the geographic axis is used. This agrees with the indications of Fig. 1, and with the assumption that the N-field is mainly produced by the ring current. The different curves for the total Z component correspond to different ratios of the contributions to Z from the internal and external fields, ranging from 0 to -1.

Suppose the change in the external uniform field during one day is h_o parallel to the geomagnetic axis. The geomagnetic north and downward vertical components of h_o are $X_o = h_o \sin \theta$, $Z_o = - h_o \cos \theta$, where θ is the co-latitude. The components of the corresponding dipole field will be of the form $X_i = h_i \sin \theta$, $Z_i = 2h_i \cos \theta$, and the total H and Z components are

$$H = (h_o + h_i) \sin \theta = h_o(1 + \lambda) \sin \theta \qquad (10)$$

$$Z = -(h_o - 2h_i) \cos \theta = -h_o(1 - 2\lambda) \sin \theta \ , \qquad (11)$$

where $\lambda = h_i/h_o$.

The two parameters h_o and λ can be adjusted to get good fits for both the H and Z observations, which shows that only the first harmonic is important in this zonal field. The H values require $h_o(1 + \lambda)$ to be about 8.5, and the Z values then determine λ. It will be seen that the best fit is obtained when λ is about 0.25. It is of interest to note that this value of λ corresponds to an internal <u>induced</u> field of about the magnitude we would expect on our present view of the distribution of conductivity within the earth. The non-cyclic variation is a relatively slowly varying field and can be regarded as the last phase of the slow exponential decay of negative Dst after magnetic storms. Studies of Dst during the earlier parts of magnetic storms and comparison with calculated induced fields in various earth models (Chapman and Price, 1930; Lahiri and Price, 1939) have indicated that the currents induced in the earth penetrate more and more deeply with time, and also the conductivity increases very markedly at a depth of about 0.1a, where a is the earth's radius. It may therefore be expected that the earth currents associated with N flow at a considerable depth and in a region of high conductivity. Suppose first for simplicity that for the varying field under consideration the earth is effectively of infinite conductivity within a sphere of radius qa, $a < 1$, and of zero conductivity above. Then, since Z must be zero at the surface of this sphere, it easily follows that the value of λ in the expression (11) above must be $\frac{1}{2} q^3$. Taking $\lambda = 0.25$ gives a value of q of about 0.8. In this case all the currents giving rise to the internal part of the field flow in an infinitesimally thin shell of radius 0.8a. The same internal part of the field could of course arise from currents flowing in a conductor of finite conductivity; in this case the currents must flow mainly at depths less than 0.2a from the surface. This accords well with the results obtained from the above-mentioned earlier studies of Dst.

Acknowledgment

I am indebted to Dr. E. H. Vestine of The RAND Corporation for most helpful discussions and comment.

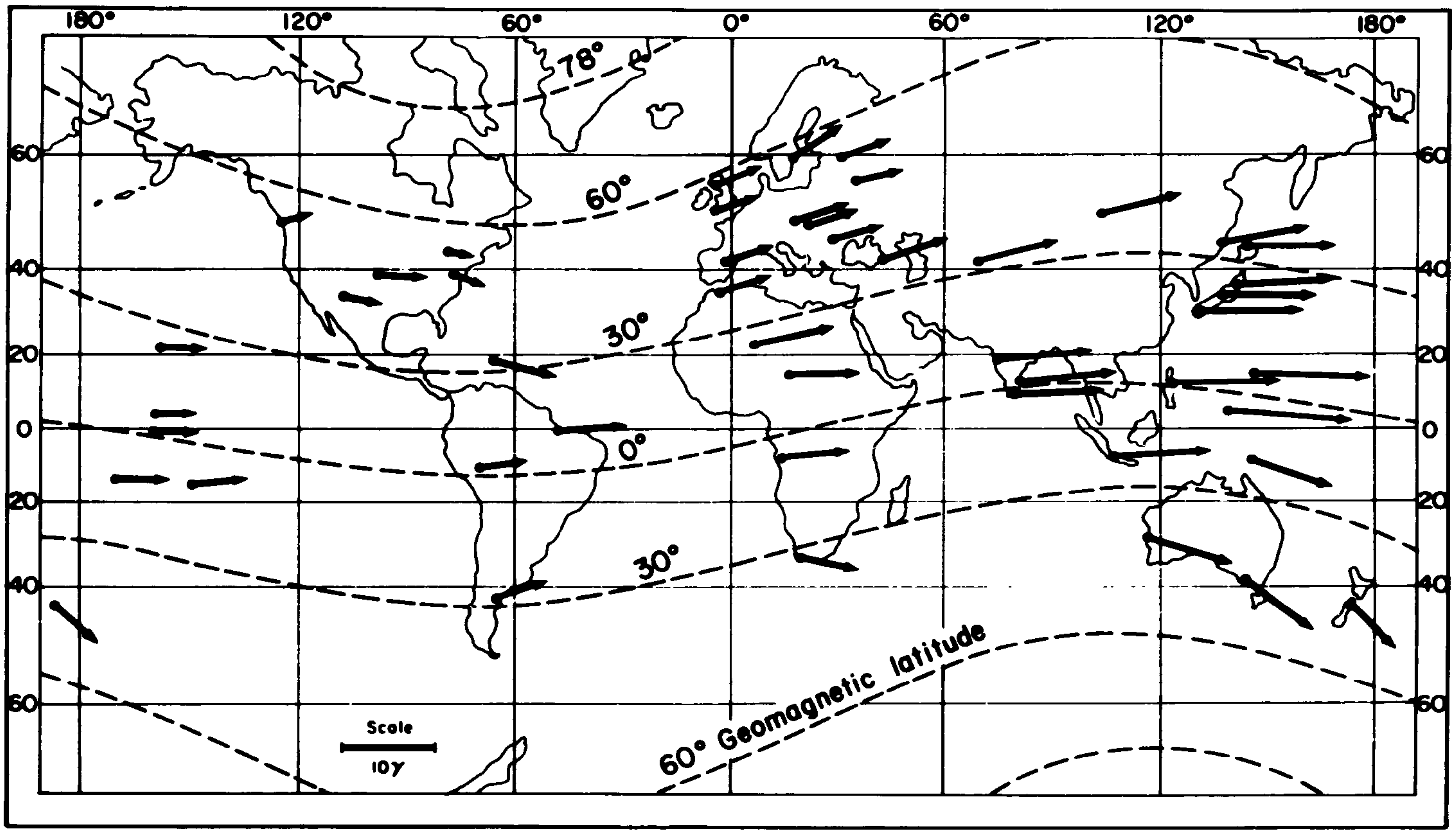

Fig. 1 Current vectors corresponding to average non-cyclic variation on quiet days of 1958

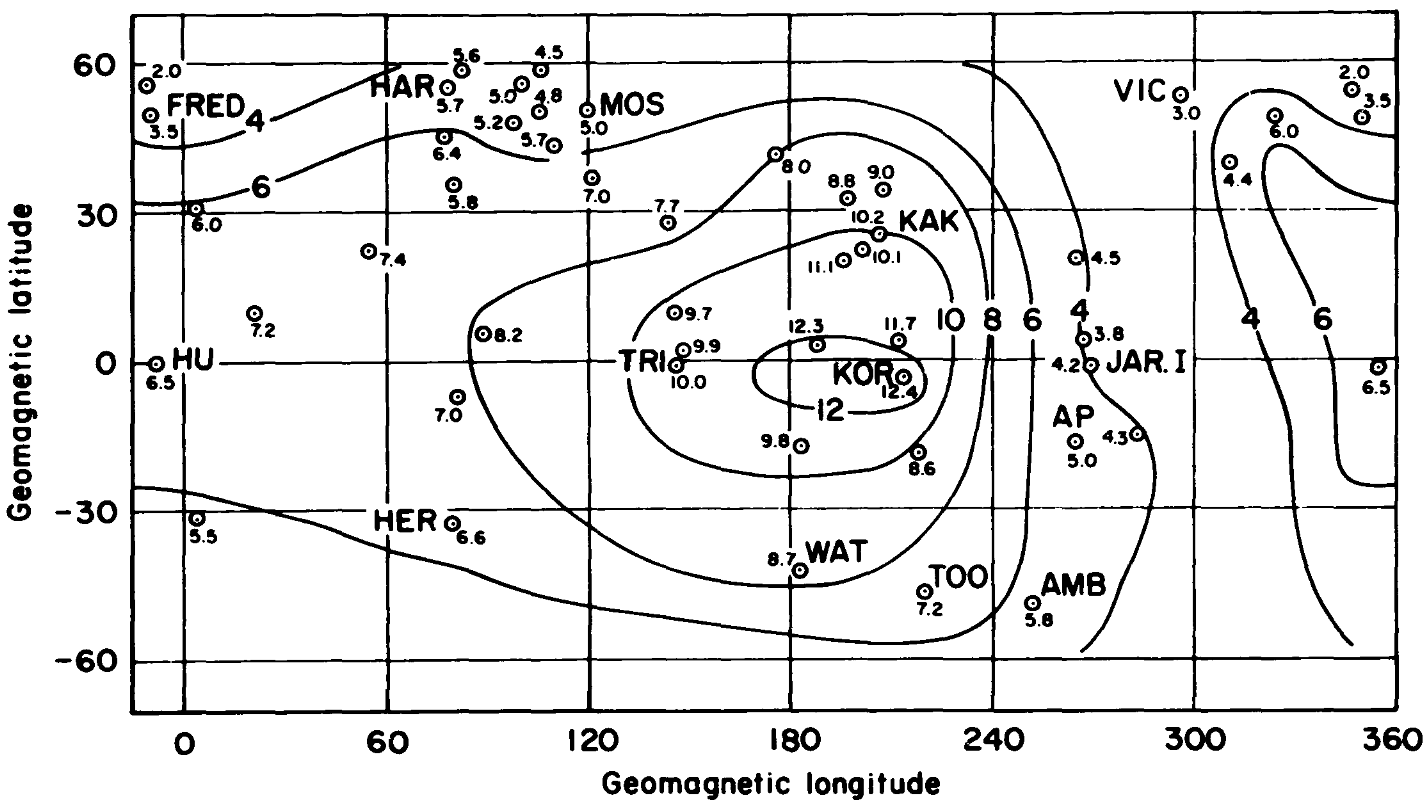

Fig. 2 Average non-cyclic change of H for quiet days of 1958

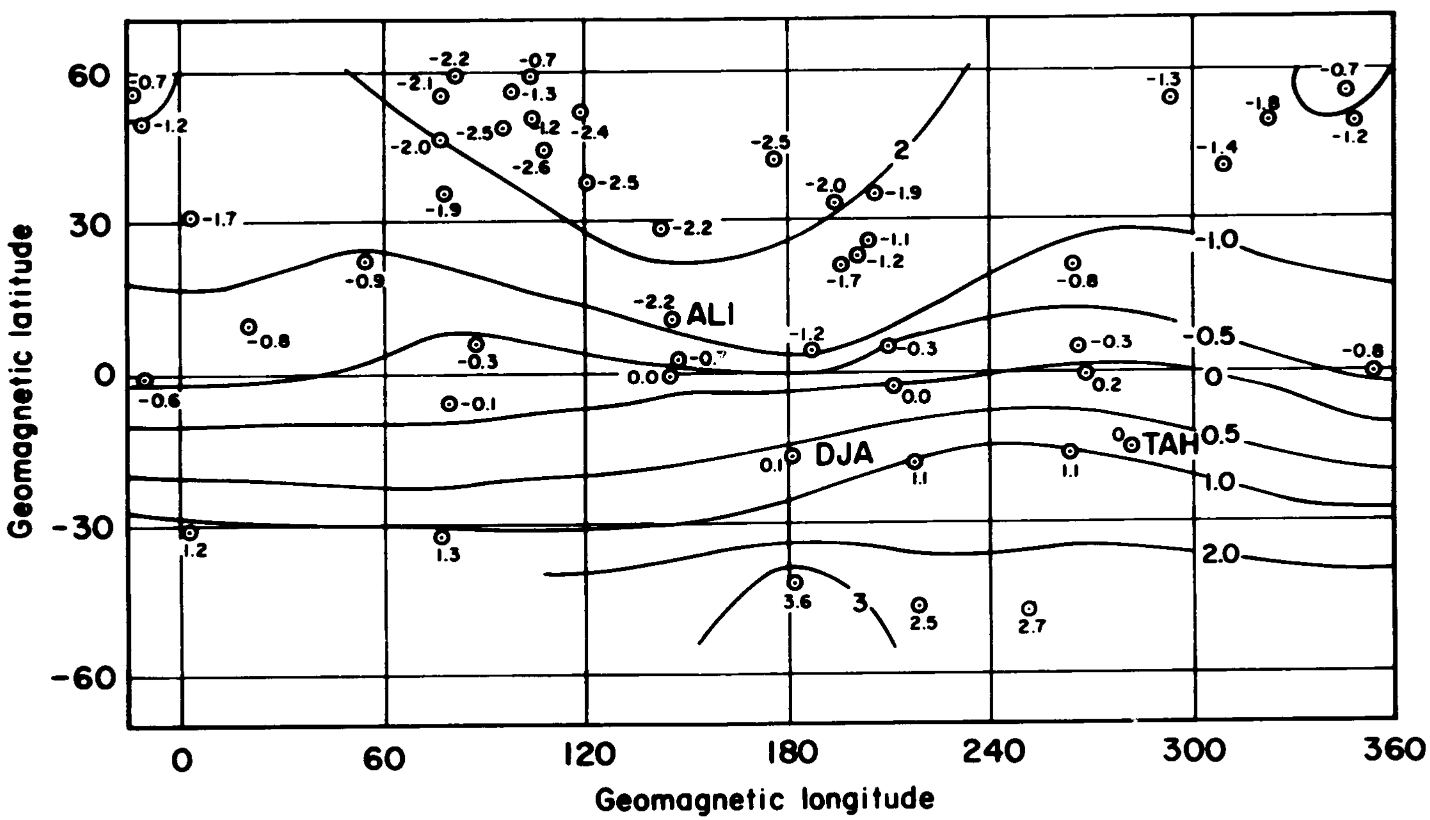

Fig. 3 Average non-cyclic change of Z for quiet days of 1958

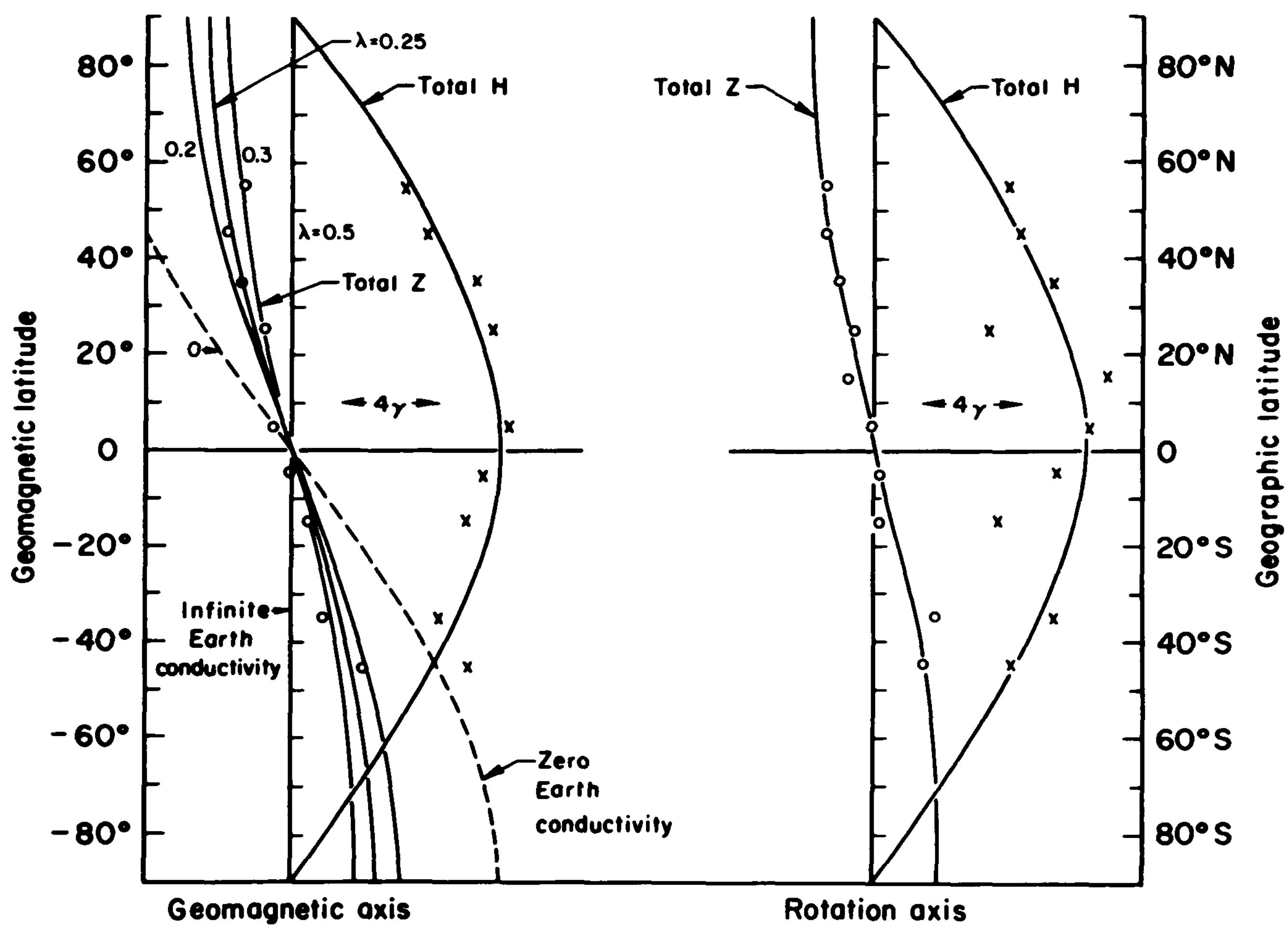

Fig. 4 Zonal part of non-cyclic variations for quiet days of 1958, and curves of total H and total Z corresponding to different assumptions about induced field

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