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Electronics Research Program

Selection of Oscillation Modes in Optical Masers

SEMIANNUAL TECHNICAL REPORT
(1 JULY - 31 DECEMBER 1962)

23 APRIL 1963

Prepared by M. HIRNBAUM
Electronics Research Laboratory

Prepared for COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
Inglewood, California

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Prepared by M. Birnbaum
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ABSTRACT

Ruby lasers formed by butting together two laser rods with plane parallel ends have been shown to possess mode selection properties. The interface between the rubies acts as a partially-transparent reflector. Calculations show that the favored axial modes of this structure correspond to the frequencies for which both rods possess a Fabry-Perot type of resonance. The experimental results are in good agreement with these predictions.

At high input energy (about 2-1/2 times threshold) a structure consisting of a 3 inch and a 1-1/4 inch rod, was observed to oscillate in only three axial modes at room temperature. The reflectivity of the internal reflector and one ruby end face is obtained by virtue of the ruby/air dielectric discontinuity. The other end, which is totally reflecting, may be either a chisel or a prism. Thus, the type of mode selectivity afforded by the segmented rod structure is well adapted for very high power applications.
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I. INTRODUCTION

The light output of optical lasers usually consists of a number of different characteristic frequencies. This occurs in general, whenever many of the closely-spaced resonances of the optical resonator fall within the linewidth of the emitting ion or atom. A typical case is that of the Ne line used in the He-Ne laser, shown in Fig. 1 (Ref. 1). Only the axial modes of the plane parallel mirror configuration (Fabry-Perot resonator) are indicated. The complete mode pattern is much more complex (Ref. 2). The relevant linewidth parameters are also indicated in Fig. 1. In the solid-state lasers, several hundred axial modes frequently fall within the linewidth of the spontaneous emission of the atom.

Fig. 1. Spectral Linewidth Factors in Laser (from Donald R. Herriott)
In most applications, laser output in a single resonator mode is the preferred type of operation. The more monochromatic the output spectrum, the greater the effective source brightness. As a consequence, the laser is particularly effective in communications and in experiments involving harmonic generation and other non-linear effects. The desirability of obtaining a laser which oscillates in either one or a few axial modes at high power levels has stimulated research in this direction. The methods employed in previous work to achieve mode selection have been of two types:

1) Use of external mirrors in addition to the end reflectors (Ref. 3), and,

2) Use of lenses and apertures between the mirrors (Ref. 4).

The first method (1) is effective in reducing the number of axial and off-axis, or non-axial, modes while the latter method (2) is effective only in the selection of non-axial modes. Method (1) has the disadvantage of lengthening the resonant structure, which increases the number of modes within the linewidth and thus aggravates the mode selection problem.

In this report, a new method of achieving mode selection in solid-state lasers is described. This new method is inherently more effective than previous methods. The new system permits operation at very high power levels because of the use of reflecting surfaces which cannot be damaged by the very intense light beams. In this system, the laser structure consists of two (or more) laser rods butted together. The interface between the two rods provides an internal reflector which produces the mode selection properties of the segmented rod. A photograph of a segmented rod laser is shown in Fig. 2.

Fig. 2. Segmented Rod Laser Showing the Method Employed to Butt together the Two Rubies
II. DISCUSSION

The resonant wavelengths of the axial modes in the Fabry-Perot resonator are given by the condition:

\[ q\lambda/2 = n_r L \]  

where \( q \) is an integer, \( \lambda \) is the wavelength of light in the medium, \( n_r \) is the index of refraction, and \( L \) is the distance between the end reflectors. The spacing between the axial modes is

\[ \Delta\lambda/\lambda = 1/q = \lambda/2n_r L \]  

Because \( q \) is a large number (\( L \) is much larger than \( \lambda \)), the Fabry-Perot resonator supports many axial modes. The axial modes do not have field variations in the transverse plane and, therefore, are sometimes referred to as plane wave modes. The non-axial modes are far more numerous than the axial modes and have characteristic field variations in the transverse plane. In solid-state lasers, identification of the off-axis modes is still in a preliminary state (Ref. 5). In this report, the major experimental efforts have been concerned with the axial modes. For this reason, and for brevity, the discussion will be limited to the axial modes, even though many of the techniques to be described would also be applicable in a discussion of the non-axial modes.

The observation of beats between the axial modes is the most accurate means of studying the spectral purity of the output of the optical maser. The first observation of axial-mode beats was made during experiments with the first He-Ne gas laser (Ref. 6). For a 1-meter structure, the beat frequency between adjacent axial modes is approximately 150 mc/sec. The beats can
be observed by allowing the light beam to impinge on the photoemissive surface of a phototube or a photomultiplier tube. The main requirements for obtaining the beat signal in a phototube output are (1) that both of the light beams strike the same surface region of the phototube, and, (2) that the electric field vectors of the two or more beams have components along a common direction. For a 3-inch ruby, the spacing between adjacent axial modes is approximately 1100 mc/sec. Beats have been observed by means of a travelling-wave microwave phototube (Ref. 8). However, it is more effective to employ a photodetector with a high-frequency response, such as the Philco photodiode type L4501, Fig. 3, (Ref. 7). When the beat frequencies of the axial modes exceed 7000 mc/sec (approximately the upper limit of the L4501), a Fabry-Perot etalon is used for the observations.
Fig. 3. Block Diagram of Apparatus Employed in Observing Optical Beats
III. EXPERIMENTAL PROCEDURE

A photograph of the supporting structure employed in butting two rubies together is shown on Fig. 2. The types of structures that can be used are severely limited by the requirements that the outer surface of the ruby rod not be obstructed from the pump light and that gas cooling for the ruby be available. The ends of the rubies are flat to approximately 1/4 to 1/20 of a wavelength of green light, and the faces are parallel to about 2 to 6 seconds of arc. The reflectivity of the interface can vary over wide limits as a function of the closeness of contact between the two surfaces. Despite this, operation of the butted rods has been found to be quite reproducible upon disassembly and reassembly with respect to total power output, peak power output, and mode selection properties.

It is usual to operate lasers with end mirrors of high reflectivity. However, this has the disadvantage of low power output. In addition, the coatings are very susceptible to damage, particularly at high power levels. A type of laser which appears to be virtually free from this limitation is one which employs a totally-reflecting silver reflector at one end (which could be replaced by a roof top or prism) and uses the reflectivity of the dielectric/air interface (about 7%) for the mirror at the other end. It was found that, with 3-inch long rubies of usual Cr^3+ ion concentration (0.05%) at 300°C, it is usually not possible to reach threshold with an input lower than 800 joules in the elliptical cylinder configuration. However, for rubies exceeding three inches in length, threshold is readily attained. The following experimental conditions are representative of those used in the investigations: 6 x 3/8 inch, 90-degree orientation, 0.04% concentration of Cr^3+ ion, 425 joules; 6 x 1/4 inch (two 3-inch rods butted together) 0-degree orientation, 0.05% concentration of Cr^3+ ions, 400 joules.

Unless otherwise stated, all the ruby configurations studied had one silvered surface for zero percent transmission; the other end was uncoated. No tests were made with coatings on the interface.
Observations of the beats between the axial modes in ruby were performed using two different techniques. The most precise observations were obtained by use of the fast photodiode (Fig. 3). A block diagram of the apparatus is shown on Fig. 3. Figure 4 shows a representative example of the appearance of the beat signals and the total laser light output. To observe beats with separation greater than about 7 gc/sec, it was necessary to use a Fabry-Perot etalon. In most cases, even for operation considerably above threshold, only a small number of lines appeared. This permitted analysis of the frequencies in the laser output and, also, provided strong evidence of the mode-selection properties of the segmented rod laser. In Fig. 5, a photograph is shown of the ring system observed in operating a laser that consisted of 3 X 1/4 inch and 1-1/4 X 1/4 inch rods with 0-degree orientation, butted together (as shown in Fig. 2).
The exposure corresponded to one flash of the ruby laser. Two No. 70 wratten-gelatin filters were in front of the lens. Kodak Tri-X 35 mm film was used in this enlargement, the white rings correspond to the position of maximum light intensity. The three rings, each of different intensity, are clearly visible; the rings correspond to three axial modes of the laser.

Analysis of the two-piece structure (Fig. 2) shows that the modes for which oscillation threshold was lowest are those that correspond to a simultaneous resonance in both pieces. A mathematical statement of this condition is

$$\Delta q = (L_1/L_2 + 1)h$$

(3)

where $\Delta q$ is an integer and is equal to the axial mode separation of the segmented structure in terms of the unit structure of identical length; $L_1$ and
$L_2$ are the lengths of the individual ruby rods; and $h$ is the smallest integer which results in an integral value for $\Delta q$. The mathematical justification of Eq. (3) is given in Section IV.

The content of Eq. (3) is best shown by some simple illustrations. Consider the case of two rubies, each 3 inches in length. If this structure behaved in the same manner as a single 6-inch piece of ruby, the frequency separation between adjacent axial modes would be approximately 550 mc/sec. However, beats are found at 1100 mc/sec. Thus, the number of modes which go into oscillation has been reduced by a factor of two. It may be argued that, because the resultant mode pattern corresponds to that of a 3-inch rod, there is a possibility that the combination functions as an oscillator/amplifier pair. The following experiments were performed to obtain more information. First, the threshold for operation of the 6-inch segmented rod was measured and found to be 496 joules. When the 3-inch ruby with the uncoated end was completely masked from the pump light by aluminum foil, the threshold rose to 619 joules. When the 3-inch ruby with the silvered end was operated by itself, a threshold value of 800 joules was obtained. The large increase in threshold for operation without the second ruby rod indicates that operation as an oscillator/amplifier unit, without requiring positive feedback from the amplifier section, cannot explain the observed thresholds. In addition, the larger threshold obtained with only the 3-inch silvered rod, indicates that the reflectivity of the interface between the two rubies is slightly greater than that of the ruby/air interface (7.5%). In another experiment, two rods of 3-inch length and 90-degree orientation were operated with parallel C-axes and with C-axes at right angles. In the parallel case, the threshold was 500 joules; and, in the perpendicular case, it was 830 joules. If the silvered rod functioned in each case as an oscillator, then approximately the same threshold for onset of oscillation would be expected.
Other experiments were performed using two rubies, one 3 inches and the other 1-1/4 inches in length. The predicted spacing between the favored modes, according to Eq. (3), is 13.2 gc/sec. A number of observations with Fabry-Perot etalons, in which two or three modes were observed, showed a frequency difference of 13.5 ± 0.5 gc/sec. This result is in satisfactory agreement with the predictions of Eq. (3).

The experimental observations are summarized in Table 1. All the measurements indicated in the table were obtained on 1/4-inch diameter rods at 300°K. The number in parentheses indicates the orientation of the C-axis relative to the rod axis. The ruby rods were excited in the elliptical

<table>
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<th>Orientation of C-Axis, degrees</th>
<th>Beat Frequencies Observed, gc/sec</th>
<th>Beat Frequencies Not Observed, gc/sec</th>
<th>Predicted Beat Frequencies, Eq. (3), gc/sec</th>
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<tbody>
<tr>
<td>3 + 3</td>
<td>(0)</td>
<td>1.1</td>
<td>0.550</td>
<td>1.1</td>
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<td></td>
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<td>2.2</td>
<td>1.65</td>
<td>2.2</td>
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<tr>
<td></td>
<td></td>
<td>3.3</td>
<td>2.75</td>
<td>3.3</td>
</tr>
<tr>
<td>3 + 1-1/2</td>
<td>(90)</td>
<td>2.2</td>
<td>1.1</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 + 1-1/4 roughened</td>
<td>(0)</td>
<td>3 modes observed with frequency separation of 13.5 ± 0.5 gc/sec</td>
<td>0.725-0.830</td>
<td>13 2a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>about 6 modes observed</td>
<td>1.070-1.140</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.560-2.720</td>
<td></td>
</tr>
<tr>
<td>3 + 2-1/16</td>
<td>(0)</td>
<td>(b)</td>
<td>(b)</td>
<td></td>
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</table>

*These observations were obtained using a Fabry-Perot etalon.

*bThe complication resulting from the presence of so many modes has made the analysis in this case more difficult. Further experiments will be required to resolve this spectrum.
cylinder configuration (length: 6-1/4 inches, distance between foci: 1-1/2 inches, 
b/a of ellipse: 0.6). Linear Kermite Xenon flashlamps (VHL6-3) were used 
throughout the observations. In each case, the observations were obtained 
for a series of exposures ranging from threshold to about 2-1/2 times 
threshold.

The data presented in Table 1 clearly demonstrate the mode selection effect 
of the reflectance provided by the interface between the two rubies. The 
data obtained also preclude the oscillator/amplifier type of operation as an 
alternate explanation of the effects. For example, in the case of the 
3 + 1-1/4 inch segmented rod (row 3, Table 1), there is no fundamental beat 
frequency corresponding to either rod by itself. In the other cases, the 
threshold for oscillation of the segmented rod is so low that it is impossible 
for part of the segmented rod to oscillate independent of the other part. An 
additional test of this point was obtained by reversing the order of the 
rubies (row 2, Table 1). No difference in operation was observed. The 
decisive observation in this case was that it is most unlikely that the 
1-1/2 inch rod should be the control oscillator when neither of its surfaces 
are silvered.

One surprising result was that the number of axial modes in the output of 
the laser was insensitive to the excitation energy. The best illustration was 
provided by observations on the 3 + 1-1/4 inch segmented rod. Near thresh-
old, only two modes were strongly excited. A very weak line corresponding 
to a third mode was observed. At higher levels, the intensity of this third 
mode became almost comparable to that of the other two. The intensity of 
light in the modes was never exactly the same; one mode was always more 
strongly excited than the others. The difference in intensity greatly facilit-
tated interpretation of the mode spectrum.

The output of the 3 + 3 inch segmented rod structure exhibited an unusually 
large amplitude compared to that observed in the operation of the 6 × 3/8 inch 
rod. The amplitude ratio was found to be between 1.5:1 and 2:1, with peak
amplitude of the segmented structure being approximately 100 kw for a 1000-joule input to the flashtube. The higher peak intensity of the spikes for the segmented rod can be understood in terms of its mode selection properties. Representative oscilloscope traces of the laser light signals from the 1-inch segmented rod at 5 μsec and at 2μsec/cm writing speeds are shown in Fig. 6. Similar data for a unit rod of 6 × 3/8 inches are shown in Fig. 7. Note that, in the segmented rod, the spikes appeared less frequently and with a shorter pulse width than those observed for the unit rod. The conditions of excitation were as alike as possible. It was found that the total power emitted by the two lasers was approximately the same. It follows, therefore, that the peak intensity of the spikes for the segmented rod must be larger. The data in Fig. 6 and Fig. 7 confirm this conclusion.

Observations obtained on gas lasers which employed external mirrors for mode selection also showed this type of behavior (Ref. 9). When the other axial modes were selected, the total laser output appeared in the single remaining oscillating mode.
Fig. 6. Laser Light Output of the 5 x 5 Inch (Zero-Degree Orientation) Segmented Rod Laser

Trace sweep speed: 5 μsec/cm

Trace sweep speed: 2 μsec/cm
Fig. 7. Laser Light Output of the 6 × 3/8 inch (90-Degree Orientation) Ruby Rod Laser with the Same End Reflectivities as the Segmented Rod of Fig. 6
IV. THEORY OF MODE SELECTION IN SEGMENTED ROD LASERS

The theory of mode selection presented in this report is intended for use in interpreting experimental observations on the segmented rod lasers and to indicate new experiments which could be performed to further illustrate the operating characteristics of these lasers. The theoretical approach used in the following discussion can be used to treat analogous cases of confocal systems and other geometries; however, these extensions will not be treated here. Similarly, no attempt is made to show the applicability of this system to the non-axial modes.

A segmented ruby rod laser will be used in the following presentation of the theory. A cross-sectional view of the laser is shown in Fig. 8. At \( z = 0 \), there is positioned a perfectly reflecting mirror; at \( z = a \), the interface between the two rubies is assumed to have a reflectivity of \( r \) and a transmissivity of \( t \); at \( z = a + b \), the ruby/air boundary is assumed to have a reflectivity of \( r \). It should be noted that the coefficient "r" describes the change in the wave amplitude and is related to the experimentally-determined reflectivity, \( R \), by the equation: \( r = R^{1/2} \); in addition, \( r^2 + t^2 = 1 \).

The waves in regions I and II of Fig. 8 are denoted respectively by: \( \exp{(ikz)} - \exp{-(ikz)} \), and, \( B \exp{(ikz)} + C \exp{-(ikz)} \). A steady-state solution is sought which characterizes the light-wave amplitudes in the rods. The common time factor, \( \exp{-(i\omega t)} \) in the expressions for the wave amplitudes has been suppressed, since it is cancelled out in the following calculations. The complex wave vector, \( k \), accounts for the amplifying properties of the medium, which are assumed to be independent of \( z \) and, of course, \( t \). Thus, \( k = 2\pi/\lambda - i\beta \) or \( \alpha - i\beta \). Unless otherwise stated, \( i = (-1)^{1/2} \).
In region I, the boundary conditions required by a perfect metallic reflector at \( z = 0 \) have been invoked in obtaining the phase change of \( \pi \) and the equality of wave amplitudes. At \( z = a + b \), the reflection occurs at normal incidence, and the amplitude of the reflected wave is given by (Ref. 10)

\[
A_r = \frac{A_i (n_1 - n_2)}{n_1 + n_2}
\]

where \( A_r \) and \( A_i \) are the amplitudes of the reflected wave and the incident wave, and, \( n_1 \) and \( n_2 \) are the dielectric constants of the ruby and air, respectively. From Eq. (4), it is clear the reflected wave does not have a phase change upon reflection, as \( n_1 \) is greater than \( n_2 \). Since it is assumed that no wave impinges upon the interface at \( z = a + b \) from the right,

\[
r_2 B \exp ik(a + b) = C \exp -ik(a + b)
\]
For normal incidence, the maximum reflection at the interface is given by (Ref. 11)

\[ A_r = \frac{2(n_1 - n_2)^2/(n_1 + n_2)^2}{1 + \left(\frac{(n_1 - n_2)(n_2 - n_1)/(n_1 + n_2)^2}{1 + (n_1 - n_2)^2/(n_1 + n_2)^2}\right)^2} = \frac{2(n_1 - n_2)/(n_1 + n_2)}{1 + (n_1 - n_2)^2/(n_1 + n_2)^2} \]  

(7)

Thus, reflection at \( z = a \) occurs without a phase change. For this reason, the wave amplitudes in region II are both positive.

Thus, at the plane \( z = a \),

\[ B \exp(ika) = t \exp(ika) = r_2 r B \exp(ik(a + 2b)) \]  

(8)

or,

\[ B = \frac{1}{1 - r_2 t \exp(ik2b)} \]  

(9)

In obtaining Eq. (8), the principle of conservation of energy has been employed by stating that the wave moving to the right in region II consists of (1) wave transmission from region I, and (2) wave reflection from the boundary at \( z = a \) from region II.
The same procedure, applied to region I, leads to the equation

\[-\exp(-ika) = r \exp(ika) + \frac{(1 - r^2)r_2 \exp ik(a + 2b)}{1 - r^2 \exp(2kb)} \quad . \quad (10)\]

The solutions to Eq. (10) are the steady-state solutions which correspond to standing waves in the segmented rod.

In the particular case considered, \( r \) and \( r_2 \) are small compared to 1; therefore, terms in \( r^2 \), \( r^2 \), \( r^2 r \) and higher powers can be discarded. All terms of \( r \) to the first power are retained. With this simplification, Eq. (10) becomes

\[ r_2 \exp i2k(a + b) + r \exp (ik2a) = -1 \quad . \quad (11)\]

The validity of Eq. (11) is shown by considering the familiar single rod case, where \( r = 0 \). Equation (11) becomes

\[ r_2 \exp i2k(a + b) = -1 \quad (12)\]

The solutions to this equation are the familiar Fabry-Perot conditions, namely,

\[ \exp (i4\alpha a) = -1 \quad \left\{ \begin{array}{l}
\exp (4\beta a) = 1
\end{array} \right. \quad (13)\]

hence

\[ r_2 \exp (4\beta a) = 1 \quad , \]

\[-18-\]
assuming that \( a = b \). The first condition of Eq. (13) states that

\[
\frac{4\pi L}{\lambda} = (2n + 1)\pi
\]

or

\[
L = \left(\frac{n}{2} + \frac{1}{4}\right)\lambda
\]

where \( n \) is an integer.

The laser will oscillate first in the mode that requires the smallest regeneration to overcome the losses. It is therefore necessary to determine the minimum value of \( \beta \). From Eq. (13), it is easy to see that the lowest value for \( \beta \) occurs when the phase factors on the left side of the equation yield the value -1. This then implies that

\[
\frac{4\pi(a + b)}{\lambda} = (2n + 1)\pi
\]

and

\[
\frac{4\pi a}{\lambda} = (2m + 1)\pi
\]

where \( n \) and \( m \) are integers. Equation (15) leads directly to Eq. (3), which was used to predict the behavior of the segmented rod laser. Thus, the modes characterized by Eq. (3) are the most favored modes.

Equation (11) also predicts an interesting property of the segmented rod laser. The favored modes given by Eq. (3) have a lower threshold than those of a
calculation shows that

$$\frac{\beta_c}{\beta} = \frac{\ln\left[-1/2 + 1/(r_2)^{1/2}\right]}{\ln\left[1/(r_2)^{1/2}\right]} = 0.9 \tag{16}$$

where $\beta_c$ refers to the segmented rod (formed of two parts of nearly equal length) and $\beta$ refers to the unit rod. The numerical value is obtained by using the value of 1.76 for $n_1$, the index of refraction for sapphire. Equation (16) also indicates that an increase in $r$ and $r_2$ would also improve threshold characteristics of the segmented rod laser. Mode selectivity can also be improved by adjusting $r$ and $r_2$. However, it should be emphasized that perhaps the most useful aspect of the segmented rod lasers is the high degree of mode selectivity that can be obtained without employing specialized coatings. This permits operation at very high power levels, including Q-switching operations, without damaging the reflecting surfaces. The mirror that was used in the experiments could be replaced by a wedge or chisel.

It is evident that the improvements indicated by Eq. (16) could be achieved by employing a three-piece segmented rod structure.

An important question which must be considered is whether there are other allowed solutions to Eq. (11) that would correspond to possible operating points. The possibility that additional modes could be excited can be determined by comparing the $\beta$'s of these modes with the $\beta$'s of the modes corresponding to solutions of Eq. (3). The resonant frequencies of these additional modes will, in general, be close to the positions of resonance of the unit rod. Numerical solution of Eq. (11) would be required. It is instructive to obtain approximate solutions for the particular structures listed in Table 1.
For simplicity, assume that \( r = r_2 \) and that \( a = b \) (row 1 of Table 1). Equation (11) can be written

\[
d^2 \cos (2aL) + d \cos (2aa) = -k
\]

and

\[
d^2 \sin (2aL) + d \sin (2aa) = 0
\]

where \( d = \exp (\beta L) \), \( L = 2a \), and \( k = 1/r_2 \). It is convenient to obtain solutions close to \((2n + 1)\pi\) and \((2m + 1)\pi\), respectively. Equations (17) and (18) can then be written

\[
d^2 \cos 2\theta + d \cos \theta = k
\]

and

\[
d^2 \sin 2\theta + d \sin \theta = 0
\]

Equations (19) and (20) have no common solutions, other than \( \theta = 0 \), \( \theta = \pi \) \((k = 3.68)\). The mode pattern of this structure and the relative gain of the allowed modes are shown in Fig. 9a.

The segmented rod structure \((3 + 1-1/2\) inches\) from Table 1, row 2, presents a more difficult case, and it appears more expedient to resort to graphical methods. Two cases were considered: (1) the 3-inch rod with the silvered surface at \( z = 0 \), and, (2) the 1-1/2 inch rod with the silvered surface at \( z = 0 \). In each case, it is found that the possibility exists for two resonances in addition to the one predicted by Eq. (3). The mode pattern is illustrated in Fig. 9b. Further work is in progress to obtain more exact values. This work will serve as a guide to the effective methods of obtaining solutions in the more
complex examples which are listed in Table 1. It is particularly necessary
to obtain close approximations of the frequencies at which these other
resonances are expected in order to search for them experimentally.

Fig. 9. Mode Pattern of the Segmented Rod Laser
(a) 3 + 3 inch; (b) 3 + 1-1/2 inch
V. SUMMARY

It is shown that, when two laser rods with plane, parallel end faces are butted together to form a longer rod, the interface between the two faces acts as a partial reflector of light. The presence of this internal reflector results in mode selection effects. The major feature of these lasers is that there is a very substantial reduction in the number of axial modes present in the output of the lasers. The improvement which results in the spectral purity of the laser output makes these types of lasers very useful in many applications.

Beat frequencies in these segmented rod lasers were found to occur at frequency separations characterized by operation in favored Fabry-Perot modes. The favored modes are those light frequencies which simultaneously satisfy the resonance condition in each rod individually. The experimental results were found to be in good agreement with the theoretical predictions.

A noteworthy feature of these segmented rod lasers is their ability to operate at very high power levels. The reflecting surfaces of the lasers are formed by the sapphire/air dielectric interface, except for a totally-reflecting silvered surface at one end which could be replaced by a chisel or a prism. Thus, these lasers are suited to very high power applications, including Q-switching.

In the case of a particular segmented rod, formed by butting together a 3-inch and a 1-1/2 inch rod, room temperature operation at lower power levels indicated the presence of only two axial modes. At high power levels, 2 to 2-1/2 times threshold, only three axial modes appeared in the output of the laser.

Experimental results are presented only for ruby lasers. It is apparent that these results should be applicable to all solid-state lasers.
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BIBLIOGRAPHY

Several excellent review articles and books have appeared recently which adequately reference published literature. These books and review articles should be consulted first for a general introduction to the subject matter and the complete referencing of the literature. These extensive lists will not be repeated here. Only those articles which specifically extend or corroborate results given in this report will be explicitly referenced.

REFERENCES


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Ruby lasers formed by butting together two laser rods with plane parallel ends have been shown to possess mode selection properties. The interface between the rubies acts as a partially-transparent reflector. Calculations show that the favored axial modes of this structure correspond to the frequencies for which both rods possess a Fabry-Perot type of resonance. The experimental results are in good agreement with these predictions. At high input energy (about 2 1/2 times threshold) a structure, consisting of a 3 inch and a (over)

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1 1/4 inch rod was observed to oscillate in only three axial modes at room temperature. The reflectivity of the internal reflector and one ruby end face is obtained by virtue of the ruby/air dielectric discontinuity. The other end, which is totally reflecting, may be either a chisel or a prism. Thus, the type of mode selectivity afforded by the segmented rod structure is well adapted for very high power applications.