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INTERIM REPORT

A. PURPOSE

To provide basic data for the evaluation of radiometer systems for both earth-based and extraterrestrial conditions, and to determine the effectiveness of such systems when used for surveillance missions.

B. DETAIL FACTUAL DATA

1. Progress - Instrumentation of Sphere Experiment

Several apparent temperature measurements have been obtained from a 10-inch sphere at various physical temperatures using the 3-cm radiometer. In addition to the sphere measurements, a few apparent temperature measurements have been obtained of more complex objects such as aircraft models, missile-like shapes, etc. The results of these measurements will be reported at a later date.

Some difficulty has been experienced with the 4-mm radiometer in obtaining a sufficient \( \Delta T \). However, this problem is being corrected, and the apparent temperature measurements conducted at 3-cm will also be performed at 4 mm.

2. Theoretical Studies and the Interpretation of Sphere Measurements

In order to interpret the results of the measurement program discussed above, and to predict the apparent temperature of any object in an arbitrary environment, it is convenient to have a theoretical expression for apparent temperature. A relatively simple formula has been derived for this purpose, although both the derivation of the formula and the application to specific cases are quite complicated.

Details of the derivation may be found in a separate report \(^1\), but the main results will be presented in a descriptive manner in the following paragraphs.

Consider a target (see Fig. 1) which is illuminated by thermal radiation with angular temperature distribution \( T(\theta) \) and which lies within the main beam of the radiometer antenna. If the antenna
and the target are both irradiated by the same thermal radiation field $T(\theta)$, then the change in the apparent temperature of the radiation coming from the target is given by

$$
\Delta T_R = \frac{e T_B}{R^2} + \frac{(\sigma T)}{R^2} - \frac{\sigma T T(\theta_0)}{R^2}
$$

where

- $R =$ distance between target and antenna
- $e =$ emission cross section of target (meters$^2$)
- $\sigma =$ total or extinction cross section of target (meters$^2$)
- $T =$ temperature of target ($^\circ$K)
- $T(\theta_0) =$ temperature of radiation ($^\circ$K) coming from $\theta_0$ (target direction) in absence of target
- $(\sigma T) =$ average value of bistatic scattering cross section of target weighted according to temperature of incoming radiation

$$
\equiv \int \sigma(\theta, \phi) T(\theta) \frac{d\Omega}{4\pi}
$$

$\sigma(\theta, \phi) =$ bistatic scattering cross section of target for plane waves coming from direction $\theta$, and scattered towards antenna.
A precise definition of the above terms is given in Reference 1. However, the three terms of Eq. (1) each have a simple physical interpretation. The first term represents the effect of the thermal radiation emitted by the target due to its own temperature and emissivity. It should be noted that the emission cross section $e$ is, by Kirchhoff's law, equal to the absorption cross section of the target, which can be determined by electromagnetic scattering experiments. The second term of Eq. (1) represents the effect of the thermal radiation incident on the target and scattered by it into the antenna. The third term represents the effect of radiation which was originally incident on the antenna (with target absent), but which is blocked out by the presence of the target. This is the term responsible for the phenomenon that the presence of a target can actually lower the apparent antenna temperature.

In order to determine how the change in thermal radiation temperature affects the "antenna temperature" $T_A$ (which is what is measured in a radiometer system), it is necessary to introduce the antenna power pattern $F(\theta)$. Then, in general, the antenna temperature is given by

$$T_A = \frac{\int F(\theta) T(\theta) d\Omega}{\int F(\theta) d\Omega}$$

However, for the purposes of this report it is convenient to introduce the idealized antenna pattern shown in Fig. 2.

This pattern has a main beam of $\Omega_o$ steradians, and a uniform side lobe level $10 \log(s) \text{ db}$ below the main beam. Thus since the fraction ($f$) of energy in the main beam is $\Omega_o$

$$f = \frac{\Omega_o}{\Omega_o + s (4\pi - \Omega_o)}$$

the antenna temperature $T_A$ for this pattern can be written, approximately, as

$$T_A \approx f T(\theta_o) + (1-f) \overline{T}$$
where
\[ T(\theta) = \text{radiation temperature along main beam direction} \]
\[ \overline{T} = \frac{1}{4\pi} \int T(\theta) \frac{d\Omega}{4\pi} \text{ = average temperature over all directions of radiation incident on the antenna.} \]

Equation (4) provides a convenient means for "calibrating" a radiometer experiment, since \( T(\theta) \), the temperature of the radiation incident on an antenna near the ground, is given approximately by the sky temperature in the upper hemisphere:

\[ T(\theta) = T_{\text{air}} \left(1 - r \cos \theta\right) \text{ for } \theta < \pi/2, \]

where \( r \) is the one-way fractional transmission coefficient of the atmosphere in the vertical direction; and \( T_{\text{air}} \) is the temperature of the atmosphere. Similarly, in the lower hemisphere, over a rough ground,

\[ T(\theta) \approx T_{\text{ground}} \text{ for } \pi \theta > \pi/2. \]
If $T_{\text{air}} = T_{\text{ground}} = 290^\circ K$, one can use Eqs. (4), (5), and (6) to calculate the apparent temperature of an antenna as a function of elevation angle for various values of $f$. Fig. 3, $T_A$ (target absent) is plotted against $\theta_0$, the angle between the antenna pointing direction and the vertical. Two values of $r$ ($r = 0.45$ and $r = 0.5$) have been used for the 4-mm case because of the dependence of $r$ on meteorological conditions.

When the target is introduced into the antenna beam, the antenna temperature changes by an amount

$$\Delta T_A = \frac{f}{\Omega_o} \Delta T_R$$

where $\Delta T_R$ is given by Eq. (1). Now Eq. (7) is in a form to provide a basis for interpreting radiometer measurements of isolated targets.

To apply Eq. (7) to the experimental conditions discussed in Section 1, one may introduce a further simplification for large spherical targets.

If it can be assumed that the sphere has radius $a$ and power reflection coefficient $\rho$, then

$$\sigma_T = \pi a$$
$$\sigma(\theta, \phi) = \rho \pi a^2$$
$$e = (1 - \rho) \pi a^2.$$ 

Thus if $\alpha = \frac{\pi a^2}{\Omega_o R^2} = \text{fraction of antenna beam occupied by target}$, then

$$\Delta T_a = \alpha f \left[ (1 - \rho) T_B + \rho \overline{T} - T(\theta_0) \right]$$

where $\overline{T}$ and $T(\theta_0)$ may be found from Eqs. (4), (5) and (6). Two cases of interest have been worked out, and the results are shown in Fig. 4. In both cases the fraction $f$ is taken as $f = 0.7$, and the target is assumed to occupy 14% of the antenna beam, i.e., $\alpha = 0.14$. In the first case the reflection coefficient of the
sphere is taken to be \( p = 0.9 \) ("bright" sphere or good reflector) and the sphere is assumed to be raised to a temperature of \( T_B = 600^\circ \text{K} \). In the second case \( p = 0.1 \), i.e., the sphere is "dark", or an excellent absorber, and it is assumed to be at ambient temperature \( T_B = 290^\circ \text{K} \). It can be seen from this figure that, in both cases, the targets should be detectable with the radiometers described in Section I.

C. PROGRAM FOR NEXT INTERVAL

During the next interval an attempt will be made to correlate the theoretical predictions outlined in this report with the apparent temperature measurements of spheres and other targets. Calculations will also be made to extrapolate these results to extraterrestrial conditions, and to targets of other configurations.

D. BIBLIOGRAPHY


Fig. 3. Antenna temperature as a function of zenith angle for various fractions of power in main beam.
Fig. 4. Change in antenna temperature caused by object in beam.
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Executive Director

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