(5) 234 300

Office of Naval Research

Contract Nonr-1866 (32)) NR-371-016

RADIATION FROM AN ELECTROMAGNETIC SOURCE IN A HALF-SPACE OF COMPRESSIBLE PLASMA – SURFACE WAVES

scale 1

By

S. R. Seshadri

March 5, 1963

Technical Report No. 396

Cruft Laboratory
Harvard University
Cambridge, Massachusetts

$2.40

(4) 2.40 (5) 234 300 17-9 NA

17-19 NA

20

21 NA

Office of Naval Research

(15) Contract Nonr-1866(32)

(16) PROJ NR 371 016

Technical Report

~~on~~

(6) RADIATION FROM AN ELECTROMAGNETIC SOURCE IN A HALF-SPACE OF COMPRESSIBLE PLASMA—SURFACE WAVES

(10) by

S. R. Seshadri

(11) March 5, 1963 (12) 21p. (13) NA

The research reported in this document was supported by Grant 9721 of the National Science Foundation. Publication was made possible through support extended to Cruft Laboratory, Harvard University, the Navy Department (Office of Naval Research), the Signal Corps of the U. S. Army, and the U. S. Air Force under ONR Contract Nonr-1866(32). Reproduction in whole or in part is permitted for any purpose of the United States Government.

(14) Technical Report No. 396

Cruft Laboratory

Harvard University

Cambridge, Massachusetts

TR396

RADIATION FROM AN ELECTROMAGNETIC SOURCE IN A HALF-SPACE OF COMPRESSIBLE PLASMA-SURFACE WAVES

by

S. R. Seshadri

Gordon McKay Laboratory

Harvard University

Cambridge, Massachusetts

ABSTRACT

The radiation characteristics of a line source of magnetic current are studied for the case in which the source is situated in a half-space of isotropic, compressible plasma which is bounded on one side by a perfectly conducting, rigid plane screen. In addition to the electromagnetic and plasma space waves, the line source excites a boundary wave. This boundary wave is a coupled wave. It has associated with it both a magnetic field component and the pressure term. This is in contrast to the space waves which can be decomposed into an electromagnetic (EM) mode with no pressure term and a plasma (P) mode with no magnetic field associated with it. The characteristics of this boundary wave are evaluated. The boundary wave propagates for all frequencies and the power carried by the boundary wave becomes smaller as the frequency is increased.

INTRODUCTION

The study of the radiation characteristics of localized electromagnetic sources in ionized gaseous media, known generally as plasmas, has application to the problem of radio communication with missiles and space vehicles passing through the ionized regions in space. In this paper, the radiation characteristics of a line source of magnetic current situated in a compressible plasma medium of semi-infinite extent and bounded on one side by a perfectly conducting, rigid planar screen, is investigated. The special case for which the line source is on the screen becomes equivalent to the problem of radiation into a plasma half-space from an infinitely long slot radiator whose width is very small in comparison to the wavelength. In a previous paper [1], the same problem was investigated for the case of an incompressible plasma in which the longitudinal plasma waves cannot be sustained. In this investigation, the previous analysis is extended to the case of a compressible plasma which is capable of supporting both the transverse electromagnetic waves and the longitudinal plasma waves.

Only recently, the radiation from sources in a compressible plasma has been studied. Hessel and Shmoys [2] have treated the problem of radiation from a point source of electric current in a homogeneous isotropic plasma. Hessel, Marcuvitz and Shmoys [3] have investigated the radiation from a line source of magnetic current in a vacuum in the presence of a plasma half-space. In this paper the radiation characteristics of a line source of magnetic current are investigated for the case in which the source is located in a half-space of isotropic, compressible plasma which is bounded on one side by a perfectly conducting, rigid plane screen. In addition to the EM and P space waves, a boundary wave is found to exist along the screen. This boundary wave is a coupled wave; it has associated with it both a magnetic field and a pressure term. This is in contrast to the space waves which can be decomposed into two distinct, uncoupled modes, namely the transverse electromagnetic (EM) mode and the longitudinal plasma (P) mode. The characteristics of this boundary wave are evaluated.

The space wave parts of the electromagnetic and the plasma modes do not propagate for frequencies less than the plasma frequency. But the boundary wave propagates for all frequencies. The power carried by the boundary wave is decreased as the frequency is increased.

FORMULATION OF THE PROBLEM

Consider a perfectly conducting and rigid, planar screen of infinite extent. Let a right-handed rectangular coordinate system x, y and z be chosen such that the screen occupies the plane $z = 0$. (Fig. 1). The half-space $z > 0$ is filled uniformly with a homogeneous, electron plasma. A line source of magnetic current is located in the plasma at $x = 0$, $z = d$; it is parallel to the y-axis and may be represented as

$$\vec{J}_m = \hat{y}\, J_m = \hat{y}\, J_o\, \delta(x)\, \delta(z-d) \tag{1}$$

It is desired to investigate the radiation characteristics of this line source with particular reference to the boundary waves that propagate along the screen.

Attention is given only to the steady-state problem. The current source is assumed to have the harmonic time dependence of the form $e^{-i\omega t}$. The frequency of the source is assumed to be sufficiently high so that the motion of the ions may be neglected. In addition, the strength of the source is assumed to be so weak that only small amplitude waves are excited, thus justifying the use of a linearized plasma theory [4] . According to the linearized theory, all the field components will have the same harmonic time dependence as that of the source, namely $e^{-i\omega t}$, which may, therefore, be conveniently suppressed. The collisions between electrons and other particles are neglected and the drift velocity of the electrons is assumed to be zero.

Let N_o be the average number density of electrons, p the pressure deviation of the electrons from the mean and $\vec{V}$ the velocity of electrons. Let $\vec{E}$ and $\vec{H}$ be the alternating electric and magnetic fields, respectively. The linearized, time-harmonic hydrodynamic equation of motion of the electrons is

$$-i\omega m\, N_o\, \vec{V} = N_o e\, \vec{E} - \nabla p \tag{2}$$

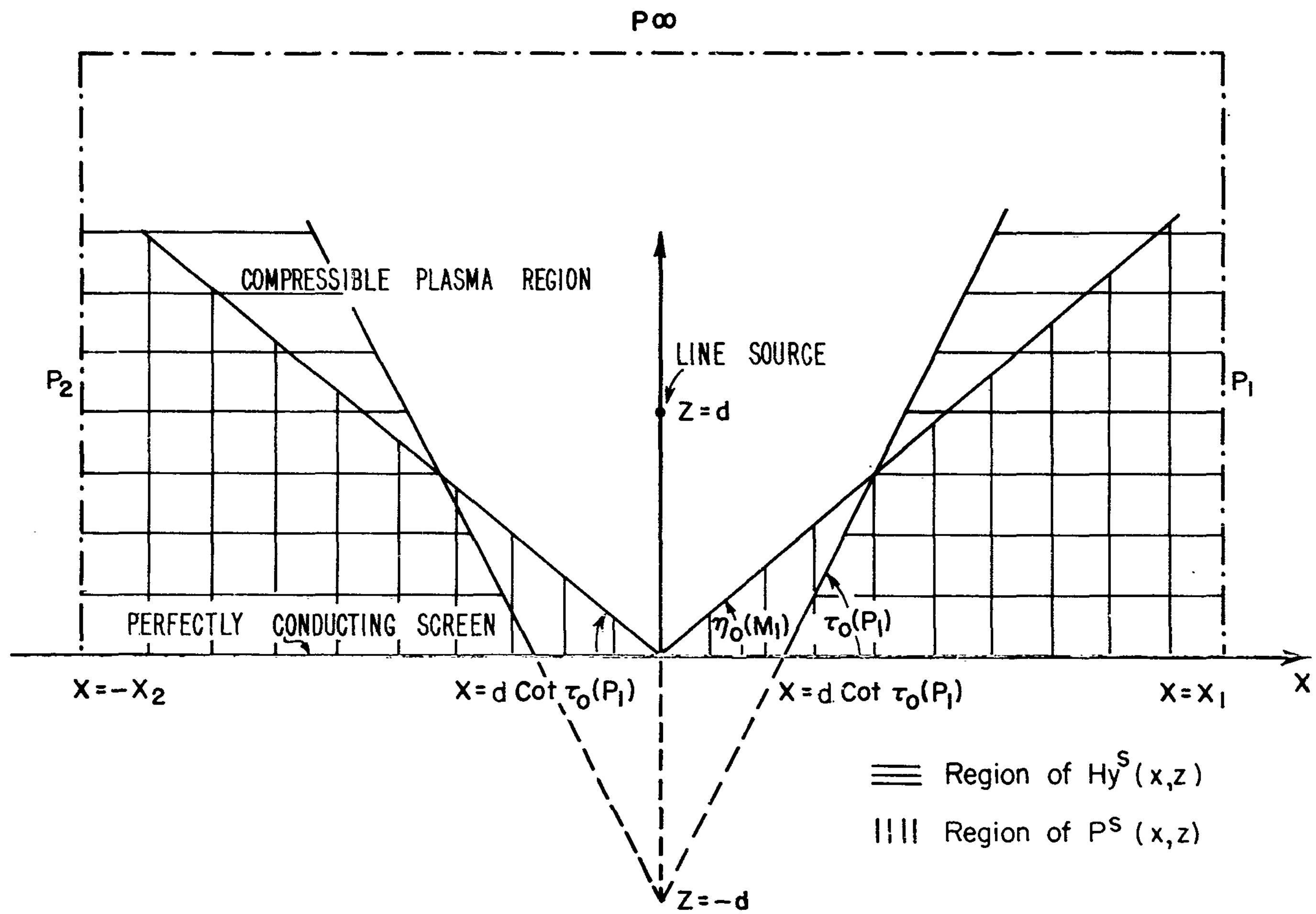

FIG. 1 GEOMETRY OF THE PROBLEM.

where e is the charge and m is the mass of an electron. The linearized equation of continuity together with the equation of state gives

$$a^2 m N_o \nabla \cdot \vec{V} = i\omega p \tag{3}$$

where a is the velocity of sound in the electron gas. In addition, the electric and magnetic fields satisfy, in the half-space $z > 0$, the following time-harmonic Maxwell's equations:

$$\nabla \times \vec{E} = i\omega\mu_o \vec{H} - \vec{J}_m \tag{4}$$

$$\nabla \times \vec{H} = -i\omega\varepsilon_o \vec{E} + N_o e \vec{V} \tag{5}$$

where μ_o and ε_o are respectively the permeability and the dielectric constant of free space.

The source and the geometry of the problem are independent of the y coordinate and, hence, all the field quantities also are independent of y. On substituting $\frac{\partial}{\partial y} = 0$ in (4) and (5), it is found that the electromagnetic field is separable into E and H modes. Since only a line source of magnetic current is present, the H mode is not excited, and, hence, $E_y = H_x = H_z = 0$. From (2), it is seen that $V_y = 0$ also. Only a single component of the magnetic field, namely, H_y is present.

By making use of (2) and (5), E_x, E_z, V_x and V_z are easily expressed in terms of H_y and p as follows:

$$E_x = \frac{1}{i\omega\varepsilon_o\varepsilon_1} \frac{\partial}{\partial z} H_y - \frac{(1-\varepsilon_1)}{N_o e\varepsilon_1} \frac{\partial}{\partial x} p \tag{6}$$

$$E_z = -\frac{1}{i\omega\varepsilon_o\varepsilon_1} \frac{\partial}{\partial x} H_y - \frac{(1-\varepsilon_1)}{N_o e\varepsilon_1} \frac{\partial p}{\partial z} \tag{7}$$

$$V_x = \frac{e}{\omega^2 m\varepsilon_o\varepsilon_1} \left[\frac{\partial}{\partial z} H_y - \frac{i\omega\varepsilon_o}{N_o e} \frac{\partial}{\partial x} p \right] \tag{8}$$

$$V_z = \frac{e}{\omega^2 m\varepsilon_o\varepsilon_1} \left[-\frac{\partial}{\partial x} H_y - \frac{i\omega\varepsilon_o}{N_o e} \frac{\partial}{\partial z} p \right] \tag{9}$$

where
$$\varepsilon_1 = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \tag{10}$$

and the plasma frequency is, $\omega_p = \sqrt{\dfrac{N_o e^2}{m\varepsilon_o}}$. The substitution of (6), (7), and (1) in (4) yields

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_e^2\right] H_y(x, z) = -i\omega\varepsilon_o\varepsilon_1 J_o \delta(x) \delta(z-d) \tag{11}$$

where,

$$k_e^2 = \frac{\omega^2}{c^2}\varepsilon_1 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right) \tag{12}$$

and c is the velocity of electromagnetic waves in free-space. In a similar fashion, the use of (8) and (9) in (3) gives

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_p^2\right] p(x, z) = 0 \tag{13}$$

where

$$k_p^2 = \frac{\omega^2}{a^2}\varepsilon_1 = \frac{\omega^2}{a^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right) \tag{14}$$

It is advantageous to rearrange the field quantities given in (6) - (9) into two distinct groups. The magnetic field component H_y and the first terms in (6) - (9) constitute the first group which is called the electromagnetic (EM) mode. The pressure p and the second terms in (6) - (9) constitute the second group which is called the plasma [P] mode. The EM mode and the P mode are written down separately as follows:

EM mode

$$H_y, \quad \vec{E}_e = \frac{1}{i\omega\varepsilon_o\varepsilon_1}\left[\hat{x}\frac{\partial}{\partial z}H_y - \hat{z}\frac{\partial}{\partial x}H_y\right]$$

$$\vec{V}_e = \frac{e}{\omega^2 m\varepsilon_o\varepsilon_1}\left[\hat{x}\frac{\partial}{\partial z}H_y - \hat{z}\frac{\partial H_y}{\partial x}\right] \tag{15}$$

P mode

$$p \ , \quad \vec{E}_p = - \frac{(1 - \varepsilon_1)}{N_o e \varepsilon_1} \left[\hat{x} \frac{\partial p}{\partial x} + \hat{z} \frac{\partial p}{\partial z} \right]$$

$$\vec{V}_p = \frac{1}{i\omega m N_o \varepsilon_1} \left[\hat{x} \frac{\partial p}{\partial x} + \hat{z} \frac{\partial p}{\partial z} \right] \tag{16}$$

The subscripts e and p denote the EM and the P modes, respectively. For the EM mode, it is seen that $\nabla x \vec{E}_e \neq 0$, $\nabla \cdot \vec{E}_e = 0$ and $\nabla \cdot \vec{V}_e = 0$. Therefore, it is obvious that the EM mode has a magnetic field component associated with it but has no charge accumulation. Moreover, the EM mode has a y component of the magnetic field and this is perpendicular to the xz-plane in which the propagation takes place. Therefore, the EM mode is a transverse mode. For the P mode, it is found that $\nabla x \vec{E}_p = 0$, $\nabla \cdot \vec{E}_p \neq 0$ and $\nabla x \vec{V}_p = 0$. The P mode obviously has charge accumulation associated with it, but no magnetic field. Since it does not contain any y component of the field, it is a longitudinal mode. Using the generalized Poynting vector for a compressible plasma given in [2] , it can be shown that in an unbounded plasma, the EM and the P modes are uncoupled in the sense that the total power can be obtained as the sum of the powers in the individual modes. However, as will be seen later, if a surface wave exists along a plasma boundary, it is a coupled wave .

It is clear from (13) that in an unbounded plasma, the line source (1) does not excite $p(x, z)$. Therefore, it is evident from (16) that the P mode is absent and only the transverse EM mode is excited. However, in a plasma half-space, on account of the presence of the boundary at $z = 0$, the longitudinal P mode is excited. Since the bounding surface $z = 0$, is a perfectly conducting and rigid screen, the following boundary conditions are to be satisfied:

$$E_x(x, 0) = 0 \tag{17}$$

$$V_z(x, 0) = 0 \tag{18}$$

INTEGRAL EXPRESSIONS FOR THE FIELDS

The solution of (11) and (13) subject to the boundary conditions (17) and (18) will yield $H_y(x, z)$ and $p(x, z)$. Once $H_y(x, z)$ and $p(x, z)$ are known, the other field quantities $E_x(x, z)$, $E_z(x, z)$, $V_x(x, z)$ and $V_z(x, z)$ are easily obtained with the help of (6) - (9). Integral expressions for $H_y(x, z)$ and $p(x, z)$ can be easily obtained in the following manner.

The geometry of the problem suggests the following representations for the field components:

$$H_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{H}_y(\zeta, z)\, e^{i\zeta x}\, d\zeta \tag{19a}$$

$$p(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{p}(\zeta, z)\, e^{i\zeta x}\, d\zeta \tag{19b}$$

Integral representations similar to (17) are assumed also for $E_x(x, z)$ and $V_z(x, z)$. Then it follows from (6), (9), (11) and (13) that

$$\bar{E}_x(\zeta, z) = \frac{1}{i\omega\varepsilon_o\varepsilon_1} \frac{\partial}{\partial z} \bar{H}_y(\zeta, z) - \frac{i\zeta(1-\varepsilon_1)}{N_o e \varepsilon_1} \bar{p}(\zeta, z) \tag{20}$$

$$\bar{V}_z(\zeta, z) = \frac{e}{\omega^2 m\varepsilon_o\varepsilon_1} \left[-i\zeta \bar{H}_y(\zeta, z) - \frac{i\omega\varepsilon_o}{N_o e} \frac{\partial}{\partial z} \bar{p}(\zeta, z) \right] \tag{21}$$

$$\left[\frac{d^2}{dz^2} + \xi_e^2 \right] \bar{H}_y(\zeta, z) = - i\omega\varepsilon_o\varepsilon_1 J_o\, \delta(z - d) \tag{22}$$

and

$$\left[\frac{d^2}{dz^2} + \xi_p^2 \right] \bar{p}(\zeta, z) = 0 \tag{23}$$

where

$$\xi_i = + \sqrt{k_i^2 - \zeta^2} \qquad k_i > \zeta$$

$$\xi_i = + i\sqrt{\zeta^2 - k_i^2} \qquad \zeta > k_i \qquad \text{for } i = e, p \tag{24}$$

The solution of (22) gives

$$\bar{H}_y(\zeta, z) = \begin{cases} A e^{i\xi_e z} + B e^{-i\xi_e z} & z > d \\ C e^{i\xi_e z} + D e^{-i\xi_e z} & z < d \end{cases} \tag{25}$$

and

$$\frac{d}{dz} \bar{H}_y(\zeta, d+) - \frac{d}{dz} \bar{H}_y(\zeta, d-) = -i\omega\varepsilon_o\varepsilon_1 J_o \tag{26}$$

Similarly, the solution of (23) gives

$$\bar{p}(\zeta, z) = E e^{i\xi_p z} + F e^{-i\xi_p z} \tag{27}$$

The usual phase radiation condition is found to be applicable for both modes. An examination of the dispersion curves ω-k_e and ω-k_p for the EM and the P modes shows that there are no backward wave regions. Therefore, the radiation condition requires that $H_y(x, z)$ and $p(x, z)$ have outward traveling phase fronts for z tending to infinity. Hence, $B = F = 0$. From the boundary conditions (17) and (18), it is clear that $\bar{E}_x(\zeta, 0) = \bar{V}_z(\zeta, 0) = 0$. It follows, therefore, from (20), (21), (25), (27) that

$$\frac{\xi_e (C-D)}{\omega\varepsilon_o\varepsilon_1} - \frac{i\zeta(1-\varepsilon_1) E}{N_o e \varepsilon_1} = 0 \tag{28}$$

and

$$-i\zeta(C+D) + \frac{\omega\varepsilon_o \xi_p E}{N_o e} = 0 \tag{29}$$

The requirement that the tangential component of the magnetic field should be continuous at z = d gives

$$A e^{i\xi_e d} = C e^{i\xi_e d} + D e^{-i\xi_e d} \tag{30}$$

The use of the jump conditions (26) in (25) leads to

$$A e^{i\xi_e d} - (C e^{i\xi_e d} - D e^{-i\xi_e d}) = - \frac{\omega \varepsilon_o \varepsilon_1 J_o}{\xi_e} \tag{31}$$

The expressions for A, C, D and E may be obtained from the solution of the simultaneous equations (28) - (31) . The results are

$$A = - \frac{\omega \varepsilon_o \varepsilon_1 J_o}{2\xi_e} \left[e^{-i\xi_e d} + \frac{\xi_e \xi_p - (1-\varepsilon_1)\zeta^2}{\xi_e \xi_p + (1-\varepsilon_1)\zeta^2} e^{i\xi_e d} \right]$$

$$B = 0$$

$$C = - \frac{\omega \varepsilon_o \varepsilon_1 J_o [\xi_e \xi_p - (1-\varepsilon_1)\zeta^2]}{2\xi_e [\xi_e \xi_p + (1-\varepsilon_1)\zeta^2]} e^{i\xi_e d}$$

$$D = - \frac{\omega \varepsilon_o \varepsilon_1 J_o}{2\xi_e} e^{i\xi_e d}$$

$$E = - \frac{N_o e \varepsilon_1 J_o i\zeta e^{i\xi_e d}}{[\xi_e \xi_p + (1-\varepsilon_1)\zeta^2]} \tag{32}$$

and

$$F = 0$$

The substitution of (32) in (25) and (27) and the use of (19) yields the following integral expressions for $H_y(x,z)$ and $p(x, z)$:

$$H_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} - \frac{\omega \varepsilon_o \varepsilon_1 J_o}{2\xi_e} \left[e^{-i\xi_e d} + \frac{\xi_e \xi_p - (1-\varepsilon_1)\zeta^2}{\xi_e \xi_p + (1-\varepsilon_1)\zeta^2} e^{i\xi_e d} \right] e^{i\zeta x + i\xi_e z} \, d\zeta$$

$$\text{for } d < z < \infty \tag{33a}$$

$$H_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} - \frac{\omega\varepsilon_o\varepsilon_1 J_o}{2\xi_e} \left[e^{-i\xi_e z} + \frac{\xi_e\xi_p - (1-\varepsilon_1)\zeta^2}{\xi_e\xi_p + (1-\varepsilon_1)\zeta^2} e^{i\xi_e d} \right] e^{i\zeta x + i\xi_e z} d\zeta$$

for $0 < z < d$. (33b)

and

$$p(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} - \frac{N_o e\varepsilon_1 J_o i\zeta}{\left[\xi_p\xi_e + (1-\varepsilon_1)\zeta^2\right]} e^{i\xi_e d + i\zeta x + i\xi_p z} d\zeta \qquad (34)$$

SINGULARITIES OF THE INTEGRAND

The contour for the integrals in (33) and (34) is along the real axis in the ζ-plane as shown in Fig. 2. It is desired to find the singularities of the integrands in (33) and (34). Branch points are seen to occur at $\zeta = \pm k_e$ and $\zeta = \pm k_p$. With the help of (12) and (14), it is obvious that the branch points at $\pm k_e$ and $\pm k_p$ occur on the real or the imaginary axis of the ζ-plane depending on whether ω is greater or less than ω_p.

The poles of the integrand are obviously determined by the roots of the equation

$$\left[(k_e^2 - \zeta^2)(k_p^2 - \zeta^2)\right]^{1/2} + (1-\varepsilon_1)\zeta^2 = 0 \qquad (35)$$

On eliminating the radical in (35), ζ is found to satisfy a biquadratic equation, the solution of which gives the following pair of possible solutions of (35) as

$$\zeta_1^2 = U + \sqrt{U^2 - W} \qquad (36a)$$

and

$$\zeta_2^2 = U - \sqrt{U^2 - W}, \qquad (36b)$$

where

$$U = \frac{k_p^2 + k_e^2}{2\left(1 - \frac{\omega_p^4}{\omega^4}\right)} \qquad W = \frac{k_p^2 \, k_e^2}{1 - \frac{\omega_p^4}{\omega^4}}. \qquad (37)$$

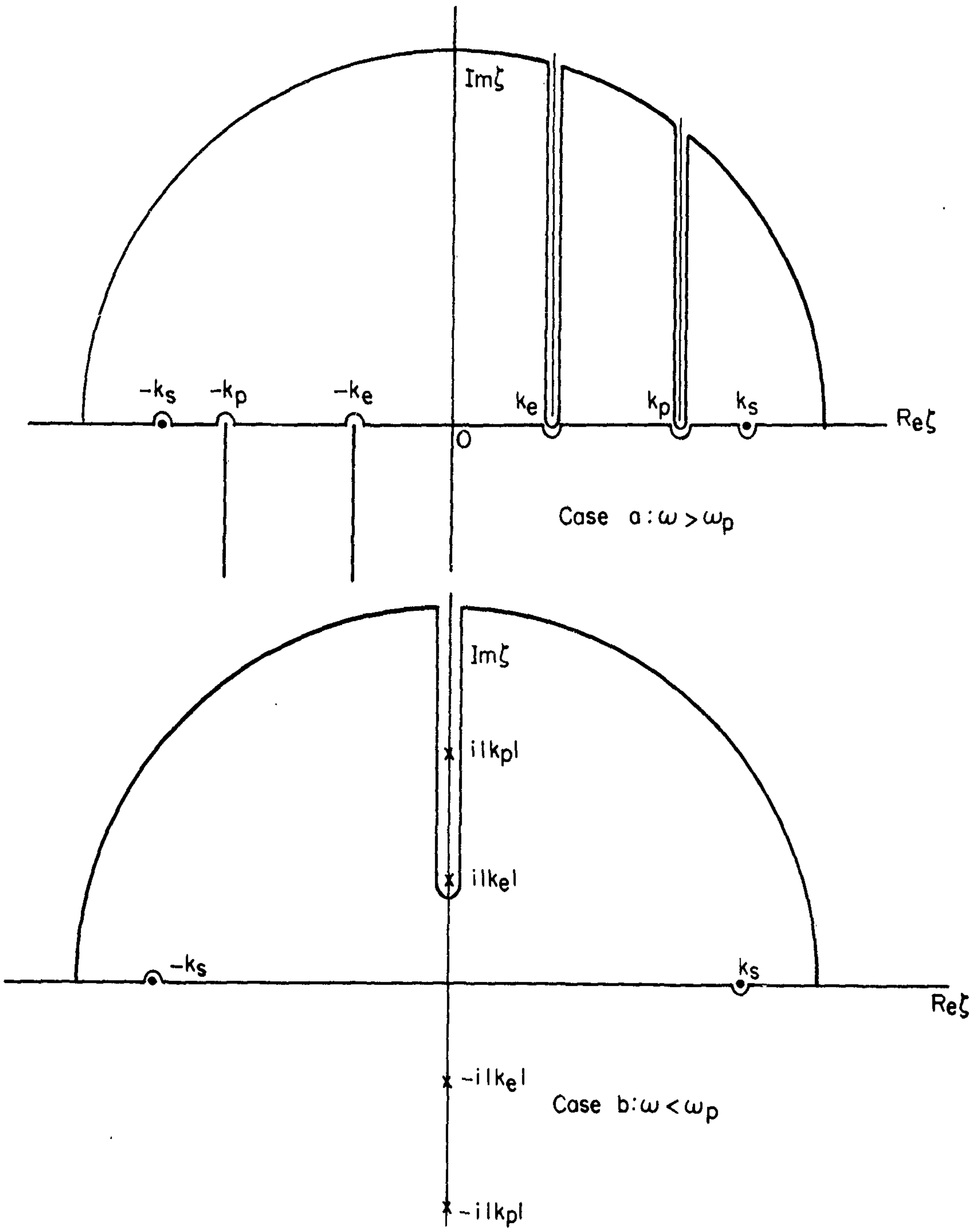

FIG. 2a, b INTEGRATION CONTOUR IN THE ζ PLANE

In obtaining (37), use has been made of (10). Only that root from (36a, b) which also satisfies (35) on the proper Riemann surface, is the proper root of (35). For finding out which one of (36a, b) is a proper root, it is advantageous to simplify (36a, b) by making use of the fact that the ratio of the plasma to the electromagnetic wave velocities $\frac{a}{c}$ is of the order of 10^{-4}. Hence, $\frac{a^2}{c^2}$ is very small in comparison with unity. It is seen from (37) that U^2 is very large compared to W and, hence, the following approximation to (36) may be used,

$$\zeta_{\substack{1\\2}}^2 = U \pm |U| \mp \frac{W}{2|U|} \mp \frac{W^2}{8|U|^3} \tag{38}$$

Case 1: For $\omega > \omega_p$, both k_e^2 and k_p^2 and, hence, U and W are positive and, hence, the two roots given in (38) may be simplified to yield

$$\zeta_1^2 = k_s^2 \doteq 2U \doteq \frac{k_p^2}{1 - \frac{\omega_p^4}{\omega^4}} \tag{39a}$$

and

$$\zeta_2^2 = k_o^2 \doteq \frac{W}{2U} + \frac{W^2}{8U^3} \doteq k_e^2 \left[1 - \frac{k_e^2}{k_p^2} \frac{\omega_p^4}{\omega^4}\right] \tag{39b}$$

The root ζ_1^2 given in (39a) is greater than k_p^2, since $\omega > \omega_p$. For this root, in view of (24) and the fact that $k_p^2 > k_e^2$, the first term in (35) is negative and the second term is positive and they can add to zero. Hence, the root (39a), which is an approximation to the root given by the upper sign in (36) is on the proper Riemann surface. Moreover, this root is real and occurs at $\pm k_s$ in the ζ-plane. With the help of (39b), the root of ζ_1^2 is seen to be slightly less than k_e^2. It follows, therefore, that on the proper Riemann surface defined by (24), the first term in (35) is positive and so is the second term. They cannot add to zero and, hence, the root (39b) is not on the proper Riemann surface. It is to be noted that for $\omega > \omega_p$, $\pm \zeta_2$ is a real pole on the improper Riemann surface.

Case 2: The root ζ_1^2 given in (36a) and approximated by k_s^2 given in (39a) is real and positive even for $\omega < \omega_p$. For $\omega < \omega_p$, k_e^2 and k_p^2 are negative and $\zeta^2 = k_s^2$ is positive, the first term in (35) is a negative real number and the second term is a positive real number and they can add to zero. Hence, the root ζ_1^2 is a proper real root of (35) and gives a pair of real poles in the ζ-plane. For $\omega < \omega_p$, with the help of (12) and (14), it is evident that $U > 0$ and $W < 0$. Since $W < 0$, $\sqrt{U^2 - W} > U$. Hence, the root ζ_2^2 is negative. Moreover,

$$k_e^2 - \zeta_2^2 \doteq k_e^2 \frac{k_e^2}{k_p^2} \frac{\omega_p^4}{\omega^4} < 0 \text{ and } k_p^2 - \zeta_2^2 = k_p^2 \left[1 - \frac{k_e^2}{k_p^2} + \left(\frac{k_e \omega_p}{k_p \omega} \right)^4 \right] < 0 .$$

Hence, the first term in (35) is negative real and the second term is also negative real and, therefore, both the terms in (35) cannot add to zero. Hence, for $\omega < \omega_p$, $\pm\, i|\zeta_2|$ gives a pair of purely imaginary poles on the improper Riemann surface.

Therefore, it is seen that the root (39a) gives rise to a pair of real poles at $\zeta = \pm k_s$ for all values of ω. The root (36b) is on the improper Riemann surface; it is real or imaginary depending on whether ω is greater or less than ω_p.

SURFACE WAVE CONTRIBUTION

The contour for the integrals in (33) and (34) is along the real axis in the ζ-plane (Fig. 2). Since the branch points at $\zeta = \pm k_e$, and $\zeta = \pm k_p$ are on the real axis for $\omega > \omega_p$, the contour of integration has to be indented suitably at these points. As was mentioned earlier, the phase-type radiation condition is applicable to the space-wave parts of the total field. Since the dispersion curves for the space waves for both the EM and P modes do not contain any backward wave regions, the radiation condition is satisfied by requiring the phase fronts of the EM and the P space waves to be traveling outward. Hence, the contour is indented from above at $\zeta = -k_p$ and $-k_e$ and from below at $\zeta = +k_e$ and $+k_p$. For $\omega < \omega_p$, no ambiguity in the integration contours at the branch points occur, since for $\omega < \omega_p$, k_e and k_p are purely imaginary.

In addition to the branch points, the integrands in (33) and (34) have poles at $\zeta = \pm k_s$ on the real axis for all ω. For $\omega > \omega_p$, this pole occurs beyond both branch points. It is obvious that the contour of integration should be indented from above at $\zeta = -k_s$ and from below at $\zeta = k_s$ (Fig. 2a, b) or vice versa. The former type of indentation will be chosen for all ω. This ensures the surface wave to have outward traveling phase fronts. As was shown in an earlier investigation [5], in a compressible plasma, the requirement of outward traveling phase fronts does not necessarily imply that the net power is traveling outward at infinity in spite of the absence of backward regions in the dispersion curve. It remains, therefore, to be verified that the total power is indeed traveling outward at infinity.

For $x > 0$, the integrals (33) and (34) may be evaluated by closing the contour in the upper half of the ζ-plane. The contribution to the integrals (33) and (34) is the sum of the residue at the pole $\zeta = k_s$ and the two branch-cut integrals. The values of the branch-cut integrals depend on some inverse power of x and, hence, for large x are negligible compared to the contribution due to the pole. Thus, for positive large x (33) yields

$$H_y(x, z) = H_y^s(x, z) = H_s \, e^{ik_s x - z\sqrt{k_s^2 - k_e^2}} \tag{40}$$

where

$$H_s = -\frac{\omega\varepsilon_o\varepsilon_1 J_o}{2k_s\sqrt{k_s^2-k_e^2}} \frac{\left[\left\{(k_s^2-k_e^2)(k_s^2-k_p^2)\right\}^{1/2} + \frac{\omega_p^2}{\omega^2}k_s^2\right] e^{-d\sqrt{k_s^2-k_e^2}}}{\left[\left\{\frac{k_s^2-k_e^2}{k_s^2-k_p^2}\right\}^{\frac{1}{2}} + \left\{\frac{k_s^2-k_p^2}{k_s^2-k_e^2}\right\}^{\frac{1}{2}} - 2\frac{\omega_p^2}{\omega^2}\right]} \tag{41}$$

Similarly, the contribution from the pole of (34) gives

$$p(x, z) = p^s(x, z) = P_s \, e^{ik_s x - z\sqrt{k_s^2 - k_p^2}} \tag{42}$$

where

$$P_s = -N_o e \varepsilon_1 J_o e^{-d\sqrt{k_s^2-k_e^2}} \left[\left\{ \frac{k_s^2-k_e^2}{k_s^2-k_p^2} \right\}^{1/2} + \left\{ \frac{k_s^2-k_p^2}{k_s^2-k_e^2} \right\}^{1/2} - \frac{2\omega_p^2}{\omega^2} \right]^{-1} \tag{43}$$

It is evident that $H_y^s(x, z)$ and $p^s(x, z)$ given respectively in (40) and (42) represent surface waves which propagate in the positive x direction and are exponentially attenuated in the z direction. The phase velocity v_s of both $H_y^s(x, z)$ and $p^s(x, z)$ is given by

$$v_s = \frac{\omega}{k_s} \doteq a\sqrt{1+\frac{\omega_p^2}{\omega^2}} \tag{44}$$

The phase velocity is infinite for $\omega = 0$, reduces monotonically as the frequency is increased and, for $\omega = \omega_p$ reaches a value which is equal to $\sqrt{2}$ times the acoustic velocity in the electron gas. As the frequency is still further increased, the phase velocity continues to decrease and asymptotically reaches the value a, the acoustic velocity in the electron gas, in the limit of infinite frequency. A plot of phase velocity v_s versus the frequency ω is shown in Fig. 3.

The attenuation factor α_h for the decay of $H_y(x, z)$ in the z direction is given by

$$\alpha_h = \sqrt{k_s^2-k_e^2} \doteq \frac{\omega}{a}\left(1+\frac{\omega_p^2}{\omega^2}\right)^{-1/2} \tag{45}$$

Similarly, the attenuation factor α_p for $p(x, z)$ is obtained as

$$\alpha_p = \sqrt{k_s^2-k_p^2} \doteq \frac{\omega_p}{a}\left(1+\frac{\omega^2}{\omega_p^2}\right)^{-1/2} \tag{46}$$

It follows, therefore, that $H_y^s(x, z)$ and $p(x, z)$ which propagate with the same phase velocity in the x direction are attenuated differently in the z-direction, as seen from (45) and (46). The attenuation factor α_h is a monotonically increasing function of frequency. It starts with the value of zero for $\omega = 0$, increases to the value $\frac{\omega_p}{a\sqrt{2}}$ for $\omega = \omega_p$ and thereafter continuously increases

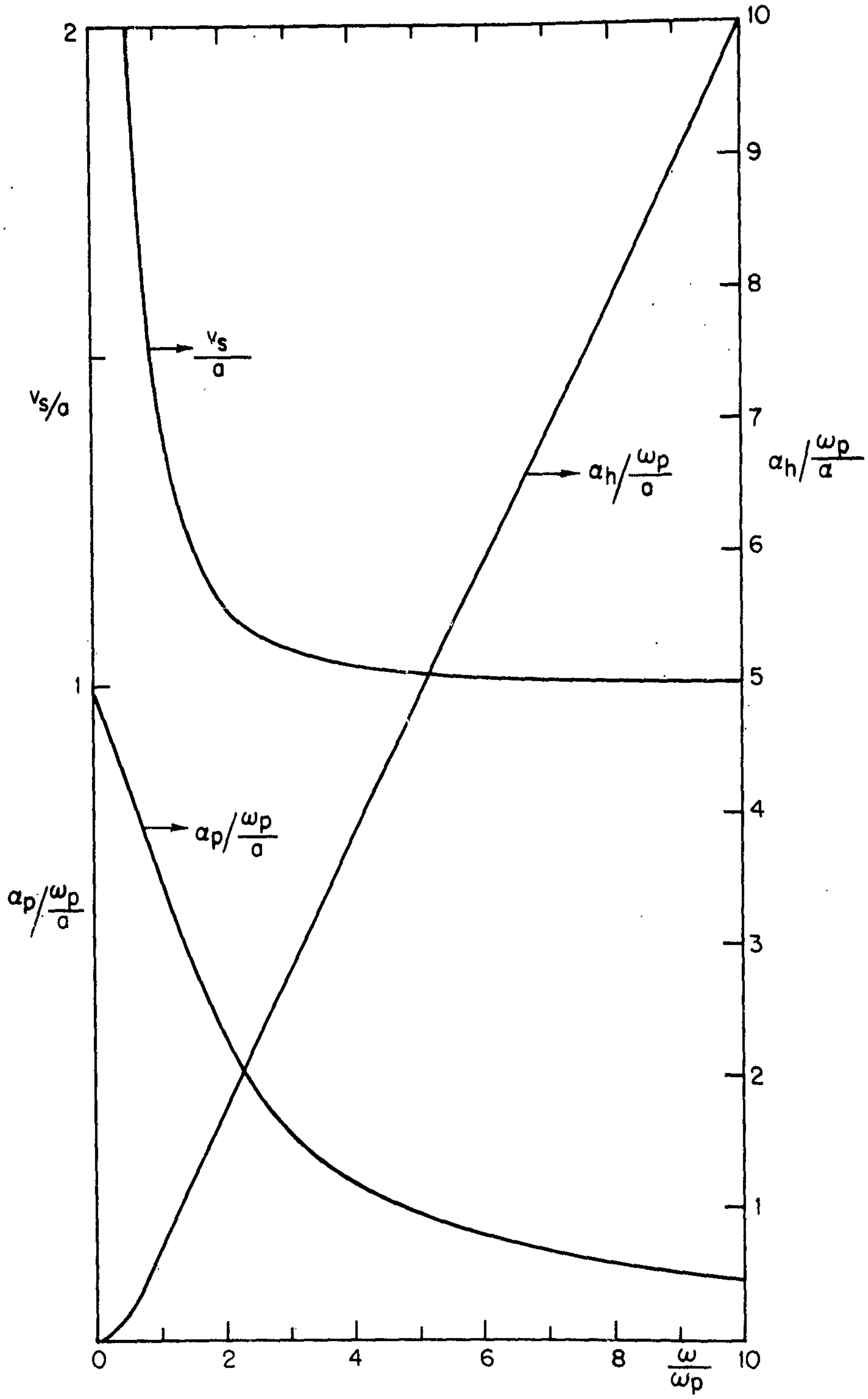

FIG. 3 PHASE VELOCITY AND ATTENUATION CONSTANTS AS A FUNCTION OF FREQUENCY

as ω is increased. However, the attenuation factor α_p for the pressure wave is a monotonically decreasing function of frequency. It starts with the value $\frac{\omega_p}{a}$ for $\omega = 0$, decreases to the value $\frac{\omega_p}{a\sqrt{2}}$ for $\omega = \omega_p$ and thereafter asymptotically approaches zero as ω is increased indefinitely. A plot of the attenuation constants α_h and α_p as a function of ω is shown in Fig. 3. For $\omega = \omega_p$, both the attenuation constants are equal. For $\omega < \omega_p$, $\alpha_p > \alpha_h$ and, therefore, the pressure field is more tightly bound to the guiding surface than the magnetic field. But for $\omega > \omega_p$, $\alpha_p < \alpha_h$ resulting in the pressure field being more loosely bound than the magnetic field.

It is desired to evaluate the total power radiated per unit length of the source in the form of surface waves. It can be shown (see Appendix) that the total power radiated by the line source is obtained as the sum of the powers in the space wave and the boundary wave separately. Hence, it is meaningful to speak of the power radiated in the surface waves alone. It has been proved [3, 5], that the time-averaged, outward normal flow of power through unit area is given by

$$\vec{S} = \frac{1}{2} \operatorname{Re} [\vec{E} \times \vec{H}^* + p\vec{V}^*] \tag{47}$$

The total power transmitted by the surface waves is, therefore, given by

$$W^s = W_+^s + W_-^s \quad , \tag{48}$$

where W_+^s and W_-^s are the net flow of power respectively in the positive and the negative x directions. With the help of (47), W_+^s and W_-^s are easily obtained as

$$W_+^s = \int_0^\infty \hat{x} \cdot \frac{1}{2} \operatorname{Re} [\vec{E}^s \times \vec{H}^{s*} + p^s \vec{V}^{s*}] \, dz$$

$$= \frac{1}{2} \operatorname{Re} \int_0^\infty [- E_z^s(x, z) H_y^{s*}(x, z) + p^s(x, z) V_x^{s*}(x, z)] dz \tag{49a}$$

and

$$W_-^s = -\frac{1}{2}\,\mathrm{Re}\int_0^\infty \left[-E_z^s(x,z)\, H_y^{s*}(x,z) + p^s(x,z)\, V_x^{s*}(x,z)\right] dz\ , \tag{49b}$$

where $H_y^s(x,z)$ and $p^s(x,z)$, for $x > 0$, are given in (40) and (42) respectively. For $x < 0$, $H_y^s(x,z)$ and $p^s(x,z)$ can be obtained easily as

$$H_y^s(x,z) = H_s\, e^{-ik_s x - z\sqrt{k_s^2 - k_e^2}} \tag{50a}$$

and

$$p^s(x,z) = -P_s\, e^{-ik_s x - z\sqrt{k_s^2 - k_e^2}} \tag{50b}$$

For $x > 0$, the expressions for $E_z^s(x,z)$ and $V_x^s(x,z)$ are found by substituting (40) and (42) in (7) and (8). The result is

$$E_z^s(x,z) = -\frac{k_s}{\omega\varepsilon_o\varepsilon_1} H_y^s(x,z) + \frac{(1-\varepsilon_1)a_p}{N_o e\varepsilon_1}\, p^s(x,z) \tag{51}$$

$$V_x^s(x,z) = -\frac{e a_h}{\omega^2 m\varepsilon_o\varepsilon_1} H_y^s(x,z) + \frac{k_s}{\omega\varepsilon_1 N_o m}\, p^s(x,z) \tag{52}$$

When (40), (42), (51) and (52) are used in (49a), it is found that

$$W_+^s = \frac{k_s}{4\omega\varepsilon_o\varepsilon_1 a_h} H_s^2 + \frac{k_s}{4\omega\varepsilon_1 N_o m a_p} P_s^2 - \frac{(1-\varepsilon_1)}{2N_o e\,\varepsilon_1} H_s P_s \tag{53}$$

In a similar manner W_-^s can be calculated and is found to be equal to W_+^s, as expected. Hence, the total power transmitted by the surface waves per unit width of the screen is obtained from (48) and (53) as

$$W^s = 2W_+^s = \frac{k_s}{2\omega\varepsilon_o\varepsilon_1 a_h} H_s^2 + \frac{k_s}{2\omega\varepsilon_1 N_o m a_p} P_s^2 - \frac{(1-\varepsilon_1)}{N_o e\varepsilon_1} H_s P_s \tag{54}$$

The expression (54) for W^s can be considerably simplified when advantage is taken of the fact that $\frac{a^2}{c^2}$ is of the order of 10^{-8} and, therefore, negligible in comparison with unity. In view of this fact, it follows from (12), (14) and (39a) that

$$k_s^2 - k_e^2 \doteq k_s^2\ , \quad k_s^2 - k_p^2 \doteq k_s^2\, \frac{\omega_p^4}{\omega^4} \tag{55}$$

The substitution of (55) in (41) immediately gives

$$H_s = -\omega \varepsilon_o \varepsilon_1 J_o \frac{\omega_p^4}{\omega^4} \left[1 - \frac{\omega_p^4}{\omega^4}\right]^{-1} e^{-k_s d} \tag{56}$$

In a similar manner, the use of (55) in (43) yields

$$P_s = - N_o e J_o \varepsilon_1 \frac{\omega_p^2}{\omega^2} \left[1 - \frac{\omega_p^4}{\omega^4}\right]^{-1} e^{-k_s d} \tag{57}$$

With the help of (45), (46), (55) - (57) in (54), it is possible to show, after considerable simplification, that

$$\widetilde{W}^s = \frac{2W^s}{\omega \varepsilon_o J_o^2} = \frac{e^{-2\frac{\omega d}{a} \frac{\Omega}{\sqrt{\Omega^2 + 1}}}}{\Omega^2 (\Omega^2 + 1)} , \tag{58}$$

where $\Omega = \dfrac{\omega}{\omega_p}$.

It is seen from (58) that $\widetilde{W}^s$ is always positive, showing that the requirement of the boundary wave to have outward traveling phase fronts does lead to the fulfillment of the radiation condition, which requires a net outflow of power at large distances from the source.

It is seen from (58) that the power delivered to the surface wave is a maximum when the line source is on the screen, i.e., when $d = 0$. Also, an increase in the frequency Ω results in a decrease in the power in the surface wave. Obviously, this analysis is not valid for very small Ω , since in that case the motion of the ions cannot be neglected. For $\Omega < 1$, the space waves are exponentially damped and all the power is carried by the surface wave.

It is well known that in an unbounded isotropic plasma the EM and the P modes are uncoupled [2] . The presence of the boundary provides a mechanism of coupling between these modes. Since the boundary surface alone provides the mechanism of coupling of power between the EM and the P modes, it is intuitively obvious that if a surface wave is present, it will be a coupled mode in the sense that the total power in the surface wave is not equal to the sum of the powers in the EM and P modes separately. That this actually turns out be the case is indicated by the presence of the cross term in (54).

ACKNOWLEDGMENTS

The author is indebted to Professor R. W. P. King for his help and encouragement with this research, to the faculty members of the electrophysics department of the Polytechnic Institute of Brooklyn for their many stimulating discussions and to Professor L. B. Felsen for enabling the author to visit the Polytechnic Institute of Brooklyn where a part of this research was carried out.

REFERENCES

1. Seshadri, S. R., "Excitation of Surface Waves on a Perfectly Conducting Screen Covered with Anisotropic Plasma," Technical Report No. 366, Cruft Laboratory, Harvard University, Cambridge, Massachusetts, May 1962.

2. Hessel, A. and Shmoys J., "Excitation of Plasma Waves in a Homogeneous Isotropic Plasma by a Dipole," Memorandum No. 63, PIBMRI-921-61, Polytechnic Institute of Brooklyn, July 1961.

3. Hessel, A., Marcuvitz, N. and Shmoys, J., "Scattering and Guided Waves at an Interface Between Air and Compressible Plasma," IRE Transactions on Antennas and Propagation, Vol. AP-10, pp. 48-54, January, 1962.

4. Oster, L., "Linearized Theory of Plasma Oscillations," Rev. Mod. Phys., Vol. 32, No. 1, pp. 141-168, January 1960.

5. Seshadri, S. R., "Excitation of Plasma Waves in an Unbounded Homogeneous Plasma by a Line Source," Technical Report No. 373, Cruft Laboratory, Harvard University, Cambridge, Massachusetts, July 1962.

APPENDIX

Let $\vec{E}$, $\vec{H}$, $\vec{V}$ and P be the total field quantities. The total field is a solution of (2), (3), (4) and (5). Also let $\vec{E}^s$, $\vec{H}^s$, $\vec{V}^s$ and P^s be the field quantities associated with the boundary wave part of the total field. The boundary wave is a solution of source-free field equations. Let

$$\vec{S}_1 = \vec{E} \times \vec{H}^{s*} + P\vec{V}^{s*} + \vec{E}^{s*} \times \vec{H} + P^{s*}\vec{V} \tag{A-1}$$

Taking the divergence of $\vec{S}_1$ and making use of (2) - (5), it can easily be shown that

$$\nabla \cdot \vec{S}_1 = -\vec{H}^{s*} \cdot \vec{J}_m \tag{A-2}$$

The integration of (A-2) throughout the volume V enclosed by the transverse planes P_1 and P_2, the surface P_∞ at infinity and the surface of the screen (Fig. 1) yields

$$\int_{P_1+P_2} \Big[-E_z(x,z)\,H_y^{s*}(x,z) + P(x,z)\,V_x^{s*}(x,z)$$

$$- E_z^{s*}(x,z)\,H_y(x,z) + P^{s*}(x,z)\,V_x(x,z)\Big]\,dz$$

$$= \int_V H_y^{s*}(x,z)\,J_m\,dv \tag{A-3}$$

The integral over the surface P_∞, which is located at infinity, vanishes since the total field is bounded and the field of the boundary wave decreases exponentially in the z direction. Also, the integral over the surface of the screen vanishes since $\hat{z} \cdot \vec{S}_1 = 0$, on the screen, in view of the boundary conditions (17) and (18).

The total field quantities may be written in the form

$$H_y(x,z) = H_y^R(x,z) + H_y^s(x^+,z), \quad P(x,z) = P^R(x,z) + P^s(x^+,z)$$

$$\vec{E}(x,z) = \vec{E}^R(x,z) + \vec{E}^s(x^+,z), \quad \vec{V}(x,z) = \vec{V}^R(x,z) + \vec{V}^s(x^+,z)$$

$$\text{for } x > 0 \tag{A-4}$$

and

$$H_y(x,z) = H_y^R(x,z) + H_y^s(x^-,z)\,, \quad P(x,z) = P^R(x,z) + P^s(x^-,z)$$

$$\vec{E}(x,z) = \vec{E}^R(x,z) + \vec{E}^s(x^-,z)\,, \quad \vec{V}(x,z) = \vec{V}^R(x,z) + \vec{V}^s(x^-,z)$$

$$\text{for } x < 0 \tag{A-5}$$

where $H_y^R(x,z)$, $P^R(x,z)$, $\vec{E}^R(x,z)$ and $\vec{V}^R(x,z)$ denote the space wave parts of the total field quantities. The field components of the boundary wave traveling in the positive and the negative x directions are denoted respectively by x^+ and x^-.

In view of (40), (42) and (6) - (8), the field components of the boundary wave traveling in the positive x direction may be represented as

$$H_y^s(x^+,z) = H_s(z)\, e^{ik_s x}\,, \quad P^s(x^+,z) = P_s(z)\, e^{ik_s x}$$

$$E_z^s(x^+,z) = E_{zs}(z)\, e^{ik_s x}\,, \quad V_x^s(x^+,z) = V_{xs}(z)\, e^{ik_s x} \tag{A-6}$$

In a similar way, with (50a, b) and (6) - (8), it is evident that the field components of the boundary wave traveling in the negative x direction, are given by

$$H_y^s(x^-,z) = H_s(z)\, e^{-ik_s x} \qquad P^s(x^-,z) = -P_s(z)\, e^{-ik_s x}$$

$$E_z^s(x^-,z) = -E_{zs}(z)\, e^{-ik_s x} \qquad V_x^s(x^-,z) = V_{xs}(z)\, e^{-ik_s x} \tag{A-7}$$

The substitution of (A-5) - (A-7) in (A-3), after some simplification, yields the following

$$\int_0^\infty dz \left[-E_z^R(x_1,z)\, H_s(z) + P^R(x_1,z)\, V_{xs}(z) - E_{zs}(z)\, H_y^R(x_1,z) + P_s(z)\, V_x^R(x_1,z)\right] e^{ik_s x_1}$$

$$+ \int_0^\infty dz \left[-E_z^R(-x_2,z)\, H_s(z) + P^R(-x_2,z)\, V_{xs}(z)\right.$$

$$+ E_{zs}(z)\, H_y^R(-x_2, z) - P_s(z)\, V_x^R(-x_2, z) \Big] \, e^{ikx_2}$$

$$+ 2 \cdot \mathrm{Re} \int_0^\infty dz \left[- E_{zs}(z)\, H_S(z) + P_s(z)\, V_{xs}(z) \right]$$

$$= - \int_V H_y^{s*}(x, z)\, J_m \, dv \tag{A-8}$$

If the position of the transverse plane P_1 is changed, the second and the third terms on the left-hand side as well as the right-hand side of (A-7) are unchanged. Hence, it follows that

$$e^{ik_s x} \int_0^\infty dz \Big[-E_z^R(x, z)\, H_s(z) + P^R(x, z)\, V_{xs}(z)$$

$$- E_{zs}(z)\, H_y^R(x, z) + P_s(z)\, V_x^R(x, z) \Big] = \text{constant} \tag{A-9}$$

In a similar manner, it may be argued that

$$e^{-ik_s x} \int_0^\infty dz \Big[-E_z^R(x, z)\, H_s(z) + P^R(x, z)\, V_{xs}(z)$$

$$+ E_{zs}(z)\, H_y^R(x, z) + P_s(z)\, V_x^R(x, z) \Big] = \text{constant} \tag{A-10}$$

The space wave has a continuous eigenvalue spectrum and so its dependence on x cannot annul the factor $e^{ik_s x}$ in (A-9) and $e^{-ik_s x}$ in (A-10). Furthermore, if the positions of the transverse planes P_1 and P_2 approach respectively $x_1 = \infty$ and $x_2 = \infty$, it is evident that $E_z^R(x, z)$, $P^R(x, z)$, $H_y^R(x, z)$ and $V_x^R(x, z)$ approach zero. Also, since k_s is real, it follows that the constants in (A-9) and (A-10) are zero. Therefore, from (A-9), (A-10), (A-3) and (A-4), it is evident that

$$\int_0^\infty dz \left[-E_z^R(x,z)\; H_y^{s*}(x,z) + P^R(x,z)\; V_x^{s*}(x,z)\right]$$

$$= \int_0^\infty dz \left[-E_z^{s*}(x,z)\; H_y^R(x,z) + P^{s*}(x,z)\; V_x^R(x,z)\right]$$

$$= 0 \tag{A-11}$$

In view of (A-11) , it is obvious that the boundary and the space wave parts of the total field are orthogonal in the sense that the total power flow is equal to the sum of the powers separately in the boundary and the space waves.

DISTRIBUTION LIST

Activity Supply Officer
Building 2506, Charles Wood Area
Fort Monmouth, New Jersey (50)
Attn: Director of Research

Commanding Officer
Office of Naval Research
Navy 100, Box 39 (25)
Fleet Post Office
New York, New York

Armed Services
Technical Information Agency
Arlington Hall Station (10)
Arlington 12, Virginia
Attn: TIPDR

The Director
Naval Research Laboratory
Washington 25, D. C. (6)
Attn: Technical Information Office

Commander, AF CRL
AFRD, ARDC, CRRLC
Laurence G. Hanscom Field (4)
Bedford, Massachusetts
Attn: Electronics Research Directorate

Commanding General
Air Research and Development Command
P. O. Box 1395 (3)
Baltimore 3, Maryland
Attn: RDTRRP

Chief of Naval Research
Department of the Navy (2)
Washington 25, D. C.
Attn: Dr. A. Shostak, Code 427

Chief of Naval Research
Department of the Navy (2)
Washington 25, D. C.
Attn: Code 427

Commanding Officer
Office of Naval Research
495 Summer Street (2)
Boston, Massachusetts

Chief, Bureau of Ships
Department of the Navy (2)
Washington 25, D. C.
Attn: Code 810

Director, Air University
Library (2)
Maxwell Air Force Base
Alabama

Chief of Naval Research
Department of the Navy
Washington 25, D. C.
Attn: Code 421

Commanding Officer
Office of Naval Research
495 Summer Street
Boston, Massachusetts

Commanding Officer
Office of Naval Research
John Crerar Library Building
86 East Randolph Street
Chicago 1, Illinois

Commanding Officer
Office of Naval Research
346 Broadway
New York 13, New York

Commanding Officer
Office of Naval Research
1030 East Green Street
Pasadena, California

Commanding Officer
Office of Naval Research
1000 Geary Street
San Francisco 9, California

Head, Document Section
Technical Information Division
Naval Research Laboratory
Washington 25, D. C.

Martin A. Garstens
Magnetism Branch, Code 6450
Solid State Division
Naval Research Laboratory
Washington 25, D. C.

Commanding Officer
U.S.N. Air Development Center
Johnsville, Pennsylvania
Attn: NADC Library

Commander
U.S. N. Air Development Center
Johnsville, Pennsylvania
Attn: AAEL

Chief, Bureau of Aeronautics
Department of the Navy
Washington 25, D. C.
Attn: EI-1

Engineering Librarian
Convair
San Diego 12, California

Dr. John E. Pippin
Applied Physics and Ferrite Devices
Sperry Microwave Electronics Co.
P. O. Box 1828
Clearwater, Florida

Engineering Library
Sperry Microwave Electronics Co.
Clearwater, Florida

Dr. Lajos Rimai
Research Division
Raytheon Company
Waltham 54, Massachusetts

Elizabeth Weeks, Librarian
Raytheon Company
28 Seyon Street
Waltham 54, Massachusetts

Report Librarian
Sylvania Electric Products Inc.
Electronic Systems Division
100 First Avenue
Waltham, Massachusetts

Document Control Center
Wayland Library
Raytheon Manufacturing Co.
Wayland, Massachusetts

J. E. Goldman
Scientific Laboratory
Ford Motor Company
Engineering Staff
P. O. Box 2053
Dearborn, Michigan

Charles C. H. Tang
Bell Telephone Labs.
Murray Hill, New Jersey

Librarian
RCA Laboratories
Princeton, New Jersey

Dr. A. Amith
RCA
Princeton, New Jersey

Commander (2)
U. S. Naval Electronics Lab.
San Diego, California

Commanding General, RCRW
Rome Air Development Center
Griffiss Air Force Base (2)
Rome, New York

Commanding General
Air Research and Development Command
P. O. Box 1395 (2)
Baltimore, Maryland
Attn: RDDDE

Commander
Air Force Cambridge Research Labs.
Laurence G. Hanscom Field (2)
Bedford, Massachusetts
Attn: CROTLS

Commander
Wright Air Development Center
Wright Patterson Air Force Base
Ohio (2)
Attn: WCLRA Library

National Security Agency
Physical Sciences Division (2)
Fort George Meade, Maryland
Attn: Dr. Alvin Meckler

Associate Prof. A. Kaprielian
Department of Electrical Engineering
University of Southern California
University Park
Los Angeles 7, California

Assistant Secretary of Defense
(Research and Development)
Research and Development Board
Department of Defense
Washington 25, D. C.

Chief of Naval Operations
Department of the Navy
Washington 25, D. C.
Attn: Op-20

Chief of Naval Operations
Department of the Navy
Washington 25, D. C.
Attn: Op-32

Chief of Naval Operations
Department of the Navy
Washington 25, D. C.
Attn: Op-413

Chief, Bureau of Aeronautics
Department of the Navy
Washington 25, D. C.
Attn: EI-4

Technical Library
U. S. Naval Proving Ground
Dahlgren, Virginia

Director
Naval Ordnance Laboratory
White Oak, Maryland

Librarian
U. S. Naval Post Graduate School
Monterey, California

Air Force Office of Scientific Research
Air Research and Development Command
Washington 25, D. C.
Attn: SRY, Physics Division

Commanding General
Rome Air Development Center
Griffiss Air Force Base
Rome, New York
Attn: RCREC-4C

Commanding General
Rome Air Development Center
Griffiss Air Force Base
Rome, New York
Attn: RCR

Commander
Air Force Cambridge Research Center
230 Albany Street
Cambridge 39, Massachusetts
Attn: CRZI

Commander
Air Force Cambridge Research Center
230 Albany Street
Cambridge 39, Massachusetts
Attn: CRZM

Commander
AF Cambridge Research Laboratories
Laurence G. Hanscom Field
Bedford, Massachusetts
Attn: Dr. Hollingsworth

Commander
Wright Air Development Center
Wright Patterson Air Force Base
Ohio Attn: WCRR

Sandia Corporation
Org. 1424, Sandia Base
Albuquerque, New Mexico
Attn: Dr. C. W. Harrison, Jr.

Sandia Corporation
Sandia Base
Albuquerque, New Mexico
Attn: Library Division 1922-1

Mr. Robert Turner
General Electric Company
Advanced Electronics Center
Cornell University
Ithaca, New York

Library
Airborne Instruments Lab.
Walt Whitman Road
Melville, Long Island, New York

Secretary, Working Group
Semiconductor Devices
346 Broadway, 8th Floor
New York 13, New York
Attn: AGET

Metals Research Laboratories
Electro Metallurgical Company
Box 580, Niagra Falls, New York
Attn: Mr. R. J. Gladieux

Librarian
General Electric Research Lab.
P. O. Box 1088
Schenectady, New York

Westinghouse Electric Corp.
Research Laboratories
Beulah Road, Churchill Boro.
Pittsburgh 35, Pennsylvania

Prof. O. E. H. Rydbeck
P. O. Box 266
Belmar, New Jersey

Dr. Melvin W. Aarons
611 East Genessee Street
Fayetteville, New York

Laboratory
Airborne Instruments
Mineola, New York

Commander
Wright Air Development Center
Wright Patterson Air Force Base
Ohio
Attn: WCRRH

Commandant
Air Force Institute of Technology
Wright Patterson Air Force Base
Ohio
Attn: MCLI Library

AF Special Weapons Center
Kirtland Air Force Base
Albuquerque, New Mexico
Attn: SWOI

Headquarters
AF Missile Test Center
MU-135, ADRC
Patrick Air Force Base
Florida

U. S. Coast Guard
1300 E Street, N. W.
Washington 25, D. C.
Attn: EEE

M. A. Krinvanich, Chief
Systems Component Branch
Electronic Warfare Division
Signal Corps Agency
White Wands Proving Ground
New Mexico

Mr. A. D. Bedrosian
Signal Corps Liaison Office
Mass. Institute of Technology
Building 26, Room 131
Cambridge 39, Massachusetts

Chief, European Office
ARDC Command
Shell Building
60 Rue Ravenstein
Brussels, Belgium

Dr. J. Anton Hofman
Ordnance Materials Res. Office
Watertown Arsenal
Watertown, Massachusetts

Acquisitions Officer
ASTIA Reference Center
Arlington Hall Station
Arlington 12, Virginia

Stanford Research Institute
Documents Center
Menlo Park, California
Attn: Mary Lou Fields

Dr. C. H. Papas
Dept. of Electrical Engineering
California Institute of Technology
Pasadena, California

Stanford Electronics Lab.
Stanford University
Stanford, California
Attn: Document Library
Applied Electronics Lab.

Department of Electrical Engineering
Yale University
New Haven, Connecticut

Librarian
Johns Hopkins University
1315 St. Paul Street
Baltimore 2, Maryland

Radiation Laboratory
Johns Hopkins University
1315 St. Paul Street
Baltimore 2, Maryland

Director, Lincoln Laboratory
Mass. Institute of Technology
Bedford, Massachusetts

Mr. John Hewitt
Document Room
Research Lab. of Electronics
Mass. Institute of Technology
Cambridge 39, Massachusetts

Professor A. Von Hippel
Mass. Institute of Technology
Lab. for Insulation Research
Cambridge 39, Massachusetts

Library, Room A 229
Lincoln Laboratory
P. O. Box 73
Lexington 73, Massachusetts

K. M. Siegel, Head
Theory and Analysis Department
Willow Run Laboratories
University of Michigan
Willow Run Airport
Ypsilanti, Michigan

Librarian
National Bureau of Standards Library
Room 301, Northwest Building
Washington 25, D. C.

Librarian
U. S. Department of Commerce
National Bureau of Standards
Boulder, Colorado

Dr. Earl Callen
National Security Agency
Physical Sciences Division
Fort George Meade, Maryland

Dr. H. Campaigne
National Security Agency
Physical Sciences Division
Fort George Meade, Maryland

Chung Kung University
Electrical Engineering Department
Tainan, Taiwan
Republic of China
Attn: Professor Chao-Hsi Chou
Head, Eng. Department

Mr. D. S. Jones
Department of Mathematics
Univ. College of No. Staffordshire
Keele, Staffordshire, England

Professor Paul Sanai Mito
Osaka City University
Dept. of Engineering Sciences
12 Nishi Ogimachi Kitaku
Osaka, Japan

Donald C. Stinson
Dept. of Electrical Engr.
University of Arizona
Tucson 25, Arizona

Professor Jerome R. Singer
Div. of Electrical Engineering
University of California
Berkeley 4, California

Professor Charles Kittel
Department of Physics
University of California
Berkeley 4, California

Serials Library
Brandeis University
Waltham, Massachusetts

Professor H. G. Booker
School of Electrical Engineering
Cornell University
Ithaca, New York

Library, College of Engineering
University Heights Library
University Heights
New York University
New York 33, New York

E. A. Chapman, Librarian
Rensselaer Polytechnic Institute
Amos Eaton Hall
Troy, New York

Robert Plonsey
Department of Engineering
Case Institute of Technology
University Circle
Cleveland 6, Ohio

Dept. of Electrical Engineering
Case Institute of Technology
University Circle
Cleveland 6, Ohio
Attn: S. Seely, Head

Dr. C. J. Falkenback
Batelle Memorial Institute
Columbus, Ohio
Attn: Electrical Engineering Division

Librarian
Engineering Library
Brown University
Providence, Rhode Island

Professor A. W. Straiton
Dept. of Electrical Engineering
University of Texas
Austin 12, Texas

Mr. William Way
Research Librarian
Tasker Instruments Corp.
3349 Cahuenga Boulevard
Hollywood 28, California

SOLID STATE ONLY

Martin A. Garstens, Head
Paramagnetic Section
Magnetism Branch
Solid State Division
Naval Research Laboratory
Washington 25, D. C.
Attn: Code 6451

Dr. Reinier Beeuwkes, Jr.
Ordnance Materials
Research Laboratory
Watertown Arsenal
Watertown, Massachusetts

Mr. A. Saika
Himeji Technical College
Himeji, Japan

Electronics Research Laboratory
Division of Electrical Engineering
University of California
Berkeley 4, California
Attn: Librarian

Johns Hopkins University
34th and Charles Street
Whitehead Hall
Baltimore 18, Maryland
Attn: Mr. J. O. Artman

Librarian
Physics Department
Amherst College
Amherst, Massachusetts
Attn: Mr. Romer

Professor I. Lowe
Department of Physics
University of Minnesota
Minneapolis, Minnesota

Professor R. E. Norberg
Department of Physics
Washington University
St. Louis, Missouri

Microwave Research Institute
Polytechnic Institute of Brooklyn
55 Johnson Street
Brooklyn, New York
Attn: Librarian

Dr. Sidney Shapiro
Arthur D. Little, Inc.
15 Acorn Park
Cambridge 40, Massachusetts

Dr. Simon Foner
Lincoln Laboratories
Box 73
Lexington, Massachusetts

Mr. William H. From
Ewen Knight Corporation
206 A Street
Needham, Massachusetts

Dr. Edward U. Condon
6604 Waterman Avenue
St. Louis, Missouri

Dr. W. M. Walsh
Bell Telephone Labs., Inc.
Murray Hill, New Jersey

Librarian
IBM Watson Laboratories
612 West 115th Street
New York 27, New York

ELECTROMAGNETIC RADIATION ONLY

Michigan State College
Department of Mathematics
East Lansing, Michigan

Microwave Research Institute
Polytechnic Institute of Brooklyn
55 Johnson Street
Brooklyn, New York

Mr. L. E. Swarts, Jr.
Building 12, Room 2617
Hughes Research Laboratories
Culver City, California

Professor O. E. H. Rydbeck
P. O. Box 266
Belmar, New Jersey

One Copy Unless Otherwise Specified