NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
A CAPSTAN-TYPE TORQUE REGULATOR FOR TIMING MOVEMENTS

David S. Bettwy

31 May 1963

HARRY DIAMOND LABORATORIES
FORMERLY: DIAMOND ORDNANCE FUZE LABORATORIES
ARMY MATERIEL COMMAND
WASHINGTON 25, D.C.
THE MISSION

The mission of the Harry Diamond Laboratories is:

(1) To perform research and engineering on systems for detecting, locating, and evaluating targets; for accomplishing safing, arming, and munition control functions; and for providing initiation signals: these systems include, but are not limited to, radio and non-radio proximity fuzes, predictor-computer fuzes, electronic timers, electrically-initiated fuzes, and related items.

(2) To perform research and engineering in fluid amplification and fluid-actuated control systems.

(3) To perform research and engineering in instrumentation and measurement in support of the above.

(4) To perform research and engineering in order to achieve maximum immunity of systems to adverse influences, including countermeasures, nuclear radiation, battlefield conditions, and high-altitude and space environments.

(5) To perform research and engineering on materials, components, and subsystems in support of above.

(6) To conduct basic research in the physical sciences in support of the above.

(7) To provide consultative services to other Government agencies when requested.

(8) To carry out special projects lying within installation competence upon approval by the Director of Research and Development, Army Materiel Command.

(9) To maintain a high degree of competence in the application of the physical sciences to the solution of military problems.

The findings in this report are not to be construed as an official Department of the Army position.
A CAPSTAN-TYPE TORQUE REGULATOR FOR TIMING MOVEMENTS

David S. Bettwy

FOR THE COMMANDER:
Approved by

Robert S. Hoff
Chief, Laboratory 400

Qualified requesters may obtain copies of this report from ASTIA.
CONTENTS

SYMBOLS .......................................................... 4
ABSTRACT ............................................................ 5
1. INTRODUCTION .................................................... 5
2. DESCRIPTION OF TORQUE REGULATOR ......................... 6
3. TEST RESULTS .................................................... 9
4. CONCLUSION ....................................................... 12

ILLUSTRATIONS

Figure
1. Schematic of torque regulator.
2. Cross section of torque regulator.
3. Torque regulator at slow moving end of train.
4. Torque regulator at fast moving end of train.
5. Time-versus-torque curves for runaway-escapement timing movement with and without torque regulation.
SYMBOLS

\[ e = 2.718... \]

\[ F_f = \text{Force due to friction} \]

\[ F_n = \text{Normal force causing frictional force} \]

\[ T = \text{Tension in an element of the torsion spring} \]

\[ T_l = \text{Tension required to wind spring tight against the fixed shaft} \]

\[ T_{in} = \text{Tension caused at input and by the torque being regulated} \]

\[ T_{out} = \text{Tension at output end, the resulting regulated torque} \]

\[ t = \text{Time per unit rotation of escape wheel} \]

\[ \beta = \text{Total angle of wrap of the torque regulator} \]

\[ \mu = \text{Coefficient of friction, (assumed a constant)} \]
ABSTRACT

A capstan type of torque regulating device is described and test results of its application to a runaway timing movement are presented.

1. INTRODUCTION

All mechanical timing mechanisms have a rate of oscillation that in some degree varies with the input force. This tendency to change rate with changes in the input torque is referred to as torque sensitivity.

In the case of the pendulum clock, an increase in driving torque increases the amplitude of swing and the period of the swing must therefore change since it is a function of the amplitude. In a spring-mass system such as the balance wheel of a detached lever movement, the period is theoretically independent of the amplitude; but this is based on the assumption that the system's restoring spring has a linear spring rate, an assumption not entirely correct. Other dynamic conditions are also involved that make the detached lever escapement torque sensitive. The runaway or verge escapement has a period that is approximately inversely proportional to the square root of the applied torque. This is probably the most torque sensitive of the ordnance escapements.

An awareness of the torque sensitivity inherent to some degree in all mechanical escapements used as time keepers has led to many devices for regulating the torque delivered to the escapement. The gravity escapement, a special type of remontoir, was devised for the pendulum. The fusee is used in ship chronometers to compensate for decay in mainspring torque with unwinding. A remontoir consisting of a spring rewound periodically by an electric motor is popular in dc-powered electric clocks.

These are all attempts to improve the accuracy of devices that have errors of much less than one percent, and involve compensating what is in many cases a slightly variable power supply. In most ordnance applications, the accuracy of the escapement over the range of torque provided by the power supply would be adequate. However, ordnance timers are usually used as a controlled-rate power package rather than purely as a timer. Instead of having a small fixed load such as minute and second hands to drive, the ordnance timer must throw switches, turn cams, bring rotors into alignment, etc. These functions not only impose relatively large loads on the timing movement but the loads are often variable throughout the cycle, and from one timer operation to another. As a result, the escapement is subjected to a relatively large variation in torque which is the difference between the torque available from the power supply and that being used for external functions.

The torque regulator described in this report was designed to allow a timing movement power supply to deliver varying amounts of torque for external work while maintaining the torque to the escapement constant.
2. DESCRIPTION OF TORQUE REGULATOR

The regulator discussed here consists primarily of two separate parts, a nonmoving post and a coiled torsion spring. The principle of operation is based on the capstan in which the frictional force on a rope is a function of the tension, angle of wrap, and the coefficient of friction. The input force is applied to one end of a torsion spring and the output is obtained from the other end. The torsion spring is coiled around the fixed post, which has a slightly smaller diameter than the inside diameter of the spring. This arrangement is shown in figure 1. If the output is restrained by a load, the input force causes the torsion spring to deflect in an angular direction that decreases the diameter of the spring. Up to the point at which the spring contracts enough to become a tight fit on the shaft, the torque at the output is approximately proportional to the product of the torsional spring rate and the angular deflection of the input end relative to the output end. Once the spring is wound tight on the shaft, the difference between the output force and the input force follows the torque attenuation characteristic of the capstan.

The equation defining the torque attenuation of the regulator is developed as follows. Consider a segment of the torsion spring subtended by an angle $\Delta \theta$ and pulled into contact with the fixed shaft as shown in figure 2. One end is subjected to a tension $T$ and the other to a larger tension $T + \Delta T$. Assuming the element to be held in equilibrium by the infinitesimal frictional force, $\Delta F_f$, which is

$$\Delta F_f = \mu \Delta F_n$$

where $\mu$ is the coefficient of friction and $\Delta F_n$ is the normal force. The normal force is given by

$$\Delta F_n = (2T + \Delta T - 2T_i) \sin \frac{\Delta \theta}{2}$$

where $T_i$ is the initial tension required to pull the spring against the shaft. Taking the limit as $\Delta \theta$ approaches zero, and assuming $\sin \frac{\Delta \theta}{2} = \frac{\Delta \theta}{2}$ and that the $\frac{\Delta \Delta \theta}{2}$ term is negligible, then equation (2) when multiplied by $\mu$ becomes

$$dF_f = (T - T_i) \mu d\theta$$

The equilibrium equation for the element is

$$T + dT = T + dF_f$$
Figure 1. Schematic of torque regulator.
Figure 2. Cross section of torque regulator.
Substituting (3) in (4) we have

\[ dT = (T - T_1) \mu d\theta \] (5)

Integrating over the total angle \( \beta \) of wrap of the spring from the end where the output tension \( T_{\text{out}} \) occurs to the end where the input tension \( T_{\text{in}} \) is applied, we have

\[ \int_{T_{\text{in}}}^{T_{\text{out}}} \frac{dT}{T - T_1} = \int_{0}^{\beta} \mu d\theta \] (6)

which when integrated gives

\[ \ln \left( \frac{T_{\text{in}} - T_1}{T_{\text{out}} - T_1} \right) = \mu \beta \] (7)

and

\[ T_{\text{out}} = T_1 + \frac{T_{\text{in}} - T_1}{e^{\mu \beta}} \] (8)

From equation (6), it can be seen that the increase in the output tension beyond the initial tension \( T_1 \) required to wind the spring tight on the shaft will be very small if the exponential term \( e^{\mu \beta} \) is very large. Some values of \( e^{\mu \beta} \) are presented in Table 1, and show that it is possible for this term to be sufficiently large, so that the \( e^{\mu \beta} \) term can be made negligible.

3. TEST RESULTS

A torque regulator of the capstan type was constructed using a spring-steel coil spring of 0.353 in. i.d. and made of rectangular wire 0.020 by 0.032 in. Several materials such as brass, bronze, aluminum, and cold-rolled and stainless steel were tried as the fixed shaft. The general configuration used is shown in Figure 3.

As shown in the figure, the regulator was at first placed at the slow moving end of a clock gear train. Although good torque regulation was achieved, the regulator operated in a stick-slip motion, so that the input moved in jerks. The stick-slip motion occurs after the input torque winds the spring tight, at which point the spring locks on the shaft; as the tension of the wound spring drives the escapement, the output end of the spring moves a small distance before the tension in the
Table 1. Values of $e^{\mu \beta}$ for several values of the coefficient of\nfriction, $\mu$, and number of active turns, $\frac{\beta}{2\pi}$, of the\nregulator spring. ($e = 2.71828$)

<table>
<thead>
<tr>
<th>$\frac{\beta}{2\pi}$</th>
<th>$\mu = .1$</th>
<th>$\mu = .2$</th>
<th>$\mu = .3$</th>
<th>$\mu = .4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6.6 x 10^{-1}</td>
<td>4.3 x 10^2</td>
<td>2.9 x 10^3</td>
<td>1.9 x 10^3</td>
</tr>
<tr>
<td>4</td>
<td>1.2 x 10^2</td>
<td>1.5 x 10^2</td>
<td>1.9 x 10^3</td>
<td>2.3 x 10^4</td>
</tr>
<tr>
<td>5</td>
<td>2.3 x 10^2</td>
<td>5.4 x 10^2</td>
<td>1.2 x 10^4</td>
<td>2.9 x 10^5</td>
</tr>
<tr>
<td>6</td>
<td>4.3 x 10^3</td>
<td>1.9 x 10^3</td>
<td>8.2 x 10^4</td>
<td>3.5 x 10^6</td>
</tr>
<tr>
<td>7</td>
<td>8.1 x 10^3</td>
<td>6.6 x 10^3</td>
<td>5.4 x 10^5</td>
<td>4.4 x 10^7</td>
</tr>
<tr>
<td>8</td>
<td>1.5 x 10^4</td>
<td>2.3 x 10^4</td>
<td>3.5 x 10^6</td>
<td>5.4 x 10^8</td>
</tr>
<tr>
<td>9</td>
<td>2.9 x 10^4</td>
<td>8.2 x 10^4</td>
<td>2.3 x 10^7</td>
<td>6.7 x 10^9</td>
</tr>
<tr>
<td>10</td>
<td>5.4 x 10^5</td>
<td>2.9 x 10^5</td>
<td>1.5 x 10^8</td>
<td>8.2 x 10^{10}</td>
</tr>
</tbody>
</table>
Figure 3. Torque regulator at slow moving end of train.
coil is reduced sufficiently to allow the input end to follow. This results in an irregular, intermittent motion at the input end that is entirely unsatisfactory when trying to measure accurate elapsed times by the angular displacement of a slow moving shaft.

No shaft material or configuration having been found that would eliminate this difficulty, it was decided to locate the regulator at the fast moving end of the train as shown in figure 4. Thus the intermittent motion is diminished by the reduction of the gear train so that the reflected jerking at the input is insignificantly small.

Previously the torque regulation had been measured with a strain-gage-instrumented torsion bar. With the regulator at the fast moving end of the train, the torque involved becomes so small that instrumentation is very difficult. For this reason the rate of a runaway escapement was chosen as a means of evaluating the performance. A runaway escapement is very torque sensitive. The time required for a specific angular rotation of the main shaft is approximately inversely proportional to the square root of the torque or

\[ t \propto \sqrt{\frac{1}{T}} \]

where \( t \) is time and \( T \) is torque.

This characteristic is shown by the no-torque-regulator curve on figure 5. For comparison, curves obtained for torque regulators having different initial clearances between the shaft and the torsion spring are shown on the same graph.

As shown by the graph, the torque regulator time-versus-torque curves approximately follow the no-regulator curve until the spring winds tight after which the increased driving torque does not significantly affect the time of rotation of the movement's main shaft. This is the performance predicted by equation 8.

The regulator curves depart from the no-regulator curves before becoming horizontal because the spring is dragging against the shaft, even before it winds tight, with a resultant loss of torque that would otherwise be delivered to the escapement.

4. CONCLUSION

These results demonstrate the torque-regulating capabilities of the capstan-type regulator. It is simple, inherently rugged, and inexpensive. All of these are desirable attributes in an ordnance item, and the regulator should be adaptable to ordnance timing movement applications.
Figure 4. Torque regulator at fast moving end of train.
Figure 5. Time-versus-torque curves for runaway-escapement timing movement with and without torque regulation.
The stick-slip problem could not be eliminated, but its effect on the accuracy of the output angular position can be minimized by putting the regulator at the fast moving end of a timer gear train.

Further tests will be conducted to determine the regulator's performance over a range of typical ordnance environmental conditions.
DISTRIBUTION

NOTE: Corrections to the following list would be appreciated and should, along with report number, be addressed to Department of the Army, Harry Diamond Laboratories, Washington 25, D.C., attn: Technical Reports Unit.

Office of the Director of Defense Research & Engineering
The Pentagon, Washington 25, D. C.
Attn: Technical Library (rm 3E1066)—2 copies

Commanding General
U.S. Army Materiel Command
Washington 25, D. C.
Attn: AMCRD-RS-CM
Attn: AMCRD-DE-W

Office of the Chief of Research & Development
Department of the Army
Washington 25, D. C.
Attn: Director, Army Research

Commanding General
U.S. Army Missile Command
Redstone Arsenal, Alabama
Attn: Asst Chief of Staff for R & D

Redstone Scientific Information Center
U.S. Army Missile Command
Redstone Arsenal, Alabama
Attn: Chief, Document Section

Commanding General
U.S. Army Munitions Command
Frankford Arsenal
Philadelphia 37, Pa.
Attn: Chief, Artillery Ammunition Components Div, 1600
Attn: Chief, Research Division (Pitmann-Dunn), 1300
Attn: Library, 0270

Commanding General
Aberdeen Proving Ground, Maryland
Attn: STEAP-TL, Tech Library (Bldg 313)—2 copies

Commanding General
U.S. Army Munitions Command
Picatinny Arsenal
Dover, New Jersey
Attn: SMUPA-TES, J. Odierno
Attn: Library, SMUPA-VA6
Attn: Glenn Johnson
Attn: Bruce Frey (3 copies)
DISTRIBUTION (Cont'd)

Commanding Officer
Chemical Research & Development Labs
Edgewood Arsenal, Maryland
   Attn: Librarian, Tech Library (Bldg 330)

Commander
Armed Services Technical Information Agency
Arlington Hall Station
Arlington 12, Virginia
   Attn: TIPDR (10 copies)

Scientific and Technical Information Facility
P.O. Box 5700
Bethesda, Maryland
   Attn: NASA Representative (S-AK/DL)-A366

Hamilton Watch Company
C/o District Chief
Philadelphia Procurement District
128 N. Broad Street
Philadelphia 2, Pennsylvania
   Attn: John James

Sandia Corporation
Sandia Base,
Albuquerque, New Mexico
   Attn: Library
   Attn: James McCutcheon

Commander
U.S. Naval Ordnance Laboratory
Corona, California
   Attn: Documents Librarian

Commander
U.S. Naval Ordnance Laboratory
White Oak, Silver Spring 19, Maryland
   Attn: Tech Library
   Attn: William Pickler
   Attn: Norman Czajkowski

General Time Corporation
2139 Wisconsin Avenue
Washington, D. C.
   Attn: C. B. Higgins (2 copies)

Raymond Engineering Corp
Berlin, Conn
   Attn: E. Blomgren
   Attn: C. Saltzman

18
DISTRIBUTION (Cont'd)

Precision Products Co. of Cincinnati
Beech at Kenilworth
Cincinnati, Ohio
Attn: Les Dehner

Hairspring Vibrating Co
406 32nd Street
Union City, N. J.
Attn: Charles Verde

Westclox
Lasalle, Illinois
Attn: George Heinard

Internal

Horton, B.M./McEvoy, R.W., Lt Col
Apstein, M./Gerwin, H.L./Guarino, P.A./Kalmus, H.P.
Spates, J.E./Schwenk, C.C.
Hardin, C. D., Lab 100
Sommer, H., Lab 200
Hatcher, B. D., Lab 300
Hoff, R. S., Lab 400
Nilson, H. M., Lab 500
Flyer, I. N., Lab 600
Campagna, J.H./Apolenis, C. J., Div 700
DeMasi, R., Div 800
Landis, P. E., Lab 900
Seaton, J. W., 260
Technical Reports Unit, 800
Technical Information Office 010 (10 copies)
HDL Library (5 copies)
Betwy, D. (25 copies)
Piper, W. M., 450
Branch 450 files (5 copies)
Comyn R./Kinzelman, G., 450
Fisher, E., 430
Frauman, J., 440
Cullinane, J., 410
Rotkin, I./Godfrey, T.B./Eichberg, R.L.
Bryant, W.T./Distad, M.F./Moorhead, J.G.

(Two pages of abstract cards follow.)
A CAPSTAN-TYPE TORQUE REGULATOR FOR TIMING MOVEMENTS -- David S. Betsey

TR-1120, 31 May 1963, 6 pp text, 5 illustrations, Department of the Army Proj. 5006-01-014, AMCS Code 5530.11.62400, HSL Proj 46200, UNCLASSIFIED Report

A capstan type of torque regulating device is described and test results of its application to a runaway timing movement are presented.

A CAPSTAN-TYPE TORQUE REGULATOR FOR TIMING MOVEMENTS -- David S. Betsey

TR-1120, 31 May 1963, 6 pp text, 5 illustrations, Department of the Army Proj. 5006-01-014, AMCS Code 5530.11.62400, HSL Proj 46200, UNCLASSIFIED Report

A capstan type of torque regulating device is described and test results of its application to a runaway timing movement are presented.