NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
MEMORANDUM REPORT NO. 1474
APRIL 1963

HIGH IMPEDANCE CATHODE FOLLOWERS FOR
PIEZOELECTRIC GAGE RECORDING IN
AIR BLAST RESEARCH

Bernard Soroka
Jacob Wenig

RDT & E Project No. 1M010501A006
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND
ASTIA AVAILABILITY NOTICE

Qualified requestors may obtain copies of this report from ASTIA.

The findings in this report are not to be construed as an official Department of the Army position.
HIGH IMPEDANCE CATHODE FOLLOWERS FOR PIEZOELECTRIC GAGE RECORDING IN AIR BLAST RESEARCH

Bernard Scroka
Jacob Wenig

Terminal Ballistics Laboratory

RDT & E Project No. 1M010501A006

ABERDEEN PROVING GROUND, MARYLAND
ABSTRACT

The design of four types of single tube cathode followers is described and selections made of the ones best suited for use with an eight-channel unit employed for air blast research.

Exact formulae as well as simplified good approximations for gain, input impedance and output impedance are derived in the Appendices. This report may serve as a reference source for cathode follower design and circuit analysis.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>3</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>7</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>9</td>
</tr>
<tr>
<td>CATHODE FOLLOWERS - GENERAL THEORY</td>
<td>9</td>
</tr>
<tr>
<td>ADVANTAGES OF RETURNING $R_k$ TO NEGATIVE SUPPLY</td>
<td>13</td>
</tr>
<tr>
<td>DRIVING CAPABILITIES</td>
<td>14</td>
</tr>
<tr>
<td>FORMULAE AND DERIVATIONS</td>
<td>15</td>
</tr>
<tr>
<td>TUBE SELECTION AND DESIGN CHARACTERISTICS</td>
<td>16</td>
</tr>
<tr>
<td>PLATE CHARACTERISTICS 12AX7</td>
<td>17</td>
</tr>
<tr>
<td>DESIGN PROCEDURE</td>
<td>18</td>
</tr>
<tr>
<td>DESCRIPTION OF EIGHT-CHANNEL CATHODE FOLLOWER</td>
<td>19</td>
</tr>
<tr>
<td>COMPOSITE RESULTS FOR FOUR TYPES OF CATHODE FOLLOWERS</td>
<td>20</td>
</tr>
<tr>
<td>CONCLUSION</td>
<td>21</td>
</tr>
<tr>
<td>FIGURE 8, EIGHT-CHANNEL CATHODE FOLLOWER</td>
<td>22</td>
</tr>
<tr>
<td>APPENDICES</td>
<td>25</td>
</tr>
<tr>
<td>A. Derivations of Gain, $R_{in}$ and $R_{out}$ for Various Cathode Followers</td>
<td>25</td>
</tr>
<tr>
<td>1. Simple Cathode Follower</td>
<td>27</td>
</tr>
<tr>
<td>2. Tapped Cathode Follower</td>
<td>29</td>
</tr>
<tr>
<td>3. Bypassed Cathode Follower</td>
<td>33</td>
</tr>
<tr>
<td>4. Bootstrapped Cathode Follower</td>
<td>34</td>
</tr>
<tr>
<td>B. Evaluation of Gain, $R_{in}$ and $R_{out}$ for Four Cases of Cathode Followers</td>
<td>39</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

A = gain
C = capacitance in farads
E = voltage*
e_g = grid voltage
e_{in} = input voltage
e_{out} = output voltage
e_m = mutual transconductance - \mu \text{ mho}
e_p = plate conductance - \mu \text{ mho}
i = current*
i_I = external generator current
i_g = grid current
I_{in} = input current
i_{\text{max}} = maximum current
i_p = plate current
Q = charge in coulombs
R_g = grid resistance
R_{C_1} = grid resistance
R_{C_2} = grid resistance
R_{in} = input resistance
R_k = cathode resistance
R_{k_1} = upper cathode resistance

* All voltages in volts; all resistances in ohms; all current in amperes.
\( R_{k_2} \) = lower cathode resistance
\( R_{k_t} \) = total cathode resistance = \( R_{k_1} + R_{k_2} \)
\( R_{out} \) = output impedance
\( r_p \) = plate resistance
\( \mu \) = amplification factor
\( \omega \) = \( 2\pi \) times frequency
INTRODUCTION

During the course of investigating air blast phenomena and the reaction of structures to shock loading, piezoelectric transducers are used frequently for measuring the pressure-time histories of air blast waves. Since piezoelectric transducers are high impedance devices, a suitable impedance matching electronic circuit usually is required between the transducer and the relatively low input impedance recording system. The cathode follower is well suited to this task as it is fundamentally a high input - low output impedance device. This report analyzes four different types of single tube cathode followers, namely the simple cathode follower, the tapped cathode follower, the bypassed cathode follower and the bootstrapped cathode follower. Exact equations as well as simplified formulae for gain, input impedance and output impedance for these four types are derived in Appendix A and may serve as a reference source for cathode follower design. Gain, input impedance and output impedance are evaluated numerically for each.

CATHODE FOLLOWERS - GENERAL THEORY

The gain of a simple cathode follower (Fig. 1) is given as

\[ A = \frac{g_m R_k}{1 + (g_m + g_p) R_k} \]

where \( g_m \) is the mutual conductance of the tube, \( g_p \) is the plate conductance of the tube and \( R_k \) is the cathode resistance. As the cathode resistance is increased the gain of the cathode follower will increase. However, in this simple cathode follower circuit, larger values of cathode resistance change the tube operating point* resulting in a lower current in the tube and a corresponding reduction of \( g_m \) and \( g_p \), which also affect gain. Also, the input impedance of this cathode follower is quite low and equal to the grid resistance, \( R_g \).

* Operating point is the static plate current and plate voltage of the tube without any external signal applied.
By using a tapped cathode follower (Fig. 2), these disadvantages are avoided.

In the tapped cathode follower (Fig. 2) $R_{k_1}$ is selected to yield the proper operating point for the selected tube current and hence $g_m$ and $g_p$ remain the same. $R_{k_2}$ is returned to the negative side of the power supply, resulting in a large value of $R_k$ and hence larger gain. Larger values of $R_{k_2}$ are thus possible but they again are limited, this time by the magnitude of the power supply it is feasible to use. For example, a 1 megohm cathode resistance ($R_{k_2}$) with 1 milliampere current would require $B-$ to be 1000 volts. Returning the grid resistor $R_g$ to the tap point, $a$, also has the effect of increasing the input resistance of the cathode follower.
If we examine the simple circuit shown in Figure 3, we see that $E_b$ provides a "bucking" voltage to the input voltage so that the net current

$$I_{in} = \frac{E_{in} - E_b}{R_g}.$$  Hence $R_{in}$, the resistance the input voltage "sees", is

$$R_{in} = \frac{E_{in}}{I_{in}} = \frac{E_{in}}{E_{in} - E_b} = \frac{E_{in}}{E_{in} - E_b}.$$  and the input resistance is, therefore,

increased. The larger $E_b$ becomes, the greater becomes the input impedance.

This is exactly what occurs in the tapped cathode follower (Fig. 2). The voltage drop $I_p R_k$ is in the correct polarity to buck the input voltage. As

derived in the Appendix, $R_{in} = \frac{R_{g}}{1 - \frac{k_1 R_{k_2}}{k_1 + k_2}}$  where $A$ is the gain of the circuit at the cathode. This is known as partial bootstrapping. If $R_g$ could be returned to a higher gain point eliminating the divider effect of $R_{k_1}$ and $R_{k_2}$, a still higher input resistance could be obtained. The highest gain point in the cathode follower is directly at the cathode. Returning $R_g$ to the cathode can be accomplished in two ways, one is by bypassing the cathode resistor with a large capacitor (Fig. 4), the second by using two grid resistors $R_{g_1}$ and $R_{g_2}$ and connecting the tap directly to the cathode (Fig. 5) through a capacitor.
FIG. 4 BYPASSED CATHODE FOLLOWER

FIG. 5 BOOTSTRAPPED CATHODE FOLLOWER
The bypassed cathode is not a very satisfactory method since an extremely large bypass capacitor is required. The second method called the bootstrap method is best and effectively returns \( R_k \) to the cathode, signalwise, and shunts \( R_{k1} \) with a large resistance \( R_{k2} \), so the resistance remains essentially \( R_k \). The circuit of Fig. 5 is fully bootstrapped and for the values used in the cathode follower designed, gives approximately twice the input impedance of the tapped cathode follower. If now the gain could be made to approach unity, the input resistance would approach infinity, \( R_{in} = \frac{R}{1 - A} \). However, the input resistance in the designed cathode follower would be high enough (1600 megohms) for piezoelectric gages. Higher input resistance (and higher gain) could be obtained by using a pentode tube as a cathode load instead of \( R_{k2} \). Still higher gain and input resistance could be obtained by using a "unity gain" amplifier which uses more than one tube for each cathode follower. This of course would add to the complexity of the circuitry especially where multiple channel cathode followers are required.

ADVANTAGES OF RETURNING \( R_k \) TO NEGATIVE SUPPLY

There are numerous advantages in returning the cathode resistance to a negative supply rather than to ground. These advantages are listed below:

1. A higher value of cathode resistance results in a higher gain which approaches but can never exceed unity.

2. The circuit becomes more stable as it becomes a constant current device.

3. There is a higher multiplication of input impedance.

4. There is a larger voltage swing capability with smaller currents so that larger input signals can be followed.

5. Larger negative signals can be followed without cutting the cathode follower off as in the case of grounded cathode resistors.

6. The output approaches zero potential. Thus, there is a minimum voltage strain across the output coupling capacitor and therefore minimum leakage.
DRIVING CAPABILITIES

The current drawn by the cathode follower is very important in the following respects:

1. It controls the maximum signal voltage that can be followed for a given cathode resistance.

2. It determines the values of mutual conductance and plate conductance and hence gain.

3. It determines the maximum capacitance load the tube can handle and still deliver its output signal.

The method of determining the current requirements of the tube with capacitance load, C, is as follows:

The charge on the capacitor is $Q = CE$ and since $C$ is constant

$$i = \frac{dQ}{dt} = C \frac{dE}{dt}.$$  

If the signal voltage is a sine wave of magnitude $A \sin \omega t$

$$i = CA\omega \cos \omega t \quad \text{..................(1)}$$

The maximum value occurs where $\cos \omega t = 1$ or $i_{\text{max}} = CA\omega \quad \text...............(2)$

For a 10 volt signal with $C = 100 \ \mu\text{F}$ and $f = 100 \ \text{KC}$

$$i_{\text{max}} = 100 \times 10^{-12} \times 10 \times 2\pi \times 10^5 = 0.63 \text{ Ma}.$$  

If $i_{\text{max}}$ is comparable to the current drawn statically by the cathode follower, then a distorted output can be produced.

From equation (2) one can see that capacity, maximum signal amplitude and frequency directly affect the maximum current requirements and hence signal output. For a given cathode follower, one must consider the required frequency response, maximum signal output, and the capacity which is to be driven.
FORMULAE AND DERIVATIONS

Formulae are derived in the Appendix A for all four cases of cathode follower. These formulae for gain, input impedance and output impedance are derived as exact forms as well as simplified close approximations. They are listed below in tabular form.

1. **Simple Cathode Follower - Exact Formulae**

   \[ A = \frac{g_m R_k}{1 + (g_m + g_p) R_k} \]

   \[ R_{in} = R_g \]

   \[ R_{out} = \frac{R_k}{1 + (g_m + g_p) R_k} \]

2. **Tapped Cathode Follower**

   **Exact**

   \[ A = \frac{g_m (R_k + R_k t + R_k 1 + R_k 2) + R_k}{R_k 2 + R_k + (g_m + g_p) (R_k + R_k t + R_k 1 + R_k 2)} \]

   \[ R_{in} = \frac{R_g \left[ 1 + (g_m + g_p) R_k t + R_k 1 + R_k 2 \right] + R_k}{1 + (g_m + g_p) R_k 1 + g_p R_k 2} \]

   **Simplified**

   \[ A = \frac{g_m R_k t}{1 + (g_m + g_p) R_k t} \]

   \[ R_{in} = \frac{R_g \left[ 1 + (g_m + g_p) R_k t \right]}{1 + (g_m + g_p) R_k 1 + g_p R_k 2} \]

   **Alternative Method**

   \[ R_{out} = \frac{R_g R_k 1 + R_k 2}{R_k 2 + R_k + (g_m + g_p) (R_k + R_k t + R_k 1 + R_k 2)} \]

   \[ R_{out} = \frac{R_k t}{1 + (g_m + g_p) R_k t} \]
3. Bypassed Cathode Follower

Results are the same as in the Bootstrapped case except the value of $R_{k2}$ is substituted for $R_{kt}$ in the Bootstrap Formulae.

4. Bootstrapped Cathode Follower

**Exact**

\[ A = \frac{(g_m R_g + 1) R_{kt}}{R_{kt} + R_g \left[ 1 + (g_m + g_p) R_{kt} \right]} \]

\[ R_{in} = \frac{R_g \left[ 1 + (g_m + g_p) R_{kt} \right] + R_{kt}}{1 + g_p R_{kt}} \]

**Simplified**

\[ A = \frac{g_m R_{kt}}{1 + (g_m + g_p) R_{kt}} \]

\[ R_{in} = \frac{R_g \left[ 1 + (g_m + g_p) R_{kt} \right]}{1 + g_p R_{kt}} \]

**Alternative Method**

\[ R_{in} = \frac{R_g}{1 - A} \]

\[ R_{out} = \frac{R_{kt}}{1 + \frac{R_{kt}}{R_g} + (g_m + g_p) R_{kt}} \]

Rout.$ = \frac{R_{kt}}{1 + (g_m + g_p) R_{kt}}$

The simplified formulae give answers within 0.1% (shown in Appendix B) of those obtained with the exact formulae and usually are used.

TUBE SELECTION AND DESIGN CHARACTERISTICS

Since the gain is essentially given by $A = \frac{g_m R_{kt}}{1 + (g_m + g_p) R_{kt}}$, it is desirable to select a tube with a high $g_m$ for a reasonable cathode current. It is also a good procedure to select a bias voltage that is not too close to zero to avoid grid current effects. The tube selected for this case was a 12AX7 having a bias of -2 volts at a current of 1 milliampere. The tube
FIG. 6.

12AX7 PLATE CHARACTERISTICS

FIG. 6.
characteristics are shown in Figure 6*. A grid resistance of 22 megohms was used as a compromise which gave very little bias shift due to grid current. Much higher grid resistances could result in unstable operation due to shift of operating point.

A high quality, low leakage ceramic tube socket was used to avoid changes in grid resistance due to leakage resistance.

The output coupling capacitor was a 25 microfarad high quality Mylar unit needed to give a good low frequency response with a minimum of leakage and drift. Minimum leakage is very desirable as the cathode followers, in this case, are coupled to high gain DC amplifiers. Any leakage through the capacitor would cause the output signal of the DC amplifier to drift.

Design procedure for all cases and a composite table are shown in the next section.

DESIGN PROCEDURE FOR CATHODE FOLLOWER WITH TAPPED CATHODE

---

Curve taken from "Conductance Curve Design Manual" by Keats A. Pullen, Jr.
In the design procedure a selection was made for a B+ of 250 volts and 
\( e_c \) of -2 volts. It can then be shown from the tube characteristic curves of
Fig. 6 that for \( e_p \) equal to 250 volts

\[
\begin{align*}
  i_p &= 1 \text{ ma} \\
  g_m &= 1700 \mu \text{ mhos} \\
  g_p &= 17 \mu \text{ mhos}
\end{align*}
\]

Evaluating \( R_{k1} \) which equals \( \frac{e_c}{i_p} \) gives a value of 2000 ohms.

Now values for \( R_{k2} \) of 150 K ohms and \( R_g \) of 22 megohms are selected. A
negative supply to give 1 ma through \( R_{k1} + R_{k2} \) which totals 152 K ohms is
required thus yielding a negative supply of -152 volts.

Evaluating gain, \( R_{in} \) and \( R_{out} \) from the simplified equations for the tapped
cathode follower give:

\[
\begin{align*}
  \text{Gain} &= 0.986 \\
  R_{in} &= 825 \text{ megohms} \\
  R_{out} &= 580 \text{ ohms}
\end{align*}
\]

The resultant input time constant is \( 0.01 \times 10^{-6} \times 825 \times 10^6 = 8.2 \) seconds.

Similar results for the other three cases are shown in Table I.

**DESCRIPTION OF 8 CHANNEL CATHODE FOLLOWER**

Two designs were used in constructing an eight channel cathode follower.
The simple cathode follower was ruled out as having too low an input
resistance, i.e., 22 megohms, and the bypassed cathode follower was eliminated
since it required an unreasonably high bypass capacitor. Thus, only the tapped
and bootstrapped cathode followers were considered.

In the unit constructed, the cathode followers and power supply are
contained on a single rack mounting chassis. Each channel is separately
shielded and only one half of each 12AX7 tube is used to prevent cross talk
or interference. The system is powered from a 115 Volt 60 cycle source so
<table>
<thead>
<tr>
<th></th>
<th>Simple Cathode Follower</th>
<th>Tapped Cathode Follower</th>
<th>Bypassed Cathode Follower</th>
<th>Bootstrapped Cathode Follower</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>0.767</td>
<td>0.986</td>
<td>0.986</td>
<td>0.986</td>
</tr>
<tr>
<td>$R_{in}$</td>
<td>22 Megas.</td>
<td>825 Megas.</td>
<td>1600 Megas.</td>
<td>1600 Megas.</td>
</tr>
<tr>
<td>$R_{out}$</td>
<td>451 ohms</td>
<td>580 ohms</td>
<td>582 ohms</td>
<td>421 ohms</td>
</tr>
<tr>
<td>$R_g$</td>
<td>22 Megas.</td>
<td>22 Megas.</td>
<td>22 Megas.</td>
<td>22 Megas.</td>
</tr>
<tr>
<td>$R_{k1}$</td>
<td>2 K</td>
<td>2 K</td>
<td>2 K</td>
<td>2 K</td>
</tr>
<tr>
<td>$R_{k2}$</td>
<td>0</td>
<td>150 K</td>
<td>150 K</td>
<td>150 K</td>
</tr>
<tr>
<td>Input Time Constant</td>
<td>0.22 seconds</td>
<td>8.2 seconds</td>
<td>16 seconds</td>
<td>16 seconds</td>
</tr>
</tbody>
</table>
that a minimum of maintenance is required for field programs as against a battery operated system which requires constant attention. A circuit diagram of a power supply and a tapped cathode follower design is shown in Figure 8.

The power supply consists of an electronically regulated system to provide B+ voltages for the plates of the cathode follower tubes as well as DC power for the tube filaments. The tubes, which are rated at 12.6 Volts, are operated at 12 Volts to keep the grid current down and input impedance up. The regulated voltage on plates and filaments insures a minimum drift of tube characteristics.

The tapped cathode resistor is returned to a -150 Volt supply, regulated by a VR 150 tube. The proper values of \( R_k \) are selected to give proper grid bias and hence operating point of the cathode follower.

The bootstrapped cathode follower, (Fig. 5) using the identical power supply, was also used successfully.

The bootstrapped cathode follower gave about twice the input impedance as the tapped cathode follower. The gain was the same in both cases, but output impedance was somewhat lower for the bootstrapped version.

Good low frequency response is necessary for long duration measurements of pressure-time histories as characterized by large explosive charges.

**CONCLUSION**

The design equations for Gain, \( R_{in} \) and \( R_{out} \) gave results which agreed with experimental test within 10%. The single tube tapped cathode follower and bootstrapped cathode follower give input resistances of approximately 800 megohms and 1600 megohms respectively with a gain of 0.986. This is quite satisfactory for most requirements of high input impedance. The bootstrapped cathode follower is best as it gives the highest input impedance with no added circuitry complications except for the bootstrap capacitor.
EIGHT CATHODE FOLLOWER TUBES

FIG. 8 - EIGHT CHANNEL CATHODE FOLLOWER
The eight channel cathode follower so designed has been satisfactorily used in the recording of blast pressures on piezoelectric gages with a multiple channel oscillograph recorder. It is characterized by simplicity of operation and reliable performance as well as a minimum of maintenance.

BERNARD SOROKA

BERNARD SOROKA

JACOB WENIG
A. Derivations of Gain, $R_{in}$ and $R_{out}$ for various cathode followers
   1. Simple Cathode Follower
   2. Tapped Cathode Follower
   3. Bypassed Cathode Follower
   4. Bootstrapped Cathode Follower

B. Evaluation of Gain, $R_{in}$ and $R_{out}$ for the four cases of Cathode followers
APPENDIX A

1. Simple Cathode Follower
   a. Gain

   \[ \mu e_g - i_p (r_p + R_k) = 0 \] .................................................(1)

   \[ e_g = e_{in} - i_p R_k \] ........................................ (2)

   \[ i_p = \frac{\mu e_{in}}{r_p + R_k (\mu + 1)} \]

   \[ e_{out} = i_p R_k = \frac{1}{r_p + R_k (\mu + 1)} \]

   \[ \frac{e_{out}}{e_{in}} = A = \frac{\mu R_k}{r_p + R_k (\mu + 1)} \]

   \[ \mu = \frac{g_m}{g_p} \quad r_p = \frac{1}{g_p} \]

   \[ A = \frac{g_m R_k}{1 + (g_m + g_p) R_k} \]

   b. \( R_{\text{input}} = R_g \)
c. Output Impedance

\[
\begin{align*}
\mu e_g - i_p (r_p + R_k) - IR_k &= 0 & (3) \\
e_g &= -(I + i_p) R_k & (4) \\
E - (I + i_p) R_k &= 0 & (5) \\
o &= i_p \left[ \frac{r_p + R_k (\mu + 1)}{\mu + 1} \right] + IR_k (\mu + 1) & (6) \\
E &= i_p R_k + IR_k & (7)
\end{align*}
\]

Solving equations (6) and (7):

\[
\frac{E}{I} = R_{\text{out}} = \frac{R_k}{r_p + (\mu + 1) R_k} = \frac{R_k}{1 + (g_m + g_p) R_k}
\]
2. **Tapped Cathode Follower**

CATHODE FOLLOWER CIRCUIT  

FIG. II - GAIN AND INPUT IMPEDANCE

a. **Gain**

The mesh equations for the equivalent circuit are:

\[ e_g = e_{in} - i_p (R_{k1} + R_{k2}) - i_g R_{k2} \]  

\[ \mu e_g = i_p (r_p + R_{k1} + R_{k2}) - i_g R_{k2} = 0 \]

\[ e_{in} - i_g (R_g + R_{k2}) - i_p R_{k2} = 0 \]

\[ e_{out} = i_p (R_{k1} + R_{k2}) + i_g R_{k2} \]
\[ \mu \text{e}_{\text{in}} = i_p \left[ r_p + (R_{k1} + R_{k2}) (\mu + 1) \right] + i_g R_{k2} (\mu + 1) \ldots (12) \]

\[ e_{\text{in}} = i_p R_{k2} + i_g (R_g + R_{k2}) \ldots (13) \]

Solving equations 12 and 13:

\[ e_{\text{out}} = i_p (R_{k1} + R_{k2}) + i_g R_{k2} \]

\[ A = \frac{\mu R_g (R_{k1} + R_{k2}) + r_p R_{k2} + R_{k1} R_{k2}}{R_g \left[ r_p + (R_{k1} + R_{k2}) (\mu + 1) \right] + R_{k2} r_p + R_{k1} R_{k2} (\mu + 1)} \]

\[ \mu = \frac{\varepsilon_m}{\varepsilon_p} \quad r_p = 1/\varepsilon_p \]

\[ A = \frac{\varepsilon_m (R_g R_{k_t} + R_{k1} R_{k2}) + R_{k2}}{R_g + R_{k2} + (\varepsilon_m + \varepsilon_p) (R_g R_{k_t} + R_{k1} R_{k2})} \]

\[ R_{k_t} = \text{total cathode resistance} = R_{k1} + R_{k2} \]

Simple Form:

if \( R_g > R_{k2} \)

\[ A = \frac{\varepsilon_m (R_{k1} + R_{k2})}{1 + (\varepsilon_m + \varepsilon_p) (R_{k1} + R_{k2})} = \frac{\varepsilon_m R_{k_t}}{1 + (\varepsilon_m + \varepsilon_p) R_{k_t}} \]

b. Input Impedance

Solving equations 12 and 13:

\[ i_g = \frac{e_{\text{in}} [r_p + (R_{k1} + R_{k2}) (\mu + 1)] - \mu e_{\text{in}} R_{k2}}{(R_g + R_{k2}) \left[ r_p + (R_{k1} + R_{k2}) (\mu + 1) \right] - R_{k2} (\mu + 1)} \]

\[ R_{\text{in}} = \frac{e_{\text{in}}}{i_g} = \frac{R_g \left[ r_p + (R_{k1} + R_{k2}) (\mu + 1) \right] + R_{k2} r_p + R_{k1} R_{k2} (\mu + 1)}{r_p + R_{k1} (\mu + 1) + R_{k2}} \]

\[ \mu = \frac{\varepsilon_m}{\varepsilon_p} \quad r_p = 1/\varepsilon_p \]
\[
R_{in} = \frac{R_g \left[ 1 + (g_m + g_p)(R_{k_1} + R_{k_2}) R_{k_2}/R_g \right] + R_{k_2}}{1 + (g_m + g_p) R_{k_1} + g_p R_{k_2}}
\]

**Simple Form:**

\[
R_g > R_{k_2} \\
R_{in} = \frac{R_g \left[ 1 + (g_m + g_p) (R_{k_1}) \right]}{1 + (g_m + g_p) R_{k_1} + g_p R_{k_2}}
\]

c. **Simplified Version of Input Impedance**

The input impedance can also be found in a simplified version.

\[
R_{in} = \frac{E}{I} = \frac{e_{in}}{e_{in} - e_2} \quad \ldots \quad (14)
\]

\[
R_{in} = \frac{e_{in}}{e_{in} - e_2} \frac{R_g}{R_k} \quad \ldots \quad (15)
\]

\[
e_2 = \frac{R_{k_2}}{R_{k_1} + R_{k_2}} \quad \ldots \quad (16)
\]

\[
e_2 = \frac{e_2}{R_{k_1} + R_{k_2}} \quad \ldots \quad (17)
\]

\[
\text{Gain} = A = \frac{e_k}{e_{in}} \quad \ldots \quad (18)
\]

Combining (15) and (16) \( A = \frac{e_2}{e_{in}} \frac{R_{k_1} + R_{k_2}}{R_{k_2}} \quad \ldots \quad (19) \)

\[
\frac{e_2}{e_{in}} = A \left( \frac{R_{k_2}}{R_{k_1} + R_{k_2}} \right) \quad \ldots \quad (20)
\]

\[
R_{in} = R_g \left( \frac{e_{in}}{e_{in} - e_2} = \frac{R_g}{l - \frac{e_2}{e_{in}}} = \frac{R_g}{1 - A \left[ \frac{R_{k_2}}{R_{k_1} + R_{k_2}} \right]} \right)
\]
d. Output Impedance

\[ \mu e_g = (i_p + I) R_k_1 - (i_p + I) \left( \frac{R_k_2 R_g}{R_k_2 + R_g} \right) \]

\[ -e_g = E = (i_p + I) R_k_1 + (i_p + I) \frac{R_k_2 R_g}{R_k_2 + R_g} \]

\[ \mu E = i_p \left( r_p + \frac{R_k_2 R_g}{R_k_2 + R_g} \right) + \left( \frac{R_k_1 + R_k_2 R_g}{R_k_2 + R_g} \right) \]

\[ E = i_p \left( \frac{R_k_1 + R_k_2 R_g}{R_k_2 + R_g} \right) + I \left( \frac{R_k_1 + R_k_2 R_g}{R_k_2 + R_g} \right) \]
Solving equations 21 and 22:

\[
R_{\text{out}} = \frac{E}{I} = \frac{r_p \left[ \frac{R_{k_2}}{R_{k_1} + R_{k_2} + R_G} \right]}{r_p + (\mu + 1) \left[ \frac{R_{k_1}}{R_{k_1} + \frac{R_{k_2}}{R_{k_2} + R_G}} \right]}
\]

\[
R_{\text{out}} = \frac{g_m}{R_{k_2} + R_g + g_m \left( g_m + g_p \right) \left( R_{k_1} + R_{k_2} \right)}
\]

**Simplified Form:**

if \( R_g \gg R_{k_2} \)

\[
R_{\text{out}} = \frac{R_{k_1} + R_{k_2}}{1 + (g_m + g_p) \left( R_{k_1} + R_{k_2} \right)} = \frac{R_{k_t}}{1 + \left( g_m + g_p \right) R_{k_t}}
\]

3. **Bypassed Cathode Follower**

![Bypassed Cathode Follower](image)

FIG. 14. BYPASSED CATHODE FOLLOWER

**Equivalent Circuit**

With C large enough so its impedance is negligible at signal frequencies, \( R_{k_1} \) is effectively out of the circuit. Results are the same as in the Bootstrapped cathode follower except \( R_{k_2} \) is used in lieu of \( R_{k_t} \) in the Bootstrap formulae.
4. Bootstrapped Cathode Follower

a. Gain

FIG. 15. BOOTSTRAPPED CATHODE FOLLOWER - GAIN

The mesh equations from the equivalent circuit are:

\[ e_g = e_{in} - i_p (R_{k1} + R_{k2}) - i_g (R_{k1} + R_{k2}) \] .............. (23)

\[ \mu e_g = i_p (r_p + R_{k1} + R_{k2}) - i_g (R_{k1} + R_{k2}) = 0 \] .............. (24)

\[ e_{in} = i_g (R_g + R_{k1} + R_{k2}) - i_p (R_{k1} + R_{k2}) = 0 \] .............. (25)

\[ e_{out} = i_p (R_{k1} + R_{k2}) + i_g (R_{k1} + R_{k2}) \] .............. (26)

\[ \mu e_{in} = i_p \left[ r_p + R_{k1} (\mu + 1) + R_{k2} (\mu + 1) \right] + i_g \left[ R_{k1} (\mu + 1) + R_{k2} (\mu + 1) \right] \] (27)

\[ e_{in} = i_p (R_{k1} + R_{k2}) + i_g (R_g + R_{k1} + R_{k2}) \] .............. (28)
Solving equations 27 and 28:

\[ e_{out} = (i_p + i_g) (R_{k_1} + R_{k_2}) \]

\[ A = \frac{e_{out}}{e_{in}} \text{ or } \]

\[ A = \frac{(R_{k_1} + R_{k_2}) (\mu R_g + r_p)}{R_g \left[ r_p (R_{k_1} + R_{k_2} + \mu + 1) \right]} + \frac{(R_{k_1} + R_{k_2}) r_p}{R_g \left[ (\mu + 1) (R_{k_1} + R_{k_2}) + (R_{k_1} + R_{k_2}) r_p \right]} \]

\[ \mu = \frac{g_m}{g_p} \quad r_p = \frac{1}{g_p} \]

\[ A = \frac{(g_m R_g + 1) R_k}{R_g \left[ 1 + (g_m + g_p) R_k + R_k \right]} + R_k \]

**Simplified Form:**

if \( g_m R_g \gg 1 \quad R_g \gg R_{k_1} + R_{k_2} \)

\[ A = \frac{g_m R_k}{1 + (g_m + g_p) R_k} \]

b. Input Impedance

\[ R_{in} = \frac{e_{in}}{i_g} = \frac{r_p (R_g + R_{k_1} + R_{k_2}) + R_g (\mu + 1) (R_{k_1} + R_{k_2})}{r_p + R_{k_1} + R_{k_2}} \]

\[ \mu = \frac{g_m}{g_p} \]

\[ R_{in} = \frac{R_g \left[ 1 + (g_m + g_p) R_k \right]}{1 + g_p R_k} \]

**Simplified Form:**

if \( R_g \gg R_{k_1} + R_{k_2} \)

\[ R_{in} = \frac{R_g \left[ 1 + (g_m + g_p) R_k \right]}{1 + g_p R_k} \]
c. **Output Impedance**

![Output Impedance Diagram](image)

Let \( R = R_g \) in parallel with \( R_{k_1} + R_{k_2} = \frac{R_g (R_{k_1} + R_{k_2})}{R_g + R_{k_1} + R_{k_2}} \)

Mesh equations:

\[
\begin{align*}
\mu e_g - i_p (r_p + R) - IR &= 0 \quad \text{(29)} \\
- e_g &= E = (i_p + I) R \quad \text{(30)} \\
- \mu E &= i_p (r_p + R) + IR \quad \text{(31)} \\
E &= i_p R + IR \quad \text{(32)} \\
I &= \frac{E [r_p + (\mu + 1) R]}{r_p R}
\end{align*}
\]
\[ R_{out} = \frac{r_p R_g (R_{k1} + R_{k2})}{r_p (R_g + R_{k1} + R_{k2}) + (\mu + 1) R_g (R_{k1} + R_{k2})} \]

\[ \mu = \frac{g_m}{g_p} \quad r_p = \frac{1}{g_p} \]

\[ R_{out} = \frac{R_{kt}}{1 + R_{kt}/R_g + (g_m + g_p) R_{kt}} \]

**Simplified Form:**

If \( R_g \gg R_{k1} + R_{k2} \)

\[ R_{out} = \frac{R_{kt}}{1 + (g_m + g_p) R_{kt}} \]
APPENDIX B

Evaluation of Gain, Input Resistance and Output Resistance

Values for gain, input resistance and output resistance of the simple cathode follower, tapped cathode follower and bootstrapped cathode follower are evaluated for exact and simplified formulae. Great care is needed in evaluating $R_{\text{input}}$ using the formula

$$R_{\text{in}} = \frac{R_g}{1-A} \quad \text{and} \quad \frac{R_g}{1-A} \left[ \frac{R_{k_2}}{R_{k_1} + R_{k_2}} \right].$$

Actually this is a very exact formula but if $A$ is known to only 1 or 2 places, a large error is introduced in subtracting $A$ from unity.

All values used in the equations are

$$R_{k_1} = 2 \times 10^3 \text{ ohms} \quad R_{k_2} = 150 \times 10^3 \text{ ohms} \quad R_{k_t} = 152 \times 10^3 \text{ ohms}$$

$$g_m = 1700 \times 10^{-6} \text{ mhos} \quad g_p = 17 \times 10^{-6} \text{ ohms}$$

$$R_g = 22 \times 10^6 \text{ ohms}$$

**Simple Cathode Follower**

$$A = \frac{g_m R_k}{1 + (g_m + g_p) R_k} = 0.767$$

$$R_{\text{in}} = R_g = 22 \text{ megohms}$$

$$R_{\text{out}} = \frac{R_k}{1 + (g_m + g_p) R_k} = 451 \text{ ohms}$$
Tapped Cathode Follower

**Exact**

\[ A = \frac{g_m (R_g R_{kt} + R_k R_{k2}) + R_k}{R_k + R_g + (g_m + g_p) (R_g R_{kt} + R_k R_{k1} + R_{k2})} \]

A = 0.986

\[ R_{in} = \frac{R_g (1 + g_m + g_p)}{1 + (g_m + g_p) R_{k1} + g_p R_{k2}} \]

\[ R_{in} = 825.2 \text{ megohms} \]

\[ R_{out} = \frac{R_g R_{kt} + R_k R_{k2}}{R_k + R_g + (g_m + g_p) (R_g R_{kt} + R_k R_{k1} + R_{k2})} \]

\[ R_{out} = 580.2 \text{ ohms} \]

**Simplified Form**

\[ A = \frac{g_m R_{kt}}{1 + (g_m + g_p) R_{kt}} \]

A = 0.986

\[ R_{in} = \frac{R_g}{1 - A} \frac{R_{k2}}{R_{k1} + R_{k2}} \]

\[ R_{in} = 825.2 \text{ megohms} \]

\[ R_{out} = \frac{R_{kt}}{1 + (g_m + g_p) R_{kt}} \]

\[ R_{out} = 580.2 \text{ ohms} \]

Bootstrapped Cathode Follower

\[ A = \frac{(g_m R_g + 1) R_{kt}}{R_{kt} + R_g + (g_m + g_p) R_g R_{kt}} \]

A = 0.986

\[ A = \frac{g_m R_{kt}}{1 + (g_m + g_p) R_{kt}} \]

A = 0.986
\[
R_{in} = \frac{R_g \left[ 1 + (g_m + g_p) R_{kt} \right]}{1 + g_p R_{kt}} + R_{kt}
\]

\[
R_{in} = 1608 \text{ megohms.}
\]

Short method of evaluating \( R_{in} \)

\[
R_{in} = \frac{R_g}{1 - A} = 1606 \text{ megohms}
\]

\[
R_{out} = \frac{R_{kt}}{1 + \frac{R_{kt}}{R_g} + (g_m + g_p) R_{kt}}
\]

\[
R_{out} = 580.2 \text{ ohms}
\]
<table>
<thead>
<tr>
<th>No. of</th>
<th>Organization</th>
<th>No. of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copies</td>
<td>Commander</td>
<td>Copies</td>
</tr>
<tr>
<td>10</td>
<td>Armed Services Technical Information Agency</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Commander</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Arlington Hall Station</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>U. S. Army Materiel Command</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Commanding General</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ATTN: AMCRD-RS-PE-Bal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Research and Development Directorate</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Frankford Arsenal</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Commanding General</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ATTN: Library Branch, 0270, Building 40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Philadelphia 37, Pennsylvania</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Picatinny Arsenal</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Commanding Officer</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ATTN: Feltman Research and Engineering Laboratories</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dover, New Jersey</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>Harry Diamond Laboratories</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Commanding Officer</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ATTN: Technical Information Office, Branch 012</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Washington 25, D. C.</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Army Research Office</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3025 Columbia Pike</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Arlington, Virginia</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Chief, Bureau of Naval Weapons</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ATTN: DIS-33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Department of the Navy</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Washington 25, D. C.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Naval Ordnance Laboratory</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>White Oak</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Silver Spring 19, Maryland</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Copies</td>
<td>Organization</td>
<td>No. of Copies</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------</td>
<td>--------------</td>
</tr>
<tr>
<td>4</td>
<td>U. S. Atomic Energy Commission</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Los Alamos Scientific Laboratory</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P. O. Box 1663</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Los Alamos, New Mexico</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>University of California</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Lawrence Radiation Laboratory</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Technical Information Division</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ATTN: Clovis G. Craig</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P. O. Box 808</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Livermore, California</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Aerojet-General Corporation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>6352 North Irwindale Road</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Azusa, California</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Armour Research Foundation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Illinois Institute of Technology Center</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chicago 16, Illinois</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Atlantic Research Corporation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Shirley Highway at Edsall Road</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Alexandria, Virginia</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Cornell Aeronautical Laboratory, Inc.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ATTN: Librarian</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Buffalo, New York</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Midwest Research Institute</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>ATTN: B. L. Rhodes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>425 Volker Boulevard</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kansas City 10, Missouri</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>The Rand Corporation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ATTN: H. L. Brode</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F. R. Gilmore</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1700 Main Street</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Santa Monica, California</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

44
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Organization</th>
</tr>
</thead>
</table>
| 10           | The Scientific Information Officer  
Defence Research Staff  
British Embassy  
3100 Massachusetts Avenue, N. W.  
Washington 8, D. C. |
| 4            | Defence Research Member  
Canadian Joint Staff  
2450 Massachusetts Avenue, N. W.  
Washington 8, D. C. |
The design of four types of single tube cathode followers is described and selections made of the ones best suited for use with an eight-channel unit employed for air blast research.

Exact formulae as well as simplified good approximations for gain, input impedance and output impedance are derived in the Appendices. This report may serve as a reference source for cathode follower design and circuit analysis.