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RADIOMETER STUDIES
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A. Radiometer Design and Performance

Figure 1 is a block diagram of the radiometer. It is basically of the type first described by R.H. Dicks (1946). A considerable effort was put into achieving stability of both the gain and noise figure of the receiver and into obtaining as low a receiver noise figure as possible. The resulting apparatus has been found to be reasonably flexible and to meet most of the original design objectives. The various parts are described in the following, in some detail where the circuits are novel.

The Microwave Circuitry

The microwave circuitry is compact. With the exception of the klystron and noise tube and their respective attenuators, it is all contained within a can 6 inches in diameter and 14 inches long. This can, which also contains the IF amplifier and second detector, is located in front of the focus of the paraboloidal antenna. The feed horn protrudes from one end of the can. Following the feed is a 20 db cross-guide directional coupler, with its side arm connected to the noise tube (located in a box on the back of the reflector) through a long length of waveguide which runs down one of the hollow feed support legs. The noise tube temperature is about 16,600°K, and its output may be varied by means of a calibrated waveguide attenuator. When the noise tube is fired, a small predetermined signal is therefore fed into the radiometer through the cross-guide coupler for calibration.

Following the coupler is a SPDT microwave switch. In one position, the feed and coupler are connected to the rest of the receiver. In the other position, the waveguide from the noise tube is coupled directly
into the receiver for high level calibration and for operation of the Automatic Noise Figure Meter (to be described later). This mechanical switch is operated by a small D.C. motor which may be actuated remotely. The switch has been found to introduce negligible losses and reflections. Its isolation is 46 db.

Following the mechanical switch is the SPDT Ferrite switch which provides the rapid temperature comparison at the input. The two terminations that have been variously connected to the reference arm are a matched load at ambient temperature or a small horn which looks out of the front of the can through a Teflon window. With the latter termination the reference temperature is close to the antenna temperature and the ill effects of change in the receiver gain are reduced. The switch works over a band of 500 mcs. centered at 36 Kmc. with insertion loss of 0.4 db, isolation greater than 22 db, and VSWR less than 1.2. The switching time is 20 μ sec.

Next is the balanced mixer which consists of a magic Tee with the two mixer crystals in standard crystal holders connected to the two side arms; the input port is the H-arm and the local oscillator feeds into the E arm. The Klystron local oscillator with its waveguide attenuator are located in the box at the back of the reflector, with the coupling waveguide passing up through one of the hollow feed support legs to the Magic Tee. The variable attenuator which follows the Klystron is controlled by a reversible D.C. motor which may be actuated remotely. The side arms of the Tee are folded together to allow close physical proximity of the crystals and hence to minimize the output capacitance of the mixer. At 36 Kmc. the separation between the E and H arms of the Tee is 39 db and the VSWR of the E-arm with matched terminations on the side arms is 1.26. The crystal mounts are tunable with a minimum VSWR which depends on the individual crystals but is typically less than 1.20.
I F Amplifier

The I F amplifier, second detector, and a wide band first audio amplifier are enclosed in an aluminum box of dimensions 1" x 2" x 4". The box is closely integrated with the crystal mixers in order to minimize stray capacitance in the coupling between mixer and I F amplifier. The box is machined from a solid aluminum block and is equipped with tight fitting cover plates in order to eliminate interference. The amplifiers employ transistors solely. The choice of transistors rather than vacuum tubes was based on a number of considerations. Transistors are small, allowing the receiver input section to be small; they are rugged, dependable, long lived, generate no microphonics, and the best ones have gain bandwidth products of approximately 1000 mcs, and, hence, have good noise figures. Although a few tube types such as WE437B and Amperex 7788 have slightly better narrow band noise figures they are only slightly better because the mixer output resistance of 300 ohms turns out to be a good match for the transistors. Because the transistors have lower effective input capacitance, more I F bandwidth is obtained than with any of the vacuum tubes. In terms of total radiometer sensitivity this offsets the slightly poorer transistor noise figure. This point is discussed in detail later in the context of optimizing the radiometer sensitivity with respect to the matching circuit between the mixer and I F input (see section C).

Five stages employing a total of 6 transistors are used in the I F amplifier to obtain a gain of 60 db and flat response from 15 to 50 mcs. The amplifier is maximally flat stagger tuned. The noise figure with a 300 source resistance is 1.2 db. This is just slightly higher than the value of 0.8 db obtainable with either of the two vacuum tube types mentioned above. Due to their large input capacitance the maximum bandwidth obtainable with either tube is 20 mcs.
The second detector is 1N616 Germanium diode. The wide band audio amplifier which follows it has a very low input impedance, about 10 ohms, which tends to fix the operating point of the diode. This in turn keeps the transfer coefficient of the crystal detector constant over a wide range of input levels. The wide band amplifier has a gain of 40 db and passes signals in the range 0-100 kc. It passes D.C., permitting the receiver to be used as a total power radiometer and also facilitating noise figure measurements.

Two ten volt supplies power the I F transistors, one supply, with its positive terminal grounded, feeds the collectors in the usual way. The other, with its negative terminal grounded, is coupled to the emitters through large resistors which are bypassed. In this way the quiescent emitter currents are held fixed, and consequently the amplifier gain is stabilized. The supplies have less than .001% ripple and vary less than .01% with line fluctuations even as large as 20 volts. The result is that the gain of the amplifier varies less than 0.1% over long periods.

Output Circuitry

The remainder of the electronic apparatus (including the transistor power supplies) is contained within a 72 inch rack. The rack is equipped with an electronic line voltage regulator and fans to remove heat generated by the vacuum tubes.

A narrow band audio amplifier precedes the synchronous detector in order to amplify the envelope resulting from the rapid switching of the input without overdriving the synchronous detector with noise. The band center is 33 cps, corresponding to the switching rate of the ferrite switch, and the bandwidth is 8 cps, which is a compromise. Narrower bandwidths decrease the spurious output noise, permitting greater dynamic range, but at the same time make the system very sensitive to small fluctuations in the switching frequency. With the 8 cps bandwidth, signals as large as 10,000^2k may be
fed into the radiometer without overdriving the detector, and at the same
time with a sample of the switching waveform fed into the amplifier the
output fluctuations remain less than 0.1%. This amplifier employs vacuum
tubes. As in the case of the IF amplifier, gain is stabilized by means of
a negative power supply and large cathode resistors, so that the quiescent
tube currents and hence their transconductances are kept fixed. The amplifier
bandpass is obtained by a bridged T R-C notch filter in a feedback loop
around the amplifier, as described on page 391 of Valley and Wallman.
Resistors and capacitors of low temperature coefficient are used in this
circuit to ensure stability. Ganged attenuators at the input and output of
this amplifier permit a range in radiometer sensitivity of 1000:1.

The synchronous detector consists of 4 computer type 1N457 diodes in
a bridge circuit, driven by high quality audio transformers from both the
signal and reference circuits. The diodes are switched by the large
reference square wave and in turn synchronously rectify the 33 cps. component
of the output from the narrow band filter. This inherently stable circuit
produces a balanced D.C. output to the integrator.

The integrator circuit is essentially two identical R-C integrators
in cascade, separated by a cathode follower. The circuit is balanced, with
two push pull cathode follower stages in cascade, an R-C integrator in each
grid. The time constants may be set at 1, 2, 5, 10, 20, and 50 seconds.
These may be changed without any offset appearing in the output. The output
cathode followers drive a Varian strip chart recorder. Actually, the
output is tapped down the cathode resistors to the point where the residual
tube current noise, appearing as a voltage across the cathode resistors, is
below the recorder dead band. The integrator employs type 6085/E0CC tubes
and it is found that after a two hour warm up the output is stable within
the dead band of the recorder. The output circuit is also provided with a
means for permitting the scale to be expanded, that is, for permitting small
temperature changes to be observable in the presence of a large temperature
differential between the input and reference. This consists of one accurate
ten-turn Helipot and a calibrated Mercury cell, so that an accurately known
voltage of up to about 1 volt may be added in series with the output. As the
recorder full scale sensitivity is 50 millivolts, this permits the full scale
radiometer sensitivity to be set at 5% of the incoming temperature differential.

The generator which provides the 33 cps signal consists of a balanced
Wienbridge feedback oscillator with a light bulb in one arm of the bridge.
The bulb, as a non-linear resistor, stabilizes the oscillator output ampli-
tude. The bridge elements have low temperature coefficients, and the bulb is
both thermally insulated and shock mounted, so that the generator is stable
in both amplitude and frequency. The output drives two separate Schmidt
Triggers, one for the ferrite switch and one for the synchronous detector.
The relative phases of the square wave outputs are adjustable to compensate
for phase shift in the narrow band audio amplifier.

These vacuum tube circuits are powered by two supplies in the same
fashion as the transistorized IF amplifier. Where a negative supply and
large cathode resistors (i.e. a current source) are employed, the relative
influence of the positive plate supply on gain is small. Therefore, a
standard Lambda type (300 v), supply is used. The negative supply, which
must be well regulated, is specially constructed. If the negative supply
is driving an amplifier of N stages, then a percentage supply voltage change
of \( \delta \) shows up as a percentage change in amplifier gain of \( N \delta \). The supply
used here has an output of 150 volts, with 1 mv ripple, and changes less
than 0.01% with line voltage changes of 20 volts. In addition, all filaments
are operated from a well regulated D.C. supply in order to insure gain
stability against line fluctuations and also to eliminate hum.
The driver for the ferrite switch, which is actuated by one of the Schmidt triggers mentioned above, is transistorized. The currents in both "positions" of the switch are adjustable for optimum operation. In addition, the switch may be held in either "position" or rapidly alternated at 33 cps.

**Automatic Noise Figure Meter**

In order to readily evaluate receiver performance and to be able to optimize it before it is to be used, the radiometer is equipped with an automatic noise figure meter. Both the mechanical switch and the ferrite switch are adjusted so that the noise tube is connected directly to the input of receiver. The tube is then switched on and off at 100 cps and from the known tube temperature one can infer the receiver noise temperature from the variation of the receiver output noise. The device to be described here does this automatically.

For the present discussion the receiver noise temperature is just the temperature of that amount of additional noise which, when added to the receiver input from the gas discharge noise tube, just doubles the receiver output noise power. In order to be effective the automatic noise figure meter must give the correct reading independent of receiver gain. This is, of course, essential, since any adjustments which change the noise figure are also likely to change the receiver gain. The conventional device accomplishes this with an automatic gain control loop. However, it was felt desirable not to use an AGC loop, and a novel device was developed which employs logarithmic amplifiers in a straightforward manner. It has the additional feature of providing an output which is linear in noise figure in dB.

A block diagram of the device is shown in figure 2, connected to the output of the receiver at point N. The D.C. amplifier with its feedback
resistor R, is the wide band audio amplifier which was discussed above. For the following discussion, refer to the symbols in figure 2. The IF output voltage is

\[ v = \sqrt{K TB R} \cdot (G^{1/2}) A \]

where 
- \( K \): Boltzman's constant
- \( B \): IF bandwidth in cps
- \( R \): IF output resistance
- \( G \): IF power gain
- \( A \): mixer voltage gain
- \( T \): equivalent temperature referred to the receiver input

If the second detector is square law, then

\[ i = cv^2 \]

and with the gain of the D.C. amplifier very high, its input impedance is low and \( v_1 \approx 0 \). Then

\[ v = iR_1 = R_1 cv^2 = \left[ R_1 KcBRGA^2 \right] T = K_1 T \]

The voltage waveform for equal on-off times of the noise tube will be as shown in figure 3. Let \( V_1 \) be the mean voltage in the interval \( t_0 < t < t_1 \) and \( V_2 \) the mean voltage in the interval \( t_1 < t < t_2 \). Also, let \( T_0 \) be the cold noise tube temperature, \( T_N \) the hot tube temperature, and \( T_R \) the receiver temperature. Then

\[ V_1 = K_1 (T_0 + T_R) \]
\[ V_2 = K_1 (T_N + T_R) \]

The average output, which goes to one log-amplifier, is

\[ K_1 \left( \frac{2T_R + T_0 + T_N}{2} \right) \]

The alternating component which is filtered out by the synchronous detector and sent to the second log-amplifier is

\[ \frac{K_1 (T_N - T_0)}{2} \]
The outputs of the log-amplifiers are by construction the logarithms of their respective inputs. The reading on the meter, which indicates the difference between the two outputs, is then

\[ R = \log \left( \frac{2T_R + T_o + T_N}{T_N - T_o} \right) = \log \left( \frac{T_N + T_o}{T_N - T_o} + \frac{2T_R}{T_N - T_o} \right) \]

Since \( T_N \approx 50 T_o \)

\[ R \approx \log \left( 1 + \frac{2T_o}{T_N} + \frac{2T_R}{T_N - T_o} \right) \]

Thus, by suitable offset and scale adjustments, the output reading gives the receiver noise figure directly in db.

The core of the device is evidently the log-amplifier. These were constructed from ordinary high gain amplifiers with semiconductor diodes in feedback loops. A large number of diodes were tested and some were found with V - I characteristics which were very closely logarithmic over 3 decades. The final device is capable of providing accurate noise figure readings from 0 to 20 db. It is built into the 72" rack and may be quickly brought into service to check receiver performance.

**Performance and Calibration**

The current flowing through each mixer crystal is monitored by a meter on the front of the rack. In addition, the operating point of each crystal may be adjusted independently. With these two controls and the klystron local oscillator attenuator control, the receiver is adjusted for minimum noise figure. This is, of course, done with the aid of the automatic noise figure meter. With IN53C mixer crystals the receiver noise temperature is about 2500°, including the contribution from the 0.4 db insertion loss of the ferrite switch.

The vacuum tube circuits require at least a two hour warm-up period.
in order to stabilize. For this reason, they are left on continuously.
When the radiometer is properly adjusted, its gain is found to be stable
within 0.1% for periods up to an hour. Greater stability is certainly possible.
However, at the present the temperature sensitivity of the second detector
will not permit it.

Calibration is done by connecting matched terminations of known
temperature to the input of the radiometer. These terminations consist of
large aluminum blocks into which have been cut waveguides containing
absorbers. The absorbers consist of thin mica sheets onto which a tenuous
layer of gold has been evaporated, and they are tapered in the usual way to
provide good terminations. Each block contains a heating element, a
thermometer, and a thermostat capable of regulating the temperature to 0.05°C,
and the block is itself enclosed in insulating material. The reflection
coefficient, $p$, of each termination was carefully measured as the emission
temperature is $(1-p^2)$ times the thermometer temperature. Two of these,
operating at a differential of about 20°C, were connected to the input of
the radiometer in succession and the resulting chart deflections compared to
those due to the noise tube feeding in through the directional coupler. In
this way the contribution of the noise tube is calibrated to an accuracy of
about 3%.

With a five second time constant, which is a convenient value for
drift measurements, the peak to peak noise on the record is about 1°, and
therefore the RMS is about 0.2°. A sample record is shown in figure 4.
This shows the result of adding in a 3.5° calibration signal from the noise
tube. It also includes a drift scan of the planet Jupiter.
B. Antenna Performance

General Description

The antenna is a ten foot diameter parabolic reflector which was machined from a solid aluminum casting in the Naval Shipyard at Norfolk, Virginia. It is mounted on a reinforced surplus SCR 584 radar pedestal located on the Southwest corner of the roof of Cory Hall. As such, it is about 125 feet above the surrounding terrain to the South and West. The mount is equatorial and is equipped with setting circles which allow positioning to within 6 minutes of arc in either declination or hour angle. Antenna motion is provided by the D.C. Servo motors that came with the radar mount. In addition, a small synchronous motor may be engaged in the hour angle drive for slow tracking. The motor is driven by a crystal controlled signal generator and will move the mount at sidereal rate or either 10% or 20% faster.

The input section of the receiver, in a cylindrical can 6 inches in diameter and 1 4 inches long, is located in front of the focal point of the reflector and supported on four legs made of hollow tubing, as shown in the photograph of figure 5. The feed horn is just behind the can. In the front of the can is a Teflon window through which the reference horn looks along the axis of the paraboloid. The klystron, wavemeter, and noise tube are in the box on the back of the reflector.

Measurement of Antenna Gain

The gain was measured by locating a transmitter on a nearby hill to the Southeast and comparing the signal received by the big antenna with that of a standard gain horn. The standard horn was obtained by constructing two identical horns of dimensions: 3" x 3.3" x 23" long, matching them, and measuring the transmission loss between them in an anechoic chamber. The measurement was repeated many times with various relative spacings of the
horns and in different locations in the chamber, in order to reduce systematic errors. The gain for either horn was found to be 27.05 dB ± .05 dB. To the standard deviations of .05 dB in the measurement must be added a 0.20 dB probable error in the precision reference attenuator used.

For calibration of the large antenna, one of the standard horns was mounted next to the receiver can and oriented along the axis of the paraboloid. The transmitter, a klystron feeding a three foot diameter dish, was located at a distance of 1.25 miles at an elevation angle of about 100°. The outputs of the horn and the large antenna were then compared with the precision variable attenuator (FXR type U164). Two feed horns were tested, one with a 20 db taper and one with a 10 db taper. The latter was adopted because it increased the gain by about 0.6 db over the former. The feed position for maximum gain was found with the help of thin waveguide shims. Taking into account the slight aperture blocking of the standard gain horn, we deduced a gain of 57.00 dB or 5 x 10^5 with a probably error of 10%, the error due primarily to inaccuracy of the precision reference attenuator.

A gain of 5 x 10^5 corresponds to an aperture efficiency of 39%. A geometrical optics calculation of the aperture blocking due to the feed system shows a resultant loss in gain of about 25%, due primarily to the feed legs. If this loss were subtracted, the efficiency would be about 50%; this is close to the gain calculated from the primary pattern and also close to what one normally expects in practice from this type of antenna system. We conclude, therefore, that little degradation of the gain results from inaccuracies in the reflector surface. Effort will be made in the future to reduce the diameter of the feed legs by using a Cassegrain feed structure.
Antenna Pointing

Accurate positioning of the antenna is provided by an optical sighting telescope which fastens to the edge of the reflector. The telescope is removed when it is not needed in order to protect it from the weather, and it is found that in removal and remounting an error of no more than 1 minute of arc results. The 8 power telescope, equipped with an illuminated cross hair, was aligned at the time of the gain measurement. The resulting pointing accuracy of one or 2 minutes of arc is well within the antenna beam width of about 12 minutes.

Measurement of the Antenna Beamwidth

A measurement of the antenna half power beamwidth was made at the time of the gain measurement. The resulting estimate of 10-11 minutes, based on interpolating between the marks of the setting circles, was somewhat inaccurate. As the solar diameter is about 2½ beamwidths and the sun has been shown to be very closely a disc of uniform temperature at this wavelength, solar drift curves provide a simple means of determining antenna beamwidth.

The drift curves may be interpreted as follows. We suppose that the antenna pattern has axial symmetry and can be approximated by a Gaussian shape. Its normalized power response is then

\[ P(x, y) = \frac{2.8}{\theta^2} \exp \left\{ - \frac{2.8}{\theta^2} \left[ \left( x - x_0 \right)^2 + \left( y - y_0 \right)^2 \right] \right\} \]

where \( \theta \) is the half power width. With the solar disc of uniform temperature \( T_s \) and semidiameter \( a_1 \) centered at the origin \( (x = 0, y = 0) \), and the antenna pointing in the direction \( (x_0, 0) \), the antenna temperature is
\[
T_A(x,0) = \frac{2.8}{\pi \theta^2} T_s \exp\left\{ \frac{2.8 x_0^2}{\theta^2}\right\} \int_{0}^{2\pi} \int_{0}^{a} \exp\left\{ -\frac{2.8}{\theta^2} \left( r^2 - 2x_0 r \cos \phi \right) \right\} r \, dr \, d\phi \\
= T_s \left[ 1 - \frac{5.6}{\theta^2} \exp\left\{ \frac{-2.8 x_0^2}{\theta^2}\right\} \right] \int_{0}^{\infty} I_0 \left( \frac{5.6 x_0 r}{\theta^2} \right) \exp\left\{ \frac{-2.8 r^2}{\theta^2}\right\} r \, dr 
\]

With \( x_0 \) near \( \alpha \) and both larger than \( \theta \), the Bessel function may be replaced by its asymptotic expansion and the integral approximated. As the earth turns with angular velocity \( \omega_s \), the beam drifts through the sun so that \( x_0 = \omega_s t \cos \delta_s \). \( \delta_s \) is the solar declination. We find that the slope of the drift curve just at the instant when the center of the beam reaches the limb is approximately given by

\[
\frac{\partial T_A}{\partial t} = \frac{\omega_s \cos \delta_s T_s(0.944)}{\theta} (1 - 0.067 \frac{\theta^2}{a^2})
\]

With the aid of (3), \( \theta \) has been found from solar drift measurements to be approximately 12.0 minutes.

C. Related Studies

Integrator Smoothing of Drift Curves

When a planet, which is in effect a point source, drifts through the antenna beam, the mean radiometer output as a function of time is just the antenna pattern. There is also noise present which may be made smaller by increasing the time constant of the integrator. However, if the time constant is made longer than the drift time, the mean trace will be distorted, its width increased and its height decreased and delayed. Further increase in the time constant will only make matters worse.
Actually, whatever time constant is employed will distort the drift curve to some extent, and a correction must be made. The following calculations predict the reduction in the maximum response due to both single pole and double pole R-C integrators, the latter being the type employed in the present radiometer.

We assume that the antenna pattern is approximately Gaussian with half width $\theta$. Then the mean input to the integrator is $e^{-t^2/\sigma^2}$ where $\sigma = \theta/(1.7 \omega \cos \delta)$, $\omega$ is the angular velocity of the earth and $\delta$ is the declination of the planet. The two forms for the integrator to be considered are the single pole and double pole R-C filters shown in figure 6. The impulse response of the single pole circuit is

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{1+j\omega T} \, d\omega = \sqrt{\frac{2\pi}{T}} e^{-t/T} H(t),$$

while that of the double pole circuit is

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{(1+j\omega T)^2} \, d\omega = \frac{\sqrt{2\pi}}{T^2} te^{-t/T} H(t),$$

where $T = RC$. With the input to the single pole integrator equal to $e^{-t^2/\sigma^2}$, the output is

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(t/\sigma)^2} \frac{2\pi}{T} e^{-\frac{(t-\tau)}{T}} \omega H(t-\tau) d\tau = \sqrt{\pi} \Re^{2R} e^{R^2} \left[1 + \text{erf}(q-R)\right]$$

where $q = t/\sigma$ and $R = \sigma/2T$. On the other hand, if the signal $e^{-t^2/\sigma^2}$ were fed into the double pole circuit, the response would be
From the equations above the outputs from the filters were plotted against \( q \) for various values of \( R \) in order to obtain the maximum response as a function of \( R \). The results are given in the following table and also plotted in figure 7. A response of unity corresponds to no degradation.

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(\frac{\tau}{\sigma}\right)^2} \frac{\sqrt{2\pi}}{T} \left( (t-\tau) e^{-\left(t-\tau\right)/T} H(t-\tau) d\tau = \sqrt{\pi} R e^{-2Rq} R^2 \right)
\]

\[
= \left\{ 2(q-R) \left[ 1 + \text{erf}(q-R) \right] + \frac{2-(q-R)^2}{\pi e} \right\} \quad (7)
\]

| TABLE I |
|-----------------|-----------------|
| Single Pole     | Doule Pole      |
| \( R \)       | max response | \( R \)       | max response |
| 0.1            | 0.3            | 0.5            | 0.527         |
| 0.6            | 0.78           | 0.75           | 0.637         |
| 1.0            | 0.96           | 1.0            | 0.752         |
| 2.0            | 0.98           | 1.5            | 0.853         |
| 5.0            | 0.98           | 2.0            | 0.907         |
| 30.0           | 0.998          | 3.0            | 0.950         |
|                |                | 5.0            | 0.98          |
|                |                | 30.0           | 0.998         |

The r.m.s. jitter present on the traces is proportional to the integral over \((-\infty, \infty)\) of the power frequency response of the filter. Hence for the single pole case

\[
\int_{-\infty}^{\infty} \left| \frac{1}{1+j\omega T} \right|^2 d\omega = \frac{\pi}{T} \quad (8)
\]
and for the double pole case

\[
\int_{-\infty}^{\infty} \left| \frac{1}{1+j\omega T} \right|^4 \, d\omega = \frac{\pi}{2T} \tag{9}
\]

As regards actual signal to noise ratio, taking into account the reduced response, the double pole filter gives an improvement of \( \frac{2}{\sqrt{3}} \). For a 5 second time constant, the double pole integrator reduces the maximum by about 5%.

**Optimum Coupling Between Mixer and I F Amplifier**

The minimum detectable signal for a radiometer is proportional to \( (F-1)B^{-\frac{1}{2}} \) where \( F \) is the overall receiver noise figure and \( B \) is the receiver bandwidth, in effect the I F bandwidth in the radiometer discussed in this report. The proper choice of coupling network between the mixer and I F amplifier will permit one to minimize this quantity by finding a compromise between I F bandwidth and noise figure. The constraints are the output capacitance and resistance of the mixer and its conversion loss and excess noise. The following calculations indicate the effect of transforming the mixer output impedance on noise figure and bandwidth of the I F amplifier and on overall radiometer sensitivity. The problem is formulated in terms of vacuum tubes for simplicity, and the numerical examples employ the type 7788 low noise tube.

The mixer resistance and capacitance, \( R_s \) and \( C_s \), give an approximate upper bound on the bandwidth which may be easily obtained.

\[
B_{\text{max}} = \frac{1}{2\pi R_s C_s} \tag{10}
\]

For simplicity an amplifier with a low pass response will be considered. In practice the very low frequencies must be rejected because of the large \( 1/f \) components from the mixer. The vacuum tube model to be used for the
noise calculations is shown below. \( e^{-2} \)

\[
\begin{align*}
R_t &= \text{resistive component of input coupling circuit} \\
R_t &= \text{input resistance of the tube due to transit time effects} \\
&\quad \text{(proportional to } f^{-2}) \\
C_1 &= \text{input capacitance of the tube} \\
\overline{v}_n &= \text{noise generator due to } R_t \text{ and } R_1 \\
\overline{v}_S &= \text{shot noise in the tube}
\end{align*}
\]

The noise generators are given by the following relations

\[
\overline{I}_1^2 = r_k (pG_t + G_1) T df
\]

\[
\overline{v}^2 = 4kT R_{eq} df
\]

where \( p \) accounts for the effective temperature of \( G_t \) (pW5),

\( R_{eq} \) is a parameter of the tube

\( df \) is the incremental bandwidth.

\( R_t \) may often be ignored in calculating the gain of a low-pass amplifier since for frequencies below 100 mcs. the reactance of \( C_1 \) is typically much less than \( R_t \). It should, however, be included in the calculation of due to its high effective temperature.

For a cascade of linear networks, the overall noise figure is given by

\[
N = N_1 + \frac{N_2 - 1}{W_1} + \frac{N_3 - 1}{W_1W_2} + \ldots + \frac{N_n - 1}{W_1W_2\ldots W_n}
\]

where \( N_n \) is the noise figure of the \( n \)th stage and \( W_n \) is the available power gain of the \( n \)th stage or network. In the present case the two relevant stages are the mixer and the I F amplifier, so that
\[ N = N_m + \frac{N_a - 1}{W_m} \]  

The effective noise figure in this calculation is the average or integrated noise figure, \( F \).

\[ F = \frac{\int_0^\infty N(f) A^2 df}{\int_0^\infty A^2 df} \approx \frac{\int_{f_2}^{f_1} N(f) df}{f_2 - f_1} \]

where \( A \) is the absolute voltage gain at frequency \( f \). The approximation results from assuming a square amplifier pass band.

The most elementary way of coupling the mixer and IF amplifier is by direct connection. The amplifier circuit which has the best gain and noise figure properties is the "cascode" circuit, consisting of a grounded cathode stage driving a grounded grid stage. It can be shown that with the mixer directly connected to this combination the noise figure of the amplifier is

\[ N_a = 1 + p \frac{G_t + G_1}{G_s} + \frac{(G_s + G_t + G_1)^2 + Y_c^2}{G_s} \left( \frac{G_t}{G_m} + \frac{1}{r_p} \right) \left[ (G_s + G_t + \frac{1}{r_p})^2 + Y_c^2 \right] \]

where \( Y_c = \omega (C_s + C_1) \), \( Y_1 = \omega C_1 \). Both tubes are the same type having parameters \( G_m \) and \( r_p \).

For the mixer, typical values are \( R_a = 300 \Omega \), \( C_s = 8 \text{ pf} \), \( F_m = 6 \), and \( W_m = \frac{1}{2} \). The minimum detectable noise temperature is then proportional to \((4 \pi a + 1)B^{-\frac{1}{2}}\). One of the best vacuum tubes for low noise circuits is the type 7788. Its parameters are

\[ G_m = 0.05 \text{ mho} \quad \mu = 58 \quad G_1 = 25 \text{ pf at } 40 \text{ ma, cathode current} \]

\[ \text{Req} = 60 \Omega \quad r_p = 1160 \Omega \quad R_t = 500 \Omega \text{ at } 100 \text{ mcs.} \]
when it is triode connected. For the mixer parameters above,

\[ B_{\text{max}} = 66 \, \text{muc.} \]

At this frequency,

\[ R_t = \left( \frac{100}{f} \right)^2 \times 500 = 1150 \, \Omega \frac{1}{f^2} \quad \text{and} \quad X_1 = \frac{1}{V C_1} = 96 \, \Omega \frac{1}{f} \]

(16)

Evidently \( R_t \gg X_1 \) for frequencies below 60 cms., although the term \( P^t / G_s \) may have some influence on \( N_a \). From the tube parameters we find the noise figure of the first stage to be dominant.

\[ \text{Req} + \frac{G_1 + G_t}{G_m} \frac{\text{Req}}{C_m} + \frac{\text{Req}}{C_m} \left[ \left( G_1 + \frac{1}{r_p} \right)^2 + \frac{\gamma^2}{G_m} \right] \approx \text{Req} + 2 \approx \text{Req.} \]

(17)

We now inquire whether any improvement will result in transforming the mixer impedance up or down. The best transformation would be by means of an ideal transformer.

\[ R_s \] \hspace{2cm} \[ R_t \]

With the transformed source resistance equal to \( R_s' \), the transformed capacitance is \( C_s' = R_s' / R_s C_s \). The bandwidth of the input circuit is then given by

\[ B = \frac{1}{2\pi R_s' (C_s' + C_1)} \]

(19)

From equation (10), it is evident that increasing \( R_s' \) will decrease \( N_a \).

On the other hand, because of the fixed (and also large) input capacitance of the tube, the bandwidth is decreased with increase in \( R_s' \) as well.

The two effects tend to offset one another. Figure 7 shows the single frequency noise figure, \( N_a \), as a function of frequency for various values of transformed source resistance. The arrows indicate the noise...
With the aid of these graphs, the integrated noise figure may be found. With this information a relative minimum temperature sensitivity, $\Delta T$, can be graphed as shown in figure 8. $\Delta T$ is shown as a function of the bandwidth which would result from any particular choice of turns ratio of the ideal transformer. It is seen to have a minimum of 30 mcs. corresponding to a reduction of input resistance from $300 \Omega$ to $120 \Omega$. The $\Delta T$ corresponding to direct connection, $R_s = 300 \Omega$, is indicated by the arrow. Only a small improvement, about 10%, results from the transformation. This is due primarily to the relatively large input capacity of the tube. It is also, of course, the result of the other parameters that were chosen. One may conclude that for matching into the I F amplifier using vacuum tubes from impedance levels of the order of $300 \Omega$, little improvement over direct connection will result. In a practical structure, the addition of circuit elements above the direct connection configuration will introduce additional capacitance to ground which in turn will reduce the bandwidth of the input circuit.
Figure 5
Receiver Feed Can Assembly
FIG. 6
ARROWS INDICATE NOISE FIGURE AT 3-DB BAND EDGE OF INPUT COUPLING CIRCUIT

FIG. 7
FIG. 8 RELATIVE TEMPERATURE SENSITIVITY AS A FUNCTION OF AMPLIFIER BANDWIDTH