FTD-TT- 63-304

409 045

TRANSLATION

STRONG INTERACTION ON A PLATE WITH CONSIDERATION OF SLIP AND NEAR-WALL TEMPERATURE JUMP

By

A. A. Bogacheva, and V. S. Galkin

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE

OHIO

FTD-TT- 63-304/1+2+4

UNEDITED ROUGH DRAFT TRANSLATION

STRONG INTERACTION ON A PLATE WITH CONSIDERATION OF SLIP AND NEAR-WALL TEMPERATURE JUMP

By: A. A. Bogacheva, and V. S. Galkin

English Pages: 13

Source: Russian book, Inzhenernyy Zhurnal, Vol. II, No. 2, pp. 231-238, 1962.

T-39
SOV/258-62-2-2-3-18

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.	PREPARED BY: TRANSLATION SERVICES BRANCH FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.

P

STRONG INTERACTION ON A PLATE WITH CONSIDERATION OF SLIP AND NEAR-WALL TEMPERATURE JUMP

A. A. Bogacheva, V. S. Galkin

We will solve the problem of the motion of a perfect gas in a laminar boundary layer on a semi-infinite flat plate at zero angle of attack, taking into account the slip and jump of gas temperature at the wall and the strong interaction between the boundary layer and a hypersonic inviscid flow. It is assumed that the viscosity coefficient μ linearly depends on temperature T, Prandtl number Pr = 1. In order to calculate pressure P we used the method of tangent wedges. The slip and near-wall temperature jump is taken into account by linearization of the solution of the relatively well-known self-similar solution with the usual boundary conditions of "adhesion" * (no slip). We will examine the case of an insulated plate and the case of a cooled ($T_w < T_0$) plate (plate temperature T_w = constant, T_0 is stagnation temperature of the free-stream flow, i.e., the temperature of the insulated plate).

---

* We are concerned with a self-similar solution for a strong interaction of zero order in the nomenclature of Hayes' work [7].

This statement of the problem was used earlier in Shidlovskiy's work [1] for the case $T_W = T_0$. Calculations were conducted at a value of the specific heat ratio $\varkappa = 1.4$ and $\varkappa = 5/3$, and the solution of the zero-approximation equation was found by interpolation of the well-known Falkner-Skan solution tables for other values of the parameter entering into this equation. Obviously, mistakes were made during this interpolation which led to qualitatively false conclusions: slip increases pressure P and the boundary-layer displacement thickness δ, whereas the nature of its effect on the local coefficient of frictional drag Cf depends on the magnitude of $\varkappa$. In this work equations of the zero and first approximations were solved simultaneously in the electronic computer which, incidentally, allowed us to refine the data of works [2], [3] for a strong interaction without regard to the rarefaction factors. The results of the calculations for $\varkappa = 1.4$ and $\varkappa = 5/3$ show that when $T_W \leq T_0$, slip and temperature jump decrease P, δ, C_f and especially the heat-transfer coefficient from the gas to the plate.

1. We will examine motion in a laminar boundary layer on a plate under conditions of strong interaction of the boundary layer with a inviscid flow. A strong interaction occurs [4] when $M_\infty^2 \ (d\delta/dx)^2 > 1$, where M_∞ is the free-stream Mach number, $\varkappa$ is the coordinate along the plate. In addition, magnitudes of the order M_∞^{-2} must be negligibly small in comparison with unity (we assume that the magnitude $2/(\varkappa - 1)$ is not one of the determining parameters when evaluating orders of magnitudes, i.e., $2/(\varkappa - 1) \sim 1$). Otherwise strong interaction occurs at the leading edge of the plate being passed by a flow with $M_\infty >> 1$, where the rarefaction effects, caused by slip and gas temperature jump on the plate, and the entropy effects, caused by

vorticity of the inviscid flow on the outer edge of the boundary-layer flow, can be particularly significant.

The investigation of these effects can be facilitated since we can study them individually by means of boundary-layer equations, [1], [5]-[7], while appropriately changing the boundary conditions, since the effects of rarefaction and vorticity are small, approximately of order one. Actually, let the terms of the boudary-layer equation be of the order of unity. Then these rarefaction effects are maximum in a case of strongly heated plate [5] when they are of the order of δ/x. The order of magnitude of entropy effects is equal [7] to $(\delta/x)^{2-4/(3\varkappa)}$.

In the zero approximation magnitudes of the order of δ/x and higher are disregarded. Here, the boundary-layer equations have a self-similar solution. Terms of the order of δ/x are taken into account in the first approximation. By virtue of their smallness we can linearize the problem with regard to the zero approximation, i.e., we can investigate individually the effects of rarefaction and vorticity, while appropriately changing the boundary conditions for boundary-layer equations.

Therefore, when investigating the effects of rarefaction the boundary conditions on the outer edge of the boundary layer remain the same as in the zero approximation: the velocity at the edge of the boundary layer is $u_\delta \approx u_\infty$ total enthalpy $H_\delta \approx u_\infty^2/2$, i.e., the temperature $T_\delta \approx 0$. The boundary conditions of slip must be fulfilled on the plate [8]

$$\Delta T = T|_{y=0} - T_w = b\sqrt{\frac{\pi\varkappa}{2}}\frac{2-\sigma}{\sigma}\left(\frac{\mu}{\rho a}\frac{\partial T}{\partial y}\right)_{y=0},$$
$$u|_{y=0} = \sqrt{\frac{\pi\varkappa}{2}}\frac{2-\sigma}{\sigma}\left(\frac{\mu}{\rho a}\frac{\partial u}{\partial y}\right)_{y=0}, \quad b = \frac{2\varkappa}{\varkappa+1}\frac{2-\alpha}{\alpha}\frac{\sigma^*}{2-\sigma}, \tag{1.1}$$

where $\underline{a}$ is the speed of sound, ρ is the density, α is the accomodation coefficient, σ is the reflection coefficient. The term $(3/4)$ $(\mu/\rho T)$ $(\partial T/\partial x)$, negligible within the limits of applicability of the boundary-layer equations is discarded in the formula for slip velocity.

When solving the problem we will look for the dependence of pressure on $\underline{x}$ in the form of $P^* = P/P_\infty = P_0\chi_1 x^{-1/2}(1 + dx^{-1/4})$, since in the zero approximation $P^* \sim x^{-1/2}$, and the correction, caused by rarefaction is of the order of $\delta/x \sim x^{-1/4}$. Here coefficient P_0 and small parameter $\underline{d}$ are subject to determination, $\chi_1 = \chi$ when $x = 1$, $\chi = M_\infty^3 \, Re_\infty^{-1/2}$, $Re_\infty = u_\infty x/\nu$, $\nu = \mu_\infty/\rho_\infty$. We will introduce now (generalizing [1]) the dimensionless dependent and independent variables by the following formulas:

$$G = \frac{H}{H_\delta}, \quad \frac{u}{u_\infty} = \left(\frac{u_\infty}{\xi\nu}\right)^{1/2}\Psi_\lambda, \quad \xi = 4P_0\chi_1 x^{1/2}, \quad \lambda = \left(\frac{u_\infty}{\nu\xi}\right)^{1/2}\int_0^y \frac{\rho}{\rho_\infty}\,dy. \tag{1.2}$$

We will note that the self-similar motions depend only on λ. Using variables (1.2) in boundary conditions (1.1), when $\lambda = 0$ we obtain

$$\Psi_\lambda = \vartheta\Psi_{\lambda\lambda}, \quad \Delta G = b\vartheta G_\lambda, \quad \vartheta = \sqrt{\frac{\pi\varkappa}{2}}\,\frac{2-\sigma}{\sigma}\,\frac{\mu_w}{\rho_\infty a_w}\left(\frac{u_\infty}{\nu\xi}\right)^{1/2}, \tag{1.3}$$

where, obviously, ϑ is the small parameter of the problem under consideration. Using further variables (1.2) in boundary-layer equation, we will reduce them by the usual method for boundary-layer theory, to

$$\begin{gathered}\frac{u_\infty}{\nu\xi}\left[2\xi\Psi_\lambda\Psi_{\lambda\xi} - \Psi_\lambda^2 - 2\xi\Psi_\xi\Psi_{\lambda\lambda}\right] = \beta\left(1 + \frac{d}{2}x^{-1/4}\right)\left(G - \frac{u_\infty}{\nu\xi}\Psi_\lambda^2\right) + \\ + \left(\frac{u_\infty}{\nu\xi}\right)^{1/2}\Psi_{\lambda\lambda\lambda}(1 + dx^{-1/4}), \\ 2\xi\left(\frac{u_\infty}{\nu\xi}\right)^{\frac{1}{2}}\left[G_\xi\Psi_\lambda - G_\lambda\Psi_\xi\right] = (1 + dx^{-1/4})G_{\lambda\lambda}.\end{gathered} \tag{1.4}$$

Here the terms of the order of d^2 and higher are discarded.

In order to obtain a system of zero-and first-approximation equations from system (1.4), we will introduce the coefficient k by the relationship $dx^{-1/4} = k\vartheta$; moreover we set

$$\Psi(\lambda, \vartheta) = \left(\frac{\nu\xi}{u_\infty}\right)^{1/2}[\varphi(\lambda) + \vartheta\zeta(\lambda)], \quad G(\lambda, \vartheta) = g_0(\lambda) + \vartheta g_1(\lambda).$$

Substiuting these formulas into relations (1.3), (1.4) discarding at first terms of the order of ϑ, and then of the order of ϑ^2 as compared to unity, we obtain a system of equations and boundary conditions corresponding to them in zero and first approximations.

$$\varphi''' + \varphi\varphi'' + \beta(g_0 - \varphi'^2) = 0, \quad g_0'' + \varphi g_0' = 0,$$

$$\varphi(0) = \varphi'(0) = 0, \quad g_0(0) = T_w/T_0, \quad \varphi'(\infty) = g(\infty) = 1; \tag{1.5}$$

$$\zeta''' + \varphi\zeta'' + (1 - 2\beta)\varphi'\zeta' + \beta g_1 = k\left[\varphi\varphi'' + \frac{\beta}{2}(g_0 - \varphi'^2)\right],$$

$$g_1'' + (\varphi g_1)' = -k g_0',$$

$$\zeta(0) = 0, \quad \zeta'(0) = \varphi''(0), \quad g_1(0) = b g_0'(0), \quad \zeta'(\infty) = g_1(\infty) = 0, \tag{1.6}$$

$$\beta = \frac{\varkappa - 1}{\varkappa}, \quad T_0/T_\infty = \frac{\varkappa - 1}{2} M_\infty^2, \quad \frac{d(\ )}{d\lambda} = (\ )'.$$

In order to solve system (1.6) it is necessary to express k in terms of unknown functions. For this we use formula $P^* = \varkappa\,[(\varkappa + 1)/2]\, M_\infty^2\,(d\delta/dx)^2$, yielded by the method of tangent wedges where the boundary-layer displacement thickness is

$$\delta = \int_0^\infty \left(1 - \frac{\rho u}{\rho_\delta u_\infty}\right) dy \approx \left(\frac{\nu\xi}{u_\infty}\right)^{1/2} \int_0^\infty \frac{\rho_\infty}{\rho}\, d\lambda$$

(physically this method of calculating pressure is justified because when calculating an inviscid flow with an error of approximately $(\delta/x)^2$ we can disregard flow through the layer, the boundary-layer thickness can be identified with the displacement thickness, and we can consider the outer edge of the boundary layer as a solid wall.

Using these formulas we will find

$$\frac{\Delta}{x} = \delta_0 \frac{\sqrt{\chi}}{M_\infty}\left\{1 + \frac{\vartheta}{I_0}(I_1 - kI_0)\right\}, \quad \delta_0 = \frac{\varkappa - 1}{\sqrt{P_0}} I_0,$$

$$P_0 = \frac{3}{4}\sqrt{\frac{\varkappa(\varkappa+1)}{2}}(\varkappa - 1) I_0, \quad k = \frac{4I_1}{7I_0}, \quad \vartheta = \frac{2-\sigma}{4\sigma}\sqrt{\frac{\pi\varkappa(\varkappa-1)}{P_0}} \frac{T_w}{T_0} \frac{\sqrt{\chi}}{M_\infty}. \qquad (1.7)$$

$$I_0 = \int_0^\infty (g_0 - \varphi'^2)\, d\lambda, \quad I_1 = \int_0^\infty (g_1 - 2\varphi'\zeta)\, d\lambda.$$

2. In order to simplify the problem we will assume, generalizing [1], that

$$\zeta = \varphi' + k\int_0^\lambda z\, d\lambda, \quad g_1 = g_0' + kg.$$

By means of (1.5) we will reduce relation (1.6) to the form

$$z'' + \varphi z' + (1 - 2\beta)\varphi' z = -\beta g + \frac{\beta}{2}(g_0 - \varphi'^2) + \varphi\varphi''. \qquad (2.1)$$

$$g'' + (\varphi g)' = -g_0'',$$
$$z(0) = z(\infty) = 0, \quad g(\infty) = 0, \quad g(0) = \frac{b-1}{k} g_0'(0). \qquad (2.2)$$

Taking (1.5) into account we have the following solution of Eq. (2.2):

$$g = g_0'\left(-\lambda + \frac{b-1}{k}\right) + [g_0'(0) + g'(0)]\, g_0' \int_0^\lambda \frac{d\lambda}{g_0'}. \qquad (2.3)$$

Here

$$k = -4(T_w/T_0)(7I_0 - 4I_2)^{-1}, \quad I_2 = \int_0^\infty (g - 2\varphi' z)\, d\lambda.$$

In order to simplify I_2 we will integrate termwise Eq. (2.1). Then

$$\beta I_2 = \varphi''(0) + z'(0) - (3/2) I_0 - \lim_{\lambda\to\infty}(\varphi z). \qquad (2.4)$$

From the obvious finite condition of I_2 and $z'(0)$ the magnitude $e = \lim (\varphi z)$ must also be finite. This is especially clear in a case of an insulated plate when

$$g = 0, \quad I_2 = -2\int_0^\infty \varphi' z \, d\lambda,$$

therefore for the existence of this integral it is necessary that

$$e = 0, \text{ because when } \lambda \gg 1 \quad \varphi \approx \lambda - m, \quad \varphi' \approx 1,$$

where $\underline{m}$ is a certain constant.

In order to determine the value of $\underline{e}$ and the magnitude of $g'(0)$ we will investigate the conduct of functions $\underline{z}$, $\underline{g}$ in the vicinity of $\lambda = \infty$, and we will examine only the principal terms of the corresponding asymptotic expansions. Since

$$g_0'(\lambda) = g_0'(0) \exp\left(-\int_0^\lambda \varphi \, d\lambda\right),$$

then the principal terms of the items of function $\underline{g}$ are

$$D(\lambda - m)\exp\left[-\frac{(\lambda - m)^2}{2}\right], \quad \frac{g_0'(0) + g'(0)}{\lambda - m}, \quad D = \text{const.} \tag{2.5}$$

Let us substitute expression (2.3) instead of $\underline{g}$ into the right-hand side of Eq. (2.1). Then, when $\varphi = \lambda - m$, $\varphi' = 1$ this equation with the usual substiutions for such an equation reduces to a Whittaker equation. The principal terms of solution of a corresponding similar equation are

$$D_1(\lambda - m)^{-2\beta} \exp\left[-\frac{(\lambda - m)^2}{2}\right], \quad D_2(\lambda - m)^{2\beta - 1};$$

the principal terms of a partial solution of a nonsimilar equation are

$$\frac{g_0'(0) + g'(0)}{2(\lambda - m)} - A(\lambda - m)^n \exp\left[-\frac{(\lambda - m)^2}{2}\right],$$

where D_1 and D_2 are arbitrary constants, A and n are known constants.

It follows from these expressions that the solution is not unique since the boundary conditions when $\lambda = \infty$ are satisfied at any values of $g'(0)$. It follows, however, that from physical considerations the thickness of total enthalpy loss be finite

$$\Lambda = \int_0^\infty (u/u_\infty)(1-G)\,d\lambda = B - k\theta \int_0^\infty g\varphi'\,d\lambda,$$

where B is the finite magnitude. It follows from this, with consideration of (2.5), that the magnitude of Λ will be finite only when $g'(0) = -g_\delta(0)$. Assuming $g'(0) = -g_\delta(0)$ and taking into account that $D_2 = 0$, follows from the finite condition e we obtain: $e = 0$, functions g and z when $\lambda \to \infty$ tend toward zero with respect to the exponent and Eq. (2.1) as the final result takes the form

$$\begin{aligned} z'' + \varphi z' + (1-2\beta)\varphi' z = \frac{\beta}{2}(g_0 - \varphi'^2) + \varphi\varphi'' + \\ + \beta g_0' \left\{\lambda - \frac{b-1}{\beta}\frac{T_0}{T_w}\left[z'(0) + \varphi''(0) - \frac{9\beta}{4} I_0\right]\right\}. \end{aligned} \tag{2.6}$$

At last, after several transformations, taking into account formulas (1.7), we obtain

$$\delta/x = \delta_0\sqrt{\chi}M_\infty^{-1}(1-\delta_1\sqrt{\chi}/M_\infty),\quad P^* = P_0\chi(1-P_1\sqrt{\chi}/M_\infty),$$

$$C_f = (2/\rho_\infty u_\infty^2)(\mu\partial u/\partial y)_{y=0} = C_{f_0}\chi^{3/4}M_\infty^{-3}(1-C_{f_1}\sqrt{\chi}/M_\infty),$$

$$\mathrm{St} = (k\partial T/\partial y)_{y=0}[\rho_\infty u_\infty c_p(T_0 - T_w)]^{-1} = \mathrm{St}_0\chi^{3/4}M_\infty^{-3}(1-\mathrm{St}_1\sqrt{\chi}/M_\infty),$$

$$C_{f_0} = \sqrt{P_0}\varphi''(0),\qquad \mathrm{St}_0 = \sqrt{P_0/4}\,g_0'(0)(1-T_w/T_0)^{-1},$$

$$\delta_1 = -3/4k\gamma,\ P_1 = -k\gamma,\ k = -4\beta(9\beta I_0 - 4z'(0) - 4\varphi''(0))^{-1}T_w/T_0,$$

$$\gamma = \frac{2-\sigma}{4\sigma}\sqrt{\frac{\pi\varkappa(\varkappa-1)}{P_0}\frac{T_w}{T_0}},$$

$$C_{f_1} = -\frac{9}{4}\frac{\beta I_0}{\varphi''(0)}k\gamma,\quad \mathrm{St}_1 = 2\gamma\frac{\varphi''^2(0)}{g_0'(0)}.$$

When estimating the limits of applicability of these formulas we must remember that the small parameter of the problem is $\varepsilon = \gamma\sqrt{\chi}/M_\infty$. We note that the coefficient C_f in this statement of the problem becomes nonintegrable (see also [1], [5]).

Equations (1.5) and (2.6) with corresponding boundary conditions were solved simultaneously on an electronic computer for $\alpha = \sigma$ in the range $0 \leq \gamma \leq 8$ with a maximum error of $\Delta \lesssim 1.10^{-4}$ for values of the magnitudes $\varphi''(0)$, $g_0'(0)$ and $\Delta < 10^{-3}$ for values of magnitude $z'(0)$.

During the calculations it was revealed that the solution of a first-approximation equation is very "sensitive" to the solution of zero-approximation equations. Tables 1-4 and Figures 1-3 show some rounded-off (to save space) results of the calculations. The data in Table 4 was computed for $\alpha = \sigma = 1$. The data in Table 3 differs somewhat from the corresponding data of Li and Nagamatsu [2], [3] mainly due to the magnitude I_0. For instance in the latter study [3] with $\varkappa = 1.4$, values are given of $I_0 = 1.322$, 0.9703, and 0.5956, respectively for $T_W/T_0 = 1$, 0.6, and 0.2. The assumption of the presence of such errors [2], [3] was stated before ([7], p. 359). In Figs. 1-3 the solid lines correspond to data when $T_W/T_0 = 1$, the dashed lines when $T_W/T_0 = 0.6$, the dot-dash lines when $T_W/T_0 = 0.2$, $\zeta_1 = \zeta'$. The magnitude ω_1 is introduced by formula

$$T/T_0 = G - u^2/u_\infty^2 = g - \varphi'^2 + \vartheta(g_1 - 2\varphi'\zeta') = \omega_0 + \vartheta\omega_1.$$

In the case of an insulated plate, the slip increases the gas velocity (especially in the near-wall portion of the boundary layer) and decreases the gas temperature which is minimum in the middle of the boundary layer (in this case $\omega_1 = 2\varphi'\zeta'$). A reduction of T_W/T_0 increases the role of the near-wall temperature jump. In the near-wall part of the boundary layer ζ' somewhat decreases, ω_1 sharply increases in the main part of the boundary-layer, which leads to an increase of g_1. The presence of extremes of these magnitudes is mainly because φ' increases from 0 to 1 when λ changes from 0 to ∞ and ζ' decreases correspondingly from value of $\zeta(0) = \varphi''(0)$ to a value $\zeta'(\infty) = 0$.

Slip and near-wall temperature jump decrease the pressure, displacement thickness, local coefficient of friction drag and, especially, the heat-transfer coefficient from gas to plate. This effect decreases with a decrease of the ratio T_W/T_0.

Analogous conclusions in the case of an insulated plate have been obtained by means of an integral form of equation of momenta in Galkin's study [5] (velocity profile in boundary layer is linear) and in Takano's work [6] (velocity profile is a fourth-degree polynomial), while in the latter only the correction for pressure due to slip when $\varkappa = 1.4$ was calculated. In case $T_W/T_0 = 1$ when $\varkappa = 1.4$, according to our data and others [5], [6] we have respectively: $P_1 = 0.312$, 0.20, 0.315, $\delta_1 = 0.23$, 0.15, $C_{f_1} = 0.34$, and 0.21. Hence, representing the velocity profile as a fourth-degree polynomial is sufficiently accurate. A linear velocity profile can be used only for very simple approximate estimations of the slip effect, especially in a case of such flows about insulated thin bodies of finite thickness (for instance, a wedge), when in the zero approximation motion is not self-similar.

The authors are grateful to A. A. Nikol'skiy, V. P. Shidlovskiy, and M. N. Kogan for a discussion of the results of this study.

TABLE 1

$\varkappa = 1,4$

λ	$T_w/T_e = 1$: φ'	$T_w/T_e = 1$: $-z$	$T_w/T_e = 0,6$: φ'	$T_w/T_e = 0,6$: g_0	$T_w/T_e = 0,6$: $-z$	$T_w/T_e = 0,2$: φ'	$T_w/T_e = 0,2$: g_0	$T_w/T_e = 0,2$: $-z$
0,1	0,075	0,034	0,067	0,620	0,039	0,059	0,239	0,039
0,2	0,147	0,066	0,132	0,640	0,076	0,118	0,279	0,078
0,3	0,216	0,097	0,195	0,661	0,112	0,173	0,318	0,115
0,4	0,282	0,127	0,256	0,681	0,147	0,229	0,358	0,150
0,5	0,345	0,154	0,316	0,701	0,179	0,284	0,397	0,184
0,6	0,405	0,178	0,373	0,721	0,210	0,338	0,436	0,216
0,7	0,462	0,201	0,427	0,740	0,238	0,390	0,474	0,245
0,8	0,516	0,220	0,480	0,760	0,262	0,441	0,512	0,271
0,9	0,566	0,237	0,529	0,779	0,284	0,489	0,549	0,294
1,0	0,614	0,250	0,576	0,797	0,302	0,536	0,585	0,313
1,2	0,698	0,266	0,662	0,832	0,326	0,623	0,655	0,340
1,4	0,770	0,268	0,737	0,865	0,333	0,700	0,719	0,352
1,6	0,829	0,256	0,800	0,894	0,325	0,768	0,777	0,346
1,8	0,876	0,234	0,852	0,919	0,302	0,824	0,828	0,327
2,0	0,912	0,204	0,893	0,939	0,269	0,871	0,871	0,296
2,4	0,960	0,136	0,949	0,970	0,187	0,936	0,933	0,216
2,8	0,984	0,076	0,979	0,987	0,109	0,972	0,970	0,130
3,2	0.994	0,036	0,992	0,995	0,054	0,989	0,988	0,067
4,0	0,999	0,004	0,999	1,000	0,008	0,999	0,999	0,010
5,0	1,000	0,000	1,000	1,000	0,000	1,000	1,000	0,000

TABLE 2

$\varkappa = 5/3$

λ	$T_w/T_e=1$: φ'	$T_w/T_e=1$: $-z$	$T_w/T_e = 0.6$: φ'	$T_w/T_e = 0.6$: g_0	$T_w/T_e = 0.6$: $-z$	λ	$T_w/T_e=1$: φ'	$T_w/T_e=1$: $-z$	$T_w/T_e = 0,6$: φ'	$T_w/T_e = 0,6$: g_0	$T_w/T_e = 0,6$: $-z$
0,1	0,083	0,033	0,073	0,621	0,040	1,2	0,735	0,231	0,692	0,836	0,306
0,2	0,163	0,065	0,144	0,641	0,078	1,4	0,803	0,226	0,764	0,868	0,307
0,3	0,238	0,094	0,212	0,662	0,114	1,6	0,857	0,211	0,824	0,897	0,294
0,4	0,310	0,120	0,277	0,682	0,148	1,8	0,899	0,188	0,872	0,922	0,268
0,5	0,377	0,145	0,340	0,703	0,179	2,0	0,930	0,160	0,909	0,942	0,234
0,6	0,440	0,166	0,399	0,723	0,208	2,4	0,970	0,101	0 958	0,971	0,157
0,7	0,499	0,185	0,456	0,743	0,233	2,8	0,989	0,054	0,984	0,988	0,088
0,8	0,554	0,201	0,509	0,763	0,255	3,2	0,996	0,024	0,994	0,995	0,042
0,9	0,605	0,213	0,560	0,782	0,274	4,0	1,000	0,003	0,999	1,000	0,005
1,0	0,653	0,222	0,607	0,801	0,289	5,0	1,000	0,000	1,000	1,000	0,000

TABLE 3

	T_w/T_e	$\varphi''(0)$	$g_0'(0)$	I_e	R_e	δ_e	C_{f_e}	St_e
$\varkappa=1,4$	1	0,7627	0	1,3093	0,5091	0,7340	0,5442	—
	0,6	0,6775	0,2025	0,9407	0,3657	0,6222	0,4097	0,1531
	0,2	0,5883	0,3946	0,5592	0,2174	0,4797	0,2743	0,1150
	0,01	0,5443	0,4817	0,3728	0,1449	0,3917	0,2072	0,0926
$\varkappa= 5/3$	1	0,8544	0	1,2194	0,9089	0,8527	0,8146	—
	0,6	0,7429	0,2063	0,8764	0,6532	0,7229	0,6004	0,2084

TABLE 4

	T_w/T_0	$-z'(0)$	$-h$	P_1	δ_1	c_{f_1}	St_1
$\varkappa = 1,4$	1 0,6 0,2	0,347 0,392 0,398	0,671 0,537 0,338	0,312 0,228 0,107	0,234 0,171 0,081	0,344 0,204 0,006	— 1,925 0,558
$\varkappa = 5/3$	1 0,6	0,343 0,406	0,682 0,531	0,334 0,238	0,251 0,178	0,429 0,252	— 2,395

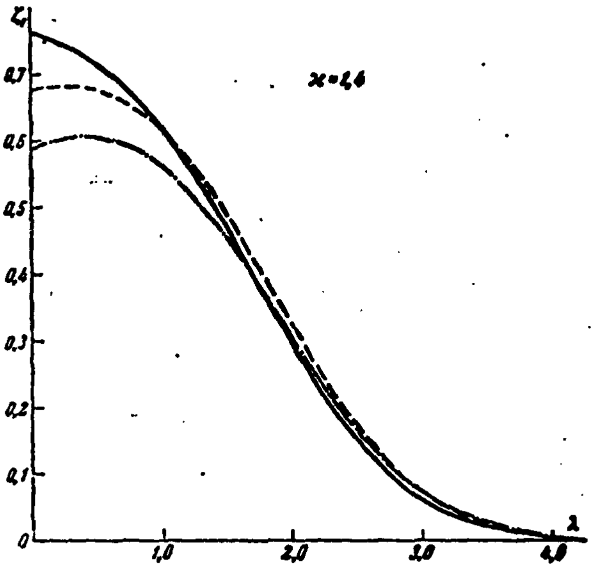

Fig. 1.

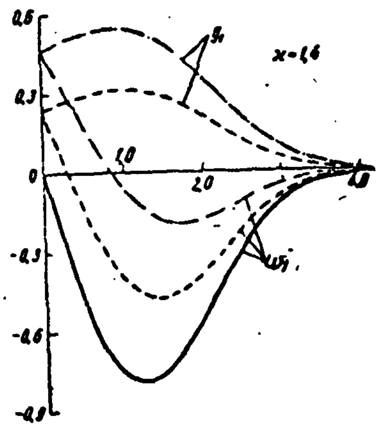

Fig. 2.

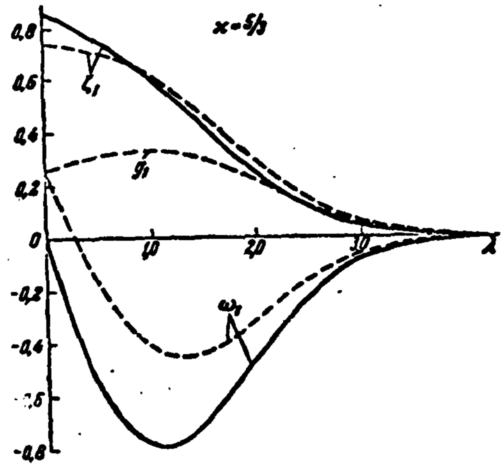

Fig. 3.

REFERENCES

1. V. P. Shidlovskiy. O vliyanii skol'zheniya na kharakteristiki laminarnovo pogranichnovo sloya v giperzvukovom potoke. Inzhenernyy zh., 1, No. 2, 1961.

2. T. Y. Li and H. T. Nagamatsu. Similar Solutions of Compressible Boundary-Layer Equations. J. Aeronaut. Sci., No. 9, 1955.

3. T. Y. Li and H. T. Nagamatsu. Hypersonic Viscous Flow on Noninsulated Flat Plate. Proc. 4th Midwestern Conf. on Fluid Mech., Purdue Univ., Lafayette Ind., 273-287, 1955.

4. L. Lees. On the Boundary-Layer Equations in Hypersonic Flow and their Approximate Solutions. J. Aeronaut. Sci., No. 2, 1953.

5. V. S. Galkin. Issledovaniye obtekaniya ploskoy plastiny giperzvukovym potokom vyazkovo slaboraztezhennovo gaza. Izv. Akad. Nauk. SSSR. Otd. Tekhn. n., Mekhanika i mashinostroyeniye, 3, 1961.

6. A. Takano. On the Effect of Rarefaction of the Air on the Hypersonic Laminar Boundary Layer over a Flat Plate Parallel to the Free Stream. Proc. 9-th Japan Natl. Congr. Appl. Mech. Tokyo, 191-200, 1959.

7. W. D. Hayes and R. F. Robstein. Hypersonic Flow Theory. Academic Press, New York, 1959.

8. T. C. Lin and R. E. Street. Effect of Variable Viscosity and Thermal Conductivity on High-Speed Flow between Concentric Cylinders. Nat. Adv. Comm. For. Aeronaut., Rep. N 1175, 1954.

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE	Nr. Copies
HEADQUARTERS USAF	
AFCIN-3D2	1
ARL (ARB)	1
OTHER AGENCIES	
CIA	1
NSA	6
DIA	9
AID	2
OTS	2
AEC	2
PWS	1
NASA	1
ARMY (FSTC)	3
NAVY	3
NAFEC	1
RAND	1
AFCRL (CRXLR)	1

MAJOR AIR COMMANDS	Nr. Copies
AFSC	
SCFDD	1
DDC	25
TDBTL	5
TDBDP	6
AEDC (AEY)	1
SSD (SSF)	2
AFFTC (FTY)	1
AFSWC (SWF)	1
ASD (ASYIM)	1