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THE EFFECT OF BOUNDARIES ON THE SPATIAL CORRELATION OF NOISE FIELDS

SS 050 000-1562

25 March 1963

by

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and

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ABSTRACT

The effect of boundaries on the spatial correlation of noise fields is investigated. Two types of noise fields are considered; namely, a volume noise model and a surface noise model. The volume noise model consists of a field of plane waves, equally probable in all directions. This homogeneous-isotropic noise field is modified by the addition of a perfectly reflecting soft surface. The addition of the boundary results in a nonisotropic, nonhomogeneous wave field. Equations for the spatial correlation are obtained and curves are drawn showing the dependence of spatial correlation on the orientation of the point receivers and the distance from the soft surface. This analysis is repeated for a hard bottom.

The surface noise model consists of directional noise sources on the surface of the ocean. A hard bottom is considered, and it is assumed that the water is deep. Equations for the spatial correlation are obtained as a function of various parameters. Curves are drawn and the spatial correlation is shown to depend on the directionality of the noise sources, the orientation of the point receivers, and the distance of the receivers from the bottom.

It is shown that for the case of vertical receivers, if the center of the line joining the two receivers is at a distance of two wavelengths or further from the boundary, there is very little change in the spatial correlation as compared to the no boundary case. This is also shown for the case of horizontal receivers and a soft boundary. It is shown that for the case of surface noise, horizontal receivers, and a hard bottom, there is very little change in the spatial correlation as compared with the no boundary case if the receivers are a distance of five wavelengths or greater from the boundary.

ADMINISTRATIVE INFORMATION

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J. Warren Horton
Technical Director

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THE EFFECT OF BOUNDARIES ON THE SPATIAL CORRELATION OF NOISE FIELDS

INTRODUCTION

The spatial correlation of a noise field is defined as the normalized correlation of the pressure at two points separated by a distance $d$. In general, the spatial correlation is a function of the distance $d$ and of the orientation and position of the two points. The spatial correlation for the case of homogeneous-isotropic noise and for the case of directional surface noise without boundaries has been obtained previously. The purpose of this study is to investigate the effect of boundaries on spatial correlation.

The model on which this investigation is based was first suggested by Marsh and by Eckart. To the writers' knowledge, however, the computation of the spatial correlation has not been carried through to the present time.

VOLUME NOISE

In general,

$$<p^2> = <p_1^2> + <p_2^2> + 2 \sqrt{<p_1^2><p_2^2>} \rho_{12},$$  \hspace{1cm} (1)

where

- $<p_1^2>$ is the mean square pressure at one point,
- $<p_2^2>$ is the mean square pressure at a second point,
- $<p^2>$ is the mean square of the sum of the pressures at Points 1 and 2, and
- $\rho_{12}$ is the normalized spatial correlation of Points 1 and 2.

*A version of the material in this report was presented at the Sixty-Fourth Meeting of the Acoustical Society of America in November 1962.

1. H. W. Marsh, Jr., Correlation in Wave Fields, unclassified article in USL Quarterly Report (CONFIDENTIAL) for period ending 31 March 1950, pp. 63-68.


We shall now compute the mean square pressures, and from the values obtained and from Eq. (1), we shall derive the spatial correlation.

Let us consider a noise field composed of equal-amplitude plane waves which are equally probable in all directions. We consider a single-frequency component of the noise and assume that the phase is random and uniformly distributed among the plane waves. This results in a homogeneous-isotropic noise field.

The spatial correlation for this case is now well established as being \( \sin kd/kd \), where \( k \) is the wave number.

![Fig. 1 - Geometry for Volume Noise Model](image)

Let us now introduce a perfectly soft surface, such as an air-water interface at the plane \( z = 0 \) as shown in Fig. 1. The pressure due to one plane wave can be expressed as the superposition of the plane wave and its reflected wave. Using spherical coordinates (see Fig. 1), we have

\[
p(x, y, z, t) = P_0 \exp i[\omega t + kx \sin \theta \cos \phi + ky \sin \theta \sin \phi - kz \cos \theta] - P_0 \exp i[\omega t + kx \sin \theta \cos \phi + ky \sin \theta \sin \phi + kz \cos \theta].
\]

Let us consider a receiver located \( I \) units below the surface with coordinates \((0, 0, -I)\). If we square the pressure and average it over time, we obtain the mean square pressure

\[
\langle p^2 (0, 0, -I) \rangle = 2P_0^2 \sin^2 (kI \cos \theta).
\]
We consider that there are \( N \) plane waves per unit solid angle. Then the total mean square pressure at a point is

\[
<p_1^2> = \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} N <p^2(0, 0, -t)> \sin \theta \, d\theta \, d\phi = 2\pi N P_0^2 \left[ 1 - \frac{\sin 2k\ell}{2k\ell} \right]. \quad (4)
\]

This result shows that the noise field is not homogeneous since the mean square pressure at a point depends on the depth of that point. The noise field is also not isotropic; if the receiver responded only to waves in a solid angle \( d\Omega \) about its axis, its response would be proportional to \( \sin^2 (k\ell \cos \theta) \, d\Omega \) when it was pointed in the direction \( \theta \). Thus the response depends on \( \theta \).

Now we consider two receivers. Let the receivers be in a horizontal plane at the points \((d/2, 0, -t)\) and \((-d/2, 0, -t)\) so that their separation is \( d \) and their orientation in the horizontal plane is irrelevant. The sum of the pressures due to one plane wave is

\[
p_1 + p_2 = 4iP_0 \exp iwt \sin (k\ell \cos \theta) \cos \left( \frac{kd}{2} \sin \theta \cos \phi \right). \quad (5)
\]

We now take the real part of Eq. (5), square it, average over time, and obtain

\[
<(p_1 + p_2)^2> = 8P_0^2 \sin^2 (k\ell \cos \theta) \cos^2 \left( \frac{kd}{2} \sin \theta \cos \phi \right). \quad (6)
\]

Integrating Eq. (6)\(^5\) over the angles \( \theta \) and \( \phi \) gives

\[
<p^2> = 4\pi N P_0^2 \left[ 1 - \frac{\sin 2k\ell}{2k\ell} + \frac{\sin kd}{kd} - \frac{\sin \sqrt{(2k\ell)^2 + (kd)^2}}{\sqrt{(2k\ell)^2 + (kd)^2}} \right]. \quad (7)
\]

---

Combining Eqs. (1), (4), and (7) now gives for the spatial correlation function

\[ \rho (d, t) = \frac{\sin kd - \frac{\sin \sqrt{(2kft)^2 + (kd)^2}}{\sqrt{(2kft)^2 + (kd)^2}}}{\frac{\sin 2kft}{1 - \frac{\sin 2kft}{2kft}}} . \]  

(8)

Note that as \( I \) increases, Eq. (8) approaches the value for the homogeneous-isotropic noise field. If the plane \( z = 0 \) were perfectly hard, the second term of Eq. (2) would be positive instead of negative and the final result would be

\[ \rho (d, t) = \frac{\sin kd + \frac{\sin \sqrt{(2kft)^2 + (kd)^2}}{\sqrt{(2kft)^2 + (kd)^2}}}{\frac{\sin 2kft}{1 + \frac{\sin 2kft}{2kft}}} . \]  

(9)

As \( I \) increases, Eq. (9) also approaches the value for the homogeneous-isotropic noise field. Furthermore, when the receivers are on the bottom \( (t = 0) \), the above equation reduces to the same value. This is characteristic of horizontal receivers and a perfectly hard bottom and will be encountered again later.

We now consider the case of a soft surface and two receivers in a vertical plane. Let the coordinates of the receivers be \((0, 0, -\frac{d}{2})\) and \((0, 0, t + \frac{d}{2})\). In this case we have symmetry about the \( z \) axis. Using Eq. (2), the pressure at these points due to one incident plane wave at an angle of incidence \( \theta \) and the reflected plane wave is

\[ p_1\left(0, 0, -\frac{d}{2}, t\right) = 2iP_o \exp i\omega t \sin \left[ k \left( \frac{t + \frac{d}{2}}{2} \right) \cos \theta \right] , \]  

(10)

\[ p_2\left(0, 0, \frac{d}{2}, t\right) = 2iP_o \exp i\omega t \sin \left[ k \left( \frac{t - \frac{d}{2}}{2} \right) \cos \theta \right] . \]  

(11)

When we take the real part of each pressure, square it, average over time, and integrate over space, we obtain results having the same form as those derived from Eq. (4).

Let us now add the pressures:

\[ p_1 + p_2 = 2iP_o \exp i\omega t \left[ \sin \left\{ k \left( \frac{t + \frac{d}{2}}{2} \right) \cos \theta \right\} + \sin \left\{ k \left( \frac{t - \frac{d}{2}}{2} \right) \cos \theta \right\} \right] . \]  

(12)
Taking the real part of Eq. (12), squaring it, and averaging over time, we obtain
\[ <(P_1 + P_2)^2> = 2P_o^2 \left[ \sin \left\{ \frac{1}{2} \left(t - \frac{d}{2}\right) \cos \theta \right\} + \sin \left\{ \frac{1}{2} \left(t + \frac{d}{2}\right) \cos \theta \right\} \right]^2. \] (13)

We now integrate this over all the contributing space and obtain
\[ <P^2> = 4\pi NP_o^2 \left( 1 - \frac{\sin 2k \left( t - \frac{d}{2} \right)}{2k \left( t - \frac{d}{2} \right)} \right) - \frac{\sin 2k \left( t + \frac{d}{2} \right)}{2k \left( t + \frac{d}{2} \right)} - \frac{\sin 2k}{2k^2} + \frac{\sin kd}{kd}. \] (14)

Substituting into Eq. (1), we obtain
\[ \rho(d, \ell) = \frac{\sin kd}{kd} - \frac{\sin 2k}{2k} \begin{bmatrix} 1 & \sin 2k \left( \ell - \frac{d}{2} \right) & \sin 2k \left( \ell + \frac{d}{2} \right) \\ \frac{2k}{2k} \left( t - \frac{d}{2} \right) & 1 & \frac{2k}{2k} \left( t + \frac{d}{2} \right) \end{bmatrix}^{\frac{1}{2}}. \] (15)

The case of a hard surface and vertical receivers is treated in the same manner, and the result is:
\[ \rho(d, \ell) = \frac{\sin kd}{kd} + \frac{\sin 2k}{2k} \begin{bmatrix} 1 & \sin 2k \left( \ell - \frac{d}{2} \right) & \sin 2k \left( \ell + \frac{d}{2} \right) \\ \frac{2k}{2k} \left( t - \frac{d}{2} \right) & 1 & \frac{2k}{2k} \left( t + \frac{d}{2} \right) \end{bmatrix}^{\frac{1}{2}}. \] (16)
SURFACE NOISE

We now treat the case where the noise originates at the surface of the water and the receivers are located near the bottom. We assume that the water is very deep as compared with the separation of the receivers and that the bottom is perfectly hard. We consider the noise sources on the surface to have equal amplitudes and random, uniformly distributed phases. In addition, we consider that these noise sources radiate with a directionality of amplitude given by \( \cos^m \theta \).

Referring to Fig. 2, we have for the mean square pressure at the point \((0, 0, t)\) due to one plane wave coming from the direction \(\theta\) and its reflected wave

\[
\langle p^2(0, 0, t) \rangle = 2A^2 \frac{\cos^2 \theta}{r^2} \cos^2 (k\ell \cos \theta),
\]

where \(A\) is an amplitude constant. When Eq. (17) is integrated over the entire infinite surface of noise sources (see Fig. 2), we obtain

\[
\langle p^1_r \rangle = 4\pi A^2 \int_0^{\pi/2} \cos^2 \theta \cos^{2m-1} \theta \sin \theta d\theta
\]

for the total mean square pressure at one receiver at distance \(t\) above the bottom.

Now we consider two receivers in a horizontal plane at the points

\[
\left( \frac{d}{2}, 0, t \right) \quad \text{and} \quad \left( -\frac{d}{2}, 0, t \right).
\]
The mean square value of the sum of the pressures at the two receivers due to one plane wave is

\[ \langle (p_1 + p_2)^2 \rangle = 8A^2 \cos^2 \theta \frac{\sin \theta \cos \phi}{r^2} \cos^2 (kt \cos \theta) \cos^2 \left( \frac{kd}{2} \sin \theta \cos \phi \right), \]  

(19)

and integrating Eq. (19) over the surface of noise sources gives

\[ \langle p^2 \rangle = 2 \langle p_1^2 \rangle + 8\pi A^2 \int_0^{\pi/2} \cos^3 (kt \cos \theta) J_0 (kd \sin \theta) \cos^{2m-1} \theta \sin \theta d\theta. \]  

(20)

When the results in Eqs. (18) and (20) are used in Eq. (1), we obtain for the spatial correlation function

\[ \rho (d, t) = \frac{\int_0^{\pi/2} J_0 (kd \sin \theta) \cos^3 (kt \cos \theta) \cos^{2m-1} \theta \sin \theta d\theta}{\int_0^{\pi/2} \cos^2 (kt \cos \theta) \cos^{2m-1} \theta \sin \theta d\theta}. \]  

(21)

Note that when the receivers are on the bottom \( t = 0 \), Eq. (21) reduces to the value obtained previously where the bottom was not considered at all.\(^6\) As \( t \) increases, it can be seen that Eq. (21) reduces also to the no-bottom case.

Next, we consider two receivers in a vertical plane located at the points \( (0, 0, t + d/2) \) and \( (0, 0, t - d/2) \). The mean square value of the sum of the pressures due to one plane wave is

\[ \langle (p_1 + p_2)^2 \rangle = 8A^2 \cos^2 \theta \frac{\sin \theta \cos \phi}{r^2} \left[ \cos \left( k \left( t - \frac{d}{2} \right) \cos \theta \right) + \cos \left( k \left( t + \frac{d}{2} \right) \cos \theta \right) \right]^2 \]  

(22)

When Eq. (22) is integrated over the surface of noise sources and when Eqs. (1) and (18) are used, we obtain

\[ \rho (d, t) = \frac{\int_0^{\pi/2} \cos \left( k \left( t - \frac{d}{2} \right) \cos \theta \right) \cos \left( k \left( t + \frac{d}{2} \right) \cos \theta \right) \cos^2 \theta \cos \left( k \left( t + \frac{d}{2} \right) \cos \theta \right) \sin \theta d\theta}{\int_0^{\pi/2} \cos^2 \left( k \left( t - \frac{d}{2} \right) \cos \theta \right) \cos \left( k \left( t + \frac{d}{2} \right) \cos \theta \right) \sin \theta d\theta}. \]  

(23)

For specified integer values of \( m \) the above integrals can be readily evaluated.

---

Fig. 3 - Effect of Soft Surface for Volume Noise with Horizontal Receivers Close to Boundary

Fig. 4 - Effect of Soft Surface for Volume Noise with Horizontal Receivers Far from Boundary

Fig. 5 - Effect of Hard Bottom for Volume Noise with Horizontal Receivers Close to Boundary

Fig. 6 - Effect of Hard Bottom for Volume Noise with Horizontal Receivers Far from Boundary
NUMERICAL RESULTS

Figures 3 through 13 show the numerical results calculated from the equations which have been derived. In all these figures, \( \lambda \) is the wavelength. Figures 3 and 4 were calculated from Eq. (8) and show the effect of a soft surface near horizontal receivers for volume noise. When the distance from the receivers to the surface is less than a wavelength, there is an appreciable effect, but when this distance exceeds one or two wavelengths, the effect becomes very small. Figures 5 and 6, calculated from Eq. (9), show that for volume noise the effect of a hard surface near horizontal receivers is very different from the effect of a soft surface. However, when the distance from the surface exceeds one or two wavelengths, the effect is again very small.

Figures 7, calculated from Eq. (15), shows the effect of a soft surface for volume noise when the receivers are vertical, and Fig. 8, calculated from Eq. (16), shows the effect of a hard surface under the same conditions. In both cases there is no appreciable effect until the receiver nearest the surface is within a few tenths of a wavelength from the surface.

Figures 9, 10, 11, and 12, calculated from Eq. (21) by numerical integration, show the effect of a hard bottom near horizontal receivers for directional surface noise with two different degrees of amplitude directionality corresponding to \( \cos^2 \theta \) and \( \cos^3 \theta \). Experimental work\(^8\) has not yet shown clearly what to expect for surface-noise directionality but suggests that it is somewhat similar to the assumptions of \( m = 2 \) and 3. It is clear from the

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Fig. 9 - Effect of Hard Bottom for Surface Noise (m = 2) with Horizontal Receivers Close to Boundary

Fig. 10 - Effect of Hard Bottom for Surface Noise (m = 3) with Horizontal Receivers Close to Boundary

Fig. 11 - Effect of Hard Bottom for Surface Noise (m = 2) with Horizontal Receivers Far from Boundary

Fig. 12 - Effect of Hard Bottom for Surface Noise (m = 3) with Horizontal Receivers Far from Boundary
results shown in Figs. 9 and 10 that the effect of the bottom on the spatial correlation is very large when the receivers are within a wavelength of the bottom. However, for \( \frac{t}{A} \geq 5 \), as shown in Figs. 11 and 12, the effects of the bottom are small.

![Diagram](image)

**Fig. 13 - Effect of Hard Bottom for Surface Noise with Vertical Receivers Far from Boundary**

Figure 13, calculated from Eq. (23) shows the effect of a hard bottom near vertical receivers for a surface noise with amplitude directionality of \( \cos^3 \theta \). For \( \frac{t}{A} = 1 \) and 2, there is very little difference between these curves and the no-bottom case.

**CONCLUSION**

A theoretical model has been constructed and analyzed for the effect of boundaries on the spatial correlation. It has been shown that the boundaries do affect the spatial correlation if the center of the line joining the two receivers is within two wavelengths of the boundary. It has also been shown that for all the cases of volume noise and for the case of surface noise and vertical receivers, if the center of the line joining the two receivers is at a distance of two wavelengths or longer from the boundary, there is very little change in the spatial correlation as compared with the no-boundary case. For the case of surface noise, horizontal receivers and a hard bottom, there is very little change in the spatial correlation as compared with the no-boundary case, if the receivers are at a distance of five wavelengths or greater from the boundary.
Navy Underwater Sound Laboratory
Report No. 570
i-ii + 11 p., figs.
UNCLASSIFIED

The effect of boundaries on the spatial correlation of noise fields is investigated. Two types of noise fields are considered; namely, a volume noise model and a surface noise model. The volume noise model consists of a field of plane waves, equally probable in all directions. This homogeneous-isotropic noise field is modified by the addition of a perfectly reflecting soft surface. The addition of the boundary results in a nonisotropic, nonhomogeneous wave field. Equations for the spatial correlation are obtained and curves are drawn showing the dependence of spatial correlation on the orientation of the point receivers and the distance from the soft surface. This analysis is repeated for a hard bottom.

The surface noise model consists of directional noise sources on the surface of the ocean. A hard bottom is considered, and it is assumed that the water is deep. Equations for the spatial correlation are obtained as a function of various parameters. Curves are drawn and the spatial correlation is shown to depend on the directionality of the noise sources, the orientation of the point receivers, and the distance of the receivers from the bottom.

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It is shown that for the case of vertical receivers, if the center of the line joining the two receivers is at a distance of two wavelengths or further from the boundary, there is very little change in the spatial correlation as compared to the no boundary case. This is also shown for the case of horizontal receivers and a soft boundary. It is shown that for the case of surface noise, horizontal receivers, and a hard bottom, there is very little change in the spatial correlation as compared with the no boundary case if the receivers are at a distance of five wavelengths or greater from the boundary.
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