EXPERIMENTAL AERODYNAMIC DATA
CONCERNING VORTEX FLOW AND THE GAS VELOCITY
IN A SMALL D-C ARC HEATER OPERATED WITH ARGON

By
L. E. Rittenhouse, E. E. Anspach, and M. H. Nesbitt
Propulsion Wind Tunnel Facility
ARO, Inc.

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a subsidiary of Sverdrup and Parcel, Inc.

July 1963
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ABSTRACT

The performance characteristics of a small d-c arc heater of the Gerdien type are analyzed with and without a vortex flow in the nozzle. Moreover, total pressure profiles taken in the plasma and static pressure distributions along the nozzle are presented and used to indicate the effects produced by the vortex in the nozzle. Injecting the working medium (argon) tangentially into the arc chamber was found to be an ineffective means for producing gas dynamic swirl due to the effects from heat addition in the nozzle. The data were obtained at chamber pressures from 1.72 to 5.15 atm and exhausting to atmosphere with power levels up to 22 kw.

Two basically different methods are used to calculate the "bulk" gas temperature, which is required to calculate the gas velocity. The first method makes use of several measured quantities to perform an energy balance on the arc heater. The second method makes use of the conventional weight-flow equation for sonic flow. The total pressure and total temperature at the sonic point (nozzle exit) were obtained from the Rayleigh heat-addition equations. A comparison of the velocities calculated by the two methods reveals agreement within ten percent at the nozzle exit.

PUBLICATION REVIEW

This report has been reviewed and publication is approved.

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NOMENCLATURE

A  Area of the nozzle, \( m^2 \)

\( A_{\text{Geometric}} \)  Geometric area of the nozzle, \( 0.0317 \times 10^{-3} m^2 \)

\( c_p \)  Specific heat at constant pressure, \( 0.124 \text{ kcal/kg} \cdot ^{\circ}\text{K} \) for argon

D  Diameter of the nozzle, \( 0.00635 \text{ m} \)

h  Total enthalpy of the gas, kcal/kg

I  Arc heater current, amp

M  Mach number

\( P_a \)  Electric power, kw or kcal/sec, as noted

p  Pressure, atm*

\( P_N \)  Nozzle static pressure, atm*

Tt  Total temperature, \( ^{\circ}\text{K} \)

V  Arc heater voltage

v  Gas velocity, m/sec

W  Weight flow, kg/sec

x  Distance measured longitudinally along the axis of the nozzle from the entrance of the nozzle, m

Z  Distance measured transversely to the jet and from the centerline of the nozzle, m

\( \gamma \)  Ratio of specific heats, 1.67 for argon

\( \eta \)  Efficiency of the arc heater as determined from an energy balance

\( \lambda \)  Rayleigh relationship, \( \sqrt{\frac{T_{t0}/T_t^*}{P_{t0}/P_t^*}} \)

SUBSCRIPTS

e  Exit conditions

g  Gas

*In this report the local atmosphere \( p = 9.79 \times 10^4 \text{ Newtons/m}^2 \) has been used rather than the standard atmosphere of \( 1.013 \times 10^5 \text{ Newtons/m}^2 \).
Chamber conditions (nozzle entrance)
Total conditions
Water

SUPERSCRIPTS

* Denotes sonic or nozzle exit conditions
1.0 INTRODUCTION

The use of an electric arc to heat a gaseous medium for the production of a high energy plasma has received considerable attention in recent years. Currently, a program is underway at the Arnold Engineering Development Center to develop an aerodynamic testing facility that would incorporate, as part of the equipment, an arc heater followed by a magnetohydrodynamic accelerator. Using this combination it is imperative to have some insight as to the pressure and temperature at the exit of the arc heater and a knowledge of the respective profiles across the plasma jet.

As a portion of the overall program, the present investigation is concerned with (1) the determination of the effect of injecting the working fluid (argon) tangentially into the arc chamber, and (2) the determination of gas velocities in the jet, which reflects on the performance and jet characteristics of a small Gerdien type d-c arc heater. The objective of injecting the argon tangentially into the chamber is to produce a swirling motion in the nozzle that would keep the arc rotating; thus hot spots in the nozzle would be eliminated. Upon consideration of the consequency of Rayleigh heat addition in a constant area passage, which is the nozzle for the present case, the concept of swirling the gas has definite limitations. A discussion of this phenomenon is presented, and experimental data are shown to substantiate the conclusion. It must be emphasized that this analysis is for a Gerdien-type heater having a high velocity in the anode passage which also serves as the nozzle.

The gas velocities are deduced by (1) performing an energy balance on the arc heater, and (2) using the conventional weight-flow equation for sonic flow with appropriate values for the total pressure and temperature at the sonic point. The values used for the total pressure and temperature were derived from the Rayleigh heat-addition equations. A comprehensive discussion of the Rayleigh heat-addition process as it applies to the present arc heater configuration is given in Ref. 1.

2.0 APPARATUS

A schematic diagram of the equipment used in the experimental investigation is given in Fig. 1, a schematic of the arc heater, in Fig. 2, and photographs of the assembled test apparatus, in Fig. 3.

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Argon was obtained from storage bottles at a pressure and temperature of about 150 atm and 290°K, respectively. It was then piped from a manifold through a pressure regulator, rotameter, and flow control valve to the arc heater.

2.1 ARC HEATER

The arc heater (Fig. 2) used in the experiment is in the 30-kw range and is of the Gerdien type (Ref. 2). The principal components of the heater are a tungsten cathode and a carbon anode which also serves as the nozzle. Both components are water cooled. The nozzle has a throat or minimum diameter of 6.35 mm (0.25 in.) and a length of about 6.7 nozzle throat diameters.

Argon was used as the working gas and injected tangentially at the rear of the arc chamber to induce flow vorticity in the nozzle. The gas was heated in the nozzle by means of an electric arc and then discharged to the atmosphere. For the test runs where flow vorticity was not desired in the nozzle, a perforated spool (see Fig. 2) was installed in the arc chamber to eliminate the vortex flow prior to entering the nozzle.

2.2 INSTRUMENTATION

The location of pressure, temperature, mass flow, and power instrumentation is shown in Fig. 1. The static pressure distribution along the nozzle was measured with ten wall orifices. These orifices were connected to a mercury-manometer board which was photographed with a 70-mm sequence camera.

A 3.17-mm-diam total-pressure probe constructed of inconel was used to traverse the effluent jet at two and four nozzle diameters downstream from the nozzle exit. A three-dimensional probe holder was used to align the probe with the geometric center of the nozzle. Since the probe was not water cooled, the traversing was accomplished as swiftly as possible. However, the response time of the instrumentation was very fast due to the shortness of the probe tubing and high response transducer. A limited number of static pressure measurements were obtained at two and four nozzle diameters with a static pressure probe. The total and static pressures were measured with a 50-psi pressure transducer, and the corresponding trace and probe position were recorded on a millivolt recorder. Temperatures of the ambient argon and water were measured with copper-constantan
thermocouples. Rotameters were used to determine the gas and water flow rates.

A rectifier type of arc welding machine was used for the d-c power supply to the arc heater. Power levels up to 32 kw were available for this unit. Current and voltages to the arc heater were recorded on millivolt recorders. A high frequency unit connected in series with the power supply initiated the arc.

2.3 PROCEDURE

When the arc heater was operated cold (P_a = 0), the chamber pressure was the only operating parameter. After the desired chamber pressure was obtained, nozzle static pressures and total pressure surveys of the jet, in addition to gas inlet temperature and mass flow data, were recorded. The runs were repeated to obtain data with and without the perforated spool in the arc chamber. Only a limited number of static pressure surveys of the jet at two and four nozzle diameters and total pressure surveys at four nozzle diameters were obtained. The majority of the total pressure surveys were made at two nozzle diameters downstream of the nozzle exit.

For the hot flow runs, the chamber pressure, water flow, and power input were set to the desired values before initiating the arc with the high frequency unit. The same pressures, temperatures, and mass flow data were recorded as for the cold flow runs with the addition of the water temperatures, power input, and water flow rates. The data were obtained by varying the arc chamber pressure at a selected power input to the heater for both the vortex and nonvortex flows. Sufficient time was allowed between each data point for the system to reach equilibrium.

2.4 PRECISION OF MEASUREMENT

To obtain an insight as to the accuracy and repeatability of the calculations, the precision of the instrumentation was estimated as:

<table>
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<th>±0.33 kw</th>
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<tr>
<td>power</td>
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<tr>
<td>pressure</td>
<td>±0.010 atm</td>
</tr>
<tr>
<td>temperature</td>
<td>±2⁰K</td>
</tr>
<tr>
<td>gas mass flow</td>
<td>±0.0002 kg/sec</td>
</tr>
<tr>
<td>water flow</td>
<td>±0.00126 kg/sec</td>
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</table>
Using the method of Ref. 3 for calculating the uncertainties, we estimated the total temperature of the gas at the nozzle exit and the gas velocity. From an energy balance equation, the uncertainty of the gas temperature was found to be ±136°K. Using the calculated total temperature and experimentally determined values for the jet Mach number, we estimated the uncertainty of the gas velocities to be ±32 m/sec.

3.0 PRESENTATION AND DISCUSSION OF RESULTS

3.1 EXPERIMENTAL EVIDENCE OF VORTEX FLOW IN THE ARC CHAMBER

As was mentioned in a preceding section, it was hoped that the vortex or swirling flow entering the nozzle from the arc chamber would be eliminated by inserting a perforated spool between the point of gas injection and the nozzle. The effect of inserting the spool on the aerodynamic characteristics of (1) the arc heater performance maps, (2) the total pressure profiles in the jet, and (3) the static pressures in the nozzle will be examined.

The performance maps for the arc heater without and with a vortex are presented in Figs. 4a and b. The data points in the upper right-hand quadrant are the measured quantities \( P_a \) and \( W_g \), whereas the results shown in the remaining three quadrants are derived from an energy balance. An examination of Figs. 4a and b reveals only minor differences in the measured and derived quantities, with and without the vortex, when the arc is struck \( P_a = 0 \). However, when the arc heater is run cold \( P_a = 0 \), the weight flows differ considerably between the two cases, particularly at the high chamber pressures. Since the nozzle is exhausting to atmosphere, the resulting pressure ratio \( (p_t_0/p_e) \) is sufficient to "choke" the nozzle. Thus, the discrepancy in the weight flow between the two cases must inherently be a result of the distribution of the total pressure through the nozzle.

Stagnation pressure profiles measured in the free jet are presented in Fig. 5. They were obtained at two nozzle diameters downstream from the exit. With a vortex in the nozzle, the total pressure profiles for the cold case (Fig. 5a) exhibit cores of low momentum gas, particularly at the high chamber pressure. Moreover, the center of the jet moves appreciably as the pressure is varied. This would be very

*The details of these calculations are presented in section 3.2.
† Except at \( p_{t_0} = 1.72 \text{ atm} \)
undesirable from the standpoint of aerodynamic testing in the jet. Without a vortex (perforated spool inserted in the chamber) the profiles are, in general, orderly with a higher total pressure in the center as compared to the vortex flow. A low momentum core still exists for the chamber pressure of 5.15 atm without a vortex, which is probably due to the wake emanating from the cathode upstream of the nozzle. The characteristics of these profiles are a direct measurement of what was inferred above from the arc heater performance maps (Fig. 4). What is of much more interest, however, is the effect of heating on the vortex. The performance maps indicate no change in the arc heater performance between vortex and nonvortex flows, which implies no change in the mass flow and in turn, no change in the pressure profiles. Thus, it would be of interest to examine the pressure distribution of the jet with the arc heater in operation.

When the arc is struck and heat added to the gas, the profiles change as shown in Fig. 5b. The apparent change between cold and hot runs with the vortex (compare Fig. 5a with Fig. 5b) results in a significantly more uniform stagnation pressure profile and the shifting of the jet centerline. The shifting of the axis of the jet must be associated with the manner in which the gas enters the nozzle. The question now arises as to why the low momentum core is affected by heat addition. This can be explained by the Rayleigh heat-addition analysis for Gerdien-type arc heaters, as described in Ref. 1.

When the arc is struck, heat is added to the gas in a constant area channel. Under these conditions, if sufficient heat is added, the flow should adjust within the channel to produce Mach number 1 at the exit. For subsonic flow entering the channel, the maximum condition would be Mach number zero at the entrance and Mach number 1 at the exit. Static pressure measurements in the nozzle (Fig. 6) reveal that heat addition significantly changes the nozzle pressure distributions compared to the cold runs. A rough extrapolation of the results indicates an entrance pressure ratio approaching 1, meaning the inlet Mach number and velocity to the constant area section approach zero. Essentially, the flow has "backed up" in the nozzle and the forward portion of the nozzle is acting somewhat as a stilling chamber. Comparison of Fig. 5a and Fig. 5b shows that the total pressure profiles are more uniform when heat is added either with or without the vortex. However, the profiles are consistently more uniform and symmetrical without the vortex.

An interesting point can be made by examining Fig. 5c, which indicates the effect of increasing the power at a given chamber pressure on the local total pressure for the cases with and without a vortex. The effect with a vortex of increasing the power input is generally opposite
the effect without a vortex. Increasing the power produces an increase in $p_{\text{tprobe}}$ with a vortex and a decrease in $p_{\text{tprobe}}$ without a vortex. The logical trend is for the $p_{\text{tprobe}}$ to decrease as the power is increased due to the heat addition in the nozzle, and this is the observed phenomena without a vortex. The same trends as just discussed regarding the effects of increasing the power input hold equally as well for a lower chamber pressure, as can be seen in Fig. 5d. The original concept for swirling the gas in the nozzle was to keep the arc moving and minimize erosion in the nozzle. However, the nozzle was in no worse condition after running without a vortex, and the arc heater performance with regard to gas temperatures and efficiency was approximately the same. Consequently, this particular arc heater could be operated without forced arc rotation. From this discussion it is apparent that gas dynamic swirl produced undesirable jet characteristics that were difficult to analyze for the anode-cathode geometry employed here. In the event arc rotation is required to protect the anode, means other than gas dynamic swirl would have to be employed to ensure favorable jet characteristics. It should be emphasized that this conclusion does not necessarily apply to arc heater geometries in which the heat addition occurs in a very low velocity region ($M_e << 1$).

A more complete analysis should simultaneously take into account the viscous (boundary-layer) and heating effects. To assess the combined friction heating effects or the effect of the vortex on the boundary layer in the nozzle would be a difficult task. The nozzle is only 6.7 nozzle-diameters long, and the use of fully developed boundary-layer profiles would be questionable. Some overall effects can be seen by examination of the static pressure distributions of Fig. 6. The static pressure decrease through the nozzle for cold flow represents the boundary-layer growth both with and without a vortex. The cold flow pressure losses with and without vortex flow are nearly the same. Slightly greater losses for vortex flow indicate that it behaves as if it had a slightly thicker boundary layer. In the case when the arc is struck, the pressure decrease through the anode nozzle reflects the combined effects of friction and heating.

It might be mentioned that the vortex flow case can be given a rough one-dimensional analysis by treating the void in the center of the jet (resulting from the centrifugal force of the gas) as a decrease of the effective flow area of the anode nozzle. The "effective" area of the nozzle can readily be calculated from the one-dimensional weight flow equation (Eq. (1)) for sonic flow ($\gamma = 1.67$), knowing the weight flow and assuming that the average $p_t$ and $T_t$ are the same at the entrance and exit of the nozzle and that the friction coefficient is unity.
\[ A_e = \frac{W_g \sqrt{T_e}}{4928 \ p_{te}} \left( M_e = 1 \right) \]  \hspace{1cm} (1)

Evaluating Eq. (1) with the geometric area of the nozzle (first case) and the measured \( W_g \) for the remaining cases, results in the following quantities for \( p_{to} = 4.41 \) and cold flow:

- Without vortex: \( W_{without \ vortex} = 0.0353 \ \text{kg/sec} \)
- With vortex: \( W_{with \ vortex} = 0.0270 \ \text{kg/sec} \)
- Geometric area: \( A_e, \text{geometric} = 0.0317 \times 10^{-3} \ \text{m}^2 \)
- Without vortex: \( A_e, \text{without \ vortex} = 0.274 \times 10^{-3} \ \text{m}^2 \)
- With vortex: \( A_e, \text{with \ vortex} = 0.0210 \times 10^{-3} \ \text{m}^2 \)

### 3.2 Calculation of Jet Velocities

Two methods may be employed to deduce the exit temperature and thus the exit velocity (assuming sonic flow at the nozzle exit). The first method is to perform an energy balance for the arc heater and nozzle. The second method is to use the conventional weight-flow equation for sonic flow with the appropriate pressures and temperatures derived from the Rayleigh heat-addition equations. Downstream Mach numbers are obtained from data similar to the total pressure surveys presented earlier. Although total pressure profiles were obtained for all the data points shown in Fig. 4, only selected profiles are presented. The velocity calculations were arbitrarily chosen for the case without vortex since there were no significant differences between the nozzle exit enthalpies with and without the vortex for the same weight flows and power levels \( (P_a > 0) \).

Several assumptions had to be made in order to make the Rayleigh heat-addition calculations possible and are summarized below:

1. The flow in the nozzle passage is one-dimensional, and the geometric area is constant.
2. All the heat is added in the nozzle, and the Rayleigh heat-addition equations are applicable.
3. The friction in the nozzle is negligible as compared to the Rayleigh heat-addition effects.
4. The flow is in equilibrium, and the ideal gas relations are adequate.
5. The Rayleigh pitot relations are adequate to determine the Mach number ahead of the shock wave at the probe (two and four nozzle diameters downstream of the exit).
6. The static pressure in the proximity of the probe is atmospheric.

7. Mach number 1 exists at the nozzle exit.

Item (3) is somewhat dubious, based on the data presented in the preceding section. Obviously, there is a boundary layer in the nozzle which causes the static pressures to decrease through the constant area nozzle passage (see Fig. 6, cold flow). The fact that the flow is assumed to be in equilibrium (Item 4) and real gas effects are neglected appears justifiable since the working fluid is argon. Reference 4 indicates that argon is thermally and calorically perfect, to a good approximation, below the onset of ionization which occurs at about 8000°K or 1000 kcal/kg. In Item (6) the static pressure in the jet downstream of the nozzle is assumed to be atmospheric. Static pressure measurements were obtained downstream of the exit at two and four nozzle diameters. Although only a limited amount of data were obtained, the results justify the assumption.

Performing an energy balance for the arc heater requires the measurement of several parameters such as: the temperature rise of the water through the cathode and anode (nozzle), weight flow of water, the electrical power into the arc heater, the gas flow rate and knowledge of the specific heats for argon and water \((c_{Pg}\) and \(c_{PW}\)). With these quantities the temperature of the gas at the exit can be computed from

\[
T_{te} = \frac{2.38 \times 10^{-4} \, VI - W_w c_{PW} \Delta T_{tw}}{c_{Pg} \, W_g} + T_{to} \tag{2}
\]

Values of \(V, I, W_W, \Delta T_{tw}\), and \(W_g\) are obtained during operation of the arc heater; thus Eq. (2) can be evaluated for \(T_{te}\). The total enthalpy of the gas can readily be obtained since

\[
h_e = c_{Pg} \, T_{te} \tag{3}
\]

Moreover, the efficiency is derived from

\[
\eta = \frac{2.38 \times 10^{-4} \, VI - W_w c_{PW} \Delta T_{tw}}{2.38 \times 10^{-4} \, VI} \times 100 \tag{4}
\]

The results presented in Fig. 4 have been derived from Eqs. (2), (3), and (4); however, they are presented in a slightly different form for convenience. It should be noted that the area of the nozzle does not enter into the calculations.
The familiar weight flow equations for sonic or "choked" flow, assuming the friction coefficient is unity, can be written as

\[ W_g = \frac{(4928) P_t^* A^*}{\sqrt{T_t^*}} \quad (5) \]

Evaluating Eq. (5) for the cold flow case (\(P_a = 0\)) poses no particular problems, since the total pressure and temperature at the exit (sonic point) usually can be assumed equal to the entrance conditions. However, when a large amount of heat is added to the gas, this assumption cannot be made, since the total pressure decreases and the total temperature increases as the sonic point in the nozzle is approached.

If Eq. (5) is manipulated to make use of the Rayleigh relationships, the equation would be as follows:

\[ \lambda = \frac{W_g \sqrt{T_{t0}}}{(4928) \frac{P_{t0}}{A^*}} \quad (6) \]

where

\[ \lambda = \frac{\sqrt{T_{t0}}}{\frac{P_{t0}}{P_{t^*}}} \]

Once the chamber conditions are known, the weight flow is measured, and the area is known, the parameter \(\lambda\) is defined. The values for \(\lambda\), as a function of the Mach number entering the nozzle, can be derived from Rayleigh tables (Refs. 5 and 6) using the appropriate value for \(\gamma\). These calculations have been performed, and the results presented in Fig. 7 as a function of the entrance Mach number. The individual ratios \(\sqrt{T_{t0}/T_{t^*}}\) and \(P_{t0}/P_{t^*}\) were obtained from the Rayleigh tables and are also presented in Fig. 7. The values shown in Fig. 7 are for one-dimensional compressible-flow in the absence of friction and area change. The maximum total pressure loss would be 23 percent of \(P_{t0}\) if the flow accelerated from Mach number zero at the entrance to Mach number 1 at the exit. However, if the initial Mach number were 0.3, the total pressure loss would still be 17.8 percent of \(P_{t0}\). Thus, with the measured weight flow of gas and initial conditions, and assuming the area is known \((0.0317 \times 10^{-3} \text{ m}^2)\), the preceding method can be used to obtain the exit temperature of the gas.

Total temperatures of the gas for several operating conditions were calculated consistent with the two methods described earlier. The temperature rise in the gas \((T_{t^*} - T_{t0})\) was then computed, and the results are shown in Fig. 8. The parameter on the abscissa, \(P_a/W_g\), is a measure of the total heat put into the gas, since the heat loss through the cathode and anode has not been subtracted. At the high values of
\( \frac{P_a}{W_g} \), the energy balance calculations predict gas temperature differentials about 20 percent higher than the Rayleigh calculations. The agreement between the two methods becomes better as \( \frac{P_a}{W_g} \) is decreased. The unsymboled line in Fig. 8 represents the heat capacity of the gas. For a given temperature rise of the gas, the difference between the energy balance calculations and the curve for the heat capacity of the gas represents the additional power required to compensate for the heat taken away by the cooling water in the cathode and anode. Data are not presented for a chamber pressure of 1.72 atm, since it is believed the flow was not sonic at the nozzle exit.

With the temperatures presented in Fig. 8, the gas velocities at the exit were calculated and are presented in Fig. 9. In general, the same trends as shown for the temperatures are manifest in the velocity calculations since \( M_e = 1 \). Again the values calculated with the energy balance are larger than those using the Rayleigh relationships at high values of \( \frac{P_a}{W_g} \). With the method described in Ref. 4 and the nozzle exit pressure extrapolated from Fig. 6, three calculations were obtained as shown by the flagged symbols in Fig. 9. The agreement appears good between the energy balance and the Rayleigh heat-addition methods.

In order to calculate the velocity of the gas at two and four nozzle diameters downstream of the exit, the temperature and Mach number must be known. Since the total temperature was not measured, it is assumed to be equal to the values calculated for the nozzle exit. The local Mach numbers were obtained from the Rayleigh-Pitot equations using the total pressure surveys (assuming the static pressure is atmospheric), and the results are shown in Fig. 10. Only a limited amount of data were obtained at four diameters aft of the nozzle. The trend is that as power is added to the gas at a constant weight flow, the Mach number decreases. However, as the power is increased, the gas velocity increases as was shown in Fig. 9. If the chamber pressure is somewhat greater than 2.45 atm, the gas expands when leaving the nozzle. The tick marks shown in Fig. 10 for the results at two nozzle diameters are intended to indicate the downstream Mach number that would be obtained if the maximum total pressure loss due to heat addition occurred in the nozzle (\( M_0 = 0 \)). In other words, the minimum downstream Mach number can be calculated by determining the Mach number for cold flow (\( \frac{P_a}{W_g} = 0 \)) and the corresponding static to total pressure ratio and then multiplying the pressure ratio by 1.299 (Rayleigh relationship) to find the minimum Mach number. This, of course, precludes the existence of shock waves in the jet before the measuring station, which would also decrease the Mach number. Calculating the minimum Mach number in this manner agrees quite well with the experimental data as shown in Fig. 10.
With the Mach number determined and the total temperature assumed equal to the values found for the nozzle exit, the gas velocity at two nozzle diameters downstream was calculated. The results are shown in Fig. 11. Gas velocities using temperatures derived from the energy balance are presented in Fig. 11a, whereas temperatures obtained using the Rayleigh heat-addition relationships were used to calculate the gas velocities shown in Fig. 11b. A comparison of Figs. 11a and 11b reveals that the use of Rayleigh relationships continues to predict good agreement with the energy balance calculations for the gas velocity except at high $P_a/W_g$ values. Although only a few data points were obtained at four nozzle diameters downstream, the agreement between the two methods is very good.

Cross plots of the calculated velocities plotted as a function of $x/D$ for several power levels and chamber pressures are given in Fig. 12. The cold-flow data are presented mainly to indicate the gas velocity when heat is not added to the gas. The trends of both the velocity calculations are nearly similar, and the differences are small at any $x/D$ for a given power setting and pressure level. Thus, it would appear that the method of velocity calculations using the energy balance would probably be the most direct procedure for estimating purposes; however, it must be kept in mind that for high $P_a/W_g$ values, there may be significant heat losses (such as radiation) that would not be detected through an energy gain to water.

4.0 CONCLUDING REMARKS

The following conclusions are based on the data obtained from a small d-c arc heater operated with argon and exhausting into the atmosphere:

1. Inserting the gas tangentially into the arc chamber (vortex flow) for the specific arc heater investigated produced undesirable characteristics in the total pressure profiles in the jet. The arc heater operation and performance were approximately the same with or without the vortex present. Therefore, it is concluded that arc rotation by gas dynamic swirl in arc heaters of the type considered here (simple Gerdien) is not required. If, however, arc rotation is desired (for example, for larger $P_a/W_g$ values), then it would probably be necessary to use other means.
2. The energy balance method of calculating the gas temperature which, in turn, can be used to calculate the gas velocity when the Mach number is known is probably the most direct approach for estimating purposes. Using the Rayleigh heat-addition relationships in conjunction with the weight-flow equation for sonic flow requires a knowledge of the pressure, temperature, and area at the sonic point. However, when the assumption is made that the area is equal to the geometric area, the gas velocities calculated by the two methods indicate relatively good agreement. Moreover, at this time it is impossible to state which method provides the best predictions for the gas velocity since absolute temperature measurements were not obtained.

REFERENCES


Fig. 1 Schematic Outline of the 30-kw Arc Heater Equipment
Fig. 2 Cutaway View of Arc Heater Indicating Major Components
Fig. 3 Installation Photographs of Arc Heater Equipment
Fig. 4 Arc Heater Performance Characteristics
\[ P_t_0, \text{atm} \]
\[ \begin{array}{c}
5.15 \\
4.41 \\
3.43 \\
2.45 \\
1.72 \\
\end{array} \]

\[ P_a, \text{kw} \]
\[ \begin{array}{c}
\eta \\
120 \\
12 \\
8 \\
6 \\
4 \\
2 \\
0 \\
\end{array} \]

\[ W_g \times 10^3, \text{kg/sec} \]

\[ W_{\text{he}}, \text{kcal/sec} \]

b. With Vortex

Fig. 4 Concluded
a. Cold-Flow Profiles at Various Chamber Pressures

Fig. 5 Total Pressure Profiles in the Jet at Two Nozzle Diameters Downstream of the Nozzle Exit with and without a Vortex
b. Profiles at Several Chamber Pressures and Power Levels

Fig. 5 Continued
c. Profiles at a Chamber Pressure of 5.15 atm and Several Power Levels

Fig. 5 Continued
Fig. 5 Concluded

**d. Profiles at a Chamber Pressure of 2.45 atm and Several Power Levels**

- **WITH VORTEX**
  - $P_{t0} = 2.45$ atm
- **WITHOUT VORTEX**
  - $P_{t0} = 2.45$ atm

Pt probe atm

$P_a$, kw

- 5.9 to 11.6
- 2.9
- 0 (COLD FLOW)
Fig. 6 Static Pressure Distributions along the Nozzle with and without a Vortex
Fig. 7 Relationships Obtained from the Rayleigh Heat-Addition Equations
Fig. 8 A Comparison between the Rayleigh and Energy Balance Methods for the Temperature Rise of the Gas Through the Nozzle

$$\Delta T_t = \frac{P_a}{\dot{m} c_{pg}} - T_{t0}$$

Fig. 9 The Velocity of the Gas at the Exit as Determined from the Rayleigh and Energy Balance Methods
Fig. 10 The Mach Number in the Jet at Two and Four Nozzle Diameters as Determined from the Probe Measurements
Fig. 11 Gas Velocity at Two Nozzle Diameters as Determined from the Energy Balance and Rayleigh Methods
Fig. 12 The Gas Velocity as Function of $x/D$ for Several Chamber Pressures and Power Levels as Determined from the Energy Balance and Rayleigh Methods
The performance characteristics of a small d-c arc heater of the Gerdien type are analyzed with and without a vortex flow in the nozzle. Moreover, total pressure profiles taken in the plasma and static pressure distributions along the nozzle are presented and used to indicate the effects produced by the vortex in the nozzle. Injecting the working medium (argon) tangentially into the arc chamber was found to be an ineffective means for producing gas dynamic swirl due to the effects from heat addition in the nozzle. The data were obtained at chamber pressures from 1.72 to 5.15 atm and exhausting to atmosphere with power levels up to 22 kw. Two basically different methods are used to calculate the "bulk" gas temperature, which is required to calculate the gas velocity. The first method makes use of several measured quantities to perform an energy balance on the arc heater. The second method makes use of the conventional weight-flow equation for sonic flow. The total pressure and total temperature at the sonic point (nozzle exit) were obtained from the Rayleigh heat-addition equations. A comparison of the velocities calculated by the two methods reveals agreement within ten percent at the nozzle exit.
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Arnold Engineering Development Center
Arnold Air Force Station, Tennessee


Unclassified Report

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