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A METHOD FOR ESTABLISHING LANDING DESIGN CRITERIA FOR CARRIER-BASED AIRPLANES

1 JUNE 1963

PREPARED UNDER NAVY, BUREAU OF NAVAL WEAPONS
CONTRACT NO. 61-0268-c

PHASE II
FINAL REPORT
REPORT NO. 2-53400/3R460

CHANCE VOUGHT CORPORATION
AERONAUTICS AND MISSILES DIVISION
P. O. BOX 5907
DALLAS 22, TEXAS
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FOREWORD

This report presents the results of the Phase II study and completes the requirements of Contract NOw 61-0268-c, Bureau of Naval Weapons, Department of the Navy. The technical monitor has been Mr. Dean C. Lindquist of the Loads and Dynamics Branch of the Airframe Design Division of the Bureau of Naval Weapons.
ABSTRACT

A method for establishing landing loads design criteria for carrier-based airplanes is presented in this Phase II report. The airplane's landing environment was mathematically defined in Phase I, and provides the initial conditions necessary for the evaluation of landing loads. The loads criteria include methods for determining design loads, fatigue spectra, and strength envelopes which are compatible with the environmental conditions at airplane touchdown. The various methods are compared relative to the time required to perform the load analyses, computer time required, and the significance of the results.
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(f) NAMC Aeronautical Structures Laboratory Report No. NAMATCEN-ASL-1035, Statistical Presentation of Fleet Operational Landing Parameters for Models F8U-1, F9F-8P, F3H-2N, A4D-1, and A3D-2 Aircraft Aboard the USS Saratoga (CVA-60) in the Mediterranean Sea Area, dtd 31 August 1961
INTRODUCTION

The development of a method for establishing carrier landing design criteria has evolved in the form of a two phase program conducted by Chance Vought under the supervision of the Bureau of Naval Weapons.

The underlying assumption for both phases has been that the design of a carrier-based airplane must consider the entire landing environment in order to obtain the landing strength and performance required by military operations.

This report presents the results of the Phase II study of that program. Utilizing the landing environment defined in Phase I, this study develops a method for determining design landing loads criteria. These criteria include procedures for defining (1) design loads, (2) fatigue spectra, and (3) strength envelopes. Fundamental to such criteria are the use of statistics and probability theory which are necessarily consistent with the procedures used in Phase I to define the landing environment. Due consideration was given to minimizing the elapsed time that would be required of a Contractor to conduct such a study for a new design. This includes the time required for the selection of a sample of initial conditions, the analysis of the mathematical model, and the determination of the design loads criteria.
1. INPUT VARIABLES AND PARAMETERS

The input variables and parameters are inclusive of those necessary to properly evaluate the landing loads expected to be experienced by an airplane during the carrier operation portion of its life. This definition presupposes that the proportion of an airplane's landings, that are carrier landings, is known. (MIL-A-8866 (ASG) requires twenty percent of the total landings to be carrier landings.)

The input variables consist of those variables, discussed in paragraph 10.12 of reference (a), which define the landing environment of the airplane at the instant of touchdown on the carrier deck. A set of values for these variables, for a particular landing, will define the initial condition of the airplane at touchdown and its position on the deck. It is these sets of values which form the initial conditions for the analysis used to evaluate the landing loads.

In addition to the initial conditions, certain other parameters should be considered for completeness in any analysis of carrier landings. Of primary interest is the deck configuration. For example, the arresting cables and their spacing will have a significant effect on the loads if contacted during touchdown or roll-out during the period of high loads on the gear. To properly assess the effects of this parameter on the
landing gear requires a probability analysis, and should definitely be included in the analytical model. (The cable and its inclusion in the analysis are discussed in more detail in Section 5.) Other carrier conditions such as deck pitch are included as inputs to the derivation of the initial conditions.

It should be noted that variations in servicing of the tires and struts are included in the initial conditions. That is, the tire pressures, the strut air pressures, and the strut oil levels can be considered as variables and have been so described in paragraph 10.12 of reference (a). The effects of tire pressure, in conjunction with cable impact or run-over, have been found to be significant in previous analyses.

It is the variables which make up the initial conditions together with the parameters which describe the deck conditions that will be the input to the load analysis.

2. SELECTION OF INITIAL CONDITIONS

Phase I of this study provides a method for establishing the carrier landing environment for current or new Navy airplanes. This environment is expressed in terms of distribution functions of the pertinent initial condition variables together with an extreme value envelope derived in accordance
with a pre-assigned probability. Hence, to establish design load criteria for a specific design, a procedure must be determined for selecting combinations of values of the initial conditions which are representative of the landing environment that is expected for the new design. These combinations of initial conditions will thus be used as inputs to the analytical model for determining the ground reactions of each of the landing gears. The procedure for selecting these combinations will be so defined as to make it readily adaptable to a high speed computer. Since the entire environment is of importance to the design, a sampling technique or weighted parameter study will be required.

Monte Carlo techniques (sampling) can be used to generate combinations of conditions which take on values in accordance with the distributions defined in paragraph 10.12 of reference (a). Digital computer routines exist in SHARE (IBM) which will generate random normal variates, or random numbers for the uniform distribution, satisfying the particular distribution function requirements. Figure 2.1 presents four hypothetical distributions and a set of initial conditions which could be generated from these distributions. Those distributions which are neither normal nor uniform, may be defined so that they may be generated randomly on a digital computer. For example, much of the empirical data in refer-
Example of a set of initial conditions which could be generated from three normal distribution functions and one uniform distribution. (i.e., $x_1 \sim f_1$, $x_2 \sim f_2$, $x_3 \sim f_3$, and $x_4 \sim f_4$)

den (a) is non-normal and, if it is to be used for a new design, it would be necessary to generate its values by use of the functions in Table X of reference (a) or by discrete representations of the curves. Changes in the method of controlling the approach of the airplanes to the carrier would result in changes to the distributions of the initial conditions which would subsequently be represented empirically or functionally. That is, a value for each of the initial
variables $x_1, x_2, \ldots, x_m$ (engaging speed, sink speed, pitch angle, roll angle, yaw angle, etc.), is generated by the computer. Each set, $(x_{11}, x_{21}, x_{31}, \ldots, x_{m1})$, of the $m$ values forms the initial condition for a landing. Combining the value $x_{11}$, with $x_{21}$, with $x_{31}$, $\ldots$, with $x_{m1}$ to form each set is possible with the assumption of mutual independence of the $m$ variables. This assumption is consistent with the relations developed in Phase I. The totality of the $n$ sets, $(x_{11}, x_{21}, x_{31}, \ldots, x_{m1})$, forms the sample of initial conditions which will be representative of the carrier landings expected to be experienced by the new design. How well, or to what degree, the sample is representative is a function of the distribution functions defined in paragraph 10.12 and the sample size. This sample will be used as the initial conditions for the equations of motion.

A weighted parameter study is a second approach which is available for determining a sample of initial conditions. While the Monte Carlo approach generated the values of each $x_i$, and combined them, in a random manner; the weighted parameter approach uses fixed values of each of the variables, $x_i$, and combines these values into sets $(x_{1i}, x_{2i}, x_{3i}, \ldots, x_{mi})$, each with an associated probability. This approach adjusts the sample size by varying either the number of variables $x_i$, or the number of values used for each $x_i$, or
both. The probability associated with each set, or combination, of the \( x_i \)'s is also assigned to the load cycle resulting from the analysis where this set was used as the initial condition.

For each variable \( x_i \), its range is subdivided into \( n_i \) intervals \( (I_{ij}, j = 1, 2, \ldots, n_i) \) each having an associated probability (weight) of the variable being within that interval (i.e. \( P(x_i \in I_{ij}) = p_{ij} \) and \( \sum_{j=1}^{n_i} p_{ij} = 1 \)). The midpoint of each interval, \( x_{ij} \), is usually selected as the "typical" value within the interval. Then all sets of the \( x_i \) variables will have an associated probability of their joint occurrence, (e.g., \( P(x_1 = x_{13}, x_2 = x_{22}, x_3 = x_{36}, x_4 = x_{44}) = p_{13} \cdot p_{22} \cdot p_{36} \cdot p_{44} \) assuming the variables are mutually independent). This is shown graphically in Figure 2.2 using hypothetical discrete distributions of the variables \( x_1, x_2, x_3 \) and \( x_4 \).) There will be \( n_1 \cdot n_2 \cdot \ldots \cdot n_m \) total sets possible with the associated probabilities \( p_{1j} \cdot p_{2k} \cdot \ldots \cdot p_{mr} \) for \( j = 1, \ldots, n_1; k = 1, \ldots, n_2; \ldots; r = 1, \ldots, n_m \). Thus, it can be noted that for large \( m \), and/or large \( n_i \)'s, the number of sets will be significant and will possibly exceed the number required for a Monte Carlo study. Reference (d) presents an application of the weighted parameter approach from the selection of initial conditions to the determination of a load distribution.
The two approaches, Monte Carlo and a weighted parameter study, have been presented separately, however, a better approach may be a combination of both. That is, a limited parameter study can be performed first to evaluate the effects of certain of the unknown initial condition variables so that their range of values might possibly be reduced to a very few selected values. For example, the effect of gear mis-servicing might be adequately defined by the use of only a finite number of values for each of the variables strut air pressure, strut oil level and tire pressure. In other
words the continuous distribution functions of these three mis-servicing variables might be approximated with discrete distribution functions if the resulting change in loads is small. The distribution of the distance from hook-deck contact to the engaged cable, \( d \), may similarly be investigated by a parameter study to evaluate the possibility of reducing it from a continuous variable to a discrete variable of a finite number of values. Having the discrete distributions for several of the variables will thus simplify the subsequent sampling. For example, the distribution of \( d \), if uniformly distributed, could be shown as

\[
f(d) = \frac{1}{D}, \quad 0 \leq d \leq D
\]

and with the results of a parameter study may be shown as

\[
P_1 = p_2 = p_3 = p_4 = .25
\]
3. OUTPUT VARIABLES

Having described the landing environment in reference (a), and having defined the input variables in Section 1, the output variables necessary for determining the design loads criteria must be defined. The ground loads, loads at the axle, or the gear member loads can be used as the output variables. For the purpose of this study, the ground loads will be used to define the criteria regarding design loads and fatigue. Adequate preliminary ground loads can be obtained using a basic cantilevered strut in the initial loads studies. These loads could be refined with a later analysis using the finalized gear design.

The subsequent evaluation of strength envelopes will involve the member loads as a part of the detail design. However, the envelopes can be expressed as functions of the ground loads and stroke. When expressed in this form, they are readily comparable with the design and fatigue loads. Thus, the vertical, side, and drag loads at the ground will be used as the output variables in the design criteria.

As noted in Section 1, the initial conditions plus several of the deck conditions, have been presented as random variables having well defined statistical properties. Procedures are presented in Section 4, which utilize these
statistical properties of the input variables to determine the corresponding statistical characteristics of the output loads.

4. PROCEDURES FOR EVALUATING THE OUTPUT VARIABLES

As previously discussed, the vertical, side and drag loads at the ground were recommended as the basic output variables to be used for establishing design criteria. The method developed in Phase I, for the derivation of a landing design criterion, considered the entire landing environment. The following criteria will provide a population of loads corresponding to the population of initial conditions. These loads at the ground can be considered as random variables having statistical properties which are dependent upon the statistical characteristics of the landing environment at airplane touchdown. The statistical properties of the output variables can thus be determined and presented in the form of distribution functions. Several procedures are proposed for obtaining design loads, fatigue spectra, and strength envelopes. The last procedure is recommended and is discussed in detail in later sections. All of them are based on samples of values of the output variables obtained as solutions to the equations of motion which correspond to initial conditions derived from the landing environment.
The first procedure develops sample distribution functions of the output variables from which design loads, a fatigue spectrum, and strength envelopes can be evaluated. In this procedure, Monte Carlo techniques are used to obtain a sample of output loads from which the distribution functions can be derived. The output will be in the form of load time histories from which the distributions can be derived. Both the fatigue spectrum and strength boundaries require the stroke that is associated with the loads. The joint, marginal, and conditional distribution functions are defined in terms of the time to exceed which is available in the time history solutions of the mathematical model. The distribution functions are derived with the following notation and equations.

**Notation**

- \( z, y, x \): vertical, side and drag load, respectively
- \( z_0, y_0, x_0 \): particular values of the \( z, y, \) and \( x \) variables
- \( s \): stroke of the landing gear
- \( s_0 \): particular value of stroke
- \( t(x > x_0) \): time that the drag load, \( x \), exceeds a particular value \( x_0 \)
- \( t(x > x_0 \land y > y_0) \): time that \( x \) exceeds \( x_0 \) and \( y \) exceeds \( y_0 \). The dot means "and".
- \( T \): total time considered for the sample of \( n \) landings
- \( P(\ ) \): probability of occurrence of the argument
P(I) conditional probability
(e.g., P(x > x₀ | y > y₀) probability that x exceeds x₀ given that y exceeds y₀)

Given; the time histories of z, y, x and s for each of n landings obtained as solutions of the mathematical model.

Let: P(x > x₀) = \( \frac{t(x > x₀)}{T} \), P(y > y₀) = \( \frac{t(y > y₀)}{T} \), P(z > z₀) = \( \frac{t(z > z₀)}{T} \)

\( P(x > x₀ \land y > y₀) = \frac{t(x > x₀ \land y > y₀)}{T} \)  \hspace{1cm} (2)

\( P(y > y₀ | x > x₀) = \frac{P(y > y₀ \land x > x₀)}{P(x > x₀)} \)

\( = \frac{t(y > y₀ \land x > x₀)}{t(x > x₀)} \)  \hspace{1cm} (3)

\( P(z > z₀ | x > x₀ \land y > y₀) = \frac{P(z > z₀ \land x > x₀ \land y > y₀)}{P(x > x₀ \land y > y₀)} \)

\( = \frac{t(z > z₀ \land x > x₀ \land y > y₀)}{t(x > x₀ \land y > y₀)} \)  \hspace{1cm} (4)

Similarly,

\( P(z > z₀ | x > x₀ \land y > y₀ \land s > s₀) = \frac{t(z > z₀ \land x > x₀ \land y > y₀ \land s > s₀)}{t(x > x₀ \land y > y₀ \land s > s₀)} \)  \hspace{1cm} (5)

Therefore, utilizing equations (1) through (5), or comparable forms, the joint distribution function of the three loads and the stroke can be evaluated.

\( P(z > z₀ \land y > y₀ \land x > x₀ \land s > s₀) = P(z > z₀) P(y > y₀ | z > z₀) \)  \hspace{1cm} (6)
These equations define the marginal, conditional, and joint distributions of the combinations of loads and stroke that are of interest. For example, Figures 4.1 and 4.2 and Table 4.1 present sample curves and a resulting table of values based on a landing of a Model F8 airplane. The time histories in Figure 4.1 are actually member loads but for illustrative purposes are presented here.

As noted in equation (1), the application of the Monte Carlo technique can provide the marginal distributions of the output variables. To define a sample size required for the analysis is difficult to do when no previous experience or data are available for estimating the variation in the output variable. However, a sequential approach is felt to be realistic for engineering applications. That is, the Monte Carlo sample of input values will be generated in blocks of $n'$ where $n'$ is small compared to the anticipated total sample size $n$ (e.g., $n' = 50$ if $n = 500$ is anticipated). The cumulative distribution function of each output variable is determined for the first sample of $n'$, the second sample of $2n'$ and so on until the last sample of $n'$ has been added to the previous sample of $n-n'$. It is anticipated that the sequence of distribution functions will approach a limit which can be
\[
P(x > x_0 \mid z > z_0 \cdot y > y_0) \cdot P(s > s_0 \mid z > z_0 \cdot y > y_0 \cdot x > x_0) \\
= \frac{t(z > z_0 \cdot y > y_0 \cdot x > x_0 \cdot s > s_0)}{T}
\]

These equations define the marginal, conditional, and joint distributions of the combinations of loads and stroke that are of interest. For example, Figures 4.1 and 4.2 and Table 4.1 present sample curves and a resulting table of values based on a landing of a Model F8 airplane. The time histories in Figure 4.1 are actually member loads but for illustrative purposes are presented here.

As noted in equation (1), the application of the Monte Carlo technique can provide the marginal distributions of the output variables. To define a sample size required for the analysis is difficult to do when no previous experience or data are available for estimating the variation in the output variable. However, a sequential approach is felt to be realistic for engineering applications. That is, the Monte Carlo sample of input values will be generated in blocks of \( n' \) where \( n' \) is small compared to the anticipated total sample size \( n \) (e.g., \( n' = 50 \) if \( n = 500 \) is anticipated). The cumulative distribution function of each output variable is determined for the first sample of \( n' \), the second sample of \( 2n' \) and so on until the last sample of \( n' \) has been added to the previous sample of \( n-n' \). It is anticipated that the sequence of distribution functions will approach a limit which can be
defined within a narrow band over the range of each variable. That is, the range of values of the output variable corresponding to a specified probability will be small.

The procedure defined by equations (1) through (7) will however be tedious to perform. It requires the reading of the sets of time histories of a significant number of landing solutions, each of which is time consuming. Thus, not because of its inadequacy, but rather because of the elapsed time required to do the job, this procedure is not recommended for general use.

A second procedure, utilizing a weighted sample (see Section 2), has its primary value in the derivation of a fatigue spectrum. It would assure the adequacy of the range of values of the input variables used in obtaining the combinations of loads used in analysis and testing. This technique requires a subdivision of each of the ranges of the initial condition variables. This presupposes that the range of each variable is finite and that enough divisions will be made. Representing the procedure symbolically we have:

\[ x_{1i} \text{ represents the midpoint of } i\text{th interval of the first variable, } x_{1}; \]
\[ x_{2j} \text{ represents the midpoint of the } j\text{th interval of the second variable, } x_{2}; \]
\[ x_{3k} \text{ represents the midpoint of the } k\text{th interval of the third variable, } x_{3}; \]
$x_r$ represents the midpoint of the $r$th interval of the fourth variable, $x_4$;

and so on for the $m$ variables used as initial conditions.

$p_{1i}$ represents the probability of $x_1$ being within the $i$th interval, or $P(x_1 = x_{1i})$

$p_{2j}$ represents the probability of $x_2$ being within the $j$th interval, or $P(x_2 = x_{2j})$

$p_{3k}$ represents the probability of $x_3$ being within the $k$th interval, or $P(x_3 = x_{3k})$

$p_{4r}$ represents the probability of $x_4$ being within the $r$th interval, or $P(x_4 = x_{4r})$

and so on for the $m$ variables. Let $n_1, n_2, n_3, n_4, \ldots, n_m$ represent the number of intervals into which the ranges of $x_1, x_2, x_3, x_4, \ldots, x_m$, respectively, have been divided.

Thus, any particular combination of values such as $x_{11}, x_{23}, x_{32}, x_{45}$ will occur with probability $p_{1325} = p_{11}p_{23}p_{32}p_{45}$ when the $x_i$ are assumed mutually independent. The load cycle corresponding to this particular set of initial conditions will have also the probability of $p_{1325}$ of occurrence. The sample of $n = n_1 \cdot n_2 \cdot \ldots \cdot n_m$ initial conditions will have then a corresponding set of $(x, y, z, s)$ loads and load cycles. This sample of $n$ load combinations and strokes will then form the spectrum for fatigue analysis and test. For example, let $m = 3, n_1 = 3 = n_2, n_3 = 2$ then the following table (4.2) will define the eighteen combinations of initial conditions and resulting loads and load cycles with the associated probabilities.
Table 4.2

<table>
<thead>
<tr>
<th>Initial Conditions</th>
<th>Ground Loads</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. x_{11} x_{21} x_{31}</td>
<td>(x, y, z, s)_{111}</td>
<td>P_{111}</td>
</tr>
<tr>
<td>2. x_{12} x_{21} x_{32}</td>
<td>(x, y, z, s)_{112}</td>
<td>P_{112}</td>
</tr>
<tr>
<td>3. x_{22} x_{31}</td>
<td>(x, y, z, s)_{121}</td>
<td>P_{121}</td>
</tr>
<tr>
<td>4. x_{22} x_{32}</td>
<td>(x, y, z, s)_{122}</td>
<td>P_{122}</td>
</tr>
<tr>
<td>5. x_{23} x_{31}</td>
<td>(x, y, z, s)_{131}</td>
<td>P_{131}</td>
</tr>
<tr>
<td>6. x_{32}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. x_{12} x_{21} x_{31}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. x_{12} x_{32}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. x_{22} x_{31}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. x_{22} x_{32}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18. x_{13} x_{23} x_{32}</td>
<td>(x, y, z, s)_{332}</td>
<td>P_{332}</td>
</tr>
</tbody>
</table>

Such a spectrum will assure that the entire range of values of the initial conditions will be included. The loading cycles will be defined in Section 7. A disadvantage of this procedure is that to adequately cover the entire range of each initial condition variable, a large number of intervals will be required. That is, the ith interval of \( x_1 \) must be sufficiently small so that the midpoint, \( x_{1i} \), properly represents it. The same is true for the other \( m-1 \) variables. Thus, \( n = n_1 \cdot n_2 \ldots n_m \) will be large especially if the \( n_i \) and/or \( m \) is large. The largeness of \( n \) may require the use of
other procedures due to the elapsed time required and/or computer costs.

A parameter study does have certain advantages however. If in the selection of combinations, they are ordered such that only one variable is allowed to change, the resulting solutions of the mathematical model when plotted would exhibit existing functional relationships. Cross-plotting such loads with the changes due to varying one or more other initial conditions would provide a family of solutions which could be used to estimate the loads for subsequent combinations of initial conditions. From such a family, the effects of delta changes in the initial values could be estimated. Also, the overall effect of one particular variable could be investigated.

This type of prediction would be most difficult when using a Monte Carlo approach. The investigator would have a distribution of loads but no well defined way of going back to check the load effects of any one particular initial condition variable.

The last procedure makes use of extreme value contours to define the design conditions and, Monte Carlo techniques to evaluate loads and load cycles for defining a fatigue spectrum and strength envelopes. Distribution functions of
the output variables are not required in this procedure. It is discussed in detail in subsequent sections and is suggested for use from a time and cost point of view.
SAMPLE MAIN GEAR LOAD
AND STROKE TIME HISTORIES
(ONE LANDING)

\begin{align*}
\text{Total Time (T)} & = 0.18 \text{ sec.} \\
\end{align*}

\begin{align*}
\text{y} & \times 10^{-3} \\
\text{x} & \times 10^{-3} \\
\text{z} & \times 10^{-3} \\
\end{align*}

\begin{align*}
\text{lb.} & = 20 \\
\text{in.} & = 180 \\
\text{sec.} & = 0.20 \\
\end{align*}
Table 4.1

DISCRETE VALUES OF JOINT DISTRIBUTION FUNCTIONS OF MG LOADS
(i.e.) $P(x\geq x_o, y\geq y_o, z\geq z_o, s\geq s_o)$
for various combinations of $(x_o, y_o, z_o, 0)$

<table>
<thead>
<tr>
<th>$x$ Load</th>
<th>$y$ Load ($10^{-3}$ lbs.) $&gt; 10$</th>
<th>$y$ Load ($10^{-3}$ lbs.) $&gt; 20$</th>
<th>$y$ Load ($10^{-3}$ lbs.) $&gt; 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>$y &gt; 0$</td>
<td>$y &gt; 40$</td>
<td>$y &gt; 60$</td>
</tr>
<tr>
<td>$&gt; 0$</td>
<td>1.00</td>
<td>.89</td>
<td>.67</td>
</tr>
<tr>
<td>$&gt; 60$</td>
<td>.95</td>
<td>.89</td>
<td>.67</td>
</tr>
<tr>
<td>$&gt; 100$</td>
<td>.78</td>
<td>.78</td>
<td>.67</td>
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<tr>
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<td>.67</td>
<td>.67</td>
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<td>$&gt; 160$</td>
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<td>$&gt; 180$</td>
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<tr>
<td>$&gt; 190$</td>
<td>.11</td>
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<tr>
<td>$&gt; 200$</td>
<td>.00</td>
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</table>
MARGINAL DISTRIBUTION FUNCTIONS
x, y, and z Loads
(Based on Table 4.1)
5. ANALYTICAL CONSIDERATIONS FOR THE MATHEMATICAL MODEL

A previous section presented a discussion of procedures for selecting combinations of initial conditions to be used in an analytical determination of ground loads and subsequent member loads of the airplane's landing gears. It is assumed that the Contractor has a basic mathematical model for his airplane (see reference (b) for the model used with the F-8 airplane and reference (c) for other gear models) with which the ground and member loads can be calculated as functions of the initial variables. The accuracy of the calculated loads, with respect to actual landing loads, will be dependent on the form of this analytical representation.

At the time of initial design the analytical representation can be simplified with the assumption of a cantilevered strut design for the gears since detail design information would not be available for the finalized gear. Subsequent analysis, which would include the actual gear configuration, would provide final loads for the detail design of the gears. Having the final gear configuration and an adequate analytical model, the analysis could provide the loads for any point in the structure.

As presented in reference (d), the effect of the impact, or roll-over, of the wheels on the arresting cables was very pronounced in the peak loads. The resulting probability distri-
bution of gear loads, determined with cable effects included, was significantly different from the distribution of loads evaluated without cable effects. The results in reference (d) were obtained from a weighted parameter study. As a side result of this study, it was noted that by varying the tire pressure, the change in loads was significant and for the F-8, the tire pressure requirement was changed to reduce the load increments in the gear due to running over a cable. It is recommended that the mathematical models defined for all future designs include the equations necessary to evaluate the effects of running over, or impacting on, the arresting cables on the carrier deck. The initial condition variables, defined in paragraph 10.12 of reference (a) include those variables necessary for calculating the cable effects. Of primary importance is the distance from hook-deck contact to the cable engaged by the arresting hook. A second problem related to the arresting cables is the proper assessment of the effects due to touching down on the deck prior to the first cable and then engaging one of the cables during the subsequent roll out, rebound or free flight.

Consider the variable $X$, the distance from the ramp to first wheel touchdown, and its distribution over the length of the landing area for a particular model airplane. The density function, $f(X)$, is assumed normal which is consistent with the data available from references (e) and (f), and is summarized in the Phase I Final report. The variable $X$ is assumed
independent of sinking speed and engaging speed. The probability of landing within the cables is equal to

\[ P(X_1 \leq X \leq X_2) \]

where \( X_1 \) and \( X_2 \) are defined in Figure 5.1.

**Figure 5.1**

**DENSITY FUNCTION OF DISTANCE FROM RAMP TO FIRST WHEEL TOUCHDOWN**

\[
f(x) = \frac{1}{2\pi c_x} e^{-\frac{(x-m_x)^2}{2c_x^2}}
\]

1 "within" is defined as the distance between the touchdown point, \( X_1 \), which is prior to the first cable and from which the gear will roll-over the first and subsequent cables under load; and the last cable located at a distance \( X_2 \) from the ramp.
The probability of landing prior to $X_1$, where the airplane can rebound and possibly be airborne at time of cable engagement, is equal to $P(X < X_1)$. The probability of landing on cable-free deck is equal to $P(X > X_2)$. Due to the differences in airplane response characteristics from model to model and the variation in the other initial variables, the hook-deck contact point would be difficult to determine. Thus, the joint probability of landing within the cable area of the deck and also having a particular distance from hook-deck contact to the engaged cable will be calculated using the marginal and conditional probabilities (i.e.) $P(X_1 \leq X \leq X_2, d_0 - \frac{\Delta d}{2} \leq d \leq d_0 + \frac{\Delta d}{2}) = P(X_1 \leq X \leq X_2)$

$P(d_0 - \frac{\Delta d}{2} \leq d \leq d_0 + \frac{\Delta d}{2} | X_1 \leq X \leq X_2)$ where $d$ is the distance from hook-deck contact to engaged cable and $d_0$ is any specified value of $d$. The marginal probability, $P(X_1 \leq X \leq X_2)$, will be evaluated from $f(X)$ and the conditional probability, $P(d_0 - \frac{\Delta d}{2} \leq d \leq d_0 + \frac{\Delta d}{2} | X_1 \leq X \leq X_2)$ will be evaluated using a uniform distribution of $d$ as defined in paragraph 10.12 of reference (a) and a preselected incremental value for $\Delta d$. However, for purposes of this study, the value of $d_0$ will be generated as a random number in the range defined by the between-cable distances, and the probabilities $P(X < X_1)$, $P(X_1 \leq X \leq X_2)$ and $P(X > X_2)$ will be fixed. Thus, if $X_1 \leq X \leq X_2$, then $d_0$ will be generated for use in the analysis.
In order that the distribution of $X$ be applicable to all aircraft carriers, a nondimensional analog will be used. Let $X'$ be the expected touchdown point and consider the variable $Y = \frac{X}{X'}$ which nondimensionalizes the variable $X$ and expresses $X$ as a proportion of the expected touchdown point $X'$. The data available from references (e) and (f) yield a mean value of $Y$ equal to 0.84 and the standard deviation, $\sigma_Y = 0.195$. Since the distribution of $X$ is assumed normal, the corresponding distribution of $Y$ is likewise normal. The value of $X'$ is a function of the relative glide path determined by the mirror, or optical landing aid, and of the airplane. Hence, for a particular carrier, mirror, and airplane, $X'$ can be determined and subsequently the distribution of $X$ can be evaluated. It is assumed that the cables are a uniform distance apart, while it is realized that the distance between cables is not always uniform for a specific carrier and certainly not uniform between carriers. However, an average distance can be used without loss of accuracy.

The arresting force, or hook load, is required when evaluating the nose gear loads and hence should be included in any analysis. The determination of the main gear loads is less sensitive to the input of the arresting force since the maximum loads have been experienced prior to the initiation
of this force.

A recently completed study, reported in reference (b), has shown the importance of several other considerations in the load analysis of a landing gear. As a result of this study, the effects of structural damping, tire coefficient of friction, tire spring rates, variations of the strut orifice coefficient, and strut efficiency were shown to be significant with regard to the landing gear loads. Of these variables, the relation of the coefficient of friction, \( \mu \), to the slip ratio*, \( r \), is of particular importance with regard to its inclusion in a design study. In the past, constant values of \( \mu \) have been required for use in analyses. However, to properly evaluate the landing gear loads, the coefficient of friction should be considered as a function of the slip ratio and within an envelope over the range of values for slip ratio of 0 to 1. As a first cut at obtaining design loads, constant values of \( \mu \) may be used but, for final loads, consideration should be given to the envelope of values of \( (r, \mu) \). The values of \( (r_1, \mu_1) \) and \( (r_2, \mu_2) \) together with

* slip ratio is defined as \( 1 - \frac{V_{\text{wheel}}}{V_E} \) where \( V_{\text{wheel}} \) is wheel tangential velocity and \( V_E \) wheel axle horizontal velocity.

\( V_{\text{axle}} - V_{\text{wheel}} = V_{\text{skid}} \)
(0, 0) and (1, 0) will define the envelope. Straight line segments will be assumed to connect these points. (See Figure 5.2)

An additional parameter is recommended for consideration in the evaluation of landing gear loads; namely, the impact due to running over a guide light or shuttle in the landing area. While there are only two shuttles to be considered, there are approximately forty guide lights in the roll out area, hence the likelihood of hitting a light with one of the gears is quite high. It may well be that the delta load to the gear, due to rolling over a light is not significant but, it should be included as a part of the parameter study for a new gear design. The delta load due to contacting a catapult shuttle could be significant especially if the gear is already heavily loaded. Of the two shuttles in the
canted deck landing area of the larger carriers, only one is sufficiently close to the center line to be considered in the normal landing area. This shuttle, within the landing area, is past the arresting cables and possibly can be contacted by the main gear during roll out. Thus, the loads due to contacting the shuttle should be investigated as a part of any analysis.

6. DESIGN LOADS

The ground loads, that are to be used for design, are functions of a set of initial conditions whose likelihood of occurrence is equal to a pre-assigned value. Extreme landing conditions will be defined in accordance with Section 11 of reference (a). The sets of these conditions are the input to the analysis from which the ground loads are obtained. These output loads are defined as the design loads for the new configuration.

The extreme landing conditions defined in reference (a) require the definition of a probability, \( p_0 \), which is associated with the extreme value of one of the initial condition variables. It is proposed that this probability, \( p_0 \), be defined in accordance with MIL-A-8866 (ASG). That is, for a specified airplane type, \( p_0 \) will equal the relative frequency of one occurrence in twice the number of carrier landings required for this airplane type. MIL-A-8866 (ASG) requires twenty percent of the total landings be carrier landings and that a factor of two, or more, be used in any subsequent use of the data for fatigue. Thus, \( p_0 = \frac{1}{2 x 0.2 x T} \) where \( T \) is the total number of landings.
As an example, Figure 6.1 presents a series of extreme value contours which provide the input for the analytical determination of landing loads. That is, the coordinates of any point on the contour are values of the initial conditions for which the joint probability of exceedance is a fixed value. The method of evaluation of the points, which define the contour, is presented in Table 12.2.1 and Figures 12.2.1 through 12.2.4 of reference (a) for a similar example. A graphic presentation can be used for four variables and tabular form for more than four variables. For example, consider the circular symbols in Figure 6.1; the first has coordinates \( x_1 = x_{11}, x_2 = x_{21}, x_3 = x_{33} \) for specified \( x_4, x_5 \) and \( x_6 \); the second point has coordinates \( x_1 = x_{12}, x_2 = x_{22}, x_3 = x_{32} \) and the same values of \( x_4, x_5 \) and \( x_6 \). These sets of coordinates would then provide two of the input conditions for the subsequent evaluation of ground loads. Thus, each of the load cycles, \((z, y, x)\), which are available from this analysis would have the same associated probability of occurrence; namely, the value of \( P \) used in determining the extreme values.

The first four coordinates used in Figure 6.1 are the initial conditions which are likely to cause significant changes in the output loads for small changes in the variable. The other input conditions can very likely be
considered as discrete variables as was indicated in Section
2. This being true, the contours in Figure 6.1 could for
example be representative of a specific set of values for
the distance to cable engaged, \( x_5 \), and a mis-servicing
condition, \( x_6 \).

The number of combinations of initial conditions used in
any analysis to determine the design loads is a function of
the change in load due to a delta change in the initial
conditions. If curves are plotted, similar to those in
Figure 6.1, enough points on the curves should be selected
to properly assess the joint effect of changes in each of
the initial conditions. The loads so determined for design
will be compared with the loads evaluated for the fatigue
spectrum.
DESIGN CONDITIONS

Combinations of Initial Conditions $x_1, \ldots, x_6$

With Joint Probability of Occurrence $P$

$P = \mathbb{E}$

$x_1 = x_{43}$

$x_5 = x_{50}$

$x_6 = x_{60}$
7. FATIGUE SPECTRUM

The spectrum of loads and load cycles to be used for a fatigue analysis and test will be obtained from the solutions of the equations of motion which correspond to the statistical sample of initial conditions. This sample of initial conditions is generated in accordance with the procedures presented in paragraph 12 of Phase I. The time histories of the ground loads, corresponding to each initial condition, will be the basis for establishing a spectrum of loadings which will be representative of the expected landing gear life.

Of importance to fatigue, will be the stroke, s, corresponding to the x, y and z loads of the gear. For each set of component load time histories corresponding to the initial condition of a sample landing, five load combinations will be considered. These five load combinations together with the corresponding stroke, will be the basis of a loading cycle which will represent this particular landing in the fatigue spectrum. The five load combinations to be determined are: 

\[(x_{max\, aft}, y_{corr}, z_{corr}, s_{corr})\], 
\[(x_{max\, fwd}, y_{corr}, z_{corr}, s_{corr})\], 
\[(x_{corr}, y_{max\, inboard}, z_{corr}, s_{corr})\], 
\[(x_{corr}, y_{max\, outboard}, z_{corr}, s_{corr})\] and 
\[(x_{corr}, y_{corr}, z_{max}, s_{corr})\]. It is assumed that all of the cycles are initiated at \((0, 0, 0, s_{\text{fully extended}})\) and are completed at
the static loads, \((x_b, y_b, z_b, s_b)\) prior to returning to \((0, 0, 0, 0)\). For each gear configuration, the order of loading can be determined from a review of the load time histories obtained from analysis. An example is presented as Figure 7.1 and indicates one such possible sequence of loadings. These seven points, of loads and corresponding stroke, will thus define a cycle. The totality of these cycles will make up the fatigue spectrum.

This spectrum can best be utilized in fatigue calculations where-in a check on the design for each of the five load combinations at the proper stroke can be made.

For fatigue testing, the cycles defined for the spectrum will provide a test block which may be used for jig or airplane drop tests. The use of a jig may require refinements regarding the gear stroke which can be obtained from the time history solutions of the equations of motion. The jig test would of course have certain advantages in time and cost when compared with the airplane drops, but it would likewise have disadvantages when considering fuselage flexibility.

The number of landings, \(N\), anticipated for the carrier life of the new airplane design will be used to define the total number of cycles required for fatigue life. The statistical sample of \(n\) landings, for which the landing loads have been
evaluated, is representative of the N landings expected during the airplane's carrier life. Therefore, to obtain a fatigue spectrum, the sample of n should be repeated kN/n times where k is the required scatter factor. Further, if the fatigue test is to be representative, the order of loading cycles applied to the gear should be random. With an automatic loader for the test machine, a random sequence of loadings is feasible.
Example of Proposed Fatigue Load Cycle

\[ (x_1, y_1, z_1, s_1) \]
\[ (x_2, y_2, z_2, s_2) \]
\[ (x_3, y_3, z_3, s_3) \]
\[ (x_4, y_4, z_4, s_4) \]
\[ (x_5, y_5, z_5, s_5) \]
\[ (x_6, y_6, z_6, s_6) \]

\( x_1, x_2, x_4, x_5, x_6 \) are x loads corresponding to the maximum y or z load.

\( y_2, y_3, y_5, y_6 \) are y loads corresponding to the maximum x or z loads.

\( z_1, z_2, z_3, z_4, z_6 \) are z loads corresponding to the maximum x or y loads.

\( s_1, s_2, s_3, s_4, s_5, s_6 \) are the strokes corresponding to the x, y and z load combinations.

*Note: The ordering, magnitude, or direction of the load combinations shown are for illustrative purposes only. The proper sequencing can be determined from the analysis for a specific gear.
8. STRENGTH ENVELOPES

The strength envelopes for the landing gear will be evaluated from the stress analysis which was based on the design loads (Section 6). To evaluate the compatibility of these envelopes with the expected usage, the sample of loadings obtained for the fatigue spectrum can be used. That is, the combinations of \((x, y, z, s)\) used in defining the fatigue cycles can be used to check whether or not they fall within the strength envelopes evaluated from the stress analysis. In order to compare the fatigue cycles and strength envelopes, the gear stroke must be considered since both the fatigue spectrum and strength envelope are functions of gear stroke. The strength envelopes are defined in terms of strength contours which are functions of the strength of the various structural parts of the landing gear. The maximum stress felt by each of these component parts is in turn a function of the gear stroke for a particular set of \((x, y, z)\) loads.

For those resultants that fall outside the strength envelope, if any, the probability of the joint occurrence of the corresponding initial conditions can be determined. If this probability is less than that value used in establishing the design conditions then it will no longer be considered.
If the probability of joint occurrence is equal to or greater than the design value then it is a point that must be considered for design and its initial conditions should fall within the envelope of design conditions.

9. CONCLUDING REMARKS AND RECOMMENDATIONS

   a) The input data to a landing loads analysis will include the initial conditions as described in reference (a), and the parameters which describe the deck conditions at the time of airplane touchdown.

   b) The initial conditions can best be selected by use of a combined Monte Carlo - Parameter Study Technique. The parameter study will result in simplified distribution functions for some of the variables to which the subsequent Monte Carlo techniques are applied.

   c) Ground loads (vertical, side and drag) are recommended as the output variables to be used for design and fatigue studies. The response loads at any point in the structure can also be evaluated as a part of the landing analysis.

   d) Several procedures for evaluating design loads and fatigue cycles are discussed. The recommended procedure uses extreme value contours to define initial conditions for design loads, and Monte Carlo techniques to generate initial conditions for evaluating a fatigue spectrum and strength envelopes.
e) The mathematical model used in a landing loads study should carefully consider (1) deck conditions (e.g., cable size, spacing and location; catapult shuttles, and landing lights) and their effect on landing loads, (2) strut and tire characteristics, and (3) the variation of the friction coefficient as a function of the slip ratio.

f) Fatigue cycles are defined in terms of seven discrete load combinations which are ordered with respect to the load and stroke time histories.

g) Strength envelopes, evaluated from the stress analysis, will be adjusted in order to be compatible with the load combinations determined from the Monte Carlo sample of initial conditions.