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THE MEASUREMENT OF ELECTRON DENSITY IN THE WAKE OF A HYPERVELOCITY PELLET OVER A SIX-MAGNITUDE RANGE

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ABSTRACT

A UHF cavity probe has been used to determine the line densities and collision frequencies of electrons deposited in the wakes of spherical aluminum pellets. Density data were obtained over a wide dynamic range of up to six orders of magnitude. These density curves are characterized by a sudden drop of about three or more orders of magnitude which occurs when the wake has cooled to a critical temperature that is a function of pressure. The sudden drop is attributed to the oxygen-molecule attachment-detachment equilibria, while the final leveling portion of the wake is regulated by the positive-to-negative-ion recombination rate. The collision frequencies throughout the entire density range are in agreement with those obtained through mobility studies.
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In order to find out something about the reaction mechanisms occurring in the ionized wakes of bodies re-entering the earth's atmosphere, experiments have been performed on the wakes produced by hypervelocity pellets. Earlier experiments by the author measured the electrons' decay in the wake down to \(10^8\) particles per linear centimeter. These same experiments yielded electron collision frequencies of dubious accuracy because of oscillator frequency drift and vibrations of the cavity. In general, wake measurements have failed to detect densities lower than \(10^8\) particles per cubic centimeter. Moreover, most experiments individually cover only a one-to-two magnitude range of densities, so that any attempt to correlate these measurements to theoretical models will most likely lead to marginal conclusions.

In the experiments presented here an improved UHF cavity probe has been used to determine the line density and collision frequency of the electrons deposited in the wake of a spherical aluminum pellet moving at 7 to 20 kft/sec. A gun-powder gun fired 0.5-inch spheres at the lower speeds; at the higher speeds a hydrogen light-gas gun fired 0.260- and 0.187-inch spheres. The spheres were fired through an atmosphere of dry air (dew point of \(-163^\circ\)F) whose pressure ranged from 5 to 80 mmHg. The pellets were fired coaxially through the cavity as shown in Fig. 1. The cavity (Fig. 2) resonated at 441 Mcps in the TM_{010} mode, exciting a longitudinal electric field. It was made of brass, cast and machined, and weighed about 450 pounds. In addition, it was heavily ribbed to eliminate vibrational problems.

The basic block diagrams of the cavity instrumentation are shown in Fig. 3(a-b). The deposition of electrons in the cavity by the pellet, in effect, changes the value of the cavity transfer impedance. In the "standard" circuit of Fig. 3(a), the output phase and amplitude of the signal coming from the cavity is measured relative to the original value obtained when no electrons are in the cavity. With the standard circuit, electron densities can be measured from a maximum of \(10^{12}\) down to about \(10^8\) electrons per linear centimeter. In the "difference" circuit [Fig. 3(b)], the output is subtracted; that is, balanced against the input to the cavity. With the difference circuit, densities can be measured from about \(10^9\) down to about \(10^5\) electrons per centimeter. By combining the two circuits with hybrid junctions, the complete range of \(10^{12}\) to \(10^5\) is obtained. This is shown in more detail in Fig. 4.

![Fig. 1. Disposition of hydrogen gun, resonant cavity, pellet, and ionized wake.](image)
Fig. 2. UHF probe cavity.

Fig. 3. Simplified block diagrams of UHF cavity. The vectors represent the input and output phase and amplitude conditions.

(a) "Standard" circuit, high density case.

(b) "Difference" circuit, low density case.
Fig. 4. Simplified block diagram of combination "standard" and "difference" circuits. Only the RF sections are shown.

The density and the collision frequency are determined by the following formulas:

"Standard" Circuit

\[
N = \frac{\pi a^2 J^2_1(\alpha) m \sin \Theta}{e^{2 \mu_0 Q_L}} \left[ \frac{(\cos \Theta_1 - S_1)^2}{\sin \Theta_1} \right] \text{electrons/meter}
\]

\[
\nu = \omega \left( \frac{\cos \Theta_1 - S_1}{\sin \Theta_1} \right) \text{collisions/second}
\]

"Difference" Circuit

\[
N = \frac{-\pi a^2 J^2_1(\alpha) m}{e^{2 \mu_0 Q_L}} \frac{S_2}{\sin \Theta_2} \text{electrons/meter}
\]

\[
\nu = -\omega \left( \frac{\cos \Theta_2 - S_2}{\sin \Theta_2} \right) \text{collisions/second}
\]

Here \(\pi a^2 J^2_1(\alpha) m/e^{2 \mu_0 Q_L}\) is a constant defined in Ref. 2, \(S\) is the normalized output amplitude, \(\Theta\) is the phase shift, and \(\omega\) is the angular frequency of the oscillator. To insure reasonably accurate results, it is essential that, initially, the cavity be tuned exactly on frequency and that the oscilloscopes and electronics be calibrated after each pellet firing. Calibration consists of feeding known amplitude changes and phase shifts into the electronics and then photographing the resulting output levels on the oscilloscopes. When the electrons are deposited in the cavity by the pellet, the time-varying output levels are also recorded on film. All the film information is read on to cards and processed with a digital computer. The output cards of the computer are then fed into an automatic curve plotter, and finally the curves (Figs. 5 through 7) are replotted by hand and condensed on semi-logarithmic paper.
Fig. 5. Density curves for 0.260-inch aluminum spheres. The numbers represent the shot identification number, the velocity (kft/sec) and the pressure (mmHg), respectively.

Fig. 6. Density curves for 0.260-inch aluminum spheres.
All the data are plotted to the same scale. Time runs from 0 to 12 msec, and the electron density runs from $10^{12}$ to $10^5$ electrons per centimeter. Occasionally, the data would extend beyond 12 msec, but are not shown in order to keep the curves to the same scale. The numerals identifying each curve are the shot number, the velocity in kilofeet per second, and the pressure in millimeters of mercury, respectively. If, for the moment, one ignores the low velocity (8 kft/sec) curves, one can note certain common features. Most of the curves start at roughly $10^{11}$/cm and then in about 1 msec experience a prominent change of slope, generally downward. The steepest slopes occur for the high-pressure cases. Then most of the curves drop about 4 to 5 orders of magnitude with assorted slopes before they start to level out.

This great variation of slope from curve to curve through more than 5 orders of magnitude merits particular attention. It is proposed here that this region is dominated by the reaction

$$2O_2 + e^- = O_2^- + O_2^-,$$

and it is further suggested that the reaction is essentially in equilibrium at all times. Phelps and Pack\(^4\) have measured the attachment and detachment coefficients of molecular oxygen as a function of gas temperature. The attachment frequency

$$v_a = 2.8 \times 10^{-30}(O_2)^2 (\text{cm}^6/\text{sec}),$$

where $(O_2)$ is the oxygen density. The attachment frequency is assumed independent of temperature. Phelps and Pack also reported values of the detachment frequency over a limited temperature range. These values can be closely duplicated by the function
Fig. 8. Relation of the various wake reactions.

Fig. 9. Density and collision frequency for 0.187-inch aluminum sphere at 19,000 ft/sec.

Fig. 10. Density and collision frequency for 0.187-inch aluminum sphere at 20,000 ft/sec.
\[
\nu_d = 1.02 \times 10^{-14} T^{3/2} e^{-5330/T} \text{ (cm}^3\text{/sec)},
\]
where \( T \) is degrees Kelvin. The differential equation for the oxygen reaction can now be written:

The rate of change of electron density

\[
\frac{dn}{dt} = \left[ \nu_d + \frac{1}{T} \frac{dT}{dt} \right] (n_o - n) - \left[ \nu_a + \frac{1}{T} \frac{dT}{dt} \right] n,
\]

where the first term represents detachment and the second attachment; and \( n_o - n \) is the negative ion concentration (\( \text{O}_2^- \)). The \( (1/T)(dT/dt) \)(terms come from the density variation with temperature at constant pressure and can be neglected here as being small compared with the other terms in the brackets. The remaining terms lead to a differential equation whose transient solution time constants are much too short compared with the observed electron density decay curves. One must conclude that only the steady state solution is significant. Setting \( dn/dt = 0 \), one obtains

\[
n = \frac{n_o}{1 + 3.57 \times 10^{-4} p(T/273)^{3/2} e^{5330/T}},
\]

where \( p \) is the pressure of the air in millimeters of mercury, and \( n_o \) is now interpreted as the positive ion concentration. From an inspection of the above equation one can note that the electron density is essentially equal to the positive ion density at pressure \( p \) until the temperature drops below the value \( T \) determined by the equation

\[
1 = 3.57 \times 10^{-4} p(T/273)^{3/2} e^{5330/T}.
\]

This critical crossover temperature is about 700° in the pressure range used. When the temperature does drop to this level, the electron density becomes proportional to a very fast function of temperature (roughly equivalent to the 34th power). This implies that the electron density is very sensitive to how fast heat is generated and dissipated in the wake. This sensitivity accounts for the variation of slopes observed from shot to shot.

The over-all picture is as follows: Initially the electrons combine with the positive ions (\( \text{NO}^+ \)). The recombination continues until the temperature drops below the value defined in Eq. (2); then the attachment-detachment reaction takes over. The electron density drops well below the positive ion concentration according to Eq. (1). When the temperature reaches ambient, the ratio between positive ions and electrons become constant. Since the positive ions can and do recombine with the negative oxygen ions, the electron decay curve now becomes a replica of the molecular ion recombination curve (Fig. 8).

Figures 9 and 10 show curves for 0.187-inch aluminum balls at 19 and 20 kft/sec. The time scale has now been extended. The collision frequencies are plotted on the same graphs. The collision frequencies were not plotted with the previous curves simply to save space and because they all look alike. Their numerical magnitudes are approximately

\[
\nu_m = 2 \times 10^8 p,
\]
Fig. 11. Density curves for a 0.5-inch aluminum sphere at a relatively low velocity.

Fig. 12. Density and collision frequency curve for a slow 0.5-inch sphere. The oscillations occur only in the density.
and are in agreement with the value observed by Phelps and Pack.\(^5\) They measured the mobility of nitrogen, which, when transformed into its equivalent electron collision frequency, is equal to

\[
\nu_m = 4.5 \times 10^8 \text{ p}
\]

at room temperature. (The electron collision frequency of nitrogen turns out to be essentially independent of temperature; therefore, one can correlate measured values obtained at different temperatures.)

Figure 11 shows curves obtained for a 0.5-inch aluminum ball at slow speeds of 8 to 9 kft/sec. The initial densities are not greater than \(10^9\) electrons per second. The most striking feature is that the "decay" curves are very irregular and may sometimes oscillate. One might first suspect that this is caused by an equipment defect such as cavity vibration. However, this is ruled out because such a defect would grossly affect the measured value of the collision frequency and yet the collision frequency remained constant throughout the range of the oscillations. In Fig. 12 an oscillating decay curve is shown along with its corresponding collision frequency curve. It is suspected that the bumps in the curves are merely lumps or kernels of ionization passing through the cavity at the wake velocity.

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REFERENCES