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RESEARCH INVESTIGATION OF
HYDRAULIC PULSATION CONCEPTS
First Quarterly Progress Report

RAC 933-1

Contract AF 33(657)-10622
Project No. 8128
Task No. 812807

REPUBLIC AVIATION CORPORATION
Farmingdale, L.I., N.Y.

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JUN 5 1963
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NOTICE

The work covered by this report was accomplished under Air Force Contract AF 33(657)-10622, but this report is being published and distributed prior to Air Force review. The publication of this report, therefore, does not constitute approval by the Air Force of the findings or conclusions contained herein. It is published for the exchange and stimulation of ideas.
FOREWORD

This First Quarterly Progress Report on the program for Research Investigation of Hydraulic Pulsation Concepts, covering the period from 5 March 1963 to 31 May 1963, was prepared by Republic Aviation Corporation, Farmingdale, New York, under USAF Contract AF 33(657)-10622 with the Flight Vehicle Power Division of the Aero Propulsion Laboratory, Aeronautical Systems Division. The program is scheduled for a period of 20 months, beginning March 1963 and ending October 1964.

USAF Contract AF 33(657)-10622 was initiated under Project No. 8128, Task No. 812807, Aeronautical Systems Division, Air Force Systems Command, Wright-Patterson Air Force Base, Dayton, Ohio. The work is being administered under the direction of Mr. B. P. Brooks of the Directorate of Aeromechanics. The program is being conducted at Republic Aviation Corporation under the direction of Mr. W. E. Mayhew, Staff Engineer - Fluid Systems, with Mr. F. H. Pollard as Principal Investigator.
ABSTRACT

During the period covered by this First Quarterly Progress Report under Contract AF 33(657)-10622, layout design of a pulsation generator was completed. The use of an electrical analogy was considered as a basis of pulsating system analysis, and a block diagram representation of a typical system was derived. To facilitate the derivation of system flow and pressure parameters, a miniaturized two-phase pulsating system is being constructed.
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SECTION I - INTRODUCTION

The fundamental objective of this program is to establish the technical feasibility of the transmission of hydraulic power through the use of pulsating flow and/or pulsating pressure concepts. To accomplish this end, research is proceeding along the following lines:

(1) Analyses and experimental component designs will be made to establish the hydraulic functional equivalents of the following electrical components: generators, transformers, rectifiers, meters, controls, and transmission and distribution lines.

(2) Analyses of pulsating flow and pulsating pressure hydraulic parameters will be performed. These analyses will at least include the following parameters: pressure attenuation, transmission line impedance, frequencies, line sizes and lengths, fluid effects, and efficiency of lines and components.

(3) Analytical investigations of a pulsating flow or pulsating pressure subsystem will be utilized for comparison of this subsystem with a representative continuous flow hydraulic subsystem which delivers power to a simulated aileron control. From the analyses will be determined the relative weight, efficiency, undamped natural frequency, and frequency response.
SECTION II - SUMMARY

Major efforts during this initial reporting period were concentrated on the design of a variable delivery pulsating flow generator. The design evolved incorporates two single-acting cylinders driven by a bellcrank which in turn is driven by one arm of a toggle linkage. The other arm of the toggle linkage is driven by a second bellcrank. The latter is driven by a connecting rod, the other end of which is connected to an eccentric drive shaft. Variable piston displacement is obtained by varying the radial position of the common toggle pivot from the bellcrank pivot. The toggle pivot is positioned by an actuator controlled by a slide valve which senses system pressure.

Preliminary system analysis using a simplified block diagram has been performed. It has been decided to use an electrical analogy in the derivation of the characteristic equations, and the application of this analogy is discussed. The equations of some of the system components have been derived.

A miniaturized pulsating hydraulic system is under construction for the purpose of establishing design parameters and for the verification of theoretical analysis. The system, which includes provisions for simulated aerodynamic and inertia loads, will be operated at room temperature using MIL-H-5606 oil.

No material or process work has been performed during this reporting period.
The type of hydraulic system currently in greatest use in aircraft is the constant pressure, flow demand system which utilizes a pressure-compensated, variable delivery pump. Republic believes that an adaptation of this type of system to the requirements of a pulsating system will produce the most desirable means of control from the standpoints of efficiency, simplicity, weight, and reliability. Major efforts during this initial reporting period have therefore been concerned with the design of such an adaptation, which is shown in Figure 1.

The generator was designed to deliver 10 gpm at 4000 psi with a pulsation frequency of 8 cps. However, these parameters can be varied if subsequent analysis indicates this is desirable. Two single-acting cylinders were chosen to deliver the alternating flow pulses; two discharge strokes or pulses are obtained for each cycle. A piston stroke of one inch was arbitrarily selected to minimize piston side forces and also to provide reasonable piston thrust forces. The piston diameter was then calculated as follows.

\[
10 \text{ gpm} = \frac{10 \times 231}{60} = 38.5 \text{ cu. in./sec}
\]

\[
38.5/8 \text{ cps} = 4.8125 \text{ cu. in./cycle}.
\]

The eccentric drive shaft speed required is:

\[
8 \text{ cps} \times 60 \text{ sec/min} = 480 \text{ rpm}.
\]

Each cylinder discharges once for each cycle or revolution of the eccentric drive shaft. Therefore, each cylinder is required to deliver

\[
4.8125/2 = 2.4062 \text{ cu. in./cycle}.
\]

Piston Area = Displacement/Stroke = 2.4062/1 = 2.4062 sq. in.
Figure 1. Variable Delivery Pulsating Flow Generator
This area is very close to that of a 1.75-inch diameter circle (2.4053). A nominal diameter of 1.75 inches was therefore established for the piston. The piston driving force required is

\[ 4000 \text{ psi} \times 2.4053 = 9621.2 \text{ lbs.} \]

A double-acting balanced cylinder could have been designed, but this would entail the use of two piston rod seals in addition to the piston head seals. Also, to avoid the detrimental effects of piston rod side forces on the rod seals, a crosshead would have been necessary.

In the single-acting cylinder design shown, the extended piston accomplishes the purposes of a crosshead and transmits the piston side forces to the bearing surfaces of the cylinder bore and end cap. The elongated pistons are then subject to the conditions imposed on the plungers of a typical aircraft hydraulic pump.

Some fluid leakage is expected to occur past the piston head seals, especially so when piston rings are used. This leakage would serve to lubricate the walls of the cylinder bore and bearing end cap. To avoid leakage past the end cap bearing, however, a seal is installed to accomplish a wiping action on the piston skirt. This leakage would be collected and conducted away by the drain port shown at the end of the cylinder.

A Scotch yoke and cam mechanism which would impart a simple harmonic motion to the pistons, and thereby result in a sinusoidal flow output, was initially investigated as a driver for the pistons. This mechanism was abandoned because of the relatively large drive forces required and the mechanical complexities introduced in attempting to vary the piston stroke.

The eccentric drive mechanism finally chosen was so proportioned that the deviation of the piston from a simple harmonic motion due to the "connecting-rod effect" has been minimized.

Variable delivery is obtained in the design shown in Figure 1 by varying the piston stroke, and is accomplished in the following manner: The eccentric drive shaft continuously oscillates the "saddle" bellcrank through the connecting rod.
The "saddle" bellcrank is connected to the cylinder bellcrank through the toggle links. The pistons and linkage are shown in the midstroke position with the toggle set for full piston stroke. The piston stroke is varied by varying the radial distance of the pivoted joint connecting the two toggle links from the "saddle" bellcrank pivot. The radial position of the toggle pivot is held and varied by the toggle cylinder. The position of the toggle cylinder, and therefore the toggle pivot, would be determined by a slide valve sensing system pressure and operating in the same manner as an aircraft pump sensing valve. Thus, piston displacement would be made to vary with system pressure. When no flow is demanded by the system, the pressure would rise and cause the toggle cylinder to pull the toggle pivot towards the "saddle" bellcrank pivot until both pivots coincide. When this occurs, oscillation of the cylinder bellcrank, and therefore piston stroking, ceases. When the system demands flow, the drop in system pressure causes the toggle cylinder to move the toggle pivot to the radial position giving the required piston displacement.

In this design, the mechanical advantage afforded by the toggle linkage helps keep down the size of the actuator required to hold and/or shift the position of the toggle pivot. The maximum force required from the toggle cylinder occurs at the maximum stroke position shown in the drawing. This force was calculated to be 2453 lbs from the geometry shown in Figure 2, as follows:

![Figure 2. Toggle Linkage Drive](image)
\[ P = \frac{9621 \times 3}{5 \times \cos 12^\circ} = \frac{28.863}{5 \times 0.9781} = 5900 \text{ lbs.} \]

\[ F = \frac{P \times 5.25}{13.94} = \frac{5900 \times 5.25}{13.94} = 2230 \text{ lbs.} \]

Allowance for bearing friction = \(10\% \times 2230 = 223 \text{ lbs.}\)

Toggle cylinder thrust required = \(2230 + 223 = 2453 \text{ lbs.}\)

Effective area of toggle cylinder = \(\frac{2453 \text{ lbs}}{4000 \text{ psi}} = 0.613 \text{ sq. in.}\)

This area may be approximated by using a piston rod diameter of 0.750 inches and a cylinder bore of 1.187 inches.
SECTION IV - ANALYSIS

A. GENERAL

As a basis for the analytical representation of a typical pulsating hydraulic positional control system, a simplified block diagram is shown in Figure 3. As the analytical work progresses, the equations for the elements of the system will be derived.

In the analysis of systems which are governed by the same equations, the mathematical procedures used in obtaining the solutions are independent of the nature of the systems. In other words, a particular mathematical means of obtaining an electrical system response can also be used to yield a mechanical system response if both systems are governed by the same equations and the excitation functions are the same.

An electrical equivalent of the mechanical system used in this study program on pulsating hydraulic systems serves the following purposes:

1. It reduces the mechanical system to a circuit diagram by using the conventional symbols for circuit elements and circuit diagrams which have been developed by electrical engineers.
2. It allows the use of electric circuit techniques of analysis for obtaining system responses.
3. It provides an easy means of setting up analog computing diagrams should situations arise which require analog computation.

In this report, both hydraulic and solid mechanical systems will be included under the general name of mechanical systems and referred to simply as "systems."

B. BASIC MECHANICAL-ELECTRICAL ANALOGIES

Two types of mechanical-electrical analogies are commonly used, the force-voltage analogy and the force-current analogy. Brief descriptions of these analogies and a comparison of them is given in the following.
Figure 3. Simplified Block Diagram of the Pulsating Hydraulic Positional Control System
1. **Force-Voltage (f-V) Analogy**

In this analogy, the force, f, or pressure, P, of a mechanical system is set to be analogous to the voltage, V, of an electrical system. The velocity U, or flow velocity, Q/A, is set to be analogous to the current, I. These and other relationships between mechanical and electrical parameters are listed in the following chart:

<table>
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<tr>
<th>Mechanical System</th>
<th>Hydraulic Mechanical System</th>
<th>Electrical System</th>
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<tbody>
<tr>
<td>Solid Mechanical System</td>
<td>Force, f</td>
<td>Pressure, P</td>
</tr>
<tr>
<td></td>
<td>Velocity, U</td>
<td>Flow Velocity, Q/A</td>
</tr>
<tr>
<td></td>
<td>Linear Displacement, X</td>
<td>Volumetric Displacement, $\int Q,dt$</td>
</tr>
<tr>
<td></td>
<td>Mass, M</td>
<td>Inertance, m</td>
</tr>
<tr>
<td></td>
<td>Damping Coefficient, b</td>
<td>Resistance, $R_h$</td>
</tr>
<tr>
<td></td>
<td>Compliance, $K_m$</td>
<td>Compliance, $K_h$</td>
</tr>
</tbody>
</table>

2. **Force-Current (f-I) Analogy**

In this analogy, the force, f, or pressure, P, of a mechanical system is set to be analogous to the current, I, of an electrical system; the velocity, U, or the flow velocity, Q/A, is set to be analogous to the voltage, V. These and other relationships between important mechanical and electrical parameters are listed below.
### Mechanical System

<table>
<thead>
<tr>
<th>Solid Mechanical System</th>
<th>Hydraulic Mechanical System</th>
<th>Electrical System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force, $f$</td>
<td>Pressure, $P$</td>
<td>Current, $I$</td>
</tr>
<tr>
<td>Velocity, $U$</td>
<td>Flow Velocity, $Q/A$</td>
<td>Voltage, $V$</td>
</tr>
<tr>
<td>Linear Displacement, $X$</td>
<td>Volumetric Displacement, $\int Q , dt$</td>
<td>Flux Linkage, $\int V , dt$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Passive Elements</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, $M$</td>
<td>Inertance, $m$</td>
<td>Capacitance, $C$</td>
</tr>
<tr>
<td>Damping Coefficient, $b$</td>
<td>Resistance, $R_h$</td>
<td>Conductance, $G$</td>
</tr>
<tr>
<td>Compliance, $K_m$</td>
<td>Compliance, $K_n$</td>
<td>Inductance, $L$</td>
</tr>
</tbody>
</table>

### C. Comparison of Both Types of Analogy

The force-current analogy seems to be more satisfactory than the force-voltage analogy from a physical point of view. For instance, a junction in a mechanical system under the force-voltage analogy consideration is equivalent to a loop in an electrical system, while under the force-current analogy consideration it is equivalent to a node. The latter is naturally more realistic in a physical sense. However, both types of analogy are used to establish the electric equivalent for a mechanical system since the systems can thereby be studied and compared with their electric equivalents from two different points of view. The velocity, flow velocity, and voltage, analogous to each other in the force-current analogy, can be measured by a vibration pickup, flow meter, and voltmeter, respectively, without disturbing the system or circuit; the force and current, however, cannot be measured without breaking into the system.

### C. SYSTEM COMPONENTS AND THEIR ELECTRIC EQUIVALENTS

#### 1. Hydraulic Actuator and its Associated Loads

Figure 4 is the schematic diagram of a servo-valve and a hydraulic actuator with its associated loads.

---

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The force equilibrium equation of the hydraulic actuator and its associated loads can be expressed as

\[ PA = (MS^2 + bS + \frac{1}{K_m}) X. \] (1)

Equation (1) can also be rewritten as

\[ PA = (MS + b + \frac{1}{K_m S}) SX, \] (2)

where
- \( SX \) = Actuator piston and load velocity
- \( M \) = Mass of load
- \( b \) = Damping coefficient
- \( K_m \) = Compliance of spring load
- \( S \) = Laplace transformation operator.

Figure 5a is the electric equivalent of the hydraulic actuator and its associated loads when the force-current analogy is used. This is a parallel RCL circuit connected to a current source.
The current equilibrium equation of the circuit is
\[ I = (CS + G + \frac{1}{LS}) V, \]  
(3)
where 
- \( C \) = Capacitance
- \( G \) = Conductance
- \( L \) = Inductance
- \( I \) = Current
- \( V \) = Voltage.

The similarity between Equations (2) and (3) shows that the characteristic behavior of the hydraulic actuator and its associated loads can be described by a parallel RCL circuit (with current as its independent variable) if \( I, C, L, \) and \( G \) are replaced by \( PA, M, K_m, \) and \( b, \) respectively.

Figure 5b is the electric equivalent of the hydraulic actuator and its associated loads when the force-voltage analogy is used. This is a series RCL circuit connected to a voltage source.
The voltage equilibrium equation of the circuit is

\[
V = (LS + R + \frac{1}{CS}) I, \tag{4}
\]

where \( R = \text{Resistance} \).

Again, the similarity between Equations (2) and (4) enables representation of the hydraulic actuator and its associated loads by a simple series RCL circuit if the quantities \( M, b, \) and \( K_m \) are replaced by \( L, R, \) and \( C, \) respectively.

2. **Hydraulic Servo-Valve**

None of the spool displacement vs. flow rate, spool displacement vs. load pressure, and flow rate vs. load pressure characteristics of commercial valves is linear. This is due to the nonlinear characteristics existing between the servo-valve variables. However, linearization of the servo-valve characteristic equations is possible under certain specific restrictions. For a closed center, four-way valve with constant pressure source, the valve performance can be represented by the following equation:

\[
P_L = P_s - \frac{2Q^2}{g^2 X_v^2}, \tag{5}
\]

where 

- \( g = C_d W \sqrt{\frac{2}{\rho}} \)
- \( C_d = \text{Discharge coefficient} \)
- \( W = \text{Valve part width} \)
- \( \rho = \text{Working fluid density} \)
- \( P_s = \text{Constant supply pressure} \)
- \( P_L = \text{Load pressure} \)
- \( Q = \text{Flow rate} \)
- \( X_v = \text{Spool displacement} \).

If the load is small and the spool is only allowed to oscillate about its neutral position with an amplitude much smaller than its total stroke, then Equation (5) can be approximated by
\[ \frac{1}{2} P_s g^2 X_c^2 = Q^2 \]  

or 

\[ \frac{Q}{A} = K_v X_v', \]  

where 

\[ K_v = \frac{g}{A} \sqrt{\frac{P_s}{2}}. \]

Equation (7) is the linear expression of a simple displacement-controlled flow source.

Figures (6a) and (6b) are the electric equivalents of the hydraulic servo-valve when the force-current and force-voltage analogies, respectively, are used.

(a) Force-Current Analogy

(b) Force-Voltage Analogy

Figure 6. Electric Equivalent of an Approximate Closed Center, Four-Way Valve
Figure 6a is an ideal voltage amplifier (a voltage-controlled voltage source), which can be described by

\[ V_i = K_a V_g, \]  \hspace{1cm} (8)

where \( V_i \) = Output voltage
\( K_a \) = Amplifier gain
\( V_g \) = Control voltage.

Figure 6b is an ideal current amplifier (a voltage-controlled current amplifier), which can be represented by the following expression:

\[ I_i = K_c V_g, \]  \hspace{1cm} (9)

where \( K_c \) = Amplifier gain
\( I_i \) = Output current.

Notice again the similarity between Equations (7), (8), and (9). A point should be mentioned here. The characteristic that the control voltage sources of both the ideal voltage and current amplifiers are independent of the amplifier output has importance insofar as the performance of the hydraulic servo-valve. In a valve of good design, the valve spool displacement, \( X_v \), which is equivalent to the control voltage source of the amplifier, will not be affected by the valve output.

3. **Compressibility Effect of the Fluid between the Servo-Valve and Hydraulic Actuator**

Equation (10) can be used to describe the compressibility effect of the fluid between the servo-valve and the hydraulic actuator, if the following conditions exist:

1. Leakage across the actuator is negligible
2. Fluid transmission line is rigid
3. Actuator piston is operated near its neutral position.

\[ Q - ASX = \frac{\nu}{2\beta} SP \]  \hspace{1cm} (10)
Equation (10) can also be written as

\[ \frac{Q}{A} - S \chi = \frac{\nu_0}{2A^2 \beta} \text{ SAP.} \]  \hspace{1cm} (11)

Parts (a) and (b) of Figure 7 are the electric equivalents of the fluid compressibility effect obtained from the force-current and force-voltage analogies, respectively. The characteristic equation of part (a) is

\[ V_I - V = LSI, \]  \hspace{1cm} (12)

while the characteristic equation of part (b) is

\[ I_I - I = CSV. \]  \hspace{1cm} (13)

Figure 7. Electric Equivalent of the Fluid Compressibility Effect
4. The Electric Equivalent of the Servo-Valve and Hydraulic Actuator with its Associated Loads

Figure 8 combines the results obtained in the previous discussion, and is the complete electric equivalent of the system shown in Figure 4. Part (a) of Figure 8 uses the force-current analogy, while part (b) uses the force-voltage analogy. Characteristic equations of the components and their electric equivalents are also given.

\[ I_1 = \frac{Q}{A} \]

\[ V_i = K_0 V_g \]

\[ V = \frac{Q}{A} \]

\[ I = U = SX \]

\[ L = M \]

\[ C = K_m \]

\[ R = b \]

\[ Q = K_v X_v \]

\[ Q = SX = \frac{V_0}{2A^2 \beta} \]

\[ V = PA \]

\[ I = SX \]

\[ I = U = SX \]

\[ L = M \]

\[ C = K_m \]

\[ R = b \]

\[ V = (LS + R + \frac{1}{CS}) I \]

\[ V = (MS + b + \frac{1}{K_m S}) X \]

\[ V_i - V = LSI \]

\[ I = (CS + G + \frac{1}{LS}) V \]

a) Force-Current Analogy

b) Force-Voltage Analogy

Figure 8. Electric Equivalent of the Servo-Valve and Hydraulic Actuator with its Associated Loads with Fluid Compressibility Effect Included
5. **Ideal Hydraulic Transformer**

Figure 9 is the schematic diagram of an ideal hydraulic transformer.

![Figure 9. Ideal Hydraulic Transformer](image)

The term "ideal" means that the power output equals the power input (i.e., no friction and leakage loss) and that compressibility and inertia effects are negligible. The function of an ideal transformer can best be described by the following equations:

**Force Balance Equation**

\[
\frac{P_2}{P_1} = \frac{A_1}{A_2}
\]

(14)

**Continuity Equation**

\[
\frac{Q_2}{Q_1} = \frac{A_2}{A_1},
\]

(15)

where \(P\), \(A\), and \(Q\) are pressure, area, and flow rate, respectively.

Figure 10 is the schematic diagram of an ideal electric transformer, which is the electric equivalent of the ideal hydraulic transformer. The characteristic equations of the electric transformer are

\[
\frac{V_2}{V_1} = \frac{N_2}{N_1}
\]

(16)

and

\[
\frac{I_2}{I_1} = \frac{N_1}{N_2}
\]

(17)
where \( V, I, \) and \( N \) are voltage, current, and coil turns, respectively.

Figure 10. Electric Equivalent of an Ideal Hydraulic Transformer (Ideal Electric Transformer)

Notice again from Figure 10 that the force-current analogy is a more satisfactory representation than the force-voltage analogy from the physical point of view.

6. **Hydraulic Transformer**

Figure 11 is the schematic diagram of a hydraulic transformer. The characteristic equations of the transformer are

**Momentum Equation**

\[
P_1 A_1 - P_2 A_2 = (MS^2 + bS + K) X
\]

(18)

**Continuity Equations**

\[
A_1 SX = Q_1 - \frac{\nu_1}{\beta} SP_1
\]

(19)

\[
A_2 SX = Q_2 + \frac{\nu_2}{\beta} SP_2,
\]

(20)

where \( \frac{K}{2} \) is the spring constant of the bellows and the other parameters are defined as before.

Combining Equations (18), (19), and (20) and neglecting small order terms, we obtain

\[
Q_1 = G_{11} P_1 + G_{12} P_2
\]

(21)

and

\[
Q_2 = G_{21} P_1 + G_{22} P_2,
\]

(22)
where

\[ G_{11} = \frac{A_1^2 S}{M^2 + bS + K} \]
\[ G_{12} = \frac{-A_1 A_2 S}{M^2 + bS + K} \]
\[ G_{21} = \frac{A_1 A_2 S}{M^2 + bS + K} \]
\[ G_{22} = \frac{-A_2^2 S}{M^2 + bS + K} \]

Equations (21) and (22) represent an interacting four-variable system, which can be represented by a four-terminal block diagram, as shown in Figure 12.

Figure 11. Hydraulic Transformer

Figure 12. Four-Terminal Block Diagram
Equations (21) and (22) can also be transformed into the following useful forms:

\[
P_2 = \frac{1}{G_{12}} \left( Q_1 - G_{11} P_1 \right) \tag{23}
\]

\[
Q_2 = \frac{G_{22}}{G_{12}} \left[ Q_1 - \left( G_{11} - G_{12} \right) P_1 \right] \tag{24}
\]
SECTION V - SYSTEM TESTS

A miniaturized pulsating hydraulic system is under construction for the purpose of establishing such parameters as pressure attenuation, transmission line impedance, frequencies, line sizes, line lengths, and fluid effects. This system, shown schematically in Figure 13, will aid materially in the verification of analytical assumptions made during the course of the theoretical study.

Tests will be conducted with the following purposes in mind:

1. To determine the effectiveness of a rectifier constructed with four check valves. Tests will include the measurement of flow loss, pressure loss, the dynamic response, and the stability of the check valves.

2. To evaluate the filter characteristics of the high pressure accumulator. Tests will include the measurement of cut-off frequency (for particular load impedance conditions) and verification of analytical assumptions.

3. To determine line losses. The test objectives will be to establish line loss characteristics as a function of pulsating frequency, flow velocity profile, and fluid viscosity.

The pump used in the system, which has been converted from an aircraft tandem actuator, will be driven by a 3 HP Varidrive unit having a speed range between 59 and 590 rpm. The pump stroke can be varied from zero to one inch by means of an adjustable eccentric drive.

The four ball-seat check valves used to form the rectifier are those used in the 1000°F Hydraulic System Investigation conducted under Contract AF 33(616)-7454. The mass of the balls and guides, as well as the spring rates, have been determined for the purpose of studying the dynamic response of the valves.

The transmission lines from the pump to the rectifier will be initially about 30 feet long. However, these line lengths and diameters will be varied to study their effect on system operation.
Although shown separately on the hydraulic schematic, the control valve is an integral part of the actuator. This actuator, which is identical to that used for the pulse generator, is connected to a mechanism for simulating aerodynamic and inertia loads. Connected in parallel to the actuator is a throttling valve, which is essentially a variable restrictor, for use at times when it is desirable to impose a constant load on the system.

Two piston-type air-oil accumulators will be used in the system, one on the high pressure side and the other on the low pressure side. The low pressure accumulator will be used for fluid make-up as well as fluid expansion, whereas the high pressure one will serve as a pulsation filter device.

Instrumentation will include six pressure transducers connected to a Brush recorder as well as a differential pressure transducer connected to a Boonshaft and Fuchs readout. A magnetic pickup with a Hewlett-Packard digital counter will be used to monitor pump cycling speed. A flowmeter in the return line and a temperature pickup will complete the instrumentation. Additional instrumentation will be included, if necessary, during the course of testing.

It is anticipated that MIL-H-5606 fluid will be used since the system will be operated at room temperature. However, it is likely that MLO-7277 will also be used in order to check the effect of fluid viscosity on system operation.
SECTION VI - MATERIALS

No materials or process work has been accomplished during the reporting period.
SECTION VII – FUTURE WORK

Future design studies will be made on a variable delivery pulsating generator in which variable flow will be achieved by valving rather than by varying the piston stroke. Design studies will also be initiated on a fluid replenishment unit.

Theoretical studies will be continued to analyze and establish the electric equivalents of system components. It is expected that the miniaturized system now under construction will be completed during the next reporting period and preliminary data on system operation will be obtained.
Research Investigation of Hydraulic Pulsation Concepts
First Quarterly Progress Report

During the period covered by this First Quarterly Progress Report under Contract AF 33(657)-10622, layout design of a pulsation generator was completed. The use of an electrical analogy was considered as a basis of pulsating system analysis, and a block diagram representation of a typical system was derived. To facilitate the derivation of system flow and pressure parameters, a miniaturized two-phase pulsating system is being constructed.

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