NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
AN AUTOMATIC SYSTEM FOR MEASURING PLASMA PARAMETERS

Prepared for:

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
LAURENCE G. HANSCOM FIELD
BEDFORD, MASSACHUSETTS

By: W. E. Scharfman
Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies should be directed to the:

DEFENSE DOCUMENTATION CENTER (DDC)
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

Department of Defense contractors must be established for DDC services or have their 'need-to-know' certified by the cognizant military agency of their project or contract.

All other persons and organizations should apply to the:

U.S. DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES
WASHINGTON 25, D.C.
AN AUTOMATIC SYSTEM FOR MEASURING PLASMA PARAMETERS

Prepared for:
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
LAURENCE G. HANSCOM FIELD
BEDFORD, MASSACHUSETTS

By: W. E. Scharfman
SRI Project No. 3977

Approved:

T. Morita, Manager
ELECTROMAGNETIC SCIENCES LABORATORY

D. R. Schedl, Director
ELECTRONIC AND RADIO SCIENCES DIVISION

May 1963

AFCRL-63-155
Technical Report 76

CONTRACT AF 19(628)-325
PROJECT 4600
TASK 460001

Copy No. 68
ABSTRACT

An inexpensive system for automatic measurement of microwave phase shift is described. The system requires for its construction only components that would be required if a manual system were used, plus a servo loop. The phase is controlled by controlling the reflector voltage on a klystron, which in turn controls the microwave frequency.

A description is given of a prototype system that was constructed and tested. Measured results of phase shift as a function of control voltage and attenuation are presented.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>iv</td>
</tr>
<tr>
<td>I INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II BRIDGE SYSTEM</td>
<td>2</td>
</tr>
<tr>
<td>III EXPERIMENTAL RESULTS</td>
<td>7</td>
</tr>
</tbody>
</table>
ILLUSTRATIONS

Fig. 1(a) Microwave System for Independent Measurement of Attenuation and Phase Shift .................. 2

Fig. 1(b) Vector Diagram of Voltages at Output Terminals of the Magic Tee ...................... 2

Fig. 2 Method of Introducing Control Voltage into the Klystron Reflector Voltage Circuit .......... 6

Fig. 3 Microwave System for Automatic Measurement of Attenuation and Phase Shift .............. 7

Fig. 4 Phase as a Function of Control Voltage for the Automatic System ............... 8
(ΔL = 3 meters)

Fig. 5 Plot of Ratio of Plasma to Wave Frequency and Ratio of Collision to Wave Frequency as a Function of Normalized Attenuation in db/Radian and Normalized Phase Shift in Degrees/Radian ... 9

Fig. 6 Automatic System Using ac Amplifiers ............. 10
I INTRODUCTION

In order to determine the electron density and collision frequency of a plasma it is necessary to make two independent measurements. Very often this is performed by measuring the absorption and phase shift, at a single frequency, of an electromagnetic wave that is transmitted through the plasma. By constructing a bridge circuit in which the plasma is one arm and a calibrated phase shifter and calibrated attenuator are another arm, a sensitive instrument for measuring absorption and phase shift can be made. In this system, in order to null the bridge both the attenuator and the phase shifter must be adjusted. Both components must be adjusted before either the phase shift or attenuation of the plasma can be determined. The phase shift cannot be found independent of the attenuation.

In order to make such a system automatic, it would be necessary to have two servo loops, one controlling the attenuator and another controlling the phase shifter. An alternative system that does not require an adjustment of both components is described below. This system has the advantage that only one servo loop (on the phase shifter) is required. Further, in a low-collision plasma, the electron density can be found in terms of the phase shift alone. Thus, to determine the electron density, only the phase shift need be measured.
II BRIDGE SYSTEM

The basic system is shown in Fig. 1(a). A klystron oscillator supplies microwave power which is divided in a directional coupler between a path that goes through the plasma (Path A) and a path that does not (Path B). The two signals are recombined in a magic tee. As shown in Fig. 1(b), when the two signals are 90 degrees out of phase the resultant fields in the crossed arms of the magic tee, $|E_1|$ and $|E_2|$, are equal. If the voltage difference between $|E_1|$ and $|E_2|$ is monitored, it will indicate when the input signals are in phase quadrature. When
a plasma changes the phase along Path A, the phase shifter is adjusted until \( |E_1| - |E_2| = 0 \). The change in the phase shifter is equal to the phase shift due to the plasma.

In this system the phase at which \( |E_1| = |E_2| \) is independent of the amplitude of the signals entering the magic tee along Paths A and B. Thus, phase shift can be measured independent of attenuation. Attenuation could be measured by monitoring the amplitude of the received signal by a directional coupler in the receiving arm.

There are several ways that a voltage-controllable phase shifter can be made. Ferrite phase shifters are commercially available, in which the phase shift is changed by varying the current in an electromagnet. However, this type of phase shifter has a hysteresis characteristic so that the phase shift is a multiple-valued function of the current. For a phase shifter with about 300 degrees of total phase shift the hysteresis error could amount to about 30 degrees. Thus for a given value of current the phase shift would be uncertain over a 30-degree range.

Traveling-wave-tube amplifiers have a phase-shift characteristic which is a function of the helix voltage. Thus a traveling-wave-tube amplifier can be used as a voltage-controllable phase shifter. The cost of such a device makes it unattractive.

Voltage-controlled crystal phase shifters have been designed and are commercially available in the lower microwave frequencies; however, the cost of these devices at X-band is still high, especially if a large phase shift is required.

A novel, inexpensive way of achieving a voltage-controllable phase shifter using only components that would be necessary for a manual system has been developed. Its operation is as follows. If a signal at frequency \( f_1 \) propagates down a transmission line of length \( L_1 \) with a wavelength of \( \lambda_{g1} \), the total phase shift down the transmission line is \( 2\pi/\lambda_{g1} L_1 \). The total phase shift down a length of line \( L_2 \) is \( 2\pi/\lambda_{g1} L_2 \). The difference in phase, \( \Delta \phi \), between the signals traveling down \( L_1 \) and \( L_2 \) is \( 2\pi/\lambda_{g1} (L_1 - L_2) \). If the frequency is changed, the guide wavelength,
and hence \( \Delta \phi \), changes. Thus, by changing the frequency the differential phase shift can be changed. If \( L_1 - L_2 \) is many wavelengths long a small change in frequency will result in a large change in differential phase.

If \( L_1 \) and \( L_2 \) are the lengths of transmission line in Paths A and B, the phase difference between the signals from A and B at the magic tee will be

\[
\Delta \phi = \frac{2\pi}{\lambda_{g1}} (L_1 - L_2) + \phi_c + \phi_a
\]

where \( \phi_c \) = Phase-shifter setting
\( \phi_a \) = Phase shift across the transmission path between the antennas.

When \( \phi_c \) is adjusted so that \( \Delta \phi = 90^\circ \), \( |E_1| - |E_2| = 0 \). If a plasma changes \( \phi_a \) so that \( \Delta \phi \) is other than \( 90^\circ \), then \( \Delta \phi \) can be made equal to \( 90^\circ \) by changing the frequency.

The amount of phase shift for a given frequency change can be calculated as follows. The differential phase shift at the two frequencies is given by

\[
\delta \phi = 2\pi \left[ \frac{1}{\lambda_{g1}} - \frac{1}{\lambda_{g2}} \right] (L_1 - L_2)
\]

For a TE\(_{01}\) mode in a rectangular waveguide, the guide wavelength is

\[
\lambda_{g1} = \frac{1}{\sqrt{\left(\frac{f_1}{c}\right)^2 - \left(\frac{1}{2a}\right)^2}}
\]

where
\( \lambda_{g1} \) = Guide wavelength
\( f_1 \) = Frequency
\( c \) = Velocity of light
\( a \) = Waveguide width.
For small frequency changes, an approximate expression for the difference in wavelength may be found. Inserting this value in the expression for the differential phase we obtain

$$\delta \varphi = \frac{2\pi}{\lambda_\Delta} \frac{(L_1 - L_2)}{\sqrt{1 - \left(\frac{\lambda_\Delta}{2a}\right)^2}}$$

where

$$\lambda_\Delta = \Delta f/c$$
$$\Delta f = \text{the change in frequency.}$$

For example, a frequency change of 10 Mc at a frequency of 10 Gc will result in a phase change of 16.3 degrees/meter.

It is true that the plasma is dispersive so that the phase shift produced by it will also be frequency-dependent. However, the rate of change of phase constant through the plasma is of the same order as for a waveguide, so that if the plasma length is made much smaller than \((L_1 - L_2)\), the dispersive effect of the plasma will be negligible.

This method of obtaining a controllable phase shift could be used as follows. Assume the system is balanced before the plasma is introduced. Then, if the plasma introduces a phase change, \(E_1 \neq E_2\). This voltage difference can serve as an error signal to drive a feedback amplifier, which in turn can be used to change the frequency. As the frequency changes, \(E_1 - E_2 = 0\). For sufficiently high loop gain, \(E_1 - E_2\) will be driven very close to zero.

The next question to consider is how to use this error signal to change the klystron frequency. The frequency can be changed by varying either the beam or reflector voltages. The reflector circuit draws practically zero current so that an external voltage applied in the reflector voltage line will not need to supply current. The reflector is several hundred volts above ground so that introducing the error signal into the circuit would raise the crystals to a high potential above ground, introducing dangerous voltages into the waveguide system.
This danger can be avoided by supplying the reflector voltage from a separate battery supply. The error signal can then be introduced at the ground side of the supply and thus be at low potential. The current drain on the batteries will be very small. The circuit is shown in Fig. 2.

FIG. 2 METHOD OF INTRODUCING CONTROL VOLTAGE INTO THE KLYSTRON REFLECTOR VOLTAGE CIRCUIT
III EXPERIMENTAL RESULT

A phase-measuring system working in this manner has been constructed and tested. The over-all system is shown in the schematic diagram of Fig. 3. The voltage out of the dc amplifier raises and lowers the total reflector voltage, thus changing the frequency. By measuring $V_1$ as a function of $\phi$, the system can be calibrated in phase in terms of voltage.

![Schematic diagram of microwave system for automatic measurement of attenuation and phase shift]

**FIG. 3 MICROWAVE SYSTEM FOR AUTOMATIC MEASUREMENT OF ATTENUATION AND PHASE SHIFT**

The attenuation is measured by the drop in the level of $V_2$. The results for phase as a function of voltage are shown in Fig. 4. Since the loop gain is finite, the ability of the control system to drive $E_1 - E_2$ to a null depends to some degree upon the levels of the signals along Paths A and B. As the attenuation in Path A is increased, the value of $V_1$ for a given $\phi$ varies somewhat. This variation is illustrated in Fig. 4 for 0, 6, and 10 db attenuation.

If the 0-db calibration curve were used when there was actually 6 db of attenuation, the inferred phase would be in error by about 6 degrees. In fact, up to 15 db attenuation the calibration curves shift about 1 degree/db of attenuation. Of course, the attenuation is measured
FIG. 4 PHASE AS A FUNCTION OF CONTROL VOLTAGE FOR THE AUTOMATIC SYSTEM (ΔL = 3 meters)

simultaneously so that the shift in calibration curve can be corrected for. Even so, when the plasma produces changes in Path A that are greater than 1 degree/db the shift would be negligible.

The range of plasma parameters for which the plasma produces 10 degrees/db can be found from Fig. 5. This chart is useful for microwave diagnostic problems. It presents the normalized plasma parameters in terms of the measured parameters, attenuation and phase shift, normalized to the electrical length of the plasma. $P$ is the normalized phase shift in degrees per radian of path length. $A$ is the normalized attenuation in db per radian. When $P = 10A$, the plasma will produce 10 degrees/db for any plasma length. From the chart it can be determined
that $P > 10A$ for $Z < 0.6$ for all $\omega_p/\omega$ up to about 0.75. Thus, plasma parameters that are to the left of the $Z = 0.6$ line will give accurate readings with this phase-measuring instrument, without correction for attenuation.

The time response of this instrument is limited by the bandwidth of the dc amplifier (about 50 cycles). Although wider-band dc amplifiers could be used they are costly. Wideband ac amplifiers are more easily available. Thus, if we could generate an error signal as an ac signal, amplify it through an ac amplifier, and detect and smooth the output before applying it to the reflector, we could increase the bandwidth of the system. This could be done by introducing a microwave switch so that $E_1$ and $E_2$ were sampled periodically. If they were different, an ac signal would result. If they were equal, the ac signal would be zero. Such a system is sketched in Fig. 6.

![FIG. 6 AUTOMATIC SYSTEM USING AC AMPLIFIERS](image)
This system would also have the advantage of eliminating the need for matched crystal detectors. The detection is by a single crystal detector. This could be an important consideration if the system were to be used to measure large attenuations, since it may be difficult to get crystals that are matched over a wide range of signal levels.

One other problem that has not been mentioned is that when the reflector voltage is varied, the power output changes. This can amount to several decibels for large frequency shifts. If the attenuation is small, this could mask the true value of the attenuation. In order to eliminate this error the servo loop could be periodically opened. Under this condition the klystron would be transmitting at the same frequency and with the same power output regardless of the plasma shift. However, this error is less than the 1 degree/db due to the servo system, so that if that error is tolerable this effect will be negligible for measurements of phase shift.
TECHNICAL REPORTS IN THIS SERIES

Reports Issued on Contract AF 19(122)-78


20. (Dropped)


Reports Issued on Contract AF 19(604)-266


42. "Radiation Pattern Measurements of Stub and Slot Antennas on Spheres and Cylinders," by J. Bain, April 1953.


Reports Issued on Contract AF 19(604)-1296


Reports Issued on Contract AF 19(604)-3458


Reports Issued on Contract AF 19(628)-325

75. "One-Sided Multipactor Discharge Modes," by E. F. Vance and J. E. Nanevicz.
| AF Cambridge Research Laboratories, Bedford, Mass, | UNCLASSIFIED | AF Cambridge Research Laboratories, Bedford, Mass, |
| AN AUTOMATIC SYSTEM FOR MEASURING PLASMA PARAM- | 1. Microwaves | AN AUTOMATIC SYSTEM FOR MEASURING PLASMA PARAM-
| An inexpensive system for automatic measurement of | I Stanford Research | An inexpensive system for automatic measurement of |
| microwave phase shift is described. The system | Institute, Menlo Park | microwave phase shift is described. The system |
| requires for its construction only components that | California | requires for its construction only components that |
| would be required if a manual system were used, | II Scharfman, W. E. | would be required if a manual system were used, |
| plus a servo loop. The phase is controlled by | III Contract AF 19(628)- | plus a servo loop. The phase is controlled by |
| controlling the reflector voltage on a klystron, | 325 Project 4600, | controlling the reflector voltage on a klystron, |
| which in turn controls the microwave frequency. A | Task 460001 | which in turn controls the microwave frequency. A |
| description is given of a prototype system that | | description is given of a prototype system that |
| was constructed and tested. Measured results of | | was constructed and tested. Measured results of |
| phase shift as a function of control voltage and | | phase shift as a function of control voltage and |
| attenuation are presented. | | attenuation are presented. |

| AF Cambridge Research Laboratories, Bedford, Mass, | UNCLASSIFIED | AF Cambridge Research Laboratories, Bedford, Mass, |
| AN AUTOMATIC SYSTEM FOR MEASURING PLASMA PARAM- | 1. Microwaves | AN AUTOMATIC SYSTEM FOR MEASURING PLASMA PARAM-
| An inexpensive system for automatic measurement of | I Stanford Research | An inexpensive system for automatic measurement of |
| microwave phase shift is described. The system | Institute, Menlo Park | microwave phase shift is described. The system |
| requires for its construction only components that | California | requires for its construction only components that |
| would be required if a manual system were used, | II Scharfman, W. E. | would be required if a manual system were used, |
| plus a servo loop. The phase is controlled by | III Contract AF 19(628)- | plus a servo loop. The phase is controlled by |
| controlling the reflector voltage on a klystron, | 325 Project 4600, | controlling the reflector voltage on a klystron, |
| which in turn controls the microwave frequency. A | Task 460001 | which in turn controls the microwave frequency. A |
| description is given of a prototype system that | | description is given of a prototype system that |
| was constructed and tested. Measured results of | | was constructed and tested. Measured results of |
| phase shift as a function of control voltage and | | phase shift as a function of control voltage and |
| attenuation are presented. | | attenuation are presented. |
Regional Offices and Laboratories

Southern California Laboratories
820 Mission Street
South Pasadena, California

Washington Office
808 17th Street, N.W.
Washington 6, D.C.

New York Office
270 Park Avenue, Room 1770
New York 17, New York

Detroit Office
1000 East Maple Road
Birmingham, Michigan

European Office
Pelikanstrasse 37
Zurich 1, Switzerland

Japan Office
911 Iino Building
22, 2-chome, Uchisaiwai-cho, Chiyoda-ku
Tokyo, Japan

Representatives

Honolulu, Hawaii
1225 Ala Moana Blvd.
Honolulu, Hawaii

London, England
19, Upper Brook Street
London, W. 1, England

Milan, Italy
Via Macedonio Melloni, 49
Milano, Italy

Toronto, Ontario, Canada
Room 710, 67 Yonge St.
Toronto, Ontario, Canada