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FIRST INTERIM REPORT ON OPTIMUM UTILIZATION OF COMPUTERS AND COMPUTING TECHNIQUES IN SHIPBOARD WEAPONS CONTROL SYSTEMS

BuWeps-Project RM1004

M88-3U1

June 1963

TRW COMPUTER DIVISION
THOMPSON RAMO WOOLDRIDGE INC.
CANOGA PARK, CALIFORNIA

CONTENTS

1. INTRODUCTION

As part of the Bureau of Naval Weapons Long Range Scientific and Technical Development Planning Program, the TRW Computer Division of Thompson Ramo Wooldridge Inc. has been asked to investigate how the unique capabilities of general purpose digital computers can best be exploited in an advanced weapons control system.

The report is the first of three reports that will be submitted to the Bureau of Naval Weapons covering the study activity.

2. PROJECT DESCRIPTION

2.1 OBJECTIVES

The objective of the study is to become familiar with present and proposed shipboard weapons systems and to determine ways to improve upon these systems, particularly through the use of digital computers and computing techniques. With the limited time available, the best approach seems to be one of a broad attack on several fronts rather than a concentrated effort on a few specific techniques. This should point out those particular areas where further investigations should be concentrated. Any areas that look particularly promising will be examined in more detail and recommendations for further study will be made if appropriate. Less promising areas will be passed over lightly.

2.2 GENERAL TASK

To investigate how the unique capabilities of general purpose digital computers may best be exploited in advanced weapons control systems.

2.3 BACKGROUND

General purpose digital computers in shipboard weapons control systems are a relatively new innovation. Understandably, current activities are primarily concerned with the problems of how to interface, integrate, maintain and use these computers. It will be a basic premise of this investigation that these near term problems are solved and that attention can be focused on exploiting the various capabilities of the computer to benefit the overall weapon system.

The general purpose computer has the inherent capabilities of flexibility, adaptability, as well as having a powerful computing potential. The flexibility offers the possibility of standardization of equipment for use in a variety of weapon control systems. The adaptability of a general purpose computer will allow it to react in an optimum fashion to a changing environment such as jamming, decoys, weather and system load. Changes in weapons characteristics can readily be accommodated by changes in the computer programs as weapons are improved. Real-time modification of the fire control program is possible, either through external sensors or through human intervention. The purpose of the study will be to investigate the possibilities of standardization of equipment for various weapons control systems and to make recommendations for the optimum utilization of general purpose digital computers in these systems.

2.4 DESCRIPTION OF WORK TASKS

a. A detailed study of several shipboard weapons control systems will be performed. These will be analyzed from an overall system viewpoint and reorganized where necessary to permit optimum "digitalization." Using these models, parameters for speed, storage, and input/output requirements will be generated for each system studied. Parameters will then be compared to determine what standardization of computing equipment is possible for these systems.

b. Using either a real or hypothetical weapons control system as derived in the previous task, a more detailed analysis will be made to determine what techniques can be used to further optimize the overall system. Among items that will be examined are the following:

(1) Digital filtering to reduce noise in overall system.

(2) Adaptive control as a function of jamming, system load, state of threat, and other variables.

(3) Real-time program modification through operator intervention or automatic feedback.

(4) Redundancy to provide system backup in case of failure of portions of the system.

These areas and others will be examined to determine which areas warrant further, more detailed study. Recommendations will be made to BuWeps for further study in those areas that look promising.

3. GENERAL DISCUSSION

The initial effort will be to gather information on the various weapons systems of concern and to generate a general system description or model for subsequent investigation. This model will incorporate those functions that are common to all the weapons systems of concern and will provide a common basis for very broad systems analysis and will provide a common basis for very broad systems analysis. It is anticipated that specific weapons systems can then be treated as deviations from the generalized model.

The study will be guided by frequent conferences with various Navy establishments to insure that the inputs are timely and to avoid duplication of effort.

In addition to the general systems analysis of the weapons systems, particular subsystems can be analyzed as well as the hardware and software associated with the systems and subsystems. Unfortunately, the hardware and software aspects of the study cannot be divorced from each other or from the systems analysis, and all must be examined simultaneously in the final analysis. As the study gradually evolves from the macroscopic to the microscopic, particular techniques will be examined in more detail.

The effort to date has been concerned primarily with gathering and digesting information considered pertinent to the study program.

3.1 VISITS AND CONFERENCES

The following activities were visited during this portion of the study period. The primary objective of these visits was to become acquainted with the work being done by these activities that is related to the study, and to acquaint the activities with the objectives of the study. NEL, NOTS China Lake, and BuWeps have been very helpful and cooperative in providing information, documents and general assistance during this period.

Activities Visited	Subjects Discussed
BuWeps	General Aspects of the Study. MK 86 GFCS
BuShips	WDE MK II and NTDS
NEL	Sampled Data Systems
NOTS (China Lake)	Digital Data Transmission Digitizing SAM Systems
NOTS (Pasadena)	ASW Weapons
APL	Tartar, Terrier
RCA	Terrier
Raytheon	MK 74
Howard Research Corp.	Digital Data Link

4. TECHNICAL DISCUSSION

4.1 GENERAL

Real-time control systems, such as fire control systems, can be considered to be made up of four basic building blocks: Sensors, Converters, Control Elements, and Effectors. These are defined as follows.

Sensor	A device or system that determines the position, size, attitude or other physical attributes of a given item or items.
Converter	A device or system for converting the basis for measurement of a quantity from one physical representation to another, e.g., analog-to-digital; or from one format to another, e.g., gray code to binary code.
Control Element	A device or system that accepts prescribed quantities and performs specified operations on these quantities.
Effectors	A device or system that effects a physical change or reorientation in response to inputs from an external source.

In many instances one or more of the elements of a control system may be a control system in itself. For example, the tracking radar can be considered as a sensor for a fire control system even though this is a control system in itself. The radar *per se* can be considered the sensor, the radar antenna and pedestal as the effector, and the tracking servo as the control element.

The sensor elements may be either open loop devices, such as a surveillance radar, or closed loop such as tracking radars. Similarly, a control device may be either open loop or closed loop.

For the purpose of this study all converters will normally be considered as part of one of the other elements in the system and will not generally be considered explicitly.

Figure 1 is a considerably simplified block diagram of a typical surface weapons control system. The diagram is separated into two parts, those elements shown to the right of the dotted line can be

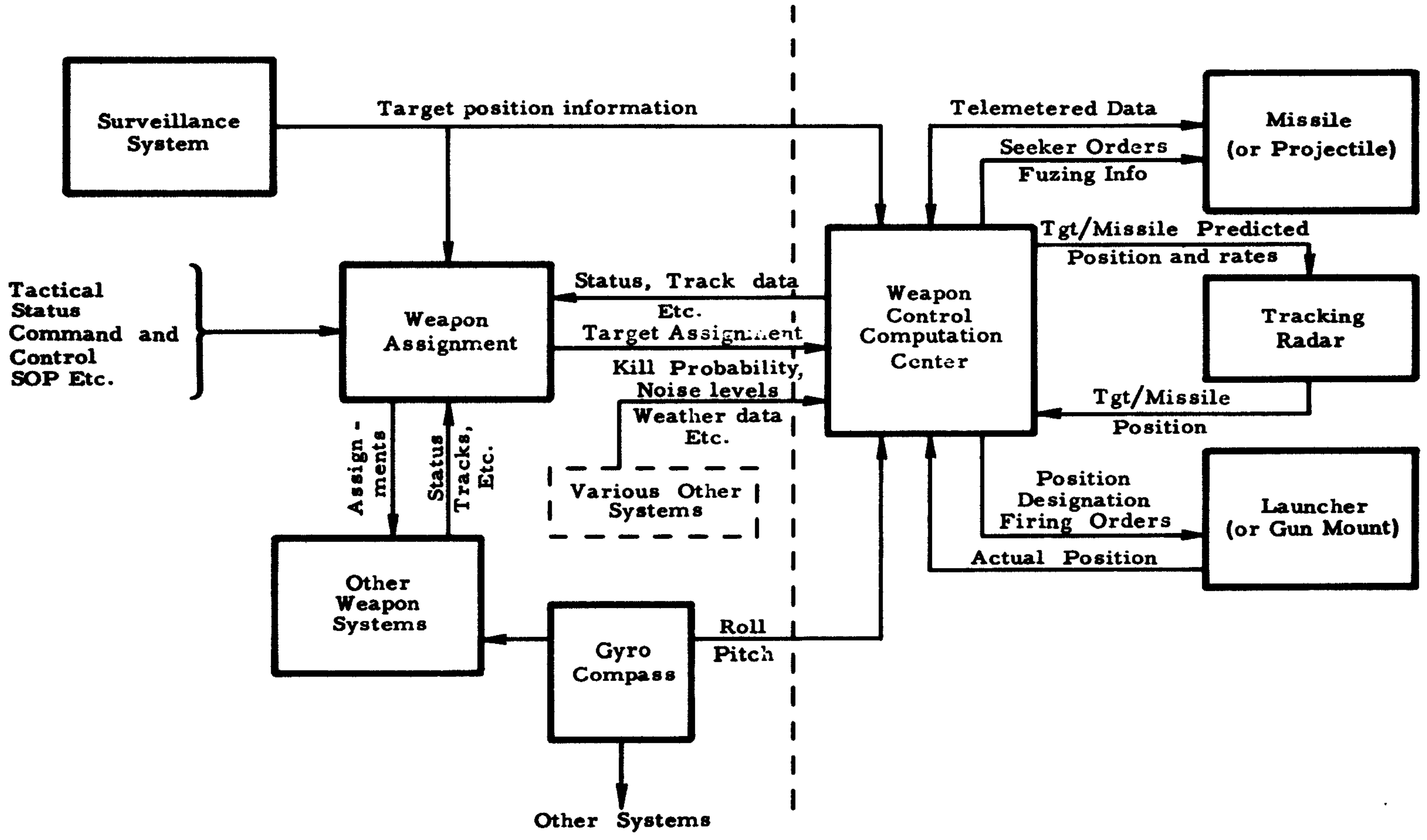


Figure 1. Simplified Surface Weapon Control System

considered to be concerned only with the weapon system while the elements on the left are considered to be external to the weapon system for various reasons.

The surveillance system consists primarily of sensors, communications, and display equipment. It provides the weapons systems with target position information and may provide other data such as identity, velocity, raid size, etc. The surveillance system may range from a single 2D radar through a NTDS type complex with multiple radar and communications inputs. Subsequent reports will look into the surveillance system in more detail.

The Weapon Assignment function is shown as a separate element in Figure 1 although it might be considered to be a part of the overall Command and Control System. It is shown as a separate box since it is a functionally distinct element of the overall system. It utilizes the inputs from the surveillance system, the weapons system, the command and control system, and other tactical status information to assist the Commander or his representatives in assigning weapons to threats.

The gyro system provides the fire control system with stable reference data for use in converting from deck to stable coordinates, and vice versa. The gyro system is shown as a separate element external to the weapons system since it may be shared by several systems aboard ships. The stabilization, rotation and smoothing problem will be discussed in more detail later.

Going on to the weapons system, it is seen to be made up of a computation center, a tracking radar, a launcher (gun mount), and the missile (projectile). In the case of a gun system, the tracking radar may also be a surveillance radar.

Figure 2 shows some basic elements of a Weapons Control Computation Center. The computational tasks are typically performed in a single computer but they may be separated.

For the initial part of this analysis, these various tasks will be investigated separately and treated as if they each had their own separate computation equipment. As the study evolves, these elements will become more and more interdependent in many cases. Also,

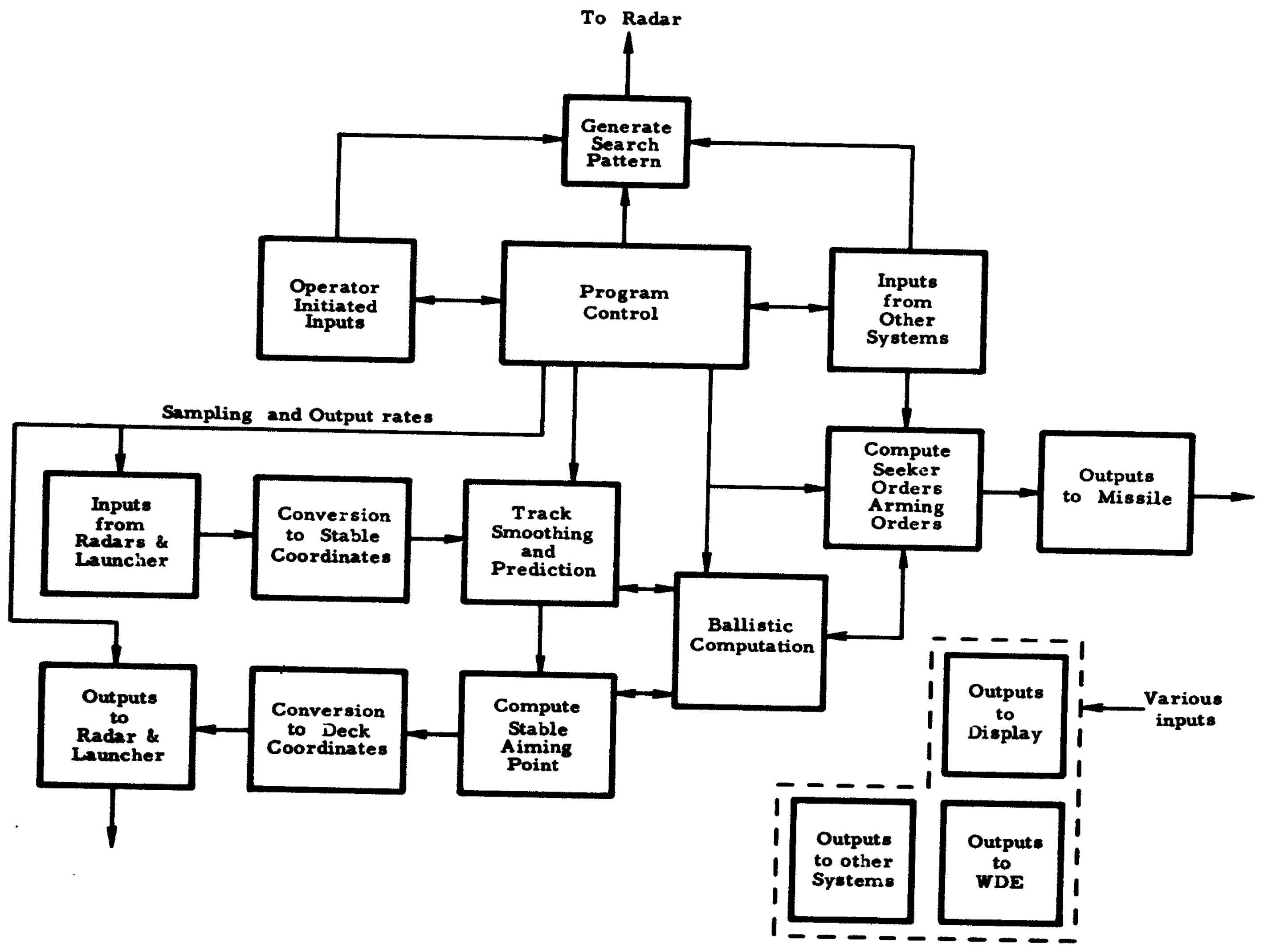

To Radar
Generate Search Pattern
Operator Initiated Inputs
Program Control
Inputs from Other Systems
Sampling and Output rates
Inputs from Radars & Launcher
Conversion to Stable Coordinates
Track Smoothing and Prediction
Compute Seeker Orders Arming Orders
Outputs to Missile
Ballistic Computation
Outputs to Radar & Launcher
Conversion to Deck Coordinates
Compute Stable Aiming Point
Outputs to Display
Various inputs
Outputs to other Systems
Outputs to WDE

Figure 2. Basic Elements of a Simplified Weapons Control Computation Center

certain trade offs between elements will become apparent which will require a broader systems analysis. Many trade offs can be handled in an "Adaptive" sense through the program control element which is assumed to have control over all elements.

As a result of this breakup into functional elements, some of the early portions of the analysis will appear, perhaps, to be somewhat unrelated. This is an unfortunate but natural result of the approach taken.

Where possible, a measure of effectiveness will be established to relate the system improvement to the cost in terms of weight, size, complexity and other factors as well as the monetary costs. Where the ratio of improvement to cost is obviously quite low, the investigation will cease. Where the ratio appears high, the investigation will continue. Many areas will of course require guidance in ascertaining what weights should be given to various factors. It may also turn out that certain avenues of investigation will be of negligible value to the overall problem or that the cost of using a certain technique is too high for the improvements that will accrue.

Referring again to Figure 2, there are three basic types of elements shown:

a. Input and Output Elements
b. Computational Elements
c. Control Elements

Much of the study effort will be concerned with the computational and control elements. Some preliminary discussion of these elements is contained in Sections 4.2, 4.3, and 4.4.

As mentioned earlier, the "Shotgun" approach will be used initially in the study. That is, many techniques and disciplines will be touched upon lightly rather than a few in detail. As these become further removed from existing techniques and hardware developments, it becomes more and more difficult to see what applications these new developments may have to weapons control systems. It is anticipated that in the course of the study there will be certain concepts and ideas that show no immediate applicability. These are included, however, where it is felt that a reasonable probability exists that they will find application in future weapons control systems.

It is very difficult to determine what to look at even when using the broad "Shotgun" approach because the field of computers and computing techniques is so large and still expanding. To limit this to a certain extent, only those hardware and software fields where TRW has a reasonably high competence will be examined.

Some of the hardware developments such as recognition memories are currently being developed and can be considered for immediate use. Other developments such as optical computers and probability state devices are still laboratory playthings and consequently, hold little promise for the immediate future. Generally, software development lags the equipment development and is easier to evaluate, although in many instances software development will cause hardware development to be biased in certain directions.

In subsequent paragraphs, several hardware and software developments will be discussed. Some of these are directly related to the general area of weapons systems and others only remotely. These and other topics will be covered in more detail in later reports.

A technique referred to as "On-Line Computation" has been used for a portion of the investigation reported in Section 4.6. This is a technique developed at TRW and has been supported by the U. S. Air Force. Since it provides such a powerful tool in investigations such as the one reported in Section 4.6 and is likely to be used frequently in the study, a more detailed description of the technique is included as an appendix to this report.

Since many fields of endeavor will be covered in this series of reports, many of the topics will be covered from an elementary "not for the specialist" viewpoint, the desire being to find application for techniques that may offer some system improvement rather than to advance the state of the art in these particular fields.

4.2 COORDINATE CONVERSION

Since a ship does not provide a stable platform for a weapons system it is desirable to convert incoming target data to a stable frame of reference to ease the smoothing and prediction computation. It also requires that orders from the computer to the launcher be converted from the stable frame of reference to a deck oriented coordinate system. The equations for performing these rotations are contained in Sections 4.2.1 and 4.2.2. Note that the sine and cosine functions come up several times in these computations but in every case both the sine and cosine are required. Techniques for computing the sine and cosine are discussed in Section 4.3.

4.2.1 Deck to Stable Coordinate Conversion

The following computations are performed in the conversion of radar deck frame spherical coordinates

$$R, B'_d, E'_d$$

to cartesion stable coordinates (north-oriented, earth-level)

$$R_{h_x}, R_{h_y}, R_v$$

using ship's roll (Z_o), pitch (E_{io}) and heading (C_{qo}).

$$R_{h_x} = \sin C_{qo} \left[Y_{u\ell} \cos E_{io} + \sin E_{io} (X_{u\ell} \sin Z_o + Z_{u\ell} \cos Z_o) \right] + \cos C_{qo} (X_{u\ell} \cos Z_o - Z_{u\ell} \sin Z_o)$$

$$R_{h_y} = \cos C_{qo} \left[Y_{u\ell} \cos E_{io} + \sin E_{io} (X_{u\ell} \sin Z_o + Z_{u\ell} \cos Z_o) \right] - \sin C_{qo} (X_{u\ell} \cos Z_o - Z_{u\ell} \sin Z_o)$$

$$R_v = \cos E_{io} (X_{u\ell} \sin Z_o + Z_{u\ell} \cos Z_o) - Y_{u\ell} \sin E_{io}$$

where $X_{u\ell}$, $Y_{u\ell}$, $Z_{u\ell}$ are target coordinates in the <u>un</u>stable deck frame referred to the <u>ℓ</u>auncher.

$$X_{u\ell} = R \cos E'_d \sin B'_d$$

$$Y_{u\ell} = R \cos E'_d \cos B'_d + P_{do}$$

$$Z_{u\ell}\ R \sin E'_d$$

P_{d0} = separation along deck center line of launcher and radar.

4.2.2 Stable to Deck Coordinate Conversion

The following computations are performed in the conversion of cartesian stable coordinates from computer a_x, a_y, a_v to deck referenced bearing B'_d and elevation E'_d angles.

$$X_{u\ell} = \sin C_{qo} \left[a_y \cos E_{io} + \sin E_{io} (a_x \sin Z_o - a_v \cos Z_o)\right]$$

$$+ \cos Z_{qo} (a_x \cos Z_o + a_v \sin Z_o)$$

$$Y_{u\ell} = \cos C_{qo} \left[a_y \cos E_{io} + \sin E_{io} (a_x \sin Z_o - a_v \cos Z_o)\right]$$

$$+ \sin C_{qo} (a_x \cos Z_o + a_v \sin Z_o)$$

$$Z_{u\ell} = \cos E_{io} (-a_x \sin Z_o + a_v \cos Z_o) + a_y \sin E_{io}$$

$$B'_d = \tan^{-1} \frac{X_{u\ell}}{Y_{u\ell}} = \sin^{-1} \frac{X_{u\ell}}{\sqrt{X_{u\ell}^2 + Y_{u\ell}^2}}$$

$$E'_d = \tan^{-1} \frac{Z_{u\ell}}{\sqrt{X_{u\ell}^2 + Y_{u\ell}^2}} = \sin^{-1} \frac{Z_{u\ell}}{\sqrt{X_{u\ell}^2 + Y_{u\ell}^2 + Z_{u\ell}^2}}$$

4.2.3 Computation

The problem of generating sine, cosine and other trigonometric functions as well as the remainder of the coordinate conversions problem will be examined from two aspects: that of faster routines within a general purpose digital computer, and that of computing these externally in DDA, analog, or other special purpose equipment. Section 4.3 gives a preliminary discussion of some of the techniques for speeding up computations with a general purpose computer. Techniques and equipment for generating this information externally to the computer are also being studied.

4.3 FAST DIGITAL TECHNIQUES FOR GENERATING TRIGONOMETRIC FUNCTIONS

It is apparent from the previous section that generation of trigonometric functions is very important in performing coordinate conversion. Trigonometric functions show up in several other equations that must be solved in the weapons control problem also. In many cases, where an angle is read in from a shaft encoder, the sine and cosine of the angle can be brought in as readily. In other cases, incremental techniques similar to DDA techniques can be used. In the most general case however, it is necessary to compute the sine and cosine or other trigonometric functions directly.

Classically, the generation of trigonometric functions is an area where analog techniques have been considerably better than digital techniques. As the state of the art in digital computers has advanced, this advantage has diminished. Three advances have contributed to this:

a. Faster Computers
b. Large Memories
c. New Techniques for Computation

Faster computers have of course resulted in the speedup of all computations and needs no further discussion. Large and relatively inexpensive memories make it possible to store entire tables of trigonometric functions or large portions can be stored with interpolation being used between entries.' In the AN/UYK-1 computer, the time required to compute the sine and cosine of an angle using a conventional power series expansion requires 1884 microseconds and 167 words of storage. Using a small 65-word table, and interpolation between entries, the computation time for the same accuracy is reduced to less than 600 microseconds. This is a significant improvement through the use of a small amount of storage. Using an additional 1000 words of storage reduces the total computation time to 300 microseconds for the pair of functions. By storing the entire table, the time can be reduced to essentially one memory access time. This is a brute force technique but has been used in many applications where time is a premium but storage is not.

Another approach to the problem is to use different computing techniques. As mentioned previously, the conventional technique for computing the sine of an angle is to use a power series approximation:

$$\sin X = X - \frac{X^3}{3!} + \frac{X^5}{5!} - \frac{X^7}{7!} + \dots .$$

The required accuracy determines the number of terms that need to be computed. There are several other approximations that can be used. One technique that is currently being examined is the use of a family of polynomials known as the CHEBYSHEV polynomials. These converge very rapidly as compared to a truncated power series. This means that fewer terms are required for comparable accuracy which in turn means fewer computations and faster sine cosine routines within the computer. Typical time savings of 30 to 40 percent are being obtained currently using these techniques. These computations will be discussed in more detail in subsequent reports.

4.4 DIGITAL FILTERING AND PREDICTION

Track smoothing and prediction is one of the most important functions performed in the weapons control computer. In an ideal noise-free system no smoothing is required. However, as the noise level increases smoothing is required to improve the accuracy of the data acquired. Smoothing is normally accomplished by making use of position information from several preceding reports. There are a large variety of techniques for assigning weights to preceding track reports. Some of the more common smoothing techniques are: exponential, least squares and quadratic. Smoothing techniques that are optimum for smoothing in a high random noise environment are not optimum for a maneuvering target in light noise. Smoothing techniques that respond rapidly to maneuvering targets are generally very susceptible to noise.

Position prediction depends, of course, upon the smoothing. Usually, tracking and prediction systems assume that the track will continue in a straight line with its present velocity. Unless there is a priori knowledge that the track will be maneuvering and in what manner, this straight line assumption is as good as any and is much easier to compute. There are, however, many instances when it would be desirable to bias the prediction as if the flight path were curved. The adaptability of general purpose computers permit such biasing to be readily accomplished.

4.5 ADAPTIVE CONTROL

The preceding sections have shown instances where it might be desirable to modify certain computations within the fire control computer based upon some knowledge of the external environment at the time of the computation. Thus it appears desirable for the computer to be able to "adapt" itself to a changing environment.

A general purpose computer is, by nature, an adaptive device in that it can be adapted to many different tasks by changing its program. Within a program, adaptations of a sort are made by "branching" or "jumping" to a new routine when certain criteria are met or an interrupt is received. It is this adaptability to changing requirements that makes the general purpose digital computer such a powerful tool in a system such as a weapons control system.

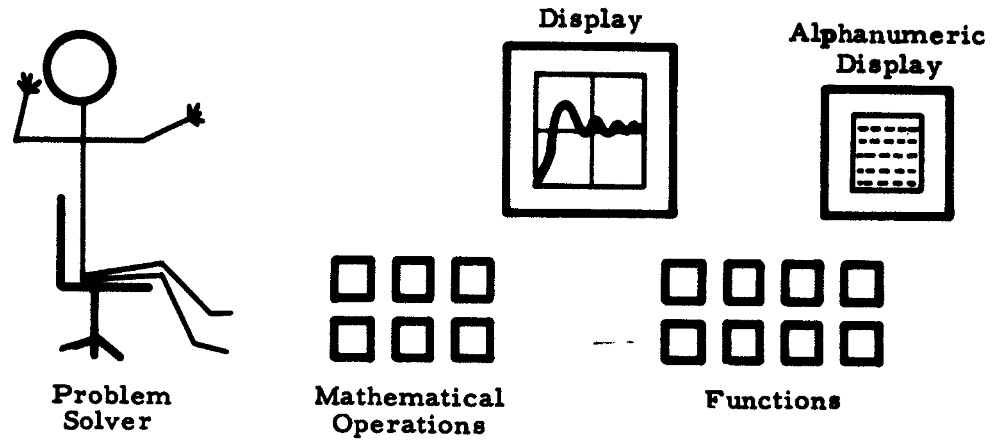
To enhance the computer's capability to adapt to the situation of the moment, it is necessary to examine various techniques for the several computing tasks. The following areas will be examined during the course of the study:

a. Smoothing Techniques
b. Prediction Techniques
c. Techniques for performing ballistic computations
d. Automatic sensing for adaptive control
e. Operator initiated adaptive control

4.6 SYSTEM ANALYSIS USING ON-LINE COMPUTING TECHNIQUES

The "On-Line" system technique is presented in detail in the Appendix so only a brief description will be given here. Basically, the system consists of a person with a problem, a set of buttons on a console which control mathematical operations which may be performed on functions rather than on discrete numbers, and some provision for displaying results. The operations to be performed depend on the results obtained and are determined and assembled by the problem solver as the solution evolves. The results are shown on a large, 17 inch, cathode-ray tube and where applicable, on an alphanumeric display tube. These displays are located at the Display Analysis Console (DAC). When a copy of the data is desired, a picture can either be taken of the scope or a printout of the function made on an electric typewriter. The use of this system is analogous to the use of a desk calculator where functions rather than numbers are used. Figure 3 is a block diagram of the system.

The insertion of a digital computer into a system such as a weapons control system immediately changes the nature of the general analysis from that of a continuous system into a sampled data system. That is, the data to be operated on in the computer is not continuous but is a sample of various quantities taken at discrete intervals. If proper sampling criteria are used and computations are treated



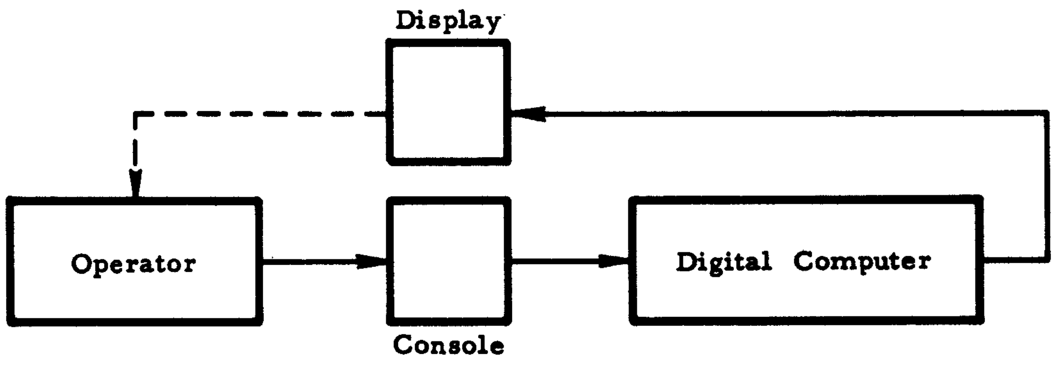


Figure 3. On-Line Computing System

accordingly, the problems are no greater than those confronted in an analog system. To assist in the analysis of sampled data systems, various functions are being simulated on the Display Analysis Console and On-Line computing equipment located at the TRW facility.

Using the DAC it is possible for an operator to simulate various aspects of the sampled data system and modify different parameters without going through the normal programming routine. This way, the operator will be able to modify transfer functions, sampling rates, compensating equations, and other quantities as he varies the noise level, sampling frequency, or other parameters.

Initially, the simple, closed loop servo system shown in Figure 4 was inserted into the system. The response to a step input and a ramp input were then obtained with different values of the parameter a. These are shown in Figures 5 and 6. The open loop frequency response is shown in Figure 7.

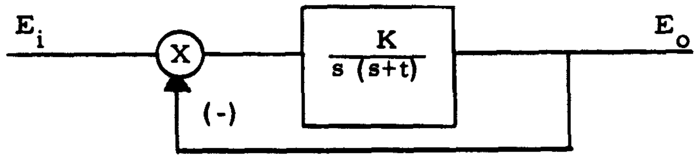
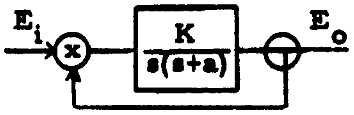


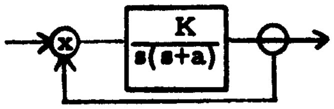
Figure 4. Closed Loop Servo System

For a preliminary examination of the effect of digitizing the system, a sampler was added to the system. To simplify the investigation initially, it was assumed that the only effect the sampler has on the system is to add a delay of e exp -sT/2 where T is the sampling period. The effect of changing the sampling rate is shown quite clearly in Figure 8. The bright dot to the left of the origin is the minus one point. Notice that with a sampling rate (T) of 1.6 seconds, the plot goes through the minus one point and will become unstable. Any slower sampling rate would result in the system becoming more unstable. This instability could be remedied by a change in the parameter a, by compensation in a computer in the loop, or elsewhere in the loop.



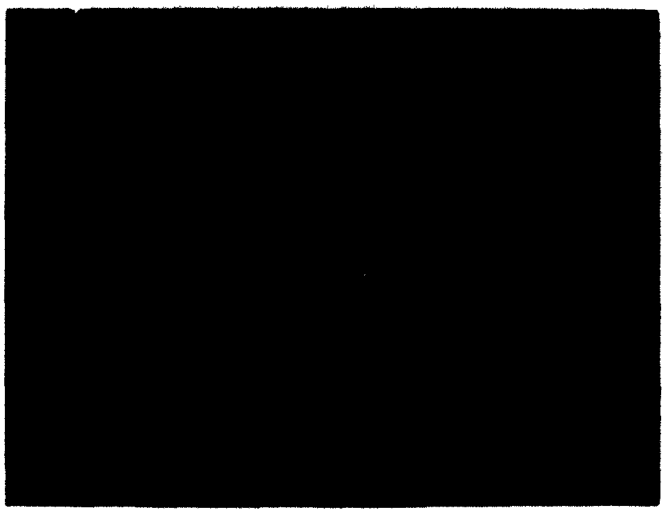
K = 1.175
a = 1.00
a = 1.25
a = 1.50

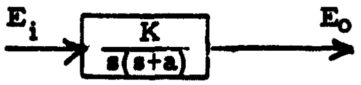
Figure 5. Response to Step Input



K = 1.75
a = 1.00
a = 1.25
a = 1.50

Figure 6. Response to a Ramp Input



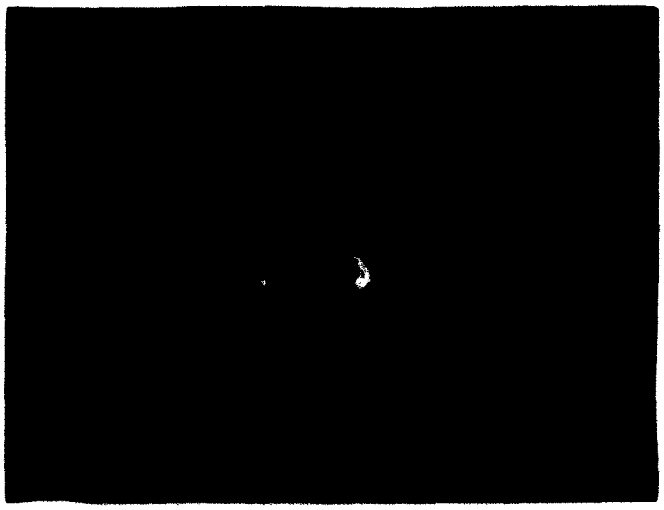


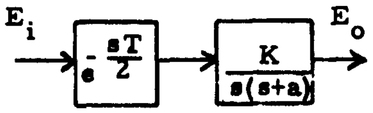
$K = 1.75$

$a = 1.25$ $a = 1.00$

$\omega = 0.5$ to 10

Figure 7. Nyquist Plot of Simple Servo





$K = 1.75$

$a = 1.25$

$T = 1.60$

$T \quad 1.40$

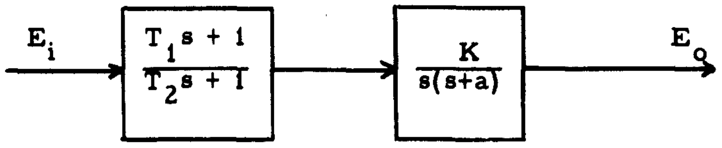
$T \quad 1.25$

$\omega = 0.5 - 10$

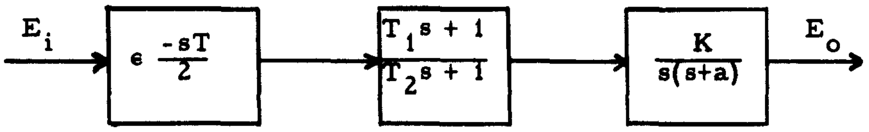
Figure 8. Nyquist Plot of Simple Servo With a Sampler

At the present time, additional plots are being obtained for the following:

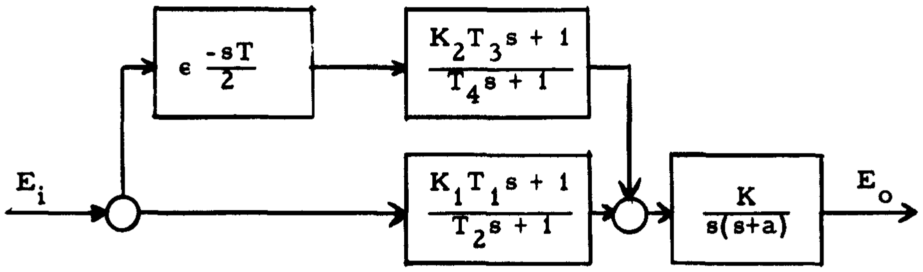
a. A simple servo with a lead lag network.



b. A sampler added to the above combination.



c. A bipass digital computer.


$\epsilon^{\frac{-sT}{2}}$
$\frac{K_2 T_3 s + 1}{T_4 s + 1}$
E_i
$\frac{K_1 T_1 s + 1}{T_2 s + 1}$
$\frac{K}{s(s+a)}$
E_o

4.7 RECOGNITION MEMORIES

Recently developed pattern recognition search techniques offer a capability of considerable consequence in various data processing applications. The technique was suggested by Dudley A. Buck as early as 1955 in what he called a "Recognition Unit." Contemporary workers have investigated this type of memory organization under such names as "Catalog Memory," "Tag Memory," "Data Addressed Memory," "Search Memory," "Parallel Search Memory," "Associative Memory," and "Content Addressable Memory."

The recognition memory becomes extremely useful in processes requiring the ordering of data such as in cataloging, compiling correlating and cross-indexing. It being a relatively new technique, comparatively little effort has been devoted to examining the full potential of its application.

4.7.1 Definition

For the purpose of the present report, the following definitions will be adhered to:

In contrast to conventional random access memories in which data is called forth from memory in terms of its address or storage location, the object of a "Recognition Memory" is to recognize the existence of a word (or bit patterns) in the memory, to select and retrieve it on the basis of a key transmitted simultaneously to all words in the memory. In a true Content Addressable Memory (CA-memory), the key may be used for recognizing the entire word or pattern rather than a portion of the word. With suitable masking techniques, a key may be employed to match any portion or portions of the words in memory. Upon recognition of the key, either the whole word (or any part of it) or the address at which the word is stored, may be retrieved nondestructively.

In a true Associative Content Addressable Memory (ACA-memory), or simply Associative Memory, a CA-memory organization is used in conjunction with a conventional memory array. Only CA-memory word bits — any or all of them — can be used in the formation of a key. If a key is recognized, the information in the conventional memory associated with the CA-memory word may be retrieved nondestructively, including or excluding the information stored in the CA-memory. (It is to be noted though that key can also be a discrete pattern such as a character or a continuous pattern, such as a spectrum.)

4.7.2 Discussion

In a recognition type memory, the objective is to locate and/or secure information from file in absence of knowledge of physical address and without resort to sequential search. This is achieved by interrogating all words in a memory file simultaneously (or in parallel) for identity with a key or descriptor word. In a recognition memory, the word to be selected from store is distinguished from all other words in terms of the information content contained in the word itself or a portion of it.

For example: consider a 10,000 word memory having an access time of 6 μsec. Searching this memory to retrieve a certain word stored in it will take an average time of 30 milliseconds, using conventional methods. However, by organizing a memory so that its entire content is addressed simultaneously, (a content-addressable memory structure) it would be possible to retrieve the desired word in less than 12 μsec. To retrieve a word in a conventionally organized memory in 12 μsec would require an access time of 2.4 nanoseconds. Such access time cannot be attained with present state of the art or even in the near future.

The simultaneous search described above may result in no match, a single match, or in the general case, in multiple matches. Multiple matches can be processed by a non-unique-to-unique match commutator such that successive searches yield in sequence every word that "matches" the key (or search) word, regardless of the number of times that word is stored in the memory file.

For example, consider a nonunique match case where the search key recognizes three words 23, 1718, and 4065 in a 4096-word CA-memory array. Upon interrogation of the memory, mismatch information (one bit) is stored on all word lines except 23, 1718, and 4065. In a subsequent interrogation cycle, the commutator jumps to word 23 and a unique selection (of word 23) occurs. When the word is read out, one bit of word 23 is temporarily altered so that in a subsequent automatic interrogation cycle, the commutator skips words 1 through 1717 and jumps to word 1718. The interrogation sequence is repeated (at the option of a program) until all match words are read out.

Thus, the CA memory furnishes data on a strict equality basis (exact match). However, it can also execute greater-than, less-than, between-limits, maximum, minimum, and mixed-search interrogations.

It should be stressed that in an associative memory, the storage elements contain not only the desired information but also an identifying key or label, which is the descriptor word. The elements containing this descriptor word are so arranged as to allow a simultaneous "content addressable" search of this portion of the memory for a given key or descriptor word. Portions of the descriptor word may be masked so that any or all descriptor word bits can be used in the formation of a key. If a descriptor word "matches" a key or search word, a signal is produced defining the physical location of that word so that the data associated with the descriptor word (and stored in a conventional core plane) may be subsequently read out by conventional methods.

One of the possible uses for a recognition memory in a surface weapons system is with a track-while-scan (TWS) surveillance radar system. In a typical TWS system, several target tracks are maintained in the track store. As reports come in from the radar, these must be correlated with the corresponding track in the track store. A rather unsophisticated way to execute this correlation is to search through all tracks in store for the one whose position best matches the incoming track from the radar. This track report is then associated with the matching track in store. If a conventional memory were employed and if the tracks are not ordered, then as an average, half the tracks in file must be searched. This typically requires a memory access and a compare for each of the tracks in store. Obviously, as the number of tracks in the track file increases in time, the time consumed in searching soon becomes excessive.

If the position information associated with each of the tracks in store is organized into the content addressable portion of an associative memory, the track association is readily accomplished with a single memory access or a between limits search (requiring two to four accesses).

High speed searching and correlation of incoming information with the proper track in the weapons direction equipment is readily accomplished with an associative track store. The application of recognition memories to decoy descrimination, digital filtering and ECCM will be examined in future reports.

4.8 MULTI-COMPUTER SYSTEMS

It is not unreasonable to assume that ships of the future like present day ships will have several different computers aboard them performing various and sundry tasks. It is also reasonable to assume that the ships of the future will have more general purpose digital computers and fewer special purpose equipments. As this is being written, several of the weapons systems in the fleet are being looked at for possible digitization. Some of these being investigated may eventually have a general purpose digital computer as the central computational element. Other newer systems are being built utilizing digital computers in the initial design.

A ship of the future may have several digital computers in various systems throughout the ship. Many of these are likely to be involved in the control of different weapons on the ship. These computers may be identical, similar, or not related at all, but they are very likely to be interconnected either directly or through some intermediate center such as the Weapons Assignment Complex. They may share inputs from a common source such as the ship's stable element or a surveillance radar and they may output to common points such as displays.

Because of this interconnection and common use of external equipment, the concept of multi-computer systems and the related communications problems need to be examined in some detail. Several approaches are being examined such as: optimum computer size and loading for various tasks, time sharing of computers, transfer of functional computations as a result of overall system load, system reliability and backup, and the related programming and system software.

4.9 OTHER HARDWARE DEVELOPMENTS

In addition to new memory organizations such as recognition memories, there are several other developments which should have a significant impact upon computer systems of the future. Three of these will be mentioned briefly here and will be covered in more detail in later reports.

The first of these is nothing new. This is the mass memory concept. Several techniques show a great deal of promise for reducing the cost and complexity of memories considerably. The reduced cost and complexity will make very large, relatively high speed memories feasible in the near future. This will allow table lookup to be substituted for computations which will result in significant time savings.

Two approaches to mass memory will be discussed in later reports. The first of these is the woven screen technique and the second is through the use of cryogenic techniques. Even though cryogenic devices have definite drawbacks for shipboard installation, the techniques look so promising in several areas that they should be touched upon.

Another hardware approach that will be mentioned briefly is the area of optical processing. Optical processors have many advantages over conventional electronic computing devices, but they also have many drawbacks at this time. Work in the field of LASERS which can provide truly coherent light and the rapidly expanding field of fiber optics should result in further optical processing capabilities. Optical processing techniques will be examined to see if any direct application to surface weapons control systems of the future is apparent.

The third hardware area that will be examined will be those hardware developments that are specifically related to Self-Organizing Systems. This will include such things as the neuristor, artificial neurons, perceptrons and other related developments. This whole area is, of course, very closely allied with software and will involve concepts of adaptive control, heuristic programming, cybernetics, majority logic, and topics that will be discussed in the next section.

4.10 OTHER SOFTWARE DEVELOPMENTS

Although it is not always the case, it is generally true that software is developed for a specific problem or class of problems. For instance, linear programming techniques were developed for a specific class of problems that did not lend themselves to more conventional techniques. The same is more or less true for queueing theory, game theory, dynamic programming, heuristic programming, and many other techniques. For this reason it is somewhat easier to evaluate the applicability of software developments for specific tasks than is the case with hardware.

Many software techniques are indispensable in the initial system analysis and design of complex weapons systems but are of little value in the actual system implementation. Linear programming, for instance, can provide a powerful tool in the optimization of a system design where there may be scores of inputs that must be considered in relation to how they are utilized to optimize various system objectives. Once designed to these optimum mixes of inputs and use of the inputs, this tool for optimizing loses much of its potency. Being a real-time, on-line system, very little can be done to optimize system performance using linear programming once the equipment is designed. Weapon assignment and warhead selection tasks could perhaps make use of these techniques in a real-time sense, however.

Several software techniques will be examined, described briefly, and discussed in subsequent reports if they appear to have any application to the problem at hand. In addition to adaptive control techniques discussed in Section 4.5, dynamic programming, heuristic programming and statistical analysis techniques appear at this time to be candidates for more extensive examination.

APPENDIX A
AN ON-LINE COMPUTING CENTER FOR SCIENTIFIC PROBLEMS

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ABSTRACT

An on-line digital system allowing an unusually direct coupling between the user (physicist, mathematician, engineer) and the computer is described. This system, which has been successfully operated during the past six months, was designed principally to provide assistance for problems whose structure is partially unknown (and frequently surprising). These typically require the development of new methods of attack, and hence an amount of program experimentation not feasible with classical computer center organizations. With the system described here, the interaction between user and computer is close enough to permit effective use of a scientist's intuition and of his detailed understanding of techniques appropriate to his special field. He is able to construct, with ease, and with no necessity for a knowledge of conventional programming techniques and procedures, machine representations of those tools he considers essential to his area, and then use these, on-line, to study or solve problems of interest. The current system is described in detail and its application to a particular illustrative problem is outlined. Implications concerning the extension of the technique to typical, large digital computer installations are given.

I. INTRODUCTION

Despite the impressive achievements in computer hardware and programming techniques in recent years, the most significant gains presently realizable are associated with new approaches to the use, the organization and the logical structure of computers. One line of research has had as its goal the assumption by the computer of certain activities generally associated with human beings; the resultant study of learning machines, adaptive machines, et al has been very fruitful. Quite a different approach has been taken by those who instead seek ways of improving the man-machine communication so that the computer can more effectively assist the man in those jobs (requiring intuition, judgment, evaluation) for which he is best suited. Although no universally accepted nomenclature seems to exist, it is sometimes characterized as a means of getting a man "on-line" with a computer[1)], as opposed to his usual "off-line" status of wading through reams of computer print-out .

Adopting this term, we describe here an operating "on-line" computer research center which provides an unusually close coupling between the man who originates a problem and a (modern, large electronic digital) computer. We hope this example of what can be accomplished using computer hardware well within the existing state-of-the-art will be useful to others concerned with the development of on-line techniques.

This work was initially motivated by the troubles which have been commonly encountered in using a computer to solve research problems whose structure is for the most part unknown and frequently surprising. It is notoriously difficult to obtain a satisfactory computer program if one does not understand, a priori, the general character of the solution. In fact, information about the general character is often what we really want, rather than quantitative details. It is possible in principle to attempt a kind of experimental mathematics, starting with some promising method of solution and the associated program and modifying one or both in the light of the results obtained. However, the lapse of time between the selection of a new method, or the modification of an old one, and the return of information from the computer to the user is in most cases so long as to make this almost infeasible.

The source of the difficulty is basically the poor communication between the "user" (by which we shall henceforth mean the scientist or mathematician who originates a problem and knows most about it) and the computer, consequent upon considerations of economy and, as well, upon the inherent difficulty of imparting to a programmer the detailed and specialized knowledge one acquires about a particular problem area after working in it for some time. One anticipates a significant improvement in a system, such as that described here, which provides for a rapid, direct, comfortable interchange of information between man and machine. In fact, however, one reaps even greater rewards; if the communication link is established in the proper way it becomes possible for the user to apply, simultaneously, to the problem his own intuition, experience, and knowledge of specialized techniques on the one hand and the tremendous computational power of the machine on the other. As we shall see, it is possible for him to build a representation, in the computer, of those analytic tools he believes valuable for a particular problem or problem area. Without any necessity for learning conventional programming techniques, he is able, using only the concepts of classical mathematics, to create his own machine language, one tailor-made to his own needs. He can freely manipulate the elements of this language, in precisely the same fashion one composes mathematical techniques, and can easily modify them to incorporate the knowledge gained from their use in problem solving, so that his computing capability grows with his understanding of the problems.

We shall describe this "on-line" system from the point of view of a typical user rather than that of a "computer expert", by which we shall henceforth mean someone skilled in the art of programming, as opposed to the "user" whom we assume to be totally unversed in such matters. The programming principles and details will be the subject of a separate article. We shall only discuss the presently existing system, as it has been operating since August, 1962. While this will inevitably entail the mention of certain specific aspects of the particular computer uses (AN/FSQ-27; RW-400), it should be kept firmly in mind, that while the detailed organizational choices were such as to take maximum advantage of the particular characteristics of this machine, the on-line techniques are in <u>no</u> <u>way</u> dependent upon these characteristics. In the last chapter we sketch a method for using these on-line techniques with a standard operational computer center, a method consistent with the usual economic constraints on computer time.

In what follows, we restrict ourselves to the use of on-line techniques in the solution of mathematical and physical problems, this being the area of principal interest to us and the only one in which we have actual experience with an on-line system. We believe, however that the techniques can be extended to quite different areas of computer applications, a point to which we return in the last chapter. Meanwhile, we shall, in the interests of clarity, confine ourselves to a very specific description of the present system.

This work is an extension of an initial effort in which a particular problem, the energy gap integral equation of the Bardeen-Cooper-Schrieffer theory of superconductivity, was solved with an on-line approach[2)]. However, in that work, which was carried out in the period July through December 1961, all of the subroutines for the problem were programmed in a conventional way. While the user was free to compose these elements in various ways using the control and display capabilities of the control console in solving the gap equation, he had no freedom to modify, on-line, the subroutines or create new ones, a freedom which is an essential characteristic of the present system. Thus, the earlier work comprised some aspects of items A and B, described below, but none of item C, the console programming.

II. THE ON-LINE SYSTEM

At the outset, we should emphasize that each aspect of the design of such a system involves a number of choices. We shall describe here our own, with no representation that they are in every case the optimum ones; in fact, in some cases experience has indicated how some of these might be improved. Nonetheless, the resulting system operates in a very satisfactory manner.

Three principal features, independent but interacting, characterize the system:

A. Functional Orientation

The programming structure is such that in the computer, as it appears to the user, functions (sets of 101 points) rather than individual numbers constitute the elements while the repertoire of "commands" consists of operations on functions (e.g., arithmetic, differential, and integral operations).

B. Control and Display Capability

Central to the operation of the system is a control console having a number of push buttons or keys, which allow for user control of the computer, and two 17 inch CRT oscilloscopes (with line-drawing capability) which provide direct graphical representation of computational results. An 8 inch CRT with alphanumeric capability and a flexwriter provide numerical output when required.

C. Console Programming

A simple procedure allows the user to construct, directly at the console, new subroutines, using as building blocks an initial set of hand programmed[3)] subroutines, plus any subroutines previously created by this "console programming"[4)] procedure.

We shall now flesh out this skeletal description with further details. The keys of the control console are divided into three groups: 24 are used for function storage; 30 to designate operations on these functions; and 11 (the digits 0 through 9, $\oplus$ and $\ominus$) for the input of individual numbers[5)]. Throughout we mean by a "function" a set of 101 points, i.e., 202 numbers, represented in the computer as 202 machine words, each word having 26 bits . (The choice of 100 intervals for the description of a function is one example of the arbitrary choices mentioned in the first paragraph. With fewer points one cannot adequately represent very much structure, while if 100 are insufficient one should probably use a different scale, or a different representation.) In addition to the 202 numbers, which all lie between -1 and 1, each function carries two scale factors (to base 2), one for abscissa values (s_x), the other for ordinates (s_y). The actual function values are thus the product of the 101 mantissas y_n, $1 \leq n \leq 101$ and the common scale factor, 2^{s_y}. For convenience, a block of 256 words is assigned to each function, the remaining 52 words being available for other labeling, for functional values (in the sense of Volterra), etc.

Because the computer module (CM) of the RW-400 has only 1024 words, functions are stored on an 80,000 word magnetic drum.

Half of the CM memory is used for two "function registers", called the C and D registers, each having a capacity of 256 words. They play a role for functions quite analogous to that which the accumulator in a computer plays for numbers: functions to be operated upon are loaded into the C and D register and the resulting function is eventually stored back on the drum.

Each of the 24 function storage keys addresses a particular section of the drum but this is of no concern to the user, who may think of the keys themselves as the storage locations. These keys initially carry some neutral labeling (e.g., the letters A through X) but as the user stores functions in them he relabels them (using any convenient nomenclature) to indicate the function stored there.

We can now describe some of the basic operator keys. LOAD and STORE bring any desired function from the drum into the D register or, conversely, store the contents of the D register into any specified function location. For example, pressing the LOAD key and then some function key, say R (or in a shorthand notation we shall employ henceforth, in which each word or symbol corresponds to the name of a particular operator or function key, LOAD R) brings the function stored in key R into the D register. Similarly, STORE F transfers whatever function may be in the D register to location F. In both cases, the words written (on the drum or in the CM memory) replace whatever was previously in that location. (The contents of the cells from which the information was taken are left unchanged so that immediately after LOAD A or STORE A both the D register and the A key contain the same function.)

The operator key J-GEN creates the identity function, $y = x$, $(-1 \leq x \leq 1)$ and puts it in the D register. The arithmetic keys (+, -, ·, ÷) cause the computer to carry out the indicated operation on the ordinates of two designated functions, assuming the abscissaes to be the same. For example, pushing the four keys

LOAD A + G

causes the computer to load whatever function is in location A into the D register; to load the function in location G into the C register; to add the y coordinates of the C and D registers, with differences, if any, in the y scale values properly taken into account; and finally to store the result in the y coordinates of the D register, leaving the x coordinates of the D register unchanged. If the user wishes the sum stored in some function storage location, say P, he then pushes STORE P. Alternatively, he can continue on with a series of arithmetic operations, all of which follow the same pattern. (Once the + button has been pushed, one may add as many functions as desired by simply pushing the + button again. This is true in general; once an operator button has been pushed that operation is continued as long as no other operator buttons have been pushed.)

Individual numbers can be put into the computer in a variety of ways. Since constant functions are sometimes required, we use them to represent also constant numbers, but this is by no means necessary. The procedure is simply: push the LOAD button; then type in the sign, followed by a mantissa less than 1, and any desired power of 10, positive or negative. (For example, 11.56 is entered as●.1156 ⊕ 02.) A constant function whose value is equal to this number is thereby loaded into the D register (i.e., the x values of the D register range as usual from -1 to +1 and the y values are all equal to the desired constant).

The DISPLAY key allows the user to see a graphical representation of any of the stored functions. DISPLAY A, for example, causes the computer to display on one of the 17 inch CRT scopes, the 101 points stored as functional values in location A, with adjacent points connected by straight line segments. Pressing the A key once more will erase the display curve from the scope (although not of course from the drum location where it is stored). Since only the mantissa values of a function are displayed on the CRT, we sometimes wish to check the scale of the whole function. This is done with the DISPLAY SCALE key, which causes the ordinate scale value, s_y, of the D register to be displayed on the alphanumeric scope. One thus has the capability of carrying out arithmetic and algebra on functions and examining the results graphically whenever desired.

The essential elements of calculus are provided by the Δ and Σ keys. The former simply takes differences of adjacent ordinate values in the D register and leaves the result in the D register, e.g., $(y_{n+1} - y_n)$ replaces y_n, $1 \leq n \leq 100$, with a special treatment at the righthand end point (for example, the first difference computed on the basis of a second or third order fit to the function values at that end replaces y_{101}). $\sum$ is just a cumulative sum of the ordinate values in the $\underline{D}$ register with the result left in the $\underline{D}$ register (0 replaces y_1

and $\left(\sum_{k=1}^{n-1} y_k \right)$ replaces y_n, $\quad 2 \leq n \leq 101$).

From these two keys it is easy to construct approximations of any desired accuracy to the derivative and (indefinite) integral operators.

At this point, the general nature of items A and B above should be clear. One has, in effect, a powerful, and exceedingly fancy, combination hand computer and plotting machine. Any desired function can be readily created in the computer (using power series, asymptotic series, etc.) and one can perform all of the operations of classical analysis upon them. Suppose, for example, one wants the sine of some function, f, which has been loaded into the $\underline{D}$ register and suppose further f is sufficiently small so that the first two terms of a power series

$$\sin f = f - \frac{f^3}{6} \tag{1}$$

suffice. Select two function keys, F and G, as "working space". The following keys would then be pushed:

$$\text{STORE F} \cdot \text{F} \quad \text{F STORE G LOAD} \ominus .166667 \oplus 00 \cdot \text{G} + \text{F} \tag{2}$$

If f was initially in the $\underline{D}$ register, sin f, to the accuracy of Eq. (1), will now be there. In precisely similar fashion one could obtain a representation of the sine function to any desired number of terms of the power series. However, it is clearly infeasible to go through this sequence of key pushes every time one wants the sine function. It is at this point that feature C, "console programming", comes in; like a giant lever (or a strong bootstrap) it provides an enormous multiplication of the capability available to the user.

Clearly, all that is required is that the computer be able to "remember" and suitably record a sequence of key pushes such as that given in the example above. Moreover, it should then, in some sense, attach this list of key pushes to some previously blank key, which thereby acquires significance. The procedure is simple: we select some key, hitherto blank, which we decide will be the SINE key, and so label it. We then "program" this key using the PROGRAM key in the following way. First push PROGRAM; then push the (hitherto blank) key which will henceforth be the SINE key; then push precisely the buttons listed in (2); finally, at the end, push the PROGRAM button again. The result is that the machine goes through a "dry run", i.e., executes the commands (2) in precisely the same fashion as if we had not pushed the PROGRAM button; examination (e.g., via the display capability) of the result of this dry run immediately provides a first check on the console program just created. In addition, however, the computer constructs a list of these key pushes, termed a "subroutine", and "inserts" it under the SINE key. If at any time in the future we push the SINE key, the computer will go through precisely this sequence of operations[6]. Furthermore, we can in the same fashion program other keys, using as component keys not only the initial hand programmed ones (such as +, etc.) but also _any keys which have been console programmed in the above fashion_. These new keys can, in turn, be used as components of other console programs, and so on, to a depth limited only by the storage volume. (The present system allows for 256 such console programs but this could easily be expanded by several factors of 2.) In this way the operator creates his own subroutines, of arbitrary complexity, and pyramids these to achieve whatever computing capability he desires.

At this point it becomes difficult to describe adequately the generality and utility of the resultant system, just as it would be difficult to explain to a college freshman (in less than a few semesters) why algebra or calculus, whose basic principles can after all be rather concisely stated, is so useful. The on-line system has a structure very close to that of mathematics in its open-endedness, its generality and the constructive capability it affords the user. We shall therefore simply use the rest of this section to describe briefly the other hand programmed keys which are initially provided to every user when he begins work, and in the next section illustrate how this capability was used for one particular, fairly illustrative problem. A detailed characterization of all the hand programmed keys is given in Appendix A.

To begin with, the 30 operator keys physically present on the control console are by no means enough to encompass the initial hand programs plus the console programs needed in a typical problem. We therefore employ the concept of "overlays". To each overlay corresponds a set of meanings for the operator keys; changing the overlay changes the significance of all of these keys. The number of overlays is limited only by the size of the large volume storage in the computer system; in our case there are 32 overlays. One key common to every overlay is OVERLAY IN. The user changes overlays by simply pressing OVERLAY IN and then typing in, on the numerical keys, the number of the overlay he wishes to use. The 256 word program (which comprises the overlay from the programming point of view) is thereupon brought from the drum into the computer and all further key pushes will be interpreted by the computer in terms of that overlay until the operator makes a change of overlay. (In addition to the OVERLAY-IN and PROGRAM keys, five others, to be described later - REPEAT, OVERLAY-OUT, DISPLAY-OV-NUMBER, INSERT and DO - are common to all overlays, leaving a total of 24 x 32 = 768 keys in the present system, each of which can have a hand program or a console program.) The existence of multiple overlays modifies the console programming procedure slightly in that we must inform the computer not only which key is to be programmed but also which overlay we want that key to be on. The latter is accomplished by simply typing in the desired overlay number (on the numerical keys) immediately after pushing the key being programmed: for example, PROGRAM SINE 10 followed by the key pushes (2) and then PROGRAM would attach the subroutine (2) to the indicated key of Overlay 10.

In a similar way, the total number of function storage keys available can be multiplied up from the 24 keys physically present to an extent again limited only by storage space. In the present system, we have 6 "banks" of these function keys, giving a total of 144 function storage locations. This too can be accomplished in many ways; at present, in place of the LOAD and STORE keys described above, we have in fact six LOAD and six STORE keys. Thus, LOAD_{I} A will load into the $\underline{D}$ register whatever function is under key A on bank I; STORE_{VI} F will store into key F of bank VI whatever function is in the $\underline{D}$ register, etc. (We use Roman numerals to label banks, Arabic to label overlays, thus minimizing a possible source of confusion.)

In typical operation, a user begins with an initial complement of hand programs and, working for a period of one to two hours[7)], creates and checks (by observing the character of displayed curves, examining individual numerical values, running test cases, etc.) the console programs he needs. When his period of operation is finished, he pushes the SYSTEM DUMP button which stores the entire system (contents of the computers and of the drum) into a designated section of magnetic tape. When he next returns to the machine his system is loaded back from this tape and the computer is in precisely the same state as when he left. In the interim another user comes to the machine, and loads his system from tape. All of the buttons, save for the initial core of hand programs, will typically be different for two users so that arguments concerning the value, efficiency or desirability of any particular console programming need never arise; each user makes his own choice, i.e., literally creates his own language.

When a user returns to the machine, SYSTEM LOAD (the inverse of SYSTEM DUMP), using his tape, restores the system (computer and drum) to precisely the state in which he left it, so that he can continue on, creating new console programs or, when he has built sufficient capability, attacking his problem. If, in the latter case, he immediately discovers a need to create new console programs or modify existing ones he is free to do so. (To reprogram a key he simply programs it as though it were blank; any previous program is simply buried.)

The SYSTEM LOAD and DUMP buttons are part of a "system" overlay which allows one to control the several components of the RW-400 system: to write out a block of data on (or read a block from) a tape unit, a buffer, the drum, etc.; aside from the tape operations, these are of interest principally to the computer expert rather than to the typical user, so we shall leave further details for the Appendix. (See Section A4, System Control Capabilities.)

The remaining hand programmed keys fall into three groups:

1. Mathematical Operations

These include first of all the arithmetic operations (on functions!) which have been mentioned already but require further comment. There are at least three ways of carrying out the function arithmetic: fixed point (with respect to the entire function), floating point (with respect to the entire function) or floating point for each point of the function. Since each has its own virtues, it is desirable to allow the user a free choice. Thus on Overlay 01 the arithmetic is done in a fixed point fashion. For example, when two functions are added, the y scales are compared and the smaller of the two is made equal to the larger one, the associated mantissa values being decreased (i.e., shifted to the right) enough times so that the functional values (mantissa plus scale) remain unchanged; the mantissa y values are then simply added together. If at some point the sum of these happens to be greater than 1, there will be an overflow. This is readily apparent if the sum is displayed, but it may happen in the course of a console program (unless one has correctly anticipated all scaling aspects of the problem) and can then be a considerable nuisance. Similar comments apply to divide, which will overflow at any points where the mantissa of the numerator exceeds the mantissa of the denominator. A second arithmetic overlay, 02, provides protection against such overflows by first floating and then contracting both summands before addition. In division, the numerator and denominator are first floated and then the numerator is contracted enough times to prevent overflow at any point, if no more than 12 contractions are required, but no more than 12 are made in any case. An Overlay 03, in which the arithmetic is done on a floating point basis for each individual point of the functions, has recently been incorporated into the system but we have not yet sufficient experience to compare it with the other two. Of the latter, Overlay 02 is generally the more convenient, but in special circumstances (larger range of variation within a single function) can lead to more loss of accuracy than would result from the careful use of Overlay 01.

In addition, we have the following: EXPAND y decreases s_y by 1 and doubles all the mantissas of the D register; CONTRACT y is its converse. (They are needed in conjunction with the arithmetic on Overlay 01 and also allow examination (on the CRT) of small amplitude structure of a curve, display of curves at common scale, etc.) FLOAT MANTISSA does EXPAND y as many times as possible without causing overflow at any point. EVALUATE picks out the value of the function in the D register at the x coordinate closest to any designated value. REFLECT interchanges the x and y coordinates of the D register; SUBSTITUTE puts the y coordinates of the C register in place of the x coordinates of the D register. (Using REFLECT and SUBSTITUTE one can create, from the real function arithmetic hand programs, console programs for complex function arithmetic. Using REFLECT and EVALUATE one can find extrema of a function.) δ-FUNCTION creates in the D register a function which is 1 at any desired point and zero elsewhere. INTEGRAL-TRANSFORM transforms the function f in the D register, using a kernel $K(x, x')$ previously stored on tape as 101 functions of x', and leaves the result, $\tilde{f}(x) = \int dx' \, K(x, x') \, f(x')$, in the x coordinates of the D register. EXPONENTIAL, SINE and COSINE operate on the function contained in the D register, leaving the result in the D register. LEFT-SHIFT and RIGHT-SHIFT perform the indicated operations on the y coordinates of the D register. RELATIVE INTERPOLATE accepts graphical point inputs (via user-controlled crosshairs on the CRT) and modifies the function in the D register so that it passes through these data points, preserving its initial shape between data points.

2. Aids to Console Programming

These include besides the PROGRAM key already described, also REPEAT, a key which allows any console program keys to be repeated any desired number of times, and two keys (TALLY and COMPARE) which provide the capability for branching within a console program.

3. Display and Output Keys

These provide the capability to display on the alphanumeric scope the number of the overlay currently in the computer; to erase all curves from the CRT; to display (on the alphanumeric scope) the binary scale of the function in the D register, as well as the value of the first point of that function; to input individual points graphically, using a movable crosshair; to print a hard copy of any desired curve on the flexwriter; to produce English language labels for kernels stored on tape, dumps stored on tape, or curves printed out on the flexwriter; to display curves on either of the two 17 inch CRT's; to use other display formats (dots along, crosses, circles, etc.) as well as the usual display of line segments.

In addition to these, there are Aids for the Hand Programmer involving the convenient input of machine words from the console, the display of sections of memory in machine language, etc.. These are described in the Appendix, together with more detailed specification of all the keys already mentioned.

In a class by itself is the SECOND-COMPUTER key which at first glance seems highly specific to the RW-400 and yet really provides an excellent illustration of how to set up an on-line system for an arbitrary computer. In the RW-400 system, there are two identical computer modules, CM-1 and CM-2. The control console is tied directly to CM-1 and this is the only one used in all of the operations described so far. Suppose however that operator key [K] on Overlay 10 is a rather long program, requiring several minutes or more to run. It is then efficient to use the SECOND-COMPUTER key, as follows: press SECOND-COMPUTER, press [K], and type in the number (in this case 10) of the overlay on which [K] is located. CM-2 then performs this program, taking the subroutines and curves needed from the drum in the same way that CM-1 would do.

While this is going on, however, the user is free to operate in the normal fashion with the control console and CM-1, doing computations; examining, if he likes, intermediate results as they are generated by CM-2; preparing new programs; setting up material for the next case; etc. In the last chapter we will explain how this serves as a model for an on-line system using a conventional, large central computer.

AN ILLUSTRATIVE PROBLEM

As an illustration of the use of the on-line system, we describe briefly one problem we have studied, a linear integral equation for a complex function of a real variable. While a single problem can no more exhibit the power and generality of the on-line system than any one application of, say, calculus, can illustrate the utility of classical analysis, we include it to show the ease with which rather sophisticated mathematical techniques can be employed in a system of this sort.

We present the problem as a mathematical one, essentially suppressing the context of physics from which it arose; outline the mathematical methods used; indicate some of the principal console programs generated to implement these methods; and show a few sample results. We can state, but not easily illustrate, one essential point: the method of solution finally adopted was itself the result of experimentation with the on-line system. Several approaches were tried; some quickly proved themselves unsuitable, but as we learned more about the nature of the solutions, we were able to develop satisfactory methods for obtaining them. Thus, quite apart from questions of mathematical convergence (e.g., of an iteration process), one sees a convergence in a kind of space of mathematical techniques.

A study of the electrostatic wave fluctuations in an electron-ion plasma subjected to an external electric field leads to the following integral equation[8] for the fluctuation electric field, of wave number k, as a function of time:

$$E(t) + \int_0^t dt' \left\{ K_e(t-t') \exp i \left[\phi(t) - \phi(t') \right] + \right.$$

$$\left. \delta\, K\,(t-t') \exp i\delta \left[\phi(t') - \phi(t) \right] \right\} E(t') = I(t), \tag{3}$$

$$0 \leq t \leq T$$

where

$$K_e(t) = te^{-k^2t^2/4} \quad K_i(t) = te^{-\delta k^2t^2/4}$$

$$\phi(t) = \mathcal{E}\, t^2/2 + ut \quad \delta = 1/1836$$

and $I(t)$ is given. (We have chosen units in which the electron thermal speed $(2T/m)^{1/2}$ and the plasma frequency, $(4\pi ne^2/m)^{1/2}$, are unity.) We shall say no more here about the physics of the problem since this is discussed elsewhere[9], and only sketch the on-line techniques used to solve it.

Using an operator notation for the integral transforms in (3),

$$\underset{\sim}{K_e} \cdot E \equiv \int_0^t dt'\, K_e(t-t')\, E(t'), \text{ etc.} \tag{4}$$

we can write (3) as

$$E + e^{i\phi} \underset{\sim}{K_e} \cdot (e^{-i\phi}E) + \delta e^{-i\delta\phi} \underset{\sim}{K_i} \cdot e^{i\delta\phi}E) = I \tag{5}$$

Over the time interval of interest ($T \lesssim 10\pi$) the norm of the first operator in (5) is so large, for $k \leq 1$, that an attempt at direct iteration proved useless. (As can be seen from the soluble special case, $k = \delta = \phi = 0$, this corresponds to computing sin t by a power series on the interval $0 \leq t \leq T$.) However, the transformation

$$F = e^{-i\phi} E \tag{6}$$

gives an integral equation for F,

$$F + \underset{\sim}{K_e} \cdot F + \delta e^{-i\psi} \underset{\sim}{K_i} (e^{i\psi} F) = J \equiv I e^{i\phi}, \qquad \psi \equiv (1+\delta)\phi \tag{7}$$

which can be solved by iterating only the last of the terms on the left hand side. The equation

$$F + \underset{\sim}{K_e} \cdot F = A \tag{8}$$

has the solution

$$F = A - \underset{\sim}{L_e} \cdot A \tag{9}$$

where $\underset{\sim}{L_e}$ is, like $\underset{\sim}{K_e}$, a translate type integral operator and hence specified by a single function L_e

$$\left[\text{i.e., } \underset{\sim}{L_e} \cdot A \equiv \int_0^t dt' \, L_e(t-t') \, A(t')\right]$$

which must obey an equation like (8) with A replaced by K_e:

$$L_e + \underset{\sim}{K_e} \cdot L_e = K_e \tag{10}$$

Having once found (for given k) the function L_e, we write (7) as

$$F = (1 - \underset{\sim}{L_e}) \cdot \left[J - \delta e^{-i\psi} \underset{\sim}{K_i} \cdot (e^{i\psi} F)\right] \tag{11}$$

and solve it by iteration

$$F_{n+1} = (1 - \underset{\sim}{L_e}) \cdot \left[J - \delta e^{-i\psi} \underset{\sim}{K_i} \cdot (e^{i\psi} F_n)\right] \tag{12}$$

With a reasonable initial guess, e.g., $F_0 = J$, we find that this converges splendidly (three iterations). From F we compute, finally, $E = e^{i\phi}F$.

The first step is to find L_e from (10). For the reasons noted above, straight iteration is ruled out. While (10) can indeed be solved with Laplace transforms, the transform of L_e involves the error function of complex argument[9] and hence is difficult to invert. Instead, we take advantage of the fact that problems which are "adjacent" in a mathematical sense are, within the on-line system, adjacent also in a computational sense. If K_e is replaced by

$$R \equiv N^2 te^{-at}, \tag{13}$$

then the inverse kernel function, L_R, satisfying

$$L_R + \underset{\sim}{R} \cdot L_R = R \tag{14}$$

is simply

$$L_R = Ne^{-at} \sin Nt. \tag{15}$$

We therefore write (10) in the form

$$L_e + \underset{\sim}{R} \cdot L_e = K_e + \underset{\sim}{D} \cdot L_e \tag{16}$$

$$D = R - K \tag{17}$$

and choose N and a so as to make the norm of D small (e.g., $a = k$, $N = 2$), thus permitting an iterative solution,

$$L_e^{(n+1)} = (1 - \underset{\sim}{L_R}) \cdot (K_e + \underset{\sim}{D} \cdot L_e^{(n)}) \tag{18}$$

This converges nicely (three or four iterations) to yield a result which will differ from the exact solution of (10) only in consequence of the approximation inherent in numerical methods. However, we can exploit the linearity of (10) to obtain a more accurate solution as follows. Let L be the result obtained by iterating (18) until it has converged. The error in L is measured by the size of

$$P \equiv K_e - L - \underset{\sim}{K}_e \cdot L \tag{19}$$

and the difference $\eta = L_e - L$ satisfies

$$\eta + \underset{\sim}{K}_e \cdot \eta = P \tag{20}$$

or equivalently

$$\eta = (1 - \underset{\sim}{L}_R) \cdot (P + \underset{\sim}{D} \cdot \eta)\,, \cdot \tag{21}$$

i.e., an equation identical with (18) save for the inhomogeneous term. If L is determined from (18) up to a percentage error of order ϵ, we can find η from (21), also with a percentage error of order ϵ, and hence get an approximation, $L + \eta$, to L_e which has an error of order ϵ^2. In a similar fashion we can find a correction to η, and so forth.

We now indicate how the on-line system was used to solve (7) by these methods. To begin with, certain function keys are assigned to the constant parameters of the problem and to the principal independent and dependent variables as shown in Table I. Locations for constant functions which one finds it convenient to have on hand are assigned as the need arises. (The notation in Table I

$\mathcal{E}$	k	u	T
δ	N	a	Δt
t	K_e	K_i	R
L_R	L_e	0	$\frac{1}{2}$
L	L'	L''	L'''
f	$\tilde{f}$	kernel source	working space

Table I

Assignment of function storage spaces (keys) on Bank I for the plasma oscillation problem. Significance of the symbols is given in Eqs. (3) through (23).

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or equivalently

$$\eta = (1 - \underset{\sim}{I}_R) \cdot (P + \underset{\sim}{D} \cdot \eta), \tag{21}$$

i.e., an equation identical with (18) save for the inhomogeneous term. If L is determined from (18) up to a percentage error of order ϵ, we can find η from (21), also with a percentage error of order $\mathcal{E}$, and hence get an approximation, $L + \eta$, to L_e which has an error of order ϵ^2. In a similar fashion we can find a correction to η, and so forth.

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is the same as in Eqs. (3) through (23); the meaning of other symbols will be explained below.) Once assigned, the labels on the function keys are used in referring to these keys rather than the neutral ones (A, B, ...X) of the preceding chapter.

Operator keys are then created, using the console programming procedure, some of the principal ones being as follows:

$[t]$ This creates the function $t = T(x+1)/2$ (assuming that the desired value of T has been previously stored in the T key of bank I) and stores it in t on bank I. It also computes Δt and stores it in Δt. As an illustration of console programming, we list the key pushes made in programming this key[10], which we suppose is to be on, say, overlay 10:

PROGRAM $[t]$ 10 OVERLAY-IN 02 J-GEN · 1/2
+ 1/2 · T STORE t Δ STORE Δt PROGRAM (22)

INITIAL SET-UP This simply displays on the alphanumeric scope the names of the constant parameters ($\mathcal{E}$, T, k, δ) and, next to each, the value presently stored for it on bank I. If the user wishes to change any of these he can, of course, do so before running the problem. This program also stores the various constants indicated in Table I and finally pushes the $[t]$ key. Thus one knows that everything is in order for the start of a calculation.

$[K_e]$ This simply computes the kernel function $K_e(t)$ and stores it on bank I.

$[R]$ This computes and stores the function, $R(t)$ defined by (13), using whatever values the user has stored in N and a on bank I.

$[L_R]$ This computes and stores the function $L_R(t)$ defined by (15). It, for example, was programmed by:

```
PROGRAM L_R 10 OVERLAY-IN 02 LOAD t · N SINE STORE L_R LOAD -1
· t a EXP · N L_R STORE L_R PROGRAM.
```

(Note that in this we have used the L_R key as a temporary working space to store one factor of the final answer.)

We frequently need to generate, on tape, the 101 functions which comprise one of our translate kernels. To produce this capability, we first program a KERNEL-GENERATE-AUXILIARY (KGA) key as follows:

```
PROGRAM KGA 10 OVERLAY-IN 02 LOAD KERNEL-SOURCE TAPE-WRITE
LEFT-SHIFT STORE KERNEL-SOURCE OVERLAY-IN 10 PROGRAM.
```

This subroutine takes whatever function is in the kernel-source space on bank I, writes it out on tape, left shifts it,(y_{n+1} replaces y_n) and stores it back in the kernel-source space. We end it by calling in the overlay (10) on which KGA has been programmed in order that it be a repeatable key. The key which will actually produce the kernel on tape is then made by simply repeating the KGA key, i.e., we make a KERNEL-GEN key as follows:

```
PROGRAM KERNEL-GEN 10 OVERLAY-IN 10 REPEAT KGA 101 PROGRAM.
```

(Repeat $[K]$ followed by a number, n, causes key $[K]$ to be repeated n times.)

It is clearly now a simple matter to make, for any desired value of k, the various functions and kernels needed for (18).

Since we will be taking many integral transforms, it proves convenient to incorporate the hand-programmed integral transform key (which produces in the x-coordinates of the $\underline{D}$ register the transform of the function in the y-coordinates of the $\underline{D}$ register) into a simple console program which will produce in the $\tilde{f}$ space of bank I the transform of whatever function has been stored in f on bank I. We designate this as INT-TRANS and, using it, program a new key, ITERATE-L_e which does one pass of (18) as follows. Assume that the kernel D has been stored on tape and, following it, the kernel L_R; that the tape is positioned to the beginning of the D kernel; and that some initial guess, or the result of a previous iteration is in L_e on bank I. We then do

PROGRAM ITERATE-L_e 10 OVERLAY-IN 02 LOAD L_e STORE f OVERLAY-IN 10 INT-TRANS OVERLAY-IN 02 LOAD $\tilde{f} + K_e$ STORE f OVERLAY-IN 10 INT-TRANS OVERLAY-IN 02 LOAD$_I$ f - $\tilde{f}$ STORE$_I$ L_e REWIND-TAPE (23)
OVERLAY-IN 10 PROGRAM

Although the program (23) is adequate, we add to it certain display and storage features which increase the convenience of operation. That is, after checking (with simple examples, special cases, etc.) the correctness of (23), we program another key, $\lceil L_e \rfloor$ which first pushes the ITERATE-L_e key (23) and then goes on to store the resultant L_e in key L, having first moved the contents of L'' into L''', L' into L'', and L into L'. Thus as the new key $\lceil L_e \rfloor$ is repeated, we are able to examine the results of the most recent four passes (and could, of course, save even earlier ones if desired, by using one of the other banks). In addition, the new key erases the scope and then displays on it the contents of L' (dotted) and the contents of L (usual dot-plus line display). Thus, each time the key is pushed the user sees, as soon as the pass has been completed, both the new result and, for comparison, the previous one, so that he can judge the convergence characteristics of the iteration process (18). (One could as well display the ratio or difference of the old and new curves, etc.)

The reader who has followed the details of the last few paragraphs can supply those omitted from what follows. Having credited the programs needed for (18), we can use them also for (21) and, by repeating the correction process, obtain a very accurate L_e. From it we make an $\underset{\sim}{L}_e$ kernel (using KERNEL-GEN) and also the $\underset{\sim}{K}_1$ kernel and are then in a position to solve (12), i.e., to make a key which will do one iteration of (12). The only new complication lies in the complex character of F, but this causes no real difficulty. We simply write out on tape two copies of $\underset{\sim}{K}_1$ kernel, followed by two of $\underset{\sim}{L}_e$. The ITERATE-F key then multiplies F by $e^{i\psi}$, uses the INT-TRANS key to transform the real part of the product with $\underset{\sim}{K}_1$, stores that in some working-space function key, transforms the imaginary part with $\underset{\sim}{K}_1$, combines the results into a single complex function, multiplies this by $\delta e^{-i\psi}$, subtracts that from J, transforms the real part with $\underset{\sim}{L}_e$, then the imaginary part, etc.

This requires, of course, that one create also keys which produce $e^{\pm i\psi}$ (used only after a change of $\mathcal{E}$ or u) and keys which give complex arithmetic capability. Using REFLECT and SUBSTITUTE, one can easily program, for example, a COMPOSE key (which, given two real functions, $f_R(t)$ and $f_I(t)$, in two standard locations composes them into a single complex function $f=(f_R, f_I)$ with the parameter t eliminated) and a DECOMPOSE key which is its inverse. For example, designate 3 keys as f_R, f_I, f and then make COMPOSE by

PROGRAM COMPOSE 10 OVERLAY-IN 02 · f_R LOAD f_I

SUBSTITUTE STORE f PROGRAM

(We use · f_R as a way of loading f_R into the $\underline{C}$ register.)

Similarly we have

PROGRAM DECOMPOSE 10 OVERLAY-IN 02 LOAD 0 + f STORE f_I

LOAD f REFLECT STORE f_R LOAD 0 + f_R STORE f_R PROGRAM

(LOAD 0 + f_R STORE f_R is just a way of restoring standard x coordinates to f_R so that it will look normal when displayed.) From these, it is then a simple matter to make the keys for complex function arithmetic.

Using the ITERATE-F key, and the L_e kernels generated by the procedure described above, it is an easy matter to obtain solutions of (3) for a variety of values of the parameters k, u and $\mathcal{E}$. As we see from Figure 2 the kernel L_e, which is a sine wave for k = 0, is increasingly damped with increasing k. (Having computed L_e one can compare it with the result obtained by keeping only the least damped pole of the Laplace transform of L_e when inverting the latter by contour integration[9]; as shown in Figure 2, the agreement is fairly good save near $t = 0$.) Understanding the somewhat exotic curves which result is assisted by comparing them with analytically soluble problems, e.g., those obtained by omitting the $\underset{\sim}{K}_1$ operator in Eq. (3) or setting $\mathcal{E}$ equal to 0. Some typical results are shown in Figure 3 through 8 for the case $I(t) = 1$. An analysis of the results and a discussion of their significance is given elsewhere[9].

DISCUSSION

The on-line system we have described is specific both as regards the computer used and the area of mathematics emphasized (classical analysis), the choice of the computer being a consequence of its availability, while the selection of problem areas was dictated by the research interests of the participants. On the basis of the experience gained in the design and operation (since July, 1962) of this particular system, the extension of these on-line techniques to other computers and to other areas of application appears rather straightforward.

We first outline a system which would be suitable for a conventional, large central computer and which would permit an operation identical, from the user's point of view, with that we have described here. Besides the central computer itself, one needs a large volume storage element, such as a disc file, and a small satellite computer, one with a memory of the order of 8,000 to 10,000 words and a 5 to 10 microsecond cycle time. As input/output equipment, the satellite computer would have two electric typewriters, whose keys take the place of the control console keys, and two CRT display scopes, each capable of displaying of the order of [illegible]000 points furnished by the satellite computer, and connecting pairs of these with line segments when desired.

The operation parallels that of the present system when the SECOND COMPUTER key is used, the control console and the first computer module of the RW-400 being replaced by the satellite computer plus its input/output equipment; the drum (where curves and subroutines are stored) being replaced by the disc file; and the second RW-400 computer module being replaced by the (larger and far more rapid) central computer. The satellite computer is used to compose and check console programs and for all trivial computing: examination and comparison of curves, formation of ratios and differences, simple test cases, etc. Only when the user has progressed to a point of having a substantial computational task which he wishes performed, is the central computer involved. He simply presses the CENTRAL COMPUTER key and then any of the keys he has previously console programmed (with the satellite computer). The CENTRAL COMPUTER is not interrupted, but when it finishes the task on which it is presently engaged and returns to its own control system for a next assignment, it is directed to take from the disc file the satellite's request, carry it out and return the results to the disc file. The central computer then proceeds to other work while the user examines the results, perhaps modifies his program or decides on a next case. The central computer is brought in only for significant computational tasks and never waits for the user. The user may occasionally have to wait a short time for the central computer[11)], but since the tasks he gives it are only those requiring a considerable amount of computation, this is not unreasonable.

In a sense, the satellite computer functions as a kind of informational impedance matching device between the man and the large central computer. Taken by themselves, these are mismatched with respect to both operations per second and dollars per hour. However, the satellite computer is economically matched to the man (i.e., rents for a figure comparable to his salary) and at the same time is sufficiently well matched to the central computer in terms of data transfer so as to be consistent with the economic constraints concerning the latter's use. Of course, many variants of this basic scheme are possible, some more suited to a particular computer center than others. Because our experience indicates that it is convenient to have of the order of 50 to 60 keys, we specify two typewriter keyboards, but in principle the necessary control capability, including that required for console programming, could be provided with far fewer keys. (Ten, representing the digits 0 through 9, plus one more ___ to, so to speak, change overlays ___ is probably the minimum required to provide a reasonable degree of operational comfort.) Because the simultaneous display of many curves on a single CRT scope gets quite confusing, it is very convenient to have two scopes, particularly for problems where one wishes to examine a mapping from one plane to another. One scope might be sacrificed, but we would argue strongly against the elimination of both, having found the rapid feedback of information in graphical form to be a tremendous asset in studying the structure of a problem and of the tools one creates, in the form of console programs, to solve it, not to mention its value in checking and trouble shooting the latter. In any case, the effective implementation of such a system will share some of the problems inherent in any time-sharing arrangement 12).

We consider briefly the generalization of on-line techniques to other problem areas. The emphasis on functional orientation is particularly important for non-local problems but it is straightforward to include also a capability for dealing with individual numbers, something which would be useful, for instance, in solving differential or difference equations. This requires only an overlay (04, say) which interprets the function keys as single numbers, i.e., allows the function keys to address individual cells of the computer memory rather than function storage blocks on the drum, the arithmetic on Overlay 04 being just the conventional single number arithmetic of the computer itself 13). Console programming would allow the composition of operations on this overlay in the usual fashion.

The extension to areas of mathematics other than classical analysis also seems feasible. To handle matrix problems, for example, one would replace the functional format by one in which matrices could be stored in the "function keys", with the basic operators being now those of matrix arithmetic rather than function arithmetic. For an algebra machine or a logic machine, the basic, hand programmed keys would correspond to the fundamental operations of these disciplines, but at present this is still somewhat speculative. In each case, the general organizational scheme of the present system, including the control and console programming aspects, would be preserved, and only those parts (actually a small fraction) of the programming associated with the functional orientation and with the graphical displays would be altered. While this is true also of other areas of computer application (e.g., those involving information processing rather than mathematics), the identification of the basic operations from which all others can be compounded by console programming appears far more difficult, there being no analog for the experience accumulated in the physical and mathematical sciences during the past 300 years.

We turn now to the system as it presently exists. We note that much of its power derives from the fact that substitution can be carried out at several levels: substitution of numbers, of functions and of programs. Substitution of different parameter values is carried out by simply writing the console programs with the parameters in question represented by certain function keys; one then has only to insert the desired constant functions into these keys before running a program. The capability for functional substitution is provided by the REFLECT and SUBSTITUTE keys. Given two functions u(x) in the C register and v(x) in the D register, the SUBSTITUTE key produces in the D register the function v(u). When displayed, this will be a curve in the (u,v) plane. Conversely, given such a curve, v(u), we can (as illustrated in the preceding chapter) easily obtain the parameterized curves u(x) and v(x).

Most important perhaps is the possibility of substitution at the program level. Suppose that we wish to make a change in a console programmed key [K] which is one of the components of another key [L], [L] in turn being a component of still a third key, [M]. If we wish to substitute a new console program for the one presently under [K], we simply program [K] in the same way as one does with a blank key; the program formerly associated with [K] will be buried. Alternatively, we may find that the console program associated with [K] is so basic and takes so long to run that it should be replaced with a hand program. (For this replacement the "user" must get the help of a "computer expert".) In either case, the program associated with key [M] will run precisely as before, save for the desired modification in [K], for the program in [M] recognizes [K] only as a key push, regardless of the significance of the subroutine it calls in.

One thus has the ability to manipulate console programs with approximately the same freedom as one juggles the mathematical operations which they represent, a feature not present in conventional programming languages. As a result, problems which are adjacent in the mathematical sense become so computationally as well; one can proceed from the simple to the more complicated, always building upon the results of what one has learned, without the necessity for redoing all of the programming as new pieces are added or old ones are modified.

While we have characterized the "user" of the on-line system as a scientist unversed in conventional programming methods, it is clear that the creation of console programs involves the very essence of programming, albeit with most of the drudgery eliminated, and that "users" would benefit from the advice of someone familiar with programming. Indeed, operation of the on-line system involves two activities which at first sight appear separable: a) the creation of those console programs needed for a problem; and b) the use of these in its solution. Why not let someone we may call a "console programmer" (since his qualifications will differ somewhat from those appropriate to programmers in the standard meaning of the word) take care of a), the "user" being involved only with b)? The point is just that a) and b) are in fact strongly coupled; as soon as one starts to use b) he typically finds that some changes or additions are needed and he must revert back to a). If the user does not actually do a) himself, he must certainly work very closely with the "console programmer" who does, in order that he thoroughly understand the significance of the keys in his system. Moreover, unless the user is familiar, from hand computation or other experience, with numerical methods, a mathematician skilled in such matters had better be available for consultation; for the privilege of having direct access to a computer, the user must pay the price of being exposed also to questions of scaling, error accumulation and all the other technical problems which are of course involved in any computational work but which are seen dimly, if at all, by the user when, as in most conventional organizations, he is insulated from the computer by several layers of intermediaries.

In conclusion, we should emphasize that there will be many computer applications for which these on-line techniques will be of little or no value. If one thoroughly understands the structure of a problem and knows a method of solution which is certain to work, then the experimentation and feedback characteristic of the on-line system are unnecessary. Indeed, such problems are handled very nicely by computer centers as presently constituted. It appears, however, that for problems whose structure is not clear, either a priori or on the basis of previous experience, and for which successful solution techniques need to be developed, an on-line system which allows the technical intuition of the user to play a central role in the solution process can be of considerable value. In this system, the user has a direct and convenient access to the computer, a fast response for computations which are essentially trivial, and a graphical representation of information where appropriate. He can build programs consisting of his own constructs within his own field, combine these in any desired way, and, if appropriate, make a trial-and-error study of the various features of his problem. Indeed, the interplay of the structural elements is often more important than the solution itself in terms of the information desired in a research problem. Since he has control over the transformations, operators and other mathematical objects involved in his problem, he is able to get hold of the pieces and study the ingredients from the point of view of validity as well as from the point of view of structure. When he has found successful methods, he can combine these into an operating program without the necessity of reprogramming. Finally, from the bulk data available after solving a problem, he is able to select only that which he really desires, either as hard copy, numerical output or in the form of pictures of curves displayed on the CRT.

ACKNOWLEDGEMENTS

We are greatly indebted to the Data Processing Laboratory at Rome Air Development Center for support of this work and for the AN/FSQ-27 portion of the equipment; to Dr. Robert Huff, George Boyd and Robert Bolman for invaluable assistance in the programming tasks; and to Professor R. P. Feynman for suggestions concerning the possible extension to an algebra machine. We acknowledge with special thanks the efforts of Professor J. R. Schrieffer, Professor Karl Menger, Professor H. W. Wyld, Jr., Professor K. A. Johnson, Fred Dion and Martin Schultz who spent much of the summer of 1962 as cooperative guinea pigs, using the on-line system

for research problems in their own fields, notwithstanding its then somewhat raw and rough-edged character, thus contributing greatly to its present state of development and to our understanding of the user's needs and desires in an on-line system.

APPENDIX - DESCRIPTION OF BASIC SUBROUTINES FOR AN ON-LINE SYSTEM

The initial hand programmed keys which at present comprise the basic system from which every user starts may be divided into five categories, save for the especially significant SECOND COMPUTER key, which stands by itself:

1. Mathematical operations.

2. Capabilities which provide assistance in the creation of console programs.

3. Programs having to do with displays or with other input/output aspects.

4. Operations involved in management of the computer system.

5. Conveniences for the computer expert who may be concerned with hand programming.

Many other items could be added to the list which follows and some of those given here could be omitted. While our set is neither exhaustive nor minimal, it has proved to be extremely convenient. Names of operator keys are in capital letters; headings not capitalized refer to groups of keys so closely related that to save space we have not listed them separately. In the description of keys we shall, in the interest of simplicity, ignore the multiplicity of function banks[14].

A1. Mathematical Keys

<u>LOAD</u> LOAD_I A brings the function in key A of bank I into the <u>D</u> register. (It also remains in A on bank I.) Similarly for LOAD_{II}, ---LOAD_{VI}.

<u>STORE</u> STORE_I puts the function in the <u>D</u> register into key A on bank I, leaving it also in the <u>D</u> register. Similarly for STORE_{II} ---STORE_{VI}.

<u>FLOAT-MANTISSA</u> The y values of the <u>D</u> register are shifted left as many times as possible without causing any one of them to overflow, and the scale value s_y is adjusted appropriately. The x values of the <u>D</u> register are unchanged.

<u>+</u> On Overlay 01, + A puts the function stored in key A into the <u>C</u> register, and then compares the y scales of the <u>C</u> and <u>D</u> registers. If the scales are equal, the y values are added together and left in the <u>D</u> register. If the y scales are unequal, the function with the smaller scale is contracted until the scales are equal and the addition is then performed. The x coordinates of the <u>D</u> register are unchanged. If the same operation is performed on Overlay 02, both functions are floated, each is contracted once, and the addition is then carried out as on Overlay 01.

<u>·</u> On Overlay 01, · A loads the function stored in key A into the <u>C</u> register, multiplies its y values by those of the <u>D</u> register, adds the scales, and leaves the result in the <u>D</u> register.

_-_ Subtraction is performed in the same fashion as addition.

_÷_ On Overlay 01, ÷ B loads the function stored in key B into the _C_ register, divides the y values in the _D_ register by those in the _C_ register, subtracts the scale values, and leaves the result in the _D_ register. When the same operation is performed on Overlay 02, each function is first floated and the numerator is then contracted enough times to prevent overflow at any point unless this requires more than 12 contractions, in which case the numerator is simply contracted 12 times.

$\sqrt{\ }$ $\sqrt{\ }$ takes the square root of the function stored in the _D_ register and leaves the result in the _D_ register.

LEFT-SHIFT The y values of the _D_ register are shifted one place to the left: y_{n+1} replaces y_n, $1 \leq n \leq 100$, and y_{101} is left unchanged.

RIGHT-SHIFT The y values of the _D_ register are shifted one place to the right: y_{n-1} replaces y_n, $2 \leq n \leq 101$, and y_1 is left unchanged.

EVALUATE This allows one to evaluate the function in the _D_ register at the value of the x coordinate nearest to any selected number, previously stored as a constant function in one of the function storage spaces. The operation is as follows: EVALUATE B loads the function stored in B into the _C_ register and subtracts the y coordinates of the _C_ register from the x coordinates of the

D register. The y value in the D register corresponding to the smallest of these differences is selected and all y coordinates of the D register are set equal to that value.

EXPAND y The y values of the D register are multiplied by two (shifted one place to the left) and the scale value s_y is reduced by 1.

CONTRACT y The y values of the D register are multiplied by 1/2 (shifted right one place) and the scale value s_y is increased by 1.

Both EXPAND and CONTRACT leave the numerical value of the function invariant since a change in scale appropriately compensates the alteration in mantissa values. However, since only the latter are displayed, the appearance of the function on the CRT is altered. One can use EXPAND to examine in detail the small amplitude structure of a curve, letting the other parts overflow being careful, of course, to retain the original representation of the function in another storage spot ; it thus complements FLOAT-MANTISSA. If one uses Overlay 01 (fixed point arithmetic) CONTRACT is necessary in order to avoid overflow in addition, subtraction. Finally, both EXPAND and CONTRACT are useful in bringing curves to a common scale for visual comparison.

δ FUNCTION This creates a Kronecker-δ type function, i.e., one which has the value 1 at one point and zero everywhere else. To create the function $\delta_a = \delta(x - a)$, load the number a into the D register then push the δ function button. The desired function then appears in the D register, i.e., all of the y coordinates in the D register are made 0, save the one corresponding to the value of x nearest to (or equal to) a, and it is set equal to 1.

EXPONENTIAL The exponential of the function in the D register is computed and the result is left in the D register.

SINE-COSINE The sine and cosine of the function in the D register are computed. The sine is put in place of the y values of the D register; the cosine is put in place of the x values of the D register. (This may alternatively be considered as a complex exponential e^{if}, where f is the function in the D register.)

J-GEN The identity function, $y = x$, $-1 \leq x \leq 1$ is put in the D register.

REFLECT The x and y values of the D register are interchanged, as are also the scale values, s_x and s_y.

SUBSTITUTE The y coordinates of the C register replace the x coordinates of the D register and similarly for the scale values. This permits, for instance, the cross plotting of two dependent variables which are functions of the same independent variable. Together with REFLECT, it allows one easily to create console programs for complex-valued functions of complex arguments using only real function hand programs (i.e., those described in the foregoing part of this section).

$\underline{\Delta}$ This is a finite difference operation on the function contained in the $\underline{D}$ register: $y_{n+1} - y_n$ replaces y_n for $1 \leq n \leq 100$, while y_{101} is computed by fitting a cubic to the values y_{98}, y_{99}, y_{100} and y_{101}.

$\underline{\sum}$ This is a running sum performed on the function contained in the $\underline{D}$ register: $\sum_{i=1}^{n-1} y_i$ replaces y_n, $2 \leq n \leq 101$ and 0 replaces y_1.

From these two keys one can, using console programming, construct "differentiate" or "integrate" keys of any desired accuracy. Thus, the identity

$$\frac{d}{dx} = \frac{1}{\Delta x} \ln(1+\Delta) = \frac{1}{\Delta x}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \dots\right]$$

shows one way of making the derivative, while its inverse provides an indefinite integral in terms of $\sum$. Of course, one simply uses the EVALUATE operation to get a definite integral.

INTEGRAL TRANSFORM The integral transform of the function

$$\tilde{f}(x) = \int_a^b dx' \, K(x, x') \, f(x')$$

f stored in the D register is computed, using a kernel $K(x, x')$ which has been stored out on magnetic tape in the form of 101 functions of x', one for each value of x. Assuming that the tape has been correctly positioned and that f is in the D register, we simply push the INTEGRAL TRANSFORM key. The first of the 101 functions, i.e., $K(a, x')$, is then read into the C register and multiplied by the function in the D register. The definite integral is computed and the resulting number is stored in the first x coordinate of the D register. The next function, i.e., $K(a + \epsilon, x')$, $\epsilon = (b - a)/100$, is then read from tape into the C register and the process repeated, the result being stored as the second x value of the D register. At the completion of the operation, which requires 7 seconds, f is still contained in the y coordinates and $\tilde{f}$ is in the x coordinates of the D register. To transform the latter into standard form one could, for example: REFLECT STORE A, LOAD 0 + A (the addition to 0 being one means of restoring the x coordinates of the D register to the canonical form used for the displays).

RELATIVE-INTERPOLATE This uses individual data points, put in with the graphical input techniques described below (Section A3), to modify the function, say f, which has been loaded into the D register. If P_a and P_b are two of the data points, the program first finds the two points, $\bar{P}_a$ and $\bar{P}_b$, of f whose x coordinates match those of P_a and P_b. If L is the straight line P_aP_b and $\bar{L}$ is the line $\bar{P}_a\bar{P}_b$, the function f is replaced by $(f - \bar{L} + L)$ for $x_a \leq x \leq x_b$. The function is left unchanged between $x = -1$ and the smallest of the x_a. (Before f is loaded into the D register, it should be displayed on the CRT scope to serve as a guide for placing the data points on the screen with the crosshair.)

A2. Aids to Console Programming

PROGRAM Press PROGRAM; then press the key to which the program to be written is to be attached; then type in the overlay number on which that key is to be located; then press the keys which will make up the desired program; at the end press PROGRAM again.

REPEAT Press REPEAT; then press any repeatable key (i.e., either a hand programmed key, or a console programmed key whose program ends on the same overlay on which the key itself is located); then type in on the numerical keys the number of times the operation is to be repeated. This repeat operation can, of course, itself be incorporated into a console program.

TALLY This is used only within a console program and is one of two capabilities for program branching. TALLY must be imbedded in some console programmed subroutine; that is to say, it must be one of the series of key pushes which make up some console programmed key. When, in the running of that subroutine, the computer comes to the point where the TALLY key was pushed, it checks the scale value s_y, of the D register. If s_y is positive the computer reduces s_y by 1 and proceeds to the next key in the subroutine; if the scale is 0 or negative, it jumps to the end of this particular subroutine.

COMPARE This operates in the same fashion as TALLY but uses as its criterion the sign of the first y value, y_1, of the D register. If this is positive, the computer continues to the next key push; otherwise, it jumps to the end of the subroutine in which COMPARE is imbedded.

(These keys make possible the incorporation of standard programming techniques - loops, tallys, etc. - at the console programming level.)

A3. Display and Output Keys

DISPLAY OVERLAY NUMBER The number of the overlay currently in the computer is displayed on the alphanumeric scope.

ERASE All curves are erased from the CRT.

DISPLAY DISPLAY A causes the function stored in location A to be displayed; pushing A again erases that curve from the scope. Subsequently pushing other keys will cause the curves stored in them to be displayed also, until some other operator key is pressed.

DISPLAY VALUE AND SCALE, BINARY The mantissa of the first y value of the D register and the y scale, s_y, of the D register are displayed on the alphanumeric scope.

DISPLAY VALUE, DECIMAL If y_1 is the mantissa of the first y value in the D register and s_y the scale of the function, the number $y_1 \cdot 2^{s_y}$ is displayed as a decimal mantissa times a power of 10.

GRAPHICAL INPUT Press POINT-INPUT; then DISPLAY-CROSSHAIR. A crosshair, whose position can be controlled by a lever, is displayed on the screen. After positioning it at any desired point, push TRANSMIT-CROSSHAIR-COORDINATE. The x and y coordinates of the selected point are then transmitted to the computer and a small crosshair symbol is displayed on the scope at that point. The points thus put in are accumulated and can be used in conjunction with the RELATIVE INTERPOLATION key described in Section A1.

PRINT Any curve loaded into the D register will be printed out on the flexwriter in conventional format, i.e., the x and y values will be listed in decimal form.

LABEL After this is pushed, the function keys serve as typewriter keys and can be used to compose any desired alphanumeric message. (Each letter or number is displayed on the alphanumeric scope as it is typed.) This is useful for generating a label to go with a kernel stored on tape, a message which is to be written on tape along with a system dump, or an identifying legend to accompany a hard copy curve when the PRINT key is used.

LEFT SCOPE A word in the display routine is set so that any curve subsequently displayed with the usual DISPLAY key will appear on the lefthand 17 inch CRT.

RIGHT SCOPE   A word in the display routine is set so that any curve subsequently displayed with the standard DISPLAY key will appear on the righthand scope.

Alternative Display Formats   There are several keys which allow the capability of displaying curves in other than the usual format. Ordinarily 100 straight line segments connecting the 101 points are displayed. However, one can instead display only the 101 dots with no connecting line, or other symbols such as crosses, circles, etc.

A4. System Control Capabilities

The keys in this group allow for convenient management of the entire computer system. Only the first few are needed by the typical user; those with an asterisk are used only by the computer expert and may be disregarded by readers not in that category.

SYSTEM LOAD   An entire system - overlays, curves, subroutines, etc. - is loaded from tape into the computer system. Whenever a user starts a period on the machine, he has his tape put on the tape unit and pushes SYSTEM LOAD, thereby putting the entire computer system into the same state it was when he last used it.

SYSTEM DUMP   This is the inverse operation to SYSTEM LOAD, and is used at the end of each user's run.

TAPE READ   This reads a block of 512 words from magnetic tape into the C and D registers of the computer.

TAPE WRITE This writes contents of the C and D registers as a 512 word block on magnetic tape.

Tape Manipulation There are keys for erasing a block on tape, skipping a block on tape, skipping to an end of file, writing an end of file; rewinding the tape, etc.

DRUM READ* DRUM READ nm reads into the C and D registers, from the drum, the computer program corresponding to Overlay nm. This allows the computer expert to examine or modify the actual machine words comprising the hand programs of this overlay.

DRUM WRITE* This is the inverse of the DRUM READ operation and is used to replace an overlay on the drum after it has been examined or modified.

SUBROUTINE READ* SUBROUTINE READ nml stores the automatic subroutine with identification number nml into the C register where it can be examined or modified by a computer expert.

SUBROUTINE WRITE* This is the inverse of SUBROUTINE READ.

A5. Aids for the Hand Programmer

All of these keys are for the computer expert alone and, like the last items in Section A4, should be ignored by readers who are not in this class. Their descriptions are included here only for completeness.

OVERLAY OUT OVERLAY OUT nm stores the overlay now in the computer onto the drum in the appropriate place. This is necessary after hand program alterations have been made in an overlay.

INSERT   This is a convenient method of inserting short hand programs into the system. Press INSERT, then a CM address (4 digits on the numerical keys), then ENTER, then a machine word (10 digits on the numerical keys), then ENTER. Any number of additional machine words can now be typed in (each followed by ENTER); they will be stored in sequence in the locations following the first one. In addition, each word (and the address of the first one) is displayed on the alphanumeric scope.

DO   This is simply a convenient means of causing the computer to execute any single instruction on command from the control console. Press DO, then type in on the numerical keys any machine command (again, 10 digits for this particular computer) followed by ENTER.

DISPLAY OF MEMORY CONTENTS   This is a convenient means of examining the contents of any portion of the CM memory. Push DISPLAY MEMORY, then type in the four digits specifying an address in the CM. That address and the (10 digit) machine word it contains are then displayed on the alphanumeric scope. If a number n is now entered on the numerical keys, the next n words of the CM memory will be displayed, an operation which can be repeated as often as desired. DISPLAY MEMORY need be pushed again only if one wishes to examine a non-contiguous portion of the memory. The incorporation of this capability into suitable console programs provides the computer expert with a convenient means of dynamically debugging a hand program.

A6. The SECOND COMPUTER Key

In a class by itself is the SECOND COMPUTER key (although in a logical sense it probably could be included in Section A4). The RW-400 system has two identical computer modules, CM-1 and CM-2. The control console normally communicates directly with CM-1, and CM-2 is not used. However, pressing the second computer key, then any previously programmed key $\lfloor K \rfloor$, followed by the number of the overlay on which $\lfloor K \rfloor$ is located, causes the SECOND COMPUTER to carry out whatever program is associated with key $\lfloor K \rfloor$, taking from the drum the subroutines, curves, and other information needed in doing this. While this is going on, the user at the control console is free to use CM-1 for any of the normal operations, e.g., to examine the results being generated by CM-2, to prepare for the next case to be run, to create new console programs, etc.

REFERENCES AND NOTES

1. J. C. R. Licklider and W. E. Clark, "On-Line Man-Computer Communication", Proc. of Spring Joint Computer Conference (May 1962). The first two pages of this contain a cogent statement of the motivation for an on-line system and of some general principles to be observed in constructing one.

2. Glen J. Culler and Robert W. Huff, "Solution of Non-Linear Integral Equations Using On-Line Computer Control", Proc. W. J. C. C., May, 1962.

   Glen J. Culler, Burton D. Fried, Robert W. Huff and J. Robert Schrieffer, "Solution of the Gap Equation for a Superconductor", Phys. Rev. Letters 8 399 (1962).

   Glen J. Culler, Burton D. Fried, Robert W. Huff and J. Robert Schrieffer, "Use of On-Line Computing in the Solution of Scientific Problems" (RW Research Laboratory Report, April, 1962, unpublished).

3. We use the term "hand program" to denote a computer program of the classical sort, i.e., a list of machine words, in contrast to a "console program", which, as described below, is essentially a list of key pushes.

4. The term "automatic programming" might be better, but unfortunately it already has a different significance.

5. The numerical keys include also ENTER, which must be pushed at the end of any sequence of numbers. When we give below illustrative lists of key pushes we shall generally omit ENTER, although in actual operation it must always be included.

6. This example is only illustrative, since in an operating system it is more sensible to make SINE a hand programmed key. In the early stages of development of the present system, however, we actually used, for SINE and COSINE, console programs based on 10 terms of the power series for functions with scale $s_y \leq 1$ and repeated use of the double angle formulas when $s_y > 1$.

7. Experience shows that operation by one user for less than one hour or more than two tends to be inefficient.

8. B. D. Fried, M. Gell-mann, J. D. Jackson and H. W. Wyld, Longitudinal Plasma Oscillations in an Electric Field", J. Nuclear Energy; Part C 1 190 (1960).

9. Glen J. Culler and Burton D. Fried, "Plasma Oscillations in an External Electric Field", (to be published).

REFERENCES AND NOTES - Continued

10. To avoid confusion between operator keys and function keys having the same label, e.g., t, we shall, in writing lists of key pushes like (22), use $\lceil \underset{\cdot}{\ } \rceil$ for any operator key whose label coincides with that assigned to some function key. Of course, in actually using the console no confusion arises since the operator keys are physically distinct (and separated) from the function keys. Also, since all functions involved in our examples are on the same bank, we omit the bank subscripts, writing simply LOAD and STORE in place of LOAD_{I} and STORE_{I}.

11. We assume the on-line work to occur during a period of the day when the central computer is being used only for short problems (maximum of a few minutes running time).

12. See, e.g., C. Strachey, "Time Sharing in Large, Fast Computers", Proc. of International Conference on Information Processing, UNESCO, p. 336 (June, 1959), and A. L. Leiner, W. A. Notz, J. L. Smith and R. B. Marimont, "Concurrently Operating Computer Systems", loc. cit., p. 353.

13. Although "complete" in the sense of being very useful for a wide range of problems, the present system is still growing in the sense that this overlay and some of the other extension described in this chapter will be incorporated in the near future.

14. The only complication which can arise concerns operations with functions stored on two different banks. Thus, in place of Load_{I} A + B which we would use to add two functions stored on I, we use Load_{I} A Load_{III} + R to add functions on banks I and III; here the Load_{III} key functions only as a bank indicator.

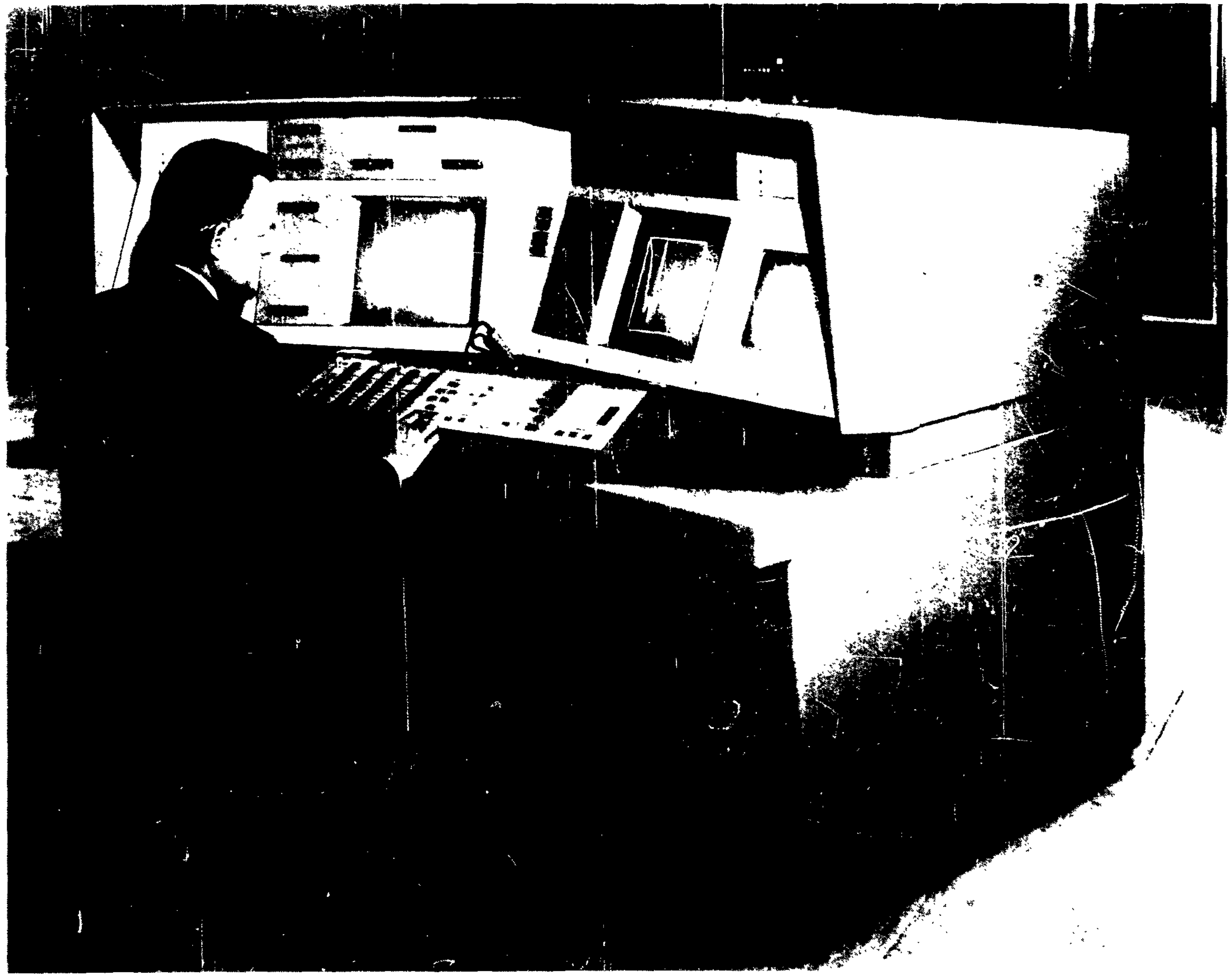

Figure 1. Control console used in the On-Line Computer Center.

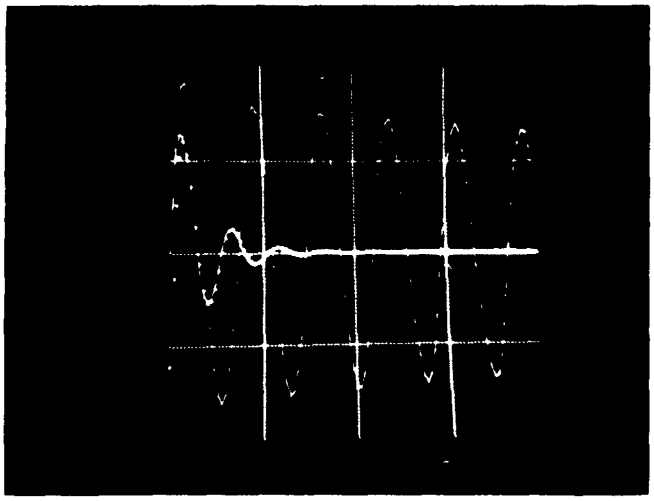

Figure 2. The kernel function L_e, defined by (10), as obtained by iterating (18), correcting the result using (21), etc. The slightly damped curve is for $k = 0.4$; the strongly damped one is for $k = 1.0$. A dotted curve shows the result for $k = 1$ obtained by retaining only the two least damped poles in the Laplace transform of L_e when inverting the transform by contour integration. Grid lines: $y = 0, \pm 1/2$; $t = 7.5, 15, 22.5$.

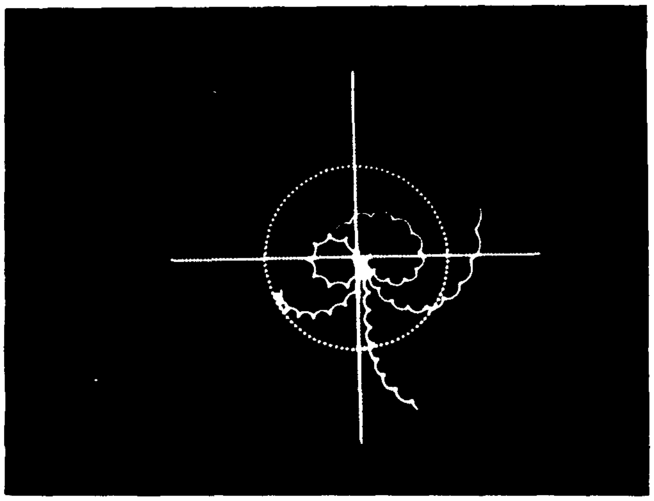

Figure 3. $E(t)$ in the complex plane for $1 \leq t \leq 30$ with $k = 0$, $\varepsilon = 0$ and (reading from left to right) $u = 0.8, 0.9, 1.0, 1.1, 1.2$. These u values bracket the region of resonance, i.e., of growing waves. The curves all start at the point $E = 1$. Also shown are the real and imaginary E axes and the circles $|E| = 8$ and $|E| = 16$.

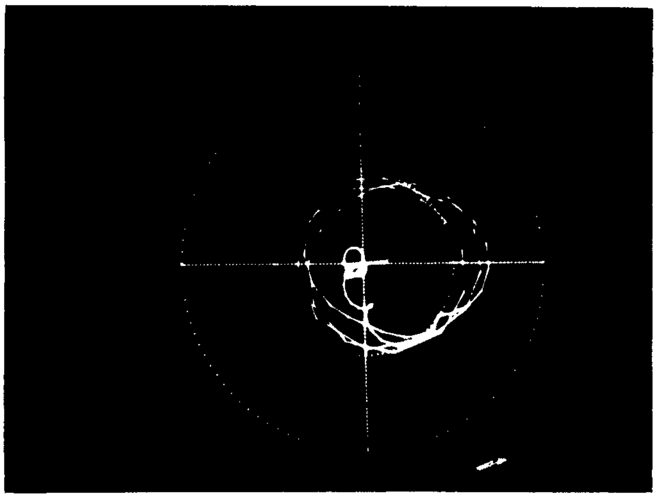

Figure 4. $E(t)$ for $k = 0$, $\varepsilon = 0.1$. The range of $d\psi/dt$, $0 \leq \dot{\psi} \leq 3$, includes the values used in Figure 3 (where $\dot{\psi}$ is constant). The approach of E to an approximate limit circle centered at $E(t = 0)$ can be predicted analytically for the single species case ($\delta = 0$). The circles $|E| = 4$ and $|E| = 8$ are shown.

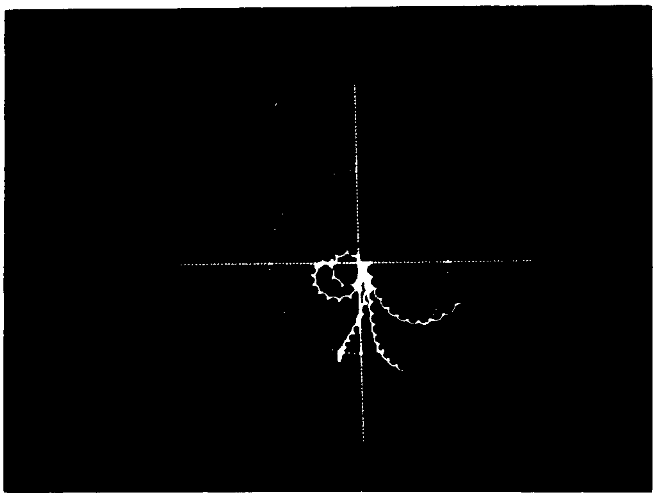

Figure 5. $E(t)$ for $k = 0.4$, $\varepsilon = 0$ and (from left to right) $u = 0.9$, 1.0, 1.1, 1.14, 1.2. Note the increased damping as compared with the $k = 0$ case. The circle $|E| = 8$ is shown.

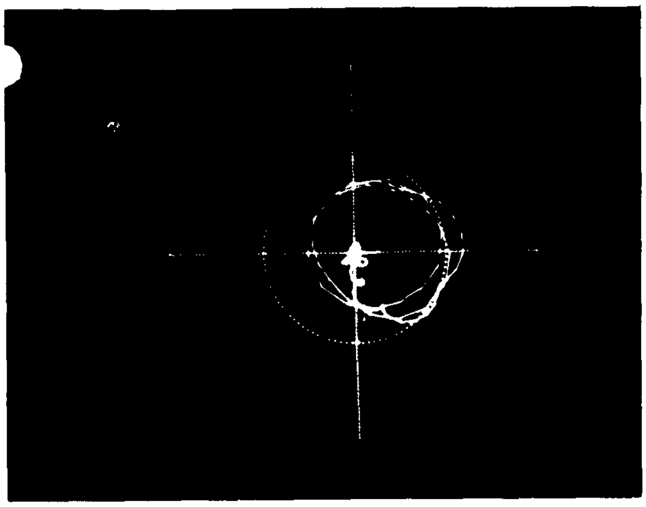

Figure 6. E(t) for k = 0.4, $\mathcal{E}$ = 0.1. The analytic solution for the single species case (δ = 0) shows that the radius of the limit "circle" approached by E should be a slowly decreasing function of t. The circles $|E|$ = 4 and $|E|$ = 8 are shown.

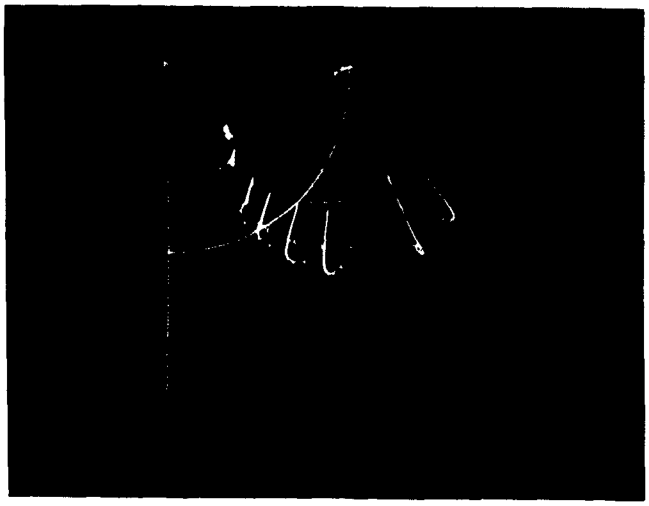

Figure 7. E(t) for k = 1.0, $\mathcal{E}$ = 0 and (left to right) u = 0.8, 1.0, 1.2, 1.3, 1.4, 1.5, 1.7, 1.8. The high frequency (electron plasma) oscillations, $\omega \approx 1$, damp out completely during the time interval depicted, leaving only low frequency waves (associated with ion motion) which appear as bright "tails" on the curves. Arcs of the circles $|E|$ = 1 and $|E|$ = 2 are shown.

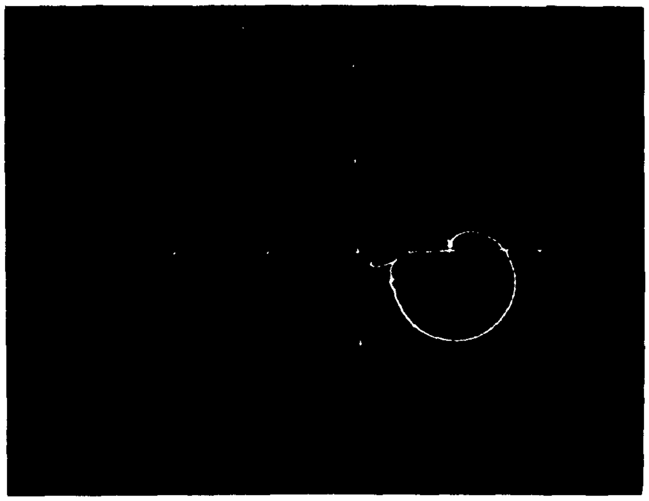

Figure 8. E(t) for k = 1, $\mathcal{E}$ = 0.1. The limit circle for δ = 0 in this case should damp as $e^{-0.4t}$. The circles $|E|$ = 1 and $|E|$ = 2 are shown.