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A VERY SENSITIVE AMPHIBIOUS LOOP MAGNETOMETER

30 APRIL 1963

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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A VERY SENSITIVE AMPHIBIOUS LOOP MAGNETOMETER

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ABSTRACT: A very sensitive loop magnetometer has been built for use at any sea depth and in air. The design uses single turn loops, a novel transformer, and a vacuum tube input. The effective frequency range is from 1 - 5000 cps but may be extended.

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The work reported herein was carried out in the Electromagnetics Division of the Physics Research Department and supported by BuWeps Task No. RUDC 4B 000.

This report contains a detailed description of the techniques which were developed to design a very sensitive underwater magnetometer. The material is revised and expanded from an earlier report, NOLTR 61-7.

A portion of the work reported herein is being published in ELECTRONICS Magazine.

R. E. ODENING
Captain, USN
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By direction
NOI.TR 63-102

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A VERY SENSITIVE AMPHIBIOUS LOOP MAGNETOMETER

Introduction and Overall Description

Project Deep Dip of the U. S. Naval Ordnance Laboratory, White Oak, has produced an undersea loop detector of exceptional sensitivity and noise figure. It was designed to be carried by the Automatic Instrumented Diving Assembly (AIDA), the Deep Dip diving vehicle, to any depth in the ocean. There it will record electromagnetic data on a tape recorder and other recording devices carried in the pressure-proof instrument sphere of AIDA.

Like the other items used with AIDA, it is designed to withstand at least 20,000 psi of sea pressure, greater than that of any known sea depth. It will take ten tons in tension on its axis.

The ultimate sensitivity or minimum detectable signal, varies with frequency, f. In air, over the range f = 1 - 5,000 cps it is given approximately by

\[ \text{Minimum Detectable signal} = \frac{2}{f} \times 10^{-9} \text{ gauss for a 1 cps bandwidth} \]  

The Deep Dip loop magnetometer can be used in air as well as in water except that without the shielding effect of a mile or more of salt water, atmospheric noise may determine the usable sensitivity. The present size was chosen as the maximum which could be readily carried in a truck. The principles of design may easily be adapted to other applications. They are best illustrated by a description and discussion of the present magnetic loop detector.

The Deep Dip electromagnetic loop antenna consists of 3 concentric, mutually perpendicular, 7 ft. dia. loops made of commercial "Mineral Insulated" #0000 copper power line cable. A single turn is used for each loop. The copper sheath over the magnesium oxide insulation (O.D. 0.70 inch) is wrapped with glass cloth impregnated with epoxy resin, giving 1.5 inches total diameter. The three loops are insulated and bonded to each other where they cross. The rigid fiberglass casing on the copper and MgO₂ core provides a structure not apt to be deformed by accidental abuse. No corrosion current can arise on the loops. A vertical axis of fiberglass tubing gives the antenna structural stability and great strength in tension.
The pressure proof amplifier case on the top end of the vertical axis holds three 1:30,000 step-up transformers designed for a 1 milliohm source impedance between 1 and 1,000 cps. Each pair of loop ends pass into an oil filled junction box under the amplifier case where the loop ends are anchored and the loop signals are led through pressure seals to the transformers inside the pressure proof case.

The copper shield of the commercial "Mineral Insulated" cable is electrically connected to the amplifier case at one end where it is led into the oil filled junction box through a commercial connector of brass. The other end of the cable is led into the junction box through a specially machined connector of fiberglass which insulates it from the case. The attached cable and connectors are then wrapped with fiberglass.

The output of each transformer is brought directly to the first grid of a 6072 low noise dual triode used in a two stage preamplifier.

The preamplifier output is taken through pressure seals to an oil filled junction box on top of the pressure case. A readily changed rubber insulated cable brings battery power from the large instrument sphere of the diving vehicle and carries back the preamplifier output to the recorders in the sphere. An opening in the junction box, capped by a rubber bag, transmits the sea pressure to the oil, and yields when the oil expands or contracts with temperature or pressure. There is very little differential pressure on the cables or on the stuffing glands or connectors, through which they enter the junction boxes.

The vector sum of the amplifier outputs of the three mutually perpendicular loops provide an omnidirectional detector giving magnitude and direction of the incident field. The result of simply adding the three outputs would be equivalent to a single loop with a new null direction intermediate to the separate nulls.
Definition of Ultimate Sensitivity

The minimum detectable signal or ultimate sensitivity of a loop detector is defined as that level of alternating magnetic field which induces a signal at the output equal to the self-noise at the output.* The field is the component along the loop's axis. Detector self noise varies with bandwidth, while induced signal is proportional to frequency so that the signal frequency and detector bandwidth must be specified when comparing loop performance.

The Deep Dip magnetometer was designed for use under miles of salt water where atmospheric and industrial noise are very low. While experimental checks on the ultimate sensitivity in air will be described, the limit to sensitivity must first be calculated and the physical factors making for sensitivity, clarified.

Basic Loop Sensitivity

A copper loop in a varying magnetic field has an inherent limit to sensitivity; the induced signal voltage $E$, must equal or exceed the thermal noise voltage $e$, of the loop's copper. By equating the expression for $E$ to that for $e$, the ultimate detectable signal is determined:

$$E = \frac{\mu_0 A}{2} f H \text{ in MKS units}$$

$$= 6.201 f H D^2 \times 10^{-6} \text{ volts rms}$$ (2)

where

$f$ = frequency of signal

$\mu_0 = 4\pi \times 10^{-7}$

$A$ = loop area $= \pi/4 \cdot D^2$

$D$ = diameter $= 7 \text{ ft.} = 2.133 \text{ meters}$

$H$ = rms magnetic field intensity

*This definition is somewhat arbitrary since a signal below the noise level may also be detected, but it is a convenient and definite standard for comparison.
The thermal noise voltage, \( e \), of any device of resistance \( R \), is an unavoidable quantity, not dependent on the composition. If \( R \) is constant over the frequency range of bandwidth \( B \), the noise is given by

\[
e = \sqrt{4KTB\ln2} \text{ volts rms} = 1.28 \times 10^{-10} \sqrt{RB}
\]

where

\[ K = \text{Boltzman's constant} = 1.38 \times 10^{-23} \]

\[ T = \text{temperature of resistor in } \degree \text{Celsius} = 273 + 298 = 298 \]

Expressing loop resistance \( R \) in terms of length, cross section, and resistivity

\[
R = \frac{4}{\pi} a^2 \pi D \rho = \frac{4D \rho}{a^2}
\]

where

\[ a = \text{wire diameter} = 0.46'' = 1.168 \times 10^{-2} \text{ meters} \]

\[ \rho = \text{specific resistance} = 1.7 \times 10^{-8} \text{ for copper} \]

Substituting (4) in (3)

\[
e = 2.56 D^2/a \sqrt{\rho B} \times 10^{-10}
\]

Setting \( E = e \) and simplifying

\[
H = \frac{4.13 \sqrt{\rho B}}{a D^{3/2}} \times 10^{-5}
\]

\[
= \frac{1.51}{f} \times 10^{-7} \text{ ampere turns/meter for a 1 cps bandwidth}
\]

\[
= \frac{1.9 \times 10^{-9}}{f} \text{ gauss}
\]

\[
= \frac{1.9 \times 10^{-4}}{f} \text{ gamma}
\]

Equation (5) shows the sensitivity to vary as \( a \) and \( D^{3/2} \). This basic sensitivity can be reduced but never improved by instrumentation after the loop.
The Transformer-Input Preamplifier

The minimum signal level of the single turn 7 ft. dia. -12 loops is very low. It equals the thermal level or $4.0 \times 10^{-12}$ volts for a 1 cps bandwidth and $4.0 \times 10^{-11}$ volts for a 100 cps bandwidth. Such a signal would be lost in the input noise of practically any amplifier or electronic instrument. A transformer with very high step up is capable of raising the voltage output of the loop (thermal noise and signal alike) well above the input noise of a vacuum tube. The additional transformer noise can be negligible. It is essential that the transformer carry little or no direct current so that the flux density in the core is low, below the hysteresis level. Otherwise, abrupt jumps in magnetization (Barkhausen effect) may generate a relatively large noise voltage. Also, eddy current losses are equivalent to a resistance at higher frequency and are capable of producing higher frequency thermal noise. Only the secondary resistance is appreciable in the Deep Dip transformer. The transformer input pre-amplifier of Figure 2 with a voltage gain of $4.5 \times 10^6$ will bring the minimum signal well above the input noise of an ordinary amplifier, and so of all electronic instruments. However, the preamplifier tubes will add appreciable noise near 1 cps.

The noise of a vacuum tube is usually expressed as that value of grid resistor which at room temperature would produce a thermal noise output equal to that observed.

The "shot noise" of a triode is inherent, its value is given by

$$r_s = \frac{2.5}{\text{transconductance}} \text{ ohms} = 1600 \text{ ohms for a 6072 triode} \quad (6)$$

Especially for oxide coated cathodes there is an additional random noise voltage $e_\phi$ due to "flicker effect" in which $e_\phi$ varies inversely with frequency. Using a narrow band harmonic analyzer, measurement shows the equivalent flicker noise resistor, $r_\phi$, to be about $5 \times 10^4$ ohms at 20 cps.

By equation (3) $r_\phi \propto e_\phi^2$

but $e_\phi \propto 1$, then $e_\phi = \frac{\text{constant}}{f}$ \quad (7)

Evaluating the constant by substituting noise value measured at $f = 20$. 

5
The total noise resistance of the tube is then

\[ r_s + r_\phi = 1600 + \frac{10^6}{f} \text{ ohms} \]  

(9)

Overall Detector Sensitivity

The overall detector sensitivity will now be calculated by setting the stepped-up loop signal at the first grid equal to the sum of the stepped-up loop thermal noise, the transformer noise and the equivalent input noise of the first tube. The second triode too adds output noise voltage, but less by a factor of 12, the gain of one stage. When unrelated noise voltages \( V_1 \) and \( V_2 \) are added, the sum is given by

\[ \sqrt{V_1^2 + V_2^2} \]

The second tube increases the tube noise voltage at the output by a factor of

\[ \sqrt{1 + (1/12)^2} \]

or only 1.0037.

The equivalent circuit brought to the tube grid is given by Figure 3. Let it be taken as a fact of measurement and calculation that up to several hundred cps, the net effect of the reactances in Figure 3a is to give an imperceptible rise in output voltage, and a decrease at higher frequencies where the step-up is more than adequate. The equivalent noise circuit may then be simplified to Figure 3b.

The transformer has a step up \( N = 3 \times 10^4 \) for voltage, impedances are therefore stepped up by a factor of \( N^2 \) or \( 9 \times 10^8 \).

By equation (2) and Figure 3 the signal brought to the grid is
NE = 0.201 \times \text{fHD}^2 \times 10^{-6} \times N = 8.47 \times 10^{-1} \text{fH}^* \tag{10}

\text{loop resistance} = 1.1 \times 10^{-3} \text{ ohms}

\text{Summing noise sources:}

\text{Transformed loop resistance} = 1.1 \times 10^{-3} \times N^2

= 9.9 \times 10^5 \text{ ohms}

\text{Transformer secondary resistance} = 1.2 \times 10^4 \text{ ohms}

\text{Equivalent triode noise resistance} = 1.6 \times 10^3 + \frac{10^6}{f} \text{ ohms}

\text{Total noise resistance} R = 10^6 + \frac{10^6}{f} = 10^6 \left(1 + \frac{1}{f}\right) \text{ ohms} \tag{11}

When \( R \) is not constant over a frequency band from \( f_o \) to \( f_1 \), the thermal noise generated in that band is

\[ e^2 = 4KT \int_{f_o}^{f_1} Rdf \tag{12} \]

Substituting equation (11) in equation (12), integrating and setting \( e = NE \)

\[ H = \frac{1.51}{f} \times 10^{-7} \sqrt{\left(\frac{f_1-f_o}{f_o}\right) + \log_e \frac{f_1}{f_o}} \text{ ampere turns/meter} \tag{13a} \]

\[ = \frac{1.9}{f} \times 10^{-9} \sqrt{\left(\frac{f_1-f_o}{f_o}\right) + \log_e \frac{f_1}{f_o}} \text{ gauss} \tag{13b} \]

\( f \) = signal frequency in the pass band \( f_1-f_o \)

For a 1 cps bandwidth the minimum detectable field is

\[ H = \frac{1.51}{f} \sqrt{1 + \log_e \frac{f_1}{f_o}} \times 10^{-7} \text{ a/m} = \frac{1.9}{f} \sqrt{1 + \log_e \frac{f_1}{f_o}} \times 10^{-9} \text{ gauss} \tag{14} \]

The quantity \( NE \times 8.7 \text{fH volts/amp-turn/meter} \) is the "voltage sensitivity" of the detector at the tube input. When multiplied by the gain of the preamplifier and succeeding amplifiers, it is the overall voltage sensitivity of the detector. It applies only to a signal above the minimum detectable level.
The quantity under the radical is the overall noise factor of the magnetometer compared to the copper loops. It is due to flicker noise in the first tube.

Between 1 and 2 cps the noise factor is 1.69 or 2.3 dB
between 10 and 11 cps 1.086 or 0.36 dB
at 100 cps 1.01 or 0.05 dB

The loops may be used at frequencies below $f_L$, the half power point, where the transformer primary impedance equals the loop resistance. To calculate the reduced sensitivity, the induced voltage $NE$ and the transformed loop resistance are both divided by

$$1 + \frac{f_L}{f}.$$ 

The resulting general expression is cumbersome, but for frequencies well below $f_L$, a good approximation is

$$H = \frac{1.9}{f} \sqrt{(f_L-f_0) + \log \frac{f_1}{f_0}} \sqrt{1 + \frac{f_L}{f}} \times 10^{-9} \text{ gauss} \ (15)$$

$$f_L \approx 0.7 \text{ cps}$$

The Verification of Sensitivity

It is usually impractical to measure sensitivity directly, by noting the obliteration of a known signal in the thermal noise background. Atmospheric and industrial noise are too high. But, if the voltage response of the transformer and loop are verified at a higher signal level and the input noise of the vacuum tube amplifier is known, the ultimate sensitivity of the combination is determined. This assumes negligible Barkhausen noise from the transformer, an attainable situation.

The gain of the transformer was checked by driving it from a 1 milliohm source as shown in Figure 4. The apparent gain of 27,500 is due to the loading of the 1 milliohm source by 11 milliohms, the transformed resistance of the 10 megohm voltmeter. The voltage ratio between the standard and 1 milliohm resistor was found by sending a heavy 60 cycle current through the pair and measuring with a calibrated multirange VTVM. The signal source was inadequate to permit direct reading on the primary.
A second check was made by raking a 7 ft. loop and transformer preamplifier far back into the wooded area of the Laboratory. A direction of minimum power line pickup was found and the amplifier output was put through a narrow band wave analyzer at frequencies between the power line harmonics. Signals were generated by smaller loops at some distance, passing known currents. Formulas are available for the magnetic field produced.\(^3\) The voltage sensitivity was verified.

Under favorable conditions a direct check of ultimate sensitivity might be possible with this procedure.

Finally, at sea with power generators off, a minimum noise level was observed corresponding to that calculated for the thermal noise of the loop.

Loop Impedance When Immersed

In addition to copper resistance and radiation resistance, a loop immersed in sea water has a resistance due to eddy currents induced in the water by the magnetic field of any current in the wire. The loss resistance has thermal noise even though negligible current flows in the loop. Also the eddy current's magnetic field partially annuls the generating field, causing a reduction in inductance compared to that in air.

The impedance of a conductor in sea water with heavy insulation is less affected by eddy current than a thinly insulated conductor, since the water is displaced from the most intense field.

The exact impedance formula for a heavily insulated loop in sea water is not available. However, the impedance lies between two limits which have been solved, a thinly insulated loop,\(^4\) and a loop lying inside a dielectric sphere.\(^5\) The first case gives the maximum possible increase in resistance and reduction in inductance.

First, the radiation resistance in air is

\[
R_{\text{air}} = \frac{\pi \omega^4 \mu \alpha^4}{6 \epsilon^3} \text{ ohms}
\]

\[= 5 \times 10^{-29} \text{ f}^4 \text{ ohms for the 7 ft. loop}
\]

\[a = \text{loop radius} = 1.067 \text{ m}
\]
The total induced resistance of a thinly insulated loop in sea water is

\[ R_{\text{sea}} = \frac{C \mu_0 a^3 \beta^2}{3} (4 - \pi \beta a) + R_{\text{air}} \beta = \frac{C \mu_0 a^3 \beta^2}{2} \]  \hspace{1cm} (17)

\( \sigma \) = sea water conductivity

\( = 4 \text{ mhos/meter} \)

The inductance of the loop in air is

\[ L_{\text{air}} = a \mu_0 \left( \ln \frac{8a}{b} - \frac{7}{4} \right) = 7.09 \mu \text{h} \]  \hspace{1cm} (18)

\( b \) = wire radius

The inductance in sea water is

\[ L_{\text{sea}} = L_{\text{air}} - \frac{C \mu_0 a^3 \beta^2}{3} \]  \hspace{1cm} (19)

The characteristics of the present loop are approximately the same in sea and air to 1KC.

Upper Frequency Sensitivity in Sea Water

When eddy current resistance is comparable to copper resistance in the loop, the ultimate sensitivity formula based on copper resistance, must be revised. Past the frequency where eddy current resistance exceeds the copper resistance, the sensitivity of a submerged loop detector will be constant:

Eddy current resistance increases (approximately at least) as \( f^2 \); Thermal noise voltage therefore increases as \( f \); Induced signal emf also increases as \( f \); therefore ultimate sensitivity of the loop is constant.
Choice of Loop Turns

The single turn of the Deep Dip detector loops may require an explanation: If a single loop of fixed copper cross section is divided into m turns, each turn will have m times the resistance of the full cross section while m turns in series will have $m^2$ times the resistance of one full cross section turn. The induced emf of m turns is multiplied by m while the inductance of m closely-wound turns is increased by $m^2$. In short, the m turn coil is equivalent to the single turn with an ideal transformer of step-up m, in voltage, resistance, inductance, and ultimate sensitivity, but not in distributed capacitance.

The distributed capacitance of an m turn coil is greater than that of an equivalent single loop, therefore the capacitance shunt reactance is lower. The resonant frequency is lower than in a single loop. In a transformed single loop the capacitative and inductive reactance are both multiplied by $m^2$ (and the capacitance by $1/m^2$) with the resonant frequency unchanged. Like the multi turn loop, an actual transformer will add distributed capacitance, but much less because of the smaller diameter on which the turns are wound.

It is assumed that the primary inductance is adequate for the lowest required frequency. In this connection, no advantage in smaller core size or fewer turns results from dividing a required step up between loop and transformer. For instance, if a 30,000 step up is required for a single loop of given diameter and cross section, then a 30 turn loop of the same diameter and copper cross section will have $(30)^2$ times the single loop impedance. Using the same core and secondary as for the single loop, the primary turns will have to be increased 30 times to have adequate impedance and the step up is reduced to 1,000.

If a transformer is used, it should best do the entire step up. The high frequency limit can be raised and a single turn, low impedance loop makes possible the use of the single turn primary.

There appears to be an advantage in eliminating the transformer and getting the necessary step up by winding the 7 ft. loops with 30,000 turns. But, such loops would each need 125 miles or 201 km. of very fine unbroken wire. The distributed capacitance would be very large and together with a 1 megohm resistance, the upper frequency response would be severely limited. With any potting method, electrical leakage between turns would be a problem.
By contrast, the 30,000 transformer turns are wound on a small diameter, making the distributed capacitance lower. The turns are protected by the thick primary and pressure case against mechanical stress and electrical leakage. The leakage problem on the immersed loops is eased by a factor of $9 \times 10^8$.

Transformer Ratio with Transistor and Tube

Vacuum tubes rather than transistors are used in the pre-amplifier; a much higher sensitivity is attainable at low frequencies. The Deep Dip loops are not matched to the vacuum tube inputs; matching is not applicable. An applicable case will illustrate the point.

If a power source has a fixed impedance and emf, then the maximum power will be dissipated in the load resistance when the load resistance equals the source resistance and the load reactance is equal and opposite to the source reactance (tuning).

Transistors are matched approximately to the signal source with transformers. The input is mainly resistive. Since transistors have semi-conductor noise in addition to the thermal, the optimum source resistance is a balance between signal loss due to mismatch and damping of excess noise by the input resistance.

Ideally, when matched, the source output voltage will drop by half. In parallel with the source, the total input resistance will drop by half and the thermal noise voltage to $1/\sqrt{2}$ of the source alone. The noise power to signal power ratio of the source will therefore increase by a factor of 2, or by 3db, in the ideal case. The excess noise of actual transistors follows a $1/f$ law like the flicker noise of vacuum tubes. Published measurements for the 2N2176 transistor, show a noise increase of 3db per octave as the frequency is decreased from a projected zero at 1,000 cps. Thus, the semiconductor noise at 1 cps is about 250 times greater than, or 24db above, the thermal noise.

The noise figure, or ratio of actual noise to unavoidable thermal noise, is not much changed by using a low impedance or a high impedance transistor circuit. When matched to the input, the transistor noise must equal and may far exceed, the source thermal noise.
The GL6072 dual triode (interchangeable with the common 12AY7) has an input impedance of about 100 megohms at 100 cps, mainly capacitative reactance. Since reactance has no thermal noise, the equivalent input noise resistance need not be correspondingly large. Together, shot noise and flicker noise total 11,600 ohms at 100 cps. At 1 cps the reactance is 10,000 megohms and the equivalent flicker noise resistance, 1 megohm.

The combination of very high capacitative reactance at the input of a vacuum tube and the relatively low equivalent noise resistance, makes it possible for a transformer to raise the signal and thermal noise of the loop far above the noise level of the tube. It could not be done with a transistor.

The 30,000:1 ratio of the present transformers, transforms the 1.1 milliohm loop into 1 megohm. The thermal noise of the loops therefore far exceeds the tube noise except at the lowest frequencies. Tube noise and loop thermal noise are equal at 1 cps. Above 100 cps a step up of about 3,000 would give an acceptable to excellent noise figure. It is for this reason that the magnetometers are not derated in sensitivity due to the capacitative transformer roll-off past 1,000 cps. The response or "voltage sensitivity" does not continue to rise with frequency at the initial rate, but that can be compensated.

Increasing the step up to 300,000 would improve the 1-2 cps noise figure from 2.3db to 0.05db, a rather small advantage. The higher frequency limit would suffer because of the added distributed capacitance. Also the grid current tolerance would be reduced. If the primary inductance was made adequate for 0.01 cps by increasing the supermalloy toroid cross section, then a 300,000 step up would give a 2.3db noise figure at 0.01 cps. However, it could not increase the basic loop sensitivity. Only more area and more copper could do that.

For ultra low frequencies an attractive alternative to higher step up is the use of a low flicker tungsten filament for this input. With the present ratio, a 20X increase in shot noise voltage could profitably be traded for a 10X decrease in flicker noise voltage. Some electrometer tubes have tungsten filaments with negligible grid current and input admittance. If desired, extreme step up could be used.

In summary, while the step up from a very low impedance to a vacuum tube can be made very high, the ratio should be no higher than that required to overcome the flicker noise at the lowest frequency of interest.
The Use of Semiconductor Amplifiers

Let \( n \) equal amplifiers be driven by a common signal generator. Each amplifier output has \( r \) ohms internal impedance and an \( \mathcal{L} \) ohm load. Figures 5a and 5b give the equivalent output circuit. With any set of output voltages, the parallel sum at any instant will be the arithmetical mean. If the signal source produces \( v \) volts across each load, then the voltage of the outputs connected in parallel (Figure 5c) will also be \( v \) volts. It does not matter if the common input signal is sinusoidal, a direct voltage, or an erratic noise source; so long as the output signals are coherent or equal at every instant, the parallel voltage is the same as the single voltage with the impedance reduced to \( r' / n \).

If, however, the r.m.s. self noise of each amplifier is \( s \) volts open circuit and \( s' \) across the load \( \mathcal{L} \), then when \( n \) loaded amplifiers are placed in parallel, the r.m.s. noise voltage will drop to \( s' / \sqrt{n} \) while a common-input-signal component will remain at the same value as for one. The difference follows because while the r.m.s. self noise is the same for each amplifier over a long time average, there is, in general, no instantaneous coincidence.

The burden of a priori proof will be shifted to an accepted similar case:

Above the flicker noise frequency range, where the amplifier noise is uniformly distributed, a resistor of \( \mathcal{L} \) ohms can be raised to a temperature to yield a thermal noise equal to \( s' \) volts r.m.s. over the bandwidth considered. Since hot resistors in parallel equal \( \mathcal{L}' / \mathcal{L} \) ohms, the thermal noise voltage (by \( e^2 = 4kTBR \)) is correspondingly reduced to \( s' / \sqrt{n} \), which was to be shown.

In the flicker noise frequency range, the noise is taken as constant over a narrow bandwidth. A resistor of \( \mathcal{L} \) ohms may be raised to a higher temperature to equal the flicker and shot noise. The previous reasoning then applies to the parallel case and may be used repeatedly to cover the range.

There is no essential difference between paralleling vacuum tubes for a low noise input as above, and combining them in one envelope to make a tube with \( n \) times the transconductance (\( G_m \)). This is equivalent to the triode noise law \( \frac{e^2}{G_m} \approx 2.5 \); because since input noise \( e^2 \propto R_{eg} \), then \( e \propto \sqrt{G_m} \approx \sqrt{n} \).
The principle of adding a number of signal sources to diminish the ratio of incoherent noise is very basic in designing for sensitivity.

No low-noise advantage results from connecting transistors in parallel. Since these are already matched to the signal source, input power must be divided between them. The number of signal carriers would not be increased.

For low-noise applications, the common transistor has an inherent disadvantage compared to a tube. However, the new "field-effect" transistors are in principle equivalent to a vacuum tube in that the input impedance can be largely reactive while the equivalent noise resistor may be considerably lower. The varactor diode too, used as a high frequency valve, can provide high input impedance with low noise. In principle, such solid state devices can combine the advantages of tube and transistor.

Constructional Details

An essential item in the performance of the Deep Dip detector is the design of the transformer, coupling heavy copper loop to vacuum tube grid.

The primary consists of a two piece copper cylinder hollowed out to fit a toroidal core of supermalloy, Figure 6. The core, a commercial item, is made of a 2 mil thick tape of supermalloy wound into a roll and annealed at high temperature. Zirconium oxide powder dusted between the layers acts as an insulator to reduce eddy current losses. It withstands the annealing temperature.

The roll is fitted into a plastic shell to protect the supermalloy from mechanical stresses. Thirty thousand turns of #42 wire are wound on the plastic case in six sections to reduce distributed capacitance and terminated in two pigtails. The windings are wrapped with thin plastic tape. The core is inserted into the cylinder with the pigtails drawn through a small hole to two terminals on the blank end of the primary.

The angle of the transformer lid is cut with the same lathe setting as used for the mating surface on the cylinder, after which the parts are silver plated for low surface resistance. As the conical lid is wedged into the mating cylinder by the twelve bolts shown, the cylinder is stretched and the J factor noticeably increased, insuring negligible contact resistance.
A stranded flexible section of welding cable connects the rigid loop to the transformer through conical connectors. With careful machining, the silver plated connectors require no mercury wetting for good contact. For some applications the flexible cable could be soldered directly into the primary before it was assembled.

The transformer is fastened to a fiberglass plate by four #10 screws which enter tapped holes in the primary.

(2) Lawrence Baker Arguimbau, Vacuum-Tube Circuits, John Wiley and Sons, Inc., 1948, Chap III, Sec. 13, p. 108, Formula (94)

(3) James R. Wait, Mutual Electromagnetic Coupling of Loops Over a Homogeneous Ground, Geophysics, Vol. XX, No. 3 July, 1955


(6) A. Silverstein, Building and Using Dielectric Amplifiers, Electronics, Feb., 1954
FIG. 1  AIDA LOOP MAGNETOMETER CONFIGURATION

- CONDUCTIVITY SWITCH
- BEACON TRANSMITTER
- BUOY, 1000 GAL. OIL
- SWIVEL HOOK
- PRESSURE GAUGE-THERMOMETER
- INSTRUMENTATION SPHERE
- JUNCTION BOX
- STEEL CABLE
- AMPLIFIER
- LOOP ANTENNA
- RELEASE MECHANISM
- WEIGHTED CARGO SLING

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FIG. 2 TRANSFORMER INPUT PREAMPLIFIER, ELECTRICAL SCHEMATIC
FIG. 3a EQUIVALENT CIRCUIT AT TUBE GRID

**E** = Induced E M F in Antenna Loop

**N** = Transformer Turns Ratio

**L_p** = Primary Inductance

**L_o** = Antenna Inductance

**L_s** = Leakage Inductance

**C_o** = Antenna Capacity

**C_s** = Secondary Shunt Capacitance

**R_s** = Secondary Resistance

**R_o** = Antenna Resistance

FIG 3b SIMPLIFIED EQUIVALENT CIRCUIT

\[ R = 1600 + 10^6 \text{ f} \]
FIG. 4 TRANSFORMER GAIN AND RESPONSE
\[ S = \text{RMS AMPLIFIER SELF NOISE OUTPUT EMF} \]
\[ S' = \text{SELF NOISE ACROSS LOAD} \]
\[ r = \text{AMPLIFIER INTERNAL RESISTANCE} \]
\[ l = \text{LOAD RESISTANCE} \]
\[ l' = \text{NET RESISTANCE OF LOADED OUTPUT} \]

\[ S' = \frac{S' l}{l + r} \]
\[ f' = \frac{lr}{l + r} \]

**FIG. 5a EQUIVALENT OUTPUT CIRCUIT**

**FIG. 5b SIMPLIFIED EQUIVALENT OUTPUT CIRCUIT**

**FIG. 5c PARALLEL CONNECTION OF OUTPUTS**
FIG. 6  TOROIDAL CORE TRANSFORMER
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US27
| 1. | Magnetometers, Loop |
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**Title**

A very sensitive amperometric loop magnetometer has been built for use at any sea depth and in air. The design uses single turn loops, a novel transformer, and a vacuum tube input. The effective frequency range is from 1 - 5000 ops but may be extended.

Abstract card is unclassified.