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DETERMINATION OF THE MECHANISMS
GOVERNING THE INFLOW OF MOISTURE PAST
A ROTARY SEAL - THEORETICAL MODEL

by

Bernard Hoffman

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DETERMINATION OF THE MECHANISMS
GOVERNING THE INFLOW OF MOISTURE PAST
A ROTARY SEAL - THEORETICAL MODEL

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ABSTRACT

This report describes the probable mechanisms governing the inflow of water and its vapor through rotary seals. The mechanisms were determined from:

1. A search of the technical literature pertaining to the mass transfer of water, its vapor and related phenomena.

2. Consultations with investigators who are active in the fields of science related to water vapor and water vapor phenomena.

3. Previously developed intuitive and statistical reasoning.

Utilizing the derived mechanisms, calculations are made to obtain engineering estimates of probable leakages through actual rotary sealing systems (systems employing rotary "O"-rings). A design of experiment is proposed for proving out the mechanisms.
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INTRODUCTION

Present fire control instruments are not properly sealed against the inflow of elements harmful to instrument performance (moisture, dirt, dust, spores, fungi growth or other foreign matter). In particular, an effective moisture barrier has never been achieved. This is evident in view of the number of unsatisfactory engineering reports received from field forces.

Various types of seals and sealants have been utilized in an attempt to minimize moisture leakage. In most instances they have been applied without much pre-knowledge of real factors affecting the leakage. The actual properties a sealing system must possess to cause an effective moisture barrier have not been defined because the mechanisms governing the transport of water and its vapor past the sealing system have not been determined. Therefore, determination of the flow mechanisms is essential to the understanding of the parameters affecting the inflow and hence to an intelligent design of an effective sealing system.

This report is a culmination of a search into the technical literature pertaining to the mass transfer of water, its vapor, and related phenomena. It is also based upon consultations with investigators who are active in the fields of science related to water vapor and water vapor phenomena. The effort was carried out with the idea that concepts and theory derived would supplement intuitive and statistical reasoning previously developed\(^1\) and form the basis for developing the moisture flow mechanisms. These hypothesised mechanisms with their mathematical equations have been presented. Calculations have been made to obtain engineering estimates of probable leakages through actual rotary sealing systems. A design of experiment is proposed for proving out these mechanisms.

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DISCUSSION

Nature of the Problem

Several means have been employed in the past and some of these are currently being used in an effort to prevent moisture accumulation in fire control instruments. Prior to the deployment of large numbers of fighting personnel and their weaponry to tropic and frigid climatic zones, little consideration was given to the deleterious effects of moisture accumulation in fire control instruments. Attempts at sealing, if any, consisted of gaskets (made of organic felt, cork or natural rubber) and threaded grease connections for static components. When some instruments were designed, no attempt was made to seal the instrument in order to allow as free an air circulation as possible, believing that this technique would minimize the moisture build-up. Dynamic seals (for rotary, oscillatory or translatory motions) consisted of grease or oil impregnated felt packing, packed grease surrounding the moving parts and in some isolated instances "O"-rings. Feed-back information from the field as to the relative merits of these schemes was meager. It was not until World War II that much feedback information was received and indicated that the sealing schemes being employed failed to prevent internal instrument moisture build-up. Immediately after World War II programs were initiated to rectify the situation and other schemes were tried. One such scheme was the use of moisture absorbing media (such as selica gel) in instruments with or without sealing means. These were so placed in the instrument that they could be removed when saturated. They generally have failed due to lack of adequate circulation within the instrument permitting very humid conditions to exist in certain parts of the instrument even though the portion containing the desiccant might be relatively dry. Another scheme was injection sealing, i.e., injection by high pressure means of a viscous substance such as EC-947, into an annular space or groove interposed between parts or members to be sealed. For static sealing purposes this technique provided a very tight seal if the grooves were properly designed. However, all attempts at dynamic sealing proved futile.

Another means that has proven successful for static seals has been the gasketed seal fabricated of non-rotting material such as the synthetic rubbers. The use of "O"-rings, quad rings, square rings and gaskets of other geometrical shapes has proven satisfactory for
static sealing purposes as long as the sealing system (gasket and mating members) was properly designed and the gasket sufficiently compressed. The use of gaskets made of various geometries as dynamic seals, however, has been limited due to their associated high torques. Only the "O"-ring (and occasionally the quad ring) has met the torque requirements when the compressive forces have been held to minimal values.

The metal bellows seal avoids the use of the conventional rotary seal and substitutes a gasketed static seal. By employing an offset wobble plate principle and taking advantage of the flexibility of the bellows, the seal is able to impart rotary motion through the metal bellows. Where geometrical considerations permit and where backlash is not a serious problem this seal is satisfactory; however, there has been very limited use for the seal in optical and other fire control instruments, mainly because of space and backlash requirements. In some instances, they have not been used due to a fear that the metal bellows may fatigue.

Rubber bellows have been employed where both translatory and rotary motion can be converted by suitable redesign to only translatory motion. This technique again avoids the use of a dynamic seal and substitutes a static seal instead. Where attempts at dynamic sealing have failed to completely prevent moisture build-up, other means have been tried. The use of desiccants was one such means. Another method which has met with some success, where applicable, has been the continuous purge method. This method requires a container of high pressure dry inert gas which is used to gradually displace the internal instrument atmosphere and, therefore, never allow it to have sufficient time to absorb harmful amounts of moisture. Of course, this solution is limited to instances where the economics, weight and physical size permits its use. It is believed that this technique has not been employed by the Army.

Today, the vast majority of Army fire control instruments are being dynamically sealed by gaskets and statically sealed by both gaskets and high pressure injection means. Dynamic seals are used only when other means to avoid their use such as metal or rubber bellows seals cannot be employed. It is to be noted that only a small fraction of the dynamic motion sealing needs can be met by these other means. Injection sealing by EC-947 is used only when parts to be sealed are rarely disassembled for maintenance purposes and certainly never normally in the field. The most commonly employed gasket geometry
(particularly for rotary motion dynamic sealing because of torque considerations) is the "O"-ring. In some instances (for large assemblies) where "O"-ring torque would prove excessive, quad rings are used. The practice is for the instruments, whether employing gaskets, injection means or both, to be first purged with a clean, dry inert atmosphere (N₂) and in most instances to be pressurized from 0.1 to 0.25 psi above the ambient environmental pressure existing at time of purging. From time to time unsatisfactory engineering reports (UER's) have been received from field forces. The reports indicate that in many instances instruments containing both dynamic and static seals and using above sealing techniques have failed to prevent moisture build-up when placed either in storage or field use. Since these UER's have not included instruments employing only static sealing techniques, it has been generally assumed that the UER's reflect a failure on the part of the dynamic seals to perform their function. Such an assumption is predicated on the belief that the instruments were initially adequately purged and that all static seals were properly designed and installed. There are, no doubt, instances where failure can be attributed to poor or improper practice when installing either or both the static and dynamic seals. However, it appears reasonable to suspect that not all the UER's reflect ineffective installation practice and since properly designed and installed static seals can greatly impede the inflow of moisture, it would appear that the present dynamic sealing techniques are inadequate.

Therefore, there is an apparent inadequacy of technique for the prevention of moisture accumulation in fire control instruments when such technique must account for dynamic motions, mainly rotary motion. Due to instrument design considerations, the present techniques employ the use of gaskets as seals or moisture penetration barriers; purging of instrument to create an initial dry (low moisture) internal instrument atmosphere; and in most cases the retention of a slight internal pressure (0.1 to 0.25 psi above ambient pressure existing at time of purging). If it is assumed that purging achieves its objective of creating an initial dry atmosphere within the instrument, then the problem reduces to one of an apparent inadequacy of sealing technique. The inadequacy consists of loss of initial dry internal instrument atmosphere and the inflow of water and its vapor in sufficient quantity (in a given period of time) to cause deleterious moisture accumulation. The present method of determining the quality of the sealing technique is to establish its tightness, i.e., its ability to limit to some arbitrary figure of merit the flow rate of a gas through the seal when a fixed total gas pressure differential is maintained across the seal. Such an
evaluation is responsive only to determining the seal's ability to retard the net total gas exchange between the internal instrument atmosphere and the existing external atmosphere. However, this evaluation is not responsive to determining the seal's ability to prevent or limit the passage of water and its vapor past the seal. It simply implies that if the net total gas exchange across the seal is smaller, then the probability of inflow of moisture past the seal is less. Therefore, sealing systems have been designed as moisture barriers and tested without much pre-knowledge of the real factors affecting the leakage past these barriers. The actual properties that a sealing system must possess to effect a moisture barrier (other than small flow paths) have never been defined because the mechanisms governing the transport of water and its vapor past the sealing system have never been fully understood. Therefore, determination of the flow mechanisms is essential to the understanding of the true role played by the various sealing technique parameters in causing the inflow. By an understanding of these parameters an intelligent design of sealing systems can be formulated. In light of the ineffectiveness of present dynamic sealing techniques, determination of the flow mechanisms is of paramount importance and constitutes the justification for the present investigation.

Extent and Limitations of Dynamic Sealing Systems to be Investigated

An apparent inadequacy of sealing technique exists for the prevention of deleterious moisture accumulation in fire control instruments when such technique must account for dynamic motion. Dynamic motion takes the form of rotation, translation and combinations of these. An effective sealing solution involving translation and rotation occurring simultaneously such as in diopter movements has been, by suitable redesign, to reduce the motion to translation and to employ rubber bellows seals. This technique avoids the use of a dynamic seal and substitutes a static seal instead. Other combinations involving translation and rotation, such as previously designed into interpupillary devices used in binocular instruments, have been changed by suitable redesign into rotary motion. Therefore, the sealing needs of combinations involving rotary and translatory motion have been reduced to static and rotary seals.

As stated above, sealing of translatory motion can be met with static rubber bellows seals. Thus, the dynamic sealing problem is restricted to the particular case where there is an apparent ineffectiveness of sealing technique for rotary motion.
There are many rotary sealing systems that have been devised and might be considered when attempting to determine the mechanisms governing the transport of water and its vapor past a rotary seal system. These systems include injection seals, gasketed seals such as "O"-rings, quad or square rings and other geometrical shapes, stuffing box seals employing metallic and nonmetallic packing, face type seals employing metallic and nonmetallic elements, labyrinth seals and combinations of these (see figure 1). However, when related to fire control instrument sealing needs, each have their own inherent advantages and disadvantages. Rotary injection seals have failed to maintain during usage (which includes storage periods) a reasonable small flow path and have introduced serious maintenance problems. Therefore, they have been discontinued and are not considered a realistic choice for fire control instrument rotary sealing needs. Commercial stuffing box seals employing suitable packing have generally not been considered because of space, wear and torque requirements. Face type rotary seals, in order to be effective, would require extremely accurate alignment and for this reason, as well as wear and space considerations (in addition to the fact that they become a poor seal when the rotary motion ceases), have not been considered for fire control instrument rotary sealing needs. Metal labyrinth rotary seals suffer from the same defects as face type seals and, therefore, are not suitable.

In determining the boundaries that must be imposed on a system, the following guidelines must be considered:

1. The format of the system must be such that it realistically considers fire control instrument design limitations, such as space and torque requirements, but not necessarily be limited to existing or presently available rotary seals.

2. Presently available rotary sealing systems that meet the above limitations are generally limited to gasketed type rotary seals (principally "O"-rings). Further, even if the other presently available systems could find use in limited applications, their format is such that the parameters used are essentially no different than those employed by gasketed type rotary seals. Therefore, gasketed type rotary seal parameters can be considered as representative of presently available rotary sealing systems that meet fire control instrument design limitations.
FIGURE 1—VARIOUS TYPES OF SEALS AVAILABLE FOR ROTARY MOTION
3. Prior to the investigation, the fact must be recognized that no firm knowledge exists as to the probable form that the flow mechanisms may take. Furthermore, the form may differ substantially for rotary sealing systems employing seal parameters which differ only in order of magnitude. Limitations must be placed on the possible magnitudes assigned to these parameters to assure that the flow mechanisms revealed during the investigation are the most probable. These limits may be arrived at by considering existing rotary sealing systems which meet design criteria.

Thus, the boundaries that must be imposed on rotary sealing systems for purposes of conducting the investigation are those imposed on gasketed type rotary seals.

Investigation

1. Prior Efforts By Others in Determining Basic Parameters

As a first step in conducting this investigation, it was believed that prior efforts by others in attempting to determine the basic parameters involved in design of rotary seals should be investigated. The information obtained would be helpful in explaining the moisture flow mechanisms. A review of these efforts indicate that former investigators concerned themselves only with the concept of net total gas exchange across the seal. The review disclosed no previous investigations, regarding determination of the mechanisms which permit water and its vapor to penetrate seals. These investigators were concerned with seal tightness in restricting total gas flow and if any thought was given to moisture flow, it was not directly expressed. Their reasoning was as follows: For similar conditions affecting total gas flow, the smaller the net total gas exchange across the seal, the smaller the net exchange of any one of the gaseous components making up the total will be. The philosophy assumed is that the seal is designed tight enough to completely exclude water. It becomes necessary only to restrict the total gas flow (by sufficiently restrictive flow path geometry) to assure an effective moisture barrier. Moisture is treated as one of the elemental or partial gases making up the total gas. Qualitatively,

\[ ^2 \text{Report, "Development of Rotary and Translatory Seals," dtd August 1954, by Syracuse University Research Institute.} \]
this approach is meaningful since size and geometry of flow path are gross effects. However, for the approach to be quantitative, it is implied that once having optimized control over the gross variables, the effects of the lesser variables are unimportant. The restrictive nature of the flow path is a function of the ratio of length of flow path to its cross sectional area. In the case of gasketed static seals, proper compression and width control can be met. However, there are limitations to the compressive force and width of gasket that can be employed for gasketed rotary seals and still meet the required fire control instrument design limitations.

Investigators in the past were not cognizant of the basic moisture flow mechanisms. Therefore, no attempt was made to account for these mechanisms in their designs of rotary seals.

2. Consultations with Investigators Active in Field (water vapor)

It was disappointing that a review of former investigators' efforts concerned with designing seals failed to reveal any direct concern with the fundamental mechanisms which permit water and its vapor to leak past the seal. Although the investigators utilized the latest scientific data, its format was not suitable for designers use. Persons active in scientific fields relating to water vapor revealed no concern and hence no knowledge, as such, relating to moisture leakage past rotary seals. Had they been cognizant of its need, they may have developed it. Their interest included permeation (by gaseous diffusion or effusion) through membranes, porous solids and hermetic sealing materials where the emphasis is placed on penetration of material composition itself. It was assumed that in any application for sealing purposes, there would be no flow paths other than the porosity of the materials.

Further, it became clear that because such knowledge did not exist (and would not exist in the classical literature) persons concerned with design of rotary seals would have to develop it for themselves. The investigators of water vapor, however, did suggest that the flow probably could occur whether or not a total gas pressure differential existed across the seal. They also suggested that the flow mechanism is most probably of a molecular nature and that gaseous diffusion plays a prominent role.
3. Search of the Technical Literature Pertaining to Mass Transfer of Water and Its Vapor

It is only when flow is thought of in terms of the exchange of mass at a very gradual rate that the literature reveals flow concepts and mechanisms which are applicable to that order of magnitude of flow suspected to take place past the rotary seals. However, before any of these flow concepts or mechanisms can be considered, it is essential that the suspected flow rates be verified. If it is considered that all mass transfer takes place as a gaseous medium and that the moisture flow rate is only a small fraction of the total net gas exchange rate, then by use of known total net gas exchange rates of existing gasketed rotary seals a rough engineering estimate of probable moisture flow rates can be made. "O"-ring rotary seals previously tested\(^3\) indicate that the total net gas exchange rate will not exceed 2.0 in.\(^9\)/year of standard density gas (i.e., gas at a pressure of one atmosphere and at a constant temperature of 60° F). Such a flow rate can only be of a molecular nature.

Before attempting to pinpoint the probable flow mechanisms, it is important that the nature of the flow path geometries in actual rotary seals be determined and carefully examined as follows:

a. Probably the most important consideration is that the flow path geometries will more probably than not continually change both in cross sectional area and length of flow path as rotary motion takes place. These changes will be of a random nature and will depend on the mode and frequency of motion as well as environmental considerations.

b. There will be both static and dynamic annular leakage flow paths (see figure 2). At any given time, each of the annular leakage flow paths will be of nonconstant cross sectional area along the length of flow path as well as of changing dimension along the periphery of the cross section.

c. When no rotary motion takes place, the same conditions will prevail except that the geometries will not continually change.

FIGURE 2. SCHEMATIC OF LEAKAGE FLOW PATHS PAST GASKETED ROTARY SEAL.
d. As seal wear occurs, further geometrical changes will occur. Therefore, leakage flow paths will not only consist of irregular geometrical shapes but of ever changing geometries. If the flow path geometries are treated as time variant, an analysis would become unduly complicated. Therefore, the flow paths may be treated as having constant average geometrical shape(s) which permit the same moisture flow to occur. Using this approach, an engineering estimate can be made of the order of magnitude of the average flow path width, cross sectional area and length. When quantitative numbers have been assigned to these three parameters, the probable moisture leakage flow mechanisms can then be more realistically investigated.

Estimation of Lump Parameters

Recent tests conducted at Frankford Arsenal as well as at other research establishments indicate that rotary "O"-ring sealing systems (from 1/4 in. to 2 in. rotary member size) permit a net total gas exchange from 0.5 to 2.0 in.\(^3\)/year-psi of standard density fluid. In addition, it has been established that after an initial wear in period, the mechanism of total net gas exchange past the seal is similar to viscous flow of a gas through long capillary flow paths. The irregular and ever changing flow paths are to be treated as having constant average geometrical shape(s). Care, however, must be exercised in choosing the shape. The choice will affect the size of cross sectional area and width of flow path determined for a given flow rate and length of flow path. The size of cross sectional area and width of flow path in turn affects the nature of total flow, i.e., streamline or molecular effusion, and the rate of moisture flow. Hence, size-wise they must be similar to actual seal configuration if reasonable theoretical engineering estimates of moisture flow rates are to be realized.

The first shape to be considered is a single capillary tube of constant diameter \(d\) and length \(L\). This approach lumps all flow paths into a single cylindrical path which will result in the minimum possible cross sectional area and maximum width of flow path for the given flow rate and length of flow path. A comparison will then be made with the other extreme where the flow path will be assumed to cover the entire annular spaces, both dynamic and static, which will result in maximum possible cross sectional area and minimum width of flow path for the given flow rate and length of flow path.
For Case I, streamline flow through a single straight capillary tube, the following formula applies (see Appendix A for discussion and derivation):

\[
\frac{dv}{d\delta} = \left( \frac{\pi r^4 g_c}{8 \mu l} \right) \Delta p
\]

Where:

- \( l \) = flow path length, ft
- \( r \) = radius of flow path, ft
- \( g_c \) = average gravity constant, ft/sec\(^2\)
- \( \mu \) = absolute viscosity, lbs/ft-sec
- \( \Delta p \) = pressure differential across seal, lb/ft\(^2\)

Determination of values to be used in above equation

For a unit pressure differential (1 psi) the flow may vary from 0.5-2.0 in. \(^3\)/year. The average value will be used or,

\[
\frac{dv}{d\delta} = 1 \text{ in.}^3/\text{year} = 1.835 \times 10^{-11} \text{ ft}^3/\text{sec}
\]

\( l = 1/64 \text{ in.} = 0.0013 \text{ ft} \) (see Appendix B)

For \( N_2 \) gas (at standard density, one atm. pressure and 60\(^\circ\) F)

- \( \mu \approx 11.8 \times 10^{-6} \text{ lb/ft-sec} \)
- \( g_c = 32.18 \text{ ft/sec}^2 \)
- \( \Delta p = 144 \text{ lb/ft}^2 \)
Using above values,

\[ \frac{dv}{d\theta} = 1.835 \times 10^{-11} \text{ ft}^3/\text{sec} = (3.1416)(r^4) \left( 32.18 \text{ ft/s}^2 \right) \left( 144 \text{ lb/ft}^2 \right) \]

or

\[ r^4 = 154.8 \times 10^{-24} \text{ ft}^4 \]
\[ r = 3.53 \times 10^{-6} \text{ ft} \]
\[ r = 4.23 \times 10^{-5} \text{ in.} \]
\[ r = 1.075 \times 10^{-4} \text{ cm} \]

Width of flow path (d) through which flow takes place

\[ d = 2r = 2 (1.075 \times 10^{-4} \text{ cm}) \]
\[ d = 2.15 \times 10^{-4} \text{ cm} \]

Cross Sectional Area (A) through which flow takes place

\[ A = \pi r^2 = (3.1416) (3.53 \times 10^{-6} \text{ ft})^2 \]
\[ A = 3.92 \times 10^{-11} \text{ ft}^2 \]
\[ A = 5.64 \times 10^{-9} \text{ in.}^2 \]
\[ A = 3.635 \times 10^{-8} \text{ cm}^2 \]

For Case 2, flow takes place through the static and dynamic flow paths (see figure 2) but with paths having constant cross sectional area along the length of flow path with the thickness (t) of both annular spaces being the same. Maximum cross sectional area and minimum width of flow path are obtained for a given flow rate and length of flow path. The flow equation is derived in Appendix C. By choosing a size of an actual "O"-ring gasketed seal, a value for \((D_1 + D_2)\) in the equation is obtained. The flow path width (t) can then be calculated. The largest "O"-ring (2 in. I.D.) gasket seal will be used to obtain the smallest value of t.
The equation is,

\[ \frac{dv}{d\theta} = \frac{\pi (D_1 + D_2) t^3 g_c \Delta p}{12 \mu \ell} \]

where:

- \( D_1 \) = diameter of dynamic annular flow path, ft
- \( D_2 \) = diameter of static annular flow path, ft

Determination of values to be used in above equation

\[ \frac{dv}{d\theta} = 1 \text{ in}^3 \text{ year} = 1.835 \times 10^{-11} \text{ ft}^3 \text{ sec} \] (same as in Case 1)

\( D_1 + D_2 = 0.3415 \text{ ft} \) (for 2 in. I.D. "O"-ring, see Appendix C)

\( \ell = 1/64 \text{ in.} = 0.0013 \text{ ft} \) (same as in Case 1)

For \( \text{N}_2 \) Gas (At standard density, one atm. pressure and 60\(^o\) F)

\[ \mu \approx 11.8 \times 10^{-6} \text{ lb/ft-sec} \]
\[ g_c = 32.18 \text{ ft/sec} \] (same as in Case 1)
\[ \Delta p = 144 \text{ lb/ft}^2 \]

Using these values,

\[ \frac{dv}{d\theta} = 1.835 \times 10^{-11} \frac{\text{ft}^3}{\text{sec}} = \frac{(3.1416)(0.3415 \text{ ft})(11.8 \times 10^{-6} \frac{\text{lb}}{\text{ft-sec}})(32.18 \frac{\text{ft}}{\text{sec}})(144 \frac{\text{lb}}{\text{ft}^2})}{(12)(11.8 \times 10^{-6} \frac{\text{lb}}{\text{ft-sec}})(0.0013 \text{ ft})} \]
\[ t^3 = 6.79 \times 10^{-22} \text{ ft} \]

Width of flow path (\( t \)) through which flow takes place

\[ t = 0.879 \times 10^{-7} \text{ ft} \]
\[ t = 1.055 \times 10^{-6} \text{ in.} \]
\[ t = 2.680 \times 10^{-6} \text{ cm} \]

Cross Sectional Area (\( A \)) through which flow takes place

\[
A = \pi (D_1 + D_2) \quad t = (3.1416)(0.3415 \text{ ft})(0.879 \times 10^{-7} \text{ ft})
\]
\[ A = 0.941 \times 10^{-7} \text{ ft}^2 \]
\[ A = 1.355 \times 10^{-5} \text{ in.} \]
\[ A = 8.74 \times 10^{-5} \text{ cm} \]

In cases 1 and 2, the values of cross sectional area and width of flow path were calculated by assuming that streamline flow occurred as opposed to molecular effusion. This assumption must be verified before the values obtained can be accepted as being valid. By comparing the calculated width of flow path with the size of the mean free path (\( \lambda \)) of \( \text{N}_2 \) gas (0.1 \( \lambda \) or smaller being considered the size of width of flow path causing molecular effusion - See Appendix A) the assumption will be verified. The molecular mean free path of \( \text{N}_2 \) gas \( \approx 6.47 \times 10^{-6} \) cm. In case 1, the width of flow path (\( d = 2r \)) is 31.7 times greater than the mean free path. In case 2, the width of flow path (\( t \)) is 0.414 times the mean free path. Since the characteristics of total gas flow through the flow paths do not change from streamline flow to molecular effusion until the size of flow path width decreases to \( \approx 0.1 \lambda \), the initial assumption of streamline flow in cases 1 and 2 is correct. It is interesting to note that had molecular effusion first been chosen to apply, the values of width of flow path obtained would have been even larger than by assuming the streamline flow. See Appendix D which shows that the diameter (\( d \)) of Case 1 recalculated using Knudsen's Law is approximately 3 times larger than by assuming Poiseuille's Law. Therefore, streamline flow applies to these two cases.
A comparison will now be made of the relative sizes of cross sectional area and width of flow path for each case. Table I illustrates these ratios.

Table I.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Sectional Area</td>
<td>3.635x10^-8 cm^2</td>
<td>8.74x10^-5 cm^2</td>
<td>2400</td>
<td>-</td>
</tr>
<tr>
<td>Width of Flow Path</td>
<td>2.15x10^-4 cm</td>
<td>2.68x10^-6 cm</td>
<td>-</td>
<td>80, 3</td>
</tr>
</tbody>
</table>

Prior to comparing relative sizes, it would be expedient to obtain a feel for the absolute size of the width of flow path in relation to the molecular sizes of the N₂ and water vapor gas molecules. Since the molecular diameter of both H₂O and N₂ is approximately 3.15 x 10^-8 cm (see footnote No. 4), the width of flow path is at least 85 times the diametrical size of the gas molecules passing through. Therefore, flow takes place through greater than molecular size apertures by approximately 2 orders of magnitude.

It is noted from Table I that there are very large size differences of cross sectional areas and widths of flow paths between cases 1 and 2. This condition can be explained as follows:

In both cases, the nature of flow was the same. Furthermore, the same condition as concerns velocity distribution was applied to each case (See Appendix C). The fact that the peripheral length of flow path [$\pi (D_1 + D_2)$] in case 2 is fixed and of a large value, causes a much larger wetted perimeter to exist than could possibly exist in case 1. Since flow rate at a fixed length of flow path is directly proportional to the ratio of cross sectional area to wetted perimeter, the

---

cross sectional area in case 2 must of necessity be much larger than in case 1. Further, since the average velocity through the cross sectional area in each case is proportional to the square of the width of flow path and is as given in the following equations:

\[
\text{Case 1 } \mu_{av} = \left( \frac{\Delta P g_c}{\mu \cdot \ell} \right) \left( \frac{1}{32} a_1^2 \right)
\]

\[
\text{Case 2 } \mu_{av} = \left( \frac{\Delta P g_c}{\mu \cdot \ell} \right) \left( \frac{1}{12} t^2 \right)
\]

the width of flow path in case 2 must of necessity be smaller than in case 1 and equal to \(\sqrt{\frac{3}{8} A_1/A_2} \cdot d_1\), where \(A_1\) and \(A_2\) denote cross sectional areas determined by cases 1 and 2, respectively. These differences when related to total flow rate are of no interest. However, an apparent anomaly exists when an attempt is made to relate them to moisture flow rate past the seal. All the possible moisture flow mechanisms suggested by the technical literature indicate that the flow rates are directly proportional to the cross sectional area of the flow path. They do not concern themselves with the periphery of the cross sectional area or to any correction factors to compensate for smallness of size or width of flow path. Since there is a difference greater than three orders of magnitude between the relative sizes of the cross sectional areas determined by cases 1 and 2 and since this difference comes about by assuming different flow path shapes, it appears that moisture flow rate will be strongly affected by the actual geometry of the flow path. Therefore, if reasonable engineering estimates are to be made of the flow rates, a much closer look must be taken to determine which case more closely represents the true situation:

1. Concerning case 2, it is highly improbable that the flow will be equally distributed through both dynamic and static annular flow paths. It is more probable that there will be a greater percentage of the flow through the dynamic portion. Hence, the total effective peripheral length of flow path through the annular spaces will be shorter than predicted by \(\pi (D_1 + D_2)\). The reduction of the effective peripheral length of flow path will require an increase in the annular width (t).
However, the percentage increase in width (t) will only be equal to the cube root of the percentage decrease in the effective peripheral length. Therefore, there will be a net decrease in effective cross sectional area of flow path.

2. Because of surface irregularities of gasket material and its mating sealing surfaces, it is highly improbable that the annular flow paths will have a constant width (t) along their periphery. It is more probable that the width (t) will be irregular and vary in size. This will further reduce the effective peripheral length of flow paths and, for the same reasoning stated above, further decrease the effective cross sectional area of flow path.

3. Some eccentricity must exist in the alignment of rotary member and gasket housing as well as some displacement due to weight and play in rotary member bearings. Therefore, there will be uneven compression of gasket material. Sections of the length of the periphery will be more tightly sealed (i.e., have lesser width (t) than other sections). Since flow rate is proportional to the cube of width (t), this effect will further reduce the effective peripheral length of flow path. Again, for the same reasoning as stated before, the effective cross sectional area will be further reduced.

4. Rotary member loading while member is in rotation, may further take up play or slack in the member support system (bearings) and cause further misalignment. This possible increased misalignment will further increase uneven compression of gasket material causing greater differences in width (t) to exist along the periphery. A reduction in the effective peripheral length of flow path will result with its attendant decrease in effective cross sectional area of flow path. The above affects all tend to decrease the effective peripheral length through which the bulk of the total gas flow takes place, increase the effective width (t) and decrease the effective cross sectional area of flow path. Therefore, the flow path shape which more closely represents the true situation shifts from case 2 toward case 1. A closer look at Table I reveals that in the more restrictive situation, case 2, the width (t) of flow path is of the order of approximately one millionth of an inch whereas in the less restrictive situation, case 1, the width (d) of flow path is of the order of approximately one ten thousandth of an inch. Further, the peripheral length of flow path (wetted perimeter) in case 2 is huge (25.8 in.) when compared to case 1(0.0027 in.). One ten thousandth of an inch is a very small number. It would not require much uneven compression of gasket material or very irregular surface
conditions to cause differences in width (t) along the peripheral length that will tend to approach that order of magnitude. Any small lengths of periphery having values of width (t) approaching case 1 width (d ≈ 0.0001 in.) will force a requirement that the effective peripheral length of the flow path also approach the case 1 peripheral length. For example, if a very small peripheral length of flow path were to open up from one millionth of an inch to one hundredth of a thousandth of an inch (a plausible situation), the increase of width (t) by a factor of 10 would require that the effective peripheral length be decreased by a factor of 1000. The effective cross sectional area would then be reduced by a factor of 100. Therefore, it appears that the probable shift from case 2 towards case 1 is appreciable and that the flow path shape which best represents the true situation is much closer to case 1 than case 2.

From the above, it may be assumed that the bulk of total gas flow probably takes place through a very small portion of the length of the peripheral sealing area. If more than one single straight capillary tube is considered to exist through which the total net gas exchange takes place (case 3), then as the number of these tubes increase, the following decrease in width (d) and increase in cross sectional area (A) will occur:

**Table II.**

<table>
<thead>
<tr>
<th>Number of Tubes</th>
<th>Size of Diameter, d</th>
<th>Size of Cross Sectional Area, A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Case 1)</td>
<td>$2.15 \times 10^{-4}$ cm</td>
<td>$3.635 \times 10^{-8}$ cm$^2$</td>
</tr>
<tr>
<td>10</td>
<td>$0.565(2.15 \times 10^{-4}$ cm</td>
<td>$3.16(3.635 \times 10^{-8}$ cm$^2$</td>
</tr>
<tr>
<td>100</td>
<td>$0.316(2.15 \times 10^{-4}$ cm</td>
<td>$10(3.635 \times 10^{-8}$ cm$^2$</td>
</tr>
<tr>
<td>1,000</td>
<td>$0.1778(2.15 \times 10^{-4}$ cm</td>
<td>$31.6(3.635 \times 10^{-8}$ cm$^2$</td>
</tr>
<tr>
<td>10,000</td>
<td>$0.1(2.15 \times 10^{-4}$ cm</td>
<td>$100(3.635 \times 10^{-8}$ cm$^2$</td>
</tr>
<tr>
<td>1,000,000</td>
<td>$0.0316(2.15 \times 10^{-4}$ cm</td>
<td>$1000(3.635 \times 10^{-8}$ cm$^2$</td>
</tr>
<tr>
<td>n→a (Case 2)</td>
<td>$0.01245(2.15 \times 10^{-4}$ cm</td>
<td>$2400(3.635 \times 10^{-8}$ cm$^2$</td>
</tr>
</tbody>
</table>

* Most probable range which best represents the true situation as concerns flow path shape
It is believed that there will be several places along the length of the peripheral sealing area which together will effectively pass the bulk of the total gas flow. Therefore, the probable flow path shapes which best represent the true situation in case 3, where the number of tubes might range from, say, 10 through 100. Examining the size of flow path width (d) and cross sectional area (A) under these conditions, it is noted that the width (d) decreases to no less than 0.316 times case 1 width and the cross sectional area (A) increases no more than 10 times case 1 cross sectional area when considering the largest probable deviation from case 1 (100 capillary tubes).

The flow shape represented by case 3 (within the boundary of 10 to 100 capillaries), which is the best probable simulation of the true seal flow path geometries, permits quantitative numbers to be assigned to average flow path widths, cross sectional areas and length of flow paths. These quantitative numbers are:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Widths of flow path</td>
<td>0.680 - 1.214 x 10^{-4} cm</td>
</tr>
<tr>
<td>Cross Sectional areas</td>
<td>1.149 - 3.635 x 10^{-7} cm^2</td>
</tr>
<tr>
<td>Lengths of flow path⁵</td>
<td>3.00 - 6.00 x 10^{-2} cm</td>
</tr>
</tbody>
</table>

The investigation of the probable moisture leakage flow mechanisms can now more realistically be determined.

**Flow Mechanisms Revealed by the Literature Search**

The investigations of both the total net gas exchange rate and the nature of the flow path geometries reveal the following facts:

1. The flow rate is very gradual and of a molecular nature.

2. The flow paths have irregular and (when rotary motion takes place) ever changing geometries.

3. The bulk of the flow takes place through flow paths which are small when compared to the length of the peripheral sealing areas. The paths are not constant and are ever shifting.

⁵For "O"-ring type gasketed seals (1/4 in. to 2.0 in. I.D.) which meet fire control instrument design limitations (See Appendix B).
4. The aperture size through which the flow takes place has (1) effective widths that range from $0.680 \times 10^{-4}$ to $1.214 \times 10^{-4}$ cm and (2) effective cross sectional areas ranging from $1.149 \times 10^{-7}$ to $3.635 \times 10^{-7}$ cm$^2$. Assuming that the mass transfer of water and its vapor occur through the same flow paths, the literature reveals transfer mechanisms which explain how the moisture leakage past the seal occurs.

The actual moisture leakage past a rotary seal is of a multifarious nature. In realistic applications, the duty cycle of the rotary member is of a random nature. Furthermore, the environmental conditions are unpredictable. Therefore, the actual conditions which cause flow cannot be predicted. Both steady state and transient conditions exist. Also, nonrotational conditions differ from operative mode. Environmental extremes (dry and humid) must be considered. It is obvious at this point that the flow pattern is complicated. Gross examination of the phenomena will ignore the attraction between gas molecules and the thermodynamic effects such as thermal currents. Their affects on the flow pattern will be considered by the use of correction factors to be applied to the more prominent flow mechanisms. There are, for purposes of analyses, principally three mechanisms which govern the flow of water and its vapor past rotary seals. These are gaseous diffusion, capillary action (which includes viscous flow due to negative differential total pressure across seal) and the mechanical pumping action due to the rotary member. These mechanisms will be considered separately, and then an attempt will be made to interrelate their affects. Prior to discussing the mechanisms, a simple model will be developed. The effect of each mechanism on the model will be investigated. This model is illustrated in figure 3. It is consistent with the flow factors previously discussed.

**Gaseous Diffusion**

The simplified model of the rotary seal shows that the seal may be considered as a series of leakage flow paths which consists of straight capillary tubes of constant diameter ($d$) and length ($l$). The only restriction that angular translation of the rotary member imposes is shifting of the capillary tubes along the rotary peripheral sealing area. This occurs without change of tube geometry. The driving forces applied to the model are environmental conditions. The environmental elements will be treated as perfect gases. Because the diameter
CROSS SECTIONAL AREA (A)

STRaight Capillary TUBE FLOW PATHS

PERIPHERAL LENGTH ALONG STATIC SEALING AREA.

PERIPHERAL LENGTH ALONG DYNAMIC SEALING AREA.

FRONTAL VIEW OF SIMULATED LUMPED PARAMETER FLOW PATHS ALONG BOTH DYNAMIC AND STATIC SEALING AREAS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>t(external)</td>
<td>-65°F to 165°F</td>
</tr>
<tr>
<td>p(external)</td>
<td>p = 29.25 inHg</td>
</tr>
<tr>
<td>p(vapor)</td>
<td>From 0 to 100% Rh for existing temp</td>
</tr>
<tr>
<td>Gases</td>
<td>N2 + water vapor</td>
</tr>
<tr>
<td>SOLIDS</td>
<td>Liquid H2O when temp below dew point</td>
</tr>
<tr>
<td>Boundary Conditions Imposed on Average Capillary Tube</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. There are from 10 to 100 similar straight capillary tubes with constant diameter (d) and length (L) interspersed randomly along peripheral sealing area. It will be assumed that the majority are along the dynamic sealing area, with only 1 or 2 located along the static sealing area.

2. During rotation, there may be a possible shifting of the tubes along the dynamic peripheral sealing area. Concerning gaseous diffusion, the tubes although shifting will remain of constant geometry. As concerns capillary action and mechanical pumping, the irregular and ever-changing geometries will be considered.

3. Capillary tube dimensions:
   - d = 0.680 X 10^-4 to 1.214 X 10^-4 cm
   - A = 1.149 X 10^-7 to 3.635 X 10^-7 cm^2
   - g = 3.00 X 10^-2 to 6.00 X 10^-2 cm

4. External atmospheric conditions although of a random nature as a function of time will for purposes of analysis assume known values.

FIGURE 3-Schematic of simplified model developed to represent nature of flow conditions to be experienced in actual rotary seals.
of the capillaries is of greater than molecular size, the gaseous media of the environments will tend to occupy the volumes of the capillaries thus causing a gaseous continuity to exist between them. No temperature sources or sinks will be considered in the model. Therefore, conditions extending from the external environment through the capillary tubes into the internal environment is isothermal. Thus the model has been defined and a discussion of gaseous diffusion follows.

Molecular diffusion is the transport of matter (mass) on a molecular scale through a fluid which is essentially stagnant or still. It is characterized by the transfer of one substance through another as a result of a concentration difference, the diffusing substance moving from a place of high to low concentration.

When molecular diffusion in gases takes place, the kinetic theory of gases provides a means of visualizing what probably occurs during the process. A molecule is imagined to travel in a straight line at a uniform velocity until it collides with another molecule. Its velocity then changes both in magnitude and direction. The average distance the molecule travels between collisions is called the mean free path, and the average velocity of the molecule is dependent upon the temperature. The molecule travels in a highly complex path, and the net distance it moves in a given time is only a fraction of the distance along its actual path. Therefore, the net rate of diffusion is a very slow process. For any given gaseous system it is dependent upon the total gas pressure, the temperature, and the length of flow path through which the diffusion must take place.

A mathematical equation to describe the above is given below. From this equation an engineering estimate of moisture leakage past the model seal will be made. This data has a constant relationship to actual rotary seals.

For a binary mixture of gases A and B (water vapor and nitrogen, respectively) which is not of uniform molecular concentration, there will be an interdiffusion of the gases which can be described in terms of the linear velocities of movement of A and B. It is assumed that the drop in concentration of gas A (−dCA) which acts as a driving force for the movement of A, is proportional to the relative linear velocity of A with respect to B (UA - UB); to the molecular concentrations of the gases (CA and CB) and to the distance (dz) through which the diffusion occurs,
\[-dC_A = \beta C_A C_B (u_A - u_B) \, dz\]  \hspace{1cm} (1)

Where:

\[
\beta = \text{factor of proportionality}
\]

For gases, the molar concentrations (C) may be expressed as \((\rho/M)\) and the concentration gradient as \((dp)\). Equation (1) becomes

\[-dp_A = \frac{\beta \rho_A \rho_B}{M_A M_B} (u_A - u_B) \, dz\]  \hspace{1cm} (2)

Where:

\[
\rho = \text{density, gm/cm}^3
\]

\[M = \text{molecular weight, gm/gm-mole}\]

\[p = \text{partial pressure, atmospheres}\]

\[N\] is defined as the number of moles of gas diffusing per unit of time, per unit area in a direction perpendicular to that of the diffusion

\[N = \rho \frac{u}{M}\]  \hspace{1cm} (3)

From equations (2 and 3)

\[-dp_A = \beta \left(\frac{\rho_A u_A \rho_B}{M_A M_B} - \frac{\rho_A \rho_B u_B}{M_A M_B}\right) \, dz\]

\[-dp_A = \beta \left(N_A \frac{\rho_B}{M_B} - N_B \frac{\rho_A}{M_A}\right) \, dz\]  \hspace{1cm} (4)

Applying the ideal-gas law

\[\frac{P}{RT} = \frac{\rho}{M}\]  \hspace{1cm} (5)
Then equation (4) becomes

\[-dP_A = \frac{\beta}{RT} \left( N_A P_B - N_B P_A \right) dz \]

Since

\[ P = P_A + P_B \]

Or

\[ P_B = P - P_A \]

Then equation (4) reduces to

\[-dP_A = \frac{\beta}{RT} \left( N_A P - N_A P_A - N_B P_A \right) dz \]

\[ \frac{R^2 T^2}{\beta P} \] is the diffusivity of gases and is represented symbolically by \( D_{AB} \).

The following form is realized,

\[-dP_A = \frac{RT}{D_{AB} P} \left( N_A P - N_A P_A - N_B P_A \right) dz \] (6)

Equation (6) is nonlinear with respect to \( P \). Both \( P \) and \( p_A \) vary randomly with respect to environmental conditions (see figure 4). Furthermore, there is an interdependence between \( P \) and \( N_B \). In view of the complex relationships, certain simplifying assumptions will be made in order to derive approximate solutions. The total gas pressure at any point along the flow path will be considered as divided into two parts. One is of constant value and equal to the lower environmental pressure. The other is the difference between the two environmental pressures existing at the point considered. Only the first portion will be assumed to be associated with diffusional flow. The second portion
Where: \( P \) = total pressure
\( P_{N2} \) = partial pressure of \( N_2 \)
\( P_A \) = partial pressure of water vapor

**CASE 1** - EXTERNAL TOTAL PRESSURE \( (P_{EXT}) \) LESS THAN INTERNAL TOTAL PRESSURE \( (P_{INT}) \)

**CASE 2** - EXTERNAL TOTAL PRESSURE AND INTERNAL TOTAL PRESSURE EQUAL

**CASE 3** - EXTERNAL TOTAL PRESSURE \( (P_{EXT}) \) GREATER THAN INTERNAL TOTAL PRESSURE \( (P_{INT}) \)

FIGURE 4 - POSSIBLE TOTAL PRESSURE, \( N_2 \) AND WATER VAPOR PARTIAL PRESSURE DISTRIBUTION ACROSS ROTARY SEAL (SIMULATED BY CAPILLARY TUBES)
of the total gas pressure will be considered to be the driving force that causes a total mass transfer in the direction of decreasing total pressure. Upon this total mass transfer will be superimposed the diffusional flow of the first portion. Under these conditions, the diffusional portion can now be considered as equivalent to equimolal counter diffusion where,

\[ N_A = -N_B \]

The flow rates are equal in magnitude and opposite in sign and equation (6) reduces to

\[ -dP_A = \frac{RT}{D_{AB}} N_A \, dz \quad (7) \]

If steady state conditions are assumed, i.e., \( N_A, D_{AB} \) as well as the concentrations at any point in the gas mixture remain constant with passage of time, then equation (7) becomes solvable,

\[ - \int_{P_{A_1}}^{P_{A_2}} dP_A = \frac{RT N_A}{D_{AB}} \int_{z_1}^{z_2} dz \]

Letting \( z_2 - z_1 = l \) (length of capillaries),

\[ N_A = \frac{D_{AB}}{RT} \left( P_{A_1} - P_{A_2} \right) \quad (8) \]

Where:

\[ P_{A_1} - P_{A_2} = \text{water vapor partial pressure differential across capillary tube.} \]

This equation is known as Fick's Law of gaseous diffusion. The flow depicted by this equation is only relative to the stationary total gas and does not include the water vapor portion of the total gas flow relative
to seal. Assuming $p_B \approx P$ and the total mass is approximately equal to nitrogen mass, the total flow may be represented as

$$N_{\text{total}} = \frac{nd^2g_c}{32\mu_n \ell R_n} P_{AV}(P_1 - P_2) \text{ (see Appendix A)} \quad (8-1)$$

Where:

- subscript "n" denotes nitrogen gas

\[ N_{\text{total}} = \text{Total flow rate} \]
\[ n = \text{number of capillary tubes} \]
\[ d = \text{diameter of capillary tube} \]
\[ g_c = \text{average gravity constant} \]
\[ \mu_n = \text{absolute viscosity} \]
\[ \ell = \text{length of capillary tube} \]
\[ R_n = \text{gas constant} \]
\[ P_1 = \text{total pressure in inside environment} \]
\[ P_2 = \text{total pressure in outside environment} \]
\[ P_{AV} = \left(\frac{P_1 + P_2}{2}\right) \]

Applying the perfect gas laws to both the average total and water vapor partial pressures the following form is derived:

$$N_A = N_{\text{total}} \times \frac{P^*_{AV}}{P^*} \times \left(\alpha\right)$$

Where:

- $P^*_{AV} \approx P_{AV}$
- $P_{AV} = \text{internal water vapor partial pressure}$
\[ P_{A2} = \text{external water vapor partial pressure} \]
\[ P_A^* = \text{average water vapor partial pressure} \]
\[ \approx \left( P_{A1} + P_{A2} \right) / 2 \]
\[ R_v = \text{gas constant for water vapor} \]
\[ \alpha = \text{contains both the ratio of the gas constants and nonlinearity equating factors.} \]

Therefore, from equations (8 and 8-1) the total flow \( (N_{A\text{total}}) \) is

\[ N_{A\text{total}} = \frac{D_{AB}}{RT} \left( P_{A1} - P_{A2} \right) + \frac{n}{2} \frac{gC P_{AV}}{N \mu N_{R N T}} \left( P_1 - P_2 \right) \frac{P_A^*}{P} \]  

(9)

If equation (6) is rearranged and the value of \( P - P_A \) assumed \( P \) as well as \( N_B \approx N_{\text{total}} \), the same type of equation develops

\[ N_A = - \frac{D_{AB}}{RT} \times \frac{dP_A}{dz} + N_{\text{total}} \times \frac{P_A}{P} \]  

(6a)

Equation (9) will be referred to as a modified form of Fick's Law.

External environmental conditions may vary in temperature, barometric pressure, and relative humidity (see figure 3 and Appendix E). Present sealing practice of initially charging instrument from 0.1 to 0.25 psi above the existing external atmospheric pressure, causes the internal pressure to exceed or be less than the external pressure. The actual valve will depend upon conditions existing at the given time. Maximum variations due to combinations of external temperature and pressure are 3.58 psi above to 3.97 psi below the existing external pressure (see Appendix G).

The individual influences of each of the environmental conditions on the moisture flow rate are discussed as follows:
1. Flow Rate Variations Due to Changes in External Barometric Pressure (Temperature and Relative Humidity Remaining Constant)

Assume temperature to be constant at 60° F, 100% R.H. to exist, and charging conditions to be, \( T = 60° F \) and \( P_1 = 14.95 \) psi. Then all factors in equation (9) will remain constant as the external barometric pressure \( (P_Z) \) varies except \( (P_1 - P_2) \). This factor varies from 0.58 psi above to 0.37 psi below existing barometric pressure (see Appendix E). Using above values (case 1 and 3, figure 4) and \( P_1 - P_2 = 0 \) (case 2, figure 4) the moisture flow rates \( (N_A) \) are calculated in Appendix E and are shown graphically in figure 5. The largest flow rates occur when the barometric pressure exceeds the internal pressure. When \( P_1 - P_2 = -0.37 \) psi, there is a 26.1% increase in flow rate above that occurring when no pressure differential exists. Maximum variations in flow rate amount to a flow rate increase of approximately 67%, due to variations in \( P_1 - P_2 \) from (+0.58 psi to -0.37 psi). This flow occurs when the barometric pressure exceeds the internal pressure by 0.37 psi. The influence of pressurizing the instrument chamber beyond that of presently employed sealing techniques was investigated by assuming the total pressure differential \( (P_1 - P_2) \) to be increased to +1 psi. The value of the moisture flow rate \( (N_A) \) was calculated in Appendix E and plotted in figure 5. The results indicate that pressurizing the instrument has a powerful retarding affect on the moisture flow rate. In fact, equation (9) suggests the flow rate can be reduced to a negligible amount if a high enough pressure in counter-direction to the moisture flow can be maintained.

2. Flow Rate Variations Due to Changes in External Temperature (Barometric Pressure and Relative Humidity Remaining Constant)

Assume initial charging conditions to be \( T = 60° F \), internal and external pressures held at 14.7 psi for this case and the relative humidity to remain at 100%. Temperature will cause all factors in equation (9) to vary with the exception of the configuration constants. The temperature limits (-65° F and +165° F) were investigated to establish boundaries (see Appendix E). Calculations indicate that large temperature excursions cause a greatly attenuated moisture flow rate. The influences on moisture flow caused by temperatures between the boundaries were calculated and are shown in tabular form in Appendix H and graphically in figure 6. As can be noted, variations of temperature
Range in variation of external environmental barometric pressures (29.25 – 31.25" Hg)

Where: Initial charging conditions, T = 60°F & P = 14.95 psi and it is assumed that temperature (T) remains constant at 60°F

\[ N_A = \frac{D_{AB}}{R_u T_L} (P_A - P_{A2}) + \alpha \left[ \frac{n a^2 q_0 P_{AV}}{32 \mu \chi R_N T} (P_1 - P_2) \right] \frac{P^\ast}{P_{AV}} \]  

See Appendix E for values to apply and solution to eq (9) for the four values of \( P_1 - P_2 \) considered.

FIGURE 5—PRESSURE EFFECTS ON MOISTURE FLOW DUE TO MECHANISM OF GASEOUS DIFFUSION
Where: Initial charging conditions, $T=60^\circ F$ & $P=14.7$ psi
and it is assumed that external barometric pressure ($B$) remains constant at 14.7 psi (1033.2 gm/cm$^2$).

$$N_A = -\frac{D_{AB}}{RVTA} (p_{A_1} - p_{A_2}) + \alpha \left[ \frac{nd^2gCP_{AV}}{32\mu RA NT} (P_1 - P_2) \right] \frac{P_{AV}}{P}$$  -- eq (9)

See Appendix H for values to apply and solution to eq (9) for the eight values of $T$.

FIGURE 6 - TEMPERATURE EFFECTS ON MOISTURE FLOW DUE TO MECHANISM OF GASEOUS DIFFUSION
from the initial charging temperature of +60° F causes a reducing effect on the moisture flow rate. Figure 6 suggests that as the temperature increases to approximately 120° F the flow ceases.

3. Flow Rate Variations Due to Changes in External Relative Humidity (Barometric Pressure and Temperature Remaining Constant)

Assume initial charging conditions to be \( T = 60° \) F and \( P = 14.95 \) psi. The external barometric pressure to remain constant at 14.7 psi and the temperature to remain constant at 60° F. All factors in equation (9) will remain constant except \( \Delta P_A \) and \( \Delta P_{A_1} - \Delta P_{A_2} \) as the relative humidity varies. The influence of humidity variation is approximately the same on both terms of equation (9). Therefore, the moisture flow rate \( (N_A) \) is directly proportional to the relative humidity. Calculations for various values of relative humidity plus a graphical plot are presented in figure 7.

As noted, the maximum moisture flow rate occurs when the following conditions exist: (1) Constant 100% R. H., (2) Constant temperature of 60° F; (3) charging conditions of \( T = 60° \) F and \( P_1 = 14.95 \) psi and (4) when external pressure exceed internal pressure by 0.37 psi (Case 3). These conditions produced a flow rate \( (N_A) = -8.36 \times 10^{-4} \text{gm/sec-cm}^2 \).

By utilizing the appropriate value of cross sectional area \( (A) \) for the particular choice of seal geometry values used in calculating all the above flow rates, a calculation is made of the worse case flow rate through the seal model \( (N^*_{A}) \) (see Appendix E). This rate is \(-9.6 \times 10^{-11} \text{gm/sec}\). The flow rate is then related to a water vapor pressure increase per week by use of the ideal gas laws assuming a 50 in. \(^3\) instrument volume/seal. The vapor pressure rise \( (\Delta P_A) \) is 0.00152 psi/week. This vapor pressure rise is then converted to an increase in relative humidity per week (at initial charging conditions of \( T = 60° \) F and \( P = 14.95 \) psi). The relative humidity rise is 0.593% R. H. change/week. Therefore, if -10° F dew point were considered the critical moisture condition within the fire control instrument, it would require slightly in excess of 10 weeks to reach this point.

By fixing the environmental conditions to that set of values which produced the largest moisture leakage flow rate, the influence of changes of seal configuration is investigated. Appendix E shows
Where: Initial charging conditions are, \( T = 60^\circ F \), \( P_1 = 14.95 \text{ psi} \) and Barometric pressure assumed to remain constant at 14.7 psi and temperature at 60°F and R.H. to vary

\[
N_A = -\frac{D_{AB}}{R_v T I} (P_{A_1} - P_{A_2}) + \alpha \left[ \frac{\pi d^2 g_c P_A V}{32 \mu L R N T} (P_1 - P_2) \right] \frac{P_A^*}{P_A^*} 
\]

assuming \( \alpha \approx 1 \)

<table>
<thead>
<tr>
<th>R.H., %</th>
<th>( P_{A_1} - P_{A_2} ) g/m²</th>
<th>( P_A^* ) g/m²</th>
<th>1st Term</th>
<th>2nd Term</th>
<th>( N_A ) g/m²/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>18.05</td>
<td>9.025</td>
<td>-6.63x10⁻⁴</td>
<td>+1.66x10⁻³</td>
<td>-5.47x10⁻⁴</td>
</tr>
<tr>
<td>75</td>
<td>13.5375</td>
<td>6.76875</td>
<td>-4.97x10⁻⁴</td>
<td>+0.89x10⁻³</td>
<td>-4.10x10⁻⁴</td>
</tr>
<tr>
<td>50</td>
<td>9.025</td>
<td>4.5125</td>
<td>-3.315x10⁻⁴</td>
<td>+0.58x10⁻³</td>
<td>-2.735x10⁻⁴</td>
</tr>
<tr>
<td>25</td>
<td>4.5125</td>
<td>2.00625</td>
<td>-1.65x10⁻⁴</td>
<td>+0.435x10⁻³</td>
<td>-1.223x10⁻⁴</td>
</tr>
</tbody>
</table>

See Appendix E for values to apply and solution to eq (9)

**FIGURE 7 - EXTERNAL RELATIVE HUMIDITY EFFECTS ON MOISTURE FLOW DUE TO MECHANISM OF GASEOUS DIFFUSION**
that if the length of flow path (l) were to assume its smallest value 
\((3.0 \times 10^{-2}\) cm), the flow rate would increase by a factor of \(4/3\) to a 
new value of \(-12.8 \times 10^{-11}\) gm/sec. Further, had the number of 
capillaries (n) been assumed = 100, fixing \(d = 0.680 \times 10^{-4}\) cm and 
\(A = 3.635 \times 10^{-7}\) cm\(^2\), then \(N_A^k\) would be increased by a factor of 4.58 
to a value of 43.9 \(\times 10^{-11}\) gm/sec. Therefore, it is possible to have 
the flow rates increase by a factor of 6.1 times the rates calculated 
by choosing \(N = 10\) and \(l = 0.04\) cm.

Capillary Action (and Special Case of Viscous Flow Due to 
Negative Total Pressure Differential Across Seal)

Under certain circumstances, the external environment may con-
sist of mass in the form of liquid water as well as gaseous nitrogen 
and water vapor. This section will concern itself with the influences 
on the moisture leakage flow rate due to the presence of the liquid 
water in the external environment. It will be assumed that the water 
comes in contact with the rotary seal. In treating this problem, the 
simplified flow model will be identical to that discussed under gaseous 
diffusion. However, cognizance must now be taken of the fact that the 
flow path geometries are irregular causing the cross sectional area 
at each point along the flow path to change (see figure 8). These 
changes cause several combination of flow mechanisms to take place. 
Their effects are discussed in detail and sample calculations are made 
in Appendix I. A summary review follows:

1. Equations are developed (see Appendix I) equating the various 
forces involved when the shaft and seal are in both the vertical and 
horizontal positions. These equations are:

(vertical position)

\[
h = - \left( \frac{4T \cos \theta}{dw} - \frac{\Delta p}{w} \right)
\]

(10)

and (horizontal position)

\[
4T \cos \theta - d\Delta p = 0
\]

(13)
Internal instrument pressure

Flow path end flaring due to decrease in gasket compression caused by gasket end effects.

Liquid water penetrating flow path capillary

Irregular shape of flow path geometry

External instrument pressure

Liquid water

$\ell$ = Effective length of flowpath as concerns capillary action

$d$ = Effective diameter of flowpath as concerns capillary action

FIGURE 8 - PROBABLE FLOW PATH GEOMETRY ENCOUNTERED IN SIMPLIFIED MODEL OF ROTARY SEAL FOR PURPOSES OF DISCUSSING MECHANISM OF CAPILLARY ACTION
where:

\[ h = \text{rise of liquid water in capillary} \]
\[ T = \text{surface tension} \]
\[ \Delta p = \text{differential pressure across seal, } (P_1 - P_2) \]
\[ w = \text{specific weight of the liquid water} \]

2. For the special case of negative total pressure differentials existing, capillary action cannot be supported and viscous flow of the liquid water through the capillaries takes place according to the following equation:

\[
N_{\text{water}}^* = \frac{w\pi d^4 g_c}{128 \mu_{H_2O} \ell} \Delta p
\]

where:

\[ N_{\text{water}}^* = \text{rate of liquid water flow through a capillary} \]
\[ \mu_{H_2O} = \text{absolute viscosity of liquid water} \]
\[ g_c = \text{gravity constant} \]
\[ d = \text{diameter of capillary} \]
\[ \ell = \text{length of flow path} \]

A calculation is made assuming initial instrument charging conditions \((T = 60^\circ F \text{ and internal pressure } 14.95 \text{ psi})\) remains constant and external barometric pressure varies from 15.32 psi to 14.37 psi. This makes possible a maximum negative \(\Delta p = 0.37 \text{ psi}\). The seal model configuration of \(n = 10, d = 1.214 \times 10^{-4} \text{ cm } (3.98 \times 10^{-6} \text{ ft})\) and \(\ell = 0.0013 \text{ ft}\) is assumed to exist and the appropriate values of \(w\) and \(\mu_{H_2O}\) for a temperature of \(60^\circ F\) are used. \(N_{\text{water}}^* \text{ (per capillary)} = 1.105 \times 10^{-11} \text{ gm/sec}\) which is of the same order of magnitude of flow occurring by mechanism of gaseous diffusion.
3. When positive total pressure differentials exist, capillary action will move the liquid water up the flow path. When shaft and seal are in the vertical position, the water advances to near the end of flow path. The flaring effectively increases the diameter \(d\) and decreases the \(\cos \vartheta\). The capillary rise ceases. Equation (10) describes the magnitude of rise of the water column. Evaporation will then take place and equimolal gaseous diffusion will thereafter be the transport mechanism. The equation to express this flow rate will be similar to equation (8) with a much decreased value of "\(t\)" and a much increased value for "\(d\)"

\[
N_A = -\frac{D_{AB}}{RTt'} (pA_1 - pA_2') \quad (8a)
\]

and

\[
N_A' = -\left[\frac{\pi (d')^2}{4}\right] \frac{D_{AB}}{RTt'} (pA_1 - pA_2') \quad (8b)
\]

where

\(t'\) = reduced length flow path

\(pA_2'\) = saturation vapor pressure

\(d'\) = increased average diameter

Equations (8a and 8b) predict that the flow rate will be directly proportional to the vapor pressure difference and inversely proportional to total internal pressure \(P_1\). Since \(pA_2'\) is fixed at saturation pressure, there is nothing that can be done to lower the value of \(pA_1 - pA_2'\). However, by increasing the internal pressure the flow rate can be reduced. Increasing \(P_1\) will also cause the liquid water level to recede. The receding water level will increase the value of \(t'\) and decrease the value of \(d'\). Therefore, the flow rate reducing power of \(P_1\) will be stronger than a first order effect. A small change in \(P_1\) will cause a significant change in \(t'\). When \(P_1\) is increased slightly, \(t' \rightarrow t\) and \(d' \rightarrow d\). Pressurizing the instrument materially reduces the flow.

---

It is to be remembered that \(D_{AB}\) has \(P\) in the denominator.
Appendix E shows $N_A$ to have a flow rate in the order of $7.62 \times 10^{-11}$ gm/sec. The value of $\Delta p$ to cause this reduced flow can be obtained by setting "h" = 0 in equation (10) and solving for $\Delta p$. Assume $\cos \theta \approx 0$

$$\Delta p = \frac{4T \cos \theta}{d} = \frac{(4)(0.04985 \text{ lb/ft})}{(0.0013 \text{ ft})(144 \text{ in./ft}^2)}$$

$$\Delta p = 1.065 \text{ psi}$$

When the shaft and seal are in the horizontal position and the maximum positive $\Delta p$ (+0.58 psi) prevails, the liquid water will travel through the capillary tube until it almost reaches the end. The flow will continue until the decrease in $\cos \theta$ and increase in "d" makes equation (13) become zero. Evaporation will take place and equimolal gaseous diffusion will be the mechanism for transporting the water vapor through the remaining length of the flow path. From this point on the same arguments hold as concerns the case for positive $\Delta p$ when the shaft and seal are in a vertical position.

**Pumping Action**

The literature search reveals no information concerning pumping action. However, the word rotary seal implies that the seal is to be used in applications in which it will be subjected to angular translation. This motion may have a pumping or sweeping action which helps move the liquid water into the capillaries. It may sweep it completely past the seal. The extent that this mechanism contributes to the total moisture leakage will require experimental verification.

**Interrelation Between the Three Principal Mechanisms**

There is some degree of interrelation between the three principal mechanisms.

Rotary pumping action may be one of the means of causing the liquid water in the external atmosphere to come in contact with the seal. Thereafter, capillary action may take over to raise the column
of liquid water to a fixed height and then, by evaporation, gaseous diffusion will be the final mechanism for transporting the mass to the internal environment.

Rotary pumping action may be combined with the viscous flow (due to negative total pressure differential across seal) thus increasing the flow rate.

Rotary motion may actually sweep part of the gas through the capillaries into the instrument. It may also slow down the rate of total gas exchange out of the instrument thus reducing the influences of the second term of equation (9). It may increase it if the total pressure differential across the seal is negative.

The degree of interrelations is unknown because of the questionable nature of the rotary pumping action.

SUMMARY AND CONCLUSIONS

A theoretical study of rotary sealing systems that meet fire control instrument design limitations indicates that there are three moisture transport mechanisms. These are (1) gaseous diffusion, (2) capillary action (or the special case of viscous flow when negative total pressure differentials exist across the seal, and (3) mechanical pumping action. The flow path geometry and the environmental conditions affect all three mechanisms. The usual procedure for designing a seal is to make the geometry such that it impedes the bulk flow of moisture. In such a design, environmental effects play a minor role. This is accomplished by designing a tight seal and is easily realized in a static seal. However, there are limitations to the tightness of seal that can be tolerated in a rotary system.

Rotary seals are subjected to a range of total gas and water vapor partial differential pressures which are a function of both the initial charging and the environmental conditions. The gas pressures may be either positive or negative. These pressure variations cause "leakage breathing" and are the major problem.
Mathematical expressions describing the flow mechanisms have been developed. Furthermore, a simplified hypothetical flow model was also developed. Using this model and the mathematical relationships, estimates were made of flow rates. The following statements summarize the results:

1. Gaseous diffusion — leakage rate = \(9.6 \times 10^{-11}\) gm/sec. In a 50 in. \(^3\) instrument volume/seal this rate yields a 0.593% relative humidity change/week (average). Worse case analysis increases rate by a factor of 6.1. Temperature variations towards extremes attenuates flow. Increased internal pressure also reduces flow.

2. Liquid flow mechanisms — multifarious in nature.
   a. Negative total pressure differentials — viscous flow applies. Leakage rate = \(11.05 \times 10^{-11}\) gm/sec-capillary.
   b. Positive total pressure differentials — capillary action moves liquid into flow paths at different levels depending on orientation of rotary member and seal. Evaporation takes place and gaseous diffusion ensues. Maximum leakage rates are less than when negative total pressure differentials exist.
   c. Increased internal pressure reduces flow.

3. Rotary seal pumping action — imparts pumping or sweeping action which helps or hinders other flow mechanisms.

It is concluded that positive total pressure differentials must be maintained in fire control instruments. Pressurizing the instrument will have a very strong influence in reducing the flow.

RECOMMENDATIONS

It is recommended that an appropriate experimental program be designed and carried out to verify the moisture flow mechanisms determined in this report.
It is proposed that this experimental program investigate the influences of variations in the following parameters:

a. Total pressure differential across the seal including negative values.

b. Water vapor partial pressure differentials across the seal.

c. Temperature.

d. Mode and frequency of rotation on the leakage rate (mechanical pumping action).

e. Flow path surface conditions.
APPENDIX A

FLOW OF GASES THROUGH CAPILLARY TUBES

If there exists a concentration difference, i.e., a pressure difference, for a gas across a gasketed path, a flow of gas past the gasket will take place. Under ordinary conditions, this is not diffusional flow in the usual sense; yet since it may be described according to the methods of diffusion, it is sometimes so considered. The structure of the gasketed flow path may be simplified. Consider it to be a series of straight capillary tubes of constant diameter \( d \) and length \( L \), reaching from the high pressure to the low pressure side of the gasketed path. At ordinary pressures, the flow of the gas in the capillaries may be streamline or turbulent, depending upon whether the dimensionless Reynolds number \( (d \rho \nu / \mu) \) is below or above 2,100. Where the diameter of the capillaries is small, and the pressure difference and, therefore, the velocity is small, flow will be streamline and as described by Poiseuille's Law for a compressible fluid obeying the perfect gas law

\[
N_N = \frac{d^2 g_c}{32 \mu N T} \left( \frac{PAV_N}{2} \right)^2 (P_{N1} - P_{N2})
\]


\[
P_{AVN} = \frac{P_{N1} + P_{N2}}{2}
\]

Where:

- \( \text{subscript } "N" \) denotes nitrogen gas \( (N_2) \)
- \( N_N \) = flow rate, lbs/sec-ft
- \( g_c \) = average gravity constant, 32.2 ft/sec

\( ^1 \)A gasketed flow path is defined as one characterized by small openings of irregular and changing geometry along length of path.

\( ^2 \)Remembering that for the gasketed rotary seal that the path not only changes as in note 1 but also randomly as a function of time.

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d = diameter, ft
\( \mu_N = \text{absolute viscosity, lbs/ft-sec} \)
\( l = \text{length of capillary, ft} \)
\( R_N = \text{gas constant, ft/}^\circ \text{R} \)
\( T = \text{absolute temperature in } ^\circ \text{R} \)
\( p_{N_1} = \text{high gas pressure, lb/ft}^2 \)
\( p_{N_2} = \text{low gas pressure, lb/ft}^2 \)

This assumes that the entire pressure difference is due to friction in the capillaries and ignores entrance and exit losses and kinetic-energy effects.

If the rate of flow is measured in terms of gas volume \((V)\) of standard density fluid flowing per unit of time \((\theta)\) then

\[
\frac{dV}{d\theta} = \left( N \right) \left( \frac{R_N T}{p_{AV_N}} \right) \left( \frac{\pi d^4}{4} \right) = \frac{\pi d^4 \gamma_c}{128 \mu_N l} \left( p_{N_1} - p_{N_2} \right)
\]

\[
\frac{dV}{d\theta} = \frac{\pi r^4 \gamma_c}{8 \mu_N l} \left( p_{N_1} - p_{N_2} \right)
\]

It is possible to compute average capillary sizes in flow paths across gasketed seals by measurements made in accordance with the above equations.

Under certain conditions, however, a different type of flow will occur. If the capillary diameters are small in comparison with the mean free path of the gas molecules \((\lambda)\), say \( d \leq 0.1 \lambda \), flow will take place by molecular effusion, following Knudsen's Law. For a single straight capillary, this becomes,
\[ N_N = K' \left( \frac{g_C}{2\pi MRT} \right) \frac{d}{\ell} \left( P_{N_1} - P_{N_2} \right) \]

where:

- \( M \) = molecular weight of gas
- \( K' \) = correction factor to take care of reflection of molecule from the capillary wall.

As concerns capillary flow paths past the gasketed seal, flow will probably occur due to both laws: in extremely fine capillaries Knudsen's flow predominates, while in less fine capillaries Poiseuille's law predominates. In any case, the equations show that the rate of flow is proportional to the pressure difference and inversely proportional to length of flow path.

An estimate of the mean free path of molecules can be made from the following relationship:

\[ \lambda = \frac{516.9 \mu}{p} \sqrt{\frac{RT}{2\pi g_C M}} \]

For \( N_2 \) (60° F, \( p \approx 14.7 \text{ psi} \))

\[ \lambda = \frac{(516.9)(11.8 \times 10^{-6} \text{ lb/ft-sec})}{(144)(14.7) \text{ lb/ft}^2} \frac{(55.18 \text{ ft/°R})(520° \text{ R})}{(6.2832)(32, 2 \text{ ft/sec}^2)(28)} \]

\[ \lambda = 6.47 \times 10^{-6} \text{ cm} \]

---

\(^3\) Mass transfer operations by R. E. Treybal, dtd 1955, Wiley.
where:

\[ \lambda = \text{mean free path, cm} \]
\[ \mu = \text{absolute viscosity, lbs/ft-sec} \]
\[ p = \text{pressure, lb/ft}^2 \]
\[ R = \text{gas constant, ft/}^{\circ}\text{R} \]
\[ T = \text{temperature, } ^\circ\text{R} \]
\[ g_c = \text{average gravity constant, } 32.2 \text{ ft/sec}^2 \]
\[ M = \text{molecular weight of gas.} \]
APPENDIX B

DETERMINATION OF AVERAGE LENGTHS OF FLOW PATH (\(t\))
FOR ACTUALLY EMPLOYED GASKETED ROTARY SEALS

The following is an analysis of forces at interface of "O"-rings and shaft employed as rotary seals in present fire control instruments. Two "O"-rings will be considered, 1/4 in. and 2 in. I.D. From this analysis an estimate of average lengths of flow path (\(t\)) will be determined.

There are two forces which seat "O"-ring to shaft, the summation of which (normal force) when multiplied by the coefficient of friction (at the interface of the "O"-ring and shaft) and the lever arm (radius of shaft) constitute the torque necessary to rotate shaft, i.e.,

\[ T = (\Sigma F_1 + \Sigma F_2) \mu r \]

where:

\( F_1 \) = Force/unit contact area caused by stretch of "O"-ring when shaft inserted.

\( F_2 \) = Force/unit contact area caused by squeeze of "O"-ring by gland.

\( \mu \) = Effective coefficient of friction at interface of "O"-ring I.D. and shaft.

\( r \) = Radius of shaft.

\( T \) = Torque.

In general:

**Determination of \( \Sigma F_1 \)**

When "O"-ring is stretched on shaft it is analogous to a ring shrunk on a shaft or to the hoop stresses created in a pressure vessel and is treated in a similar manner.°

*For simplicity of analysis, correction for slight distortion in cross sectional area will be neglected (except as it influences the area of contact),
For purposes of analysis the cross sectional width of "O"-ring will be considered small in comparison with I.D., i.e., the "O"-ring will be considered a thin ring with large diameter or essentially a continuous rod.

The "O"-ring may be considered split into two halves with one half exerting the force (P) at each of the cross sections A and B. Counterbalancing this force is the force/unit area (w) exerted by the shaft on the interface area of contact. Summing the vertical forces:

$$2P = 2 \int_0^{\pi/2} (S \times t) w \cos \theta$$

where:

- $S = \text{differential area length of contact}$
- $t = \text{thickness of contact area}$
- $S \times t = \text{differential area of contact}$
\[ 2P = 2 \int_{0}^{\pi/2} (r \times d\theta \times t) \cos \theta \cos \theta \]

\[ 2P = 2rtw \int_{0}^{\pi/2} \cos \theta \, d\theta \]

\[ 2P = 2rtw \left[ \sin \theta \right]_{0}^{\pi/2} \]

\[ P = rtw \]

but

\[ P = S_t A \]

and

\[ S_t = (E_t) \frac{\Delta D}{D} \]

Where:

- \( S_t \) = tensile stress in "O"-ring
- \( \Delta D \) = increase of "O"-ring I.D. or amount of stretch
- \( D \) = "O"-ring I.D.
- \( E_t \) = "O"-ring modulus of elasticity for tension
- \( A \) = "O"-ring cross sectional area, \( \frac{\pi}{4} t^2 \)
- \( t \) = width of "O"-ring

therefore,

\[ r tw = E_t \times \frac{\Delta D}{D} \times A \]
or
\[ w = \frac{E_t \Delta D}{r \frac{r}{D}} \times A \]

\[ \Sigma F_1 = (w)(2\pi r \times l) \]

where:
\[ r = \frac{1}{2} D \]

\[ \Sigma F_1 = 2\pi A \left( E_t \times \frac{\Delta D}{D} \right) \]

**Determination of \( \Sigma F_2 \)**

When "O"-ring is squeezed by the gland there is developed a compressional stress,

\[ S_c = E_c \frac{\Delta t}{t} \]

where

\[ E_c = "O"-ring modulus of elasticity for compression \]

\[ S_c = \text{compressional stress in } "O"-ring \]

\[ \Delta t = \text{decrease in } t \text{ or amount of squeeze} \]

\[ \Sigma F_2 = S_c \left( 2\pi \times r \times l \right) \]

\[ \Sigma F_2 = 2\pi r \times l \left( E_c \times \frac{\Delta t}{t} \right) \]

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Determination of $T$

$$T = 2\pi \left[ AE_t \frac{AD}{D} + r \times t \times E_c \times \frac{At}{t} \right]$$

For Garlock 1/4 Inch Dia. Buna "N" Rubber "O"-ring

$r = 0.125$ in.
$t = 0.070$ in.
$E_c = 4000$ psi (estimated)*
$E_t = 3000$ psi (estimated)*
$\mu = 0.16*$

$\ell = 0.012$ in. or 0.0305 cm

$D = 0.250$ in.
$A = 0.7854(0.070 \text{ in.})^2 = 0.00385 \text{ in.}^2$
$\Delta D = 0.009$ in.**
$\Delta t = 0.0145$ in.***

*Recognizing that both $E_c$ and $E_t$ are not constant but will vary under different stretched or compressed conditions and that $\mu$ will increase under increased normal force.

**Probable average stretch of manufacturer's allowable 0.003 in. to 0.015 in.

***Probable average compression of manufacturer's allowable 0.015 in. to 0.0185 in.
\[ T = (6.2814) \left( \frac{0.00385}{0.250} \right) (3000)(0.009) + \]
\[ \left( \frac{0.125}{0.012} \right) \left( \frac{4000}{0.0145} \right) (0.16)(0.125) \]

\[ T = (6.28)(0.416 + 1.242)(0.16)(0.125) \]
\[ T = 1.958 \text{ lb-in. or } T = 3.13 \text{ oz-in.} \]

For Garlock 2 Inch Dia. Buna "N" Rubber "O"-ring

- \( r = 1.000 \text{ in.} \)
- \( t = 0.200 \text{ in.} \)
- \( E_c = 4000 \text{ psi} \)
- \( E_t = 3000 \text{ psi} \)
- \( \mu = 0.3 \)

- \( \ell = 0.020 \text{ or } 0.0508 \text{ cm} \)

- \( D = 2.000 \text{ in.} \)

- \( A = (0.7854)(0.2 \text{ in.})^2 = 0.0314 \text{ in.}^2 \)

- \( \Delta D = 0.022 \text{ in.} * \)

- \( \Delta t = 0.024 \text{ in.} ** \)

*Probable average stretch of manufacturer's allowable 0.011 in. to 0.033 in.

**Probable average compression of manufacturer's allowable 0.018 in. to 0.030 in.
\[ T = 6.28 \left[ \frac{(0.0314)(3000)(0.022)}{(2.000)} \right] + \left[ \frac{(1.000)(0.020)(4000)(0.024)}{(0.200)} \right] (0.3)(1.000) \]

\[ T = (6.28)(0.3)(1.0362 + 9.60) \]

\[ T = 20.5 \text{ lb-in.} \]

The above values of torque agree within 5% with those determined experimentally. Other rotary seals thusly calculated also show similar agreement. The length of flow paths \( l \) appear to range from 0.030 cm to 0.060 cm.
APPENDIX C

FLOW OF GASES THROUGH ANNULAR SPACES

In the total net gas exchange a problem presents itself whereby a choice must be made of approximating the real geometrical shape of the flow paths past rotary gasketed seals with simplified constant geometrical shape(s). This is done for the purpose of estimating width and cross sectional area. In Appendix A a choice was made to treat these shapes as straight capillary tubes of constant diameter (d) and length (ℓ). In this Appendix the flow path shape will be treated as having annular cross sections (see figure 2) but different than actual flow paths in that the cross section is constant along the length of flow path with thickness (t) of both annular spaces being the same.

Because $D_1$ or $D_2 >> t$ we can consider the flow as occurring between parallel plates a width (t) apart, $\pi \left[ D_1 + D_2 \right]$ long and $t$ deep.
If a force balance on a volume of flow path $2x$ wide, $\pi(D_1 + D_2)$ high and $t$ deep is considered, the following is obtained:

$$P \left[ \pi(D_1 + D_2) \right] 2x - (p + dp) \left[ \pi(D_1 + D_2) \right] 2x - \pi(D_1 + D_2) t 2\tau = 0$$

$$-xdp - t\tau = 0$$

$$\tau = -\frac{xdp}{t} \quad (a)$$

A relationship between shear stress ($\tau$) and velocity distribution ($\mu$) must be obtained if flow rate is to be determined. This is possible by considering the relationship which applies to a thin fluid film of uniform thickness $h$, between two concentric cylinders, one fixed and one having a peripheral velocity $v$.

Now,

$$\tau = \frac{\mu}{g_c} \frac{v}{h}$$

but

$$\frac{\partial u}{\partial y} = \frac{v}{h}$$

where $\frac{\partial u}{\partial y}$ is the rate of change of velocity with distance from stationary surface.

In this instance, although the shear stress is not constant, it is given by the following equation
\[ \tau = \frac{\mu}{\delta c} \frac{\partial u}{\partial y} = - \frac{\mu}{\delta c} \frac{\partial u}{\partial x} \] (b)

where:

\[ u = \text{velocity} \]

\[ y = t' - x \]

Equating equation (a) and (b) we obtain,

\[ -\int_{u=0}^{u=u} \frac{\partial u}{\partial x} \, dx = \frac{\Delta p g c}{\mu} \int_{0}^{t'} x \, dx \]

\[ u = \frac{\Delta p g c}{2 \mu t'} t'^2 \left[ 1 - \left( \frac{x}{t'} \right)^2 \right] \] (c)

The volume flow rate \( \left( \frac{dv}{d\theta} \right) \) follows:

\[ \frac{dv}{d\theta} = 2 \left[ \int_{0}^{t'} ud \left[ \pi (D_1 + D_2) x \right] \right] \]

\[ \frac{dv}{d\theta} = 2\pi (D_1 + D_2) \int_{0}^{t'} u \, dx = 2\pi (D_1 + D_2) t' \int_{0}^{t'} u \left( \frac{x}{t'} \right) \]
\[
\frac{dv}{d\theta} = 2\pi (D_1 + D_2) \left( \frac{\Delta p g_c}{2\mu t} \right) t^3 \int_0^t \left[ 1 - \left( \frac{x}{t'} \right)^2 \right] d\left( \frac{x}{t'} \right)
\]

\[
\frac{dv}{d\theta} = \frac{2}{3} \frac{\pi (D_1 + D_2) t^3 g_c}{\mu} \Delta p
\]

since
\[t' = \frac{1}{2} t\]

\[
\frac{dv}{d\theta} = \frac{1}{12} \frac{\pi (D_1 + D_2) t^3 g_c \Delta p}{\mu}
\]
APPENDIX D

SAMPLE CALCULATIONS OF DIAMETER (d) AND CROSS SECTIONAL AREA (A) OF A STRAIGHT CAPILLARY TUBE ASSUMING KNUDSEN'S LAW TO APPLY

Knudsen's Law → \[ N_N = K' \left( \sqrt{\frac{g_c}{2\pi MR_N T}} \right) \frac{d}{l} (\Delta p) \]

Where:

- \( N_N \) = Flow rate, gm moles/sec-cm²
- \( K' \) = Correction factor to take care of reflection of molecule from capillary wall.
- \( g_c \) = Gravity constant, 980 cm/sec²
- \( M \) = Molecular weight, gm/gm mole, for \( N_2 = 28 \)
- \( R_N \) = Universal gas constant, 84,780 gm(cm)/gm mole (*K)
- \( T \) = abs. temp, °K
- \( d \) = Diameter of capillary, cm
- \( l \) = Length of capillary, cm
- \( \Delta p \) = Total pressure difference, gm/cm²

\[ \frac{dv}{d\theta} = \left( \frac{N_N}{PAV} \right) \left( \frac{R_N T}{PAV} \right) \left( \frac{\pi d^2}{4} \right) \]

where:

- \( PAV \) = Total pressure of standard density fluid, 1033.2 gm/cm²

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\[
\frac{dv}{d\theta} = \frac{K'}{P_{AV}} \left( \sqrt{\frac{\frac{R \cdot N \cdot g_c}{2\pi M}}{2\pi M}} \right) \frac{\pi d^3}{4l} (\Delta p)
\]

For \( N_2 \) gas,

\[ M = 28 \]

for

\[ \Delta p = 70.2 \text{ gms/cm}^2 \]

\( K' \) assumed less than 1

\[ l = 0.0397 \text{ cm} \]

\[ \frac{dv}{d\theta} = 5.20 \times 10^{-7} \text{ cm}^3/\text{sec} \]

Using above values we obtain,

\[ 5.20 \times 10^{-7} \text{ cm}^3/\text{sec} = \left( \frac{K'}{1033.2 \text{ gms/cm}^2} \right) \left( \frac{\pi d^3}{(4)(0.0397 \text{ cm})} \right) (70.7 \text{ gms/cm}^2) \]

\[ \left( \sqrt{\frac{84.780 \text{ gm/cm}}{\text{gm-mole} (\text{K})}} \left( \frac{520^\circ \text{K}}{1.8} \right) \left( \frac{980 \text{ cm}}{\text{sec}^2} \right) \right) \]

\[ (2)(3.1416) 28 \text{ gms/gm-mole} \]

\[ 5.20 \times 10^{-7} \text{ cm}^3 = \frac{(3.1416)(70.2)(K')d^3}{(1033.2)(4)(0.0397)} \sqrt{\frac{(84.780)(520)(980)}{(6,2832)(28)(1.8)}} \]

\[ 5.20 \times 10^{-7} \text{ cm}^3 = 1.344K'd^3 \sqrt{1.364 \times 10^8} \]

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\[ d^3 = \frac{(5.20 \times 10^{-7}) \text{ cm}^3}{(1.344)(1.168 \times 10^4) K'} \]

\[ d = \frac{3.215}{\sqrt[3]{K'}} 10^{-4} \text{ cm} \]

if \( K' = .8 \), \( \sqrt[3]{K'} = 0.935 \)

\[ d = \frac{3.215 \times 10^{-4}}{0.935} \text{ cm} \]

\[ d = 3.44 \times 10^{-4} \text{ cm} \]

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SAMPLE CALCULATIONS OF MOISTURE FLOW RATES DUE TO MECHANISM OF GASEOUS DIFFUSION

The modified form of Fick's Law of gaseous diffusion as developed in the body of the report,

\[ N_A = - \frac{D_{AB}}{RVT} \left( P_{A_1} - P_{A_2} \right) + \alpha \left[ \frac{nd^2 g_c P_{AV}}{32 \mu l RNT} \left( P_1 - P_2 \right) \right] \frac{P_A}{P_{AV}} \]  

(9)

Where:

\[ P_A^* = \frac{P_{A_1} + P_{A_2}}{2} \]

will be employed to estimate probable moisture flows to be experienced by existing actual rotary seal systems. For simplicity we will assume a constant temperature of 60° F (520° R or 288.8° K) and a standard atmospheric pressure of 14.696 psi (1033.2 gm/cm²). In Appendix F, an average value of \( D_{AB} \) was found to be \( 0.000216 \text{ ft}^2/\text{sec} \) (0.0001755 gm/cm-sec). The worse humidity conditions will be assumed to prevail such that \( P_{A_1} = 0.2563 \text{ psi (18.05 gm/cm}^2) \text{) — 100% R.H.} \) and \( P_{A_2} = 0.000 \text{ psi (0.000 gm/cm}^2) \text{) — 0% R.H.} \). The other values are as follows:

\( (N_2)_\mu = 11.8 \times 10^{-6} \text{ lb/ft-sec (0.0001755 gm/cm-sec}) \)

\( R_V = \frac{84,780 \text{ gm-cm/gm-mole} \cdot ^\circ \text{K}}{18 \text{ gm/gm-mole}} = 4710 \text{ cm/}^\circ \text{K} \)

\( R_N = \frac{84,780 \text{ gm-cm/gm-mole} \cdot ^\circ \text{K}}{28 \text{ gm/gm-mole}} = 3030 \text{ cm/}^\circ \text{K} \)
\[ gc = 980 \text{ cm/sec}^2 \]

\[ P_{AV} = 1033.2 \text{ gm/cm}^2 \]

Seal configuration values of length \( l \), diameter \( d \) and cross sectional area \( A \) will be considered fixed for the first approximation and the environmental effects investigated. Then the environmental effects will be fixed at some set of values and the seal geometry configuration effects investigated. For the first approximation, the following seal configuration values will be assumed:

\[ n = 10 \text{ capillary tubes} \]
\[ d = 1.214 \times 10^{-4} \text{ cm} \]
\[ A = 1.149 \times 10^{-7} \text{ cm}^2 \]
\[ l = 4.00 \times 10^{-2} \text{ cm (1/64 in.)} \]

Substituting above values in equation (9),

\[
N_A = - \frac{(2.00 \text{ cm}^2/\text{sec})(18.05 \text{ gm/cm}^2)}{(4710 \text{ cm/}^\circ\text{K})(288.8^\circ\text{K})(4.0 \times 10^{-2} \text{ cm})} + \alpha \left( \frac{9.025 \text{ gm/cm}^2}{1033.2 \text{ gm/cm}^2} \right) 
\]

\[
\left[ (10)[(1.214)^2(10^{-8} \text{ cm}^2)](980 \text{ cm/sec}^2)(1033.2 \text{ gm/cm}^2)(P_1 - P_2) \right] 
\]

\[
\left[ (32)(0.0001755 \text{ gm/cm-sec})(4.0 \times 10^{-2} \text{ cm})(3030 \text{ cm/}^\circ\text{K})(288.8^\circ\text{K}) \right] 
\]

\[
N_A = - \frac{2.00(18.05) \text{ gm/sec-cm}^2}{(4710)(288.8)(4.0 \times 10^{-2})} + \alpha \frac{(9.025)(1.214)^2(10^{-7})(980)(P_1 - P_2) \text{ gm/sec-cm}^2}{(32)(0.0001755)(4.0 \times 10^{-2})(3030)(288.8)} 
\]
\[ N_A = -6.63 \times 10^{-4} \text{ gm/sec-cm}^2 + \]
\[ (\alpha) 6.63 \times 10^{-6} (P_1 - P_2) \text{ gm/sec-cm}^2 \]

Now, since under ambient conditions (see figure 3) the temperature may vary from -65° F to +165° F and the barometric pressure from approximately 29.25 - 31.25 in. Hg (≈ 1 psi) and with the present sealing technique practice of charging instrument from 0.1 to 0.25 psi above ambient atmospheric pressure existing at time of charging, the possibility exists that the total internal instrument pressure at any given time may either exceed or be less than the external atmospheric pressure depending on the actual ambient conditions existing at the particular given time. As noted in Appendix G, this difference could range from 3.58 psi above to 3.97 psi below the existing external pressure. Therefore, due to these pressure variations, the second term of equation (9) can assume either positive or negative values and also vary in magnitude. However, it must be remembered that the wide range of pressure differences are caused mainly by the large temperature variations from the assumed charging temperature of 60° F. These temperature variations not only affect internal instrument pressures but also the viscosity (\( \mu \)) of the nitrogen gas, the values of \( D_{AB}, P_A^* \) and \( (P_{A1} - P_{A2}) \). These accumulative effects, particularly changes in \( p_A^* \) and \( (P_{A1} - P_{A2}) \), tend to more strongly affect the magnitudes of both terms (hence the total moisture flow rate) than does the change in the magnitude of the second term due to the total pressure variation. For example, a large drop in temperature will make the second term become negative, and the \( P_1 - P_2 \) plus \( \mu \) effects will tend to increase its magnitude; however, the decrease in \( p_A^* \) will overcome these effects and cause the magnitude of the second term to actually decrease. Further, changes in \( D_{AB} \) and \( P_{A1} - P_{A2} \) will cause the first term to also decrease. Therefore, the decrease in magnitudes of both of these terms could cause an actual decrease in flow rate even though the second term becomes additive. Thus, for initial analysis only the effect of varying external barometric pressure will be considered when no change in temperature occurs. Under this condition, the internal pressure may vary from 0.58 psi above to 0.37 psi below the possible existing external barometric pressure. Using above values, the \( P_1 - P_2 \) affects on moisture flow rate will be investigated.
(Case 1)\(^1\)

When,

\[ P_1 - P_2 = +0.58 \text{ psi (40.7 gms/cm}^2) \]

where:

\( P_1 = \text{total pressure inside environment, gms/cm}^2 \)

\( P_2 = \text{total pressure outside environment, gms/cm}^2 \)

The moisture flow rate becomes,

\[
N_A = \left[ -6.63 \times 10^{-4} + (x)(6.63 \times 10^{-6})(40.7) \right] \text{gms/sec-cm}^2
\]

Assuming \( x \approx 1 \),

\[ N_A = -3.93 \times 10^{-4} \text{ gms/sec-cm}^2 \]

(Case 2)\(^1\)

When,

\[ P_1 - P_2 = 0 \]

The moisture flow rate becomes,

\[
N_A = \left[ -6.63 \times 10^{-4} + (x) \left(6.53 \times 10^{-7}\right)(0) \right] \text{gms/sec-cm}^2
\]

Assuming \( x \approx 1 \),

\[ N_A = -6.63 \times 10^{-4} \text{ gms/sec-cm}^2 \]

\(^1\)See figure 4.
When, 
\[ P_1 - P_2 = -0.37 \text{ psi} (26.0 \text{ gm/cm}^2) \]

The moisture flow rate becomes,
\[ N_A = \left[ -6.63 \times 10^{-4} - (\alpha) (6.63 \times 10^{-6}) (26.0) \right] \text{ gms/sec-cm}^2 \]

\[ N_A = \left[ -6.63 \times 10^{-4} - (\alpha) (1.73 \times 10^{-4}) \right] \text{ gms/sec-cm}^2 \]

Assuming \( \alpha \approx 1 \)
\[ N_A = -8.36 \times 10^{-4} \text{ gm/sec-cm}^2 \]

It is obvious from the above that the worse conditions prevail when the outside barometric pressure exceeds internal total pressure, there being a 26.1% increase in flow rate above that occurring when no pressure differential exists. Leakage breathing effects as concerns only changes in barometric pressure, with no temperature changes occurring, amount to a moisture flow rate variations of approximately 67% with the greater flow occurring when the external barometric pressure exceeds the internal total pressure.

Prior to making calculations concerning affects of large temperature changes, the effect of pressurizing the instrument above that of presently employed sealing techniques will be investigated. It will be assumed that the pressure differential is now increased to +1 psi (+70.2 gm/cm²).

The moisture flow rate then becomes,
\[ N_A = \left[ -6.63 \times 10^{-4} + (\alpha) (6.63 \times 10^{-7}) (70.2) \right] \text{ gms/sec-cm}^2 \]

\[ N_A = \left[ -6.63 \times 10^{-4} + (\alpha) (4.66 \times 10^{-4}) \right] \text{ gms/sec-cm}^2 \]

\(^1\text{See figure 4.}\)
Assuming \( \alpha \approx 1 \),

\[
N_A = -1.97 \times 10^{-4} \text{ gms/sec-cm}^2
\]

Therefore, pressurizing the instrument appears to have a powerful affect in reducing the effective moisture flow rate. In fact, equation (9) suggests that it can be stopped completely if a high enough pressure differential in counter-direction to the moisture flow were maintained. It is believed, however, that the retarding effect of the second term of equation (9) will not be as strong as it appears at first glance. This is so because as the pressure increases, the value of the viscosity \( (\mu) \) will, although slowly, also increase. Also, the shape factor \( (\alpha) \) may have a slight pressure dependancy and thus influence the second factor.

Two sample calculations have been made concerning large temperature variations, one at \(-65^\circ F\) (ignoring the fact that the water vapor will be in solid form) and one at \(+165^\circ F\). They are as follows:

1. At \(-65^\circ F\) (219.4 K)

\[
D_{AB} = (2.00)(219.4/288.8)^{1.5} = (2.00)(.76)^{1.5} = (2.00)(.663) = 1.33 \text{ cm}^2/\text{sec}
\]

\[
P_{A1} - P_{A2} < 0.0020 \text{ psi} (0.141 \text{ gm/cm}^2)
\]

(Due to low temperature exact quantitative value unavailable)

\[
P_A^* > 0.0705 \text{ gm/cm}^2
\]

\[
\mu = 9.8 \times 10^{-6} \text{ lb/ft-sec} (0.000146 \text{ gm/cm-sec})
\]

\[
P_1 - P_2 = -3.97 \text{ psi} (-279 \text{ gm/cm}^2) \text{ (see Appendix G)}
\]
Therefore,

\[ N_{A}^{(2)} = - \frac{(1.33)(0.141)}{(4710)(219.4)(4.00 \times 10^{-2})} - \frac{(0.0705)(1.473)(10^{-7})(980)(279)}{(32)(0.000146)(4.0 \times 10^{-2})(3030)(219.4)} \]

\[ N_{A} = -0.0453 \times 10^{-4} \text{ gm/sec-cm}^2 - 0.2280 \times 10^{-4} \text{ gm/sec-cm}^2 \]

\[ N_{A} = -0.2733 \text{ gm/sec-cm}^2 \]

Therefore, the effect of large drops in temperature (as previously estimated) is to reduce the moisture flow rate.

2. At +165° F (347° K)

\[ P_{A1} - P_{A2} > 5.000 \text{ psi (351 gm/cm}^2) \]

(Due to high temperature exact quantitative value unavailable)

\[ P_{A}^{*} > 175.5 \text{ gm/cm}^2 \]

\[ \mu \approx 14.90 \times 10^{-6} \text{ lb/ft-sec (0.000221 gm/cm-sec)} \]

\[ P_{1} - P_{2} = +3.58 \text{ psi (+251 gm/cm}^2) \] (see Appendix G)

Assuming \( \alpha \approx 1 \)
$D_{AB} = (2.00) (351/288.8)^{1.5} = (2.00)(1.215)^{1.5}$

$= (2.00)(1.339) = 2.678 \text{ cm}^2/\text{sec}$

Therefore,

$N_A = -\frac{(2.678)(351)}{(4710)(347)(4.0 \times 10^{-2})} + \frac{(175.5)(1.473)(10^{-7})(980)(251)}{(32)(0.000221)(4.0 \times 10^{-2})(3030)(347)}$

$N_A = -143.5 \times 10^{-4} \text{ gm/sec-cm}^2 + 214.0 \times 10^{-4} \text{ gm/sec-cm}^2$

$N_A = 0 \text{ gm/sec-cm}^2$

Therefore, the effect of large rises in temperature is to reduce the moisture flow rate.

The condition producing the maximum rate of moisture flow/unit area has been investigated as concerns flow through actual rotary sealing systems using the cross sectional area under the first approximation condition, i.e., $A = 1.149 \times 10^{-7} \text{ cm}^2$. The investigation has produced the following results:

$N_A \times A = (-8.36 \times 10^{-4} \text{ gm/sec-cm}^2)(1.149 \times 10^{-7} \text{ cm}^2)$

$N_A^{(3)} = -9.6 \times 10^{-11} \text{ gm/sec} = -2.12 \times 10^{-13} \text{ lb/sec}$

$N_A^{*} = -(2.12)(10^{-13}) \text{ lb/sec} \times (3600) \text{ sec/hr} = -7.63 \times 10^{-10} \text{ lb/hr}$

$\frac{3}{3} N_A^{*} = \text{ gm/sec} = N_A/\text{seal}$
By introducing the ideal gas law \( p_AV = w_ARVT \) and assuming for engineering estimate purposes that one seal serves a volume of about 50 in.\(^3\), then \( V = \frac{50}{1728} \text{ ft}^3 \) and the vapor pressure change/week,

\[
\Delta p_A = \frac{w_ARVT}{V} = \left( \frac{1.28 \times 10^{-7} \text{ lb/week}}{50 \text{ ft}^3} \right) \left( \frac{85.80 \text{ ft}^3}{520^\circ \text{R}} \right) \left( \frac{1728}{1 \text{ week}} \right)
\]

\[
= 2.19 \times 10^{-1} \text{ lb/ft}^2\text{-wk}
\]

\[
\Delta p_A = 1.52 \times 10^{-3} \text{ psi/week}
\]

\[
\Delta p_A = 0.00152 \text{ psi/week}
\]

or

\[
\% \text{ R.H. change} = \frac{\Delta p_A}{p_{AS}} \times 100
\]

where:

\[
p_{AS} = \text{saturation pressure}
\]

\[
\% \text{ R.H. change} = \frac{0.1520}{0.2563} = 0.593\% \text{ R.H. change/week}
\]

Thus, under maximum external total barometric pressure conditions and 100\% R.H. for actual rotary sealing systems at the temperature of initial charging (worse possible combination of conditions), each seal (for a volume of 50 in.\(^3\)) will have a moisture leakage causing a 0.593\% R.H. increase per week.

The environmental effects have been fixed at a set of values and the seal configuration effects investigated.
Effects of Change in Length

Range of lengths 3.0 - 6.0 x 10^{-2} cm. If length were actually only 3.0 x 10^{-2} cm in lieu of 4 x 10^{-2} cm, then flow rate would increase by a factor of 4/3 or \( N_A = -8.36 \times 10^{-4} \text{ gm/sec-cm}^2 \times 4/3 = -11.15 \text{ gm/sec-cm}^2 \).

Effect of Change in Assuming Number of Capillary Tubes

Had \( n \) been assumed to be 100, then \( A = 3.635 \times 10^{-7} \text{ cm}^2 \) and \( d = 0.680 \times 10^{-4} \text{ cm} \) and,

\[
N_A^* = AN_A = (3.635 \times 10^{-7} \text{ cm}^2) \left[ -6.63 \times 10^{-4} - \frac{(10)}{\sqrt{10}} 1.73 \times 10^{-4} \right] \text{ gm/sec-cm}^2
\]

\[
N_A^* = (3.635 \times 10^{-7} \text{ cm}^2)(-12.10 \times 10^{-4} \text{ gm/sec-cm}^2)
\]

\[
N_A^* = -43.9 \times 10^{-11} \text{ gm/sec}
\]

which is slightly over 4 times larger than assuming \( n = 10 \) capillary tubes.

Therefore, it is possible to have flow rates increase by a factor of \( 4/3 \times 43.9/9.6 \) or 6.1 above the rates calculated by choosing \( n = 10 \) and \( \ell = 0.04 \text{ cm} \), if \( n \) were actually = 100 and \( \ell \) were = 0.03 cm.
APPENDIX F

DETERMINATION OF DIFFUSION CONSTANT FOR H₂O VAPOR IN N₂

Expressions for estimating $D_{AB}$ in the absence of experimental data are ordinarily based on considerations of the kinetic theory. Hirschfelder, Bird and Spotz¹ have summarized what appears to be the best method in the following equation

$$D_{AB} = 0.0009292T^{1.5} \left( \frac{1}{M_A + M_B} \right)^{5} \frac{1}{P(r_{AB})^2 \left[ f(KT/\epsilon_{AB}) \right]}$$

where

$D_{AB} =$ diffusivity, cm²/sec

$T =$ abs. temp., °K (288.8° K)

$M_A, M_B =$ molecular weight of A and B, respectively

$P =$ absolute pressure, atm. (1.0 atm.)

$r_{AB} =$ molecular separation at collision, Å

$= (r_A + r_B)Z$

$\epsilon_{AB} =$ energy of molecular interaction, ergs

$= \sqrt{\epsilon_A \epsilon_B}$

$K =$ Boltzmann constant = 1.38 x 10^{-6} ergs/°K

\[ f(KT/\epsilon) = \] a collision function given in chart, Figures 2 & 3\(^{(2)}\)

\[ T_{\text{crit.}} = \text{critical temperature, } ^\circ\text{K} \]

\[ v_{\text{crit.}} = \text{critical volume, cm}^3/\text{gm mole} \]

For H\(_2\)O vapor and N\(_2\), the above take on the following values\(^2\):

\(\text{(H}_2\text{O vapor) } \epsilon/K = 0.75 \quad T_{\text{crit.}} = (0.75)(647.2^\circ\text{K}) = 485^\circ\text{K} \)

\(\text{(N}_2\text{) } \epsilon/K = 0.75 \quad T_{\text{crit.}} = (0.75)(126.1^\circ\text{K}) = 94.6^\circ\text{K} \)

\(\text{(N}_2\text{) } r_B = 0.835 \quad v_{\text{crit.}}^{1/3} = 0.835 \left(\frac{28}{0.311}\right)^{1/3} \)

\[ = 3.681 \text{ cm}^3/\text{gm-mole} \]

\(\text{(H}_2\text{O vapor) } r_A = 0.835 \quad v_{\text{crit.}}^{1/3} = 0.835 \left(\frac{18}{4}\right)^{1/3} \)

\[ = 2.98 \text{ cm}^3/\text{gm-mole} \]

\[ \frac{\epsilon_{AB}}{K} = \sqrt{\frac{\epsilon_{\text{H}_2\text{O}}}{K} \times \frac{\epsilon_{\text{N}_2}}{K}} = \sqrt{(485) 94.6 (\circ\text{K})^2} = 214^\circ\text{K} \]

\[ \frac{KT}{\epsilon_{AB}} = \frac{288.8^\circ\text{K}}{214^\circ\text{K}} = 1.345 \]

\[ f \left( \frac{KT}{\epsilon_{AB}} \right) = 0.620 \]

\[ r_{AB} = \frac{r_A + r_B}{2} = \frac{2.98 + 3.681}{2} \text{ cm}^3/\text{gm-mole} \]

\[ = 3.3305 \text{ cm}^3/\text{gm-mole} \]

Substituting in equation (e),

\[
D_{AB} = \frac{(0.0009292)(288.8)^{1.5}(1/18 +1/28)^{5}}{(1.00)(3.3305)^{2}(0.62)}
\]

\[
D_{AB} = 0.196 \text{ cm}^2/\text{sec}
\]

According to Gilliland\textsuperscript{3}

\[
D_{AB} = 0.0043 (T)^{1.5} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}
\]

\[
P \left( \frac{v_A^{1/3} + v_B^{1/3}}{2} \right)^2
\]

Where:

\[
D_{AB} = \text{diffusivity, cm}^2/\text{sec}
\]

\[
T = \text{abs. temp.,} \ ^\circ\text{K}
\]

\[
M_A, M_B = \text{molecular weights of A and B}
\]

\[
P = \text{total pressure, atm}
\]

\[
V_A, V_B = \text{molecular volumes of A and B, cm}^3/\text{gm-mole}
\]

\[
D_{AB} = \frac{(0.0043)(288.8)^{1.5} \sqrt{1/18 + 1/28}}{(1.000)((21.0)^{1/3} + (21.7)^{1/3})^2}
\]

\[
D_{AB} = 0.2035 \text{ cm}^2/\text{sec}
\]

Therefore, a good average value of $D_{AB}$,

$$D_{AB} \approx 2,000 \text{ cm}^2/\text{sec}$$
APPENDIX G

CALCULATIONS TO DETERMINE RANGE OF INTERNAL INSTRUMENT TOTAL PRESSURE DUE TO CHANGES IN EXTERNAL ENVIRONMENTAL CONDITIONS

If it is assumed instrument is initially charged at 0.25 psi when standard atmospheric conditions ($T = 60^\circ F$, $B = 14.7$ psi) exist, then variations in external environmental conditions of $T = -65^\circ$ to $+125^\circ$ F and $B = 29.25$ in. to 31.25 in. Hg will cause internal total pressure variations as follows:

Ideal gas law $PV = wRT$

where:

- $P = p + B =$ total pressure, lb/ft$^2$
- $p =$ total gage pressure, lb/ft$^2$
- $B =$ barometric pressure, lb/ft$^2$

Initial state

(1) \[(14.95 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)\,(V) = w(55.18 \text{ ft/}^\circ\text{R})(520^\circ \text{R})\]

State when

$T = -65^\circ F$

and

$B = \begin{cases} 31.25 \text{ in. Hg} \\ 15.32 \text{ psi} \end{cases}$

(2) \[\left[ (P) \text{ lb/in.}^2 \right] (144 \text{ in.}^2/\text{ft}^2) \, V = w(55.18 \text{ ft/}^\circ\text{R})(395^\circ \text{R})\]

Dividing equation of state (1) by (2),

\[\frac{14.95 \text{ lb/in.}^2}{P} = \frac{520}{395}\]

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Or

\[ P = \left(14.95\right)\frac{395}{520}\ psi = 11.35\ psi \]

But

\[ p = P - B \]

Therefore,

\[ p = 11.35\ psi - 15.32\ psi \]

\[ p = -3.97\ psi \text{ (Case 3, figure 4)} \]

State when

\[ T = +165^\circ\ F \]

and

\[ B = \begin{cases} 29.25\ in.\ Hg \\ 14.37\ psi \end{cases} \]

\[ \left[\left(P\right)\frac{\text{lb}}{\text{in.}^2}\right] \left[\frac{144\ \text{in.}^2}{\text{ft}^2}\right] V = w\left(55.18\ \text{ft/}^\circ\text{R}\right)\left(625^\circ\ R\right) \]

Dividing equation of state (1) by (3)

\[ \frac{14.95\ \text{lb/in.}^2}{P} = \frac{520}{625} \]

or

\[ P = \left(14.95\right)\frac{625}{520}\ psi = 17.95\ psi \]

\[ p = P - B = 17.95\ psi - 14.37\ psi \]

\[ p = +3.58\ psi \text{ (Case 1, figure 4)} \]
APPENDIX H

TEMPERATURE EFFECTS ON MOISTURE FLOW DUE TO
MECHANISM OF GASEOUS DIFFUSION

<table>
<thead>
<tr>
<th>Temperature °F</th>
<th>D&lt;sub&gt;AB&lt;/sub&gt; cm&lt;sup&gt;2&lt;/sup&gt;/sec</th>
<th>P&lt;sub&gt;A1&lt;/sub&gt; - P&lt;sub&gt;A2&lt;/sub&gt; gm/cm&lt;sup&gt;2&lt;/sup&gt;</th>
<th>P&lt;sub&gt;A&lt;/sub&gt; gm/cm&lt;sup&gt;2&lt;/sup&gt;</th>
<th>P&lt;sub&gt;1&lt;/sub&gt; - P&lt;sub&gt;2&lt;/sub&gt; gm/cm&lt;sup&gt;2&lt;/sup&gt;</th>
<th>μ&lt;sub&gt;N&lt;/sub&gt; gm/sec-sec</th>
<th>N&lt;sub&gt;A&lt;/sub&gt; gm/sec-cm</th>
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</thead>
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<tr>
<td>-65</td>
<td>219.4</td>
<td>1.33</td>
<td>0.141</td>
<td>0.0705</td>
<td>-235</td>
<td>0.000146</td>
</tr>
<tr>
<td>-30</td>
<td>239</td>
<td>1.505</td>
<td>0.2515</td>
<td>0.1258</td>
<td>-163.5</td>
<td>0.000155</td>
</tr>
<tr>
<td>-10</td>
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<td>1.612</td>
<td>0.787</td>
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<td>0.000162</td>
</tr>
<tr>
<td>0</td>
<td>255.5</td>
<td>1.665</td>
<td>1.400</td>
<td>0.700</td>
<td>-103</td>
<td>0.000165</td>
</tr>
<tr>
<td>+60</td>
<td>288.8</td>
<td>2.00</td>
<td>18.05</td>
<td>9.025</td>
<td>0</td>
<td>0.0001755</td>
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<td>305.5</td>
<td>2.11</td>
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<td>25.625</td>
<td>+78.6</td>
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<td>322.5</td>
<td>2.235</td>
<td>119.6</td>
<td>59.8</td>
<td>+140</td>
<td>0.000196</td>
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<td>+165</td>
<td>347</td>
<td>2.678</td>
<td>175.5</td>
<td>122</td>
<td>+228</td>
<td>0.000221</td>
</tr>
</tbody>
</table>

Where:

Initial charging conditions, T = 60° F and P<sub>1</sub> = 14.7 psi and it is assumed that external barometric pressure (B) remains constant at 14.7 psi (1033.2 gm/cm<sup>2</sup>)

\[
N_A = \frac{D_{AB}}{R_V T \ell} (P_{A1} - P_{A2}) + \alpha \left[ \frac{n d^2 g c P_{AV}}{2 \mu c R_N T} (P_{1} - P_{2}) \right] \frac{P_{A}^*}{P_{AV}}
\]  \( (9) \)

Assuming \( \alpha \approx 1 \)

\[
R_V = 4710 \text{ cm/°K}
\]

\[
R_N = 3030 \text{ cm/°K}
\]

\[
\ell = 4.00 \times 10^{-2} \text{ cm}
\]

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\[ d = 1.214 \times 10^{-4} \text{ cm} \]

\[ n = 10 \]

\[ \text{gc} = 980 \text{ cm/} \text{sec}^2 \]
APPENDIX I

SAMPLE CALCULATIONS OF POSSIBLE PENETRATION OF LIQUID WATER THROUGH ROTARY SEAL DUE TO CAPILLARY ACTION AND VISCOUS FLOW DUE TO NEGATIVE PRESSURE DIFFERENTIAL ACROSS SEAL

Assumptions:

a. The relative humidity outside of instrument is very high causing moisture condensation or rainy weather causes water in liquid form to accumulate outside periphery of seal.

b. The liquid water wets down and is drawn into the capillary tubes by surface tension.

\[ h = \text{HEIGHT LIQUID WATER RISES, FT} \]
\[ d = \text{EFFECTIVE DIAMETER OF CAPILLARY FLOW PATH} \]
\[ T = \text{SURFACE TENSION} \]
c. The shaft is in a vertical position. The vertical component of surface tension \((T \cos \theta)\) acting on the peripheral length \((\pi d)\) must balance the weight of the water column rise \((wh \frac{\pi d^2}{4})\) and the differential pressure across seal \((\Delta p)\) acting on capillary cross sectional area \((\pi d^2/4)\) at the meniscus or,

\[
\pi d \cos \theta = \frac{wh \pi d^2}{4} + \frac{\Delta p \pi d^2}{4}
\]
or

\[
h = -\frac{4T \cos \theta}{w} - \frac{\Delta p}{w} \tag{10}
\]

where:

\(w\) = specific weight of the liquid water, \(\text{lb/ft}^3\)
\(T\) = surface tension, \(\text{lb/ft}\)
\(\Delta p\) = differential pressure across seal, \(\text{lb/ft}^2\)

For liquid water and air

\(T \approx 0.004985 \text{ lb/ft}\)

\(w \approx 62.4 \text{ lb/ft}^3\)

Therefore,

\[
h = \frac{(4)(\cos \theta)(0.00495 \text{ lb/ft})}{(d)(62.4 \text{ lb/ft}^3)} - \frac{\Delta p}{62.4 \text{ lb/ft}^3}
\]

\[
h = -\left(\frac{\cos \theta (0.000317) \text{ ft}^2}{d} - (0.01602)(\Delta p) \text{ ft}^3/\text{lb}\right) \tag{11}
\]
If it is assumed first that instrument charging conditions, 
\( T = 60^\circ F \) and internal pressure \( (P_1) = 14.95 \text{ psi} \), remain constant and that barometric pressure varies from 15.32 psi to 14.37 psi (see figure 3), then \( \Delta p \) can vary between two extreme values, -0.37 psi and +0.58 psi.

a. If the negative condition is treated first, \( \Delta p = -0.37 \text{ psi} \), and the following values assumed,

\[
d^{(1)} \approx 0.001 \text{ in.} \approx 0.0000833 \text{ ft}
\]

\[
\theta \approx 30^\circ, \quad \cos \theta = 0.866
\]

\( h \) becomes,

\[
h = \left[ \frac{(0.866)(0.000317) \text{ ft}^2}{0.0000833 \text{ ft}} - (0.01602)(-0.37)(144) \text{ ft}^3/\text{lb x lb/ft}^2 \right]
\]

\[
h = -(3.29 + 0.854) \text{ ft}
\]

\[
h = -4.144 \text{ ft}
\]

but, this is impossible, as the length of flow path \( l \) is only approximately 1/64 in. Therefore, the first term of eq (11) must not exist as the second term alone is sufficient to cause flow. Thus, viscous flow of the liquid water will take place which is similar in nature to viscous total gas flow treated previously in Appendix A. A calculation of this flow rate is made,

\[
N_{\text{water}}^* = \frac{wd^4gc}{128\mu H_2O l} \Delta p
\]  \hspace{1cm} (12)

Where:

\[
N_{\text{water}}^* = \text{rate of liquid water flow through seal in lb/sec}
\]

\[
d = \text{diameter, ft}
\]

\footnote{This value is 21.7 greater than maximum model estimate of "d" and assumed due to the flaring of ends of flow path.}
\[ \mu_{H_2O} = \text{absolute viscosity of liquid water, lb/ft-sec} \]
\[ \ell = \text{length of flow path, ft} \]
\[ g_c = \text{gravity constant, 32.2 ft/sec}^2 \]
\[ w = \text{specific weight of water, lb/ft}^3 \]
\[ d^{(2)} = 1.214 \times 10^{-4} \text{ cm (3.98 x 10^{-6} ft)} \]
\[ w = 62.4 \text{ lb/ft}^3 \]
\[ \mu_{H_2O} \approx 6.77 \times 10^{-4} \text{ lb/sec-ft} \]
\[ \ell = 0.0013 \text{ ft} \]
\[ N_{\text{water}} = \frac{(-0.37)(144 \text{ lb/ft}^2)(62.4 \text{ lb/ft}^3)(3.98)(10^{-24})(\text{ft}^4) (32.2 \text{ ft/sec}^2)}{(128)(6.77 \times 10^{-4})(\text{lb/sec-ft})(0.0013 \text{ ft})} \]
\[ N_{\text{water}} = \frac{(-0.37)(144)(62.4)(250.9)(10^{-24})(32.2) \text{ lb/sec}}{(128)(6.77)(10^{-4})(0.0013)} \]
\[ N_{\text{water}} = 2.39 \times 10^{-13} \text{ lb/sec} = 11.05 \times 10^{-11} \text{ gm/sec} \]

There are 10 capillary tubes for the chosen value of diameter (d). Therefore, if liquid water were to get into all of the capillaries, the flow rate would be increased by a factor of 10.

\[ ^2\text{Because of flow conditions, "d" will be that depicted by flow model, figure 3.} \]
b. If the positive condition is treated, $\Delta p = +0.58$ psi

\[ d \approx 0.001 \approx 0.000833 \text{ ft} \]
\[ \theta \approx 30^\circ, \quad \cos \theta = 0.866 \]

\[ h \text{ becomes,} \]
\[ h = -\left[ \frac{(0.866)(0.000317)}{(0.000833)} - (0.01602)(0.58)(144) \right] \text{ ft} \]
\[ h = -\left[ 3.29 - 1.57 \right] \text{ ft} = 1.72 \text{ ft} \]

Again, this is impossible, as the length ($l$) is only 1/64 in. Actually, the first term is not a constant but will diminish in value because as the driving force ($T$) moves the fluid along the flow path and it approaches the path inner end, the value of $\theta \rightarrow 90^\circ$ and hence $\cos \theta \rightarrow 0$. Therefore, the column will reach a point near the end of the flow path where the increase\(^3\) in diameter ($d$) and decrease in $\cos \theta$\(^3\) will force the first term of eq (11) to decrease in magnitude until the magnitude indicated by eq (11) actually agrees with rise in capillary tube. This magnitude will always be less than the length of flow path ($l$). Evaporation will take place and equimolal\(^4\) gaseous diffusion will then be the mechanism for transporting the water vapor through the remaining length of flow path into the instrument. The rate of flow equation will be similar to eq (8) of the text only with a much decreased value of "\(l\)" and much increased value of "d".

\[ N_A = \frac{D_{AB}}{RT \cdot l'} \left( p_{A_1} - p_{A_2} \right) \quad (8a) \]

\(^3\)Due to flaring or belling at the end of flow path.
\(^4\)This is so if we treat $P - p_{A_1} \approx P_1$.
where,

- $t' = \text{reduced length flow path}$
- $p_{A_1}' = \text{saturation vapor pressure}$
- $d' = \text{increased average diameter}$

and

$$N_A^* = \left[ \frac{\pi (d')^2}{4} \right] \left[ \frac{D_{AB}}{RT \cdot t'} \right] \left( p_{A_1}' - p_{A_2} \right)$$  \hspace{1cm} (8b)

if,

$$D_{AB} = \frac{D_{AB}'}{P_1}$$

where:

- $D_{AB}' = \text{new diffusivity associated only with physical properties of gases involved.}$
- $P_1 = \text{internal total pressure}$

Then eq (8a) becomes

$$N_A = \frac{D_{AB}'}{RT P_1 t'} \left( p_{A_1}' - p_{A_2} \right)$$  \hspace{1cm} (8c)

Equation (8c) predicts that the flow will be directly proportional to the vapor pressure differential and inversely proportional to the internal pressure. Since $p_{A_1}'$ is fixed at saturation pressure, there is nothing that can be done to lower this value. However, by increasing the internal pressure the flow rate can be reduced. Increasing $P_1$ will cause the liquid level to recede correspondingly, increasing the magnitude of $t'$ and decreasing the magnitude of the average diameter $(d')$. The flow rate reducing power of $P_1$ will be stronger than just a linear effect. The increase in magnitude of $+\Delta p$ in eq (11) need not be very
great (even if \( \cos \theta \) is assumed equal to unity) before \( h \) becomes a small fraction of \( \ell \) and for all practical purposes \( \ell' = \ell \) and \( d' = d \). Therefore, it becomes very desirable to pressurize the instrument.

If assumption (c) does not exist and the shaft (and seal) are in a horizontal position, the horizontal component of the surface tension acting on the peripheral length \( (\pi d) \) must balance the differential pressure across the seal \( (\Delta p) \) acting on the capillary cross sectional area \( (\pi d^2/4) \) at the meniscus or,

\[
\pi d T \cos \theta = \frac{\pi d^2}{4} \Delta p
\]

\[
4T \cos \theta - d \Delta p = 0 \quad (13)
\]

a. If the negative condition is treated first, the equation will not balance and the situation will be similar to that discussed under shaft being in a vertical position, i.e., \( \Delta p \) will become the driving force and the flow will be viscous and equal in magnitude to that discussed for the vertical position.

b. If the positive condition is treated, the following develops:

As the fluid attempts to enter the capillary, the term \( d \Delta p \) can either exceed or be less than the term \( 4T \cos \theta \). If it is greater, even when \( \cos \theta \approx 1 \), then the fluid will never enter the capillary. If it is less, the fluid will travel along the capillary until the value of \( \cos \theta \) and \( d' \) make eq (13) valid.

For

\[ \Delta p = +.58 \text{ psi} \]
\[ d = 3.98 \times 10^{-6} \text{ ft} \]
\[ T \approx 0.004985 \text{ lb/ft} \]

and

\[ \theta \approx 1 \]
\[ 4T \cos \theta = 0.01994 \text{ lb/ft} \]

\[ d \Delta p = (3.98 \times 10^{-6} \text{ ft})(144)(0.58) \text{ lb/ft}^2 = 0.000573 \text{ lb/ft} \]

Hence, \( d \Delta p \) is less than \( 4T \cos \theta \) and fluid will travel through capillary until the decrease in \( \cos \theta \) and increase in "d" will make eq (13) valid. It will stop at some given point near the inner end, evaporation will take place and equimolar gaseous diffusion will be the mechanism for transporting the water vapor through the remaining length of flow path into the instrument. From this point on the same arguments hold as concerns the case when the shaft (and seal) are in a vertical position.
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