ESD-TDR-63-178 W-5785

RELATIVISTIC AND CLASSICAL DOPPLER ELECTRONIC TRACKING ACCURACIES

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-178

June 1963

J. Hoffman

Prepared for

DIRECTORATE OF INSTRUMENTATION AND ADVANCED DEVELOPMENT

ELECTRONIC SYSTEMS DIVISION

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

L. G. Hanscom Field, Bedford, Massachusetts

Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts
Contract AF33(600)-39852 Project 705

ESD-TDR-63-178 W-5785

RELATIVISTIC AND CLASSICAL DOPPLER ELECTRONIC TRACKING ACCURACIES

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-178

June 1963

J. Hoffman

Prepared for

DIRECTORATE OF INSTRUMENTATION AND ADVANCED DEVELOPMENT

ELECTRONIC SYSTEMS DIVISION

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

L. G. Hanscom Field, Bedford, Massachusetts

Prepared by

THE MITRE CORPORATION

Bedford, Massachusetts

Contract AF33(600)-39852 Project 705

FOREWORD

This paper was presented at the AIAA Space Flight Testing Conference held March 18-20, 1963 at Cocoa Beach, Florida.

ABSTRACT

The first order Doppler effect has in general been used in electronic tracking. With the emphasis upon accuracies of tenths of a foot per second and less, this approach is no longer permissible. This study has taken a fundamental look at the Doppler effect. Five steps have been developed which enable the exact derivation of the Doppler equations for any system. Six different configurations of transmitter, receiver, and vehicle have been investigated and the results applied to a number of present and future operational Doppler systems. It has been determined that for velocity accuracies of a foot per second, or better, the second order relativistic or classical equations must be used. The receipt of a zero Doppler shift has also been investigated and it does not necessarily imply zero line of sight velocity.

TABLE OF CONTENTS

1.0 INTRODUCTION

1.1 General

In the operation of a space or missile test program one of the critical aspects is the receipt of trajectory information. This is true for both real-time control and for post-flight evaluation. The velocity information is usually obtained by some type of Doppler effect. The Doppler effect is the shift in frequency of the transmitted signal caused by the relative motion of the vehicle and the station. The Doppler shift is received and then transformed into vehicle velocity information.

The usual practice has been to use the first order classical Doppler equations to relate the frequency shift to the velocity data. This was satisfactory as long as the accuracy requirements were given in feet per second. However, with the emphasis now upon accuracies of tenths of a foot per second and less, this approach is no longer permissible. What is required is at least the second order classical Doppler equation. In fact, for certain Doppler systems the second order relativistic Doppler equation must be used.

Because of the past sufficiency of the first order equations there did not exist the need for accurate derivations of these equations. This is because the un-needed second order terms usually contained any inaccuracies in the derivations. However, with the present need for more precise accuracies these second order terms become significant. Therefore, a fresh and fundamental look at the Doppler effect in electronic tracking is of considerable value.

1.2 Objectives and Approach

It is therefore the objective of this study to establish the fundamental steps that should be used in a correct derivation of the Doppler equation for any type of Doppler system.

Then, to use these steps to derive the Doppler equations (classical and special relativistic) for a number of representative systems. Next the inaccuracies of using the first order equations are determined. With these equations in hand, several present and future operational systems are evaluated. The specified accuracies are compared with the inaccuracies inherent in the first order equations. The term inaccuracy rather than error is used, since the inaccuracy is correctable if the proper equation is used to relate Doppler frequency shift to vehicle velocity.

In addition, the phenomenon of zero Doppler shift is investigated. It is necessary, since this does not always imply zero line of sight velocity when the second order equations are used.

1.3 Scope

The scope of this study includes the fundamental derivation philosophy of any Doppler equation. It includes the actual derivation of five different Doppler systems containing 6 different configurations of transmitter, receiver and vehicle. It also includes the analysis of the zero Doppler shift effect for these 6 different configurations. Finally, it includes an evaluation of the approximation accuracies of four Doppler systems.

It does not include any equipment or operational accuracies.

For clarity of understanding the derivations it is stated here that capital letters denote vector quantities. This is also mentioned at the onset of the derivations, section 2.1.

2.0 FUNDAMENTAL DOPPLER EQUATIONS AND APPROXIMATIONS

2.1 Derivation Philosophy

The key to the derivation of the Doppler effect for any configuration of transmitter, receiver, and beacon (or reflector) lies in the following sequence of steps.

1) Equate the phase of the radar wave as observed by the transmitter to that observed by the receiver, and so forth. This is so, since the phase is an invarient and is identical in any coordinate system; transmitter, receiver, or beacon.

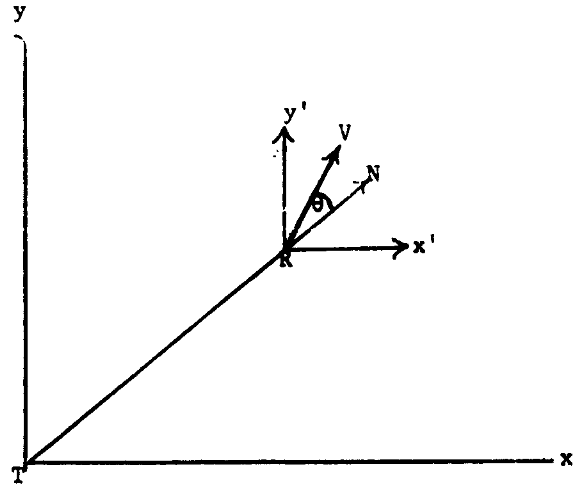
2) Select the coordinate system to which all measurements will be referenced.

3) Establish the relative velocity parameters between sequential portions of the configuration - transmitter and beacon, beacon and receiver, etc. The Doppler effect depends only on relative velocity.

4) Apply the Galilean transformation (non-relativistic) or the Lorentz Transformation (relativistic) to the phase equalities.

5) Solve the resulting equations for the Doppler effect.

Consider the application of the above to the configuration depicted in the figure below.



The transmitter (T) is fixed on earth. The receiver (R) is fixed on a vehicle travelling with a velocity V relative to the earth. (Capital letters denote vectors, lower-case letters denote magnitude). N is the wave normal of a plane wave which is described in the transmitter reference system by one or several wave functions of the form,

$$A \cos 2\pi f \left[t - N.X/w \right]$$

where t is the time parameter, X is the position vector, w is the phase velocity, and f is the frequency.

For step 1, the phase is equated.

$$f \left[t - N.X/w \right] = f' \left[t' - N'.X'/w' \right] \tag{2-1}$$

where the primed parameters are the measurements in the receiver's reference system.

For step 2, the earth based transmitter reference system is chosen. This means that the final equation relating f and f' will contain no primed parameters.

For step 3, the relative velocity for this simple configuration is just V.

For step 4, the Lorentz Transformation, which is given in the appendix as equation (6-4), is applied. Equation (6-4) is the inverse transformation and it is applied to the left hand side of equation (2-1). To have applied the direct transformation, equation (6-3), would result in the final Doppler equation containing primed variables which is contrary to step 2. Thus,

$$f\left\{\gamma\,[t' + V.X'/c^2] - N.X'/w - N.V\,[(\gamma - 1)\,X'.V/v^2 + \gamma t']/w\right\} =$$

$$f'\left\{t' - N'.X'/w'\right\} \tag{2-2}$$

where c is the velocity of light,

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

and

$$\beta = \frac{v}{c} \; .$$

Equation (2-2) must be satisfied for all values of the independent variables t', X'. This is possible only if the coefficients of t', X', respectively, on both sides are equal. Equating the coefficients of t' gives

$$f\,[\gamma - N.V\gamma/w]\,t' = f'\,[1]\,t' \; . \tag{2-3}$$

Therefore, the Doppler frequency f' observed by the receiver on the vehicle is

$$f' = f\,\frac{1 - N.V/w}{\sqrt{1 - \beta^2}} \tag{2-4}$$

where N.V is the scalar product of the two vectors, and N is a unit vector. From the previous figure,

$$N.V = v\cos\theta \; .$$

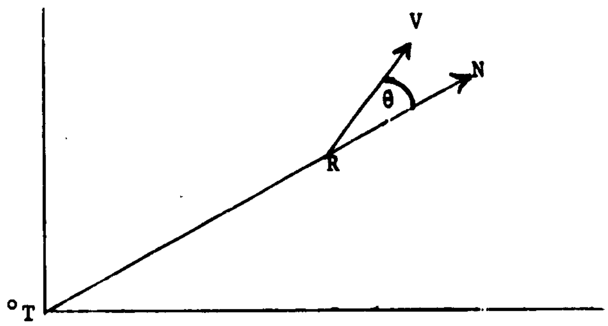
Appendix 6.2 discusses some pitfalls which occur when steps 2 and 3 are not judiciously applied.

Equation (2-4) is the relativistic Doppler effect. Allowing $\beta \to 0$, results in the classical Doppler effect, which would of course result from using the Galilean Transformation.

It is interesting to note that equating the coefficients of the vector components of the X' variable on both sides of equation (2-2) would result in the determination of the remaining primed parameters as functions of the unprimed ones.

2.2 Specific Configurations

2.2.1 Configuration A-Transmitter Stationary, (Vehicle) Receiver in Motion



Earth Fixed Reference

Figure 1.0 Configuration A

Configuration A is identical to that discussed in Section 2.1 where

$$f_T = f,\ f_R = f',\ V = V. \tag{2-5}$$

Therefore, from equation (2-4) the relativistic Doppler frequency is

$$f_R = f_T \frac{1 - N.V/w}{\sqrt{1 - \beta^2}} \tag{2-5a}$$

where

$$N.V = v \cos\theta \ .$$

Allowing $\beta \to 0$ results in the classical Doppler frequency

$$f_R = f_T \, [1 - N.(V + \Delta V)/w] \qquad (2\text{-}5b)$$

where $(V + \Delta V)$ is the inaccurate velocity obtained by using equation (2-5b) instead of (2-5a).

Equation (5) contains earth-fixed reference parameters.

Expanding equation (2-5a), and retaining up to second order terms only, results in

$$f_R \approx f_T \, [1 - N.V/w + \tfrac{1}{2}\beta^2] \qquad (2\text{-}6)$$

Subcontracting equation (2-5b) from (2-6) results in

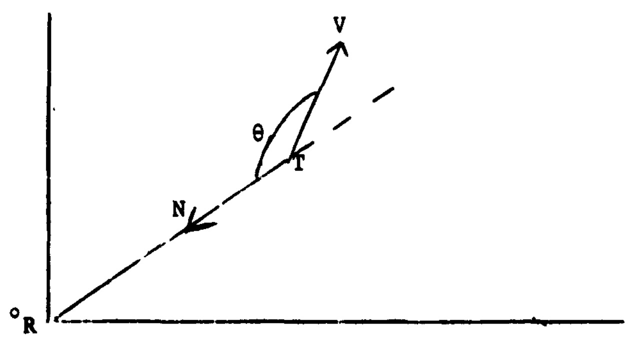
$$N.\Delta V = \Delta v \cos\theta \approx - \tfrac{1}{2} \frac{v^2}{c} \qquad (2\text{-}7)$$

where it has been assumed that

$$\frac{w}{c} \approx 1 \ .$$

Equation (2-7) is the inaccuracy in the velocity determination using the classical Doppler equation.

2.2.2 Configuration B - (Vehicle) Transmitter in Motion, Receiver Stationary



Earth Fixed Reference
Figure 2.0 Configuration B

In configuration B the vehicle's reference system is still considered the primed systemsas in Configuration A. Therefore,

$$f_T = f', \; f_R = f, \; V = V$$

and from equation (2-4)

$$f_R = f_T \frac{\sqrt{1 - \beta^2}}{1 - N.V/w} \tag{2-8a}$$

Again, allowing $\beta \rightarrow 0$ results in

$$f_R = f_T \; \frac{1}{1 - N.(V + \Delta V)/w} \quad . \tag{2-8b}$$

Equation (2-8) also contains earth-fixed reference parameters.

Expanding equation (2-8) results in

$$f_R \approx f_T \left[1 + N.V/w + (N.V/w)^2 - \tfrac{1}{2}\beta^2 \right. \quad . \tag{2-9a}$$

and for $\beta \rightarrow 0$

$$f_R \approx f_T \left\{1 + N.(V + \Delta V)/w + [N.(V + \Delta V)/w]^2\right\} \; . \tag{2-9b}$$

Subtracting equation (2-9b) from (2-9a) results in

$$N.\Delta V = \Delta v \cos\theta \approx -\tfrac{1}{2} \frac{v^2}{c} \;\; \frac{1}{1 + 2N.V/c} \tag{2-10a}$$

where it has been assumed that

$$\frac{w}{c} \approx 1$$

and where $(N.\Delta V/w)^2$ has been neglected.

Comparing equation (2-10a) with (2-7) shows that the inaccuracy of Configuration B is approximately that of A since $2N.V/c \ll 1$ in equation (2-10a).

The usual practice is to neglect all second order terms. It is of interest to note the resulting inaccuracy. Neglecting the second order terms in equation (2-9a), results in an inaccuracy ΔV_1 given by

$$N.\Delta V_1 = \Delta v_1 \cos\theta \approx -\tfrac{1}{2}\frac{V^2}{c} [1 - 2\cos^2\theta] \qquad (2\text{-}10b)$$

where it has been assumed that

$$\frac{w}{c} \approx 1.$$

Since

$$-1 \le [1 - 2\cos^2\theta] \le 1,$$

the procedure of neglecting second order terms reduces the inaccuracy associated with the use of the classical Doppler equation. In fact, when $\theta = \pi/4$, $3\pi/4$ there is effectively no inaccuracy as equation (2-10b) indicates. However, the sign of the inaccuracy may change.

2.2.3 Configuration C - Transmitter in Motion, Receiver in Motion

Two cases are considered -- the tracked vehicle first containing the receiver, and then containing the transmitter.

1) <u>Receiver in Tracked Vehicle</u>

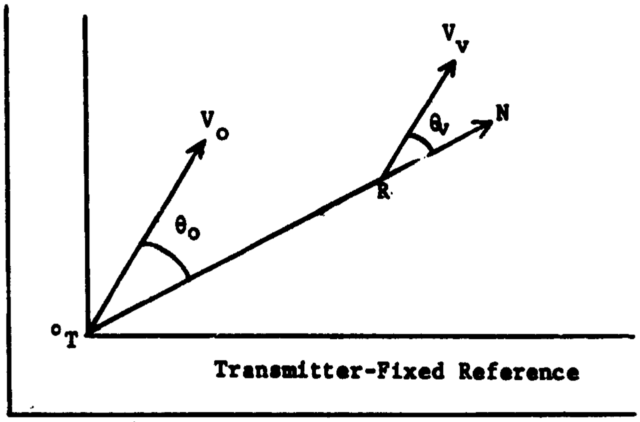


Figure 3.1 Configuration C - Case 1

This case is identical to Configuration A with

$$V = V_v - V_o \qquad (2\text{-}11a)$$

$$\Delta V = \Delta V_v \qquad (2\text{-}11b)$$

$$N.V = N.(V_v - V_o) = v_v \cos\theta_v - v_o \cos\theta_o \qquad (2\text{-}11c)$$

$$N.\Delta V = N.\Delta V_v = \Delta v_v \cos\ _v \qquad (2\text{-}11d)$$

$$v^2 = v_v^2 + v_o^2 - 2\, V_v.V_o \qquad (2\text{-}11e)$$

$$\beta = \frac{v}{c} \qquad (2\text{-}11f)$$

Equations (2-5) through (2-7) apply directly with the modifications given by equation (2-11). These adapted equations now contain transmitter-fixed reference parameters.

It is of interest to consider the situation where V_o is antiparallel to V_v, $\theta_o = 180 - \theta_v$. From equation (2-7), modified by (2-11), the inaccuracy using the classical Doppler equation is

$$N.\Delta V_v = \Delta v_v \cos\theta_v \approx -\tfrac{1}{2}\frac{v_v^2}{c}\left[1 + 2\left(\frac{v_o}{v_v}\right) + \left(\frac{v_o}{v_v}\right)^2\right] \qquad (2\text{-}12)$$

For a satellite tracking station, travelling antiparallel to the tracked vehicle at approximately the same velocity, the inaccuracy would be four times that of an earth-based tracking station, as indicated by equation (2-12).

2) Transmitter in Tracked Vehicle

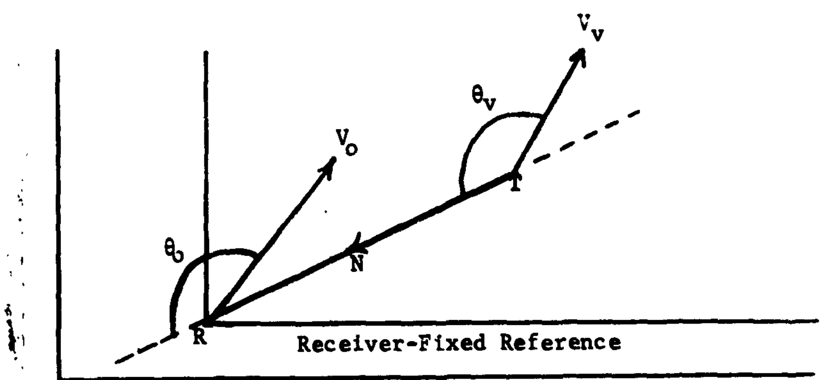


Figure 2.3 Configuration C - Case 2

This case is identical to Configuration B with the modifications given by equation (2-11).

Equation (2-8) through (2-10) apply directly with the modifications given by equation (2-11). These adapted equations now contain receiver-fixed reference parameters.

Consider again V_o antiparallel to V_v, $\theta_o = 180 - \theta_v$. From equation (2-10a), modified by (2-11), the inaccuracy using the classical Doppler equation is

$$N.\Delta V_v = \Delta v_v \cos\theta_v \approx$$

$$-\tfrac{1}{2}\frac{v_v^{\,2}}{c}\left[1 + 2\left(\frac{v_o}{v_v}\right) + \left(\frac{v_o}{v_v}\right)^2\right]\frac{1}{1 + 2N.V/c} \qquad (2\text{-}13)$$

Since 2N.V/c << 1, the discussion in Case 1 applies here as well.

2.2.4 Configuration D - Transmitter Stationary, Receiver Stationary, Vehicle in Motion

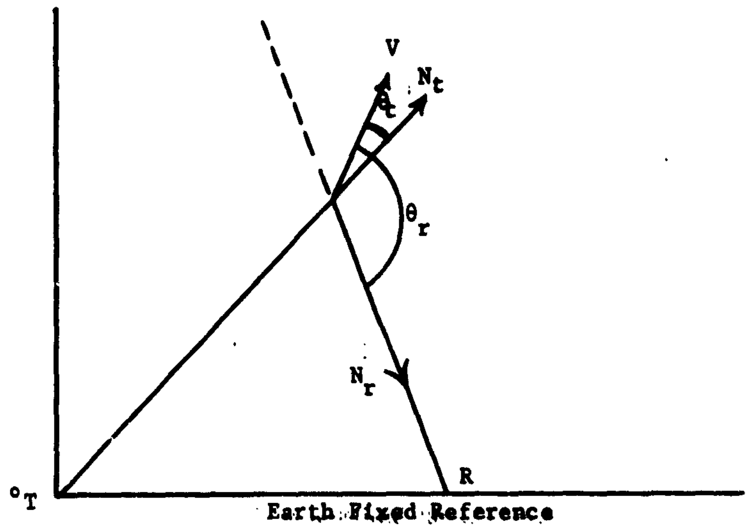


Figure 4.0 Configuration D

In the general case, the vehicle receives the Doppler frequency f_o, and the beacon retransmits $(af_o + b)$ to the receiver. For the special case of skin tracking $a = 1$, and $b = 0$.

Consider the Doppler frequency f_o received by the vehicle. This portion is identical to Configuration A with f_o substituted for f_R in equation (2-5a). Therefore,

$$f_o = f_T \frac{1 - N_t.V/w}{\sqrt{1 - \beta^2}}$$

The retransmitted (or reflected) frequency is $(af_o + b)$. The Doppler frequency received by the receiver is identical in form to Configuration B with $(af_o + b)$ substituted for f_T in equation (2-8a). Therefore,

$$f_R = (af_o + b)\frac{\sqrt{1 - \beta^2}}{1 - N_r.V/w}$$

Eliminating f_o from the above two equations results in

$$f_R = af_T \frac{(1 - N_t.V/w) + h\sqrt{1 - \beta^2}}{1 - N_r.V/w} \tag{2-14a}$$

where

$$h = \frac{b}{af_T}$$

Allowing $\beta \to 0$ results in the classical Doppler equation

$$f_R = af_T \frac{1 - N_t.\ (V + \Delta V)/w + h}{1 - N_r.\ (V + \Delta V)/w} \tag{2-14b}$$

Equation (2-14) contains earth-fixed reference parameters.

Consider the skin tracking case where $a = 1$, and $b = 0$. Comparing equations (2-14a) and (2-14b) shows that the relativistic and classical Doppler equations are identical, and $\Delta V = 0$. This is also true for beacon tracking where $b = 0$ and "a" has any value.

Expanding equation (2-14) through second order terms yields

$$f_R = af_T \left\{(1 + h) + (1 + N_r.V/w)\left[(1 + h)\ N_r.V/w - N_t.V/w\right] - \tfrac{1}{2}\beta^2 h\right\} \quad (2\text{-}$$

and for $\beta \rightarrow 0$

$$f_R = af_T \left\{(1 + h) + \left[1 + N_r \cdot (V + \Delta v)/w\right] \cdot \right.$$

$$\left. \left[(1 + h)N_r \cdot (V + \Delta V)/w - N_t \cdot (V + \Delta V)/w\right] \right\} \qquad (2\text{-}15b)$$

Subtracting equation (2-15b) from (2-15a) results in

$$(N_t - N_r) \cdot \Delta V = \Delta v(\cos\theta_t - \cos\theta_r) \approx$$

$$\frac{1}{2} \frac{v^2}{c} h \frac{(\cos\theta_t - \cos\theta_r)}{(1 + 2N_r \cdot V/w)\left[\cos\theta_t - (h + 1)\cos\theta_r\right]} \qquad (2\text{-}16a)$$

where $N_r \cdot \Delta V N_t \cdot \Delta V/w^2$ and $(1 + h)\ (N_r \cdot \Delta V/w)^2$ have been neglected and it has been assumed that $w \approx c$.

Equation (2-16) is the inaccuracy using the classical Doppler equation for beacon tracking where $b \neq 0$ $(h \neq 0)$.

Let us examine the inaccuracy in the usual practice of dropping the second order terms for skin tracking or beacon tracking $(b = 0,\ h = 0)$. Letting $h = 0$ in equation (2-15a) results, of course, in the identical relativistic and classical Doppler equation as discussed previously. Dropping second order terms and solving for the inaccuracy ΔV_1 yields.

$$(N_t - N_r) \cdot \Delta V_1 = \Delta v_1(\cos\theta_t - \cos\theta_r) \approx$$

$$\frac{v^2}{w}\left[\cos\theta_t \cos\theta_r - \cos^2\theta_r\right] .$$

Solving for Δv_1 gives

$$\Delta v_1 \approx \frac{v^2}{w} \cos\theta_r \qquad (2\text{-}16b)$$

which is independent of the transmitter angle θ_t.

2.2.5 Configuration E - Transmitter in Motion, Receiver in Motion, Vehicle in Motion

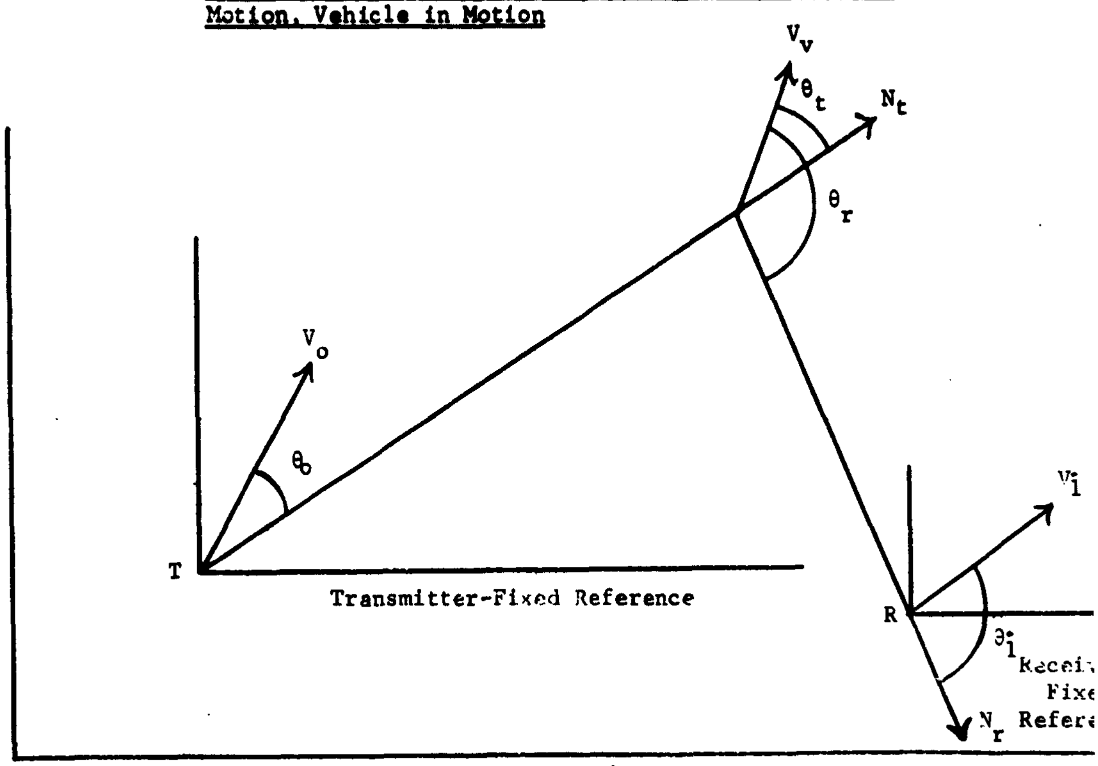


Figure 5.0 Configuration E

Consider the case of skin tracking or beacon tracking with no beacon frequency shifts (a = 1, b = 0; h = 0 of Configuration D). The derivation is identical to that of configuration D with the modifications as indicated.

The Doppler frequency f_o as seen by the vehicle is

$$f_o = f_T \frac{1 - N_t.V_t/w_t}{\sqrt{1 - \beta_t^2}}$$

The Doppler frequency seen by the receiver is

$$f_R = f_o \frac{\sqrt{1 - \beta_r^2}}{1 - N_r.V_r/w_r}$$

Eliminating f_o from the above results in

$$f_R = f_T \frac{1 - N_t.V_t/w_t}{1 - N_r.V_r/w_r} \frac{\sqrt{1 - \beta_r^2}}{\sqrt{1 - \beta_t^2}} \tag{2-17a}$$

where

$$V_t = V_v - V_o$$

$$V_r = V_v - V_1$$

$$N_t.V_t = v_v \cos\theta_t - v_o \cos\theta_o$$

$$N_r.V_r = v_v \cos\theta_r - v_1 \mathrm{vos}\theta_1$$

$$v_t^2 = v_v^2 + v_o^2 - 2V_v.V_o$$

$$v_r^2 = v_v^2 + v_1^2 - 2V_v.V_1$$

Allowing β_r, $\beta_t \rightarrow 0$ results in the Classical Doppler equation

$$f_R = f_T \frac{1 - N_t.\ (V_t + \Delta V)/w_t}{1 - N_r.\ (V_r + \Delta V)/w_r} \tag{2-17b}$$

where

$$\Delta V = \Delta V_v$$

If the transmitter and receiver have the same velocity vector ($V_o = V_1$) then the radicals cancel in equation (2-17a) and the classical and relativistic equations are identical.

The subscripts "t" and "r" refer to transmitter and receiver fixed reference systems respectively.

Expanding equation (2-17a) yields

$$f_R \approx f_T \left\{1 + (1 + N_r.V_r/w_r) \left[N_r.V_r/w_r - N_t.V_t/w_t\right] + \tfrac{1}{2} (\beta_t^2 - \beta_r^2)\right\} \qquad (2\text{-}18a)$$

where

$$\tfrac{1}{2} (\beta_t^2 - \beta_r^2) = \tfrac{1}{2} \frac{v_o^2 - v_1^2}{c^2} + \frac{V_v}{c} . \frac{(V_1 - V_o)}{c}$$

$$= (V_o - V_1). \left[(V_o + V_1)/2 - V_v\right]/c^2$$

and for $\beta_t, \beta_r \rightarrow 0$

$$f_R \approx f_T \left\{1 + \left[1 + N_r.(V_r + \Delta V)/w_r\right] \left[N_r.(V_r + \Delta V)/w_r - N_t/(V_t + \Delta V)/w_t\right]\right\} \quad (2$$

Subtracting equation (2-18b) from (2-18a) yields the inaccuracy

$$(N_t - N_r).\Delta V = \Delta v(\cos\theta_t - \cos\theta_r) \approx$$

$$- \frac{(V_o - V_1)}{c} . \left[V_v - \tfrac{1}{2}(V_o + V_1)\right] \frac{1}{(1 + N_r.V_r/c) - \dfrac{N_t.V_t/c - N_r.V_r/c}{1 - \cos\theta_t/\cos\theta_r}} \quad (2$$

where it has been assumed that $w_t \approx w_r \approx c$, and where $N_r.\Delta VN_t.\Delta V/w^2$ and $(N_r.\Delta V/w)^2$ have been neglected.

Equation (2-19a) is the inaccuracy using the classical Doppler equation. This inaccuracy is zero when $V_o = V_1'$, and when $V_v = \frac{1}{2}(V_o + V_1')$.

Let us examine the case where $V_o = V_1'$, ($V_r = V_t$). From equation (2-18a), with $w_t \approx w_r = w$,

$$f_R \approx f_T \left\{1 + (1 + N_r.V_t/w) \cdot \left[N_r.V_t/w - N_t.V_t/w\right]\right\}$$

which is identical to equation (2-15a) of Configuration D with h = 0 and V replaced by V_t.

The inaccuracy ΔV_1, in dropping second order terms is therefore given by equation (2-16b) with v replaced by v_t or v_r since for this case $v_t = v_r$. Therefore

$$\Delta v_1 \approx \frac{v_r^2}{w} \cos\theta_r =$$

$$\frac{v_v^2}{w}\left[1 + \left(\frac{v_1'}{v_v}\right)^2 - 2V_v.V_1'/v_v^2\right]\cos\theta_r \qquad \text{(2-19b)}$$

If the vehicle is travelling antiparallel to the receiver (also transmitter) then

$$\Delta v_1 \approx \frac{v_v^2}{w}\left[1 + 2\left(\frac{v_1'}{v_v}\right) + \left(\frac{v_1'}{v_v}\right)^2\right]\cos\theta_r$$

For $v_1' = v_v$, the inaccuracy is four times that of the similar earth-fixed system given by equation (2-16b) in Configuration D.

2.3 Zero Doppler Shift Effect

2.3.1 General

In the use of the Doppler systems, it is generally assumed that the receipt of a zero Doppler shift implies that the line of sight velocity of the vehicle is identically zero. This is, of course, inaccurate and the degree of inaccuracy is dependent upon the particular configuration in use. This section will analyze the degree of inaccuracy for Configuration B. It will also briefly indicate the inaccuracies associated with other Configurations. For completeness it should perhaps be stated that there is a zero Doppler shift for the trivial case of zero relative motion for all configurations.

2.3.2 Configuration B - (Vehicle) Transmitter in Motion, Receiver Stationary

Configuration B is discussed in Section 2.2.2 and is illustrated there in Figure 2.0.

From equation (2-8a), the zero Doppler shift occurs when

$$1 = \frac{\sqrt{1 - \beta^2}}{1 - N.V/w} \qquad (2\text{-}20a)$$

or

$$N.V/w = 1 - \sqrt{1 - \beta^2} \qquad (2\text{-}20b)$$

From an examination of Figure 2.0 in conjunction with equation (2-20b) it is clear that the zero Doppler shift occurs only when the vehicle is approaching the earth-fixed receiver.

Expanding equation (2-20b) and letting $w \approx c$ yields

$$N.V = v \cos \theta \approx \tfrac{1}{2}\frac{v^2}{c} \qquad (2\text{-}20c)$$

Equation (2-20c) is the line of sight velocity inaccuracy when the zero Doppler shift is implied to mean zero line of sight velocity.

2.3.3 Other Configuration

Configuration A

Identical to Configuration B except that the effect occurs when the vehicle is receding from the transmitter.

Configuration C

Case 1) Identical to Configuration A

Case 2) Identical to Configuration B

Configuration D

Setting equation (2-14a) equal to $(af_T + b)$, expanding, and letting $w \approx c$, results in

$$\left[N_t - (1 + h)\, N_r\right].V \approx - h\tfrac{1}{2}\frac{V^2}{c} \tag{2-21a}$$

For the case of skin tracking (or beacon with $b = 0$), $h = 0$ and the zero shift effect occurs when

$$N_t.V = N_r.V \quad . \tag{2-21b}$$

The Doppler shift produced by the vehicle receding from the transmitter is exactly cancelled by that produced by the vehicle approaching the receiver.

For colocation of receiver and transmitter ($N_t = -N_r$) the zero shift occurs only when the line of sight velocity is identically zero.

For the general case exhibited by equation (2-21b) the inaccuracy is thus $N_t.V$ (or $N_r.V$).

Configuration E

Setting equation (2-17a) equal to f_T, expanding and letting $w_t \approx w_r \approx c$ yields

$$N_t \cdot V_t - N_r \cdot V_r \approx \tfrac{1}{2} (v_t^2 - v_r^2)/c. \qquad (2\text{-}22a)$$

For the case where $V_o = V_1$,

$$(N_t - N_r) \cdot V_v = (N_t - N_r) \cdot V_o \qquad (2\text{-}22b)$$

3.0 ACTUAL DOPPLER SYSTEMS - RELATIVISTIC AND CLASSICAL APPROXIMATION INACCURACIES

3.1 General

This section consists of an examination of a number of Doppler Systems which are either operational or being planned for the future. The examination is limited to a comparison of the system accuracy requirements with the inaccuracies resulting from either the use of classical equations (in place of relativistic) or from the practice of neglecting second order terms in the equations. In addition, the effect of receiving a zero Doppler shift will be investigated. The equations pertinent to the above considerations have been developed in Section 2.0 and will be referred to as needed. This is also true for the figures which illustrate the particular configurations or systems under discussion. However, special forms of these equations will be developed, as needed, in the course of analyzing these systems. Therefore, for an understanding of the symbols used, and for illustrations of appropriate configurations, Section 2.0 should be referred to.

It should be noted that the inaccuracies determined here are predictable, and therefore, are not classified as errors. These inaccuracies may be avoided by the use of the relativistic Doppler equations rather than the classical, and by retaining the second order terms in these equations. This is especially true when the accuracies called for by the system are greater than that obtainable by first order classical equations.

For purposes of numerical computations, the following values are used:

Velocity of light, $c \approx 10^9$ ft/sec,

escape velocity, $v \approx 3.5 \times 10^4$ ft/sec,

orbital velocity, $v \approx 2.5 \times 10^4$ ft/sec.

	Escape Velocity	Orbital Velocity
$\frac{v}{c}$	3.5×10^{-5}	2.5×10^{-5}
$\frac{v^2}{c^2}$	1.2×10^{-9}	0.6×10^{-9}
$\frac{v^2}{c}$	1.2 ft/sec	0.6 ft/sec

TABLE 1.0 Numerical Values Used

3.2 <u>GLOTRAC - Global Tracking Network</u>

GLOTRAC is being designed and constructed to meet the tracking requirements of advanced satellite and space-probe programs. GLOTRAC contains a number of measurement subsystems and techniques. The Doppler principle is used in range rate measurements. A minimum of three range rate stations operate simultaneously on the signals radiated by the transponder. One of the three stations sends either 5060.194 mc (type C transponder) or 5052.0833 mc (type G transponder) signals to the transponder as interrogation signals. Range rate data obtained at a station equipped with a transmitter and receiver will have an accuracy of 0.09 ft/sec. Stations equipped only with receivers will have an accuracy of 0.51 ft/sec. Reference 1 should be referred to for a more detailed description.

The GLOTRAC Doppler subsystem is a configuration D system, analyzed in Section 2.2.4.

Type C Transponder

1) Transmitter interrogation frequency f_T = 5060.194 mc

2) Beacon receives Doppler frequency f_o

3) Beacon retransmits the frequency $af_o + b$ where $a = 1$, $b = -60.194$ mc.

4) Receivers receive Doppler frequency f_R.

The parameter h is therefore,

$$h = \frac{b}{af_T} \approx -.01.$$

From equation (2-16a), for h and $N_r.V/C \ll 1$, the inaccuracy resulting from the use of the classical Doppler is

$$\Delta v\ (\cos\theta_t - \cos\theta_r) \approx \tfrac{1}{4}\frac{v^2}{c}\, h \tag{3-1a}$$

For the master station (receiver and transmitter) $\cos\theta_r = -\cos\theta_t$ and equation (3-1a) becomes

$$\Delta v\ \cos\theta_t \approx \tfrac{1}{4}\frac{v^2}{c}\, h \tag{3-1b}$$

For the remote stations (receiver only) for the case where $\theta_t \approx \theta_r$ equation (2-16a) yields

$$\Delta v \cos\theta_r \approx - \frac{1}{2}\frac{v^2}{c} \qquad (3\text{-}1c)$$

which is independent of the 'h' parameter.

From equation (2-21a), for the master station, a zero Doppler shift is received when

$$\Delta v \cos\theta_t \approx - \frac{1}{2}\frac{v^2}{c}\, h. \qquad (3\text{-}2a)$$

For the remote stations, for the case when $\theta_t \approx \theta_r$ equation (2-21a) yields

$$\Delta v \cos\,\theta_r \approx \frac{1}{2}\frac{v^2}{c} \qquad (3\text{-}2b)$$

which is independent of the 'h' parameter.

The numerical results are listed in Table 2.1. The numerical values from Table 1.0, Section 3.1, have been employed . Included in Table 2.1 are references to the specific equations used.

	MASTER STATION (Transmitter and Receiver)			REMOTE STATION (Receiver Only)		
	Equation Used	Escape Velocity	Orbital Velocity	Equation Used	Escape Velocity	Orbital Velocity
Classical Inaccuracy	3-1b	-0.003ft/sec	-0.0015ft/sec	3-1c	-0.6ft/sec	-0.3ft/sec
Zero Doppler Shift	3-2a	0.003ft/sec	0.0015ft/sec	3-2b	0.6ft/sec	0.3ft/sec
Specified Accuracy		0.09 ft/sec.			0.51 ft/sec.	

Table 2.1 GLOTRAC (Transponder C) Inaccuracies

The specified accuracies include uncorrectable propagation errors and atomic frequency standard offset errors. From Table 2.1 it appears that the GLOTRAC (Transponder C) Range Rate System should utilize the Relativistic rather than the Classical Doppler equations at the Remote Stations. In addition, allowance should be made for the fact that the receipt of a zero Doppler frequency does not necessarily imply zero line of sight velocity.

Type G Transponder

1) Transmitter interrogation $f_T = 5052.0833$ mc

2) Beacon receives Doppler frequency f_o

3) Beacon retransmits the frequency $af_o + b$ where $a = 96/97$, $b = 0$.

4) Receivers receive Doppler frequency f_R.

The parameter h is therefore

$$h = \frac{b}{af_T} = 0$$

Comparing equations (2-14a) and (2-14b) shows that the relativistic and Classical Doppler equations are identical. Therefore, there is no inaccuracy using the Classical equation and $\Delta v = 0$.

However, there is an inaccuracy Δv_1 that results from the practice of neglecting second order terms in the classical expansion which is given by equation (2-15b) with $h = 0$. From equation (2-16b) this inaccuracy is

$$\Delta v_1 \approx \frac{v^2}{c} \cos\theta_r \quad . \tag{3-3a}$$

For the master station $\cos\theta_r = -\cos\theta_t$ and equation (3-3a) becomes

$$\Delta v_1 \approx - \frac{v^2}{c} \cos\theta_t \tag{3-3b}$$

or the line of sight inaccuracy is

$$\Delta v_1 \cos\theta_t \approx -\frac{v^2}{c}\cos^2\theta_t . \tag{3-3c}$$

For the remote stations, from equation (3-3a), the line of sight inaccuracy is

$$\Delta v_1 \cos\theta_r \approx \frac{v^2}{c}\cos^2\theta_r \tag{3-3d}$$

From equation (2-21b), for the master station, a zero Doppler shift is received when

$$\theta_r = \theta_t = \pi/2 \tag{3-4a}$$

and the line of sight velocity is identically zero.

For the remote stations, from equation (2-21b), a zero Doppler shift is received when

$$v \cos\theta_t = v \cos\theta_r \tag{3-4b}$$

and the zero Doppler received implies a line of sight velocity equal to $v \cos\theta_t$ (or $v \cos\theta_r$ since $\theta_r = \theta_t$ from equation 3-4b).

The numerical results are listed in Table 2.2, and follow the same procedure as Table 2.1.

	MASTER STATION (transmitter and Receiver)			REMOTE STATION (Receiver Only)		
	Equation Used	Escape Velocity	Orbital Velocity	Equation Used	Escape Velocity	Orbital Velocity
Approximation Inaccuracy	3-3c	-1.2ft/sec	-0.6ft/sec	3-3d	1.2ft/sec	0.6ft/sec
Zero Doppler Shift	3-4a	0	0	3-4b	$\theta_r = \theta_t$	$\theta_r = \theta_t$
Specified Accuracy	0.09 ft/sec			0.51 ft/sec		

Table 2.2 GLOTRAC (Transponder G) Inaccuracies

The approximation inaccuracies have been calculated for $\theta = 0$.

From Table 2.2 it is clear that in using the classical equations the second order terms should be retained to achieve the specified accuracies.

3.3 ANNA - Army, Navy, NASA, and Air Force

ANNA is a geodetic satellite which is expected to obtain fine measurements concerning the shape of the earth and to relate major geodetic datum to each other and to the earth's center of mass. Specifically, the satellite experiments will produce precision measurements related to angle, range and range-rate.

Range-rate information will be obtained by observation of the Doppler shift of ultra-stable transmissions from the satellite and four frequencies will be broadcast continuously for this purpose. Frequencies for geodetic measurements will be 162-324mc with a 54-216mc pair reserved for refraction studies and as a back-up in event of failure of prime tracking frequencies.

All four frequencies will be coherent, so that tracking can be accomplished using any two. Transmitters will have low power drain, level will be left on continuously to be available to observations throughout the world. Reference 2 should be referred to for further description.

The ANNA Doppler subsystem is a Configuration B system, analyzed in Section 2.2.2.

From equation (2-10a) for $2N.V/c \ll 1$, the inaccuracy resulting from the use of the classical Doppler equation is

$$\Delta v \cos\theta \approx - \frac{1}{2} \frac{v^2}{c} \qquad (3\text{-}5a)$$

From Table 1.0 for orbital velocity

$$\Delta v \cos\theta \approx -0.3 \text{ ft/sec} \tag{3-5b}$$

Equation (3-5b) is then the inaccuracy in the line of sight velocity when the relativistic equations are not used.

However, if the classical Doppler is used and the second order terms neglected, this inaccuracy tends to reduce the inaccuracy produced by not using the relativistic Doppler. From equation (2-10b) the total inaccuracy is

$$\Delta v_1 \cos\theta \approx -\tfrac{1}{2}\frac{v^2}{c}[1 - 2\cos^2\theta]. \tag{3-6a}$$

Comparing with equation (3-5) shows that

$$0 \leq |\Delta v_1 \cos\theta| \leq 0.3 \text{ ft/sec}, \tag{3-6b}$$

and when $\theta = \pi/4$, $3\pi/4$ there is no inaccuracy. However, from equation (3-6a) it is seen that the sign of the inaccuracy is changed.

From equation (2-20c), a zero Doppler shift is received when

$$v \cos\theta \approx \tfrac{1}{2}\frac{v^2}{c} \approx 0.3 \text{ ft/sec}. \tag{3-7}$$

Therefore, the receipt of a zero Doppler shift implies a line of sight velocity of 0.3 ft/sec and not zero ft/sec.

3.4 MISTRAM - Missile Trajectory Measurement System.

MISTRAM is a precision missile trajectory measurement system which will operate independently of other range systems at AMR, to acquire a launched vehicle, track its flight through space, and accurately measure its position and velocity vectors.

The range rate data is obtained by Doppler techniques at X-band with a specified accuracy of 0.02 ft/sec. (Refer to reference 3 for further information.)

1) The transmitter at the master or central station generates two CW-X-Band frequencies, nominally 8148mc and 7884mc to 7892mc. The higher frequency (the range signal) is very stable while the lower frequency (the calibrated signal) is swept periodically over the indicated range. Therefore f_T = 8148mc.

2) The airborne transponder receives the signals, amplifies frequency shifts by 68mc, and retransmits back to earth.

Therefore, the received Doppler frequency (the range signal) is f_o and the retransmitted frequency is $af_o + b$.

3) If the 68mc is a proportional shift then

$$a = \frac{8216}{8148} = \frac{2054}{2037},$$

$b = 0$, and $h = 0$.

4) If the 68mc is a constant shift then $a \cong 1$ and b = 68mc. The parameter h is therefore,

$$h = \frac{b}{af_t} \approx .01.$$

5) Receivers receive Doppler frequency f_R.

The MISTRAM Doppler subsystem is a configuration D system analyzed in Section 2.2.4.

The analysis is therefore similar to that of GLOTRAC presented in Section 3.2. The results tabulated there also apply here. For the constant shift (item 4 above) Table 2.1 applies with the specified accuracies changed to 0.02 ft/sec. The GLOTRAC 'h' was negative while the MISTRAM 'h' is positive. Therefore, the tabulated results (Table 2.1) for the Master Station are positive for MISTRAM. For the proportional shift (item 3 above) Table 2.2 applies, again with the specified accuracies changed to 0.02 ft/sec. Therefore, refer to the analysis in Section 3.2.

3.5 Ultra - Precise Trajectory Measurement System Study

In the Project 5930. AFMTC Range Instrumentation Project Card listing is task No. 5930.08, Ultra-Precise Trajectory Measurement System Study. The following is abstracted:

> "The proposed study will investigate the feasibility of and will determine the best approach to the expansion or development of a trajectory measurement system capable of measuring velocity to an accuracy of 0.01 ft/sec. at a range of 500 nautical miles. This accuracy cannot be attined by any trajectory measurement technique now known."

The purpose of abstracting the above information is but to indicate the accuracies desired in certain measurement systems. In doing so, it further illustrates the basic requirement of progressing from the use of the first order classical to the use of the relativistic Doppler equations.

4.0 SUMMARY AND CONCLUSIONS

This study has taken a fundamental look at the Doppler effect in electronic tracking. Five steps have been developed which enable the derivation of the Doppler equations for any Doppler system. These steps have then been used to derive six different configurations of transmitter, receiver, and vehicle. It has been determined that for velocity accuracies of a foot per second or better, the second order relativistic equations must be used. In some Doppler systems, the relativistic and classical equations are identical. In those cases the second order equations must also be used.

The effect of receiving a zero Doppler shift has also been investigated It has been determined that this effect does not imply zero line of sight velocity. Allowance for this fact must be made when the Doppler information is transformed into velocity information.

A number of present and future operational Doppler systems have been examined (GLOTRAC, MISTRAM, etc.). The evaluation clearly indicates that to obtain the desired accuracies, the second order equations must be utilized.

Finally, in the appendix there is a discussion of some pitfalls which occur when the equations are not carefully derived from the fundamental Doppler effect. It is shown that the approximate first order equations for different Doppler systems are identical in appearance. However, the second order equations do differ when correctly derived.

Jerome Hoffman

Jerome Hoffman

JH:bmm

Attachments: References
Appendix A

6.0 APPENDIX

6.1 Galilean and Lorentz Transformations

Consider two arbitrary systems of inertia described by Cartesian coordinates $X = (x, y, z)$ and $X' = (x', y', z')$ respectively. According to the Newtonian Conceptions of space and time, the connection between the time parameters and the coordinate vectors X and X' for the same space point in the two coordinate systems is given by

$$X' = X - Vt \tag{6-1a}$$

$$t' = t \tag{6-1b}$$

where V is a vector denoting velocity and direction of motion of the primed system. 't' is the time and, for simplicity, it is assumed that the origin of the two systems coincide at the time $t = 0$. Thus, the time is considered an abosolute quantity. Equation (6-1) is often referred to as the "Galilean Transformation."

Since the two systems of coordinates are completely equivalent, and since the unprimed system moves with the velocity -V relative to the primed system, the inverse transformation to (6-1) is simply obtained by interchanging the primed and the unprimed variable and simultaneously replacing V by -V.

$$X = X' + Vt' \tag{6-2a}$$

$$t = t' \tag{6-2b}$$

According to the special theory of relativity this same connection is given by

$$X' = X + V\left[(\gamma - 1)\, X.V/v^2 - \gamma t\right] \tag{6-3a}$$

$$t' = \gamma\left[t - V.X/c^2\right] \tag{6-3b}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

and

$$\beta = \frac{v}{c}$$

Equation (6-3) is usually called the "Lorentz Transformation."

The inverse transformation is again obtained by interchanging (X', t') and (X, t) and replacing V by -V.

$$X = X' + V\left[(\gamma - 1)\ X'.V/v^2 + \gamma t'\right] \tag{6-4a}$$

$$t = \gamma\left[t' + V.X'/c^2\right] \tag{6-4b}$$

In the limit as $\beta \rightarrow 0$ $(c \rightarrow \infty)$, the Lorentz transformation goes over into the Galilean transformation.

Refer to reference 4 for further discussion.

6.2 Clarification of Apparent Ambiguities

In Section 2.1 a list of five steps was given for the derivation of the Doppler equations for any configurationof transmitters and receivers. It was also mentioned that the improper application of steps 2 and 3 in the process of deriving a specific Doppler equation may result in an incorrect or only an approximate solution. For example, in reference 5, there is derived a Doppler equation for a specific configuration. In the derivation, the velocity terms were treated in an absolute sense. This is contrary to step 3 which states that the Doppler effect depends only on the relative velocity. As a result, equation (3) in Reference 5 is incorrect.

The practice of expanding the Doppler equations and retaining only first order terms usually eliminates the incorrect portions of the equations since these are carried in the higher order terms. However, with the requirements for more precise accuracies the second order terms must be retained. This of course necessitates that the Doppler equations be correct, at least to second order terms. Therefore, the five steps outlined in Section 2.1 must be correctly applied in the derivations.

The significance of step 3 was discussed above. Let us now consider the outcome of an improper application of step 2. Step 2 concerns the selection of the coordinate system to which the measurements are to be referenced.

Consider Configuration A analyzed in Section 2.2.1. There is an earth-fixed transmitter and a receiver in the vehicle. (See Figure 1.0, Section 2.2.1). The Doppler equations derived contain earth-fixed reference parameters. The vehicle is traveling with a velocity V with respect to the transmitter. Therefore, from step 2 the earth-fixed transmitter is travelling with a velocity -V relative to the vehicle.

From step 1, equation (2-1), the phases are equated.

$$f_T\,[t - N.X/w] = f_R\,[t' - N'.X'/w'] \qquad (6\text{-}5)$$

where the primed parameters are the measurements in the receiver's (vehicle) reference system.

Applying the direct transformation, equation (6-3), to the right hand side of equation (6-5) yields

$$f_T\,\{t - N.X/w\} =$$

$$f_R\left\{\gamma\,[t - V.X/c^2] - N'.X/w' - N'.V\Big[(\gamma - 1)\,X.V/v^2 - \gamma t\Big]/w'\right\} \qquad (6\text{-}6)$$

Equating the coefficients of t, as in Section 2.1, gives

$$f_T[1]\ t = f_R\ [\gamma + N'.V\gamma/w']\ t \qquad (6\text{-}7)$$

Therefore, the Doppler frequency observed by the receiver on the vehicle, in its own reference system, is

$$f_R = f_T \frac{\sqrt{1 - \beta^2}}{1 + N'.V/w'} \qquad (6\text{-}8)$$

where

$$N'.V = v \cos\theta' \quad .$$

Here we see the interesting ambiguity of two apparently different Doppler equations for the same system, Configuration A. Compare equation (6-8) with (2-5a). In fact, equation (6-8) is in appearance similar to a different system, Configuration B. Compare equation (6-8) with (2-8a). Why? The answer lies in the fact, as stated before, that equations (2-5a) and (2-8a) contain earth-fixed reference parameters, whereas equation (6-8) contains vehicle-fixed reference parameters.

Let us pursue this interesting phenomenon further. Let us now compare the two Configuration A equations, (2-5a) and (6-8) uder first order expansions.

From equation (2-6) in earth-fixed coordinates.

$$f_R \approx f_T\ [1 - N.V/w]\ . \qquad (6\text{-}9a)$$

From equation (6-8) in vehicle-fixed coordinates.

$$f_R \approx f_T\ [1 - N'.V/w'] \qquad (6\text{-}9b)$$

Again we encounter the first order approximation phenomenon which apparently eliminates the necessity for strict adherence to the five steps of derivation given in Section 2.1. The earth-fixed and vehicle fixed forms of the approximate Doppler equation, equations (6-9a) and (6-9b) respectively, are identical in appearance.

It is beyond the scope of this paper to pursue this subject any further. The purpose of discussing some common pitfalls is to draw attention to the fact that the requirements for more precise measurement accuracies leads directly to a more precise derivation of the appropriate Doppler equations - specifically the adherence to the five steps listed in Section 2.1.

It should, perhaps, be noted that in the process of equating phases, step 1, the primed (vehicle-fixed) parameters may be determined as functions of the unprimed (earth-fixed) parameters. Thus equation (6-8) may be transformed into equation (2-5a) and vice versa. This was stated in the last paragraph of Section 2.1.

five steps of derivation given in Section 2.1. The earth-fixed and vehicle fixed forms of the approximate Doppler equation, equations (6-9a) and (6-9b) respectively, are identical in appearance.

It is beyond the scope of this paper to pursue this subject any further. The purpose of discussing some common pitfalls is to draw attention to the fact that the requirements for more precise measurement accuracies leads directly to a more precise derivation of the appropriate Doppler equations - specifically the adherence to the five steps listed in Section 2.1.

It should, perhaps, be noted that in the process of equating phases, step 1, the primed (vehicle-fixed) parameters may be determined as functions of the unprimed (earth-fixed) parameters. Thus equation (6-8) may be transformed into equation (2-5a) and vice versa. This was stated in the last paragraph of Section 2.1.

5.0 REFERENCES

1) GLOTRAC - Global Tracking Network, General Dynamics, Report No. AE2-0304.

2) ANNA-1 Scheduled for Launch This Week, Aviation Week and Space Technology, May 7, 1962, Pages 30,31.

3) Introduction to MISTRAM - A Precision Missile Trajectory Measurement System, General Electric.

4) Moller, C., "The Theory of Relativity," Oxford University Press, 1960.

5) William, W., Jr., "A Doppler Navigation System Using Earth Satellites," IRE Transactions Fifth National Symposium, 1960 on Space Electronics and Telemetry, September 19-20-21, 1960, Washington, D. C.

Hq. ESD, L.G. Hanscom Field, Bedford, Mass.
Rpt. No. ESD-TDR-63-178. RELATIVISTIC AND CLASSICAL DOPPLER ELECTRONIC TRACKING ACCURACIES. Final report, June 1963, 40p. incl. illus., tables, 5 refs.

Unclassified Report

The first order Doppler effect has in general been used in electronic tracking. With the emphasis upon accuracies of tenths of a foot per second

1. Doppler tracking
2. Doppler systems
I. Project No. 705
II. AF33(600)-39852
III. The MITRE Corporation, Bedford, Mass.
IV. Hoffman, J.
V. W-5785
VI. In DDC collection

Hq. ESD, L.G. Hanscom Field, Bedford, Mass.
Rpt. No. ESD-TDR-63-178. RELATIVISTIC AND CLASSICAL DOPPLER ELECTRONIC TRACKING ACCURACIES. Final report, June 1963, 40p. incl. illus., tables, 5 refs.

Unclassified Report

The first order Doppler effect has in general been used in electronic tracking. With the emphasis upon accuracies of tenths of a foot per second

1. Doppler tracking
2. Doppler systems
I. Project No. 705
II. AF33(600)-39852
III. The MITRE Corporation, Bedford, Mass.
IV. Hoffman, J.
V. W-5785
VI. In DDC collection

Hq. ESD, L.G. Hanscom Field, Bedford, Mass.
Rpt. No. ESD-TDR-63-178. RELATIVISTIC AND CLASSICAL DOPPLER ELECTRONIC TRACKING ACCURACIES. Final report, June 1963, 40p. incl. illus., tables, 5 refs.

Unclassified Report

The first order Doppler effect has in general been used in electronic tracking. With the emphasis upon accuracies of tenths of a foot per second

1. Doppler tracking
2. Doppler systems
I. Project No. 705
II. AF33(600)-39852
III. The MITRE Corporation, Bedford, Mass.
IV. Hoffman, J.
V. W-5785
VI. In DDC collection

Hq. ESD, L.G. Hanscom Field, Bedford, Mass.
Rpt. No. ESD-TDR-63-178. RELATIVISTIC AND CLASSICAL DOPPLER ELECTRONIC TRACKING ACCURACIES. Final report, June 1963, 40p. incl. illus., tables, 5 refs.

Unclassified Report

The first order Doppler effect has in general been used in electronic tracking. With the emphasis upon accuracies of tenths of a foot per second

1. Doppler tracking
2. Doppler systems
I. Project No. 705
II. AF33(600)-39852
III. The MITRE Corporation, Bedford, Mass.
IV. Hoffman, J.
V. W-5785
VI. In DDC collection

and less, this approach is no longer permissible. This study has taken a fundamental look at the Doppler effect. Five steps have been developed which enable the exact derivation of the Doppler equations for any system. Six different configurations of transmitter, receiver, and vehicle have been investigated and the results applied to a number of present and future operational Doppler systems. For accuracies of a foot per second, second order relativistic or classical equations must be used.

and less, this approach is no longer permissible. This study has taken a fundamental look at the Doppler effect. Five steps have been developed which enable the exact derivation of the Doppler equations for any system. Six different configurations of transmitter, receiver, and vehicle have been investigated and the results applied to a number of present and future operational Doppler systems. For accuracies of a foot per second, second order relativistic or classical equations must be used.

and less, this approach is no longer permissible. This study has taken a fundamental look at the Doppler effect. Five steps have been developed which enable the exact derivation of the Doppler equations for any system. Six different configurations of transmitter, receiver, and vehicle have been investigated and the results applied to a number of present and future operational Doppler systems. For accuracies of a foot per second, second order relativistic or classical equations must be used.

and less, this approach is no longer permissible. This study has taken a fundamental look at the Doppler effect. Five steps have been developed which enable the exact derivation of the Doppler equations for any system. Six different configurations of transmitter, receiver, and vehicle have been investigated and the results applied to a number of present and future operational Doppler systems. For accuracies of a foot per second, second order relativistic or classical equations must be used.