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TRANSLATION

BASES FOR CALCULATING THE BREAKING OF ROCKS WITH AN EXPLOSION

By

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AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE

OHIO

MAY 31, 1963
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English Pages: 18

CHAPTER II

THE EHDA METHOD FOR SCALE MODELS

The Method of Electrohydrodynamic Analogy

The method of using models as an aid to solving differential equations has become widespread in scientific research as a result of the simplicity of realization, the visualization, and the sufficient accuracy of the results obtained. The method of electrohydrodynamic analogy is one of the most used methods of electrical modeling. Its application is based on the analogy between the nonturbulent motion of the ideal liquid and the motion of electrical current in conductive bodies, summarized by the fact that both of the phenomena are described by the same differential equations.

The EHDA method was proposed by the Academician N.N. Pavlovskii in 1922 (3) for the solution of hydraulic problems and at present it is widely used in studies in various branches of physics, mechanics, hydraulics, etc. In hydraulics, for example, the EHDA method plays an exceptionally important role in the study and calculation of complicated three-dimensional filter flows in hydraulic equipment, the mathematical calculation of which is either not always possible, or of extreme difficulty.

The application of the EHDA method is scientific studies has been the subject of a number of works (4-6) in which are described the various systems of EHDA installations, detailed procedures and methods are worked out for the solution of problems under consideration, the accuracy of the solutions is studied, and ways of reducing error when using this method are indicated.

In the present work are exposed the results of the application of the EHDA method in solving a number of problems of blasting.
As was shown in the first chapter, the assumptions made permitted the construction of a simple model of the action of an explosion in which the behavior of the environment after the explosion is characterized by the equations of hydrodynamics. In particular, the potential of initial speed is described by the Laplacian equation

$$\Delta \varphi = 0$$

By the magnitude of speed potential from correlations (2) it is possible to find in every point of the environment the magnitude of the speed vector, determining per unit of mass the initial charge of kinetic energy received by the environment under the effect of the explosion. Thus, mathematically finding the original field of speeds leads to the solution of the differential equation in partial derivatives of the second order. However, the analytical solution of the Laplacian equation is difficult and laborious even in the case of plane problems when it is possible to apply a conformal reflection. The EHDA method, then, makes it possible to exchange the calculation of speed potential by means of the analytical solution of the Laplacian equation for the direct measurement of the electrical potential in a model.

The EHDA Installation and the Methodology of the Work

The EHDA installation includes an electrolytic tank, an electrical system, and a mechanical apparatus for noting the results of the measurements (fig. 10). The electrical diagram of the EHDA installation consists of two circuits -- the feed circuit and the measuring circuit. The feed circuit contains the power source and the devices for regulating the voltage of the model. In the measuring circuit there are a voltage separator and the indicator of system balance. As is seen in figure 10, the measuring system of the EHDA installation is based on the principle of the Wheatstone bridge. Two arms of the...
Figure 10. Electrical diagram of the EHDA installation in the blasting laboratory of IGS (Inst. of Mining)

bridge, \( R_1 \) and \( R_2 \) act as voltage separator, and the other two, \( R_3 \) and \( R_4 \), the resistance of the electrolyte between the electrodes and the sound.

At balance, \( \frac{R_1}{R_2} = \frac{R_3}{R_4} \), there is no current in the diagonal bridge, which is indicated by the minimum reading of the measuring device in the diagonal of the bridge. A general view of the EHDA installation in the blasting laboratory of the IGD AN SSRS (Institute of Mining, Academy of Science, USSR) is shown in figure 11. In the electrical diagram of the installation, two IMS-1 resistance boxes serve as the \( R_1 \) and \( R_2 \) arms. Rheostats \( r_1 \) and \( r_2 \) compensate the resistance effect of the feeder leads. As a balance indicator, a VG vibration galvanometer, outstanding for its compactness and high sensitivity, is used.

The installation is supplied by the light circuit, the voltage of which is reduced to 20-25 v. by an autotransformer LATR-1. The use of an alternating current supply reduces measuring error because of diminished polarity of the electrodes (of the model). The tank of the installation measures 100 X 100 X 150 centimeters. The electrolyte may be water from the ordinary supply, or a weak (0.01%) FTD-TT-62-1846/1r2.
Figure 11. General view of an EHDA installation solution of copper sulfate. The electrodes (of the model studied) may be of iron or stainless steel in work with water, or of brass or copper when using the copper sulfate solution. Such combinations assure sufficient reduction of the intensity of a number of detrimental processes occurring in the tank and inducing error in the measuring results. Primary among such deleterious phenomena are capacitive currents, electrolysis, diffusion potential, galvanic effect, etc. Various procedures for reducing harmful effects and assuring necessary accuracy of the measurements are covered in the literature on the EHDA method.

The pantograph serves as a mechanical setup for notation of the results in measuring the potential in the tank. By the sound, anchored in the pantograph, points of equal potential are located, so that it passes for a voltage separator. The sound, a thin, insulated metal bar bared for 2 mm. at the tip, is secured in the pantograph holder and manipulated manually. As they are found, points of equal potential are noted on the chart by the pencil or stylus of the pantograph imitating the motion of the sound. The points of equal potential thus obtained are joined in a smooth
curve forming the family of equipotential lines which serve as references for the behaviour of a distinct type of calculations or structures necessary for the solution of one problem or another and described below.

In the EHDA method of studying the distribution of electrical potential, the values obtained are not absolute, but corrected potential, i.e. potential inferred from the full voltage drop on the model. Thus, on the surface of the charge model, the value of the corrected potential is kept equal to one and that of the open surface equal to zero. For determining absolute magnitudes of equipotential curves or of determined currents, their relative values are multiplied by the absolute magnitude of the speed potential on the surface of the charge which, in general, is calculated by the balance of energy (7).

Analytically it is possible to express the speed potential on the surface of the charge in elementary functions of the case of a single charge located in an unlimited medium and having the form of an ellipsoid of revolution (7).

In this case, the speed potential on the surface of the charge is expressed by the formula

$$
\varphi_{\text{surf}} = \sqrt{\frac{Q \ln \left( \frac{b + \sqrt{b^2 - a^2}}{a} \right)}{2 \pi \rho \sqrt{b^2 - a^2}}},
$$

in which $\varphi_{\text{surf}}$ is the speed potential on the surface of the charge in $m^2$ per sec.; $\rho$ is the density of the medium in kg·sec$^2$/m$^4$; $Q$ is the energy of the charge in kg·m (kilogrammeters); $a$ and $b$ are respectively the short and long axes of the ellipsoid $m$.

In the majority of practical instances of blasting it is possible to calculate the values of $a$ and $b$. Then the formula (52) is simplified:

$$
\varphi_{\text{surf}} = \sqrt{\frac{Q}{2 \pi \rho \ln \left( \frac{b}{a} \right)^2}},
$$

where $l = 2b$, length of charge, m; $d = 2a$, diameter of charge, m.
Although formulae (52) and (53) are intended for the case of the effect of a charge in an unlimited medium, they may be used as good approximations for calculating the actual pattern of the effect of an explosion, e.g., on a bare surface, for example, for solving problems leading to a comparative evaluation of the charge distribution patterns, or establishing the influence of charge shapes (various ratios of length to diameter) on the result of the explosion.

In more complex instances, the magnitude of the speed potential on the surface of the charge may be determined by measuring the electrical resistance of a model in the tank of the EHDA installation (8).

As shown above (see Chapter I), the total magnitude of the kinetic energy received by the environment under the effect of the blast is expressed by the formula

\[ Q = -\frac{1}{2} \Phi_\text{sur} \frac{\partial \Phi}{\partial n} dF \]  

(7)

Here \( \rho \) is the density of the medium; \( \Phi_\text{sur} \) the speed potential on the surface of the charge; \( n \) the direction of normal to the surface of the charge; \( F \) the surface area of the charge.

On the other hand, the magnitude of the current flowing through the model in the tank of the EHDA installation is determined by the expression

\[ I = -\frac{1}{r} \int \frac{\partial \Phi_\text{sur}}{\partial n} dF \]  

(54)

or

\[ I = \frac{\Phi_\text{sur} - \Phi_0}{R} \]  

(55)

where \( r \) is the specific resistance of the electrolyte; \( R \) the resistance of the model in the tank; \( \Phi_0 \) the potential on the free surface.

If we consider \( \Phi_0 = 0 \), formula (55) takes on the aspect

\[ I = \frac{\Phi_\text{sur}}{R} \]  

(56)
From formulae (54) and (56) we arrive at

$$\phi_{\text{max}} = -\frac{R}{t} \int \frac{\partial \phi_{\text{max}}}{\partial t} dF.$$

Equating expressions (7) and (57) we get

$$\frac{\phi_{\text{max}}}{R} = \frac{2Q}{\rho \phi_{\text{max}}}.$$

whence

$$\phi_{\text{max}} = \sqrt{\frac{2\pi}{\rho t}} \sqrt{R}.$$

It follows that the speed potential on the surface of charges of the same energy but with different $1/d$ ratios will be related as the magnitude of $\sqrt{R}$, measured for the models of these charges.

The most important condition for obtaining a correct result when using models of physical processes is the assurance of the similarity to nature of the manifestations studied on the models. To achieve this it is necessary in all cases to observe the geometrical correspondence of the models to nature.

In the present work, while using EHDA to study various patterns for the distribution of electrodes in order to simplify the modelling and reduce the work, a method was employed based on the use of the following phenomena.

If the plane surface of a dielectric is placed in the electrolytic tank in the vicinity of the electrode, the electric field becomes the same as if there were present a second electrode of the same charge as the first and located a distance from it equal to twice the distance from the electrode of the dielectric surface. In other words, the dielectric surface creates a mirror image of the picture of the electric field with respect to itself. The surface of the electrolyte in the tank (the air-electrolyte boundary) acts in an analogous fashion, as do the bottom and the sides of the tank. Thus, in order to eliminate distortion of the field it is necessary when using models to calculate
the influence of the tank walls, choosing the size of the model with respect to the tank so that the size of the areas in which the field will be studied are smaller with respect to the distance from this area to the walls of the tank. If, however, instead of a nonconductor we insert a flat conductive surface, the character of the field becomes the same as it would be in the presence of two oppositely charged electrodes located from each other at a distance equal to twice the distance from the electrode to the conductive surface.

These properties of dielectric and conductive surfaces permit, under conditions of symmetrically distributed electrodes (of the model), studying instead of the whole pattern, that part located between the symmetry planes of the system.

Figure 12, as an example, shows the distribution of the speed potential for a model of a long (concentrated) charge perpendicular to the plane of the open surface.

![Figure 12](image)

Figure 12. The initial speed field of a model of a separate, concentrated charge

The model for the charge, a copper or brass bar corresponding to the scale of the dimensions of the model, is placed perpendicularly to the plane of a flat sheet of copper (modelling the open surface) and separated from that sheet by a distance equal to the length of the
stemming. Thus the stemming is simulated by the electrolyte which means the equality of its properties to those of the medium to be exploded.

Out of the relationships (2) in any point of the field it is possible, by the magnitude of the speed potential, to figure the magnitude of the speed vector \( \mathbf{v} \), the direction of which coincides with normal on an equipotential curve passing through a given point.

It is convenient to plot the initial field of speed potential across identical intervals. Then equal distances between equipotential curves will correspond to identical values of speed (on the strength of the relationships /2/), i.e., the spacing of the equipotential curves will give an idea of the density of the kinetic energy in the medium around the charge.

In those areas where disintegration of the medium is possible, the formation of funnels of discharge material necessitates the fulfillment of one more condition: particles of the medium in a given spot must have the possibility of movement, i.e., the vectors of their speed must be directed toward the free surface, for if these speed vectors of the particles are directed towards the undestroyed parts of the mass, there will be no ejection at this point.

Having defined by calculation, or on the basis of experimental data, the contours of the zone of disintegration on the free surface, it is then possible to determine the dimensions of the crater and, as a geometric spot in the depth of the mass, the points the speed vectors of which pass by the end of the zone of destruction on the free surface. The particles lying within this contour will, after the explosion, have been thrown out.

Beyond the limits of the ejection zone there is a zone of crumbling within which the particles of medium may move while remaining inside the zone. The boundary of the crumbling zone in the depth of the mass will
be seen as a line of current. The medium beyond the bounds of the crumbling zone will basically only be compacted by the blast.

As is seen in the present example, the ejection of the medium takes place only in the area of the mouth of the bore hole. The rest of the bore hole remains intact, in spite of the high density of the energy in the neighborhood of the charge. This agrees quite well with the destruction map obtained in practice with an explosion of uniformly concentrated charges and may be easily interpreted physically.

Figure 13 represents the initial speed field for a model of a cut consisting of two parallel blast holes, one of which is charged (cross-hatched) and the other blank. The greatest density of equipotentials, which means the greatest energy density, is obtained in the wall between the blast holes. The speed vectors of the rock particles in this place are directed toward the uncharged blast hole, which, together with the high density of energy, creates favorable conditions for destruction of the wall between the blast holes. With sufficient explosive energy in the area of the mouth of the blast hole, a crater is produced as well.

Figure 13. Initial speed field of a model of a cut with a hollow and a charged (cross-hatched) blast hole

Figure 14 presents the initial speed field, taken from a model, of two simultaneously exploding blast-hole charges. Unlike the preceding example, the density of the energy in the wall between the
bore holes is relatively little with respect to the energy density in other areas of the field. Besides, the particles of the medium in this spot are unable to move. Thus, when both charges are exploded simultaneously the conditions affecting the wall between the charges are less favorable than when a charged bore hole is exploded parallel to an empty hole.

Figure 14. The initial speed field of a model of a cut with two charged bore holes

This qualitative demolition map, set up on the basis of modeling, accords very well with practice. It is known, for example, that in extensive straight cuts it is better not to charge a part of the bore holes and use them as supplemental bare surfaces, thus improving the conditions for destruction of the rock and increasing the coefficient of use of the bore holes in the cut.

The examples considered show that the initial speed field obtained through use of the EHDA method provides a visual map of the energy distribution of the blast in the surrounding medium on the basis of which it is possible to solve a wide circle of problems of blast stemming.
Blast Modelling in Stratified Media

The use of the EHDA method of modelling also makes it possible to calculate the effect of a blast in stratified media. This has practical value for planning mass blasting in nonuniform rock and for calculating parameters for the disposition of charges in stemming ores in shallow veins when the destruction zone in surrounding rock must be minimal, and in other cases.

Ordinarily, stratified media are modelled by a separate degree of conductivity in corresponding regions (4), however, the preparation of such models, particularly in three dimensions, is extremely difficult.

A much simpler solution in a number of cases is the preparation of three-dimensional models of stratified media from a uniform electrolyte in which, to create the potential difference on the boundary between strata it is necessary to introduce a surface resistance sufficient to satisfy the condition

\[ \gamma_1 \phi_1 = \gamma_2 \phi_2 \]

where \( \gamma_1 \) and \( \gamma_2 \) are the specific weight of the strata in nature; \( \phi_1 \) and \( \phi_2 \) are the values of the speed potential in two neighboring strata.

Within each stratum, the speed potential \( \phi \) must satisfy the Laplacian equation \( \Delta \phi = 0 \).

To the point that the medium is incompressible, no discontinuity of speed can occur on the boundaries between strata, i.e., the following condition must be satisfied at the boundary:

\[ \frac{\partial \phi_1}{\partial n} = \frac{\partial \phi_2}{\partial n} \]

where \( n \) is the direction of normal to the surface of the strata discontinuity. With a specific resistance of the surface of the strata discontinuity \( R \), the density of the current flowing between the two strata is determined by the expression

\[ \frac{\phi_1 - \phi_2}{R} = \frac{1}{12} \frac{\partial \rho}{\partial \sigma} \]
where $\tau$ is the specific resistance of the electrolyte.

From the expressions (a) and (b) we find that

$$R = -\tau \frac{A}{\ln r} = \left(\frac{r_1}{r} - 1\right) \frac{\tau_{\text{in}}}{\ln r}.$$

As a simple example, let us look at the action of a spherical charge of the radius $r_1$, surrounded by two concentric layers of medium with radii $r_2$ and $r_0$ and volumetric weights $\gamma_1$ and $\gamma_2$ (fig. 15). By spherical symmetry, the expressions for the speed potential of strata I and II may be written in the form

$$\varphi_1 = A + \frac{k}{r}, \quad \varphi_2 = B + \frac{k}{r}.$$

The constants $A$ and $B$ are determined by a look at the boundary conditions:

a) on the surface of the charge, i.e., at $r = r_1$, the speed potential must be equal to one,

$$A + \frac{k}{r_1} = 1, \quad \text{whence} \quad A = 1 - \frac{k}{r};$$

b) on the free surface, i.e., at $r = r_0$, the speed potential is equal to zero. It follows that $B + \frac{k}{r_0} = 0$, whence $B = -\frac{k}{r_0}$. Besides that, $-\frac{\partial \varphi}{\partial n} = -\frac{k}{r_0}$. From this we get

$$R = \left(\frac{1}{r_1} - 1\right) \tau_{\text{in}} \left(\frac{1}{r_0} - r_1\right).$$

The surface resistance is formed out of silk material or netting impregnated with electroconductive paint made of graphite powder mixed with BF glue. The specific electroconductivity of such
partitions may be varied in wide limits by introducing a varied quantity of graphite into the paint.

Calculating the Break-up by the Modelling Data

Calculating the break-up of the media by the initial field of speeds makes it possible to theoretically calculate the effectiveness of various charge distribution patterns in relation to coarseness and uniformity of breaking at the blast.

We have introduced several variations of the break-up calculations by initial speed field either obtained by calculation or read out on the EHDA. Calculations gave the distribution of speed potential for two simple cases: the blast of a concentrated charge near a bare plane surface and the application of a concentrated impulse loading to the spherical model. In both cases the distribution of the speed potential is expressed by simple formulae, making it possible to calculate it for any point with the necessary degree of accuracy.

![Figure 16. Diagram for a method of calculating break-up](image)

For computing the second derivatives of the speed potential $\Phi$ by the coordinates entering into the expression of the breakability criteria $D$, it is convenient to figure the speed potential $\Phi$ in junctions of a grid square, after which the magnitudes of the second derivatives may be determined by the following diagram (fig. 16):
\[
\left( \frac{\partial \phi}{\partial x} \right)_0 = \frac{\phi_{11} - \phi_{11}}{\Delta x}, \quad \left( \frac{\partial \phi}{\partial y} \right)_0 = \frac{\phi_{11} - \phi_{11}}{\Delta y},
\]

\[
\left( \frac{\partial^2 \phi}{\partial x \partial y} \right)_0 = \frac{\phi_{11} - 2\phi_{11} + \phi_{11}}{\Delta x \Delta y}, \quad \left( \frac{\partial^2 \phi}{\partial x^2} \right)_0 = \frac{\phi_{11} - \phi_{11} - \phi_{11} + \phi_{11}}{\Delta x^2}.
\]

and analogously

The calculation of breakage by the initial speed field, read out on an EHDA installation, was derived for an oblong charge located in the center of a cube, and for a number of models of well charges. The calculations showed that for studies of breakage coarseness by this method it is necessary to have a high degree of accuracy in reading the distribution of speed potentials, inasmuch as a twofold differentiation leads to a significant increase of errors, which crept in when determining the speed potentials. In sports where the pattern under study shows only insignificant variations of potential the calculation of coarseness becomes laborious, since in order to obtain the desired results it was necessary to compute the magnitudes of \( D \) at a great number of points.

When working with models on EHDA, may be somewhat reduced by an increase of the model scale which leads to a reduction of relative errors when reading out the distribution of potential. This was done when studying the breakage of the cube (above mentioned) modelled on the scale of 6:1 (full-scale dimensions 150X150X150 mm.). However, this bit of assistance is difficult to realize when modelling practical patterns of the action of blasts--of bore hole or well-hole charges.

Thus, with the practical application of the EHDA method to the calculation of the breakage coarseness for commercial patterns of
charge placing it is necessary to tend toward a direct measurement of secondary derivatives of potential.
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