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Final Report on Scoring Methods Study

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DIRECTORATE OF ARMAMENT DEVELOPMENT
Det 4, AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND • UNITED STATES AIR FORCE

EGLIN AIR FORCE BASE, FLORIDA

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FINAL REPORT ON SCORING METHODS STUDY

March 1963

AFSC Project No. 7844
FOREWORD

This report presents the results of a fourteen-month study program conducted under Contract AF 08(635)-2631, sponsored by Detachment 4, ASD, Target Development Laboratory, Applied Research Branch (ASQTR), Eglin Air Force Base, Florida.

The general purposes of the study program were to determine the applicability of various physical phenomena to the problem of trajectory scoring, to assemble all necessary background information, and to establish basic procedures and criteria for planning and developing future scoring systems and for evaluating concepts proposed for scoring. It is felt that this report satisfactorily demonstrates the realization of these objectives. Despite the large variety of problems related to scoring treated under this program, many more remain. A continuing program of studies of the types reported here will be needed to keep abreast of technological advances and new information which may be useful in scoring.

It was understood from the beginning that the program was not to be one of gadgeteering in which efforts would be directed toward the discovery of novel arrangements of system components. It was recognized at the outset, however, and even hoped, that some unique method for scoring might evolve naturally from these studies of basic phenomena. A certain amount of visionary speculation by research personnel was permitted and, fortunately, a promising idea was conceived for an angle-measuring device which would sense the line-of-sight direction to a light source. Supplemental funding (Supplement Nr 1) to the contract to support experimental studies of this idea led eventually to the development of the Photo-Electric Rotating Slit Elevation and Azimuth Sensor (PERSEAS). Details of this device are presented in the final report covering the experimental studies under this contract (Reference 5.1).

Many individuals contributed time and talent to the performance of the research and to the subsequent documentation of results contained in this final report. Unfortunately, not all of these individuals can be recognized, and it is disappointing that the credits indicated herein do not necessarily reflect accurately the value of the contributions nor the amount of effort expended.

Of special value in the production of this report were the skillful efforts of Jerry A. Hawkins who devoted much time to the technical editing of a major portion of its contents. In some instances extensive
rewriting and reorganization of material were involved. His contributions included the editing and rewriting of all six appendices, much of the composition of Chapters One and Two, the discussion of pressure waves in Chapter Three, and revision of the discussions of inertial systems and gravitational fields.

Dr. Roy Pietsch provided helpful consulting services and advice about the numerous problems in optics and laser devices. In addition, he offered some welcome critical evaluation of, as well as some modification to, the treatments of electrostatic and magnetic fields, and of nuclear radiation.

Dr. Mark O. Glasgow conducted the original mathematical studies covered in final form in Appendices 1, 2, 3, 5, and 6 and wrote the original trajectory-synthesis discussions of Chapter Two. He also provided answers to many of the mathematical questions which arose during the course of the studies, especially those questions pertaining to data handling. The one-, two-, and three-station geometry studies of Appendix 4 were carried out by Dr. Hugh A. Williamson.

Mr. Charles R. Longwell, who was associated full time with the program from its beginning, carried out the initial studies of magnetic fields and nuclear radiation as applied to scoring. Mr. Charles H. Hayes made similar studies of electrostatic fields, gravitational fields, and inertial systems, and helped assemble and classify the publications listed in the bibliography. Messrs. Hayes and Longwell collaborated to produce the major portion of Chapter Four, Electromagnetic Radiations, and in general contributed in many other ways that were necessary to the successful completion of a study program of this type.

During the first few months of the program Mr. J. B. Oliphint shared the responsibility of directing the study efforts and was engaged in the mathematical studies of the encounter geometries.

In order to facilitate the publishing and subsequent handling and use of this report, it is being presented in two volumes. Volume I contains the basic discussions and results related directly to the study of scoring methods. Volume II contains the six appendices which support elements of the discussion presented in Volume I.

The report is unclassified. This classification has necessitated the omission of some material which would have revealed the anticipated performance characteristics and expected tactics of future anti-
satellite and anti-ICBM weapon systems. Data of the type omitted will help to determine the look-angles, data-taking rates and time responses to be required of an eventual scoring system and must surely be included in any subsequent studies of the type conducted here.

William H. Purdy
Research Scientist Associate V
ABSTRACT

The results of a study of the applicability of seven classes of physical phenomena to trajectory scoring - measuring the relative trajectory of a munition with respect to its target - are presented. The seven phenomena (classes) considered are:

1. electrostatic fields
2. magnetic fields
3. nuclear radiation
4. gravitational fields
5. inertial systems
6. pressure waves
7. electromagnetic (optical) radiation

In addition, several mathematical studies which treat the encounter geometry, the relationship of measurement accuracy to position errors, data-handling problems, and the influence of own-ship angular motion on the accuracy of position determination are presented, in Appendices 1 through 6. Included also is an extensive bibliography. Scoring-system recommendations are made for the three general target classes: satellites, intercontinental ballistic missiles and aerodynamic-type vehicles.

PUBLICATION REVIEW

This technical documentary report has been reviewed and is approved.

L. S. KARABLE
Lt Colonel, USAF
Chief, Target Development Laboratory
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1. INTRODUCTION

This report gives the results of a scoring methods study performed under Contract AF 08(635)-2631. This was a study of phenomena-sensing methods and measurement techniques for obtaining complete data on the trajectory of a munition in the vicinity of and with respect to the following types of targets: (1) upper-atmosphere vehicle, (2) satellite, and (3) ICBM. This chapter gives the study requirements outlined in the contract and discusses both general and specific scoring problems in order to establish the terminology used in the remainder of this report and to justify the approach used in the study. The next chapter gives the results of mathematical studies, and Chapter Three discusses the potential value of phenomena other than electromagnetic radiation for trajectory scoring. Chapter Four considers optical methods of obtaining trajectory data. Radio and microwave systems were not considered specifically in this study because numerous organizations specializing in these fields have already studied such devices. Chapter Five summarizes the findings of this study and recommends possible solutions to the various scoring problems considered. A list of the references referred to in each chapter is given at the end of that chapter. A master list of all references used in this study appears at the end of this report. For conciseness and clarity in the main body of this report, six of the mathematical studies done under the contract are given as appendices, and the results obtained are merely summarized in Chapter Two.

2. SCORING SYSTEMS

Evaluation of the effectiveness of a weapons system in attacking a particular type of target requires information about the relative trajectories of the attacking missiles or projectiles with respect to the given type of target which is moving in a specified manner at the time of intercept. The complete specification of the problem includes the nature of the target trajectory and the attack conditions as well as the characteristics of the target and missile.

The method by which the trajectory data are made available is said to be a scoring system. By definition, a scoring system includes the techniques, procedures, and equipment for sensing and/or measuring weapon-target environment interaction phenomena, reporting the data to a receiver, reducing the data, and computing the trajectory
information. By this definition, scoring requires an actual attack by a specified missile or projectile against a real or substitute target: a simulation system is not a scoring device although some scoring systems can be made to control and/or evaluate simulated attacks (no missile fired).

Scoring systems are usually classed according to the type of trajectory data obtained as follows: (1) trajectory (relative range $R$, relative azimuth $A$, and relative elevation $E$ vs time $t$, or equivalent data), (2) spherical firing error (values of relative Cartesian coordinates $X, Y, Z$ and of time $t$ corresponding to the minimum value of range $R$), (3) planar firing error (frequently $X, Y$ at time $t$ when $Z = 0$), (4) scalar-type miss-distance ($R$ vs $t$), (5) miss-distance (minimum value of $R$ and corresponding $t$), and (6) proximity ($R$ less than or equal to some specified value during intercept). Relative coordinates are usually not obtained directly unless the sensing devices are mounted on the missile or on the target; type (3) data can be obtained by a scoring system with sensors mounted on the interceptor which fires the missile(s).

The equipment which collects the data (and transmits it to a receiving station if required) is sometimes called a scorer. The definition places no restriction on scorer location; it may be (1) interceptor-borne, (2) target-borne, (3) ground-based, (4) cooperative (divided between target and missile), or (5) missile-borne. The missile normally carries a dummy warhead and is not recoverable; the target usually survives the attack and can be recovered and reused. Scorers of types (2) through (6) will normally be target-borne or cooperative and will never be interceptor-borne or ground based.

Since the missile is normally destroyed and frequently has very little available space, class (5) systems have not been used. In class (4) systems the missile usually carries only a radiation source (light, flare, transmitter, reflector, etc.) and is said to be augmented. The target is frequently relatively small and must be augmented to be realistic or even visible to detectors on the interceptor and/or missile.

The cost of a missile large enough to carry scorer equipment in the warhead space is more than sufficient to justify a class (5) system if this is advantageous. If both missile and target are non-recoverable, loss of the scorer is not a consideration.

A class (5) scorer cannot be used, of course, if its operation interferes with the guidance of the missile or if installation of the sensing device(s) changes the flight characteristics significantly. Some missiles, notably the Sidewinder, are obviously unsuitable for a class (5) system because of their overcorrected tracking. If these objections do not apply, the following possible advantages may be
considered: (1) a sensing system with hemispherical coverage or less (often considerably less) will be able to see the target up to nearly minimum range, and the problem of obtaining spherical coverage will be avoided if extrapolation is an acceptable method of obtaining the remainder of the trajectory; (2) the missile may have wings, fins, or canard surfaces suitable for mounting sensing devices; and (3) the guidance system of the missile itself may provide some of the data required by the scorer.

3. FUTURE USAF SCORING REQUIREMENTS

Three future USAF scoring requirements are specified by the contract, and one requirement was added later by oral agreement. The relative trajectory \( (X, Y, Z \text{ or } R, A, E \text{ vs } t) \) of the missile with respect to the target is wanted in each case for all \( R \leq R_{\text{Max}} \). The following missile-target combinations were to be considered: (1) a projectile whose closing rate on a target is 6,000 to 10,000 ft/sec at altitudes of 70,000 to 200,000 ft, \( R_{\text{Max}} = 3,000 \text{ ft} \), accuracy = \pm 100 ft; (2) a munition launched from a satellite at a target satellite orbiting at any altitude from 100 to 1,000 miles, \( R_{\text{Max}} = 500 \text{ ft} \); (3) a munition launched from the ground at the target satellite specified by item (2); and (4) a munition directed at an ICBM or its warhead during any phase (boost, mid-course, or re-entry) of its flight, \( R_{\text{Max}} = 2,000 \text{ ft} \). It was specified orally that emphasis should be placed on requirements (2) and (3) since these have the highest priority.

The position accuracy required was not specified for cases (2), (3), and (4). The values assumed for this study were 50 ft, 50 ft, and 100 ft, respectively; these are taken to be probable errors as is the usual practice. The range from the intercept point to the nearest ground station will be as much as 1,000 miles in case (1). This range is not specified for cases (2), (3), and (4) but obviously could be 1,000 miles or more; it may be necessary to provide relay stations for telemetered data.

Since the three phases of an ICBM trajectory have significantly different characteristics, they are considered separately. It is also necessary to distinguish at least two types of targets--conventional supersonic bombers and vehicles of the Dyna-Soar type--in the 70,000 to 200,000 ft altitude regime. Thus the four cases expand to at least six or seven types of targets. The problem is further complicated by assumptions about the characteristics and tactics of the attacking missile and about the closing rates expected for all except case (1).

4. TACTICAL CONSIDERATIONS
The scoring requirements are not completely formulated until the tactics to be employed are given. This information determines the closing rates and acceleration characteristics of the trajectories to be scored, the fields of view required for the scoring devices, and the time available for taking data. Since this information was not specified by the Contract and was not later supplied, it was necessary to formulate reasonable assumptions for use in the study.

It is fairly evident that for satellite-to-satellite encounters the timing, control, velocity, and acceleration requirements are considerably less stringent for coorbital or near-coorbital encounters in which both satellites are traveling in the same general direction. In this case the relative velocity of the munition with respect to its target will be fairly low; for purposes of this study, it will be assumed that the minimum closing rate is 2,000 ft/sec.

If an aircraft, a satellite, or an ICBM in the mid-course phase of its trajectory is to be attacked by a missile launched from the ground, the simplest tactic is a holding attack in which the missile is put in the path of the oncoming target and kept there as nearly as possible; the target then simply runs into the blast or particle cloud created by detonating the missile's warhead. The thrust and maneuver requirements on the missile for correcting initial aiming errors would be relatively light. The missile would be near the peak of a steep if not near-vertical trajectory and would have relatively little velocity; the closing rate would be determined by the velocity of the oncoming target.

Attacks on an ICBM during boost and re-entry phases would probably have to be nose attacks with relatively high closing rates. The attack during the boost phase could be made by a munition launched from a satellite. A ground-launched missile would be required for attack during re-entry; severe timing problems and aerodynamic heating rule out other attacks.

It is evident from the above discussion that the minimum closing rate for an attack on an ICBM could not be much less than 10,000 ft/sec; the maximum closing rate would certainly be 17,000 to 22,000 ft/sec or more, depending on which phase of the trajectory was being considered. The maximum closing rate for a satellite attack would be of the order of orbital velocity, about 25,000 ft/sec.

The closing rates for the air-vehicle scoring requirement were established by the Contract. The attacks on this vehicle could conceivably include tail chases and lead-collision courses. The 6,000 ft/sec minimum closing rate may be a little high for such attacks, but catching up with a high-speed vehicle is not easy, and fairly high closing rates are needed for reasonable success in such attacks.
5. LIMITATIONS IMPOSED BY THE SCORING REQUIREMENTS

Until more information can be obtained on sizes, shapes, and other pertinent characteristics of the missiles and targets (or substitute targets) which will be used, final selection of a scoring system for a particular scoring requirement cannot be made. A scoring system which is satisfactory in one situation may be completely unusable for another application.

Seven limitations are imposed by the scoring requirements given previously: (1) passive detection systems (no active tracking) will be required for all except ground-based scoring systems; (2) ground-based scoring systems will not be used except possibly for covering the boost and re-entry phases of an ICBM trajectory; (3) multi-station scoring systems and complete trajectory coverage by use of electromagnetic-radiation devices will be difficult to obtain except in ground-based and satellite-based scoring systems; (4) sensing devices, beacons, reflectors, etc. on vehicles traveling inside the atmosphere will usually have to be non-protruding and covered; (5) extremely high data recording rates will not be required except for satellite intercepts; (6) corrections for own-ship angular motion and possibly for bending will probably be required; and (7) augmentation is usually acceptable for the target only, and ranging systems based on signal strength of electromagnetic radiation will not be used.

The first limitation—that detection and/or measuring systems located in the vicinity of the intercept must operate without active tracking—is not new; reasons such as very high angular rates and excessive power and space requirements could be advanced for much simpler scoring problems than the ones considered in this study. This limitation does not mean that the scoring system cannot have moving parts such as mechanical scanning devices. It may be that electronic tracking, such as is used in phased-array radar, is capable of the required angular rates, but such systems would obviously present a severe antenna-mounting problem.

Limitation (4) is also not a new requirement. It is commonly necessary to protect sensing devices from damage or warping by the air stream. At the high supersonic and hypersonic speeds of the vehicles covered by the new requirements, it will also be necessary to consider the consequences of and provide protection against aerodynamic heating; aerodynamically clean shapes will be required for both missile and target.

The problem of scoring an attack against a satellite by a munition launched either from the ground or from another satellite can be solved by the development of a scorer satellite. The scorer system would have weight and size limitations as usual and might have to be
folded for protection until actually in orbit. No limitations on scoring method except items (1), (5), (6), and (7) are apparent, however; multi-station scoring systems with sensors mounted on folding booms could be employed, for example. A multi-station system is defined to be one which uses data from two or more detectors (or sets of detectors) which are physically separated in space.

In contrast, it seems likely that the targets employed in the air-vehicle scoring situation specified above will frequently be substitutes (drones) considerably smaller, lighter, and (most important) cheaper than real targets; they will probably be rocket-powered and will probably have relatively small and thin fins, canard surfaces, and/or wings for stabilization and control. The VKD2B-1 Mach 2, delta-wing drone is a radar- and infrared-augmented, rocket-powered target of this type. Even if it is assumed that a multi-station scoring system with detectors in the wing tips could be developed for such a target, the wingspan is too small (3'-3") for good accuracy, and the coverage would be inadequate because the body of the target would limit the field of view. Since a multi-station system with detectors in the body would not cover the nose and tail attacks which are to be expected, it can be concluded that multi-station methods will not be used unless the targets are designed for such systems.

Similar remarks apply to ICBM intercepts when a substitute target such as a solid-fuel sounding rocket is used. A full-size ICBM is large enough to have a forward-looking multi-station scoring system installed in its nose, but the range capability and field of view would be limited as before unless the sensors were mounted on extensible booms. It is doubtful if the system could survive re-entry with or without the booms, however.

If a holding attack (defined previously) is planned, it may be better to mount the scoring system in the missile. The missile will have a lower speed than the target and it may be possible to mount a multi-station scoring system in the body of the missile since the missile will have its axis roughly perpendicular to the target trajectory.

The small drone targets do not have the size or appearance of a real target in general, and augmentation is necessary. Augmentation of the missile would not be required for a class (5) (missile-borne) scorer, but in other cases might be desirable in order to improve the signal-to-noise ratio; alternately, the missile could be illuminated by a source on the target. Unfortunately, augmentation of the missile by a beacon, antenna, or reflector is undesirable since it may change the flight characteristics; in some cases it may not be possible to mount a suitable source or reflector. For example, where could a light source be mounted on a rocket so as to be visible in all directions yet protected from damage?
Thus a scoring system should usually not be cooperative or require augmentation of any but the scorer vehicle. The observed vehicle may emit radio-frequency or microwave radiation for control, tracking or telemetering purposes, however, and may naturally radiate strongly in the infrared region. The possibility of obtaining trajectory information from such signals should not be ignored in developing a scoring system; the requirements are severe enough without wasting any opportunities for obtaining data.

It would be convenient to assume a constant-strength source of electromagnetic energy radiating uniformly in all directions, preferably monochromatic, but this would be a mathematical fiction, and no suitable approximation is likely to be obtained under the scoring conditions being considered. It follows that ranging devices based on measurement of received signal strength will not be used.

It is fairly evident from the above discussion that complete trajectory coverage by use of electromagnetic radiation will be difficult if not impossible to obtain except in ground-based and satellite-based scoring systems. Most instruments are directional, and the severe mounting problems expected will not improve this situation. Thus instruments mounted to obtain the first half of the trajectory (more or less) will probably not be able to follow the last half of the trajectory, and vice versa.

It will frequently be extremely difficult or impossible to mount the instruments needed for $4\pi$ steradian coverage. Moreover, the design of the scorer will usually be simplified if less than hemispherical coverage is required. It will probably be desirable in general to limit the field of view to that required by tactical considerations for obtaining most of the first half of the trajectory. If the mathematical nature of the trajectory is known, the missing parts can be obtained satisfactorily by extrapolation.

The coverage problem could be solved, of course, by observing the motions of the missile and target from a third vehicle following the missile or target or stationed near the expected intercept point. This solution is obviously impractical in general.

The feasibility of using ground-based (or ship-based) equipment to satisfy future scoring requirements can be estimated from information contained in the scorer conference report (Reference 1.1). This report states that the accuracy of the phase-comparison system MATTS (Multiple Airborne Target Trajectory System) and presumably also of MIDAS (Missile Intercept Data Acquisition System) is about 100 ft at 100 nm (nautical miles) with a sampling rate of 20 samples/sec. The accuracy of the tracking radars at Eglin Air Force Base is not given, but it is certainly less than the resolution of the angular
data (0.04 mil or 24 ft at 100 nm); the recording rate is as high as 100 samples/sec.

Various values are given for the approximate bounds of the three phases of an ICBM trajectory. One reference gives 400,000 ft as the altitude at which the boost phase ends and also as the altitude at which re-entry begins. During the mid-course phase, a missile with a 5,500 nm range rises to 500 nm and falls back; if the range is reduced to 500 nm, apogee is at 3,500 nm. These figures and those of the preceding paragraph show that ground-based and ship-based scoring systems are capable of giving boost and re-entry and parts of the mid-course phase to the nearest 100 ft. If the same accuracy is required for the relative trajectory of a missile attacking an ICBM, however, the trajectories of the missile and the ICBM must each be measured to the nearest 50 ft. In this case the phase-comparison systems are adequate only to 304,000 ft (50 nm). The tracking radars would presumably cover the boost and re-entry phases adequately except in the near vicinity of the intercept; a telescopic camera slaved to the radar could probably be used to obtain the missing data. These estimates neglect down-range distance and are thus somewhat liberal but they are adequate for discussion. Evidently the present ground-based scoring systems would cover or could be made to cover two phases of the ICBM-missile intercept scoring requirement; however, at least an order of magnitude improvement in accuracy would be required for adequate coverage of the mid-course phase. This improvement in accuracy is probably feasible, but the cost of the development and of the necessary tracking ships rules out this solution.

Ground coverage of the satellite-intercept problem can be ruled out by reasoning similar to that given above, by the more stringent accuracy requirement, and by the inconvenience of having to arrange for the intercept to occur nearly overhead. Observation of an aircraft from a ground station 1,000 miles away is, of course, impossible; the maximum altitude given in the requirement (200,000 ft) would be about 440,000 ft below the horizon. It should be added, however, that one solution to the latter scoring problem would be to arrange for the intercept to occur above one of the existing tracking stations. This might be inconvenient but not compared to the difficulty of a similar arrangement for satellite intercepts.

If ±100 ft is taken to be the accuracy requirement for ICBM and air-vehicle scoring, there is not much point in obtaining more than about one point for every 200 ft of trajectory or 30 points on the maximum-length 6,000 ft trajectory. Taking 20,000 ft/sec as the maximum closing rate for the ICBM intercept gives (20,000/6,000)30 = 100 observations/sec as the required sampling rate. The maximum rate for the air-vehicle scoring requirement is (10,000/6,000)30 = 50 samples/sec. For the satellite-intercept problem, a sampling rate of
(25,000/1,000)20 = 500 samples/sec to obtain a maximum of 20 points. For comparison, the maximum frame rates and picture sizes for three Benson-Lehner range cameras are as follows: (1) Model HS-70, $2\frac{1}{4} \times 2\frac{3}{4}$ in. pictures, 80 frames/sec; (2) Model HS-35, $0.920 \times 0.723$ in. pictures, 300 frames/sec; (3) Stereo-flex, half-frame 16 mm, 15,000 frames/sec. The recording rate for the Eglin radars was given previously as 100 samples/sec.

These sampling rates are estimates offered for discussion, and the final scoring requirements may well specify different values. The rates of 50 and 100 samples/sec are both relatively low, but some data systems would require improvement to attain either of these rates. The sample rate for the satellite problem is a fairly severe limitation on the scoring system.

Three out of the six types of scorers (proximity, miss-distance, and scaler-miss-distance types) have spherical symmetry and are unaffected (neglecting possible unintentional instrument sensitivity) by own-ship angular motions and bending of the vehicle due to aerodynamic loading, heat stresses, etc. The planar firing error indicator is somewhat sensitive to rotation about an axis perpendicular to the plane of measurement; it is relatively insensitive to rotation about an axis in the plane of measurement provided the angle remains small. The spherical firing error indicator is, of course, sensitive to rotation about all three coordinate axes, but in practice the effects of angular motion are unimportant if the vehicle does not depart too far from the desired orientation. It will usually be unnecessary to correct the data obtained from either of the firing error indicators for aircraft angular motions in straight-and-level flight.

The trajectory scorer alone is quite sensitive to angular motions and to bending, and the sensitivity increases as the maximum range for scoring is increased. If corrections for these motions are not made, the trajectory determined by the device may be severely distorted in a manner that cannot be corrected properly by smoothing or curve-fitting operations. If the scoring method is based on assumptions about the nature of the relative trajectory (such as constant relative velocity or straight-line relative course), the results obtained neglecting angular motion could well be pathetic.

It can be said in summary that showing that a proposed method of scoring is capable of obtaining the necessary data under idealized conditions is not sufficient. A realistic appraisal of the method should be made, and it should include the following items: (1) estimated size and weight, (2) mounting problems, (3) interference problems, (4) effects of instrument errors and of assumptions and approximations, (5) background noise, dropped-data, and spurious-signal problems, (6) data-reduction problems, and (7) computational problems. Not all
problems can be identified without building and testing a prototype system, but careful study should reveal most of the important difficulties.

This study was not concerned with hardware or telemetry-interference problems since these considerations should be left to people who are entirely familiar with scorer construction and operation problems. The involvement in sensor-mounting, data-reduction, and computational problems has been limited to efforts to foresee any unusual difficulties which might develop. Thus items (4) and (5) in the above list were the only ones considered in much detail. In short, an effort was made to keep the studies realistic without becoming involved in work which would be a duplication of effort or could better be done elsewhere.

6. METHODS OF OBTAINING TRAJECTORY DATA

Scoring data can be obtained by measuring the acceleration or velocity of each vehicle with respect to a coordinate system fixed in inertial space by use of so-called inertial instruments. If the initial conditions are known, the trajectory of each vehicle can then be obtained by integration, and the desired relative trajectory is obtained by subtraction.

Most scoring devices function by measuring appropriate properties of fields or waves reflected from, radiated by, or resulting from the motion of the vehicle under observation; the waves have vector properties and the fields are vector fields. Scalar fields such as ionization, temperature, index of refraction, carbon dioxide content, etc. can indicate the space paths taken by missile and target, but the paths are not related to each other (i.e., no point-by-point time history can be obtained). Moreover, a scalar field cannot be surveyed by either missile or target unless it has reflective properties for some type of radiation (a contrail, for example, reflects light).

The measurable properties of a wave are amplitude, frequency, phase, orientation of wave front, and time of travel from scorer to reflecting (or transponding) object and back; the measurable properties of a vector field are magnitude and direction at the point of observation. Shock waves produced when a body moves through air at supersonic speed constitute a special case; their useful properties are the amplitude and period of the pressure disturbance and the conical shape of the wave itself. Not all wave properties are useful in a given situation. The magnitude of a reflected wave for example, can seldom be used to determine range.

The most useful fields and waves appear to come from a single point source and have spherical symmetry; i.e., the amplitude or magnitude at any point is a function of distance from source only.
Multiply-reflected signals are usable, but except in optical systems, some loss of accuracy is to be expected unless the observed body is far enough away from the detector to appear to have the essential properties of a single source. Phenomena which have rotational symmetry—a bow shock wave, for example—are also usable if a straight-line trajectory can be assumed.

The properties of radial fields and waves which can be used to determine distance are (1) size or density of image (in photographic-type systems), (2) wave amplitude or field strength, and (3) signal travel time from scorer to observed object and back. All of these methods require only one observation station; if two or three relatively well-separated stations can be used, and if orientation angles of the line-of-sight are measured at each station, range can be determined by triangulation.

There are four types of detectors for measuring orientation of a line-of-sight: (1) tracking, (2) direction-cosine, (3) phase-front or wave-front, and (4) amplitude. Tracking devices and direction-cosine detectors (including cameras) obtain data at a single station. Phase-front and wave-front devices use three or more detectors to determine two phase differences or two differences in arrival time (of a pulse). An amplitude system would use ranges computed from amplitudes measured at three points to determine the line-of-sight orientation angles at any of the points.

By definition, a trajectory scorer determines $R$, $A$, $E$ (or equivalent quantities) as functions of time; however, there is no requirement that these data be determined directly. For example, the velocity of the observed object with respect to the scorer could be obtained as a function of time by observing the line-of-sight angular rate and the doppler frequency shift. If the range at one point is known—and this could be determined by triangulation if two angle-measuring systems on the scorer have overlapped fields of view—the velocity data can be converted to range data by numerical integration.

Present scoring devices are frequently based on one or more assumptions about the trajectory; (1) plane wave (or phase) front (very distant source), (2) straight-line relative trajectory (target and missile have constant velocities throughout the region of interest), (3) path of missile parallel to path of target, and (4) speed of missile is known or can be determined separately. The only one of these assumptions which seems likely to be acceptable under any of the scoring requirements given previously is (2); this was one of the problems considered in the mathematical studies reported in the next chapter.

A scoring system based on assumptions about the trajectory is not a general-purpose system, of course, but the object of this study
was to determine suitable systems for each scoring requirement, and there was no restriction to general-purpose systems. A special-purpose system may or may not be better than a general-purpose system suitable for the same application; there is no reason for rejecting such systems in advance. The advantages of special-purpose systems is that trajectory information is extracted from measured data which would otherwise be insufficient; data reduction and/or processing may be considerably more complicated than usual, but the scorer is simplified. Even if assumptions about the trajectory are not used in the scorer itself, it is always helpful to know the expected characteristics of the trajectories to be measured; this information can be put to good use in smoothing, interpolating, extrapolating and curve-fitting operations.

7. SCOPE OF THE STUDY PROGRAM

In brief the objective of the studies reported herein was to investigate all physical phenomena which might possibly be applied to the solution of the scoring requirements given above. The phenomena of concern were those resulting from the interaction of the vehicles with their encounter environment. The studies were to determine which of these phenomena have real potential for use in a scoring system and what techniques would be suitable for trajectory scoring. The phenomena and associated techniques were to be placed in one of two categories. Category one was to include those phenomena and techniques usable for trajectory scoring without any advances in the state of the art; category two would include those techniques which would be feasible only after further research on and/or further development of measuring instruments, sensors, or other equipment. Any phenomena not included in one of these categories have zero potential for scoring.

A further objective of the study was to determine the relationships among the variables characterizing each feasible technique and the effects of measurement errors on scoring accuracy.

The study was restricted to target-borne and/or missile-borne systems with emphasis placed on non-cooperative systems and on solutions to the satellite-intercept problem. There was no indication that interceptor-borne systems would be of any practical interest; a study of ground-based systems would have amounted to a duplication of effort.

8. ORGANIZATION OF THE STUDY PROGRAM

Initial efforts under the program led to the grouping of the studies into two general areas: (1) Mathematical Studies and (2) Sensing Devices and Techniques. The mathematical studies were conducted to determine the geometrical relationships which would characterize the various encounters and to determine how the various
parameters are related through the geometry. For the most part, though, the interest lay in the error studies which were concerned basically with the effect of an error in measuring one quantity on the accuracy required in measuring other quantities. Ultimately, however, it was desired to determine and have readily available relationships for the errors in the position components as functions of measurement errors. The results of these studies are discussed in Chapter Two of this report. Principally because of the quantity of material generated through these studies, their details have been included in Appendices 1 through 6.

The study of sensing devices and techniques was started with a survey of the literature on hand in the MPRL library and in the University libraries. Those reports which were pertinent to the program were abstracted and a reference list and card file were developed to facilitate use of the reports. Additional reports were ordered and a general effort was made to collect data on the state-of-the-art in sensors and line-of-sight measuring techniques. As a result of this initial search for information, a relatively complete bibliography was compiled which, it was recognized, would be a valuable aid to subsequent programs of the type conducted here. For this reason, this bibliography is included in this report.

The principal concern in the study of sensing devices and techniques was the basic physical phenomena which might characterize the encounters between advanced vehicles. After some consideration, it appeared that all of the phenomena which might have potential utility in scoring techniques could be included in seven classes. The classes established were: (1) electrostatic fields, (2) magnetic fields, (3) gravitational fields, (4) inertial effects, (5) pressure waves, (6) nuclear radiations, and (7) electromagnetic radiations.

Only slight consideration was needed to become convinced that the phenomena having the greatest scoring potential were electromagnetic (EM) radiations. In fact, there is such a variety of problems involving sensors, sources, backgrounds, component combinations, etc., that each of the other phenomenon studies is dwarfed by that on EM radiations. For that reason one entire section (Chapter Four) is devoted to optical phenomena. The other six classes are treated in Chapter Three. Radio-frequency and microwave devices were not studied in detail since organizations specializing in these fields have already made such studies. All that is required in this field is a continued monitoring of new developments to locate any new devices which are potentially useful for scoring.

The results obtained in the phenomena studies are summarized in Chapter Five and recommendations are made for solving each of the
scoring problems considered. These recommendations are of course based on the findings of the studies made to date and are subject to revision as advances are made in the state-of-the-art and as the scoring requirements themselves become better defined.

9. REFERENCES

CHAPTER TWO
MATHEMATICAL ASPECTS OF TRAJECTORY SCORING

1. INTRODUCTION

Trajectory scoring may be defined as the techniques and procedures for sensing the occurrence of weapon-target environment interaction phenomena, reporting the data to a receiver, and computing the relative trajectory of the weapon with respect to the target during the time of intercept. This definition should be construed in a broad sense, general enough to include the purely mathematical aspects of planning a scoring mission, choosing the instrumentation, reducing the data, and computing the relative trajectory. In particular, the techniques of error analysis should be included in the definition.

The mission of the scoring methods study has been to investigate those phenomena which have potential for use in trajectory scoring and to determine how they can be used to obtain trajectory data. This chapter considers the mathematical aspects of the problem. Discussions of error analysis, planning for scoring encounters, and selecting sampling rates for recording data are given in the following sections. The last section summarizes six mathematical studies made under this contract. Detailed reports of the results obtained in these studies are given in Appendices 1 through 6, each of which is self-contained and could be issued as a separate memorandum or report. All studies except Appendix 2 have been reported previously in papers transmitted with the monthly progress reports, but the papers given here have been thoroughly revised and some additional material has been added.

2. REMARKS ON ERROR ANALYSIS

A standard procedure is used for assessing the effects of small random errors. The functional relations between the quantities to be computed (such as the relative position components) and the observed quantities (such as angles, distances, and their rates) are first obtained; in a complete error analysis the methods used in data reduction must be considered in developing these relations. These functional relations may be differentiated to obtain the linear relations for the differential errors in the computed quantities in terms of the differential errors in the data. Statistical averaging may be used to obtain the variance relations. These mathematical procedures are discussed in the last portion of Appendix 4.
The implicit assumption was made in the preceding discussion that systematic errors have been eliminated from the observed data. Corrections for instrument and other known errors (such as refraction) should always be made in the data reduction. Corrections for refraction are discussed in References 2.1, 2.2, and 2.8; no refraction corrections are needed if the scorer is mounted on either the missile or the target. Instrument corrections could be made not only for static but also for dynamic errors as well. Use of instrument transfer functions might be necessary to make corrections for dynamic errors. Systematic errors which are not corrected have a much more serious effect on the reliability of trajectory data than random errors of the same magnitudes. The latter would tend to average out in the data reduction while the former would not.

While a complete error analysis necessitates consideration of data-reduction methods, it is not always required for every purpose. For engineering work approximate error bounds are usually satisfactory. Such bounds are easier to obtain and to interpret, and they are generally conservative; also approximate relations or simplifying assumptions may be used in their development. A simplifying assumption that has been used in some of the investigations reported in the appendices is that the relative velocity is constant over the range interval of interest for scoring. An approximate relation for the maximum position error due to uncorrected angular-velocity effects was used in obtaining the bounds for instrumentation accuracy discussed in Appendix 2. Other examples of justifiable simplifying assumptions could be cited.

The distributions of the various errors may be important considerations. While normality is often assumed, it may not always be justified. Experimentation may be necessary to determine the distributions of the errors. Random errors in instrument observations tend to be normally distributed; also as the number of independent sources of error increases, the distributions of the resulting errors in computed quantities tend to normality under mild restrictions as shown by the Central Limit Theorem of statistics. Even if nothing is assumed about the distribution, the probability that the errors will not be greater than a given size can be obtained by use of a theorem due to Tschebyscheff which is quoted in Appendix 4. The accuracy is, of course, not as good as it would be if the nature of the distribution were known.

When all the data are combined to give a trajectory, the reliability will be better than that obtained for a single point or for a lesser number of points. Smoothing operations tend to average out the random errors and reduce noise effects. The data-handling operations of smoothing, interpolation, extrapolation,
and possibly differentiation and integration will be required in most trajectory-synthesis problems. Since many excellent references on these procedures are available in standard texts and in reports of MPRL and other laboratories, these procedures were not investigated for the present study. However, two data-reduction methods of general applicability for combining redundant data from a wide variety of sources were briefly investigated. One of these is a method for least-squares adjustment of data which is developed in Appendix 5. This method has had important applications in the determination of orbits and other trajectories (Reference 2, 8); it is well adapted to the scoring problem of position fixing using redundant data as well as to instrument calibration. The other method investigated was the use of weighting factors, discussed in Appendix 4. This method is considerably simpler, though less general, than the method of least-squares.

Bad observations should, of course, be rejected. Rather arbitrary criteria dictated by experience are frequently used in deciding to keep or reject data. A rule of fair generality is that a data point may be rejected if the difference between the observed value and the adjusted value is greater than or equal to three standard deviations. For any error distribution function, the probability of an error of this size is quite small when the number of observations is limited.

The subject of error analysis is not discussed in any separate appendix although it enters into the investigations of each of these appendices in a natural context. In particular, the allowable position errors for the three kinds of scoring encounters specified in the contract are used in Appendix 1 to obtain equations for the allowable errors in velocity and acceleration. Bounds for the angular rates of the relative range vector are discussed in Appendix 3; these bounds will be useful in selecting sensing equipment with a suitable scan rate. Criteria for neglecting own-ship angular motions are discussed in Appendix 2 for the scoring encounters of interest. A special application of the least-squares method for adjustment of data is made in Appendix 5, and the resulting gain in reliability is determined. Error relations for various scoring geometries are given in Appendices 1 and 4.

3. PLANNING FOR SCORING ENCOUNTERS

The portion of a relative trajectory during which data are taken for scoring is only a small part of a much longer trajectory. In order to arrange for the weapon-target encounter, study of the probable target trajectory is necessary. With most missile systems some sort of terminal guidance would be required. Standard trajectories may be sufficient for many purposes in planning;
certain approximate trajectory relations are discussed in Appendix 1. For the actual encounter, the latest and most accurate trajectory information may be needed to arrange last-minute details. Orbit predictions from trajectory observations in real time are practical (see Reference 2.2) and might be used as inputs for making these arrangements. For satellite-vs-satellite encounters, study has shown that mission requirements are less stringent for co-orbital or near-co-orbital encounters, with both satellites orbiting in the same general directions (Reference 2.7).

Tactics to be employed profoundly influence the functional relations for the relative trajectory variables during the time interval of interest for scoring. The instrumentation required, data sampling intervals, etc. will also be affected. Instrumentation that would be acceptable for one application may be totally unsuitable for another scoring situation. The expected relative trajectory should be analyzed for probable time behavior of the relative-trajectory variables, and bounding relations; the results would be useful in choosing the instrumentation, arranging the initial conditions, etc.

The most accurate methods presently used for satellite orbit determination are optical methods employing cameras. Such methods are capable of position determination for near-earth satellites with a standard deviation of the order of a few seconds of arc (Reference 2.8). The data require a lengthy reduction process, however, so that the results are excellent for analysis, but they are not available for short-time predictions. Predictions of satellite orbits for a few revolutions ahead by use of radar observations are said to have probable errors of the order of a few miles. Radar distances and range rates may be measured quite accurately, but the angular errors are large enough to have a serious effect on the accuracy of the predictions.

The use of optical and radar observations from ground stations to establish the relative trajectories for all of the scoring situations described in the contract would put accuracy requirements on the instrumentation beyond present capabilities. Since these well-known techniques are not new, ground-based scoring systems were not investigated in this study. The emphasis has been put on relative-trajectory determination from observations made aboard either the target or the weapon, or both.

4. ON TIME INTERVALS BETWEEN DATA POINTS

If instrument readings are recorded continuously during a scoring encounter, there is no problem of setting a time interval between data points. If the sensing equipment is of a scanning type so that data are only obtained periodically, or if data must be
sampled periodically for telemetering purposes, the problem of selecting a suitable sampling frequency becomes important. No exact methods were found to solve this problem. However, some arbitrary guiding principles should prove useful in applications.

A permissible error in relative position is generally specified. If the data are taken frequently enough to obtain position fixes separated by no more than this allowable position error, such data-taking frequencies should be acceptable. If the maximum closing rate is $R$, the maximum permissible error in position is $\ell$, and the time interval between the position fixes is $\Delta t$, this formula would have $\Delta t \leq \ell/R$. The assumption is made that any errors due to interpolation between position fixes are negligible in comparison to the errors in the data points.

In practice, matters may be not quite so simple. A position is not normally determined from one instrument alone, but must be computed from combined readings of several instruments reduced to simultaneous values. For the better kinds of numerical smoothing and interpolation formulas to be used, as many as ten to twenty data points may be needed. This bound on the total number of observations to be taken can be used to determine the sampling frequency when $R_{\text{Max}}$ and $R_{\text{Max}}$ are known.

It may be neither necessary nor desirable to sample all the data at the same frequency. Some of the information may have considerably greater weight than other data as far as the accuracy of the resulting position determination is concerned. A rough quantitative estimate of the relative weights may be obtained from the variance relations between computed position and observed quantities. If the product of the variance of an observed quantity and the square of the partial derivative of the computed quantity with respect to the observed quantity is comparatively large, then the observed variable is comparatively important to the accuracy of the computed quantity. Sampling of this observed quantity at more frequent intervals than other variables of less importance would be indicated for optimum results. This principle indicates that when the weighting factors are equal in the variance relations, the variables with the greater variance should be sampled more frequently; more than one telemetering channel may be desirable for such data.

Another principle is that other things being equal, the observed quantity which is changing most rapidly should be sampled most often. This should allow more accurate interpolation for this variable and result in greater accuracy for the computed position.

It is thus seen that the required data-recording frequencies
vary greatly with the attack situation. It appears that there is little difficulty to be expected from getting data at too frequent intervals; the problem may be to get data at as many points as needed. The problem is more acute for relative trajectories with high closing rates, and small maximum range. The maximum relative range for the attack of a ground-launched missile against a satellite is specified as 500 ft. If twenty position fixes are considered sufficient for establishing the trajectory, and a closing rate of 25,000 ft/sec is assumed, the 1000-ft range interval would be traversed in 0.04 sec; the time interval between data points would be 0.002 sec, corresponding to a data-taking frequency of 500 position fixes/sec. Such rates may be practical for some instrumentation and impractical for others.

5. SUMMARIES OF THE APPENDICES

5.1. Appendix 1. Trajectory Analysis and Synthesis - The use of relative-velocity or relative-acceleration data to obtain the relative trajectory is discussed. Equations and graphs are given relating the permissible errors in velocity and acceleration to the permissible position error and the duration of the trajectory. The linear-relative-trajectory (or constant-relative-velocity) assumption is investigated, and graphs are given for determining whether a given constant acceleration can be neglected. Expressions for range rate, angular rate, minimum range, and maximum angular rate are developed for linear relative trajectories; differential error relations are obtained for range, minimum range, and angular rate. The velocity and acceleration characteristics of a rocket are determined neglecting gravity and aerodynamic forces; the results can be used to determine the validity of the linear-relative-trajectory assumption when either the missile or the target or both have non-zero thrust during the encounter. Finally, the problem of non-zero relative acceleration is considered, and a particularly useful method for smoothing, interpolating, extrapolating, and integrating or differentiating trajectory data is discussed briefly.

5.2. Appendix 2. Effects of Own-Ship Angular Motion on Relative Trajectories - The transformations required to correct position, velocity, or acceleration data for own-ship angular motion and for bending are discussed, and an error bound for the effects of uncorrected own-ship angular motions on the trajectory is given. The use of inertial instruments to obtain trajectory data is considered, and it is shown that such instruments must be mounted on a stabilized platform in each vehicle. Criteria for neglecting own-ship angular motions are developed for each of the scoring requirements considered in this study; these criteria are expressed in terms of the permissible error in position, the
maximum range, and the duration of the attack.

If the effects of own-ship angular motion are likely to be too large to neglect, the preferred method of correction is to stabilize either the entire vehicle or a platform containing the measuring instruments; in either case, correction of the data will be unnecessary. It is shown that the accuracy of modern stabilization equipment is more than adequate; the selection of equipment can therefore be based on considerations other than accuracy such as size, weight, cost, etc. If the instruments cannot all be mounted on the stabilized platform, it will be necessary to measure and telemeter the platform angles and to correct for angular motions in the data reduction and/or processing. It may also be necessary to measure and correct for bending. If a stabilized platform is not used, the roll, pitch, and yaw angles can be determined by integrating measured values of the angular rates; this procedure is not recommended in general since it complicates the data reduction and is likely to have poor accuracy if the angular data are needed over any extended period of time. The above conclusions about the adequacy of stabilization equipment apply only to the short-duration attacks considered; the use of such equipment for long periods of time, as might be necessary in order to obtain initial conditions for a purely inertial scoring system, for example, is not considered in this paper.

5.3. Appendix 3. Rate Characteristics of Linear Relative Trajectories - In designing a trajectory-scoring system to satisfy a given scoring requirement, it is desirable to know at least the ranges of variation and the maximum time rates of change of the relative angular coordinates. It will also be useful to have some information on the general behavior of the angles and angular rates as functions of time. This information establishes the required field of view, helps to determine the instrumentation requirements, and will be useful in error analyses. This paper develops equations for the azimuth and elevation angular rates and bounds for these rates for the special case of a linear relative trajectory. A particular linear relative trajectory is taken as an example, and graphs are given of the relative range $R$, the Cartesian coordinates $X$, $Y$, and $Z$, the azimuth $A$, the azimuth rate $A$, the elevation $E$, and the elevation rate $E$, all as functions of time. Expressions for the angular rates and for bounds on the angular rates are also developed for the angles $\alpha$ and $\beta$ which would be measured by a photopotentiometer scoring system, for example.

5.4. Appendix 4. Minimum Data Requirements and Variance Considerations for One-, Two-, and Three-Station Measurements - This paper considers minimum data requirements and mean-square-error (variance) relations for one-, two-, and
three-station measurements of fields and waves which, for point sources, have spherical symmetry and appear to radiate from a point. The complications which may result when the source (observed object) is too large or too close to appear as a point are ignored. A few general remarks are made concerning one-station measurements, while all possible combinations of minimum sets of data (three distance and/or angle measurements) are considered in a systematic manner for two-station measurements. Only a few selected cases of minimum data are developed for three-station measurements. The equations for computing range, azimuth, and elevation are given without derivation for each case considered. Equations and procedures are developed for two specific cases of two-station angle-only measurements to show how the variance in the computed range is related to the variances in the measured data and how either the ranges of variation or the variances of the measured quantities are limited by the requirement that the variance in computed range should not exceed a specified value. The graphs which are included may be used to obtain numerical results. A method is also developed for weighting the measured quantities so that the variance of a computed quantity can be minimized if redundant data are available; a numerical example is given to illustrate the method.

One important conclusion of this study is that two- and three-station triangulation systems have field-of-view limitations determined by the overall accuracy requirements and the accuracy of the instruments used; improving the instrument fields of view beyond a certain point is useless unless their accuracy is also improved. The system viewing angle is also limited by the station separation; equations of the type developed can be used to select a base-line length consistent with the accuracies and fields of view of the instruments used. The accuracy and fields of view of triangulation systems used to satisfy the scoring requirements considered in this study may well be limited by the maximum length of the base line in each case rather than by instrument deficiencies. The accuracy can be improved, however, if redundant data are obtained.

5.5. Appendix 5. Least-Squares Adjustment of Two-Station Angle-Only Position Fixes With Reliabilities of Adjusted Data - If two-station simultaneous azimuth and elevation angle data on the position of an observed point are available, there is a redundancy of data for determining the position of the observed point by triangulation. A method for least-squares adjustment of such data is presented, and expressions are developed for the variances of the adjusted measurements and computed quantities of interest. In an average sense the variance of the range is reduced to approximately one fourth of its value before the least-squares adjustment, and
the standard deviation and probable errors are reduced by a factor of two. The original angular data are assumed to have equal variances and to be uncorrelated and unbiased.

One advantage of this process over certain other least-squares procedures is that preliminary estimates of position are not required. A disadvantage is that one condition equation is required to force the adjusted rays from two stations to intersect and an additional condition equation would have to be used for each added station; as a result, the adjusted data have non-zero covariances. The process could be readily generalized to include correlated data of differing reliabilities and to include more stations and more observational data; the process is not limited to angular data.

5.6. Appendix 6. Application of Polynomial-Based Smoothing and Interpolating Formulas to Scoring Problems - It is shown that relative Cartesian coordinate data for short-duration weapon-target encounters are probably well adapted to polynomial-based numerical methods of extrapolating, interpolating, etc., whereas relative polar coordinate data do not appear to be so well adapted. However, certain functions of the polar coordinates (viz., \( R^2 \), \( \tan A \), \( 1/\sec^2 E \), and \( 1/A \)) are shown to be well approximated by polynomials of low degree and are therefore suitable for use with polynomial-based formulas. A reasonable procedure for performing operations such as smoothing and interpolation on relative polar coordinate data would be to first convert to Cartesian coordinates, perform the operations required, and then convert the resultant values back to polar coordinates.

6. REFERENCES


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CHAPTER THREE
INTERACTION PHENOMENA

1. INTRODUCTION

Environmental characteristics attributed to or associated with the encounter or near encounter of two aerospace vehicles are treated in this section, with the exception of electromagnetic radiation which is treated later. The treatment is not exhaustive; it is directed toward scoring methods applications.

The approach in handling each of the phenomenon classes was to consider first the basic characteristics of each class and to determine and specify mathematical descriptions relating the physical phenomenon to scoring. This was done without first considering the target class or encounter regime since each phenomenon was either independent of the altitude, or the variations that did occur with altitude were included in the relationships describing the phenomenon. After this initial study was made, the phenomenon was related directly to scoring encounters with one of the target classes.

For these studies, rather ideal conditions, not to be expected in nature, have been assumed. When it is shown that even under these ideal conditions, a particular phenomenon has little if any scoring potential, it has been dismissed from further consideration.

2. ELECTROSTATIC FIELDS

2.1 Effects of Motion on the Field Components - Electric and magnetic fields at a point in space as measured by two observers in motion with respect to each other (such as two satellites in orbit traveling at different velocities with respect to the earth) are not equal. An electromagnetic field produced on a missile will in general appear differently to lets say the target of the missile and to the scoring device, than it will to the missile. It appears then that some care should be exercised in scoring a missile by observing an augmented field on the missile. Let us now examine the problem closer. Let $\vec{E}$ and $\vec{B}$ represent the electric and magnetic induction vectors. A subscript || or \perp will denote components parallel and perpendicular to the relative velocity vector between the missile and its target. The components of the field measured by the satellite are given by

$$B_{\parallel}' = B_{\parallel}; \quad B_{\perp}' = \gamma(\vec{B} - \vec{v} \times \vec{E}/c^2)_{\perp}$$

$$E_{\parallel}' = E_{\parallel}; \quad E_{\perp}' = \gamma(\vec{E} + \vec{v} \times \vec{B})_{\perp}$$
where
\[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \]
and the primes refer to quantities measured by the satellite, and unprimed quantities refer to those measured in the missile's coordinate system. For \( v \ll c \) where \( c \) is the velocity of light, terms involving \( c^2 \) may be safely dropped.

The above relations become
\[
\begin{align*}
B'_\parallel &= B_\parallel, \quad B'_\perp = B_\perp \\
E'_\parallel &= E_\parallel, \quad E'_\perp = E_\perp + (\vec{v} \times \vec{B})_\perp
\end{align*}
\]

Should the missile carry a permanent magnet, then the satellite would measure not only the magnetic field produced by the magnet but also an electrostatic field \( (\vec{v} \times \vec{B})_\perp \). The first set of relations also indicates that a magnetic field would be produced by a missile in motion if it carried a static charge. The above relations apply to instantaneous values; thus they are valid for time dependent fields as well as for static ones.

2.2 Electric Charge in Motion - The electromagnetic field of a charged particle in motion is different from the field due to a stationary charge. In the case of the charge moving with a constant velocity and the observer fixed, the electric field \( \vec{E} \) is given by

\[
\vec{E} = \frac{Q\vec{r}}{4\pi \varepsilon_0 r^3} \cdot \frac{1 - (u/c)^2}{[1 - (u/c)^2 \sin^2 \psi]^{3/2}}
\]

where \( \vec{u} \) is the velocity of the charged particle and the angle \( \psi \) shown in Figure 3.1 is the angle between the velocity vector and present range vector \( \vec{r} \). The first term in (3.1) is the field due to the static charge; the term in brackets which is obtained from Lienard-Wiechert potential expresses the effect of motion on the field. The same relation could be obtained from relativistic considerations.
For $u/c$ small and $\psi = 90^\circ$

$$|\vec{E}| \approx \left( Q/4\pi \varepsilon_0 \right) \left( 1 + (3/2)(u/c)^2 \right)$$

and if $u$ should equal 10 kilometers/second, by this relation we see that the effect of motion on the electric vector is to change $|\vec{E}|$ by less than one part in $10^9$ over that for the static case. The direction of $\vec{E}$ is also virtually unchanged.

The moving charge also gives rise to a magnetic field for which the magnetic induction, $\vec{B}$, is given by $\vec{u} \times \vec{E}/c^2$, a value which is indeed small for orbital velocities. As an example if $E$ were 1 volt/meter (a rather high value) and $u$ were 10 kilometers/sec, then $B$ would be of the order of $10^{-7}$ weber/meter$^2$ (or $10^{-13}$ gauss). For constant velocities at orbital speeds, it becomes apparent that the effects of motion on the electromagnetic field vectors can be neglected.

When the charged particle is accelerated, the field relations become more complex through addition of terms involving acceleration. Special cases of the field dependence upon the orientation of $\vec{u}$ are sometimes of interest. One instance is the radiation produced by a decelerating electron—bremsstrahlung.

Stratton divides the field into two parts—a velocity field which contains no acceleration and an acceleration field which goes to zero as acceleration goes to zero. The velocity field varies as $1/r^2$ while the acceleration field, which predominates at large range, varies as $1/r$. The acceleration field will not be pursued further since several different specific cases would have to be considered, each depending upon the orientation of $\vec{u}$ (for example, see Chapter 20, "Radiation From
\[ Q = Q_0 e^{-t/\tau} \]
\[ = \varepsilon E_{\text{max}} 4\pi r_0^2 e^{-t/\tau} \]

where \( t \) represents elapsed time and where \( \tau \) is the relaxation time of air obtained by dividing the permittivity \( \varepsilon \) by the conductivity; that is

\[ \tau = \frac{\varepsilon}{\sigma} \]  

(3.4)

At a distance \( r \) from the munition, the target-borne detector will attempt to measure the electric field which is given by

\[ E = \frac{Q}{4\pi \varepsilon r^2} \]
\[ = \frac{E_{\text{max}} r_0^2 e^{-t/\tau}}{r^2} \]  

(3.5)

If the detector has the capability of measuring a certain minimum field \( E_0 \), then the maximum measurable range \( r_{\text{max}} \) will be found by substituting this value for \( E \) in the equation above, and this leads to

\[ r_{\text{max}} = \sqrt{\frac{E_{\text{max}}}{E_0}} e^{-t/2\tau} r_0 \]

\( E_{\text{max}} \), the dielectric strength of air at ground level, is known to be (Reference 3.1) \( 3 \times 10^6 \) volt/meter, and the required range is 3,000 ft or 915 meters.

The permittivity of air will be taken to be that of free space since the dielectric constant is very nearly equal to unity and changes so little with pressure that it can be regarded as a constant (\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ coul}^2/\text{nm}^2 \)). The conductivity, however, increases greatly with altitude. If as an average value of conductivity we choose the value at 65,000 ft, which, according to Reference 3.2, is

\[ \sigma = 180 \times 10^{-14} \text{ mho/meter} \]

we get from equation (3.4) for the relaxation time 4.63 sec.

If the radius, \( r_0 \), of the munition is taken to be 1 meter and if 100 sec are required for the munition to intercept the target, then
the required sensitivity of the detector as obtained from equation (3.5) is \(10^{-9}\) volts/meter.

**Electric Dipole Fields** - Since the charge leaks off the munition so fast, it is reasonable to look into the case of the charge resulting from a continuous redistribution of charge within the munition during its flight.

As a highly idealized case, let the munition be represented by two spheres of radius \(r_o\) separated by a distance \(l\) and let charge be moved from one sphere to the other as fast as it is leaking off through the atmosphere in the opposite direction, so that the sphere maintains constant charges of \(+Q_o\) and \(-Q_o\). The magnitude of the electric field at a distance along any line perpendicular to their axis of symmetry midway between the two spheres will be

\[ E = l Q_o / 4\pi r^3 \]

The amount of charge \(Q_o\) that can be stored in the spheres is determined by the dielectric strength of the atmosphere \(E_{\text{max}}\) at the altitude of the munition. It will also be given by the equation

\[ Q_o = \varepsilon E_{\text{max}} 4\pi r_o^2 \]

with the understanding that \(E_{\text{max}}\) is not necessarily the same as before. From these two relations

\[ E = l E_{\text{max}} r_o^2 / r^3 \]  \hspace{1cm} (3.7)

For an approximation of the field to be expected at the range of 3,000 ft (or 915 meters), let \(l = 3\) m, \(r_o = 0.5\) m and, for lack of data at the actual altitudes encountered, let \(E_{\text{max}} = 3 \times 10^6\) v/m, the value at sea level, then from (3.7)

\[ E = 3 \times 10^{-3}\] volts/meter.

**Slowly-Varying Electric Dipole Fields** - A varying field could be produced by varying the distance between the two spheres mentioned above or by spinning the dipole about any line not collinear with their axis of symmetry. These motions will give rise to an
oscillating dipole. A dipole can also be produced by a current element on a wire.

The field produced by the oscillating electric dipole has two electric and one magnetic field components. At zero frequency, the static case, while the magnetic field vanishes, it becomes significant for the oscillating dipole at large distances from the dipole. For the static case the electric field possesses radial and transverse components; these vary as $1/R^3$. The radial component becomes significant when compared to the transverse component for the oscillating dipole at large ranges since the radial component will then vary as $1/R^2$ while the longitudinal component varies as $1/R$ (Reference 3.7). Slowly varying electric fields are difficult to measure and for this reason the magnetic component produced as a result of the varying electric field is measured instead. A magnetometer would be employed for this purpose.

Airborne instruments, capable of measuring electric fields of about 100 volts/meter, have been successfully used in study of the electric fields surrounding storm clouds (Reference 3.3). However, meters with the sensitivity required for such small fields as will exist for the representative situations described above and have the further necessary requirements of being able to give a vectorial presentation of the field and having a fast response time, do not appear to exist at present. Therefore scoring by the measurement of electrostatic fields must be considered unfeasible and will be placed in Category 2 pending development of more sensitive instruments.

2.4 Orbital Space - An earth satellite will acquire a charge through collision with ions and electrons in space. Assuming that the ions and electrons have the same mean kinetic energy, the electrons move faster than the ions; consequently, a greater number of electrons strike the satellite, leaving the satellite negatively charged. As the satellite acquires a charge, the accretion rates for electrons and ions will change and when these rates become equal an equilibrium potential will be established. For instance at equal ion and electron temperatures of 0.15 electron volt, the equilibrium potential is 0.8 volt.

As the energy of incident charged particles increases, the satellite may emit secondary electrons or ions. Electrons with energies of 10-20 ev emit secondary electrons. Positive ions with energies above 100 ev may emit electrons, negative ions or neutral atoms. Sputtering of neutral particles does not alter the charge on the satellite.

Ultraviolet light and X-rays will cause electrons to leave the surface of the satellite, leaving it positively charged. Consequently,
a satellite may be either positively or negatively charged. The equilibrium potential is determined by the equalization of the positive and negative charging rates. The body becomes negatively charged if the electron flux exceeds the number of photoelectrons emitted per second and positively charged if the electron flux is less than the photoelectrons emitted per second.

A conductor can also acquire a surface charge due to rotation in a magnetic field and this charge in turn will give rise to a potential. For a satellite in the earth’s magnetic field, however, the potential due to reasonable satellite spin is very small and, can therefore be neglected. (For a satellite of 25 cm radius spun at 0.75 rad/sec in a field of 0.3 gauss, the potential is $4.7 \times 10^{-7}$ volts at the surface.)

A negatively charged body in an ionized atmosphere will be surrounded by a shell of positive ions. For a body in motion this shell becomes distorted with a concentration of ions in front (in the direction of motion) and an ion trail behind. When the speed of the satellite exceeds the thermal speed of the ions, the satellite will move out of the space charge and proceed unshielded. At an altitude of 795 km for a gas temperature of 4000 K the thermal velocity of the positive ions is $2.5 \times 10^5$ cm/sec. If a charged munition at this altitude were traveling at a velocity less than $2.5 \times 10^5$ cm/sec, an observer on the scoring satellite would not be able to detect this charge.

In essence should the velocity of the munition drop below that of the thermal velocity of the positive ions along a segment of the trajectory, then it cannot be detected over this portion of the trajectory by measuring an electrostatic field. In most attacks the munition will be near an apogee during the encounter or at its minimum velocity. The foregoing discussion leads to the conclusion that electrostatic phenomena have little potential for development of scoring systems.

The conductivity of the atmosphere is known to increase with altitude (References 3.2 and 3.4). Therefore the arguments expressed under Electric Fields due to a Net Charge are probably applicable to the case of a munition charged on one satellite and launched toward another; the charge would leak off too fast. A quantitative treatment of this problem was not attempted because good data on conductivity at satellite altitudes was not available.

The satellites themselves may develop an equilibrium potential due to collisions with ions (References 3.5 and 3.6). Let us suppose that a satellite develops such a potential $V$ and that it has no shield of orbiting ions to detract from its distant electric field $E$. Let $Q$ be
its charge and its radius. Then
\[ V = \frac{Q}{4\pi \varepsilon r_0} \]

and
\[ E = \frac{Q}{4\pi \varepsilon r^2} \]
is its electric field at a distance r; thus
\[ E = \frac{V r_0}{r^2} \]

Suppose that a satellite with a radius of 1 meter has a potential of 100 volts; then the electric field at a distance of 500 feet is
\[ 4.3 \times 10^{-3} \text{ volts/meter} \]

If the munition is made to sustain a dipole field by a continuous redistribution of charge, the magnitude of the field at the required range is given by equation (3.7).
\[ E = \frac{t E_{\text{max}} r_0^2}{r^3} \]

Taking the values \( t = 3 \text{ m}, E_{\text{max}} = 3 \times 10^6 \text{ v/m}, r_0 = 0.5 \text{ m} \) as was done in the discussion of Electric fields due to a net charge, but using \( r = 500 \text{ ft} (= 152 \text{ m}) \) for the range gives
\[ E = 0.64 \text{ v/m} \]

This places electrostatic field methods in Category II for the same reasons given above.

3. MAGNETIC FIELDS

3.1 Passive Scoring Using Earth's Field

Description of Earth's Magnetic Field - From satellite measurements and measurements made on the surface of the earth, the magnetic field associated with the earth approximates the field of a magnetic dipole centered in the earth. Close to the surface of the earth the field is badly distorted because of variations in permeability of the earth from location to location and because of surface electrical currents. At extreme altitudes (100 miles and greater) the magnetic field is distorted because of the solar winds (plasma winds) with some distortion caused by the Van Allen Belts. Some of the distortions are
constant in time while others are periodic and aperiodic. The time-varying distortions are of small to moderate amplitude and usually of long period. That is to say the time rate of change of the magnetic field is small and except for electrical storms may be considered to be zero for periods of a few seconds (Reference 3.8).

First-Order Approximations - For upper atmosphere encounters (an altitude of 100 miles or more) an assumption will be made that the earth's magnetic field is a constant value $B_0$ throughout the region containing the encounter. Further assumptions will be that the earth's field is uniform (parallel lines of flux) and homogeneous (no sources).

Perturbation caused by a Sphere - Let us consider now what happens to a homogeneous parallel magnetic field when a sphere is introduced into the magnetic field. For this discussion we shall assume that the permeability of the sphere $\mu_2$ is greater than the permeability of the surrounding environment $\mu_1$. Figure 3.2 illustrates the distortion of the lines of flux of the magnetic field when a ferromagnetic sphere is introduced.

![Figure 3.2. Distortion of a Magnetic Field by a Ferromagnetic Sphere](image)

The field strength at some point $P$ where $r$ is large compared to $a$ is given in polar coordinates by the relations (Reference 3.9)
Equations (3.8) and (3.9) are observed to be composed of two parts. One part is due to the unperturbed field $B_0$, and the second part is due to the field of a magnetic dipole positioned parallel to $B_0$ and of strength

$$m = KB_o a^3$$

(3.11)

We may rewrite equations (3.8) and (3.9) in terms of the perturbation amplitude:

$$\Delta B_r = \frac{2KB_o a^3 \cos \theta}{r^3} = b_r$$

(3.12)

$$\Delta B_\theta = \frac{KB_o a^3 \sin \theta}{r^3} = b_\theta$$

(3.13)

where $b_r$ and $b_\theta$ are the polar components of the magnetic field of a magnetic dipole of strength $KB_o a^3$ in the absence of any external magnetic field. It is possible then to view the problem as though there were no earth's magnetic field and that the missile (or target as the case may be) has been magnetized with the axis of magnetization in some known constant direction. In other words, the orientation of the dipole is known at all times. The strength of the dipole is, of course, dependent on the value $B_0$.

In a scoring encounter $r$ and $\theta$ are not known and since the values of $b_r$ and $b_\theta$ depend on the value of $\theta$ these will not be calculable quantities. The only calculable quantities will be $b_T$, the total field
strength at the point in question, and some angle, say $\beta$, which is the angle between the vector $\vec{b}_T$ and some axis taken parallel to the known direction of $\vec{m}$. Rewriting equations (3.12) and (3.13) and solving for $r$, we have the two relations

$$r = \left( \frac{2KB_o a^3 \cos \theta}{b_T b_{0r}} \right)^{1/3} = \left( \frac{2KB_o a^3 \cos \theta}{b_T b_{0r} \cos (\beta - \theta)} \right)^{1/3}$$

(3.14)

from equation (3.12) and

$$r = \left( \frac{2KB_o a^3 \sin \theta}{b_T b_{0\theta}} \right)^{1/3} = \left( \frac{KKB_o a^3 \sin \theta}{b_T b_{0\theta} \sin (\beta - \theta)} \right)^{1/3}$$

(3.15)

from equation (3.13). If we assume that $K$, $B_o$, and $a$ are known and that $b_T$ and $\beta$ can be calculated from equations (3.12) and (3.13), then $r$ and $\theta$ can be solved for from the two relations equations (3.14) and (3.15). Figure 3.3 illustrates the vector relations between the above-mentioned quantities.

From the symmetry of the dipole field, a plane passing through $\vec{b}_T$ and the x-axis will also pass through $\vec{m}$. Knowing the relation between this plane and the coordinate system on the sensor (direction cosines of $\vec{b}_T$) and being able to calculate $r$ and $\theta$ an elevation angle, we can determine an azimuth angle and range from one observation station. This is, of course, theoretically speaking. For actual feasibility, one must determine the present capabilities of measuring the various parameters and variables and how the capabilities match the requirements of a particular scoring system.

As an example let us consider for the moment that the parameters $K$, $a$, and $B_o$ are known. Suppose that $\mu_2 \gg \mu_1$, so that $K = 1$ and that $a$, $B_o$ and $\theta$ are 150 cm, 0.20000 gauss, and 45°; then from equations (3.14) and (3.15), $\beta \approx 70^\circ$ and $\cos (\beta - \theta) \approx 0.9$. From these same equations for

(a) $r = 10^5$ cm $\sim$ 3000 feet, $b_T = 1.07 \times 10^{-9}$ gauss,
(b) $r = 3 \times 10^4$ cm $\sim$ 1000 feet, $b_T = 4 \times 10^{-8}$ gauss
(c) $r = 1.5 \times 10^4$ cm $\sim$ 500 ft, $b_T = 3.2 \times 10^{-8}$ gauss
Figure 3.3. Field Components of Magnetic Dipole

Figure 3.4. Field Components of Oscillating Magnetic Dipole
Without going into an error study, it is clear that if a charge of $10^{-7}$ gauss in the earth's magnetic field is to be detected, the earth's field must be known to seven significant figures at the position in space where the encounter takes place. The field, of course, is not constant and cannot be predicted with this accuracy. Perhaps measurements of the earth's magnetic field could be taken immediately before and after the encounter so that the earth's field during the encounter could be approximated by extrapolation, but even this would be questionable because of local spatial variations in the field and because of the accuracy of field measurements.

3.2 Passive Augmented Scoring Using an Electromagnet - As seen in the previous section, the induced dipole moment was extremely small, too weak to be detected at the desired ranges. Here we shall consider a missile or a target, as the case may be, augmented with an electromagnet in an attempt to increase the effective detection range of the augmented vehicle. The earth's magnetic field will be ignored.

The source of the field will be a magnetic dipole, either static such as a bar magnet or oscillating such as that produced by an induction coil.

Let a source with a dipole moment $\mathbf{m}$ be located at the origin of a XYZ coordinate system as shown in Figure 3.4, with $\mathbf{m}$ pointing in the direction of the positive Z axis. The magnetic field at field point P with coordinates $(r, \theta, \phi)$ has two components $B_r$ and $B_\theta$ which are in the same plane containing $\mathbf{m}$. An oscillating dipole will furthermore give rise to an electric field which has only one component. This component is perpendicular to the plane containing $B_r$ and $B_\theta$. The components of the field vectors at P in MKS units are (Reference 3.7):

$$E_\phi = \frac{k^2}{4\pi} \sqrt{\frac{\mu}{\varepsilon}} \left( \frac{1}{r} + \frac{i}{kr^2} \right) \sin \theta \ |m| \ e^{-i\omega \tau}$$

$$B_r = \frac{\mu}{2\pi} \left( \frac{1}{r^3} - \frac{ik}{r^2} \right) \cos \theta \ |m| \ e^{-i\omega \tau}$$

$$B_\theta = \frac{\mu}{4\pi} \left( \frac{1}{r^3} - \frac{ik}{r^2} - \frac{k^2}{r} \right) \sin \theta \ |m| \ e^{-i\omega \tau}$$

where $\tau = t - r/v$, $v$ is the velocity of the wave produced by the dipole and $k = 2\pi/\lambda$.

In the static case, $\omega$ and $k$ are zero, the electric field vanishes,
and the two components of the magnetic field vary as $1/r^3$. The field produced by the oscillating dipole is rather complex for small ranges as is seen by the relations above; however, for large values of $r$, the radial component of the magnetic field vanishes, leaving only one component for the magnetic field and one for the electric. And the magnitude of these components varies merely as the inverse of range with an added feature of increasing with the square of frequency.

Suppose that the direction of the dipole moment, $\vec{m}$, is known (perhaps by telemetering orientation data from the vehicle carrying the dipole); then a reference frame can be established in which components of the field can be measured.

The total field $B_T$ is given by

$$B_T^2 = B_r^2 + B_\theta^2$$

and

$$B_r = B_T \cos (\beta - \theta)$$
$$B_\theta = B_T \sin (\beta - \theta)$$

But for $\omega = 0$

$$B_r = \frac{\mu}{2wr} |m| \cos \theta$$
$$B_\theta = \frac{\mu}{4wr} |m| \sin \theta$$

From these relations one obtains

$$\tan \theta = \frac{1}{2} \tan (\beta - \theta)$$

which can be solved for $\theta$ when $\beta$ is given. With the angles $\theta$ and $\beta$ and $B_T$ known, the above relations can be solved for $r$. For $\vec{B}_T$ not parallel to $\vec{m}$ there will be two values of $\theta$ for each $B_T$ and $\beta$. The angle $\phi$ is known from the direction of $\vec{m}$ and the position of $P$. The multiplicity of $\theta$ now implies that two computed missile trajectories are possible.

When $\omega$ is not zero and when $r$ is large, the magnitude of the total magnetic field measured at $P$ is $B_\theta$ and of the electric field is $E_\phi$. 
In this case it is not necessary to know the direction of \( \vec{m} \) if \( E_\phi \) can be measured. For \( \vec{E} \times \vec{B} \) will give the direction of \( \vec{m} \) (with \( \vec{E} \) and \( \vec{B} \) measured in the scoring system).

No energy is radiated in the static case. For \( \omega = 0 \), the energy radiated by the dipole is (Reference 3.7)

\[
W = 10k^4 |m|^2 \text{ watts}
\]

If the field is produced by a current \( I \) passing through a single loop of wire of radius \( R_1 \), then \( |m| = rR_1^2I \) and the radiated energy becomes

\[
W = 160\pi^6 (R_1/\lambda)^4 I^2 \text{ watts}
\]

3.3 Magnetic Sensors - For extremely sensitive detectors there are two major groups or techniques of measuring magnetic field strength. One method makes use of the natural magnetic moment of the atomic nucleus and is called a precessional magnetometer. The second technique uses the resonant characteristics of a magnetic-electronic circuit in a magnetic field. Another method which is not widely used is based upon the deflection of an electron beam in a magnetic field.

The first method is based upon the following principle. The atomic nucleus has a magnetic moment which causes the angular-momentum vector of the nucleus to precess when placed in a magnetic field. This precession is about an axis parallel to the applied field (say the \( Z \) axis). An oscillating magnetic field is now applied normal to the \( Z \) axis. When the frequency of this field is equal to the Larmor frequency, the nucleus flips over. The Larmor frequency being proportional to the applied field then gives a measurement of the applied field.

The variable \( \mu \) magnetometer falls under the second method. This magnetometer is based upon magnetic alloys with permeabilities that change strongly with magnetized fields. Mumetal is one such material. The variable \( \mu \) alloy is employed as a core of a coil. When a current is passed through the alloy, the circular magnetic field about the alloy causes \( \mu \) to change. Pulsating this current produces an oscillating \( \mu \). The magnetic field responsible for the changing \( \mu \) is at right angles to the coil surrounding the core and does not couple with it. When a field \( H \) is applied parallel to the core, a magnetic induction, \( B \), is produced where \( B = \mu H \). And since \( \mu \) is an oscillating quantity, \( B \) is time varying and couples with the coil producing a voltage across its terminals.
The Electro-Mechanics Company produces a three-component variable-μ magnetometer which can measure fields of the order of 1 to 0.1 γ. A model built for rocket applications had a sensitivity of 5γ. The present model varies μ at 10 cycles per second but this rate of variation can be changed. The upper limit being set by the ferrite core is around 1 to 10 kc. The response of the unit will be determined by this frequency. An off-the-shelf magnetometer by Electro-Mechanics with a time response (including telemetry) does not exist. A complete redesign would be required for the high time response in order to be sure that the associated electronics would pass the proper frequencies.

An electron-beam magnetometer has been built which can measure fields of the order of $8 \times 10^{-6}$ gauss with a signal-to-noise ratio of 9.7 to 1. The beam is accelerated down a one-meter tube and collected by two electrically separated plates. The signal from these plates is sent to a differential amplifier. For an undeflected beam, both plates receive the same number of electrons per second and the output of the amplifier is zero. A magnetic field deflects the electron beam, thus resulting in a greater electron current on one plate. The disadvantage of this magnetometer is that the accelerating voltage of 600-1400 volts must be held constant to one part in $10^4$, and since the output current from each plate is of the order of $10^{-8}$ amperes, the differential amplifier must detect current differences of $10^{-12}$ amperes.

4. NUCLEAR RADIATION

Radiation from radioactive decay has been used in the past for miss-distance scoring devices. We shall now examine the possible use of this type of radiation in complete trajectory scoring.

Two major types of radiation are involved in radioactive decay. One type of radiation is electromagnetic radiation in the form of hard X-rays and gamma rays, with the second type being particles such as electrons, protons, neutrons, etc. Both types of radiation occur naturally through radioactive decay and can also be produced by bombarding certain materials with high energy particles.

Scoring through the use of these radiations will be considered only for altitudes greater than 100 miles; thus the results will be valid for scoring the encounters with satellite targets or the mid-course phase of ICBM trajectories. At these altitudes it is possible to ignore atmospheric effects completely.

By placing a radioactive source on the missile, a sensor on the target and knowing the intensity of the source, the range from the missile to the target can be determined by measuring the intensity
at the sensor. The degree of accuracy of the computation of the range will depend on the sensitivity of the sensor. It has been assumed here that the source radiates isotropically.

If we consider that charged particles are being radiated we must take into account the fact that a magnetic field, the earth's magnetic field, exists between the source and the sensor and that it will affect the trajectories of the particles. For neutrons and gamma rays, the magnetic field will have no effect.

The intensity of the radiation from the source will fall off as $1/r^2$ for the altitudes considered here since there would be no attenuation due to atmosphere. If we were to consider lower altitudes, the intensity as a function of range would be proportional to an exponential function such as

$$I/I_0 \propto e^{-Rx}$$

where $R$ is the absorption coefficient of the surrounding medium. The absorption coefficient will vary from particle to particle and to some extent with particle energy. For the alpha particle, $R$ is very large; for gamma rays, $R$ is very small.

To obtain the missile's position with respect to the scoring satellite, one records the flux density of impinging charged particles and the angles of arrival. The present position of the satellite represents one point on the trajectory for the charged particle and the angles of arrival are tangent to the trajectory at that point. The point and tangent can now serve as a set of initial conditions to solve the equations of motion for the charged particle backwards in time. By knowing the flux density at the initial point (at target satellite) and the manner in which the flux varies with range (or arc length) one continues to compute back along the particle trajectory until the computed flux density becomes equal to that prescribed to the source.

4.1 Electrons and Protons - A charged particle moving in a magnetic field will have a force exerted on it due to the field (as long as the velocity vector is not parallel to the magnetic field). The exact relation is given by the Lorentz force equation

$$\mathbf{F} = e\mathbf{v} \times \mathbf{B}/c$$

where $e$ is the charge on the electron in e.s.u.

$c$ the speed of light cm/sec

$\mathbf{v}$ is the velocity of the electron cm/sec
$\mathbf{B}$ is the magnetic induction in e.m.u.

$m$ is the rest mass of the electron in grams

The trajectory of a charged particle moving parallel to the magnetic field will not be influenced by the field while a particle traveling normal to the field will have its trajectory strongly altered. When $\mathbf{v}$ is perpendicular to $\mathbf{B}$, $\mathbf{F}$ has only a radial component and the trajectory becomes a circle. For no other external forces, one obtains from the Lorentz relation

$$\frac{e}{c} v B = \frac{mv^2}{r}$$  \hspace{1cm} (3.16)

where $r$ is the radius of the charged particle's path. Neglecting relativistic effects, the particle kinetic energy is written as

$$T = \frac{1}{2} mv^2 \text{ ergs}$$  \hspace{1cm} (3.17)

Solving this relation for $v$ and substituting the result in (3.16), one obtains

$$r = \frac{c}{eB} \sqrt{2Tm}$$  \hspace{1cm} (3.18)

At altitudes of 100 miles the earth's magnetic field can be approximated by that at sea level. With this approximation suppose that $B = 0.3$ gauss; then since $e = 4.8 \times 10^{-10}$ e.s.u, $m = 9.1 \times 10^{-28}$ gr,

$$r = 0.89 \times 10^7 T^{1/2}$$  \hspace{1cm} (3.19)

For a 10 kev electron ($T = 1.6 \times 10^{-8}$ ergs), $r = 1.12 \times 10^3$ cm. (The energy of one electron volt is equivalent to $1.6 \times 10^{-12}$ ergs.) The maximum distance perpendicular to the magnetic field this electron can attain from the source is $2r$, or 22.4 meters. A 1-mev electron can travel 224 meters from the source.

These figures indicate that the effects of the earth's magnetic field would make it difficult to use electrons in a scoring device for distances beyond 250 meters perpendicular to the magnetic field. If no magnetic field were present, the position of the source would be obtained by measuring the direction at which the electrons are arriving. However due to the field the paths of the electrons are curved, and the position of the source is no longer a simple function of the angles at which the electrons arrive. For a non-monoenergetic source (a source
emitting electrons with kinetic energies distributed over a band of energies) the situation becomes worse because the electrons will arrive at more than one angle. Since there are no radioactive monoenergetic electron emitters, one would then almost rule out the electron for scoring.

The proton and alpha particle appears to be more favorable in this respect, however, since they are emitted from radioactive materials with monochromatic energies. The proton and alpha particle have another advantage over the electron in that the magnetic field has a smaller effect on its trajectory due to the increase in mass. For the velocity vector at right angles to the magnetic field, the radius of the trajectory of the proton is about 43 times as large as that for the electron for the same particle energy.

Charged particles whose velocity vector is neither normal nor parallel to the earth's magnetic field will travel a spiral trajectory with the axis of the spiral parallel to the magnetic lines of flux. To compute the position of the particle, the earth's magnetic field must be known at every point along the trajectory. For short distances the trajectory may be approximated by assuming that the magnetic field is constant over the trajectory, but even so should the scoring vehicle accumulate a charge, the resulting force on the charged particle would have to be factored into the Lorentz relation. It appears that one must record flux density, angle of arrival, scoring vehicle potential and space position in order to compute the location of the source. An extremely difficult task in obtaining all this data would be recording the angle or angles at which the electrons strike the scoring vehicle; but even if this data were known it appears that this method of scoring would require far too much knowledge about the environment and far too much input data to be practical.

4.2 Neutrons and Gamma Rays - The direction of travel or propagation of neutrons and gamma rays is not affected by magnetic fields. The neutron and gamma ray travels in a straight line in free space; consequently the orientation of the vector position of their source is directly related to the direction of propagation. The particle density decreases as $1/r^2$ along a line containing the source. By measuring the intensity at a point and knowing the intensity of the source, the distance to the source can be computed.

Now let's investigate the intensity of the source that would be required for an isotropic radiator in order to detect a charge of $\Delta r$ at a range $r$. Let $q$ be the flux density at a detector located a distance $r$ from a source emitting a total flux $Q$; then

$$q = Q/4\pi r^2 \text{ particles/cm}^2\text{-sec}$$

(3.20)
The total count, \( \bar{q} \), per second crossing an area \( A \) is

\[
\bar{q} = Aq
\]  

(3.21)

The count crossing the same area at ranges \( r_1 \) and \( r_2 \), denoted by \( \bar{q}_1 \) and \( \bar{q}_2 \), satisfies the relation

\[
\frac{\bar{q}_1}{\bar{q}_2} = \frac{r_2^2}{r_1^2}
\]  

(3.22)

For \( r_2 = r_1 - \Delta r \) where \( \Delta r \ll r_1 \)

\[
\frac{q_1}{q_2} \approx 1 - 2 \frac{\Delta r}{r_1}
\]

As an example, let \( r_1 = 1000 \) feet, \( \Delta r = 50 \) feet; then

\[
\frac{q_1}{q_2} = 0.9
\]

Since particle counters count only whole particles, \( \bar{q}_2 \) must be at least 10 counts/sec. In practice radioactive sources are not isotropic; consequently a better value for \( \bar{q}_2 \) would be 100 counts/sec. In this case

\[
|\bar{q}_1 - \bar{q}_2| = 10 \text{ counts/sec}
\]

From equations (3.20) and (3.21)

\[
Q = 1.17 \times 10^{10} \frac{q}{A}
\]  

(3.23)

which for \( \bar{q} = 10^2 \) leads to a 32-curie source for \( A = 1 \text{ cm}^2 \) (1 curie = \( 3.7 \times 10^{10} \) disintegrations/sec). The size of this source makes it highly impractical.

There are no radioactive substances that decay by the emission of neutrons. Neutrons can be produced only by induced emission. This emission is produced either by an interaction of two particles or by fission. These processes require a particle accelerator or an atomic reactor, both of which are too large for missile augmentation.
Gamma rays are easier to obtain than neutrons. There are many radioactive substances that emit gamma rays upon disintegration. Large fluxes are available with short half lives such as that from Co$_{60}$.

Gamma rays cannot be focused or reflected in the manner similar to longer wavelength light; consequently it is difficult to measure the direction of propagation. Gamma rays can be absorbed by thick layers of lead, gold, thorium, tungsten, etc., which means that they can be collimated. The direction of the source is located by pointing the collimator such that the flux down the collimator is a maximum.

A thick lead rotating drum with a narrow slit parallel to the axis of symmetry could measure one angle by noting the position of the slit when the radiation detected at the interior of the drum is a maximum. A sketch of such a detector is shown in Figure 3.5. A detector with a slit width "$w$" and a drum thickness "$d$" will for the two-dimensional case receive radiation from all sources within an angle $2 w/d$ radians for $w$ small (as shown in Figure 3.6).

For a resolution of one milliradian, $2 w/d = 0.001$. Suppose that the drum is one inch thick; then the slit would be 0.0005 in. wide and if the length of the drum is $10/2.54$ cm ($\approx 4$ cm) the area of the slit, $A$, is $5 \times 10^{-3}$ cm$^2$. Continuing with the previous example by (3.23), $Q = 2.34 \times 10^{14}$ disintegrations/sec which is about 6300 curies. This source is fantastic for scoring and must be ruled out for the present.

5. GRAVITATIONAL FIELDS

A gravity meter or gravimeter is a very sensitive inertial accelerometer used to measure the gravity field of the earth. Since it is an inertial instrument, it would not measure the acceleration of a vehicle due to the gravitational field (see section on Inertial Systems); but if its motion is appropriately constrained or if the total acceleration is determined by another method, it can be used to determine either gravity or the gravitational field.

Except for its great sensitivity, the gravimeter is thus, fundamentally no different and no better than any other inertial instrument. Moreover, its sensitivity is achieved at the expense of relatively long response time; and it is thus not very well suited for scoring applications. Since a gravimeter could conceivably be used to detect the presence of a nearby vehicle, it is worthwhile demonstrating that this is impossible except at very short ranges.
Figure 3.5. Angle Measuring Detector

Figure 3.6. Geometry Affecting Resolution of Detector

2 \tan^{-1} \frac{w}{d}
The gravitational attraction between two bodies with masses $m_1$ and $m_2$ separated by a distance $r$ is given by

$$F_G = \frac{Gm_1m_2}{r^2}$$

where $G = 6.66 \times 10^{-8}$ dyne-cm$^2$/gm$^2$ is the universal gravitational constant. This force will give $m_1$ an acceleration $a_1 = \frac{Gm_2}{r^2}$.

Then the acceleration at 500-ft range (15240 cm) due to an 11-ton munition (about $10^7$ gm) would be

$$a_1 = \frac{6.66 \times 10^{-8} \times 10^7}{(1.524 \times 10^5)^2} = 2.87 \times 10^{-9} \text{ cm/sec}^2$$

This represents a considerably smaller field than even stationary gravimeters are presently capable of detecting, and airborne instruments are restricted to fields no larger than about $10^{-2}$ cm/sec$^2$ (10 milligals) according to References 3.10 and 3.11. Even these meters require several seconds for a reading.

Aside from any question of instrument sensitivity, this field perturbation would have to be detected against the background of the vehicle's non-gravitational acceleration and the earth's field which would be about 963 cm/sec$^2$ at 200,000 ft according to Reference 3.12. For the perturbation to be detected, the background would have to remain effectively constant during the passage of the perturbing body; this would be, to say the least, an unlikely occurrence.

These arguments indicate that gravitational measurements should be classified as having zero potential for scoring, regardless of any future improvements in the instruments.

6. INERTIAL SYSTEMS

6.1 Introduction - A purely inertial scoring system would use three inertial accelerometers (or velocimeters) in each vehicle. Each set of accelerometers (velocimeters) would be mounted on a stabilized platform with the sensitive axes of these instruments mutually orthogonal; each set would be equivalent to the sensing part of an inertial guidance system. Given the necessary initial conditions, the integration needed to determine the vehicle's trajectory could be done by a computer aboard the vehicle or the acceleration (velocity)
data could be telemetered to the ground for later computation.

The two stabilized coordinate systems would be non-rotating and preferably parallel but at least of known orientations so that the relative trajectory could be determined from the trajectories of the individual vehicles. The same result could be obtained from hybrid systems containing non-inertial as well as inertial sensors. Hybrid systems frequently use a slowly rotating platform with one axis vertical; a second axis may be pointed in the direction of motion or to geodetic north. Rotating coordinates of this type are acceptable for the short-duration trajectories of present interest if the alignment is maintained with sufficient accuracy.

The probable error in the relative trajectory obtained by inertial methods is a function of the probable errors in the individual vehicle trajectories and would be given by the square root of the sum of the squares of the errors. The principle of the inertial scoring system is sound, but it remains to be determined whether the instruments available at present or those that will be available in the near future have sufficient accuracy for the conditions to be expected in the various types of encounters. The question to be answered in some cases may be: "Does the existing guidance system of the vehicle have sufficient accuracy for use in scoring?"

Calculation of Trajectories from Inertial Data - An inertial accelerometer measures the force acting on a mass suspended in such a way that it has one degree of translational freedom. Some accelerometers have an output which is proportional to the change in velocity and are therefore described as integrating accelerometers or velocimeters, but the principle of operation is the same.

An inertial instrument reacts to the gravitational field (Newtonian attraction) as well as to the acceleration of the vehicle, and this reaction is equal and opposite to the reaction of the instrument to the gravitational component of the acceleration. Hence these instruments measure only accelerations due to non-gravitational forces acting on the vehicle which carries them. The gravitational field can be determined by use of these instruments if the total acceleration is determined by some other method or if the motion of the vehicle is suitably constrained.

The total acceleration acting on the vehicle is given by the equation
\[ \vec{a}_T = \vec{a}_a + \vec{G} \]
where \( \vec{a}_T \) is the total acceleration (or the acceleration of transport)
of the vehicle with respect to inertial space, \( \ddot{a} \) is the acceleration vector measured by the accelerometers, and \( \ddot{g} \) is the gravitational acceleration. It is assumed here that the platform carrying the accelerometers is non-rotating with respect to inertial space. If the platform is stabilized in some other way (with one axis vertical, for example), the small rotational motion may be detected by the accelerometers, but the error is unimportant for present purposes. Integrating the above equation gives

\[
\dot{V} = \int_0^t \ddot{a} \, dt + \int_0^t \ddot{g} \, dt + V_0
\]

where \( \dot{V} \) is the total velocity at time \( t \) measured with respect to inertial space, and \( V_0 \) is the value of \( \dot{V} \) at \( t = 0 \). Integrating again gives the displacement

\[
\Delta = \int_0^t \int_0^t \ddot{a} \, dt \, dt + \int_0^t \ddot{g} \, dt \, dt + \dot{V} \, t + \Delta_0
\]

where \( \Delta \) is the total displacement at time \( t \) and \( \Delta_0 \) is the value of \( \Delta \) at \( t = 0 \). The acceleration of the center of the earth with respect to inertial space is negligible for most purposes, and the center of the earth is usually taken as the origin of the inertial system for purposes of measuring displacement \( \Delta \).

It is evident that if inertial measurements are to be used to compute the trajectory of a vehicle, the value of \( \ddot{g} \) must be known as a function of the space coordinates. In some areas, such as around Cape Canaveral, the value of \( \ddot{g} \) has been carefully determined. Then if the vehicle is to follow a standard path, the effects of \( \ddot{g} \) can be precomputed and allowed for in the guidance settings; somewhat more accurate results will be obtained if the computer is supplied with time-varying coefficients for differential corrections to be used if the actual trajectory deviates appreciably from the precomputed course.

Another scheme, used in hybrid systems, involves keeping the platform horizontal (approximately perpendicular to \( \ddot{g} \)) and using only two accelerometers; altitude and vertical velocity are obtained by some other method such as radar. The accelerometer data are used to keep up with the latitude and longitude of the point on the sea level surface of the earth which is "under" the vehicle. Even if there were no error due to the inertial instruments, this scheme is necessarily imperfect.
due to gravitational anomalies and curvature of the vertical; the position of the vehicle must therefore be corrected from time to time by sightings on objects of known location such as the stars or prominent landmarks.

The alternative to the above procedures is to compute the gravitational field using the best available mathematical model. It is found that the actual field at the surface of the earth seldom agrees with the model. The rms average discrepancy or anomaly can be as large as 20 to 30 milligals (20 milligals = 0.020 cm/sec$^2$ = 0.00065616 ft/sec$^2$) over areas several hundred miles in each direction with anomalies at internal points of two to three times the average. Anomalies of the order of 100 milligals are found on some islands, and anomalies up to 200 milligals are found in fairly narrow areas over some ocean deeps. Strictly speaking, the anomalies are gravity anomalies since gravity is the quantity measured, but they are certainly gravitational anomalies as well.

The size and extent of an anomaly decrease with increasing altitude above sea level; moreover the effects of anomalies will tend to average out over long trajectories. Since relative trajectories are wanted, some of the effects of an anomaly will cancel out when the space-referenced trajectories are subtracted; in some instances nearly complete cancellation can be expected. Finally, it may be noted that a 20-milligal anomaly is equivalent to the value quoted later for accelerometer bias. In short, the accuracy of purely inertial systems is limited by imperfect knowledge of the gravitational field, but this is not at present the most important limitation.

6.2 Calculation of Errors - The sources of trajectory error vary from one inertial system to another and are functions of the design and construction of the accelerometers, the type of platform stabilization, etc. The chief sources of error for a typical system are given in Table 3.1; the rms values listed were taken from Reference 3.13. The accelerometer errors appear to be typical of those given in the unclassified literature, but the gyro drift rates are somewhat larger than the values given for some instruments.

Table 3.2 gives equations (taken from Reference 3.13) which are to be used in computing the approximate trajectory errors at burnout resulting from gyro, accelerometer, platform-misalignment, and computer errors. The coordinate system referred to is space-fixed with the origin at the center of the earth as shown in Figure 3.7. The initial position of the missile is ($x_o$, $y_o$, $z_o$) with $x_o = y_o = 0$ and $z_o = \text{radius of earth at the launch site}$. The trajectory lies in the $x$-$z$ plane.
Table 3-1. The Main Error Sources and Representative Values for Inertial Guidance System

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Symbol</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accelerometers</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Bias (each)</td>
<td>$a$</td>
<td>$5 \times 10^{-4} \text{ft/sec}^2$</td>
</tr>
<tr>
<td>2. Scale factor (each)</td>
<td>$\varepsilon$</td>
<td>$3 \times 10^{-5} \text{g}/\text{g}$</td>
</tr>
<tr>
<td>3. Misalignment (each on other two axes)</td>
<td>$\beta$</td>
<td>10 sec of arc or $0.4848 \times 10^{-4} \text{rad}$</td>
</tr>
<tr>
<td><strong>Gyrosopes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Constant torque drifts</td>
<td>$\phi$</td>
<td>0.10 deg/hr</td>
</tr>
<tr>
<td>2. Mass unbalance drifts</td>
<td>$c$</td>
<td>0.10 deg/hr/g</td>
</tr>
<tr>
<td><strong>Platform</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Initial Misalignment</td>
<td>$\phi$</td>
<td>20 sec of arc</td>
</tr>
<tr>
<td>Azimuth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td></td>
<td>10 sec of arc</td>
</tr>
<tr>
<td><strong>Computer</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Resolution (down &amp; cross-range)</td>
<td></td>
<td>0.1 ft/sec</td>
</tr>
</tbody>
</table>
Table 3.2
Error Approximation Formulas

<table>
<thead>
<tr>
<th>Error Source</th>
<th>$\Delta x$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accelerometers</strong></td>
<td></td>
</tr>
<tr>
<td>1. Bias</td>
<td>$a_x \frac{t_{bo}^2}{2}$</td>
</tr>
<tr>
<td>2. Scale factor</td>
<td>$t_x [x_{bo}]$</td>
</tr>
<tr>
<td>3. Misalignment</td>
<td>$\beta_z x [z_{bo} + g_o \frac{t_{bo}^2}{2}]$</td>
</tr>
<tr>
<td><strong>Gyrosopes</strong></td>
<td></td>
</tr>
<tr>
<td>1. Constant torques</td>
<td>$-\phi_y c [t_{bo} (z_{bo} - z_o)^2 - \int_0^{t_{bo}} (z - z_o) dt]$</td>
</tr>
</tbody>
</table>
| 2. Mass unbalance drifts | $\int_0^{t_{bo}} \Delta \dot{x} dt$ where \[
\Delta \dot{x} = C_{1h} \left[ \int_0^{t_{bo}} (v_z + g_o t)(a_z + g_o) dt \right]$ |
<p>| <strong>Platform Misalignment</strong> | $-\phi_y o [z_{bo} - z_o + g_o \frac{t_{bo}^2}{2}]$ |
| <strong>Error Source</strong>      | $\Delta y$                                      |
| <strong>Accelerometers</strong>    |                                                 |
| 1. Bias               | $a_y \frac{t_{bo}^2}{2}$                        |
| 2. Scale factor       | 0                                               |</p>
<table>
<thead>
<tr>
<th>Error Source</th>
<th>$\Delta y$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accelerometers</strong></td>
<td></td>
</tr>
<tr>
<td>3. Misalignment</td>
<td>$\beta_{y,x}[x_{bo}] + \beta_{y,z}[z_{bo} + g_o z]^{t_{bo}}$</td>
</tr>
<tr>
<td><strong>Gyroscopes</strong></td>
<td></td>
</tr>
<tr>
<td>1. Constant torques</td>
<td>$\phi_{x,c}[t_{bo}(z_{bo} - z_o) - 2 \int_0^{t_{bo}} (z - z_o) dt]$</td>
</tr>
<tr>
<td></td>
<td>$+ g_o / 6^{t_{bo}^{3}} - \phi_{z,c}[t_{bo}^{x_{bo}} - 2 \int_0^{t_{bo}} x dt]$</td>
</tr>
<tr>
<td>2. Mass unbalance drifts</td>
<td>$\int_0^{t_{bo}} \Delta y^* dt$ where</td>
</tr>
<tr>
<td></td>
<td>$\Delta y^* = C_{3s} \left[ \int_0^{t_{bo}} v_x (z_o + g_o) dt \right]$</td>
</tr>
<tr>
<td></td>
<td>$+ C_{2s} \left[ \int_0^{t_{bo}} (v_z + g_o t) v_x dt \right]$</td>
</tr>
<tr>
<td><strong>Platform Misalignment</strong></td>
<td>$\phi_{x,0}[z_{bo} - z_o + g_o z]^{t_{bo}} - \phi_{z,0}[x_{bo}]$</td>
</tr>
<tr>
<td><strong>Error Source</strong></td>
<td>$\Delta z$</td>
</tr>
<tr>
<td><strong>Accelerometers</strong></td>
<td></td>
</tr>
<tr>
<td>1. Bias</td>
<td>$a_z^{t_{bo}^{2}}$</td>
</tr>
<tr>
<td>2. Scale factor</td>
<td>$e_z[z_{bo} + g_o z]^{t_{bo}}$</td>
</tr>
</tbody>
</table>
Table 3.2 (continued)

<table>
<thead>
<tr>
<th>Error Source</th>
<th>$\Delta z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometers</td>
<td></td>
</tr>
<tr>
<td>3. Misalignment</td>
<td>$\beta_{yx} [x_{bo}]$</td>
</tr>
<tr>
<td>Gyroscopes</td>
<td></td>
</tr>
<tr>
<td>1. Constant torques</td>
<td>$\phi_{yc} [t_{bo} x_{bo} - 2 \int_{0}^{t_{bo}} x , dt]$</td>
</tr>
<tr>
<td>2. Mass unbalance drifts</td>
<td>$\int_{0}^{t_{bo}} \Delta z , dt$ where $\Delta z = - C_{1h} \left[ \int_{0}^{t_{bo}} (v_{z} + g_{o} t) a_{x} , dt \right]$</td>
</tr>
<tr>
<td>Platform Misalignment</td>
<td>$\phi_{yo} [x_{bo}]$</td>
</tr>
</tbody>
</table>
Figure 3.7. Inertial Coordinate System
When the values for the error sources given in Table 3.1 are substituted into the formulas of Table 3.2, and if the other parameters given in Reference 3.13 for a 5500-nautical-mile, 70°-burnout-angle trajectory are also used, the errors listed in Table 3.3 result at burnout, $t_{bo} = 200$ sec. The radial error if given by

$$\Delta r = \left[ (\Sigma \Delta x)^2 + (\Sigma \Delta y)^2 + (\Sigma \Delta z)^2 \right]^\frac{1}{2}$$

$$= \left[ 355^2 + 546^2 + 458^2 \right]^\frac{1}{2} = 796 \text{ ft}$$

This fairly large error could be reduced by at least several hundred feet by picking the most favorable values given in the available literature for the error sources, but even an order-of-magnitude improvement would leave an error larger than desired. Since each type of inertial system has its own particular sources of error, and since these errors may contribute to the overall error in different ways, a detailed mathematical treatment of each of the many systems presently available is beyond the scope of this report.

An approximate formula can be developed for estimating the error at any time $t$ given the error at some other time such as burnout time $t_{bo}$. This formula can be developed by assuming that before burnout the error is proportional to the square of the time and that after burnout it is proportional to time. The first of these assumptions results from the relation connecting the error in distance ($\Delta x$) with time ($t$) and a constant error in the measurement of acceleration ($\Delta a$):

$$\Delta x = \frac{1}{2} (\Delta a) t^2$$

The second assumption comes from treating the missile as having no forces acting on it after burnout other than the gravitational field. In this case the accelerometers would register zero, and the trajectory would be extrapolated from the data at burnout. Neglecting the effects of gravitational anomalies the error accumulated after burnout would be $\Delta V (t - t_{bo})$ where $\Delta V$ is the velocity error at burnout time $t_{bo}$.

A sketch illustrating the approximation based on these two assumptions is given in Figure 3.8.

6.3 Encounters in the Upper Atmosphere - If the inertial system of the previous example is taken to be representative of one used in a missile launched from the ground at a target maintaining some altitude
Table 3-3. Inertial System Errors for 5500-Nautical-Mile Missile Trajectory at Point of Burnout (t_{bo} = 200 sec)

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Δx ft</th>
<th>Δy ft</th>
<th>Δz ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer bias</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Accelerometer scale factor</td>
<td>48</td>
<td>0</td>
<td>41</td>
</tr>
<tr>
<td>Accelerometer misalignment</td>
<td>67</td>
<td>155</td>
<td>88</td>
</tr>
<tr>
<td>Constant torque drifts</td>
<td>40</td>
<td>16</td>
<td>56</td>
</tr>
<tr>
<td>Mass unbalance drifts</td>
<td>103</td>
<td>277</td>
<td>156</td>
</tr>
<tr>
<td>Platform misalignment</td>
<td>67</td>
<td>68</td>
<td>87</td>
</tr>
<tr>
<td>Computer</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Maximum total error</td>
<td>355</td>
<td>546</td>
<td>458</td>
</tr>
</tbody>
</table>
Figure 3.8. Approximate Error Accumulation Function for a Rocket
between 70,000 and 200,000 ft, then the error in position of the missile can be estimated by use of the approximation that error is proportional to time squared. The time of flight of the missile, and therefore the error in position, would be least for a vertical trajectory that encountered the target directly over the launch site, and would be increasingly greater for interceptions at longer distances from the launch site. The approximate errors for various times of encounter are shown in the graph of Figure 3.9. The accuracy requirement is that the distance separating the missile and target be known to within ±100 ft for relative ranges from zero to 3,000 ft. The relative-range requirement does not limit the use of inertial systems since each vehicle is independent of the other as regards sensing. However, the ±100-ft error will be seen to limit the use of such systems in these encounters. If both missile and target are assumed to contribute equally to the ±100-ft allowable error, the permissible error for each will be $\sqrt{100/2} = \pm 71$ ft.

It will be seen at once that a purely inertial system (inertial sensing in both munition and target) is not practical if the position errors given in Table 3.3 are correct. The target would be in flight longer than 60 sec, the time at which an error of ±71 ft is accrued by the system. If the target's position and velocity were accurately determined just prior to the intercept by auxiliary means such as ground-based tracking, however, this objection would be removed. The 60-sec limit would still restrict the missile times-of-flight. This system is at least marginally feasible but it is not a pure inertial system.

6.4 Satellite-to-Satellite Encounters - An inertial system accrues errors with time, and it is evident that after only one orbit the inertial system of a satellite would be in error by a sizable amount. Scoring to the required accuracy by purely inertial methods is therefore impossible.

A hybrid system incorporating ground-based tracking of the satellites and an inertial system mounted in the munition might be feasible, especially if the time of flight of the munition were 60 sec or less. In this case the scoring error would be the sum of the inertial-system error and the two tracking errors. The latter errors could be reduced by additional data from radar installed in either satellite.

Accelerometers on a non-spinning body in free fall in empty space read zero. Therefore unless the satellites are expected to maneuver or take evasive action, it is difficult to see how accelerometers would be useful.

6.5 ICBM Encounters - Since errors build up with time in an inertial system, those cases in which both vehicles have a short time of flight prior to the encounter are the most promising. For the purely inertial systems, only the case of a munition launched from the ground
Figure 3.9. Approximate Maximum Error in Position for Rocket-Based Inertial Guidance System
against an ICBM fits this criterion. The guidance system in the
ICBM would have to be very much better than the one used in the
example, however, since it is unlikely that a ground-launched missile
could attack an ICBM during its boost phase. Thus the purely inertial
system is apparently not feasible for this application. If the position
and velocity of the ICBM at burnout could be determined with suffi-
cient accuracy by ground-based tracking systems, however, a suffi-
ciently accurate mid-course trajectory could be computed. It might
then be feasible to determine the trajectory of the attacking munition
by purely inertial means.

6.6 Conclusions - From the foregoing discussion it can be con-
cluded that purely inertial scoring is not practical at this time, but
that hybrid systems may be useful in encounters where the munition
has a relatively short time of flight.

The numerical approximations used in this study were based on
1958 data (revised 1960). Reference 3.14 is a later work that attempts
to classify various inertial systems. It is difficult to compare sensors
that are based on different principles of operation, but it can be seen
that some systems offer a reduction in error from some of the values
used in the treatment above. Further development in this field may
cause purely inertial scoring to become feasible. On the basis of the
evidence available at the time of writing, however, it must be consider-
ed in Category 2.

Some very recent developments were not assessed in this study
because reports which had been ordered were not received. Among
these developments is the nuclear gyro developed by General Precision,
Inc. which is said to have virtually zero drift. Improvement of a single
component does not solve the problem as will be seen from Table 3.3,
and perfection of all measuring components would not eliminate the
gravitational field problem, the computing error, or the misalignments.
Nevertheless Reference 3.15 must be awaited with some eagerness
since the advertising for the HIPERNAS system very nearly claims less
than zero overall error.

7. PRESSURE WAVES AND EFFECTS

It would appear at first glance that sound waves radiated from a
natural or artificial source on a vehicle in subsonic flight would be
quite useful for trajectory scoring, but this is not the case for numerous
reasons. In the first place nearly all sound sources are more or less
directional so that the amplitude of the wave is not a simple function of
distance from the source only. Moreover, sound waves in air are
attenuated fairly rapidly, the medium is far from homogenous and
isotropic, a considerable amount of bending (refraction) can take place
in a fairly short distance, etc.
Shock waves coming off a body in supersonic flight have considerably more potential for scoring purposes, but it can again be demonstrated by purely qualitative arguments that their actual utility is negligible in general. The pressure profile across a shock wave is N-shaped consisting of a sharp rise in pressure followed by an approximately linear decline to a pressure about equally far below ambient atmospheric pressure and then a sudden return to atmospheric pressure. DuMond (Reference 3.19) develops equations for the amplitude and period of this wave; these quantities are functions of Mach number, density, size and shape of the body, and probably other variables.

Marks (Reference 3.16) used stationary pencil-type Barium Titanate piezoelectric pressure gauges for miss-distance and velocity measurements on a firing range. These gauges had good pressure sensitivity and flat frequency responses to 80,000 cps and were essentially non-directional for angles up to at least 45 degrees off-axis. His values for the exponents of distance d in the N-wave amplitude and period equations showed good but not exact agreement with DuMond. The velocity determination depended only on measuring the time of arrival of the shock wave at two gauges and were quite successful. Miss-distance determination based on either the amplitude or period of the N-wave were less successful; the latter method was found to be superior, but both methods required calibration and exhibited large deviations. In fact, the supposed power-law variations with distance were not well confirmed. It was estimated that the maximum miss-distance measurement range for a 37 mm shell would be about 150 ft. The oscilloscope traces showed the N-waves followed by long rough and irregular transient responses of undetermined origin. Since these transient responses have been observed with other types of detectors, the method is not well-suited to rapid-fire weapons.

Shock waves have been used with indifferent success for planar miss-distance determination. The acoustical firing error indicator (FEI) was a glider containing three microphones (one in the nose and two in the vertical tail) and necessary telemetering equipment. The telemetered microphone responses and a timing signal were recorded by an oscilloscope at the receiving station. The time of arrival of the shock wave of each bullet at each of the three microphones was obtained in the data reduction. It was observed that the noise level of the system was high, that some pulses were apparently missing, and that the N-waves were not very sharp (poor high-frequency response); the transient response following the N-wave made it very difficult to obtain good data for any except the first bullet of a burst. Frequently it was difficult if not impossible to identify the pulses (and arrival times) associated with each bullet; this was because the pulse from the n-th bullet sometimes arrived at the i-th microphone after the pulse from the (n + 1)-th bullet as a result of variations in miss distance.
The equations developed in References 3.17 and 3.18 give the X and Z miss distances in the vertical plane containing the microphones and with respect to the X and Z axes drawn through the two lower microphones and through the two tail microphones respectively. It was assumed in developing the equations that the bullet trajectory was a straight line in the vicinity of the glider; the shock wave was taken to be a perfect cone with vertex on the center of mass of the bullet and with axis coinciding with the straight-line trajectory. The speed of the glider and of the bullet with respect to the air mass together with the X and Z direction cosines of the bullet velocity vector were used in the solution directly. The velocity of the bullet was also needed for determining the Mach number which in turn was used in determining the shock cone half-angle from wind tunnel data. These bullet velocity data had to be computed from approximate ballistic equations involving the velocity of the gun-carrying aircraft (assumed straight and level and parallel to the FEI velocity vector), the gun azimuth and elevation angles, the azimuth and elevation angles of the FEI as seen from the gun-carrying aircraft, the altitude (used to determine air density from the standard-atmosphere tables), and the range from aircraft to FEI. A limited number of test calculations indicated that the equations of Reference 3.17 are not very sensitive to errors in the bullet velocity or in the direction cosines provided the direction cosines are small. It was also found that the equations have poor resolution (relatively large errors) except in small areas close to the microphones; the equations give an extraneous answer which is not always easily identified. Sensitivity was limited by the noise level and by the transient responses so that data were not obtained for large miss distances (greater than perhaps 50 ft). Moreover, a missile crossing sufficiently far behind the scorer vehicle will not be detected because the shock wave will either not be able to catch up with the scorer at all or will not catch up before it's energy has been attenuated to the noise level.

The acoustical FEI could be improved of course by making the scorer larger and by using better microphones and other equipment. It might also be helpful to add additional microphones both in and out of the measurement plane. These changes would not remove the basic deficiencies of the system, but if fairly good miss distances could be determined from period or amplitude measurements, an improved measurement scheme could be developed to remove some of the objections. It should be noted, however, that the attenuation of sound waves increases with altitude; and at high altitudes acoustic scoring systems become inoperative; the useful range would be undesirably small at any altitude because of the assumptions which are used.
From the above discussion it will be seen that acoustical methods would be limited to vehicles moving in the lower part of the atmosphere. None of the intercepts considered in this study would be fully covered, and satellite intercepts would not be covered at all. The range requirements could not be met by a scorer vehicle of any reasonable size.

More important, it seems certain that both missile and target would be supersonic during most of the trajectories which could be covered (the ICBM would not be supersonic for several seconds after launch).

Under these conditions it would be useless to mount an acoustic device on either missile or target since the shock waves coming off the scorer vehicle itself would interfere with (probably would block altogether) the reception of shock waves from the observed vehicle. Conceivably the scoring device could be mounted on a subsonic vehicle stationed in the near vicinity of the intercept, but this would be expensive and unreliable; moreover, there would be no way to distinguish missile and target pulses in the data or to eliminate pulses due to shock waves other than bow waves. Acoustic devices must therefore be classified as having zero potential for all future scoring requirements.

8.0 REFERENCES


3.16 Miss-Distance and Velocity Measurements on Projectiles Using Piezoelectric Gauges, BRL Memorandum Report No. 815, Spence T. Marks, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, May 1954.

3.17 Derivation of Equations for Reducing Time Difference Data from the Firing Error Indicator, Robert W. Schmied, Military Physics Research Laboratory, The University of Texas, Austin, Texas, 18 March 1954 (Confidential).

3.18 Ballistic Equations and Computational Procedure for Reducing Data from the Firing Error Indicator, Jerry A. Hawkins, Military Physics Research Laboratory, The University of Texas, Austin, Texas, 19 July 1954, (Confidential).

CHAPTER FOUR
ELECTROMAGNETIC RADIATION

1. INTRODUCTION

The variety of devices and combinations of devices and components which function on the basis of sensing electromagnetic radiation is almost unlimited. In recognizing this and attempting to be consistent with the original intent of this study, the investigations related to this class of phenomenon were directed to establishing some fundamental characteristics for use in the choice of components for a scoring system. It was believed that once the basic environmental condition and resulting requirements were established, this work would serve as a helpful guide and reference in future planning and design problems.

No consideration of a basic nature was given to sensors of radiation at the long wavelength end of the spectrum since such study involves radio frequencies in general, including microwave and radar. So much work has already been done by organizations that specialize in this area of study that it would have been a duplication of effort to consider the problem in detail here. Of special interest, however, was the availability of completed, miniaturized rf systems which would provide information to supplement data taken by other scoring components. Monitoring of work in this area is recommended so that any breakthrough may be taken advantage of.

2. BACKGROUND RADIATION

2.1 Sun - The radiance of the sun is so great that it will probably be impractical to attempt to detect even a beacon-augmented vehicle when the sun is in the sensor's field of view. Nevertheless, the spectrum of the sun figures as an important factor in background radiation since the sun is the initial source of most of the radiation that comes from the planets, the earth, and the moon.

In the optical regions, the solar spectrum roughly approximates that of a 6000°K black body. The spectral irradiance from the sun at the edge of the earth's atmosphere is given in Figures 4.1 and 4.2, which were taken from Reference 4.1.

2.2 Earth - Next to the sun, the earth is the most radiant object that will be observed at the altitudes at which the encounters are to take place. Since there appears to be no possibility of avoiding encounters in which the earth appears in the background, a thorough investigation of its radiance must be made in an effort to discover characteristics that will permit a sensor to discriminate between the...
Figure 4.1. Solar Spectral Irradiance Above the Earth's Atmosphere at the Earth's Mean Distance from the Sun
Figure 4.2: Solar Ultraviolet Spectral Irradiance above the Earth's Atmosphere.
radiation received from the munition to be scored and the radiation from the earth in the background.

As a source of background noise the radiation emanating from the earth can be considered to consist of three principal components: that reflected from the surface of the earth; that thermally radiated from the earth and its atmosphere; that reflected by the atmosphere. From Figure 4.3, it can be seen that the thermal radiation of the earth and atmosphere is something less than that of a 288° black body. This radiation is insignificant compared to that due to reflection except for wavelengths longer than 8 microns. At present, no reliable data on reflection from the atmosphere has been found, but it is expected to be of much less importance than the reflection from the surface of the earth. It is therefore concluded that the major contribution is that due to reflection from the surface of the earth. We will chiefly be concerned then with a quantitative measure of the radiation received above the atmosphere as a result of solar radiation passing through the earth's atmosphere, being reflected by the earth, and passing back through the atmosphere.

The absorbing effects of the atmosphere on the solar radiation are illustrated in Figure 4.4. The radiation traverses the atmosphere twice, once before and once after reflection, and suffers atmospheric absorption both times. Therefore, the smallest optical air mass traversed is \( m = 2 \), for which the radiation is normal to the earth's surface, whereas the average air mass will be about \( m = 3 \). The solar radiation is altered by reflection at the earth's surface as well as by transmission through the atmosphere. Figure 4.5 shows the effect on the solar spectrum following those processes. The reflecting characteristics of water were used in attendant calculations inasmuch as about three-fourths of the earth's surface is covered by water. The rise in the curves between 8 and 8.5 microns is due to the earth's radiation. Figure 4.6 compares the radiation received at the top of the atmosphere from the sun to that received from the earth.

No data were available for the reflectance of water outside the wavelength region of 4,000-8,000A; however, Figure 4.7 shows that it is reasonable to expect the reflectance in the IR region to be approximately constant at 0.008. The absorption of IR radiation by the atmosphere is illustrated in Figure 4.8. This low transmission in particular bands combined with low reflectance throughout the IR region results in practically no irradiance in these bands.

Since the reflectance of the earth's surface varies from one type of terrain to another, the usefulness of these data is confined to encounters in which water surfaces comprise the background. Figure 4.9 shows how a scoring system could take advantage of the absorption characteristics of the earth and atmosphere to reduce the undesirable
Figure 4.3. Typical Spectral Radiance curve for Thermal Radiation leaving the Earth
Figure 4.4. Solar Irradiance After Passing Through Various Air Masses
Reflectance, $r_\lambda$, is that of water.

$Q_\lambda$ is the solar irradiance after passing through the air mass specified.

Figure 4.5. Spectrum of Sunlight Reflected from Earth as seen from Above Atmosphere
Figure 4.6. Irradiance at Top of Atmosphere; Due to sun, $m = 0$,
Due to earth, $m = 3$
Figure 4.7. Spectral Reflectance of Water
background effects.

2.3 Stars - Stars are customarily rated according to their apparent visual magnitude. This means essentially that they are classified according to their ability to stimulate the sensation of brightness in the human eye, but, unfortunately for the purposes at hand, the human eye is far from being a linear detector. The spectral response of the eye is given in Figure 4.10.

The characteristics of the brightest stars are listed in Table 4.1. Except for the last column, these data were taken from Reference 4.4. The figures in the last column were derived from the approximation that the visual magnitude is directly proportional to the total irradiance (see Figure 4.11). If more accurate values are subsequently found to be necessary, the temperature of each star must be taken into account.

2.4 Moon and Planets - The irradiance at the detector due to the moon and planets will be chiefly reflected solar radiation, the spectral distribution of which is altered only slightly. Thus, in the case of an unaugmented munition irradiated by the sun, discrimination between the munition and the various planets that may appear in the background cannot be based on spectral differences. Moreover, unlike the radiation from the earth the planets appear as discrete objects of small angular size in the background. One means of discriminating between the munition and the planets, however, would be through a measurement of the relative radiances. For a planet the radiance is given by the product of the solar constant and the albedo of the planet divided by $2\pi$. For the preparation of Table 4.2, in which is presented some characteristics of the moon and the planets, the albedo was assumed to be the same for all wavelengths. The albedos listed in the various references are not quite in agreement; those in Table 4.2 were taken from Reference 4.5.

3. SOURCES OF RADIATION

3.1 Natural Thermal Radiation - A possibility exists for the scoring of an unaugmented munition through the sensing of thermal radiation emitted from either the rocket plume or, in the absence of this, from the heated missile body where the heat is derived from aerodynamic friction. However, no thorough treatment of scoring techniques based on sensing this radiation will be attempted here for the following reasons. First, very little data was available in which the radiation characteristics for these situations were given. Since the radiation would, presumably, vary greatly from one type of munition to another, thorough treatment in this area would require the specific description of the particular munitions to be used in the various encounters. Further, it appears that such radiation offers less practicable means for detection than the other types of electromagnetic
Figure 4.10. Luminous Efficiency of Monochromatic Radiant Flux as a Function of Wavelength
Table 4.1 Characteristics of Brightest Stars
(adapted in part from Ref. 4.4)

<table>
<thead>
<tr>
<th>Star</th>
<th>Visual Magnitude</th>
<th>Effective Temperature ($^\circ$K)</th>
<th>Approximate Total Irradiance Outside Temperature (watts/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Sirius</td>
<td>-1.60</td>
<td>11,200</td>
<td>$1 \times 10^{-11}$</td>
</tr>
<tr>
<td>2. Canopus</td>
<td>-0.82</td>
<td>6,200</td>
<td>$5 \times 10^{-12}$</td>
</tr>
<tr>
<td>3. Rigel Kent (double)</td>
<td>0.01</td>
<td>4,700</td>
<td>$3 \times 10^{-12}$</td>
</tr>
<tr>
<td>4. Vega</td>
<td>0.14</td>
<td>11,200</td>
<td>$2 \times 10^{-12}$</td>
</tr>
<tr>
<td>5. Capella</td>
<td>0.21</td>
<td>4,700</td>
<td>$2 \times 10^{-12}$</td>
</tr>
<tr>
<td>6. Arcturus</td>
<td>0.24</td>
<td>3,750</td>
<td>$2 \times 10^{-12}$</td>
</tr>
<tr>
<td>7. Rigel</td>
<td>0.34</td>
<td>13,000</td>
<td>$2 \times 10^{-12}$</td>
</tr>
<tr>
<td>8. Procyon</td>
<td>0.48</td>
<td>5,450</td>
<td>$1 \times 10^{-12}$</td>
</tr>
<tr>
<td>9. Achenar</td>
<td>0.60</td>
<td>15,000</td>
<td>$1 \times 10^{-12}$</td>
</tr>
<tr>
<td>10. β Centauri</td>
<td>0.86</td>
<td>23,000</td>
<td>$1 \times 10^{-12}$</td>
</tr>
<tr>
<td>11. Altair</td>
<td>0.89</td>
<td>7,500</td>
<td>$1 \times 10^{-12}$</td>
</tr>
<tr>
<td>12. Betelgeux (variable)</td>
<td>0.92</td>
<td>2,810</td>
<td>$1 \times 10^{-12}$</td>
</tr>
<tr>
<td>13. Aldeberan</td>
<td>1.06</td>
<td>3,130</td>
<td>$1 \times 10^{-12}$</td>
</tr>
<tr>
<td>14. Pollux</td>
<td>1.21</td>
<td>3,750</td>
<td>$8 \times 10^{-13}$</td>
</tr>
<tr>
<td>15. Antares</td>
<td>1.22</td>
<td>2,900</td>
<td>$8 \times 10^{-13}$</td>
</tr>
<tr>
<td>16. α Crucis</td>
<td>1.61</td>
<td>2,810</td>
<td>$5 \times 10^{-13}$</td>
</tr>
<tr>
<td>17. Mira (variable)</td>
<td>1.70</td>
<td>2,390</td>
<td>$5 \times 10^{-13}$</td>
</tr>
<tr>
<td>18. β Cruis</td>
<td>2.24</td>
<td>2,810</td>
<td>$3 \times 10^{-13}$</td>
</tr>
<tr>
<td>19. R. Hydrae (variable)</td>
<td>3.60</td>
<td>2,250</td>
<td>$1 \times 10^{-13}$</td>
</tr>
</tbody>
</table>
Based on the irradiance of a zero magnitude star being taken as $2.27 \times 10^{-12}$ watt/cm$^2$ and the relation

$$\log_{10} \frac{I_0}{I} = 0.4m$$

where $m$ = apparent visual magnitude
$I$ = irradiance
$I_0$ = irradiance of zero mag star

Figure 4.11. Total Irradiance From Individual Stars Outside Earth's Atmosphere
Table 4.2 Characteristics of Moon and Planets

<table>
<thead>
<tr>
<th>Planet</th>
<th>Albedo, a</th>
<th>Solar Constant, L (watts/cm²)</th>
<th>aL (watts/cm²)</th>
<th>Radiance, aL/2π (watts/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moon</td>
<td>0.073</td>
<td>0.14</td>
<td>0.01022</td>
<td>0.00163</td>
</tr>
<tr>
<td>Venus</td>
<td>0.59</td>
<td>0.256</td>
<td>0.151</td>
<td>0.0241</td>
</tr>
<tr>
<td>Mars</td>
<td>0.154</td>
<td>0.058</td>
<td>0.00893</td>
<td>0.00142</td>
</tr>
<tr>
<td>Jupiter</td>
<td>0.41</td>
<td>0.00517</td>
<td>0.00212</td>
<td>0.000337</td>
</tr>
<tr>
<td>Mercury</td>
<td>0.06</td>
<td>0.935</td>
<td>0.0562</td>
<td>0.00895</td>
</tr>
<tr>
<td>Saturn</td>
<td>0.42</td>
<td>0.00153</td>
<td>0.000642</td>
<td>0.000102</td>
</tr>
</tbody>
</table>
radiation sources to be described below. Finally, the limited data that is available is classified (see, for example, Reference 4.7) so that such treatment of the subject would necessitate a classified section of this report. The limited quantity of data presently available does not appear to justify the additional complication at this time.

3.2 Diffusely Reflected Radiation - The irradiance \( I \) at a distance \( R \) from a diffusely reflecting sunlit sphere of radius \( b \) is given by

\[
I = \frac{2aI_o b^2 [\sin \phi + (\pi - \phi) \cos \phi]}{3\pi R^2}
\]

where \( I_o \) is the incident solar irradiance, \( a \) is the albedo of the sphere, and \( \phi \) is the angle between the sun and the observer. This equation is valid only for cases in which \( R \gg b \). The angle dependence is illustrated by the examples

\[
I_{\phi = 0} = \frac{2aI_o b^2}{3R^2}
\]

\[
I_{\phi = 90^\circ} = \frac{2aI_o b^2}{3\pi R^2}
\]

in which the irradiance is seen to have fallen off by a factor of \( \pi \) when \( \phi \) is changed from zero to \( 90^\circ \).

3.3 Specularly Reflected Radiation

Polished Sphere - The irradiance at a distance \( R \) from the center of a specularly reflecting sphere is given by

\[
I = \frac{b^2 I_o}{4R^2} \tag{4.1}
\]

where \( b \) is the radius of the sphere and \( I_o \) is the irradiance at the surface of the sphere. This equation is valid only for \( R \gg b \) and where \( I_o \) is incident in the form of plane waves. Also assumed is a reflectivity of 100%.

If the incident radiation on the sphere is due to the sun, \( I_o \) is the solar constant, 0.14 watt/cm\(^2\). As an example of the irradiance to be
expected at a detector a distance $R$ from the sphere, we shall consider a sphere of radius two feet, while $R$ takes the values of the different maximum scoring ranges. The results of these calculations appear in Table 4.3.

<table>
<thead>
<tr>
<th>Range (ft)</th>
<th>Irradiance (watt/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>$5.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>2000</td>
<td>$3.5 \times 10^{-9}$</td>
</tr>
<tr>
<td>3000</td>
<td>$1.6 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Specular Reflection from a Cylinder - It is expected that the form of the detected vehicle will not in general be that of the sphere treated above. If the detected vehicle is the munition, a more realistic approximation would probably result from taking the form to be cylindrical.

In Figure 4.12 the incident solar irradiance $I_o$ is taken to be normal to the axis of the right circular cylinder and the range of detection $R$ is assumed to be much larger than the radius $b$ of the cylinder. The power incident on the area $dA$ is given by

$$P_1 = (I_o \cos \alpha) dA,$$

$$= (I_o \cos \alpha) (lb \, da)$$

where $l$ is the length of the cylinder. All this radiation will pass through an area $dA_2$ on an imaginary co-axial cylinder of radius $R$. The power incident on this area will be

$$P_2 = I dA_2$$

where $I$ is the value of the irradiance at $dA_2$. Then

$$P_2 = I (2lR \, da)$$

since $R >> b$. If the power is conserved, $P_1 = P_2$ so that
Figure 4.12. Irradiance from a Specularly Reflecting Cylinder
This should be multiplied by the reflectance, $\rho$, to account for power loss at the surface of the cylinder. Thus

\[ \frac{I_0 \rho b \cos \alpha}{2R} \]

Thus we see that, unlike irradiance due to a specularly reflecting sphere, here it falls off inversely with the first power of the range and is now dependent on the angle of observation.

Plane Mirrors - The solar radiation reflected from a plane mirror is equivalent to that incident on the mirror except for a factor due to reflectance. That is, if $I_s$ is the incident solar irradiance and $I_m$ is the irradiance at the detector due to reflection, then

\[ I_m = \rho I_s \quad (4.2) \]

where $\rho$ is the reflectance of the mirror. The above equation is valid, however, only at distances from the mirror short enough so that the detector receives radiation from all parts of the sun. If, for example, the detector is the human eye, the irradiance at the eye will be constant out to the range at which the eye no longer sees the whole image of the sun in the mirror. From that point onward, the irradiance falls off as the inverse square of the range. The maximum distance, $R_{\text{Max}}$, from the mirror at which the eye will see the whole solar image is

\[ R_{\text{Max}} = \frac{d}{a} \cos \theta \quad (4.3) \]

where $a$ is the angular diameter of the sun, $d$ is the diameter of the mirror, and $\theta$ is the angle of incidence.

Equation (4.3) is useful for determining the minimum diameter of a circular mirror that will irradiate the detector with the total available irradiance, $\rho I_s$. This diameter is

\[ d_w = \frac{a R_{\text{Max}}}{\cos \theta} \quad (4.4) \]
where the subscript $w$ indicates that the whole image of the sun is visible in the mirror and where $R_{\text{Max}}$ now represents the maximum scoring range for the particular type of encounter. This is illustrated in Figure 4.13. Maximum range for minimum diameter is realized for normal incidence ($\theta = 0^\circ$). Such mirror diameters for the required scoring ranges are listed in Table 4.4.

It is not necessary that the mirror reflect the maximum available radiation, $pI_s$, into the sensor in order that detection take place. The equation relating the irradiance at the detector due to solar reflection from a mirror of diameter smaller than $d_w$ of equation (4.4) is readily derived with the aid of Figure 4.14. The ratio of irradiances at the detector will simply be the ratio of the areas of the two mirrors. If the eye is the detector again, then we can let $I_w$ be the irradiance due to the larger mirror, $I_p$ be that due to the smaller, in which only part of the sun is visible, and $A_w$ and $A_p$ be the respective areas of the two mirrors. Thus

\begin{align}
\frac{I_p}{I_w} &= \frac{A_p}{A_w} \\
I_p &= \frac{A_p}{A_w} I_w \\
I_p &= \frac{d^2}{d_w^2} I_w \\
I_p &= \frac{d^2 I_w}{a^2 R^2} \cos^2 \theta \\
I_p &= \frac{d^2 \rho I_s \cos^2 \theta}{a^2 R^2}
\end{align}

Also, from (4.2)

\begin{align}
I_p &= \frac{d^2 \rho I_s \cos^2 \theta}{a^2 R^2}
\end{align}

The average value of $\cos^2 \theta$ is $1/2$ so that the average irradiance at the detector due to a mirror of diameter $d$ will be given by
Figure 4.13. Plane Mirror Illuminated by Sun
Figure 4.14. Relation Between Mirror of Diameter $d_w$ and Smaller Mirror
I = \frac{d^2 \rho I_s}{2a^2 R^2} \quad (4.9)

so long as \( d \leq aR \). Values of \( I \) at the required scoring ranges are given in Table 4.4 in which \( \rho \) was taken to be one and the mirror diameter to be \( d = 3.38 \) in, giving the same area as a square mirror three inches along the side.

<table>
<thead>
<tr>
<th>R (ft)</th>
<th>( d_w ) (ft) (for ( \theta = 0 ))</th>
<th>( d ) (in)</th>
<th>( I ) (watts/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>4.95</td>
<td>3.38</td>
<td>( 5.15 \times 10^{-4} )</td>
</tr>
<tr>
<td>2000</td>
<td>18.6</td>
<td>3.38</td>
<td>( 3.22 \times 10^{-5} )</td>
</tr>
<tr>
<td>3000</td>
<td>27.6</td>
<td>3.38</td>
<td>( 1.43 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

Some scoring systems may use two detectors. It may be useful to know their maximum separation if each is to receive radiation from a single mirror. At the maximum range to receive the maximum available irradiance for a given mirror diameter (see Figure 4.15), there will be only one point, A, at which this full value of irradiance will be detected. As the detector is moved from point A to point B, it receives less radiation since rays from only a limited area of the sun's image are incident at that point. Point C represents the point at which only rays from the extreme edge of the image are incident; beyond this point the irradiance drops to zero. Because of the great distance to the sun, angle AEC is equivalent to angle \( \alpha \), so that the distance AC is the same as the diameter of the mirror. At this range, therefore, the detectors cannot be separated by a distance greater than twice the diameter of the mirror.

The more general case of an arbitrary range and a mirror of arbitrary diameter is illustrated by Figure 4.16. In this case the maximum separation of the detectors is \( D \), which is found to be

\[
D = 2aR - D_2
= 2aR - a(R - R_1)
= aR + d
\]
Figure 4.15. Maximum Separation of Detectors, Special Case
Figure 4.16. Maximum Separation of Detectors, General Case
Plane Mirrors on a Sphere - We now take up the question of how many mirrors of what size, shape, and disposition over the surface of a scored vehicle would be required to furnish some reflected light at all points throughout the region where a sensor might be located. Further investigation would be needed to determine the energy increase and to find answers to other related questions.

In this preliminary investigation, simplified mathematical models will be used which are based on fundamental laws of optics for the reflection of light from a plane mirror. The light source is the sun, which has an angular diameter at the earth of some 32 seconds of arc. The slight variations that occur with season and as a result of atmospheric refraction are ignored in the following treatment.

The first matter to be looked into is the boundaries of the region into which sunlight reflected from a given plane mirror would fall. The sensor will be assumed to be located at a distance, \( R \), from the target object, with \( R \) infinitely large in comparison to the linear dimension, \( d \), of the mirror. Figure 4.17 shows a sketch of the plane of the incident and reflected rays, with the region illuminated at \( R \) by the reflected rays. Since the sun is so far away, rays from the same point on the sun to any point on the mirror have, essentially, the same direction. A ray from the center of the sun to the center of the mirror is shown. The radius of the sun's disk subtends an angle of 16 minutes at the mirror. Therefore, the two lines drawn from the mirror to the center of the sun and to a point on the circumference of the sun diverge by this angle \( \Delta a/2 \), where \( \Delta a \) is the angular diameter of the sun. At distances \( R \) large in comparison to \( d \) (the mirror diameter), the linear dimension or diameter of the region in which reflected light falls is essentially \( 2R \sin \Delta a/2 \), or \( R\Delta a \). This diameter of the boundary zone of reflected light is, of course, normal to a central direction. From symmetry, the two-dimensional region at \( R \) in which reflected light falls is a circle normal to \( R \). These boundaries of the region of reflected light at distance large in comparison to the dimensions of the plane mirror are then independent of the size and shape of the mirror used, and form a circular cone of central angle \( \Delta a/2 \).

Next consider the change in direction of a reflected ray from the center of the sun to the mirror as the mirror is rotated in the plane determined by the ray and the mirror normal. If the angle between the mirror normals is \( \delta \), the angle between the reflected central rays is \( 2\delta \). Thus, if the patterns of illumination from two adjacent mirrors are to overlap, the angle \( \delta \) between the mirror normals must be such that

\[ 2\delta < \Delta a, \quad \delta \leq \Delta a/2 \]

For future reference, one may note that when the plane of the
Figure 4.17. Reflection of Sunlight from a Plane Mirror
mirror is rotated through an angle $\delta$ about an axis which is normal to the mirror normal and lies in the plane determined by the mirror normal and the incident ray, then the reflected ray is rotated about this same axis, and is then rotated only through the angle $\delta$ instead of $2\delta$. One may then conclude that the problem of obtaining some reflected light at all exterior points from a spherical surface requires closer placement of mirrors to give coverage at points in space which have the same direction as the sun. For such directions, the requirement $2\delta \leq \Delta a$ applies. On the other hand, such requirements must cause the amount of overlap of lighted regions from adjoining mirrors to progressively increase as the direction from the target sphere opposite to the sun's direction is approached by the sensor.

Next consider the two-dimensional problem of the number of plane mirrors needed to give some reflected illumination at all points in an arbitrary plane determined by the radii from the observed object to the sun and to the sensor. The observed object will be idealized as a sphere of radius $p$, with the mirrors assumed to be of some uniform linear dimension $d$, and tangent to this sphere at their centers. If $N$ is the number of such mirrors required in the given plane, these mirrors form a regular polygon of $N$ sides. The angles between normals of adjacent mirrors is then some constant $\Delta \theta$, with $N \Delta \theta = 2\pi; \Delta \theta = 2\pi/N$. From the discussion of the boundaries of the region illuminated by sun rays reflected from a single plane mirror, it may be seen that the requirement

$$\Delta \theta \leq \Delta a/2$$

is necessary and sufficient for all points in the given plane, lying outside the observed object and its small shadow, to receive some sunlight reflected from the mirrors. Using the value of $\Delta \theta$,

$$2\pi/N \leq \Delta a/2$$

$$N \geq 4\pi/\Delta a \sim 1350$$

While this might appear to be twice as many as are needed, actually only half the mirrors are available for reflectors at any one time, the other half being in the shadow. Thus some 1350 mirrors of equal size arranged about a circle containing the line-of-sight to the sun would be needed to give the required coverage with reflected radiation in the plane of this circle. If the linear dimension of such a mirror is $d$, the sum of these lengths, $Nd$, is a good approximation of the circumference of this circle, $2\pi p$,

$$2\pi p \sim Nd$$

$$p \sim Nd/2\pi$$
For example, if \( d = 1/16 \) inch, then \( \rho \), the radius of the sphere, should be at least 13.43 inches.

The coverage with reflected radiation may be obtained for any size \( d \); the essential requirement to obtain the desired coverage is that the angles between normals of adjacent mirrors be less than \( \Delta \alpha /2 \), or the sun's angular radius. Thus, for the given value of \( d \), and an arbitrary object shape in the given plane, the given coverage is also obtained so long as \( \rho \), the radius of curvature of the reflecting surface is the given plane, is such that \( \rho \geq 13.43 \) inches (\( \rho \) continuous). Again, the minimum number 1350 would be required; more may be needed if the mirrors are to be touching their neighbors. The solution to the two-dimensional problem for the mirrors arranged about a circle also provides the answer for the mirrors arranged about an arbitrary plane curve of the required radius of curvature.

Looking now at the three-dimensional problem, let us again assume that the target object is spherical and covered with plane mirrors, each with differential area. It is seen that a single mirror will illuminate a circular cross-sectional surface patch, at distance \( R \) from the mirror, of area \( \pi \left( \frac{R \Delta \alpha}{2} \right)^2 \). If \( M \) such plane mirrors are needed, one must have the sum of these surface patches greater than or equal to the surface of radius \( R \),

\[
M \pi \left( \frac{R \Delta \alpha}{2} \right)^2 \geq 4\pi R^2
\]

\[
M \geq \frac{16}{(\Delta \alpha)^2} \sim 184,658
\]

At least twice this number of mirrors would be needed, however, as half are always in shadow. Also, twice this number would not give complete coverage, as some of these surface patches at the radius \( R \) would overlap, leaving other regions without coverage.

In order to get some notion of a minimum amount of overlap required to give complete coverage, it will be necessary to choose a simple model for the arrangement of the mirrors, or rather of their normals. Figure 4.18 is a sketch of a differential plane surface element of the sphere of radius \( R \) showing three central rays \( A, B, C \), from three adjacent mirrors on the target object illuminating a surface patch in the direction of the sun. As stated previously, this direction makes the most severe demands on mirror placement. These central rays are here supposed to be arranged at the corners of an equilateral triangle. Each such corner of this triangle is the center of a circle of reflected illumination with radius \( R \Delta \alpha /2 \). For complete coverage, these three circles have a common point at \( P \), the center
Figure 4.18. Surface Patches at Radius R for Minimum Overlap of Illuminated Regions
of the inscribed circles of triangle ABC.

The areas common to two or more circles in Figure 4.18 represent regions of overlap of the zones of reflected sunlight from adjacent mirrors on the target object. With this symmetric arrangement of mirrors on the target object, each of these boundary circles of reflected light on the sphere of radius R will have six such equal regions of overlap. For computing a factor for this loss in coverage, half of the region of overlap must be deducted from the area, $\pi(R \Delta a/2)^2$, of each surface patch at R illuminated by a single mirror. Each plane mirror then is required to illuminate a regular hexagon of side $(R \Delta a/2)$, and area $0.6495(R \Delta a)^2$. Taking into account this minimum overlap, the number of mirrors required is determined by

$$0.6495 M(R \Delta a)^2 \geq 4\pi R^2$$

$$M > 223,290$$

However, it will be seen that this number must be multiplied by four.

From the geometry of Figure 4.18, one may compute the lengths AB, BC, AC to be $0.866R \Delta a$. If the target (the vehicle being scored) is spherical of radius $\rho$ and the points of contact (tangency) of the mirrors are symmetrically arranged to form equilateral triangles, the distance separation of normals at the points of tangency of neighboring mirrors is $0.433\rho \Delta a$. This factor of two change results because the reflected light is here rotated through twice the angles between the normals. This arrangement of the normals is an equilateral triangular pattern could be achieved by use of hexagonally shaped plane mirrors, with side $d$ such that $d = \rho \Delta a/4$. The surface area of such a mirror is $0.6495 (\rho \Delta a/2)^2$. The minimum number, $M$, of such mirrors required is such that $0.6495 M (\rho \Delta a)^2 \geq 4\pi \rho^2$ or $M \geq 893,160$. Thus if $d = 1/16$ in., a sphere of radius 26.86 inches completely covered with some 893,160 such hexagonal mirrors would provide some reflected illumination for all orientations of the target at all sensor locations outside a small shadow region. Mirrors of such size would also serve for covering a target object (scored vehicle) of arbitrary shape so that all plane curves on the surface would have a continuous radius of curvature $\rho \geq 26.86$ inches. Likewise, plane mirrors of arbitrary shape, but such that each mirror could be enclosed by the hexagon of side 1/16 inch would also serve for such targets. At least this number of mirrors would be required for vehicles of arbitrary shape and size; more would be needed if the proper disposition could not be obtained.
It has been stated that the overlap of illuminated areas must progressively increase as the sensor approaches a direction from the target which is opposite to the sun's direction. One may compute an average figure for the amount of overlap from a spherical target covered with some 893,160 hexagonal mirrors of a uniform size. Here, half of this total number of mirrors must illuminate the entire spherical surface at radius $R$, while each such mirror illuminates a surface patch of area $\pi \frac{R \Delta a}{2}^2$. This gives an average ratio of coverage,

$$\frac{893,160}{2} \frac{\left(\frac{R \Delta a}{2}\right)^2}{4 \pi R^2} \approx 2.42$$

One may obtain an approximation which should be a generally better estimate of an average ratio of overlap at any point by considering the reflection from a differential latitude belt, the sun's direction being located above the north polar axis of the scored vehicle. If the north colatitude ($90^\circ$ - north latitude) of this reflecting differential belt is $a$, then this belt reflects onto the sphere of radius $R$ into a belt of north colatitude $2a$. If one ignores the small overlap of reflected radiation in the north-south or latitude direction and considers only the overlap of reflected light in the east-west, or longitude direction, one may then compute an average figure for the coverage ratio which will apply to sensor locations having the general colatitude coordinate, $2a$.

Thus, a differential reflecting belt of mirrors, 1 mirror high, at colatitude $a$ has approximately $K$ reflecting hexagonal mirrors, each of side $\frac{\rho \Delta a}{4}$, with the centers of adjacent mirrors thus being separated by some $1.732 \frac{\rho \Delta a}{4}$. The circumference of this colatitude parallel on the reflecting sphere is $2\pi \rho \sin a$, so that $K \approx \frac{2\pi \rho \sin a}{1.732 \frac{\rho \Delta a}{4}}$. Each such mirror illuminates a portion of the surrounding spherical surface of radius $R$, at the colatitude $2a$, each such portion having an east-west dimension of $R \Delta a$. On the surrounding sphere of radius $R$, the circumference at this north colatitude $2a$ is $2\pi R \sin 2a$. Thus, an average coverage ratio at sensor locations of north colatitude $2a$ is given approximately by

$$KR \Delta a / 2\pi R \sin 2a \approx 2.3 \frac{\sin a}{\sin 2a}$$

This coverage ratio applies only as an average value, with some points...
having coverage from multiple mirrors, other points having reflected light only from a single mirror. Also, interpreted as an average ratio, the ratio provides a conservative estimate of the coverage because the small amount of overlap from one direction has been ignored and the reflecting target may be non-spherical, thus requiring a larger value for K. This value is only approximate since, for simplicity, the analysis has been greatly simplified.

The approximate numerical values obtained in this analysis must be regarded as lower bounds for the number of mirrors required to give complete coverage for all orientations. In practice, the placement of the mirrors on the surface so that they are tangential at their centers can never be done exactly; therefore appropriate factors must be used to increase the number of mirrors used, thus decreasing the mirror size. For targets covered with such a large number of mirrors, the limiting case of the specular spherical reflector should provide estimates of the intensity of reflected radiation at sensor locations. For the case of a target covered with a finite number of plane mirrors, a more detailed investigation of the overlap would probably be required to obtain the intensity of reflected radiation at the sensor.

3.4 Radiant Sources - If the natural radiation from the missile or the reflectivity of the missile is not adequate to produce a large enough signal-to-background-noise ratio, it may be possible to increase the signal level by augmenting the missile. Various methods are available for augmentation. If, for example, the missile is being illuminated by a radiant source near the sensor (an "active" type system), a set of corner reflectors might be used to enhance the reflective characteristics of the missile. A corner reflector has the property of reflecting incident light along a path parallel to the incident path. One principal disadvantage of the active system, however, is that the signal strength at the sensor or receiver varies inversely as the fourth power of the range to the missile. For natural illumination from sunlight or earth albedo, either flat or curved-surface reflectors can be used. Expected values of reflected solar power for various reflectors have been given in the previous section entitled Specularly Reflected Radiation.

Another augmentation scheme which appears to offer some advantages is the placement of fluorescing materials, chelates in particular, on the missile. The optical chelate (Reference 4.8) reportedly has the ability to absorb radiation over a wide wavelength band and to re-emit the absorbed energy in a very narrow wavelength band. Through the use of a narrow band pass filter to pass the characteristic wavelength of the chelate, the background noise could be suppressed without reducing the signal level significantly. The chelate would be effective in absorbing either natural radiation from the sun and earth or that transmitted by an active radiant system.
These materials have been known to exhibit an 85% quantum efficiency which means that 85% of the total energy absorbed by non-thermal processes (electronic or molecular excitations) is re-emitted in the narrow wavelength band characteristic of the chelate material.

For encounters in which the signal-to-background-noise ratio is not sufficient even through the use of passive aids—this could arise as a result of high background noise level or with a small missile of low reflectivity—augmentation might require the addition of a radiant source on the missile. The characteristics of the particular sensor being used would determine the type of light source desired. The four main types of light sources are: (1) filament lamps and (2) high-pressure arc lamps, (3) low-pressure arc lamps (vapor lamps), and (4) lasers.

Of the possible methods which might be used to decrease the effects of background noise, the first obvious one is that of brute-force, whereby the signal strength is increased greatly through the addition of a very intense black-body type source on the missile. If the missile to be scored is located directly between the scorer and the sun, that is, if it is superimposed on the solar disk, it would be futile to try to increase the radiancy over and above that of the sun. For this particular situation obscuration techniques would be more effective. Since this alignment would seldom occur, it would be more logical to plan the test to minimize the probability of such an occurrence. Even if the missile is situated with the sunlit earth or moon in the background, much power would be required to raise the signal level above the background by this first method.

A second method might involve augmenting the missile with a vapor arc-lamp whose output power lies in specific spectral lines or bands. Particular wavelengths desired can usually be obtained by the proper choice of vapor or vapor mixtures. Although greater power outputs are available from the tubes with high internal pressure, the more efficient sources are the low-pressure arc-lamps. The most efficient source appears to be the sodium vapor lamp whose greatest intensity lies at the yellow doublet spectral lines. The larger sodium lamps are capable of an 11,000 lumen output which corresponds to about 25 watts in the visible spectrum, much of which is contained in the yellow doublet. This technique would require the use of an optical band-pass filter which would be transparent to wavelengths associated with the most intense output of the lamp but which would be opaque to all other wavelengths. Effectively then, the signal-to-background-noise ratio would be increased because the filter would suppress much of the black-body type background radiation without substantially reducing the signal level.

Another method of overcoming background noise would be through the use of a pulsed (flasher) light source or strobe lamp. The improvement results because of the much higher power output that exists.
during the "on" period as compared to the cw power level that would exist for continuous operation, where both situations involve the same input power. The result is a higher signal level with no change in background noise level. A disadvantage would arise in the use of this technique if a scanner or framing type device were used in conjunction with the sensor. For then synchronization between the pulsing of the source and the "on" interval of the scanner would be required.

If missile augmentation in the form of a light source on the missile is to be considered, the light source must be the most efficient possible to minimize weight and power consumption. The particular type of light source needed will be determined by the requirements of the scoring device. From a study of the present state-of-the-art of radiant sources, it has been determined that the arc-discharge lamp is the most efficient in converting electrical energy into non-thermal electromagnetic radiation.

For the production of black-body type radiation, the high-pressure Xenon short arc lamp seems to exhibit the greatest luminous efficiency (lumens/watt input). Both the General Electric and the Hanovia Xenon arc lamps are very similar in their spectral output and luminous efficiency. The high-pressure (~20 atmospheres) Xenon arc lamps are generally for high power inputs (500 to 5000 watts). The color temperature of all of the Xenon lamps is about 6000°C but with the output curve exhibiting very pronounced resonance lines between 0.8 and 1.0 microns (infrared). For the visible region (8000 Å to 4000Å), the radiation curve is almost identical to the radiation curve of the sun.

Greater luminous efficiency can be attained with a Xenon arc by the addition of a small amount of mercury to the lamp. Since most of the added radiation is towards the blue end of the spectrum, however, the lamp no longer approximates a black-body source. A typical 1000-watt Xenon-Mercury arc lamp emits approximately 205 watts of power in the visible, approximately 95 watts of power in the ultraviolet (down to wavelengths of 2000 Angstroms) and approximately 200 watts in the IR (wavelengths of 1.4 microns and greater). The total energy radiated between 2000 Å and 14,000Å, therefore, is approximately 50% of the total input energy. Graphs illustrating the spectral output for both the Xenon and Xenon-Mercury arc lamp are presented in Figures 4.19 and 4.20. For the pure Xenon lamp, the output in the visible is approximately 11.5% of the total input energy as compared to 20.5% for the Xenon-Mercury. The arc lengths of these various arc lamps range between 3 and 6 mm. For sensing devices that function on image intensity rather than total power, the short-arc lamp will appear as a point source at long ranges and the image of the source in the focal plane of some optical system will be
Figure 4.19, Spectral Energy Distribution of Xenon Compact Arc Lamp
Figure 4.22. Generalized System
a Fraunhofer diffraction pattern. This means that even though the total power arriving at the lens may be extremely small, the intensity in the diffraction pattern is quite high.

A possible method of decreasing the power input to the lamp without diminishing the power output level would be to pulse-operate the lamp. The pulsating output would also make the signal a little more distinguishable from the background.

A stroboscopic lamp (strobe-lamp) having a black-body type output is produced by General Radio Co. It is rated at 35 watts input, has flash durations of 0.8, 1.2 and 3.0 \(\mu\)sec and an output of approximately \(10^6\) candlepower at flash duration of 0.8 \(\mu\)sec. The estimated weight of a system designed for augmentation would less than 10 pounds.

If monochromaticity is required, a vapor lamp can be used in conjunction with filters. The most efficient of the vapor lamps producing light in the visible region of the spectrum appears to be the low-pressure sodium arc discharge lamp. However, this lamp is limited to low vapor pressures (0.0009 mm Hg) and consequently to maximum input powers of about 250 watts. A 250-watt sodium arc lamp is capable of producing 25,500 lumens or about 50 watts output in the sodium doublet 5890\(\AA\) and 5896\(\AA\). (The doublet is approximately 17 times as powerful as the next brightest line in the visible spectrum.)

A very efficient arc-discharge lamp with a resonance line in the ultra-violet is the low-pressure mercury vapor arc lamp. The Hanovia Co. produces a low-pressure mercury arc lamp rated at 30 watts input and it has an output of 10.4 watts in a region about 30\(\AA\) wide at 2537\(\AA\). As the power or the pressure increases in the various mercury lamps the efficiency in converting input power into output power in a given spectral line decreases since more and more lines begin to appear in the spectrum. The high-pressure mercury arc lamps generally run at inputs from 100 watts to 7500 watts whereas the low-pressure lamps generally run below 35 watts. In the high-pressure lamps the resonance line has shifted from 2537\(\AA\) to 3660\(\AA\); the 2537 line becomes reversed (absorbed).

The laser or optical maser is either a solid state or gaseous device that emits highly monochromatic coherent light. The only solid state and gaseous lasers that emit in the visible spectrum are ruby and a neon-helium mixture. The wavelength emitted by the ruby laser is 6943\(\AA\) at room temperature while 6934\(\AA\), 7009\(\AA\), and 7041\(\AA\) have been reported at lower temperatures. The neon-helium
laser emits at 6328Å in the visible region. All other solid-state and gaseous materials useful as lasers emit in the infra-red region.

Most solid-state lasers are pulsed devices—the output lasting on the order of a millisecond with a pulse repetition rate of about one pulse per 25 seconds. The gaseous lasers on the other hand are continuous operating. By lowering the operating temperatures to that of liquid oxygen, Bell Laboratories have operated a single crystal of calcium tungstate (CaWO$_4$;Nd$^{3+}$) continuously for 20 minutes in the IR region (10, 650Å). The power output of this device is about 1 milliwatt.

The gaseous lasers operating at room temperatures have an output of the order of 1 to 10 milliwatts with 40 watts input. The tube of the gaseous lasers is generally longer than the flash heads for the solid-state lasers. One neon-helium laser (available from Perkin-Elmer Corporation) is 26 inches long and weighs 13 pounds.

The weight of the power supply and storage capacity for the solid-state laser will depend in part on the energy requirements of the pumping source. Hughes Aircraft has a 780 joule power supply that weighs 28 pounds. The output energies for the solid-state lasers vary considerably, depending upon the crystal, doping agent and degree of doping, length and diameter of crystal, optical imperfections, reflectivity of end plates, efficiency of pumping, etc. One ruby crystal 1-1/2 inches long and 1/2 inch in diameter has an output of one to two joules. The American Optical Company reported an output of over 100 joules for a neodymium-doped glass laser. The power output varies for different lasers as is readily seen by looking at the average power which is the energy output divided by the duration of the pulse. The power output, however, is not constant throughout the pulse; a graph of power versus time will show numerous spikes with millisecond half-widths. The peak power of these spikes can range from 10 kilowatts to one megawatt depending upon the laser.

The polarization of the light emitted depends upon the orientation of the z-axis of the crystal with respect to the optical axis; the polarization may be either plane or elliptical. Gaseous lasers emit plane polarized light.

One property of the laser is the high degree of spatial coherency across the emitting surface. The image of a perfectly coherent source is limited only by the diffraction pattern which means for visible light the beam divergence can be made smaller than that obtainable from a non-coherent source. (The beam divergence is given by $1.22\lambda/a$ radians for a circular aperture where $"a"$ is the
aperture diameter and \( \lambda \) the wavelength.) The small beam divergence is important in tracking for an active system for if the target is larger than the radiating beam, the light intensity received back at the source will vary merely as \( 1/r^2 \) whereas if the beam is larger then the target the signal will vary as \( 1/r^4 \) where \( r \) is the distance from the source to the target.

If the output of the laser is to be spread over \( 4\pi \) steradians, there will be no advantage in using the laser over other light sources as far as intensity is concerned. The laser on the other hand being monochromatic does possess some merit since narrow band interference filters with their transmission bands centered about the laser frequency can be used in conjunction with the laser to increase the signal-to-noise ratio in detection.

Another source to be considered is the diffused junction GaAs diode. Philco produces one diode that emits at 9000 Å with a power output of one milliwatt at room temperatures while at 77\(^\circ\)K the power output can be increased to 25 milliwatts. The efficiency of this device increases from 5% at room temperature to almost 100% at 77\(^\circ\)K. The light emitted by the Philco diode is incoherent while General Electric Company reports having produced a coherent light with the GaAs p-n junction at 7100 Å with a half width of 12 Å. The spectral width is a function of current density; a current density of 16,000 amperes/cm\(^2\) produced a half width of 125 Å, 19,000 amperes/cm\(^2\), 12 Å. General Electric expresses the opinion that the output wavelength can be selected at any value between 6200 Å and 8400 Å. The Lincoln Laboratory has reported a spectral width of 5 Å at 4.2\(^\circ\)K with currents from 6 to 190 amperes. The peak radiated power is believed to be 280 watts for their particular diode. The coherent mode (with a 5-microsecond pulse) was obtained by pulsing the diode at a 13-cps repetition rate.
4. SENSORS

4.1. Bolometers - The word "bolometer" technically refers to a particular instrument devised by Langley for the measurement of radiation. It will be used here, however, in the general sense to designate any radiation-measuring device that utilizes the temperature coefficient of resistance of the detecting element. Some of the bolometers in current use are the metal, Polaroid, Strong, columbium nitride, evaporated-gold, and thermistor types. The characteristic common to all these bolometers that is most liable to limit their use in scoring systems is their rather long time constant. The time constant of the columbium nitride bolometer is, according to Reference 4.9, about 0.5 millisecond for a 63 percent response, while that of the evaporated-gold type is 7 milliseconds and that of the thermistor type is also a few milliseconds.

4.2. Photoconductors - Photoconductive detectors are constructed from semiconductor materials and they have their widest applications in the intermediate-I.R region. The characteristics of some typical photoconductors are given in Table 4.5. In this table, the detectivity, \( D^* \), is defined as

\[
D^* = (A \Delta f)^{1/2} D
\]

where \( D \) is the usual detectivity; that is, the reciprocal of the noise equivalent power. \( A \) is the cell area and \( \Delta f \) is the noise-equivalent bandwidth (Reference 4.9, page 147).

A device developed by the Electro-Optical Systems is called an Infrared Radiation Tracking Transducer. The transducer is a junction photo-device whose output is sensitive to the position of a light spot on its surface. The response of the cell is a function of total power in the light spot and the position of the spot. The cell will respond in proportion to the location of the center of gravity of a general intensity distribution on the surface. Since the zero response of the cell is in the geometric center of the cell a uniform illumination by background radiation has no effect on the determination of the position of a light spot falling on the surface of the cell. This is assuming, of course, that the background level is below the saturation level of the cell. If the background is not uniform, such as a star field, the output of the cell will be a linear superposition of the effects of the individual spots.

At the present state of development the linearity exhibited by the cell is not satisfactory and the responsivity is quite low, but the Electro-Optical Systems Company is trying to improve these characteristics.
### Table 4.5. The Characteristics of Some Photoconductors (Reference 4.9)

<table>
<thead>
<tr>
<th>DETECTOR TYPES</th>
<th>DETECTOR SENSITIVITY</th>
<th>Operating Temp °C</th>
<th>Resistance Megohms</th>
<th>Spectral Response Limits, μ</th>
<th>Time Constant μ sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>PbS</td>
<td>$1 \times 10^{12}$</td>
<td>25</td>
<td>1</td>
<td>0.72 - 4.7</td>
<td>5 - 1,000</td>
</tr>
<tr>
<td>PbS</td>
<td>$2.5 \times 10^{12}$</td>
<td>-78</td>
<td>20</td>
<td>0.72 - 5.5</td>
<td>1,000 - 3,000</td>
</tr>
<tr>
<td>PbS</td>
<td>$1.5 \times 10^{13}$</td>
<td>-196</td>
<td>1 - 100</td>
<td>0.72 - 5.9</td>
<td>250 - 3,000</td>
</tr>
<tr>
<td>PbTe</td>
<td>$5 - 8 \times 10^9$</td>
<td>-196</td>
<td>100</td>
<td>0.72 - 6.0</td>
<td>&lt; 30</td>
</tr>
<tr>
<td>PbSe</td>
<td>$1.5 \times 10^9$</td>
<td>-196</td>
<td>0.2 - 20</td>
<td>0.72 - 6.5</td>
<td>10 - 40</td>
</tr>
<tr>
<td>InSb (PEM-type)</td>
<td>$2 \times 10^9$</td>
<td>25</td>
<td>$1 \times 10^{-6}$</td>
<td>0.72 - 7.0</td>
<td>1 - 5</td>
</tr>
<tr>
<td>InSb (PEM-type)</td>
<td>$7 \times 10^9 - 1.5 \times 10^{10}$</td>
<td>-196</td>
<td>$2 - 7 \times 10^{-6}$</td>
<td>0.72 - 7.0</td>
<td>2 - 10</td>
</tr>
<tr>
<td>InSb (n-type)</td>
<td>$3 \times 10^7$</td>
<td>-196</td>
<td>5 - 6</td>
<td>0.72 - 7.0</td>
<td>2 - 5</td>
</tr>
<tr>
<td>InSb (p-type)</td>
<td>$0.8 - 4 \times 10^9$</td>
<td>-196</td>
<td>0.2 - .5</td>
<td>0.72 - 7.0</td>
<td>2 - 5</td>
</tr>
<tr>
<td>InSb (diffused junction)</td>
<td>$2 \times 10^{10}$</td>
<td>-196</td>
<td>$9 \times 10^{-3}$</td>
<td>0.72 - 7.0</td>
<td>1 - 2</td>
</tr>
<tr>
<td>Ge:Au</td>
<td>$2 \times 10^{10}$</td>
<td>-196</td>
<td>--</td>
<td>0.72 - 11</td>
<td>---</td>
</tr>
<tr>
<td>Ge:Au, Sb</td>
<td>$2 \times 10^{10}$</td>
<td>-196</td>
<td>--</td>
<td>0.72 - 6</td>
<td>---</td>
</tr>
<tr>
<td>Ge:Au, Zn</td>
<td>$4 \times 10^9$</td>
<td>-223</td>
<td>0.3</td>
<td>0.72 - 45</td>
<td>&lt; 0.01</td>
</tr>
</tbody>
</table>

$D^*$ is the detectivity $D$ reduced to unit bandwidth of the noise and unit area of the detector.

$D^* = (A \Delta f)^{1/2} D$, where $D$ is the detectivity measured with a noise-equivalent bandwidth $\Delta f$ and $A$ is the area of the detector.
A device for determining the position of a light spot along a line is the Photopot developed by the Giannini Controls Corp. This device has been described in detail in the Technical Supplement to the Proposal on Purchase Request ASQT 62-200, Supplemental Agreement No. 1 to Contract AF 08(635)-2631, and under experimental research has been found to be unsatisfactory in its present state for a scoring system. It is felt, however, that with further development the photopotentiometer could show feasibility as a sensing device in a scoring system.

4.3. Photoemitters - The term photoemitter as applied here will refer both to multiplier phototubes and TV camera tubes. Single unit phototubes will not be discussed here since they have no advantage over the multiplier phototube.

The operation of the multiplier phototube is a function of the total radiant power falling on the cathode; therefore the only purpose of an optical system associated with a multiplier phototube arrangement would be to increase the aperture of the tube and decrease the field of view. If a moveable slit is added to the system at the focal plane of the optics, the image can be scanned and the output of the multiplier phototube is a function of radiant power distribution in the image. In general, the response time of multiplier phototubes is less than $10^{-8}$ second. The wavelength response is a function of the photoemissive surface and is below 12000 A wavelength. There are no operative phototubes for the IR region since the photon energy is less than the work function of any known photoemissive substance. The minimum detectable signal will be determined by the noise characteristics of the tube and these are listed for various tubes in Table 4.6. The dark current in the tube is a function of temperature and can be minimized by cooling. The maximum signal strength that can be handled by the tube without damage will have to be obtained from the individual manufacturers. The multiplier phototubes appear to give the best responsivity and detectivity of any of the sensors yet mentioned. One possible drawback might be the fact that the majority of the multiplier phototubes have their peak response in the violet or ultraviolet region of the spectrum. If augmentation in scoring is considered, the multiplier phototubes are well adapted to the use of either high or low-pressure mercury vapor lamps.

Another photoemitter device that is extremely sensitive is the TV camera tube. The advantage of the TV tube over the multiplier phototube is the ability of the TV tube to determine the location of a light spot in the focal plane of some optical system. That is, the field of view is scanned electronically rather than with a rotating slit as mentioned above under multiplier phototubes.

There are three types of TV tubes either presently on the
Table 4.6. Some Characteristics of Photomultiplier Tubes

<table>
<thead>
<tr>
<th>Spectral Response Types</th>
<th>Wavelength of Maximum Response</th>
<th>Sensitivity $\mu$amp/$\mu$watt</th>
<th>Equivalent Input Noise (lumens)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1</td>
<td>8000</td>
<td>420</td>
<td>$1.5 \times 10^{-10}$</td>
</tr>
<tr>
<td>S-4</td>
<td>4000</td>
<td>24,000 to 78,000</td>
<td>$10^{-11}$ to $10^{-13}$</td>
</tr>
<tr>
<td>S-8</td>
<td>3650</td>
<td>750</td>
<td>$7.5 \times 10^{-12}$</td>
</tr>
<tr>
<td>S-11</td>
<td>4400</td>
<td>6,000 to 800,000</td>
<td>$3 \times 10^{-12}$ to $4 \times 10^{-12}$</td>
</tr>
<tr>
<td>S-19</td>
<td>3300</td>
<td>65,000</td>
<td>$7.5 \times 10^{-13}$</td>
</tr>
<tr>
<td>S-20</td>
<td>4200</td>
<td>1,200,000</td>
<td>Developmental</td>
</tr>
<tr>
<td>---</td>
<td>2350</td>
<td>40</td>
<td>Developmental</td>
</tr>
</tbody>
</table>
market or under development. The three types are (1) the image orthicon, (2) the vidicon, and (3) the image-intensifier tube. The vidicon tube is the most rugged tube of the three and it has a large signal-to-noise ratio. The sensitivity is, however, the lowest of the three. The orthicon, while less rugged, is more sensitive than the vidicon. The most sensitive of the three TV tube types is the image-intensifier tube. This tube generally consists of an image intensifier placed in front of a regular vidicon tube. The electrons given off by the photoemissive surface are accelerated through a high voltage before they are allowed to strike the target material of the vidicon. By this means there is essentially a photomultiplier effect intensifying the image falling on the face of the tube. For the visible region response the tube is generally referred to as the EBICON. Westinghouse is presently developing a UVICON tube which is essentially the same as the EBICON with the exception that the photoemissive surface is most sensitive in the UV region of the spectrum. The UVICON and the EBICON manufactured by Westinghouse have a zoomar capability built electronically into the tube. Normally the whole image plane is focused onto the target material in the vidicon. Through the electronic zoomar technique, a small portion of the center of the image plane is magnified to cover the entire target material, increasing the resolution and magnifying power of the tube. Westinghouse engineers have indicated that it might be possible to zoom in on other areas of the image plane rather than just in the center as the tube now does.

The resolving power of any TV tube is a function of the number of line pairs scanning the target material. As seen from Table 4.7 this resolving power may run from a total of 115 lines to over 1000 lines. One of the disadvantages of the use of a TV tube in a scoring device is the large amount of electronic circuitry associated with the system.

5. SUMMARY AND CONCLUSION

5.1. Background Radiation - The selection of components for a scoring system for a particular encounter will be based in part upon the input signal-to-noise ratios to be expected. To make a reasonable estimate as to whether a system will be acceptable, correct knowledge of the background radiation (noise) must be available. Since the atmosphere is very absorbing for some optical wavelengths, the influence of the atmosphere on the background noise level must therefore be studied.

The lowest encounter altitude will be 70,000 ft, so that in all cases there will be approximately one air mass between the sensor-bearing vehicle and the earth. From this lowest scoring altitude up to about 100 miles, the approximation used above of regarding the
### Table 4.7. Some Characteristics of T. V. Camera Tubes

<table>
<thead>
<tr>
<th>Detector Type</th>
<th>Spectral Response</th>
<th>Sensitivity</th>
<th>S/N</th>
<th>Image Plane (sq. in.)</th>
<th>Equivalent ASA</th>
<th>Resolution (lines)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orthicon</td>
<td>Visible UV</td>
<td>$10^{-8}$ to $10^{-11}$ watts/cm$^2$</td>
<td>4:1 to 60:1</td>
<td>2.25</td>
<td>$10^4$ to $10^7$</td>
<td>115 to 700</td>
</tr>
<tr>
<td>Vidicon</td>
<td></td>
<td>$10^{-5}$ to $10^{-8}$ watts/cm$^2$</td>
<td>~300:</td>
<td>0.25</td>
<td>1.0 to 1600</td>
<td>300 to 1200</td>
</tr>
<tr>
<td>Ebicon</td>
<td></td>
<td>$10^{-11}$ to $10^{-13}$ watts/cm$^2$</td>
<td>~4:1</td>
<td>~4.0</td>
<td>$10^7$ to $10^9$</td>
<td>250 to 500</td>
</tr>
<tr>
<td>Uvicon</td>
<td></td>
<td>DEVELOPMENTAL ITEM SIMILAR TO EBICON (See Text)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
earth's surface as an infinite plane is reasonably accurate. For greater altitudes, the mathematical models developed in References 4.5 and 4.6 are recommended. However, these models make the possibly equally serious approximation that the reflectance of the earth is constant across its surface.

Those encounters at altitudes of less than 100 miles above the ocean should offer good scoring conditions since the earth then appears as a wide area of constant reflectance and the absorption characteristics of the air will eliminate almost all reflection in some spectral regions.

5.2. Radiant Sources - A completely unaugmented missile would be detectable only by the radiation reflected from its surface into the sensing instruments of the scoring vehicle. This reflection could be either diffuse or specular. In the case of a sun illuminated spherical object, specular reflection is independent of the angle of observation whereas diffuse reflection is not. Specular reflection becomes dependent on the angle of observation as the shape deviates from spherical.

The simplest kind of augmentation would be the painting of the munition with highly reflecting, fluorescent, or other special purpose paint. It appears that coating the munition with a chelate material would increase the signal-to-noise ratio due to its property of absorbing radiation over a wide wavelength band and re-emitting this energy within a narrow band.

More complicated augmentation incorporates the use of mirrors. Plane mirrors can increase the irradiance at a given point of detection, but this is achieved at the cost of lost spherical distribution of the radiation. An even spherical distribution can be approached only by the use of a large number of very small mirrors, but in this case the irradiance at no point will be as great as that due to a larger, single mirror.

Finally, there is augmentation in the form of radiating beacons mounted on or within the missile. Such augmentation, although the most complicated, can give the greatest signal-to-background noise ratio at the detector.

5.3. Sensors - It is expected that detectors of relatively long time constants, such as bolometers, will prove to be unsuitable for scoring purposes. The photoconducting detectors are of established efficacy as infrared detectors and the photo-emissive devices, such as photomultipliers and television tubes, appear to be well suited for detection in the visible region.
A generalized system illustrating the combined effects of background, source, and sensor is shown in Figure 4.22.

6. REFERENCES


CONCLUSIONS AND RECOMMENDATIONS

1. INTRODUCTION

This chapter presents conclusions drawn from studies presented in previous chapters. For clarity, the conclusions are kept as brief as possible. It should be remembered, however, that these results cannot be taken out of context; that is, a conclusion is valid only for the particular situation and conditions considered in this study. The situations were made as realistic as possible (some conditions were established by contract requirements, e.g., the range over which scoring was to be accomplished) but the actual scoring problems which will present themselves in the future surely will not be identical in all respects to the situations considered here.

It would be presumptuous to prescribe now the exact system configuration for solving future scoring problems when the operational characteristics of the munitions to be scored can only be estimated. Hence no finished scorer design is included. The results of these studies, however, permit sound recommendations as to the characteristics or qualities of scoring-system components. These recommendations are presented in Table 5.1 together with brief supporting arguments and data.

2. SUMMARY OF RESULTS

In the studies covering the seven classes of phenomena it was found as expected that techniques using electromagnetic radiations showed by far the greatest potential for use in trajectory-scoring systems. Since feasible techniques for scoring through the use of electromagnetic radiations have been devised, they must be considered as Category I phenomena. A summary of the results for the other six classes of phenomena treated in Chapter Three, categorized with supporting reasons follows:

a. Electrostatic Fields -- Category II
   (1) Detectors lack required sensitivity.
   (2) Rapid leakage of charge from munition.

b. Magnetic (Magnetostatic) Fields -- Category II
   (1) Detectors lack required sensitivity.
   (2) Severe field-distortion problems.
Table 5.1 Recommendations

<table>
<thead>
<tr>
<th>ENCOUNTER TYPES</th>
<th>WAVELENGTH REGION (Microns)</th>
<th>DETECTOR TYPE</th>
<th>SENSING SYSTEM</th>
<th>RANGING METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>SATELLITE:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ground-launched munition</td>
<td>1.33 - 1.47</td>
<td>Photoconductor (InSb)</td>
<td>PERSEAS</td>
<td>Triangulation</td>
</tr>
<tr>
<td>Satellite-launched munition</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SATELLITE:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Satellite-launched munition</td>
<td>1.33 - 1.47</td>
<td>Photoconductor (InSb)</td>
<td>PERSEAS (4π Steradian coverage)</td>
<td>Triangulation</td>
</tr>
<tr>
<td>ICBM, Boost Phase:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Satellite-launched attacks</td>
<td></td>
<td></td>
<td>Ground-based and/or Inertial</td>
<td></td>
</tr>
<tr>
<td>ICBM, Mid-course Phase:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ground-launched munition</td>
<td>1.33 - 1.47</td>
<td>Photoconductor (InSb)</td>
<td>PERSEAS</td>
<td>Triangulation</td>
</tr>
<tr>
<td>ICBM, Mid-course Phase:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Satellite-launched munition</td>
<td>1.33 - 1.47</td>
<td>Photoconductor (InSb)</td>
<td>PERSEAS (4π Steradian coverage)</td>
<td>Triangulation</td>
</tr>
<tr>
<td>ICBM, Re-entry Phase:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ground-launched munition</td>
<td></td>
<td></td>
<td>Ground-based and/or Inertial</td>
<td></td>
</tr>
</tbody>
</table>
c. Nuclear -- Zero potential for trajectory scoring
   (1) Source would be prohibitively large for neutral particles.
   (2) Earth's magnetic field interferes with charged particles.

d. Gravitational Fields -- Zero scoring potential
   (1) Detectors incapable of required sensitivity.
   (2) Unreasonable assumptions required.

e. Inertial Systems -- Category II
   (1) Available systems lack required accuracy.
   (2) Some hybrid systems have marginal Category I status.

f. Pressure waves -- Zero scoring potential for advanced-vehicle encounters
   (1) Insufficient atmospheric density at orbital altitudes.
   (2) Not usable if both missile and target are supersonic.

3. RECOMMENDATIONS

Table 5.1 contains the recommendations appropriate to the critical areas of concern in the development of a scoring system. The tabular form has been used for brevity and for easy reference. Although most of these recommendations were implied as a result of the studies reported in a previous section, brief supporting arguments are included below. Additional recommendations are also made.

Figure 4.8 indicates that the atmosphere completely absorbs radiation in several infrared wavelength regions. The problem of background radiation can at least be partially solved by choosing an optical band-pass filter which blocks out all radiation except that lying in one of these regions. Sunlight reflected from the earth will not contribute to the background because the atmosphere blocks out wavelengths passed by the filter. It would be necessary, therefore, to select a sensor that can detect radiation in the wavelength band passed by the filter. The most preferable wavelength region is at 1.33 to 1.47 microns because a larger proportion of solar energy is radiated in this band than in the bands at longer wavelengths.

The selection of sensors is limited in this region because the photon energy is too low for photo-emission, thus ruling out television camera tubes and photo-multipliers. Moreover, most photo-conductors require cooling to be useful in this band. However, Reference 4.9 indicates that InSb at 250 C performs satisfactorily in this wavelength band.
It is therefore recommended that this wavelength region be used together with the InSb detector.

The thermal radiation from the body of a munition as well as that from burning rocket engines and hot rocket nozzles will add to the reflected solar radiation. Further study should be made to determine the contribution of the rocket exhaust to the received energy.

Further study should also be made of background noise (especially that contributed by the earth) since the data used so far have often been based on approximate mathematical models rather than on experimental results. Such experimental measurements should be available from some of the many instrumented orbital vehicles and space-probes which have been launched. In these studies, attention should be given to regions of partial absorption lying in the near IR or visible, such as the one at 0.91 microns, since some very sensitive photo-multipliers can be used at these wavelengths.

The system to be used with the photo-conductive detector could be any one of several types of scanning devices. The PERSEAS is recommended, however, because of its demonstrated feasibility (Reference 5.1). A cut-away drawing of the PERSEAS device is presented in Figure 5.1; Figure 5.2 is a reproduction of a photograph of the instrument. Details of the construction and performance of this system are included in the Final Report covering the supplemental experimental studies under this program (Reference 5.1).

Ranging can be accomplished by triangulation methods utilizing two PERSEAS devices separated by a baseline of some 10 feet. This may require that the instruments be supported on booms extending from the detecting vehicle (satellite or ICBM) as illustrated in Figure 5.3. To range to the accuracy of ± 50 ft at a 500 ft range with the 10 ft baseline would require that the angles be measured to an accuracy of one milliradian. A 3 ft baseline would require angular accuracies of about 0.3 milliradian. Although it is believed that the PERSEAS ultimately will be capable of this latter angular accuracy, it is recommended that further study be made of the feasibility of developing a miniaturized, pulse-transit-time measuring device that would obviate the use of the extended booms and permit some relaxation of angular accuracy requirements.

The choice of wavelength regions, sources, sensor types, and angle-sensing techniques will be the same for satellite-to-satellite and
Figure 5.1. Photo-Electric Rotating Slit Elevation and Azimuth Sensor

- Resolver
- Rotating Shaft
- Slits
- Levelling Screws
- Drive
- Lens Mount
- Lens
- Prism
- Light-Exclusive Housing
- Photo-Multiplier
- Base
ground-to-satellite encounters. Much of the relative trajectory of the
ground-launched munition can be expected to lie within a small cone
projected ahead of the target satellite; whereas the munition launched
from another satellite could conceivably approach from any direction,
depending on the relative positions of the satellites at the time of launch.
Unless some restriction can be placed on geometry, a $360^\circ (4 \pi \text{ steradian})$
field of view is required for the latter type of encounter. Complete cover-
age could be obtained through the use of a large number of complete
scoring systems with overlapping fields of view, but it would obviously
be better to use a single system with provision for rotating the satellite
to face the expected attack.

The most reasonable procedures for scoring during the boost and
re-entry phases of ICBM trajectories utilize the long-range, ground-
based systems presently available (tracking telescopes, radar, MATTS)
at instrumented test ranges. It is recommended that further study be
made of the use of inertial systems in ground-launched munitions for
supplementing data taken with other scoring systems, particularly in the
boost and re-entry phases of ICBM trajectories. It is anticipated that
advancements in the state-of-the-art in inertial systems will soon bring
at least hybrid inertial systems (and possibly pure inertial systems) to
full Category I status.

Since the environment and encounter geometries associated with
the mid-course phase of ICBM trajectories are essentially those of
satellite-to-satellite encounters, scoring systems designed for the latter
class of encounter can also be used, with minor modifications, for scor-
ing the ICBM mid-course encounters.

Although scoring systems designed for use in the satellite encoun-
ters would no doubt perform satisfactorily when adapted for use on aero-
dynamic vehicles, little advantage is gained over the use of existing,
ground-based systems. For scoring attacks against these targets, then,
it is recommended that combinations of tracking telescopes, radar, and
MATTS equipment now available be used if possible; the encounters
would of course have to take place over existing test ranges.

The use of radio and microwave devices for measuring distances,
angles, velocities, etc. has not been considered in any detail during
this study for the reason mentioned in the last chapter. It should be
obvious, however, that any such devices which are usable in the vehicles
considered may be substituted for the corresponding optical devices. No
mention has been made in this summary of some related problems such as corrections for bending and own-ship angular motions which were considered in Chapter Two and in the Appendices. The reader is expected to refer to these discussions as needed.

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### Table 1: Firing Error Indicators

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### Table 2: The Results of a Study of the Applicability of Seven Classes of Physical Phenomena to Trajectory Scoring - Measuring the Relative Trajectory of a Munition with Respect to Its Target

The results of a study of the applicability of seven classes of physical phenomena to trajectory scoring - measuring the relative trajectory of a munition with respect to its target are presented. The seven phenomena (classes) considered are: (1) electrostatic fields, (2) magnetic fields, (3) nuclear radiation, (4) gravitational fields, (5) inertial systems, (6) pressure waves, and (7) electromagnetic (optical) radiation. In addition, several mathematical methods which treat the encounter geometry, the relationship of measurement accuracy to position errors, data-handling problems, and the influence of own-ship angular motion on the accuracy of position determination are presented in Appendices 1 through 6. Included also in an extensive bibliography. Scoring-system recommendations are made for the three general target classes: satellites, intercontinental ballistic missiles and aerodynamic-type vehicles.

### Table 3: The Results of a Study of the Applicability of Seven Classes of Physical Phenomena to Trajectory Scoring - Measuring the Relative Trajectory of a Munition with Respect to Its Target

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