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**CONSEQUENCES OF SYMMETRY  
IN PERIODIC STRUCTURES**

by

**P. J. Crepeau and P. R. Mc Isaac**  
**Research Report No. PIBMRI-1113-63**

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**POLYTECHNIC INSTITUTE OF BROOKLYN**  
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**ELECTROPHYSICS DEPARTMENT**

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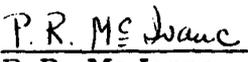
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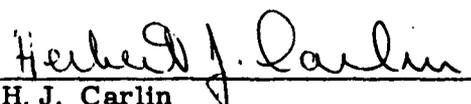
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ABSTRACT

Periodic guiding or radiating structures at microwave frequencies frequently possess symmetry properties in addition to their axial periodicity. These include rotation and reflection symmetries, wither occurring alone or in conjunction with translations. These symmetries influence the characteristics of the electromagnetic fields associated with the structures. Therefore, useful information concerning the fields can be obtained from the symmetry properties without resorting to detailed field solutions or to equivalent circuit analogs. These symmetry properties are conveniently analyzed by introducing symmetry operators under which the structure is invariant.

This paper shows that two symmetries, the screw and the glide, are particularly important in determining the characteristics of the fields. Some of the implications of these symmetries for leaky wave antennas and microwave tube interaction circuits are explored. The consequences of screw and glide symmetries, together with five other possible symmetries, are examined to facilitate the analysis and synthesis of periodic microwave structures.

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## I. INTRODUCTION

Periodic structures for guiding or radiating electromagnetic waves have a wide range of application in modern microwave technology. They are used in leaky and surface-wave antennas, microwave filters, and linear accelerators as well as microwave tubes, such as traveling-wave tubes, backward-wave amplifiers and oscillators, crossed-field amplifiers and oscillators, and beam-type parametric amplifiers. When analysis has been undertaken in the past, the usual procedure has been to treat each structure as a separate problem and to attempt to obtain the propagation or radiation characteristics and any other desired information by an exact or approximate solution of the particular problem. The approach has been to seek general field solutions (perhaps using approximation techniques for computational purposes) or to introduce equivalent networks. For many structures of current interest, however, the geometry is such that a general field solution may be obtained only after lengthy computations, if at all. Moreover, the field solution often yields more information than is required for a particular application. On the other hand, an equivalent network approach requires the choice of approximate equivalent circuits, and this choice is often difficult without some prior knowledge of the characteristics of the structure. In addition, the equivalent network approach generally yields considerable less information than a field solution, often less than is desired for a particular application.

A relevant question then, is whether any of the performance characteristics of periodic structures can be predicted without resorting to detailed calculations for each structure of interest. Furthermore, given desired performance characteristics, can one predict a structure or class of structures having these properties? A promising approach to these questions is a consideration of the symmetry properties of each structure. This paper demonstrates that several of the salient propagation characteristics of periodic structures can be derived from the symmetry properties.

Two groups of general symmetry types are investigated, and their relation to the performance characteristics of structures explored. One group includes those symmetries whose main effect is restricted to influencing the occurrence of certain of the space-harmonic components of the electromagnetic fields and to determining the relative phases of pairs of space-harmonic components. The second group of symmetries in addition to influencing the occurrence of space-harmonic components also controls important characteristics of the dispersion curve for the structure. The symmetries of this second group are of most interest to this discussion, since they have the greatest influence on the properties of periodic structures. Therefore, a major portion of the paper is devoted to exploring the consequences of these symmetries,

and a brief discussion is given of the other symmetries.

The discussion will be restricted to lossless, reciprocal structures, which are periodic along a rectilinear direction (taken parallel to the  $z$  axis) with period  $L$ . Re-entrant periodic structures, such as magnetron circuits, will not be considered, although parts at least of this analysis could be readily extended to include them. For convenience, the fields are expressed in terms of a circular cylindrical co-ordinate system  $(r, \theta, z)$  and the  $z$  axis is taken as the symmetry axis of the structure, if one exists. Single-frequency excitation at radian frequency  $\omega$  is assumed throughout, and all fields are understood to vary as  $\exp(j\omega t)$ . Typical periodic structures are illustrated in Figure 1.

## II. PROPERTIES OF PERIODIC CIRCUITS

The basic character of the electromagnetic fields of any periodic structure is determined by Floquet's theorem.<sup>1</sup> The statement of the theorem relevant to this discussion is that for a given mode of propagation at a given steady-state frequency, the fields at one cross section differ from those one period away only by a complex constant (of modulus unity for lossless propagating structures). For the electric field, this can be expressed as

$$E(r, \theta, z+L) = e^{+j\beta_0 L} E(r, \theta, z). \quad (1)$$

The most general function that can satisfy this requirement is the product of  $\exp(\pm j\beta_0 z)$  and a function of  $(r, \theta, z)$  that is periodic in  $z$  with period  $L$ . This latter function can be expressed as a Fourier series in  $z$  with the Fourier coefficient being functions of  $(r, \theta)$ . The significance of having two Floquet constants in Equation (1) lies in the fact that the structure is reciprocal and can support a corresponding mode in the backward direction for each mode in the forward direction. For ease of notation, the discussion will be restricted to modes with the Floquet constant  $\exp(-j\beta_0 L)$ . The final results, however, will be generalized to include both classes of modes. The magnetic field, of course, satisfies an equation analogous to (1).

In cylindrical co-ordinates, the electric and magnetic fields can be separated into transverse and axial components, and a knowledge of the axial components of both the electric and magnetic fields is sufficient to determine the total fields (see Appendix). Each of the field components must, of course, satisfy Floquet's theorem. With no loss of generality the ensuing discussion is confined to  $E_z(r, \theta, z)$ , since the same discussion applies virtually unchanged to  $H_z(r, \theta, z)$ , and from these two longitudinal components the transverse components can be derived.

In addition to periodicity along the  $z$  axis, these structures must also necessarily have periodicity in the  $\theta$  direction with a period  $2\pi$ . Therefore, the field variation in the  $\theta$  direction can also be expressed as a Fourier series. As a consequence, any field component can be expressed as a double Fourier series whose coefficients are functions of  $r$  only (see Appendix), for example,

$$E_z(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} E_{znm}(r) e^{-jm\theta} e^{-j\beta_0 z} e^{-j2\pi n z/L}, \quad (2)$$

where  $E_{znm}(r)$  is the Fourier coefficient of the  $m^{\text{th}}$  angular and the  $n^{\text{th}}$  axial space harmonic. It is customary to combine the Floquet phase factor,  $\beta_0$ , into the axial Fourier expansion

sion by defining

$$\beta_n = \beta_0 + 2\pi n / L ; \quad (3)$$

then

$$E_z(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} E_{znm}(r) e^{-jm\theta} e^{-j\beta_n z} \quad (4)$$

There is a wave component corresponding to each integer,  $n$ , which is traveling in the  $z$  direction with phase velocity  $\omega/\beta_n$ . These wave components are referred to as axial space harmonics, and  $n=0$  is the fundamental space-harmonic component. The numbering of the axial space harmonics is arbitrary, and the fundamental space harmonic may be chosen such that  $-\pi/L < \beta_0 < \pi/L$ , or as the one with the largest amplitude coefficient, or in some other way. For convenience, the fundamental space harmonic here is usually taken as that one for which  $|\beta_0|$  has the least value.

The properties of periodic structures that are of interest to microwave engineers include the general dependence of  $\beta_0$  on  $\omega$ ; that is, the frequency bands where  $\beta_0$  is real (pass bands) or imaginary (stop bands), as well as the more specific variation of  $\beta_0$  versus  $\omega$  within the pass bands. Clearly if  $\beta_0$  versus  $\omega$  is known, all the  $\beta_n$  are determined. It is also of interest to know whether axial and/or angular space-harmonic field components occur as well as to have some estimate of the relative amplitudes of the space harmonics that are present. For some modes the symmetry characteristics of the structure will provide information about some of these properties of interest.

The character of the electromagnetic fields in periodic structures is profoundly influenced by the periodicity of the structure in both the  $z$  and  $\theta$  directions. The fields are essentially influenced by the periodic boundary conditions associated with the structure; they are directly periodic in  $\theta$  and have an underlying periodicity in  $z$ . That is, if the Floquet constant,  $\exp(-j\beta_0 L)$ , is removed, the fields are periodic in  $z$ . Thus, in a certain sense, the fields can be said to have the axial symmetry (or periodicity) of the structure.

The phrase "fields with the symmetry of the structure", or its equivalent, will be used to describe the situation existing when the structure symmetry has appreciably influenced the geometry of the electromagnetic field. Only rarely will the electromagnetic fields have precisely the identical symmetry of the structure, but often, and in every case of interest in this discus-

sion, the structure symmetry does determine the underlying, or basic, symmetry of the fields. That is, for these modes the fields at two points in space that are related by the combined translation, rotation, and reflection operations appropriate to the structure symmetry will differ, at most, by a complex constant. These fields will be referred to as having the symmetry of the structure. For a few structure symmetries, it is not possible for the underlying field symmetry to have a symmetry identical with that of the structure, and such case will be pointed out in the discussion to follow.

For convenience, the function  $F(r, \theta, z)$  is introduced to describe the various structure symmetry properties that are discussed. For example, the basic axial periodicity of the structure is described by the equation,

$$F(r, \theta, z + L) = F(r, \theta, z) . \quad (5)$$

To describe the various field symmetries resulting from the structure symmetries, it is convenient to introduce corresponding operators. Thus the translation operator,  $T$ , is used to represent the translation symmetry of the electromagnetic fields that is a consequence of the axial periodicity of the structure. This translation operator is defined by

$$TE_z(r, \theta, z) = E_z(r, \theta, z + L) ; \quad (6)$$

and from Floquet's theorem,

$$TE_z(r, \theta, z) = e^{+j\beta_0 L} E_z(r, \theta, z) . \quad (7)$$

This is an eigenvalue equation with the two Floquet constants,  $\exp(\pm j\beta_0 L)$ , being the eigenvalues of the translation operator. As other symmetries are introduced, and where the fields can have the underlying symmetry of the structure, additional symmetry operators for the fields will be introduced.

There are seven common symmetry properties that periodic structures may possess, the structure may have only one of these symmetry properties, or any combination of them. These symmetry properties are characterized by the translation, rotation and reflection operations (which can occur singly or in combination), that cause the structure to coincide with itself. Two of the symmetry types, the screw and the glide, have much broader implications for the propagation characteristics of a structure than do the other five types of symmetry. Therefore, the screw and glide are explored in detail, and a major portion of this paper is devoted to the consequences of these two symmetry properties on the propagation characteristics. The other five symmetry types are examined more briefly.

III. SYMMETRY OPERATORS FOR SCREW AND GLIDE SYMMETRY

A. Screw Symmetry:  $F(r, \theta + \psi, z + \delta) = F(r, \theta, z)$

Screw symmetry, or rotation-translation, consists of a combined angular rotation plus an axial translation causing the structure to coincide with itself. A typical form that this symmetry might take is illustrated by the turnstile structure of Figure 1a. There a rotation of  $\frac{\pi}{2}$  radians plus a translation of a quarter of a period causes the structure to coincide with itself, so that  $\psi = \frac{\pi}{2}$  radians and  $\delta = L/4$  meters.

Given a structure with screw symmetry, suppose that repeated applications of this symmetry property are made; that is, rotation of  $\psi$  radians with translation by  $\delta$  meters, the structure coinciding with itself after each pair. Since the structure is periodic both in  $\theta$  (with period  $2\pi$ ) and in  $z$  (with period  $L$ ), then both  $2\pi/\psi$  and  $L/\delta$  must be integers, say  $p$  and  $q$  respectively. If  $q = L/\delta$  pairs of rotations and translation are made, then

$$F(r, \theta + q\psi, z + q\delta) = F(r, \theta + q\psi, z) = F(r, \theta, z) . \quad (8)$$

Further consideration of the symmetry leads to the conclusion that  $q < p$ , since for  $p$  pairs of rotations and translations,  $\theta + p\psi = \theta + 2\pi$ , and the structure has been rotated one complete revolution. Then the real period is less than  $L$ , contradicting the original assumption, unless

$$z + p\delta > z + L = z + q\delta . \quad (9)$$

In fact,

$$z + p\delta = z + \sigma L , \quad (10)$$

where  $\sigma$  is a positive integer, since after a net rotation of  $2\pi$  radians, the structure can, at most, be displaced axially by an integral number of periods. Therefore,  $p/q = \sigma$ , where  $\sigma$  is an integer greater than or equal to one. If  $\sigma$  is greater than one, Equation (8) states that the structure has angular rotation symmetry in addition to the screw symmetry, since rotation by  $q\psi = q2\pi/p = 2\pi/\sigma$  radians causes the structure to coincide with itself (see Section VIII B). The structure illustrated in Figure 1a has  $p = 4$ ,  $q = 4$ , and  $\sigma = 1$ .

If the fields have the symmetry of the sense discussed above, then there are restrictions on the Fourier coefficients,  $E_{znm}(r)$ , in the double Fourier series of Equations (2) or (4). Introduce the screw operator,  $S_{pq}$ , where the subscripts  $p$  and  $q$  are characteristic numbers for the structure:

$$S_{pq} E_z(r, \theta, z) = E_z(r, \theta + 2\pi/p, z + L/q) = s_{pq} E_z(r, \theta, z) , \quad (11)$$

where  $s_{pq}$  is an eigenvalue of the screw operator. It is clear from the nature of screw symmetry that the repeated application of the screw operator  $p$  times is equivalent to the repeated application  $\sigma = p/q$  times of the translation operator,  $T$ :

$$S_{pq}^p E_z(r, \theta, z) = T^{p/q} E_z(r, \theta, z) . \quad (12)$$

It is also clear that repeated applications of the screw operator  $q$  times is equivalent to the combined operation of a translation of one period with a rotation of  $2\pi q/p = 2\pi/\sigma$  radians:

$$S_{pq}^q E_z(r, \theta, z) = R_\sigma T E_z(r, \theta, z) , \quad (13)$$

where  $R_\sigma$  is the rotation operator associated with the symmetry of the structure (see Section VIII B).

Since the operators commute, they have common eigenfunctions and (12) can be written in terms of the eigenvalues,

$$s_{pq}^p = (e^{-j\beta_o L})^{p/q} = e^{-j\beta_o pL/q} , \quad (14)$$

$$s_{pq} = (e^{-j\beta_o pL/q})^{1/p} = e^{-j\beta_o L/q} e^{-j2\pi(\frac{\alpha}{p} + \nu)} , \quad (15)$$

where  $\alpha = 0, 1, 2, \dots, p-1$ , and  $\nu$  is any integer (positive or negative). There are a total of  $p$  separate eigenvalues for  $S_{pq}$ . If (14) and (15) are applied to the fields written in terms of the double Fourier series and the appropriate orthogonality conditions are applied, then

$$e^{-j\beta_o L/q} e^{-j2\pi n/q} e^{-j2\pi m/p} = e^{-j\beta_o L/q} e^{-j2\pi(\frac{\alpha}{p} + \nu)} , \quad (16)$$

or

$$\frac{m}{p} + \frac{n}{q} = \frac{\alpha}{p} + \nu . \quad (17)$$

Thus, for each value of  $n$ , only a restricted set of  $m$  values are possible, these values being given by

$$m = -\frac{p}{q} n + p\nu + \alpha . \quad (18)$$

For fixed  $n$ , the allowed values for  $m$  are separated by  $p$ , since  $\nu$  takes on all integer values from  $-\infty$  to  $+\infty$ . Therefore in Equation (4) only those Fourier coefficients,  $E_{znm}(r)$ ,

will be nonzero for which Equation (18) is satisfied.

Equation (13) is a statement of the angular rotation symmetry of the fields for  $p > q$ , and it does not furnish new information about the space harmonics. It fixes the requirements on the  $m$  values for any  $n$  that

$$m = \frac{p}{q} \nu + \alpha' \quad , \quad (19)$$

where  $\alpha' = 0, 1, 2, \dots, \frac{p}{q} - 1$ , and  $\nu$  is any integer. The  $m$  values for a given  $n$  in Equation (18) form subsets of  $m$  values in Equation (19).

Each of the modes of the structure which has the underlying screw symmetry of the structure in the sense discussed will have associated with it a value of  $\alpha$  lying between 0 and  $p-1$ , that is,  $\alpha$  is a number that is characteristic of a mode. If screw symmetry is the only symmetry the fields possess, then at most there can be  $p$  modes which have this symmetry. However, because of boundary conditions on the structure, not all the various values of  $\alpha$  may represent modes which can be excited in practice. On the other hand, the presence of other symmetries may influence the occurrence of modes with particular values of  $\alpha$ , and there may be several modes whose fields include screw symmetry as well as other symmetries and which have the same value of  $\alpha$ . In any case, any mode that exists and has underlying screw symmetry will have a particular value of  $\alpha$  lying between 0 and  $p-1$  associated with the  $E_z$  for that mode. For that same mode, there will be a corresponding characteristic integer between 0 and  $p-1$  associated with the  $H_z$  for that mode, but this characteristic integer is not necessarily identical with  $\alpha$  and often differs.

B. Glide Symmetry:  $F(r, 2\theta_k - \theta, z + \delta) = F(r, \theta, z)$

The second important symmetry is glide symmetry, or reflection-translation. Here combined angular reflection and axial translation causes the structure to coincide with itself. In this case  $\delta$  must be equal to  $L/2$ , half a period of the structure. There are no restrictions on the number of angular reflection planes except that their total must be an even number, and that they must be equally spaced in  $\theta$ . If more than two angular reflection planes are present, then the structure also has angular rotation symmetry, where the characteristic rotation angle is equal to  $4\pi$  divided by the total number of angular reflection planes. Let  $2M$  be the total number of reflection planes, these being located at  $\theta_k = \pi k/M$  ( $k$  is an integer in the range,  $0 < k < 2M-1$ ); then  $2\pi/M$  is the characteristic rotation angle. Figure 1b illustrates this symmetry with  $M=1$ .

For fields with the symmetry of the structure, there are restrictions on the angular and

axial space harmonics. Let  $G$  be the glide operator, defined by

$$GE_z(r, \theta, z) = E_z(r, 2\theta_k - \theta, z + L/2) = gE_z(r, \theta, z) \quad (20)$$

where  $g$  is an eigenvalue associated with this operator. The successive application of the glide operator twice is equivalent to an axial translation of the field by one period:

$$G^2 E_z(r, \theta, z) = TE_z(r, \theta, z) \quad (21)$$

and since  $G$  and  $T$  commute,

$$g^2 = e^{-j\beta_o L} \quad (22)$$

Thus there are two eigenvalues:

$$g_1 = e^{-j\beta_o L/2} \quad (23)$$

$$g_2 = -e^{-j\beta_o L/2}$$

Applying this operator to the fields written in terms of the angular and axial space harmonics, and using the orthogonality properties, one obtains the condition,

$$E_{zn, -m}(r) e^{j(2m\theta_k - \pi n)} = +E_{znm}(r) \quad (24)$$

The plus sign corresponds to  $g_1$  and the minus sign to  $g_2$ . This equation must be true regardless of which angular reflection plane,  $\theta_k$ , is chosen. Since  $\theta_k = \pi k/M$ , then

$$2m\theta_k = 2\pi mk/M \quad (25)$$

If  $m$  is restricted to  $0, +M, +2M, +3M, \dots$ , then the dependence of Equation (24) on  $k$  is removed. This restriction on the  $m$  values is not really a consequence of the glide symmetry of the structure, but rather a consequence of the angular rotation symmetry which the structure also possesses.

The glide symmetry does affect the space harmonics, however, since with the values of  $m$  properly restricted by the angular rotation symmetry,

$$E_{zn,-m}(r) e^{-j\pi n} = E_{znm}(r) \quad (26)$$

for  $g_1$ , and

$$E_{zn,m}(r) e^{-j\pi n} = -E_{znm}(r)$$

for  $g_2$ . This implies that the angular harmonics associated with a particular axial space harmonic must have the proper odd or even character, as indicated in Table I. An examination of the field expressions shows that the corresponding relations for the axial magnetic field components lead to a  $\theta$  variation of  $\sin m\theta$  and  $\cos m\theta$ , respectively, when the axial electric field components have  $\cos m\theta$  and  $\sin m\theta$ .

$$\begin{aligned} \text{C. Combined Screw and Glide Symmetry: } F(r, \theta + \psi, z + \delta) &= F(r, \theta, z) \text{ and} \\ F(r, 2\theta_k - \theta, z + \delta) &= F(r, \theta, z) \end{aligned}$$

Many structures have both screw and glide symmetry simultaneously, as, for example, the ring-bar structure shown in Figure 1c. The glide symmetry imposes the condition that  $\delta = L/2$ , or in terms of the screw symmetry indices,  $q = 2$  only. Since  $p/q$  must equal an integer, then  $p$  must be even integer, greater than or equal to two. It is also necessary for the structure to have angular rotation symmetry with a characteristic rotation angle equal to  $4\pi/p$  for the screw and glide symmetries to be compatible. This restricts the total number of glide planes to be equal to  $p$ .

This combination of screw and glide symmetry does not introduce any new conditions on the space harmonics that were not imposed by glide symmetry alone. One may note that for screw symmetry alone,  $q = 2$  leads to a space-harmonic distribution similar to that for glide symmetry, since for  $q = 2$ , the sets of allowed  $m$  values are either the even or the odd integers.

In the discussion of screw symmetry, it was noted that any mode which possessed the screw symmetry of the structure has a characteristic number,  $\alpha$ , associated with it that lies in the range  $0 < \alpha < p-1$ . On the other hand, there are only two eigenvalues associated with glide symmetry. Therefore if a mode is to have simultaneously both screw and glide symmetries, then the values of  $\alpha$  are restricted to 0 and  $p/2$  to correspond with the two glide symmetry eigenvalues. Modes with other values of  $\alpha$  might exhibit screw symmetry, but not glide symmetry.

Table 1.  $\theta$  variation for axial electric field components with glide symmetry.

$g$	$n$	$E_{znm}$	$\theta$ variation
$g_1$	even	$E_{zn, -m} = E_{znm}$	$\cos m\theta$
$g_1$	odd	$E_{zn, -m} = E_{znm}$	$\sin m\theta$
$g_2$	even	$E_{zn, -m} = -E_{znm}$	$\sin m\theta$
$g_2$	odd	$E_{zn, -m} = E_{znm}$	$\cos m\theta$

#### IV. MODE COUPLING

The important feature of the screw and glide symmetries is that each impose restrictions on the relationship between the angular ( $m$ ) and axial ( $n$ ) space harmonic indices. For screw symmetry this relationship is given by Equation (18) and for glide by Equation (26), which is summarized in Table 1. The introduction of one of these relationships into an adaption of coupled mode theory leads to interesting conclusions concerning the dispersion characteristic, or  $\omega$  versus  $\beta$  diagram, of a structure with the corresponding symmetry.

The  $\omega$  versus  $\beta$  diagram for one of the modes of a periodic structure is periodic in  $\beta$ , since for fixed  $\omega$  there are an infinite set of  $\beta_n$  separated by  $2\pi/L$ . Because of the restriction to reciprocal structures, there exists for each mode with energy propagating in the  $+z$  direction a corresponding mode with energy propagating in the  $-z$  direction. In the discussion of symmetry operators, it was convenient to treat only one of the pair, that for which the Floquet constant was  $\exp(-j\beta_0 L)$ . The other mode, with Floquet constant  $\exp(+j\beta_0 L)$ , can be obtained by taking the negative of  $\beta_0$  for the first mode. Treating the other mode does not change any of the conclusions that have been reached concerning symmetry.

Several typical classes of  $\omega$  versus  $\beta$  diagrams for periodic circuits for microwave tubes are illustrated in Figure 2. Modes with energy propagating in the  $+z$  direction are indicated by solid lines and those with energy propagating in the  $-z$  direction by dotted lines. The axial space harmonic indices,  $n$ , are shown on the curves for each of the modes for an arbitrarily chosen numbering system. The  $\omega$  versus  $\beta$  diagrams in Figure 2 fall into two types. In 2a and 2c, there is no crossing of the mode curves, while in 2b and 2d, the mode curves do cross. The conditions for crossing, or noncrossing, of the mode curves are of considerable

practical importance.

In terms of coupled mode theory, one can say that a crossing will occur when the two modes do not couple, that is, when, in some sense, neither mode "sees" the other. It is assumed here that coupling between two modes at a particular frequency will occur whenever the electromagnetic fields of the modes are not orthogonal over a volume characteristic of the structure. This characteristic volume is defined as being one period long in the  $z$  direction and extending over the total transverse cross section (out to the surrounding shield for a closed-boundary structure and out to infinity for an open-boundary structure). If the fields of the two modes are orthogonal over this volume, then the modes will not couple. Therefore, the condition for the absence of coupling between the modes is given by

$$\int_0^L \int_{A_t} \int (\underline{E}_a \cdot \underline{E}_b^* + \underline{E}_a^* \cdot \underline{E}_b) r dr d\theta dz = 0 \quad (27)$$

$$\int_0^L \int_{A_t} \int (\underline{H}_a \cdot \underline{H}_b^* + \underline{H}_a^* \cdot \underline{H}_b) r dr d\theta dz = 0 \quad (28)$$

where the electric and magnetic fields for the two modes are  $\underline{E}_a$  and  $\underline{H}_a$ , and  $\underline{E}_b$  and  $\underline{H}_b$ , where  $A_t$  represents the transverse cross section of the structure. The left-hand side of Equations (27) and (28) is proportional to the time-average stored electric and magnetic energy shared between the two modes. If these shared energies vanish, then the modes do not couple.

The integration in the  $z$  direction gives a non-zero result, in general. Writing the electric field in terms of the axial space harmonics, and letting

$$D_{nn'} = \int_{A_t} \int \underline{E}_{an}(r, \theta) \cdot \underline{E}_{bn'}^*(r, \theta) r dr d\theta, \quad (29)$$

one obtains for the left-hand side of Equation (27):

$$\begin{aligned} & 2 \operatorname{Re} \left\{ \sum_{n=-\infty}^{+\infty} \sum_{n'=-\infty}^{+\infty} D_{nn'} \int_0^L e^{-j(\beta_{oa} - \beta_{ob})z - j2\pi(n-n')z/L} dz \right\} \\ & = 2 \operatorname{Re} \left\{ \sum_{n=-\infty}^{+\infty} \sum_{n'=-\infty}^{+\infty} D_{nn'} \left[ jL \frac{e^{-j(\beta_{oa} - \beta_{ob})L - j2\pi(n-n')} - 1}{(\beta_{oa} - \beta_{ob})L + 2\pi(n-n')} \right] \right\} \end{aligned}$$

$$= 2 \operatorname{Re} \left\{ \sum_{n=-\infty}^{+\infty} \sum_{n'=-\infty}^{+\infty} D_{nn'} L \left[ \frac{\sin \eta}{\eta} + j \frac{\cos \eta - 1}{\eta} \right] \right\} \quad (30)$$

with  $\eta = (\beta_{oa} - \beta_{ob})L + 2\pi(n - n')$ . This is not zero, in general, if the  $D_{nn'}$  are not zero. For the particular cases of interest here where the two modes are basically identical but traveling in opposite directions,  $\beta_{ob} = -\beta_{oa}$ . Initially, let us consider the case where the crossover occurs at  $\beta_{oa} = \pm\pi/L$ . For this case,  $\eta = \pm 2\pi + 2\pi(n - n')$ , and all the terms in expression (30) are zero except for  $n = n' + 1$  for  $\beta_{oa} = \pi/L$ , and  $n = n' - 1$  for  $\beta_{oa} = -\pi/L$ . Examination of Figures 2b and 2d shows that it is just for these pairs of axial space-harmonic indices that the crossover occurs on the  $\omega$  versus  $\beta$  diagrams.

The integrals of Equations (27) and (28) will be zero only if the integration over the transverse cross section yields zero. While the integration in  $r$  might yield zero in some cases, it is believed that this would occur only rarely, and therefore, this possibility is ignored. Thus the integration in  $\theta$  plays the primary role in determining whether the modes couple. Since the  $\theta$  variation can be written in terms of a Fourier series of angular space harmonics,  $\exp(-jm\theta)$ , orthogonality can exist only if the intersecting branches possess completely distinct values of  $m$ , or if the Fourier coefficients are such that the exponentials can be combined so that one branch contains only  $\sin(m\theta)$  and the other contains only  $\cos(m\theta)$ . Since for the coupling situation considered here, contributions to the coupling integrals, (27) and (28), occur only for neighboring values of the axial space-harmonic indices of the two modes, the possible  $m$  values for neighboring  $n$  values must be examined. For screw symmetry, by Equation (18) the  $m$  values must be different for neighboring values of  $n$ , for glide symmetry, neighboring values of  $n$  have a  $\theta$  dependence which shifts from  $\sin m\theta$  to  $\cos m\theta$ , or vice versa (Table 1). Thus screw and glide symmetries have the requisite conditions for orthogonality of the two crossing modes, and coupling does not exist.

Structures with screw and/or glide symmetry would be expected to exhibit  $\omega$  versus  $\beta$  diagrams of the type shown in Figures 2b and 2d, and this is indeed the case. For example, the helix<sup>2</sup>, Figure 6a, which might be considered the limiting case of screw symmetry (with both  $\psi$  and  $\delta$  being infinitesimal), has a dispersion curve similar to Figure 2d. The cross-wound helix<sup>3</sup> and the ring-bar family<sup>4</sup>, Figure 9, are structures which have both screw and glide symmetry and also exhibit a dispersion curve similar to Figure 2d. Coupled cavity structures, typified by the Hines<sup>5</sup> structure of Figure 3a and the structure of Figure 3b which have both screw and glide symmetry, are examples of structures with dispersion curves

of the type shown in Figure 2b. Screw or glide symmetry is a sufficient condition for mode orthogonality, zero coupling, and hence, the possibility of mode crossings. The question of whether one of these symmetry types is a necessary condition has not been examined, but every case of mode crossing that we have observed has involved either screw or glide symmetry, or both.

The preceding discussion represents a departure from the customary application of coupled mode theory<sup>6</sup>, in which circuits of known characteristics are coupled, and the characteristics of the composite circuit are derived as a perturbation of the uncoupled characteristics. In the present instance, only the composite structure is known, and the nature of the hypothetical uncoupled circuits is of no interest or importance to the problem. It is assumed that the branches of the  $\omega$  versus  $\beta$  diagram for the hypothetical circuits have the same angular harmonic content as the corresponding branches of the actual circuit, and hence, predict the proper coupling and crossing behavior.

Consider now somewhat more general mode-coupling possibilities. In general, the possible mode crossings for a reciprocal pair of modes could occur for  $\beta_{oa} = -\beta_{ob} = \mu\pi/L$ , where  $\mu$  is any integer, positive or negative ( $\mu = \pm 1$  was considered above). Now,  $\eta = 2\pi\mu + 2\pi(n - n')$ , and all the terms in expression (30) will be zero except for  $n' = n + \mu$ . First, the implications of this for screw symmetry will be investigated. Equation (18) states that for a given mode (fixed value for  $\alpha$ ); the  $m$  values for any  $n$  are separated by  $p$ ; the  $m$  values increase by  $-p/q$  when  $n$  increases by 1; and the set of  $m$  values for every  $q^{\text{th}}$   $n$  are identical. Therefore, there can be coupling between the reciprocal modes only if  $\mu$  is an integer multiple of  $q$ . As a consequence, there can be no mode crossings at locations where  $\mu$  is an integer multiple of  $q$ , but mode crossings can occur for all other values of  $\mu$ .

Next, the implications for glide symmetry are examined. Table 1 states that for all given mode (corresponding to one of the eigenvalues,  $g_1$  or  $g_2$ ) the  $\theta$  variation changes alternately from  $\sin(m\theta)$  to  $\cos(m\theta)$ , or vice versa, for each unit increase in  $n$ . Therefore, mode coupling between reciprocal modes will occur when  $\mu$  is an even integer, but not when  $\mu$  is an odd integer. And as a consequence, there can be no mode crossings if  $\mu$  is an even integer, while mode crossings will occur if  $\mu$  is an odd integer.

This theory would appear to predict that coupling might also occur between completely different modes. At the present time, this must be regarded as conjecture, and has not been experimentally confirmed. These couplings, however, appear to be of little practical significance in comparison to the coupling between space harmonics of the reciprocal modes.

### V. APPARENT PERIODICITY FOR SCREW SYMMETRIC STRUCTURES

The fields along the symmetry axis of a screw symmetric structure exhibit an underlying periodicity which is different from the real period of the structure because of the restrictions on the allowed values of  $m$  given by (18). If it is assumed that there is a region surrounding the symmetry axis that is empty (almost always the case for microwave tube structures, at least), then the expressions for the electromagnetic fields in the neighborhood of the symmetry axis are those given in the Appendix. Because the radial variation of the axial space-harmonic components of the longitudinal fields,  $E_z$  and  $H_z$ , varies as  $I_m(\gamma_n r)$  where  $I_m$  is a modified Bessel function of order  $m$ , these axial space harmonic components can be non-zero on the axis ( $r = 0$ ) only if  $m = 0$  is one of the allowed values for the particular  $n$  being considered.

For a given mode, that is, a given value of  $\alpha$ , only those axial space harmonics which have  $n$  values satisfying

$$\frac{n}{q} - \frac{\alpha}{p} = \text{integer} \quad (31)$$

will include  $m = 0$ , and, hence, have non-zero  $E_{zn}$  or  $H_{zn}$  on the symmetry axis. This can be satisfied for only a portion of the modes, those having values of  $\alpha$  that are zero or an integer multiple of  $p/q$ . And for the modes which can satisfy (31), only  $1/q$  of the total possible  $n$  values will satisfy (31). Because of this limitation on  $n$ , the apparent period for the structure which would be determined by examining the longitudinal fields along the symmetry axis is  $L/q$ . It may be noted that the characteristics of traveling-wave-tube interaction structures are commonly investigated by exploring the longitudinal electric field along the symmetry axis. For example, the ring-bar circuit with  $q = 2$  (Figure 1c) has an apparent periodicity of  $L/2$ , while a helix supported by three rods has  $q = 3$  (Figure 6b) and an apparent periodicity of  $L/3$ . An unsupported helix has  $q \rightarrow \infty$ ; in this case the longitudinal fields on the symmetry axis are non-zero only for  $n = 0$ , and as far as these fields are concerned, the structure appears to be uniform, not periodic.

Thus, if one determines the  $\omega$  versus  $\beta$  diagram for a periodic structure with screw symmetry by exploring only the longitudinal fields on the symmetry axis, the complete  $\omega$  versus  $\beta$  diagram for the structure may not be found. In particular, those branches of the  $\omega$  versus  $\beta$  diagram corresponding to space-harmonic fields that are zero on the symmetry axis will not be located. As a consequence, the diagram determined in this manner may indicate an apparent periodicity for the structure which is shorter than the real period. As an illustration, the folded waveguide of Figure 4a has an  $\omega$  versus  $\beta$  diagram for the longitudinal electric field

on the symmetry axis as shown in Figure 4b, but the complete diagram for the structure is shown in Figure 4c.

A similar discussion applies to the transverse fields on the symmetry axis. The radial variation of the space-harmonic components of the transverse fields is given by a combination of  $I'_m(\gamma_n r)$  and  $mI_m(\gamma_n r)/\gamma_n r$ . These will be non-zero at  $r = 0$  only for  $m = \pm 1$ . From (17),

$$\frac{n}{q} - \frac{\alpha + 1}{p} = \text{integer}, \quad (32)$$

for  $m = \pm 1$ . Again, only part of the modes will satisfy this, those having  $\alpha + 1$  equal to zero or an integer multiple of  $p/q$ . And again, for those modes, only  $1/q$  of the total possible  $n$  values will satisfy (32), in general. For  $p/q = 1$  or  $2$ , however, both  $\alpha + 1$  and  $\alpha - 1$  could be an integer times  $p/q$  (for all  $\alpha$  in the first case and for odd  $\alpha$  in the second case), therefore the number of  $n$  values for non-zero transverse fields on the axis would be  $2/q$  of the total in these special cases.

## VI. LEAKY WAVE ANTENNAS

Structural symmetries have important consequences for leaky wave antennas<sup>7</sup>. Consider the idealized  $\omega$  versus  $\beta$  diagram shown in Figure 5a for an open-boundary structure suitable for radiation. The triangular regions with horizontal shading are regions where propagation without radiation may occur. The unshaded region is the region where the space harmonics will be leaky waves, and radiation may occur. The leaky wave region is defined by the relation  $|k| > |\beta|$ , where  $k = \frac{\omega}{c}$  (see Appendix). The encircled point represents broadside operation of an antenna with a radiating  $n = -1$  axial space harmonic. If coupling between the reciprocal modes occurs at this point, then the modes will not cross and the group velocity of the structure will be nearly zero there (since  $\frac{\partial \omega}{\partial \beta}$  will be nearly zero). As shown by Hessel<sup>8</sup>, the amplitude of the radiating space harmonic decreases sharply under these circumstances. Thus the broadside radiation characteristics of an antenna are strongly influenced by the structure symmetry through the possible mode coupling.

In the leaky wave region  $\beta$  is complex and a complete description of the coupling would require a three-dimensional plot of  $\omega$  versus complex  $\beta$ . The desired information can be obtained, however, from the  $\omega$  versus  $\text{Re}\beta$  diagram. In this instance, the possible coupling might occur at  $\beta_{oa} = -\beta_{ob} = 2\pi/L$ , and  $\mu = 2$ . Since  $\mu$  is even, for glide symmetry the reciprocal modes couple and there can be no mode crossing. Hence structures with glide symmetry, for example the slotted waveguide shown in Figure 1b, will not radiate exactly at broadside. For screw symmetry, the discussion of the last section showed that mode coupling will occur if  $\mu$  is an integer multiple of  $q$ . In this case, mode coupling will occur for  $q = 1$  or  $2$ , and for these values mode crossing will not occur and the structure will not radiate exactly at broadside. For  $q \geq 3$ , however, mode coupling will not occur, mode crossing will be present, and the structure will radiate broadside.

In summary, a structure with glide symmetry will not radiate exactly at broadside with the  $n = -1$  axial space harmonic, while a structure with screw symmetry will radiate broadside only if  $q \geq 3$ . The turnstile antenna shown in Figure 1a illustrates a structure with four-fold screw symmetry ( $p = q = 4$ ) which accordingly radiates broadside. The  $\omega$  versus  $\text{Re}\beta$  for the turnstile will have the form shown in Figure 4b; the first mode coupling occurs for  $\beta_{oa} = 4\pi/L$ . The helix possesses the highest-order screw operator ( $p, q \rightarrow \infty$ ), the differential screw operator. Therefore, there is no mode coupling for any value of  $\mu$ , and all possible mode crossings will occur, as shown in Figure 4a. As a result, the helix is capable of broadside radiation. It is interesting to note that for a helix supported by dielectric rods

to radiate broadside, there must be at least three equally spaced supporting rods. For equally spaced supporting rods, the value of  $q$  for the supported helix is equal to the number of rods, as we have seen,  $q$  must be at least three for broadside radiation.

It is also of interest to note than an open structure with combined screw and glide symmetry will not radiate exactly at broadside. As shown above, this type of structure must have  $q = 2$ , and hence, will not radiate broadside with the  $n = -1$  axial space harmonic.

## VII. MICROWAVE TUBE CIRCUITS

Microwave tubes which use extended interaction between an electron beam and a propagating circuit, such as traveling-wave tubes and backward-wave amplifiers and oscillators, use periodic structures for the interaction circuits. Most of the periodic circuits which are currently employed in these microwave tubes have screw and/or glide symmetry. The reasons for the extensive utilization of these symmetries are not entirely clear because, presumably, periodic structures with other symmetries might yield comparable bandwidths and interaction impedances. And at high power levels, certain of the properties possessed by structures with screw and/or glide symmetry may be undesirable.

When the electron beam diameter is small in terms of guide wavelength, screw symmetry does have an advantage because of the characteristics of the electromagnetic fields which lead to the reduced apparent period on the symmetry axis (Section V). This reduction in apparent period is caused by the zero amplitude of certain of the space harmonics on the symmetry axis. That is, an electron beam which is filamentary "sees" fewer space harmonic fields than a thick beam. The helix, either unsupported or supported by a concentric dielectric shell is, perhaps, the most striking example of this. Figure 6a shows an idealized  $\omega$  versus  $\beta$  diagram for a helix. Only the fundamental space harmonic is non-zero on the axis of the helix; this space harmonic is denoted by the solid line on the diagram. Thus a filamentary electron beam along the axis of the helix "sees" only a single space harmonic field, and as far as the electron beam is concerned, the structure is uniform, not periodic. For a filamentary electron beam there will be no possibility of interacting with the  $n = -1$  space harmonic and producing a backward-wave oscillation in this case since this space harmonic will have zero amplitude at the electron beam.

When a helix is supported by three dielectric rods equally spaced azimuthally, the apparent period as seen by a filamentary electron beam on the axis is  $L/3$ , as indicated by the solid lines in Figure 6b. Again, for a filamentary electron beam the  $n = -1$  space harmonic will not interact to cause backward-wave oscillation.

A thick electron beam which fills an appreciable fraction of the structure cross section will "see" the complete space harmonic spectrum for the structure. Since both screw and glide symmetries can have mode crossings (see Section IV), traveling-wave tube structures with one or both of these symmetries may have difficulties with backward-wave oscillations in the operating frequency band if the electron beam diameter is appreciable. This problem can be minimized by limiting the beam diameter and by choosing an operating

band well below the frequency at which the modes cross. However, this limits the power handling capability of the tube since the total beam current will be restricted. Traveling wave tubes using helices are an example of this. The power handling capability of these tubes is restricted by the onset of backward-wave oscillations as the power level is raised.

One solution to this problem is to start with circuits with screw symmetry and destroy the screw symmetry by introducing appropriate asymmetric structures. Then mode coupling would take place and no mode crossings could occur. For example, if a helix supported by three dielectric rods had rods which differed in shape, size, or dielectric constant, or which were oriented asymmetrically about the helix, then the screw symmetry would be destroyed. This would alleviate the backward-wave oscillation problem. Of course, other types of oscillation, such as band edge or higher passband oscillations, may be of equal or more importance in particular cases.

Another solution to the backward-wave oscillation problem is to develop circuits which have neither screw nor glide symmetry. The meander line<sup>9</sup> is an example of this approach; see Figure 7. Since there are no mode crossings within the lowest passband there is no possibility of backward-wave oscillation there. This accounts at least partially for its success as a wide band, high power traveling-wave tube interaction circuit. In the future a fruitful approach to the development of interaction circuits for high power traveling-wave tubes may be to develop appropriate structures which have neither screw nor glide symmetry.

### VIII. OTHER SYMMETRIES

A. Axial Reflection:  $F(r, \theta, 2z_k - z) = F(r, \theta, z)$ .

If reflection planes perpendicular to the  $z$  axis exist at  $z = z_k$  about which the structure has reflection symmetry, there are two, and only two, reflection planes per period of the structure, and these must be separated by a distance  $L/2$ . This restriction follows from the fact that the real period of the structure is taken as  $L$ , and only two reflection planes per period separated by  $L/2$  can be consistent with this period. Thus for this condition,  $k = 1, 2$  and  $z_2 = z_1 + L/2$ . This symmetry is illustrated in Figure 8a.

In this case, for a single propagating wave, it is not possible for the electromagnetic fields to have the symmetry of the structure. If this field symmetry were possible, the axial space harmonics would have to be related by  $E_{zn}(r, \theta) = E_{z, -n}(r, \theta)$ . This can be true only if  $\gamma_{-n} = \gamma_n$ , which occurs at  $\beta_0 = 0$ , only. Thus, in general, one may conclude that a single mode cannot have fields with the axial reflection symmetry of the structure. This conclusion is not correct if a standing wave is present (a pair of propagating waves moving in opposite directions); here the fields will have the axial reflection symmetry of the structure.

B. Angular Rotation:  $F(r, \theta + \Psi, z) = F(r, \theta, z)$ .

Figure 8b illustrates this type of symmetry with a ring-line circuit (which also has axial reflection symmetry.) Since successive rotations of  $\Psi$  radians cause the structure to coincide with itself, and a rotation of  $2\pi$  radians must also produce this, then  $\sigma = 2\pi/\Psi$  is an integer. There are no restrictions on the value that  $\sigma$  may assume. In Figure 8b, the example has  $\sigma$  equal to 2.

In this case it is possible for the fields to have the symmetry of the structure in the sense discussed above. Define the rotation operator,  $R_\sigma$ , such that

$$R_\sigma E_z(r, \theta, z) = E_z(r, \theta + \Psi, z) = r_\sigma E_z(r, \theta, z), \quad (33)$$

where  $r_\sigma$  is the associated eigenvalue. Since  $\sigma$  repeated rotations cause the structure to be rotated one complete revolution, then

$$R_\sigma^\sigma E_z(r, \theta, z) = E_z(r, \theta, z) = r_\sigma^\sigma E_z(r, \theta, z), \quad (34)$$

$$r_\sigma = e^{-j2\pi k/\sigma}, \quad (35)$$

where  $0 \leq k \leq \sigma - 1$ . Each separate value of  $k$  in the given range corresponds to a mode of the structure whose fields have the rotational symmetry.

The  $\theta$  variation of the fields may be described by a Fourier series in  $\theta$ , and by using the orthogonality properties of the angular space harmonics one can see that Equation (33) implies that

$$e^{-jm\Psi} = e^{-j2\pi k/\sigma} \quad (36)$$

for all values of  $m$  that have non-zero Fourier coefficients. Since  $\Psi = 2\pi/\sigma$ , then for each value of  $k$  there is a set of values of  $m$  which are allowed:

$$m = k, k + \sigma, k + 2\sigma, k + 3\sigma, \dots \quad (37)$$

Thus only a portion of the total possible number of angular space harmonics will be present for those modes which have the rotational symmetry of the structure.

C. Angular Reflection:  $F(r, 2\theta_j - \theta, z) = F(r, \theta, z)$ .

For this structure symmetry, there may be any even number of reflection planes in the range  $0 \leq \theta < 2\pi$ , because if  $\theta_j$  is such a reflection plane, then  $\theta_j + \pi$  must also be a reflection plane. In addition, all of the reflection planes must be equally spaced in  $\theta$ . Thus if the total number of reflection planes is  $N$  ( $N$  must be even), these reflection planes will be spaced at  $2\pi/N$  radians. Figure 8c illustrates this symmetry with  $N = 2$ .

Again, it is possible for the fields to have the symmetry of the structure. Define the angular reflection operator,  $A$ ,

$$A E_z(r, \theta, z) = E_z(r, 2\theta_j - \theta, z) = a E_z(r, \theta, z), \quad (38)$$

where  $a$  is the associated eigenvalue. Two successive angular reflections give the original field,

$$A^2 E_z(r, \theta, z) = E_z(r, \theta, z) = a^2 E_z(r, \theta, z), \quad (39)$$

and there are two eigenvalues,  $a = \pm 1$ .

Consider first  $a = +1$ , for which the electric field is symmetric about the planes  $\theta = \theta_j$ . Using the orthogonality of the space harmonics, one can write Equation (38) as

$$e^{j2m\theta_j} E_{zn, -m}(r) = E_{znm}(r). \quad (40)$$

For a given mode, this must be independent of  $\theta_j$ , and, as a consequence, only certain  $m$  values can have non-zero Fourier coefficients. Choosing  $\theta = 0$  so that  $\theta_j = 2\pi j/N$ , then  $2m/N$  must be an integer:

$$m = 0, N/2, 2N/2, 3N/2, \dots, \quad (41)$$

$$E_{zn, -m}(r) = E_{znm}(r). \quad (42)$$

In this case it turns out that  $H_z$  is antisymmetric about the  $\theta_j$  planes so that  $H_{zn, -m}(r) = -H_{znm}(r)$ .

For  $a = -1$ , the second eigenvalue,  $E_z$  is antisymmetric and  $H_z$  is symmetric about the  $\theta_j$  planes:

$$E_{zn, -m}(r) = -E_{znm}(r),$$

$$H_{zn, -m}(r) = H_{znm}(r). \quad (43)$$

The allowed values of  $m$  are those given in Equation (41). These restrictions, of course, lead to azimuthal variations of  $\sin m\theta$  or  $\cos m\theta$  for each of the angular space harmonic fields.

If a structure has  $N$  angular reflection planes, then it also has angular rotation symmetry with  $\sigma = N/2$ . The converse need not be true; a structure with angular rotation symmetry need not have angular reflection symmetry.

D. Skew Symmetry:  $F(r, 2\theta_j - \theta, 2z_k - z) = F(r, \theta, z)$ .

In skew symmetry, a combined reflection in an angular reflection plane with reflection in an axial reflection plane causes the structure to coincide with itself, as illustrated in Figure 8d. To satisfy the axial periodicity of the structure, there can be two, and only two, axial reflection planes per period, separated by  $L/2$ . Thus  $k = 1, 2$  only and  $z_2 = z_1 + L/2$ . There can be only an even number of angular reflection planes, say  $N$ , and these must be equally spaced azimuthally at  $2\pi/N$  radians.

Because of the axial reflection planes, it is not possible for the fields of a single mode

to have the full symmetry of the structure (this would again imply that  $\gamma_{-n} = \gamma_n$ , which occurs only for  $\beta_0 = 0$ ). It would be possible for the fields to have angular symmetry, however. If there is angular reflection symmetry of the fields, then the allowed  $m$  values will be restricted, and the Fourier coefficients,  $E_{znm}(r)$  and  $E_{zn, -m}(r)$ , will be related.

E. Rotation-Reflection:  $F(r, \theta + \Psi, 2z_k - z) = F(r, \theta, z)$ .

Combined angular rotation with reflection in an axial reflection plane will cause the structure with this symmetry to coincide with itself, as illustrated in Figure 8e. Again, to satisfy the axial periodicity of the structure, there can be two, and only two, axial reflection planes ( $k = 1, 2$ ) separated by a half a period,  $L/2$ . There are no restrictions on the angular rotation parameter,  $\Psi$ , except that  $\sigma = 2\pi/\Psi$ , where  $\sigma$  is any integer, since the structure is periodic in  $\theta$ . As in every symmetry case involving axial reflection planes, it is not possible for the fields of a single mode to have the full symmetry of the structure, except possibly at  $\beta_0 = 0$ . The fields might have angular rotation symmetry, if so, the allowed values of  $m$  will be restricted.

#### F. Combined Symmetries

The connected-ring class of structures possess all seven symmetries discussed in this paper simultaneously. The ring-bar circuit shown in Figure 1c is the simplest example of this class, and several others are shown in Figure 9. For each of these structures  $q = 2$ , and  $p$  is an even integer. There are two modes for each of the structures (corresponding to  $\alpha = 0$  and  $\alpha = p/2$ ) for which the underlying symmetry of the fields is that of the structure.

On the symmetry axis, the apparent periodicity of the longitudinal fields is  $L/2$  for these modes. For the mode with  $\alpha = 0$ , the fundamental axial space harmonic component of  $H_z$  is zero on the symmetry, while for  $\alpha = p/2$ , the fundamental axial space component of  $E_z$  is zero on the symmetry axis. Because of the presence of glide symmetry, there will be no mode coupling, and hence mode crossing, when  $\beta L/\pi$  is an odd integer, while when  $\beta L/\pi$  is an even integer mode coupling will occur, and there will be no mode crossings. This latter situation prevents this class of structure from radiating exactly at broadside.

## IX. CONCLUSIONS

Considerable useful information concerning the electromagnetic fields associated with periodic microwave guiding or radiating structures can be derived from the symmetry properties of these structures. It has been shown that screw and glide symmetries are particularly important in determining the characteristics of the fields. For example, these symmetries control the occurrence of mode crossings which establish the broadside radiation characteristics of leaky wave antennas and influence the capability for backward-wave oscillations of microwave tube interaction circuits. Also, in structures with screw symmetry the fields in the neighborhood of the symmetry axis may indicate an apparent structure period which is less than the real period of the structure.

It is possible that the structure symmetry will influence other characteristics of the electromagnetic fields of periodic structures. The authors believe that consideration of the consequences of the symmetries of periodic microwave structures may provide a fruitful approach to the solution of many analysis and synthesis problems involving these structures.

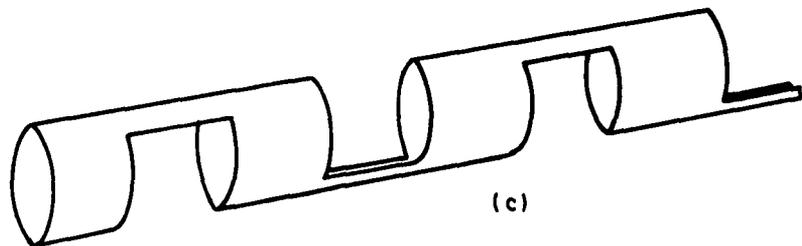
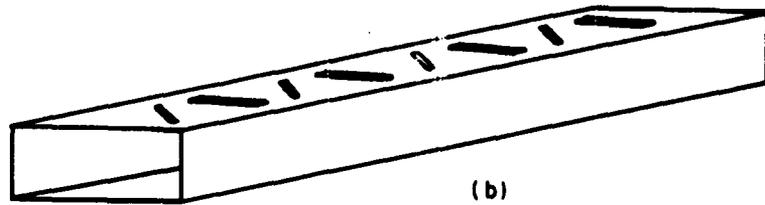
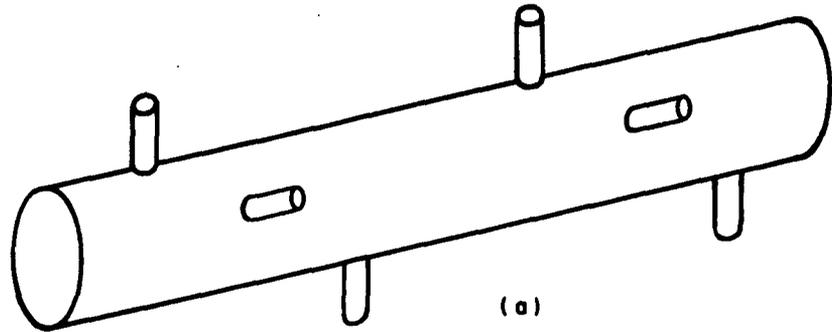


Figure 1 a) Turnstile antenna with screw symmetry.  $p=q=4$   
b) Slotted waveguide with glide symmetry.  $M=1$  c) Ring-bar circuit with combined screw and glide symmetry  $p=q=2$

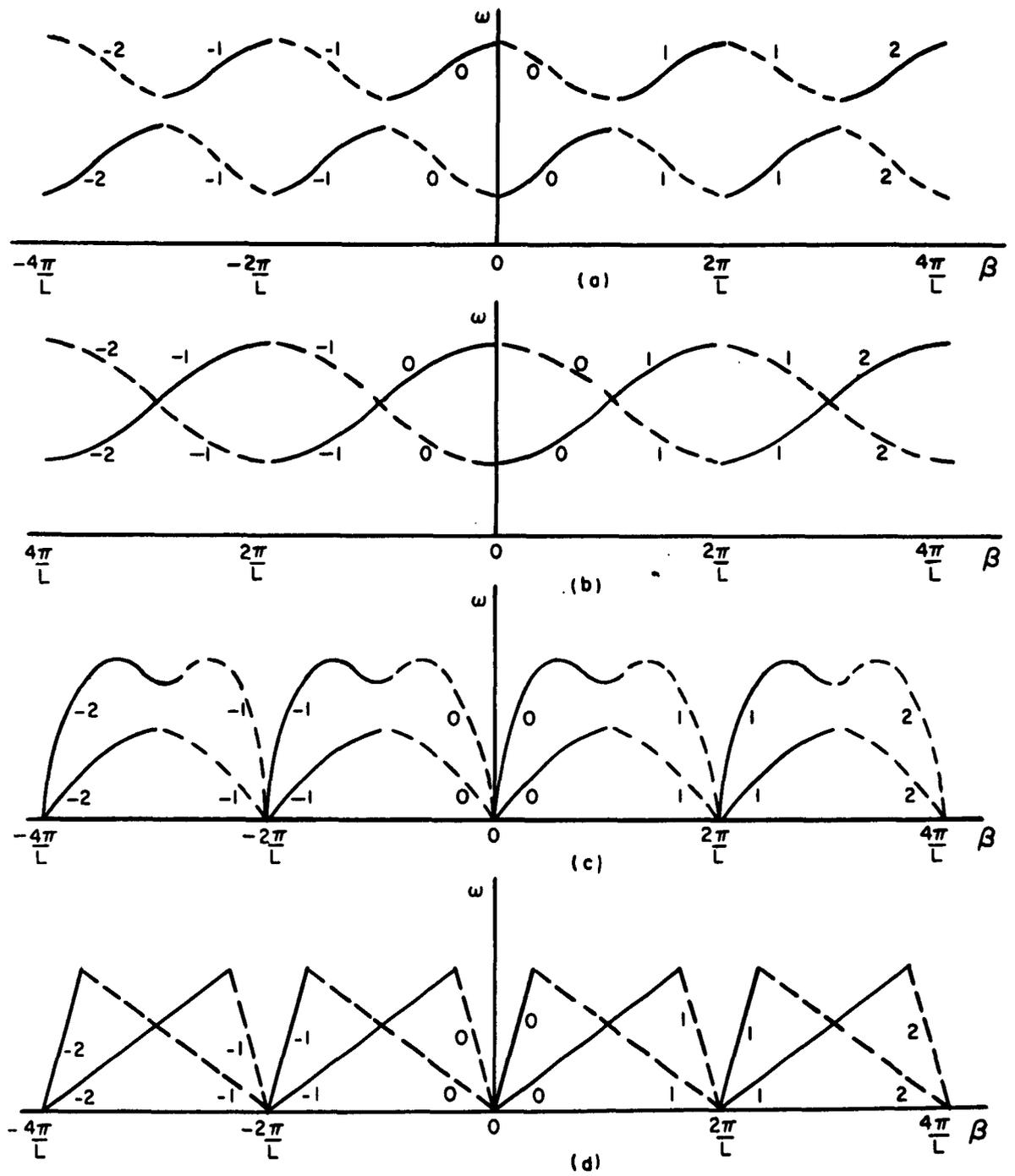
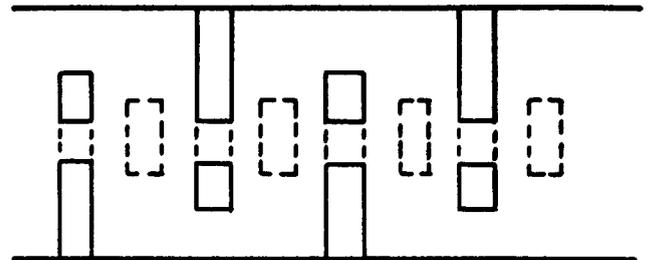
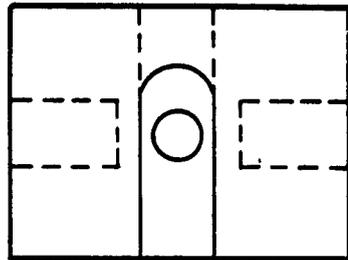
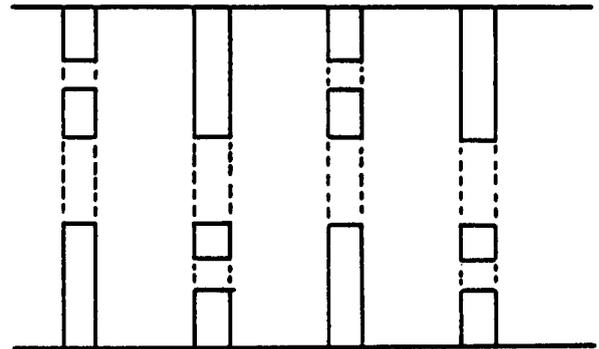
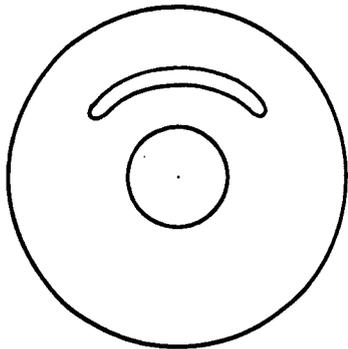


Figure 2 Typical  $\omega$  versus  $\beta$  diagrams for periodic structures.

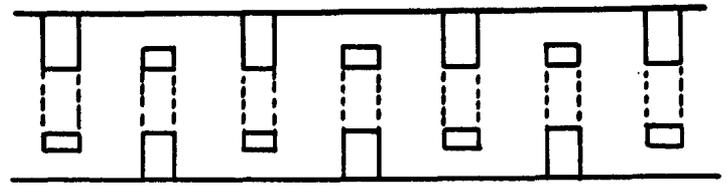
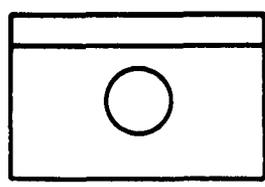


(a)



(b)

Figure 3 Periodically loaded waveguide structures with combined screw and glide symmetry. a) Hines Structure. b) Long slot coupled structure.



(a)

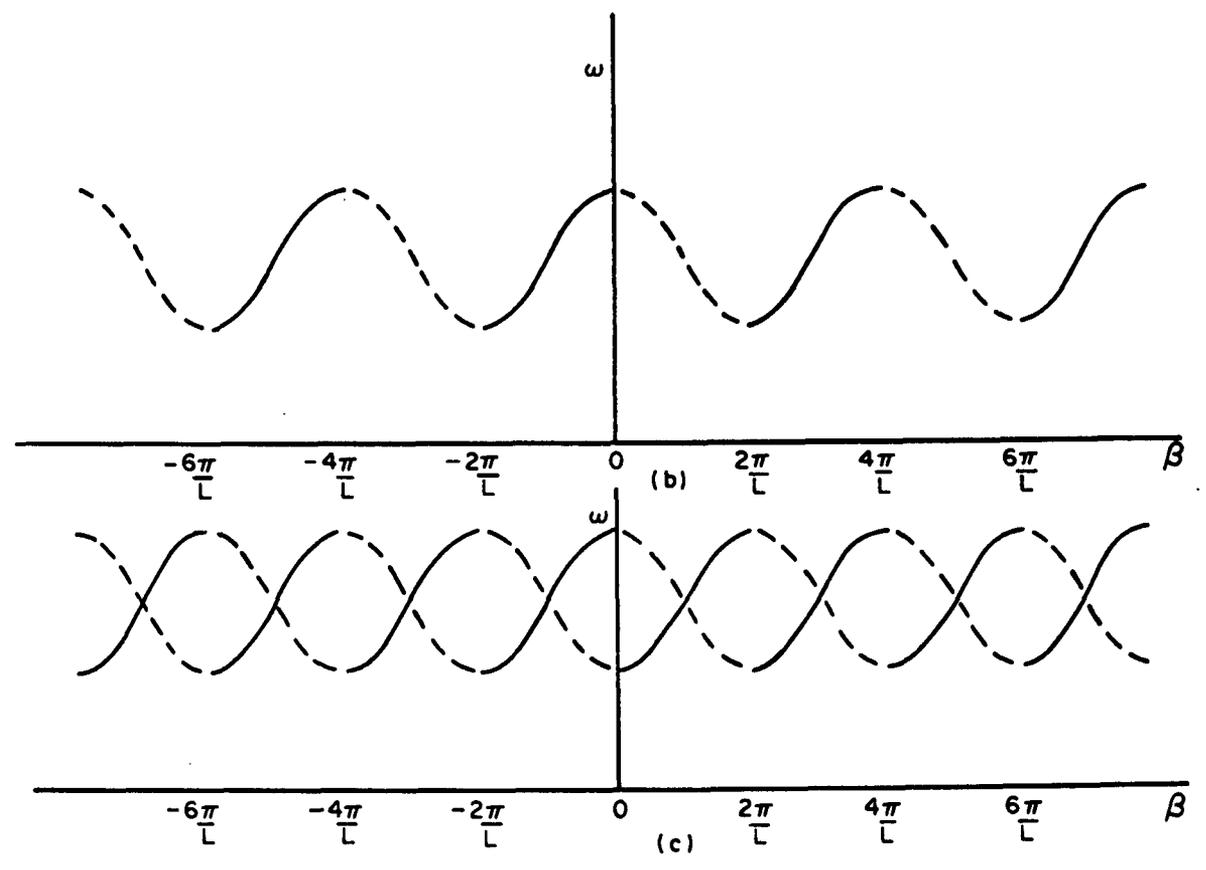


Figure 4 a) Folded waveguide with aperture along symmetry axis. b)  $\omega$  versus  $\beta$  diagram for longitudinal electric field on symmetry axis. c) Complete  $\omega$  versus  $\beta$  diagram.

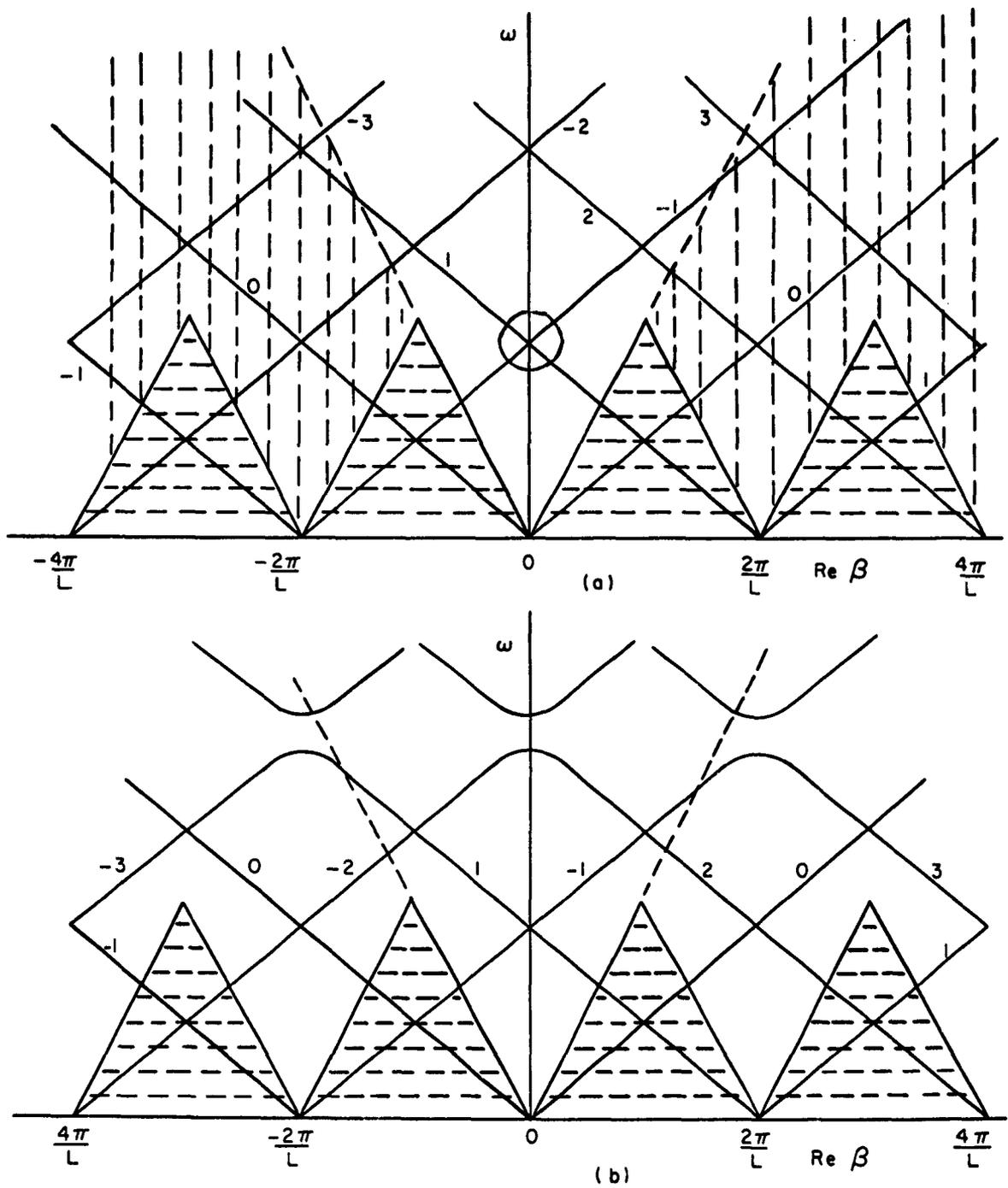


Figure 5 a) Idealized  $\omega$  versus  $\text{Re } \beta$  diagram for a leaky wave antenna. Leaky wave region is unshaded. b) Idealized  $\omega$  versus  $\text{Re } \beta$  diagram for a turnstile antenna,  $p = q = 4$ .

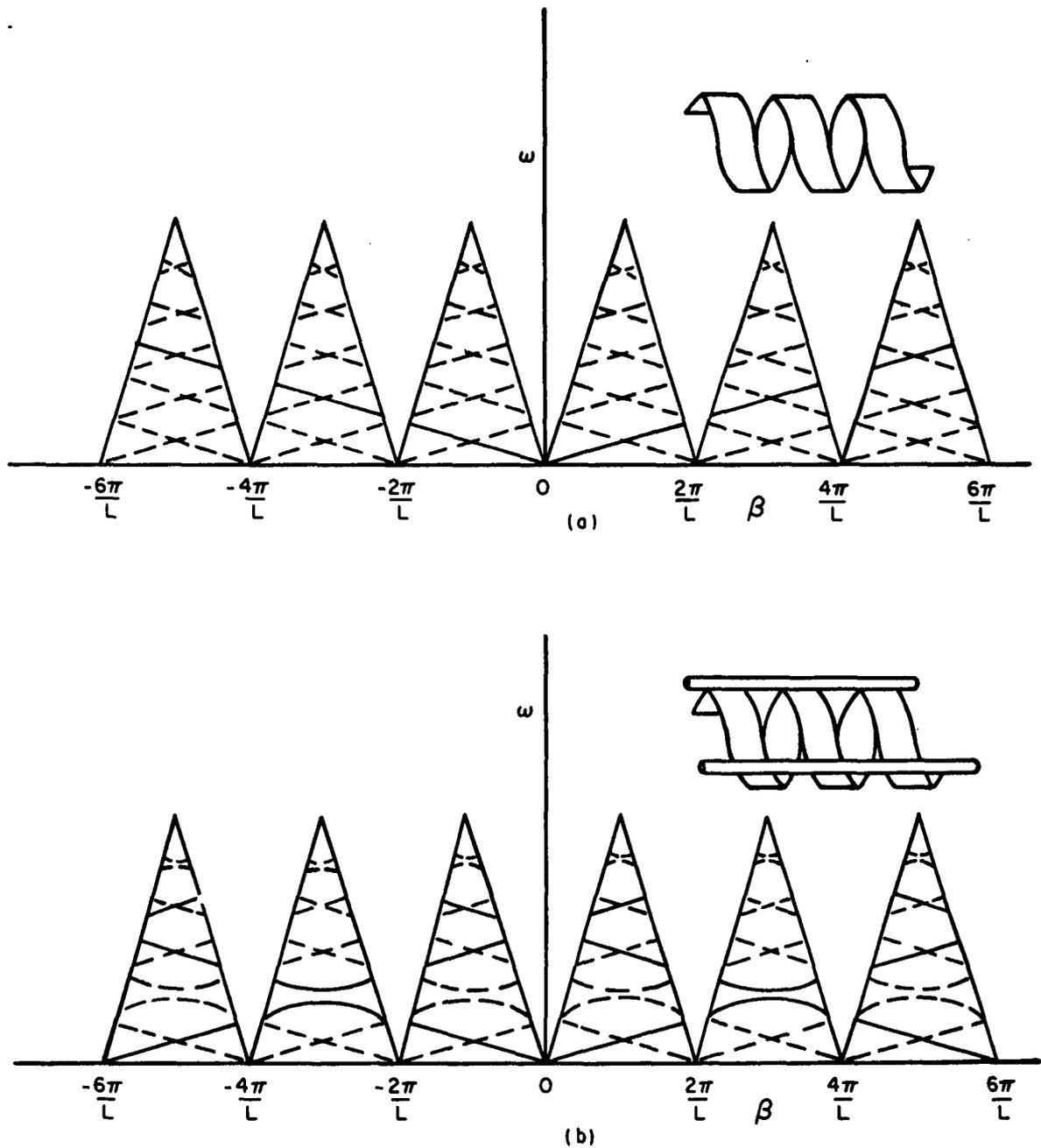
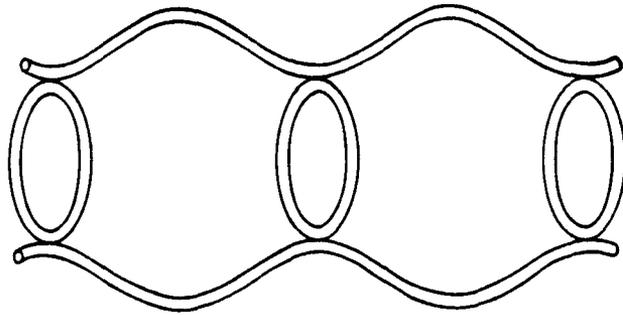
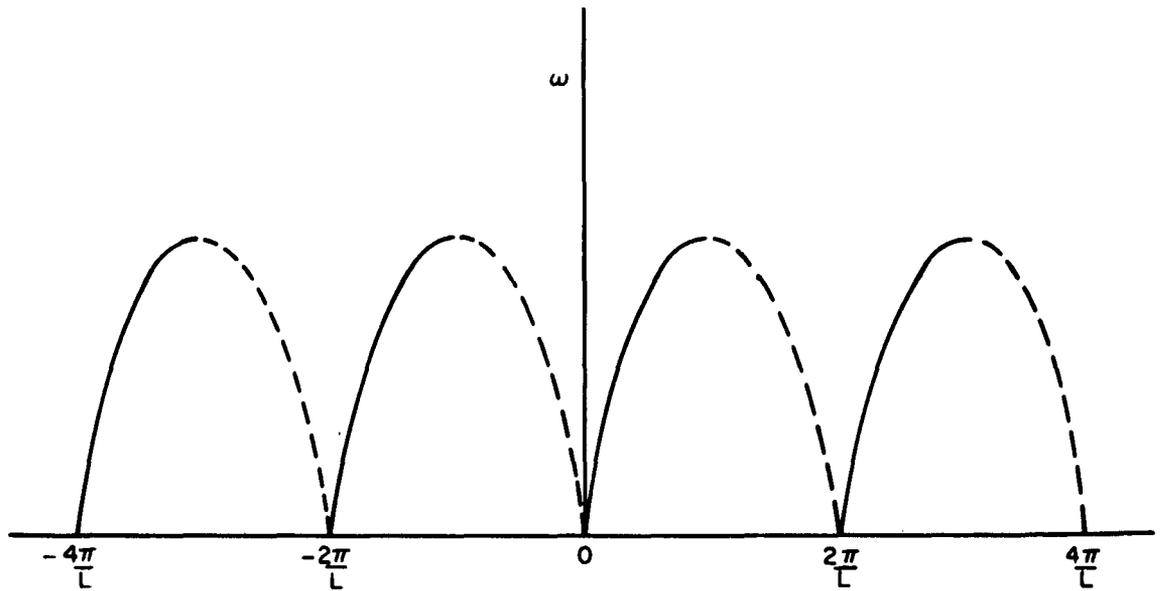


Figure 6 a) Idealized  $\omega$  versus  $\beta$  diagram for a helix; propagation region only shown. b) Idealized  $\omega$  versus  $\beta$  diagram for a helix supported by three dielectric rods; propagation region only shown.



(a)



(b)

Figure 7 a) Meander line. b)  $\omega$  versus  $\beta$  diagram for a meander line.

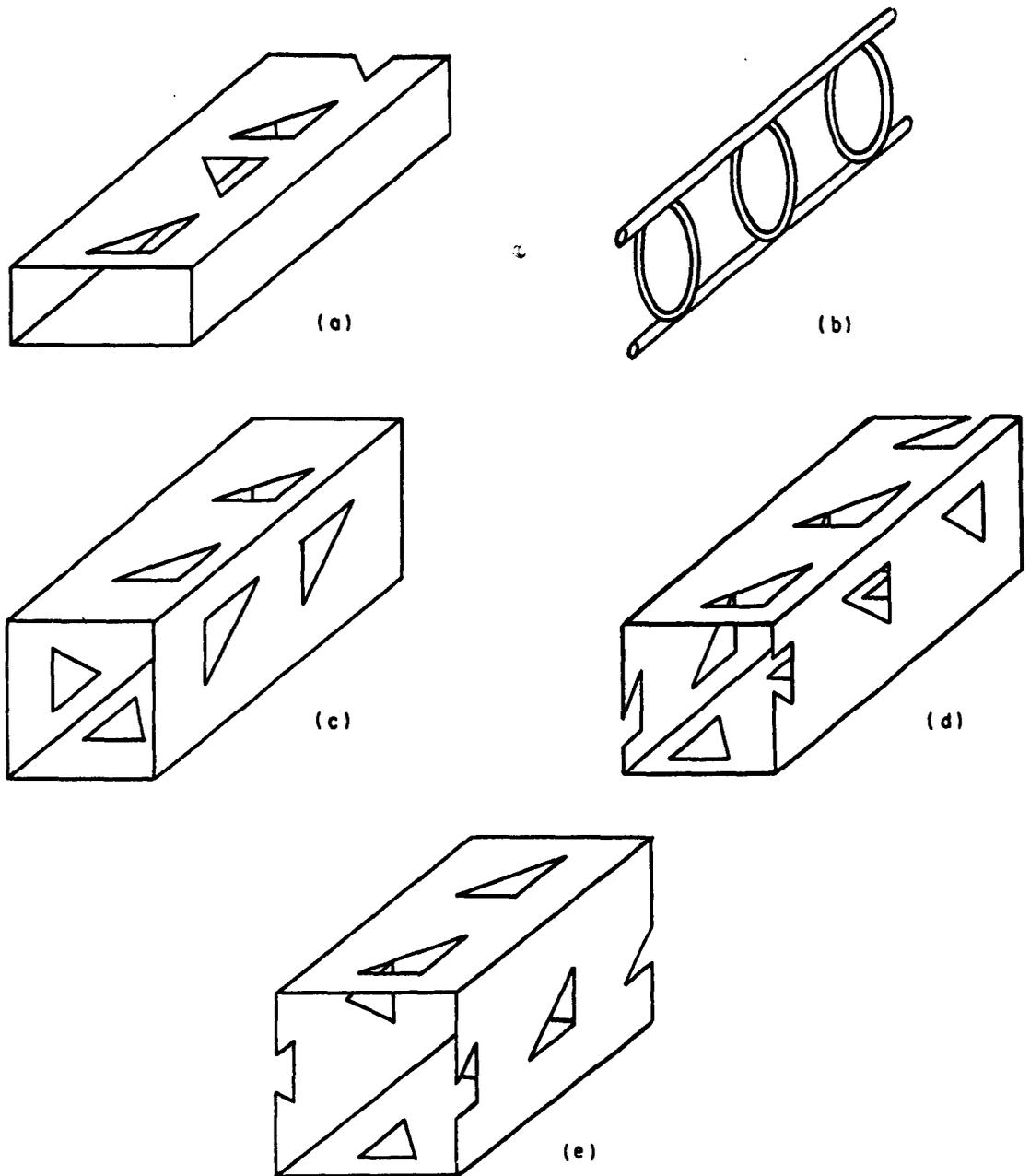
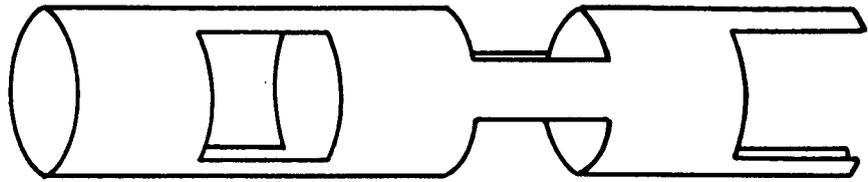
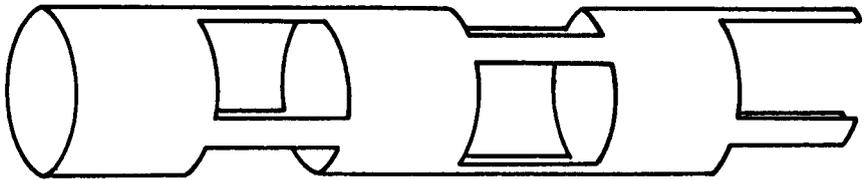


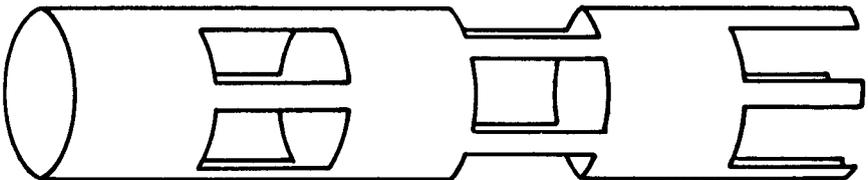
Figure 8 a) Slotted waveguide with axial reflection symmetry. b) Ring-line circuit with angular rotation symmetry.  $\sigma = 2$ . c) Slotted waveguide with angular reflection symmetry. d) Slotted waveguide with skew symmetry. e) Slotted waveguide with rotation-reflection symmetry.



(a)



(b)



(c)

Figure 9 Connected-ring structures with all seven symmetries.

Appendix

For lossless, reciprocal structures which are periodic along a rectilinear direction (the z direction) there are certain useful relationships involving the electromagnetic field components. Floquet's theorem holds for both the electric and magnetic fields so that these can be written (assuming a time variation of  $e^{j\omega t}$ ) as

$$\underline{\underline{E}}(r, \theta, z) = e^{-j\beta_0 z} \underline{\underline{E}}_1(r, \theta, z), \quad (\text{A-1})$$

$$\underline{\underline{H}}(r, \theta, z) = e^{-j\beta_0 z} \underline{\underline{H}}_1(r, \theta, z). \quad (\text{A-2})$$

here  $\underline{\underline{E}}_1(r, \theta, z)$  and  $\underline{\underline{H}}_1(r, \theta, z)$  are functions periodic in z with period L equal to the structure period. Because of this periodicity, the z variation of the fields can be expressed using a Fourier series. In addition to the axial periodicity, the electromagnetic fields must also be azimuthally periodic with period  $2\pi$  in  $\theta$ . Thus the azimuthal variation of the fields can also be represented by a Fourier series. The fields therefore can be written as

$$\underline{\underline{E}}(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \underline{\underline{E}}_{nm}(r) e^{-jm\theta} e^{-j\beta_n z} \quad (\text{A-3})$$

$$\underline{\underline{H}}(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \underline{\underline{H}}_{nm}(r) e^{-jm\theta} e^{-j\beta_n z} \quad (\text{A-4})$$

where

$$\beta_n = \beta_0 + 2\pi n/L. \quad (\text{A-5})$$

$\underline{\underline{E}}_{nm}(r)$  and  $\underline{\underline{H}}_{nm}(r)$  are the Fourier coefficients for the  $n^{\text{th}}$  axial and  $m^{\text{th}}$  angular space harmonic components of the fields.

It is possible, and often convenient, to represent the transverse electric and magnetic fields in terms of the longitudinal components. For simplicity these relationships will be developed here assuming that the permittivity and permeability associated with the structure do not vary with z. They may vary, however, in a transverse plane, either continuously or discontinuously. These relationships are based on Maxwell's curl equations.

$$\nabla \times \underline{E} = -j\omega\mu\underline{H}, \quad (\text{A-6})$$

$$\nabla \times \underline{H} = j\omega\epsilon\underline{E}. \quad (\text{A-7})$$

Let  $\underline{E}_T$ ,  $\underline{H}_T$  be the transverse components of the electric and magnetic fields and define the transverse del operator,  $\nabla_T$ , as

$$\nabla_T = \nabla - \underline{a}_z \frac{\partial}{\partial z}, \quad (\text{A-8})$$

where  $\underline{a}_z$  is a unit vector in the z direction. The transverse components of Equations (A-6) and (A-7) are

$$-\underline{a}_z \times \nabla_T \underline{E}_z + \underline{a}_z \times \frac{\partial \underline{E}_T}{\partial z} = -j\omega\mu\underline{H}_T, \quad (\text{A-9})$$

$$-\underline{a}_z \times \nabla_T \underline{H}_z + \underline{a}_z \times \frac{\partial \underline{H}_T}{\partial z} = j\omega\epsilon\underline{E}_T. \quad (\text{A-10})$$

Solving for  $\underline{E}_T$  and  $\underline{H}_T$ , and setting  $k = \omega\sqrt{\mu\epsilon}$  and  $Z = \sqrt{\frac{\mu}{\epsilon}}$  (k and Z can vary in the transverse plane),

$$\underline{E}_T + \frac{1}{k^2} \frac{\partial^2 \underline{E}_T}{\partial z^2} = \frac{1}{k^2} \frac{\partial}{\partial z} (\nabla_T \underline{E}_z) + j \frac{Z}{k} (\underline{a}_z \times \nabla_T \underline{H}_z), \quad (\text{A-11})$$

$$\underline{H}_T + \frac{1}{k^2} \frac{\partial^2 \underline{H}_T}{\partial z^2} = \frac{1}{k^2} \frac{\partial}{\partial z} (\nabla_T \underline{H}_z) - \frac{j}{kZ} (\underline{a}_z \times \nabla_T \underline{E}_z). \quad (\text{A-12})$$

Taking account of the z variation imposed by Floquet's theorem and using the orthogonality of the axial space harmonics,

$$\underline{E}_{Tn}(r, \theta) = j \frac{\beta_n}{\gamma_n} \nabla_T \underline{E}_{zn}(r, \theta) - j \frac{kZ}{\gamma_n^2} (\underline{a}_z \times \nabla_T \underline{H}_{zn}(r, \theta)), \quad (\text{A-13})$$

$$\underline{H}_{Tn}(r, \theta) = j \frac{\beta_n}{\gamma_n} \nabla_T \underline{H}_{zn}(r, \theta) + j \frac{k}{\gamma_n^2 Z} (\underline{a}_z \times \nabla_T \underline{E}_{zn}(r, \theta)), \quad (\text{A-14})$$

where

$$\gamma_n^2 = \beta_n^2 - k^2. \quad (\text{A-15})$$

Thus if the longitudinal field components are known, the transverse field components can be found.

For many periodic structures, particularly those developed for microwave tube applications, there will be a region surrounding the symmetry axis which is empty and contains no conducting or dielectric material. In this region the field components have a particularly simple form. The differential equation for each of the axial space harmonic components of the longitudinal electric field in this region is

$$\nabla_T^2 E_{zn}(r, \theta) - \gamma_n^2 E_{zn}(r, \theta) = 0. \quad (\text{A-16})$$

Using polar coordinates and applying the orthogonality conditions for the angular space harmonics

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} E_{znm}(r) \right) - \left( \gamma_n^2 + \frac{m^2}{r^2} \right) E_{znm}(r) = 0, \quad (\text{A-17})$$

with a similar equation for  $H_{znm}(r)$ . The solutions to this differential equation are modified Bessel functions of order  $m$  and argument  $\gamma_n r$ . Since the symmetry axis,  $r = 0$ , is included in the region only the modified Bessel functions of the first kind,  $I_m(\gamma_n r)$ , are used. Employing the solution to Equation (A-17) and the relations between the transverse and longitudinal field components given by Equations (A-13) and (A-14), the complete fields in this region are:

$$E_z(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} A_{nm} I_m(\gamma_n r) e^{-jm\theta} e^{-j\beta_n z} \quad (\text{A-18})$$

$$H_z(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} B_{nm} I_m(\gamma_n r) e^{-jm\theta} e^{-j\beta_n z} \quad (\text{A-19})$$

$$E_r(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \left[ j \frac{\beta_n}{\gamma_n} A_{nm} I'_m(\gamma_n r) + \frac{k_0 Z_0}{2} \frac{m}{r} B_{nm} I_m(\gamma_n r) \right] e^{-jm\theta} e^{-j\beta_n z} \quad (\text{A-20})$$

$$H_r(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \left[ j \frac{\beta_n}{\gamma_n} B_{nm} I'_m(\gamma_n r) - \frac{k_0}{Z_0 \gamma_n} \frac{m}{r} A_{nm} I_m(\gamma_n r) \right] e^{-jm\theta} e^{-j\beta_n z} \quad (\text{A-21})$$

$$E_{\theta}(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \left[ \frac{\beta_n}{\gamma_n^2} \frac{m}{r} A_{nm} I_m(\gamma_n r) - j \frac{k_0 Z_0}{\gamma_n} B_{nm} I'_m(\gamma_n r) \right] e^{-jm\theta} e^{-j\beta_n z} \quad (\text{A-22})$$

$$H_{\theta}(r, \theta, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \left[ \frac{\beta_n}{\gamma_n^2} \frac{m}{r} B_{nm} I_m(\gamma_n r) + j \frac{k_0}{Z_0 \gamma_n} A_{nm} I'_m(\gamma_n r) \right] e^{-jm\theta} e^{-j\beta_n z} \quad (\text{A-23})$$

In these equations,  $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$ ,  $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ ,  $I'_m(\gamma_n r)$  is the derivative of  $I_m(\gamma_n r)$  taken with respect to the argument, and  $A_{nm}$ ,  $B_{nm}$  are constants.

For open boundary structures, if attention is restricted to the region exterior to the structure then the electromagnetic field components are given by expressions similar to those in Equations (A-18) through (A-23). The only difference is that  $I_m(\gamma_n r)$ , the modified Bessel function of the first kind is everywhere replaced by  $K_m(\gamma_n r)$ , the modified Bessel function of the second kind. For a real argument,  $K_m(\gamma_n r)$  decreases exponentially toward zero as  $r$  increases. For an imaginary argument, however,  $K_m(\gamma_n r)$  decreases only as  $r^{-1/2}$  for large  $r$ . Physically, an imaginary argument corresponds to electromagnetic power being radiated radially. Under these circumstances the structure is no longer propagating in the usual sense, and is operating in the leaky wave region. This occurs whenever  $\gamma_n$  is imaginary, and hence from equation (A-15), whenever  $k > \beta_n$ . Thus  $|k| = |\beta|$  forms the boundary between the leaky wave region and the propagation region.

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