NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
TRANSLATION
TURBULENT CIRCULAR AND FLAT STREAMS DEVELOPING
IN A COUNTERFLOW

By
E. Timma

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE
OHIO
TURBULENT CIRCULAR AND FLAT STREAMS DEVELOPING IN A COUNTERFLOW

By: E. Timma

English Pages: 13

TURBULENT CIRCULAR AND FLAT STREAMS DEVELOPING IN A COUNTERFLOW*

E. Timma

Until now there have been no analytical articles concerning turbulent streams developing in a counterflow which would permit us to satisfactorily determine the change in velocity on the stream axis.

The experimental investigations of L. A. Vulis, T. P. Leont'yeva and V. P. Kashkarov [1,2] and of Yu. V. Ivanov and Kh. N. Suy [3,4] do not include the entire stream length.

Certain authors [1,2] experimentally study the change of the dimensionless range $L/d$ of a circular stream with parameter values $\mu = \frac{u_H}{u_{0m}} = 0.2$ to $-0.48$ ($u_{0m}$ and $u_H$ are the maximum velocity in the initial section of the stream and the flow velocity, respectively) other authors [3,4] examine the velocity field in the main section of a circular and a flat stream within bounded limits of a change of

* This work was carried out under the supervision of Yu. V. Ivanov, Doctor of Technical Sciences.
the determining parameter.

We know of no experimental data for determining the values of turbulent characteristics in the initial section and in sufficiently distant sections in the main section of the stream. Heated streams developing in a counterflow have not been studied at all.

In this article an attempt is made to experimentally study the development of turbulent streams in a counterflow. This article is a sequel to the investigations conducted earlier at the Power Engineering Institute of the Academy of Sciences of the Estonian SSR [3,4].

It is expeditious to investigate by sections a turbulent flow, developing in a uniform counterflow. We can, for example, divide a stream into the following three characteristic sections:

1. the initial section in which exists a potential flow core;

2. the main section in which the flow acquires a structure such that the stream can be considered as developing from a point source or from a source of insignificant size;

3. the transient section, included between the initial and main sections, in which rearrangement of the stream structure ends.

The initial and transient sections were experimentally investigated in greater detail; to a considerably lesser degree we investigated the main sections of a turbulent circular and flat stream developing in a counterflow. This is due to the fact that the main section of an isothermal stream developing in a counterflow has been studied in greater detail earlier [3,4]. To conduct the tests we used the experimental arrangement pictured in Fig. 1, a detailed description of which is presented in an earlier article [5].
In the tests the stream velocity was varied from 13 to 20 m/sec while the average velocity of the counterflow remained constant, 6.7 m/sec.

To form the circular stream a nozzle of 40 mm in diameter was used; for a flat stream we used a nozzle 14 x 200 mm (with orientation perpendicular to the larger side). The maximum temperature difference between the unheated stream and the flow did not exceed 8 degrees, and therefore we can consider that the ratio of the maximum absolute temperature of the stream ($T_{om}$) to the absolute temperature of the flow ($T_H$) $\theta = T_{om}/T_H$ is approximately equal to unity. The temperature of the flow throughout the experiments equaled room temperature.

In stream cross sections the velocity in the direction of the stream axis and the temperature $T$ were measured using the following parameter values:

1. for a flat stream

   a) in an isothermal stream when $\theta = 1$ and with ratios of flow velocity to stream velocity $\mu = u_H/u_{om} = -0.33$; $-0.50$.

Fig. 1. External view of the arrangement.
b) in a nonisothermal stream when $\theta = 1.48$ and $\mu = 0.3$.

2. For a circular stream

a) in an isothermal stream ($\theta = 1$) when $\mu = -0.33; -0.50$.

b) in a nonisothermal stream $\theta = 2$ and $\mu = 0.33$. Since the ratio of the flow and nozzle diameters is comparatively large, then in agreement with the formula presented earlier [4] we can establish that with a parameter value $\mu = -1$ to $-0.3$ a stream is developed in a counterflow, but when $\mu > -0.3$ there is interaction of the stream not with the flow but with a counterstream.

The temperature and velocity fields in stream cross sections were symmetrical; therefore it was possible to decrease the number of measurements. For determining velocity or temperature field in cross section of a circular stream it was sufficient to conduct measurements with respect to a horizontal line cutting across the center. Figures 2a and 2b show the velocity profiles in various cross sections of a circular stream when $\theta = 1$ and $\lambda = 1/\mu = -3$, -2 and a flat stream when $\theta = 1$ and $\lambda = -2$. From these figures it is obvious that with the indicated parameter values in the initial section of the stream, there is a noticeable deformation of the velocity field of the flow.

The relative error when measuring the velocities in the initial and transient sections of the flows did not exceed $\%$. The relative accuracy was higher at the stream axis and decreased toward the outside boundaries of the stream.

Close to the line of zero velocity the relative error is larger and increases with distance from the nozzle. Close to the point on the axis where velocity $u = 0$, we noted velocity pulsations which noticeably hindered the measurements. We can say that the velocity
pulsations increase at the end of the main section of the stream. During temperature measurements the relative error did not exceed 4%.

It is known that one important property of a turbulent stream developing in a counterflow or in an immobile medium is the similarity of velocity profiles in the initial, transient, and main sections. By profile similarity, we mean the possibility of representing velocity profiles in different cross sections of the streams by means of the same function:

\[
\frac{u - u_m}{u_m - u_n} = f_k(\eta),
\]

where \(u\), \(u_H\), and \(u_m\) are the velocities at an arbitrary point of the flow, in the flow, and at the flow axis, respectively;

\(f_k(\eta)\) is the universal function in cross sections of the stream, which can have varying aspects in the initial, transient, and main sections;

\(\eta\) is the dimensionless ordinate of the point, defined in the initial section by the formula

\[
\eta = \frac{y - y_1}{y_2 - y_1},
\]

and in the transient and main sections of the stream by

\[
\eta = \frac{y}{\delta},
\]

where \(y\) is the distance of the point from the stream axis; \(y_1\) and \(y_2\) are the ordinates of the point of the internal and external boundary, respectively, of the displacement zone in the initial section of the stream;

\(\delta\) is the ordinate of the stream boundary in the transient and main sections. In order to verify the above mentioned similarity of profiles of a turbulent stream developing in a counterflow we must consider that in Eq. (1) the velocity of the counterflow \(u_H\) is
negative. Since experimental determinations of the boundary point where the dimensionless velocity $u-u_H$ equals zero are possible only with large errors, the experimental data are presented in Fig. 3 in the following coordinates:

$$\frac{u - u_H}{u_m - u_H} \text{ and } \frac{y_{0.5} - y}{y_{0.5} - y_{0.9}}$$

where $y_{0.5}$, $y_{0.9}$ are the ordinates of the point; the dimensionless excess velocity $\frac{u - u_H}{u_m - u_H}$ is equal to 0.5 and 0.9, respectively. A simple conversion shows that if the experimental data lie on a single curve, the condition of similarity (1) is fulfilled.

From study of the similarity of the velocity profiles $\frac{u - u_H}{u_m - u_H}$ in different cross sections of a stream it follows, as is obvious from Fig. 3, that the similarity in the initial and transient sections of circular and flat streams is observed only within dimensionless velocities from 0.3 to 1. For determining the profiles we can use formulas introduced in a previous article [5].

Thus, according to Schlichting's formula

$$h_i(\eta) = [1 - \eta^{n_\alpha}]$$

and according to the polynomial formula

$$h_2(\eta) = 1 - 6\eta^2 + 8\eta^3 - 3\eta^4$$

after the substitution

$$\eta = \eta_0.5 - \frac{y_{0.5} - y}{y_{0.5} - y_{0.9}} (\eta_0.1 - \eta_0.5)$$

we get the solid and dashed lines, respectively, in Fig. 3. Here $\eta_{0.5}$, $\eta_{0.1}$ and $\eta_{0.9}$ are constants defined from the corresponding equations

$$i_5(\eta_{0.5}) = \frac{1}{2}; \quad i_4(\eta_{0.1}) = 0.1; \quad i_4(\eta_{0.9}) = 0.9.$$ 

It is obvious in Fig. 3 that the calculation according to Polynomial (5) better agrees with the experimental data than does the calcula-
tion according to Schlichting's Formula (4).

Fig. 2. Velocity profiles in different cross sections of a turbulent stream developing in a counterflow: a) flat stream; b) circular stream.
Fig. 3. Profiles of dimensionless excess velocity in cross sections of a turbulent stream developing in a counterflow: a) circular stream; b) flat stream.
Figure 4 compares the profiles of dimensionless excess velocity in different cross sections of a circular stream according to experiments by Kh. N. Suy [4], L. A. Vulis, and the author. Obviously in the main section of a turbulent circular stream developing in a counterflow the velocity profiles are similar and they can be satisfactorily determined with the aid of Polynomial (5) by the substitution

$$\eta = \eta_{0.5} \frac{y}{y_{0.5}}. \quad (7)$$

Figure 5 gives the temperature profile of the stream in dimensionless coordinates. Here it is obvious that the profiles of dimensionless excess temperature $\frac{T - T_H}{T_{m} - T_H}$ in different cross sections of a circular and a flat stream are similar and can be calculated from Formula (5).

$T$, $T_H$ and $T_{m}$ is the temperature at an arbitrary point of the stream, in the flow, and at the stream axis, respectively.
$y'_{0.5}$ and $y'_{0.9}$ are the distances of the points from the stream axis, at which $\frac{T - T_H}{T_m - T_H}$ equals 0.5 and 0.9 respectively.

For the determination of the long range $L$ of a circular and flat stream the following simple formulas are obtained:

$$\frac{L}{R} = -\frac{5.5}{\mu},$$  \hspace{1cm} (8)

$$\frac{L}{b_0} = \frac{1 - 28 \mu}{\mu^2} - 47,$$ \hspace{1cm} (9)

where $R$ is the radius of the circular nozzle;

$b_0$ is the half width of the flat nozzle.

Figure 6a compares test data with calculations according to the formula of Kh. N. Suy [4], L. A. Vulis [1,2], and Formula (8). In the L. A. Vulis formula

$$\frac{L}{R} = \frac{2k}{|\mu|},$$

where the coefficient $k = 3 - 3.5$.

When comparing the calculations in Fig. 6a we used the averaged value

$k = 3.25$. 

-10-
In Fig. 6a the dashed line is given according to Formula (8), the broken line according to Formula (8a) and the solid line according to Suy's formula [4]. As is obvious from Fig. 6a, Formula (8) gives the better agreement.

![Diagram of test data comparison](image)

**Fig. 6.** Comparison of test data according to the range of the stream with a calculation according to formulas of various authors; a) circular stream; the dotted line is from Formula (8), the solid line is from Kh. N. Suy [4], and the broken line is from the L. A. Vulis formula (8a). b) flat stream; the solid line is from Formula (9).
It is obvious in Fig. 6b that the solid line drawn according to Formula (9) well describes the test data within the limits of change of the parameter $\mu$ from 0 to 0.5.

Conclusions

1. There is a similarity of profiles of dimensionless excess velocity $\frac{u - u_H}{u_m - u_H}$ within limits from 1 to 0.3 in different cross sections throughout a stream, beginning at a distance of one nozzle diameter from the initial section for a circular stream and at one nozzle width for a flat stream, depending on the dimensionless excess ordinate $\frac{y_{0.5} - y}{y_{0.5} - y_{0.9}}$. Profiles of dimensionless excess velocity can be determined with the aid of Formulas (4) or (5).

2. In the main section of a circular or flat stream the profiles of dimensionless excess velocity, depending on the dimensionless ordinate $\frac{y}{y_{0.5}}$, can be determined with the aid of Polynomial (5) with substitution of (7).

3. There is similarity of the profiles of dimensionless excess temperature $\frac{T - T_H}{T_m - T_H}$ within limits of 1 to 0.3 in different cross sections throughout the stream, beginning at a distance of one nozzle diameter from the initial section for a circular stream and at one nozzle width for a flat stream, depending on the dimensionless excess ordinate $\frac{y_{0.5} - y}{y_{0.5} - y_{0.9}}$. The profiles of dimensionless excess temperature can be determined from Formula (5).

4. The long range of circular and flat streams developing in a counterflow is represented by Formulas (8) and (9).

FTD-TT-63-372/1+2+4  -12-
REFERENCES


2. L. A. Vulis, V. P. Kashkarov, and T. P. Leont'yeva. Issledovaniye slozhnykh turbulentnykh struyakh techeniy. Tssoledo-

3. Kh. N. Suy and Yu. V. Ivanov. Issledovaniye razvitiya krug-


5. E. P. Timma and Yu. V. Ivanov. Kruglaya i ploskaya turbu-

Power Engineering Institute
Submitted March 5, 1962
Academy of Sciences of the
Estonian SSR

FTD-TT-63-372/1+2+4
### DISTRIBUTION LIST

<table>
<thead>
<tr>
<th>DEPARTMENT OF DEFENSE</th>
<th>Nr. Copies</th>
<th>MAJOR AIR COMMANDS</th>
<th>Nr. Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFSC</td>
<td></td>
<td>SCFDD</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DDC</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TDBTL</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TDBDP</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AFFTC (PTY)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ASD (ASYIM)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BSD (BSF)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SSD (SSF)</td>
<td>2</td>
</tr>
</tbody>
</table>

**HEADQUARTERS USAF**

<table>
<thead>
<tr>
<th></th>
<th>Nr. Copies</th>
<th></th>
<th>Nr. Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFCIN-3D2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARL (ARB)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**OTHER AGENCIES**

<table>
<thead>
<tr>
<th>Agency</th>
<th>Nr. Copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIA</td>
<td>1</td>
</tr>
<tr>
<td>NSA</td>
<td>6</td>
</tr>
<tr>
<td>DIA</td>
<td>9</td>
</tr>
<tr>
<td>AID</td>
<td>2</td>
</tr>
<tr>
<td>OTS</td>
<td>2</td>
</tr>
<tr>
<td>AEC</td>
<td>2</td>
</tr>
<tr>
<td>PWS</td>
<td>1</td>
</tr>
<tr>
<td>NASA</td>
<td>1</td>
</tr>
<tr>
<td>ARMY (FSTC)</td>
<td>3</td>
</tr>
<tr>
<td>NAVY</td>
<td>3</td>
</tr>
<tr>
<td>NAFEC</td>
<td>1</td>
</tr>
<tr>
<td>RAND</td>
<td>1</td>
</tr>
</tbody>
</table>

**FTD-TT-63-372/1+2+4**