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ABSTRACT

A shock test is frequently specified as an acceleration pulse of controlled shape and magnitude and which has a desired shock spectrum. A description is given as to how the Navy High-Impact Shock Machine for Medium weight Equipment can be made to provide acceleration pulses of a half-sine or sawtooth shape by inserting elastic or plastic materials between various impacting surfaces. Details as to the shape of the inserts are given for nominal 6-millisecond sawtooth pulses having amplitudes up to 60 g, and for relatively long (10-20 milliseconds) low-amplitude half-sine pulses. A maximum velocity-change up to 8 ft/sec is possible. The acceleration amplitudes involved in stopping the anvil are generally small compared with the initial pulse. Displacements are limited to 3 inches.

PROBLEM STATUS

This is an interim report on one phase of the problem; work is continuing.

AUTHORIZATION

NRL Problem F03-02
Project RR 009-03-45-5754 (RN)
and
NRL Problem F03-08
BuShips Project SF 013-10-01, Task 1804

SAWTOOTH AND HALF-SINE SHOCK IMPULSES FROM THE NAVY SHOCK MACHINE FOR MEDIUMWEIGHT EQUIPMENT

INTRODUCTION

Specifications for shock tests are sometimes given in terms of simple acceleration pulses. Two of the most commonly used acceleration pulses are the half-period sine pulse and the sawtooth pulse. The "half-sine" pulse is, perhaps, the easiest type to generate inasmuch as a mass which is made to impact against one end of a linear spring whose opposite end is fixed experiences a half-sine acceleration pulse.

The residual shock spectrum of any symmetrical shock pulse is periodically zero at certain multiples of the pulse period. This leads to less damage potential for the shock pulse at corresponding frequencies. Moreover the positive and negative values of shock spectra are different for any given symmetrical pulse. This makes it necessary to test in both directions along any one axis. It has been pointed out (5) that an asymmetrical shock pulse, such as the sawtooth pulse, has equal positive and negative spectra and that the residual spectra remain at a relatively uniform value for frequencies higher than the reciprocal of the pulse length. For these reasons the sawtooth pulse is frequently preferred as a test specification. The sawtooth pulse can be obtained by impacting a mass against a suitably shaped plastic material. For a nonelastic impact the force drops abruptly to zero as soon as the relative impact velocity becomes zero.

TECHNIQUE FOR OBTAINING PULSE SHAPES

The Shock Machine and Its Operation

The Navy High-Impact Shock Machine for Mediumweight Equipment can be used to generate both half-sine and sawtooth type pulses. As is required, this can be done with no modification to the machine itself; hence the techniques must be considered merely as improvisations to broaden the original intended use of the machine. Different techniques would be used, of course, if new designs or modifications of the machine were permitted.

A previous report (1) describes the Shock Machine for Mediumweight Equipment. The shock-table (anvil) of the machine is constrained to move in a vertical direction for a maximum distance of 3 inches between stops. This is shown schematically in Fig. 1. For normal operation a steel hammer impacts solidly against the steel anvil and the anvil attains its full upward velocity in about 1 millisecond. It travels through the clearance distance until it impacts against the top stops. It then rebounds against the bottom stops, upon which it gradually comes to rest.

*The pulses are only nominally of simple shape. Many factors that cannot easily be controlled affect their shapes. When the distortions are small compared with the magnitude of the pulse, and when the setup is such that the pulse can be duplicated and repeated with precision, and it is impractical to make further improvements, then the pulse must be considered acceptable.
†Since NRL Report 5618 describes the Navy high-impact shock machines and shock spectra, these descriptions will not be given here. For additional information on the subject of shock spectra see Refs. 1-5.
Fig. 1 - Schematic diagram of the anvil table of the Shock Machine for Mediumweight Equipment. The table is constrained to move in a vertical direction between the bottom and top stops. The total clearance is about 3 inches. Plastic or elastic elements are placed in positions 1, 2, and 3, called the hammer-impact, the top-stop, and the bottom-stop positions respectively.

For normal operation an equipment under test is not attached directly to the anvil, but is attached to suitable channels or other standard fixtures that are sufficiently flexible to provide some degree of isolation from the effects of the hard impacts.

When the shock machine is used as a generator of simple acceleration pulses, the equipment is attached rigidly to the anvil table. No flexible fixtures intervene unless they are considered part of the equipment.

Plastic Elements

The plastic elements used are shown in Fig. 2, where (a) illustrates the anvil plastic element, (b) the four bottom-stop elements, and (c) the four top-stop elements. The plastic material used was 50-50 (lead-tin) solder. Pure lead was too soft. The solder was more suitable and was easily available. The anvil plastic element is a cylinder with a conical top as shown in Fig. 2a and Fig. 3.

Fig. 2 - Plastic elements. The lower part of the figure illustrates the undeformed plastic elements for (a) the anvil, (b) the bottom stops, and (c) the top stops. The upper part illustrates the deformed elements.
As the hammer drops become higher, the dimension $a$ must be increased in order to maintain a constant pulse duration. Since the plastic element is a casting, it is convenient to determine the weight of material required and pour this into a mold, leaving the dimension $a$ unspecified.

The bottom stops, (b) of Fig. 2, are halves of diagonally cut 1-inch cubes. They are placed with their diagonal surface on top of the shock machine air-jack piston, and secured by tape. The top stops are 1·1·2 inch blocks. Both the 1-inch cubes and the 2-inch blocks were cut with a band saw from long cast bars of 1-inch-square cross section. It is possible to substitute good quality rubber or equivalent material for the bottom stops.

The appearances of the plastic elements after shock are shown above their corresponding undeformed sections in Fig. 2. Some care is required to insure that the impacting surfaces of the hammer and anvil, and the anvil plastic element, are clean. Grease on these surfaces greatly increases the ease with which the plastic elements deform and causes large variations of shock motions.

Elastic Elements

Rubberlike materials were used for elastic elements. Pulses of half-sine shape could be obtained only for relatively low amplitude pulses. For relatively high hammer drops the material became nonlinear and inelastic, so that the pulses were more of the sawtooth type. If the use of a larger effective area were possible, which could be accomplished by reasonable effort, half-sine pulses could be obtained for considerably higher amplitudes.

Polyurethane was the only material tried of the rubberlike family that was able to survive successfully the stresses imposed by the shock machine. The specimens used are shown in Fig. 4. Since polyurethane elements have indefinitely long lives, whereas the plastic elements must be recast for reuse, polyurethane is more convenient to use for the top and bottom stops for all types of pulses. Except for very small shocks (a few g peak acceleration) the top and bottom stops cushion other shock impulses to relatively small values. For hammer drops of several feet or more the life of the element in the hammer-impact position was relatively short.

Positioning of Elastic or Plastic Elements

To obtain a pulse of desired shape an elastic or plastic material is attached to the underside of the anvil as shown in position 1 of Fig. 1, or it may be attached to the center of the hammer impact surface. Elastic or plastic materials must also be placed in positions 2 and 3 of Fig. 1 in order to reduce the shock excited by impacts at these locations to values which are small compared with the desired pulse.

---

* A diamine (Dupont Moca) catalyst was added 11 parts per hundred to a liquid urethane elastomer (Dupont Adiprene) at 250°F. The materials were mixed, poured into molds of the proper shape, and cured for 30 minutes at 320°F. This treatment provided a Shore A hardness of about 65.
Fig. 4 - Elastic elements used between the impact surfaces shown in Fig. 1: (a) hammer-impact, (b) top-stop, and (c) bottom-stop elastic elements. All elements are 1 in. thick; (a) and (c) are 4 in. in diameters, and (b) is 5.2 in. O.D. and 2.3 in. I.D.

Figures 5 and 6 illustrate the positioning of plastic materials on the shock machine. Figure 5 shows a plastic element taped to the anvil impact surface together with two of four top-stop plastic elements. Figure 6 illustrates how one of the four bottom-stop elements is taped into position between the jack and the mating anvil-pad. In order that the stopping pulses be small compared with the starting pulse it is necessary that the effective stiffness of the stopping elements be small compared to that of the hammer-impact element, and that their thicknesses be greater.

If a considerable amount of work is involved which would require the shock machine to be used as a shock-pulse generator, then it may be advisable to use elastic elements in the top-stop and bottom-stop positions. This can be done by using split polyurethane donuts for the top stops. Three donuts (Fig. 4) were used, spaced 120 degrees apart, being placed around the shanks of three of the twelve bolts. These elastic elements were at locations corresponding to that of the plastic top-stop elements shown in Fig. 5. Four polyurethane disks (Fig. 4) can be used as bottom-stop elastic elements, instead of the plastic elements shown in Fig. 6.
When an elastic element was used in the hammer-impact position, it was usually held in position on the hammer (rather than on the anvil surface) by means of some spring clips that were attached to the hammer and clipped into the groove (Fig. 4a) which had been machined around the circumference of the element.

RESULTS

Sawtooth Acceleration Pulses

Attempts were made to determine the shape of the plastic element required for a nominal 6-millisecond sawtooth pulse. With 50-50 solder, and with element shapes and dimensions as previously given, the motions of the shock table are as shown in Figs. 7 through 11. Both the velocity and acceleration of the table were independently measured. In one case (Fig. 9) the displacement of the table was calculated by integrating the velocity curve. The accelerometer signal was unfiltered and contained all frequencies passed by the Visicorder galvanometers. These were reasonably flat up to 5000 cps. The velocity meter records were corrected by disregarding the “bottoming” of the pickup, by requiring the initial and final velocity to be zero, and by requiring the slope of the velocity curve to be equal to the acceleration of gravity under all free-fall conditions. The displacement curves will terminate with a negative value of displacement because of the crushing of the bottom stops. The overall displacement must be less than 3 inches.

![Fig. 7 - Anvil table motion for a 1-ft hammer drop with a 12-oz plastic (solder) element](image1)

![Fig. 8 - Anvil table motion for a 2-ft hammer drop with a 21-oz plastic (solder) element](image2)
Fig. 9 - Anvil table motion for a 3-ft hammer drop with a 21-oz plastic (solder) element

Fig. 10 - Anvil table motion for a 4-ft hammer drop with a 21-oz plastic (solder) element

Fig. 11 - Anvil table motion for a 5-ft hammer drop with a 21-oz plastic (solder) element
The acceleration curves have the following characteristics. The acceleration increases linearly up to the peak value and then rapidly decreases to zero. The rise-time varies from 6 to 8 milliseconds. The drop-time varies from 1 to 2 milliseconds. The rise-time can be increased or decreased by increasing or decreasing the weight of the anvil plastic element. It cannot be made shorter than the drop time, as this represents an elastic contribution of the machine. The acceleration pulse starts the table moving in an upward direction. After about 1.5 inches displacement the top stops become effective. They gradually stop the table over a distance of about 3/4 inch. Since the original hammer-anvil impact was inelastic, the hammer has continued to follow closely behind the anvil. When the anvil is stopped, the hammer again impacts against it. Because the plastic material on the anvil surface has already been deformed, this impact is relatively hard (short duration); however it has a small amplitude compared with that of the first pulse. The accelerations caused by the bottom stops are shown on the last part of the acceleration curves. These are more irregular in shape and again are small in amplitude compared with the original pulse. This brings the table to rest with as little shock as possible.

The peak value of the nominal 6-millisecond pulse as a function of hammer drop height is shown in Fig. 12. If the pulse remained the same length (time duration), and had straight sections, then the peak value would vary directly as the hammer impact velocity. However, because of the limited number of shapes of the hammer-impact plastic elements that were used, the pulse-length for the higher hammer drops became shorter, and the acceleration curve became more peaked. This condition could be eliminated by using hammer-impact specimens that were thicker and of greater diameter.

半正弦加速脉冲

因为制造聚氨酯形状和大小所需的相当大的工作量，所以并未尝试控制脉冲的持续时间。结果仅报告了其中一个尺寸和硬度的聚氨酯。图中所示的尺寸在图4中。所有元素的硬度为Shore A硬度计读数65。

由于聚氨酯的变形，加载在弹性体上的力相当大，因此对于一次冲击中超过几英寸高度的锤击，结果是相当非线性的。此外，硬质时间脉冲的恢复时间较长。结果仅报告了其中一种尺寸和硬度的聚氨酯。图中所示的尺寸在图4中。所以硬度

加载在弹性体上的力非常大，因此对于一次冲击中超过几英寸高度的锤击，结果是相当非线性的。在图4中，所有元素的硬度为Shore A硬度计读数65。

加载在弹性体上的力非常大，因此对于一次冲击中超过几英寸高度的锤击，结果是相当非线性的。
the polyurethane is quite long. As a result it behaves much more inelastically than one might expect for the dynamic loads applied. This accounts for the somewhat sawtooth shapes of these curves.

The accelerations experienced by the shock table for a series of hammer blows of different height are shown in Fig. 13. Unless it is desired to provide a very low acceleration for a relatively long time (20 to 30 milliseconds) it is suggested that the sawtooth pulse be used. To obtain a reasonable approximation for a half-sine acceleration pulse of short duration (6 to 12 milliseconds) it would be necessary to increase the diameter of the hammer-impact plastic element to greater than 12 inches with the specimen hardness used.

CALCULATION OF VELOCITY CHANGE AND PEAK VALUE OF ACCELERATION

If the form of an acceleration pulse is assumed and if the velocity change caused by the pulse is determined, then the peak value of the acceleration can be calculated. Consider the masses of the hammer and of the anvil together with its load, \( M_h \) and \( M_a \), respectively. If we assume no rotations, then from momentum conservation we obtain

\[
M_h(V_f - V_i) = M_aV_a
\]
where $V$ is the velocity (velocity change) attained by the anvil and its load, $v_i$ is the hammer velocity at impact, and $v_r$ is the hammer rebound velocity. The velocities should be regarded as those of the centers of gravity of the two systems, with local vibrational velocities being disregarded.

If there is energy loss involved in the impact, then the hammer rebound velocity will become less. This is expressed as

$$v_r = -kv_i + V$$

in which $k$ is the coefficient of restitution.

The rebound velocity is eliminated from the two equations, and the table velocity change is found to be

$$V_a = \frac{M_a(1 + k)v_i}{M_a + M_h}.$$

For the case of perfect plasticity, which is the case for the sawtooth pulse, the value of $k$ is zero and

$$V_{a_{\text{saw}}} = \frac{M_hv_i}{M_a + M_h}.$$

For the perfectly elastic case, which applies for a half-sine pulse, the value of $k$ is unity, and

$$V_{a_{\text{sin}}} = \frac{2M_hv_i}{M_a + M_h}.$$

To find the maximum acceleration involved for the two cases one equates the area of the acceleration pulse to the velocity change. For the sawtooth pulse (Fig. 14a) this is

$$a_p T/2 = V_{a_{\text{saw}}}$$

and for the half-sine pulse (Fig. 14b) this is

$$2a_p T/\pi = V_{a_{\text{sin}}}.$$

where $a_p$ is the peak acceleration and $T$ is the pulse duration.

If the acceleration is expressed in terms of units of gravity so that

$$G = a_p/g$$

Fig. 14 - (a) Sawtooth pulse of time duration $T$ and peak value of acceleration $a_p = G_{\text{sew}}$. (b) Half-sine pulse of time duration $T$ and peak value $a_p = G_{\text{sin}}$. 
where \( g \) is the acceleration of gravity, then one obtains

\[
G_{saw} = \frac{2V_{hi}V_i}{gT} = \frac{2M_hV_i}{(M_a + M_h)gT}
\]

and

\[
G_{sin} = \frac{\pi V_{ein}}{2gT} = \frac{\pi M_hV_i}{(M_a + M_h)gT}
\]

It can be seen from these equations that for a given pulse shape and load the amplitude of the acceleration pulse will vary directly as \( V_i \). For a given thickness of hammer-impact element, nonlinear behavior will cause the pulse to get shorter as the load gets heavier, and the slope of the pulse will increase as its peak value is approached. This will cause the acceleration to be greater than estimated under linear conditions.

A comparison of the velocities of the anvil as determined by calculations from the measured accelerations, by direct velocity measurements, and by momentum considerations for a sawtooth pulse where only \( \Gamma \) is experimentally determined is given in Table 1. A curve showing the maximum amplitude of the sawtooth acceleration pulse, as a function of hammer impact velocity, or drop height, is shown in Fig. 12. The curve is slightly concave upward, since there is a tendency for the pulse length to be shorter for the greater impacts.

<table>
<thead>
<tr>
<th>Hammer Drop Height (ft)</th>
<th>Anvil Velocity (ft/sec)</th>
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<th>From Velocity Pickup</th>
<th>From Momentum Calculation</th>
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<td>5.7</td>
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</tr>
<tr>
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<tr>
<td>5</td>
<td>7.7</td>
<td>7.7</td>
<td>7.7</td>
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SHOCK SPECTRA REPRESENTATION

Shock spectra of half-sine and sawtooth pulses are shown in Figs. 15 and 16. The older and simpler method of presentation given on these figures provided only the acceleration spectra. Figures 17 through 20 provide a simultaneous representation of acceleration, velocity, and displacement spectra. The additional amount of information that is easily available by this method of simultaneous plotting more than compensates for the nondimensional expression of frequency in terms of pulse length. The other three coordinate systems are all in units of acceleration, but \( S_a \), \( S_v \), and \( S_d \) are respectively in units of acceleration, velocity, and displacement. The value of \( G = a/g \) is the acceleration in gravity units. It is nondimensional.

To obtain the acceleration, velocity, or displacement shock spectral values for a given frequency, one multiplies the corresponding coordinate values by \( G \), \( GT \), or \( GT^2 \) respectively. The value of \( g \) has been taken as 386 in./sec², hence the units for \( S_a \) and \( S_d \) will be in in./sec and inches respectively. The value of \( S_v \), being expressed in terms of \( g \), does not require an expression of particular units unless \( g \) is evaluated numerically.
Fig. 15 - Acceleration shock spectra, $S_a$, for a half-period sine-wave acceleration pulse. The pulse has a duration of $T$ seconds and an amplitude of $G_g$ (G represents the number of gravity or g units in the peak acceleration). The primary spectrum is the maximum response during the pulse interval. The residual spectra are the maximum responses after the pulse interval.

Fig. 16 - Shock spectra for inset sawtooth acceleration pulse. Primary and residual refer, respectively, to maxima that occurred during and after the pulse period. (After Morrow and Sargeant, Ref. 5.)
Fig. 17 - Residual and overall shock spectra of a half-sine acceleration pulse shown inset. The "overall" spectrum is the usual shock spectrum, i.e., the maximum response regardless of when it occurs. $S_a$, $S_v$, and $S_d$ are, respectively, acceleration, velocity, and displacement shock spectra expressed in units of in./sec$^2$, in./sec, and inches. $G$ is the acceleration expressed in units of gravity, $g$. $T$ is the pulse duration; $f$ is frequency. If the pulse length is 0.006 second and the amplitude is 200 g, then for a frequency of 100 cps ($fT = 0.6$) the (overall) shock spectral values of $S_a$, $S_v$, and $S_d$ are 340 g, 200 in./sec and 0.32 inch respectively.

Fig. 18 - Shock spectra of a symmetrical triangular acceleration pulse
Specifications which are given in terms of acceleration pulses ignore the fact that a unidirectional acceleration pulse results in a continued velocity and infinite displacement. It must be assumed that accelerations in the opposite directions will have been applied, either before and/or after the pulse, so that the final velocity will be zero and the relative displacement will be reasonable. The specifications can ignore these factors as long as their effects are small compared with that of the pulse. When the maximum displacement is limited to about 3 inches, as for the present case, it may be that the stopping pulses cannot always be ignored. In fact if the starting pulse be long, so that the acceleration maximum is only a few g, then the stopping pulse may be of greater value (and also involve a greater velocity change), so that the latter pulse may be preferred as the test pulse.
The following information can be immediately obtained from the four-coordinate shock spectra representation (see Fig. 20).

1. For very high frequencies, region A, the value of the shock spectrum, \( S_a \), becomes equal to the maximum value of acceleration of the shock motion.

2. For some intermediate frequencies, region B, the shock spectra may have maxima of accelerations. These are responses to dominant frequencies associated with the shock motion, and may involve accelerations much greater than those of the actual motions.

3. For some intermediate frequency range, region V, which is lower than region B, the effect of the pulse can be considered to be the same as a simple change in velocity. The value of \( S_v \) is the value of the velocity change. It will remain at a constant value as the frequency is decreased until the displacement involved in the response exceeds the displacement of the shock motion.

4. For very low frequencies, region D, the value \( S_d \) of the shock spectrum becomes equal to the maximum displacement involved in the shock motion. If the sudden displacement consists of a motion away from and back to an initial position there will be no residual spectrum. If the sudden displacement consists of a motion away from an initial position and back to a different final position there will be an overall spectrum as before and a residual displacement spectrum equal to the difference between the initial and final position.

REFERENCES


