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LOCATION AND DETERMINATION OF THE LOCATION OF THE ENTRANCE PUPIL (CENTER OF PROJECTION) OF PC-1000 CAMERA IN OBJECT SPACE

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ABSTRACT

A technical discussion on the location of the entrance pupil (center of projection) of a metric lens is presented and certain misconceptions concerning the relationships between nodal points, the entrance pupil and the center of projection are revealed. Following this, practical techniques for determining the location of the entrance pupil of a lens and the object space coordinates of this point in the field are presented.

REFERENCES

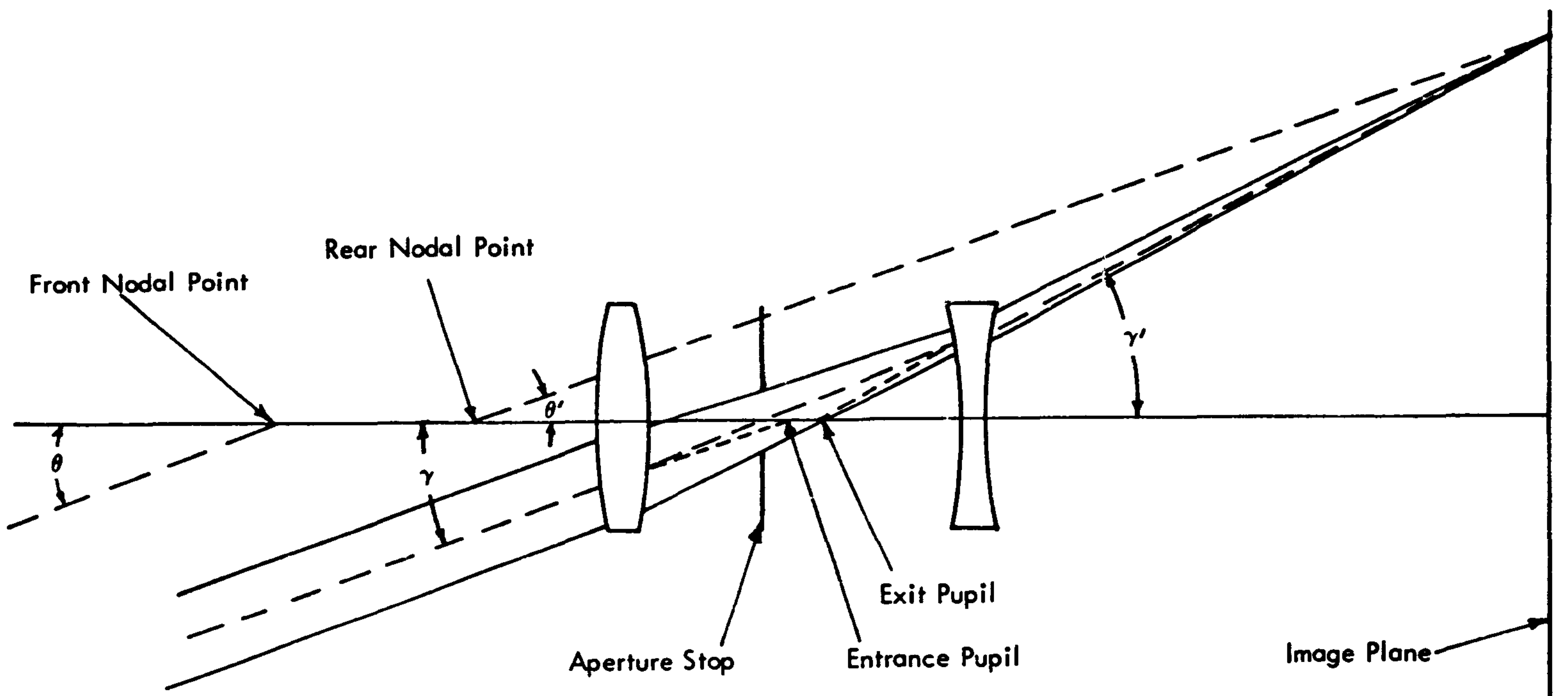
1. Duane C. Brown "The Simultaneous Determination of the Orientation and Lens Distortion of a Photogrammetric Camera", RCA Data Reduction Technical Report No. 33, Nov. 1956.

2. Duane C. Brown "A Treatment of Analytical Photogrammetry with Emphasis on Ballistic Camera Applications", RCA Data Reduction Technical Report No. 39, August 1957.

LOCATION AND DETERMINATION OF THE LOCATION OF THE ENTRANCE PUPIL (CENTER OF PROJECTION) OF PC-1000 CAMERA IN OBJECT SPACE

In ballistic camera data reduction it is necessary to establish the location of the center of projection in object space relative to a local survey mark. For any camera whatever, the center of projection in object space is located at the center of the entrance pupil and the center of projection in image space is located at the center of the exit pupil. In photogrammetry there is a fairly widespread misconception found in a few textbooks and in some technical papers to the effect that the front and rear nodal points constitute the center of projection in object and image space respectively. This is strictly true only when the lens has perfect symmetry, for then the nodal points coincide with the centers of the entrance and exit pupils. Inasmuch as early photogrammetric lenses were generally symmetric, it is understandable how this misconception originated. The fact that it persists in some quarters is probably attributable to the fact that in most conventional photogrammetric problems, the center of projection in object space is resected in the normal course of the reduction and its physical relationship to the camera itself is usually of no practical consequence. Moreover, most modern mapping lenses, while not symmetric, are direct outgrowths of the classic symmetric forms so that the separation of the front nodal point from the entrance pupil is often only a few millimeters. Here again this is usually of no practical consequence (except possibly in rare instances of close-up photogrammetry wherein the center of projection in object space is enforced rather than being recovered in the reduction).

With the PC-1000 which has a telescopic lens the separation between the front nodal point and the center of the entrance pupil is very considerable amounting typically about 520mm. The geometry of the projection is illustrated in the accompanying figure



In general: $\theta \quad \theta'$ for distortionless lens
$\theta \quad \gamma$ only for points at infinity
$\gamma \quad \gamma'$
Only with perfectly symmetric lenses do front and rear nodal points coincide with entrance and exit pupils respectively.

Illustrating entrance and exit pupil projection versus nodal point projection for simple telephoto lens. Physical centers of projection in object space and image space are at centers of entrance and exit pupils respectively, not at front and rear nodal points.

in which the lens is idealized as consisting of a single positive and a single negative element. As with all telephotos, both the front and rear nodal points are physically located in front of the objective.

The entrance and exit pupils consist of the images of the aperture stop in object and image spaces respectively. Any lens has a unique aperture stop and hence also unique entrance and exit pupils. The iris diaphram of a camera is invariably located at the aperture stop. The ray in object space passing through the center of the entrance pupil is termed the principal ray of object space. Similarly the corresponding ray in image space passing through the exit pupil is the principal ray in image space. If the aperture stop is reduced to the point where only the principal rays can pass through the stop, one sees that the center of the entrance pupil constitutes the actual center of project in object space and the center of the exit pupil constitutes the center of project in image space.

Although the angles θ and θ' (see figure) subtended at the front and rear nodal points are equal for a distortionless objective, the front and rear nodal points do not constitute the physical centers of projection. For points at infinity the angle γ subtended at the entrance pupil is equal to θ. In general the angle γ' subtended by the image at the exit pupil will neither be equal to γ nor to θ, the only exception being for a symmetric lens.

Because of the importance of knowing the location of the entrance pupil, a practical laboratory technique for determining the location of this point on the PC-1000 camera (or any other camera for that matter) has been developed. The procedure for applying this technique consists of the following:

1. The camera is mounted, aligned, and leveled, with the lens axis in a horizontal position, on any surface possessing the capability of providing precise measurable horizontal translation of the lens (in the direction of the lens axis). The bed of a milling machine is ideal for this purpose although a number of suitable alternatives can be conceived.

2. A telescopic lens system, possessing the capability of being focused at a rather short finite distance (10 feet) and a magnification of at least 15x is set up with the axis of the telescope aligned approximately coincident with the axis of the lens under test and with the objective of the telescope facing the lens objective. A Wild T-2 theodolite is ideal for this purpose.

3. The objective and inside of the lens under test is illuminated by a brilliant light source, such as a 500 watt photoflood system.

4. The telescope is focused on some external physical point on the lens, such as the front objective retainer rings, and the setting of the translation dial of the bed upon which the lens is resting is recorded. Inasmuch as the depth of focus of the telescope is extremely small at this close range (on the order of a millimeter or less), this arrangement is quite sensitive to detecting small horizontal translations of the lens.

5. The lens is translated horizontally toward the telescope, using the bed on which the lens rests, while observation is made through the telescope. This translation is stopped when the internal iris diaphram snaps into sharp focus in the telescope, and the setting of the translation dial is recorded.

6. The difference between the two translation dial settings is the distance, along the optical axis of the lens, from the external physical point on the lens upon which the initial focus of the telescope was made to the entrance pupil. With this information, the location of the entrance pupil is easily referenced to some more meaningful point on the camera, such as the plumb-bob-hook.

Inasmuch as this technique is rather subjective in nature, several independent determinations of the location of the entrance pupil should be made, by different observers if possible, and the average of these adopted as the location of the entrance pupil.

Although the above procedure is sufficiently accurate for most applications, an analytical technique would yield a more accurate determination of the entrance pupil. Cursory analysis indicates that a practical analytical technique could be developed and should be given consideration in the future.

The Gaussian focal length resulting from a calibration of distortion represents the distance from the image plane to the rear nodal point. In the particular case of the PC-1000 the front and rear nodal points are virtually coincident and lie about 200 mm in front of the objective. Inasmuch as the exit pupil lies 321 mm behind the objective, an error of over 500 mm would be introduced if the front nodal point were taken as the physical center of projection in object space. Such an error would be particularly serious in aircraft tests where baselines of only 10 to 15 miles would be typical. It is for this reason that considerable attention has been given here to the matter of the correct determination of the physical center of projection. This subject was first brought to the attention of photogrammetrists by Dr. W. Roos over 20 years ago in a German technical paper. This paper is abstracted in the December 1957 issue of Photogrammetric Engineering under the title, "On the Location of the Centers of Projection of an Objective and its Significance in Photogrammetry". Another discussion of the problem by Roos is to be found in Photogrammetria No. 3 (1951 - 1952) in a article entitled, "On the Definition of Fundamental Concepts in Photogrammetry".

It is our opinion that stellar techniques provide the only completely satisfactory calibration of distortion for cameras of relatively long focal length and wide aperature. A major disadvantage of conventional laboratory methods for such lenses is that the aperture of the collimating telescopes is generally considerably smaller than that of the camera so that only a small portion of the lens is actually used in each measurement. With the PC-1000, for instance, the collimating telescope would have to have at least an 8 inch objective in order to intercept the entire bundle of rays emanating from the camera. A telescope with a 2 inch objective would utilize only 1/16 of the camera objective for each measurement. The stellar calibration method, on the other hand, utilizes the entire camera lens and moreover, being a photographic technique, is not subject to the chromatic errors which degrade visual methods of calibration.

Inasmuch as the center of the entrance pupil does not coincide with the center of rotation of the 3-axis mount of the PC-1000, the location of the center of projection varies according to the azimuth and elevation of the camera. Reduction of data from the cameras normally requires that the location of this point, relative to a specified reference system, be accurately known. This may be determined in the following manner:

1. The reference system is defined by a Cartesian system 0-XYZ situated with the XY plane tangent to the spheroid at 0, the Y axis pointing due north, the X axis due east and the Z axis normal to the spheroid at 0.

2. The camera is set up directly over 0, and, prior to the mission, levelled with all dials on zero and the azimuth zero reference aligned with true north. The vertical distance D_z from 0, the reference monument, to the camera roll ring (immediately adjacent to the plumb bob hook) is measured and recorded.

3. The XYZ coordinates of the center of projection when the camera has been rotated through any azimuth a and elevation ω (roll of the camera does not influence the location of the center of projection) may be computed from

$$X = D_c \sin a \cos \omega$$
$$Y = D_c \cos a \cos \omega$$
$$Z = D_z + 155\,\text{mm} + D_c \sin \omega,$$

in which D_c, the distance from the center of rotation of the camera mount to the center of the entrance pupil, is determined from

$$D_c = D_r - D_0$$

where D_r is the distance from the center of rotation (which is 155mm above the plumb bob hook) to the front surface of the objective, and D_0 is the distance from the center of the entrance pupil to the front surface of the objective. For all PC-1000's $D_0 = 321$ mm. The distance D_r varies from camera to camera by as much as ±10mm and should be measured directly for each camera.