OPTICAL GENERATOR PROGRAM

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Contract Number AF19(604)-8065
Project Number 7600
Task Number 760003

FINAL REPORT
February 1963

Prepared for
GEOPHYSICAL RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS
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INTRODUCTION

The program described herein was devised primarily to provide acquisition data to a ground-based instrumentation net consisting mainly of various types of cameras. The cameras, in this case, are employed to photograph a flashing strobe light that is mounted on each pole of the ANNA geodetic satellite that is magnetically stabilized along its spin axis.

Provisions have been made to compute a satellite ephemeris by a differential correction procedure as the satellite is stepped around the orbit in some desired increment of time.

It is assumed that the density distribution of the Earth is axially symmetric and that the force field is represented by the principle term and the zonal terms 2 through 4. Provision is also made for accommodating the parameters of a model atmosphere. At each step computations are made to determine:

1) If a given observing site is in darkness (elevation angle to the Sun less than some desired $\epsilon$).
2) If the elevation angle from the observing site to the satellite is positive or greater than some desired $\epsilon'$. 
3) The components of the magnetic field (North, East, vertical, horizontal and total field) and the dip and declination.
4) Which strobe, if either, is visible to an observing site.
5) If the recorded image size on the photographic plate will be larger than some $\epsilon''$.

If the above conditions are satisfied, the program continues and computes the following additional information for each observing site:

1) Time of the observation.
2) The azimuth and range and the topocentric hour angle and declination to the satellite.
3) The latitude and longitude of the sub-satellite point.
4) The angle between the observer-satellite vector and the observer-moon vector.

5) If the satellite is illuminated by the Sun.

6) The angle between the observer-satellite vector and the center of the light cone.

Other features of the program are:

1) The selection of the most valuable observations for a station to make from series of possible observations. This is achieved by considering the azimuths at which the station has previously recorded data. On this basis, a final selection of flash times and the associated "look angles" from each site is made.

2) The preparation of teletype messages (see Appendix 1).

3) The capability to compute acquisition data for observers not concerned with the light (range or range-rate stations).

4) A station designated as a "share station" will not have flashes scheduled specifically for it. However, acquisition data will be computed for any scheduled flashes that may be observed by the "share station."
The standard elements (Figure 1)

- a = semi-major axis
- e = eccentricity
- i = inclination
- Ω = right ascension of the ascending node
- ω = argument of perigee
- M = mean anomaly
- v = true anomaly
- u = ω + v, the argument of latitude

are used in the computation of the ephemeris. Time is considered the independent variable and, as the satellite is stepped around the orbit, computations are made to determine the perturbative effects of the Earth's oblateness and drag on the satellite's position.
The radial, transverse, and normal perturbative accelerations (see Figure 2) due to oblateness are:

\[
\begin{align*}
\dot{r}_B & = - \frac{\mu}{r^2} \left\{ \frac{J}{r^2} \left[ 1 - \frac{3}{2} \sin^2 i (1 - \cos 2u) \right] \\
& + \frac{4H}{5r^3} \sin i \left[ 3(1 - \frac{5}{4} \sin^2 i) \sin u + \frac{5}{4} \sin^2 i \sin 3u \right] \\
& + \frac{K}{r^4} \left[ \frac{1}{2} (1 - 5 \sin^2 i + \frac{35}{8} \sin^4 i) + \frac{5}{2} \cos 2u \sin^2 i (1 - \frac{7}{6} \sin^2 i) \\
& + \frac{35}{48} \cos 4u \sin^4 i \right] \right\} \\
\dot{r}_B & = \frac{\mu}{r^2} \left\{ - \frac{J}{r^2} \sin^2 i \sin 2u + \frac{3H}{5r^3} \sin i \left[ (1 - \frac{5}{4} \sin^2 i) \cos u + \frac{5}{4} \sin^2 i \cos 3u \right] \\
& - \frac{K}{r^4} \sin^2 i \left[ \sin^2 i (1 - \frac{7}{6} \sin^2 i) + \frac{7}{12} \sin^2 i \sin 4u \right] \right\} \\
\dot{r}_B & = \frac{\mu}{r^2} \left\{ - \frac{2J}{r^2} \sin i \cos i \sin u + \frac{3H}{5r^3} \cos i \left[ 1 - \frac{5}{2} \sin^2 i (1 - \cos 2u) \right] \\
& - \frac{K}{r^4} \sin i \cos i \left[ 2 \sin u (1 - \frac{7}{4} \sin^2 i) + \frac{7}{6} \sin^2 i \sin 3u \right] \right\}
\end{align*}
\]
The radial, transverse and normal perturbative accelerations due to drag are:

\[ \mathbf{a}_D^r = \frac{A}{m} \rho a e V \sin E \mathbf{E} \]

where:

\[ E = \frac{n}{1 - e \cos E} \]

\( \rho \) = air density

\[ r^v_e = -\frac{A}{m} \rho a (1 - e^2)^{\frac{1}{2}} V \left[ 1 - d (1 - e \cos E)^2 / (1 - e^2) \right] \mathbf{E} \]

\[ r^v_b = -\frac{A}{m} \rho a \omega \left( \frac{1 - e \cos E}{n} \right)^2 \]

where:

\[ V = \left( \frac{\mu}{a} \right)^{\frac{1}{2}} \left( \frac{1 + e \cos E}{1 - e \cos E} \right)^{\frac{1}{2}} \left[ 1 - d \left( \frac{1 - e \cos E}{1 + e \cos E} \right) \right] \]

\[ d = \left( \frac{\omega_s}{n} \right) (1 - e^2)^{\frac{1}{2}} \cos i \]

\( \omega_s \) = rotational rate of the Earth

Expressions for the perturbative effect on the elements:

\[ a^p = -\frac{2}{3} a^p \frac{n^p}{n} \]

\[ \frac{n^p}{n} = -\frac{3}{1 - e^2} \left[ \frac{r^p}{\sqrt{\mu p}} \sin \nu + \frac{r^v}{\sqrt{\mu p}} \frac{p^2}{r} \right] \]

where

\[ p = a (1 - e^2) \]

\[ e^p = \frac{r^p}{\sqrt{\mu p}} \left( \frac{p}{r} \sin \nu \right) + \frac{r^v}{\sqrt{\mu p}} \left[ \left( \frac{p}{r} + 1 \right) \cos \nu + e \right] \]

\[ i^p = \frac{r^b^v}{\sqrt{\mu p}} \cos u \]
\[ \Omega' = \frac{r^2 b'}{\sqrt{\mu \, p}} \sin \frac{u}{\sin i} \]

\[ \omega' = u' - \nu' \]

\[ u' = -\Omega' \cos i \]

\[ \nu' = \frac{1}{e} \left[ \frac{r^2}{\sqrt{\mu \, p}} \left( \frac{p}{r} \cos \nu \right) - \frac{r^2 v'}{\sqrt{\mu \, p}} \left( \frac{p}{r} + 1 \right) \sin \nu \right] \]

\[ E' = \left( 1 - e^2 \right)^{1/2} (u' - \omega') - \frac{a e}{2a} \sin E - \frac{r' r v'}{\sqrt{\mu \, a}} \]

Perigee height, \( h_{\pi} = a(1 - e) - Re_{\pi} \)

Radial distance, \( r = a(1 - e \cos E) \)

The difference between the height of the satellite above the Earth (\( h'' \)) at any time and the height of the satellite above the Earth at perigee (\( h_{\pi} \))

\[ h'' - h_{\pi} = (Re_{\pi} - Re_{h''}) + ae \left[ 1 - \cos E \right] \]

The air density, \( \rho = \rho_{\pi} e^{-Kd_{h}} \)

where

\[ K = \frac{1}{h} = \frac{1}{\text{scale height}} \]

\( \rho_{\pi} = \text{air density at perigee} \)

The total perturbative effect of bulge and drag on the elements at any time

\[ a = a' + \left( a_D + a_B \right) dt \]
\[ e = e' + \left( e_D + e_B \right) dt \]
\[ i = i' + \left( i_D + i_B \right) dt \]
\[ \Omega = \Omega' + \left( \Omega_D + \Omega_B \right) dt \]
\[ \omega = \omega' + \left( \omega_D + \omega_B \right) dt \]
\[ E = E' + (E + E_D + E_B) \text{d}t \]
\[ u = u' + (u + u_D + u_B) \text{d}t \]

where:

\[ \dot{E} = \frac{n}{1 - e \cos E} \]
\[ \dot{u} = \frac{\sqrt{1 - e^2}}{1 - e \cos E} \dot{E} \]

Dots denote the two body changes.
Primes denote the value at the previous step.
ACQUISITION DATA

At each step values of \( a, e, i, \Omega, \mu \) and \( \nu \) are computed after which the program enters the "acquisition data" phase. The computations for this phase break into two groups: those which are made only once and those which are made once for each observation station.

When a coordinate system is not specified in what follows, an "Earth-fixed" system is to be assumed, i.e., \( \mathbf{i} \) and \( \mathbf{j} \) in the equatorial plane with \( \mathbf{i} \) pointed toward Greenwich, \( \mathbf{k} = \mathbf{i} \times \mathbf{j} \).

**Group 1: Computations made only once**

1) Compute \( \lambda_n \), west longitude of node.
   
a) \( \theta_G = \) right ascension of Greenwich.
      \[ \theta_G = \theta_{G_0} + (t - t_0) \dot{\theta}_1 + (t - t_0) \frac{\theta_2}{\text{days}} + \frac{\theta_2}{\text{frac}} \]
      \[ \lambda_n = \theta_G - \Omega \]
      \[ -\pi < \lambda_n \leq \pi \]

2) Compute \( \lambda_s \), west longitude of sub-satellite point.

   \[ \Delta \lambda = \tan^{-1} \left( \frac{\cos \phi}{\cot u} \right) \]
   \[ \lambda_s = \lambda_n - \Delta \lambda \]
   \[ -\pi < \lambda_s \leq \pi \]

3) Compute \( \phi_s \), the geodetic latitude of sub-satellite point.
\[ \delta = \text{declination of satellite} \]

\[ \tan \delta = \frac{\sin i \sin u}{\sqrt{1 - \sin^2 i \sin^2 u}} \]

\[ \tan \phi_\circ = \frac{1}{1 - e^2} \tan \delta; \quad e^2, \text{ spheroid eccentricity} = 0.0067665579 \]

\[ \phi_\circ = \tan^{-1} \left\{ \frac{1}{\left( 1 - e^2 \right)} \tan \delta \right\} \]

4) Compute \( r \), distance from center of Earth to satellite.

\[ r = a(1 - e \cos E) \]

**Group 2: Computations made for each station at each time step**

5) Elevation angle to the Sun

\[ \text{Elev} = \sin^{-1} \left[ \sin \phi \sin \delta_\circ + \cos \phi \cos \delta_\circ \cos \beta \right] \]

where \( \phi = \text{observers latitude} \)

\( \delta_\circ = \text{declination of the Sun} \)

\( \beta = \text{LST} - \alpha_\circ \)

\( \text{LST} = \text{local sidereal time} \)

\( \alpha_\circ = \text{right ascension of the Sun} \)

where

a) \( \alpha_\circ = \ell_\circ - C_{16} \sin 2\ell_\circ ; \quad \delta_\circ = C_{17} \sin \alpha_\circ \)

b) \( L_\circ = \text{longitude of Sun at } t_\circ \) (Jan 0.0 of year of interest)

c) \( \ell_\circ = L_\circ + C_2 (t - t_\circ) + C_{15} \sin (C_2 [t - t_\circ] - C_{14}) \)

\( \phi_\circ = \tan^{-1} \left\{ \frac{1}{\left( 1 - e^2 \right)} \delta_\circ \right\} ; \quad \lambda_\circ = \theta_G - \alpha_\circ \)

6) If the satellite is illuminated by the Sun, the angle \( I \) must be positive (Figure 4a).
I = 180 - d - A

\[ A = \sin^{-1}\left(\frac{Re}{r}\right) \]

\[ d = \cos^{-1}\left(\sin \phi \sin \phi_c + \cos \phi \cos \phi_c \cos \beta\right) \]

where: \( \cdot \) denotes satellite

\[ \beta = |\text{RA} - \text{RA}_0| \]

7) Magnetic dip and declination are computed by evaluating the gradient of the magnetic potential at altitude (details in Appendix II) to obtain the x, y and z components of the magnetic field. These quantities permit the computation of (Figure 5).

a) Horizontal component of Earth's magnetic field (H)
b) Total field vector (F)
c) Vertical component of Earth's magnetic field (V)
d) Horizontal component of Earth's magnetic field (H)
e) Total field vector (F)

Magnetic dip, \( a_m = \cos^{-1} \left[ \frac{H}{F} \right] \)

Magnetic declination, \( \delta_m = \cos^{-1} \left[ \frac{Z}{H} \right] \), plus Easterly

8) The light angle, \( \Theta \), (Figure 6) is the angle between the station-satellite vector and a unit vector in the direction of the center of the light cone. Orientation of \( \hat{S} \), a unit vector in direction of the North axis of the light cone as a function of its latitude (\( \phi \)), longitude (\( \lambda \)), magnitude dip (\( a_m \)) and magnetic declination (\( \delta_m \)), is

\[ \hat{S} = i \left\{ \cos a_m \sin \delta_m \sin \lambda - \cos \lambda \left[ \cos \delta_m \cos a_m \sin \phi + \sin a_m \cos \phi \right] \right\} \]

\[ + i \left\{ \cos a_m \sin \delta_m \cos \lambda + \sin \lambda \left[ \cos \delta_m \cos a_m \sin \phi + \sin a_m \cos \phi \right] \right\} \]

\[ + k \left\{ \cos \delta_m \cos a_m - \sin a_m \sin \phi \right\} \]

\[ \Theta = \cos^{-1} \left\{ \frac{\text{ST} \cdot \hat{S}}{\text{ST}} \right\} \]
A vector from the center of the Earth to the satellite, $\overline{OS}$, is

$$\overline{OS} = r \left[ (\cos \phi_s \cos \lambda_E) \hat{i} + (\cos \phi_s \sin \lambda_E) \hat{j} + \sin \phi_s \hat{k} \right]$$

$$\overline{OT} = (R_e + h) \left[ (\cos \phi_T \cos \lambda_E) \hat{i} + (\cos \phi_T \sin \lambda_E) \hat{j} + \sin \phi_T \hat{k} \right]$$

$$\overline{OS} - \overline{OT} = \overline{TS}$$

8) Elevation angle to the satellite
Define a unit vector \( \mathbf{Z} \) normal to the spheroid at the observer (Figure 7)

\[
\mathbf{Z} = \cos \phi \cos \lambda \mathbf{i} + \cos \phi \sin \lambda \mathbf{j} + \sin \phi \mathbf{k}
\]

A vector \( \mathbf{N} \), in the meridian and pointing North

\[
\mathbf{N} = -\sin \phi \cos \lambda \mathbf{i} + \sin \phi \sin \lambda \mathbf{j} + \cos \phi \mathbf{k}
\]

\( \mathbf{E} = \mathbf{N} \times \mathbf{Z} \)

\( \mathbf{u} \) is a unit vector from the station to the satellite

\[
\mathbf{u} = \frac{\mathbf{TS}}{|\mathbf{TS}|} \quad \text{(see Figure 6)}
\]

Elev Angle = \( \sin^{-1} (\mathbf{Z} \cdot \mathbf{u}) \)

10) Azimuth from the station to the satellite

\[
\text{Azimuth} = \cos^{-1} \left\{ \frac{\mathbf{Z} \times \mathbf{u}}{|\mathbf{Z} \times \mathbf{u}|} \cdot (-\mathbf{E}) \right\}
\]

11) Topocentric hour angle and declination.

\( \mathbf{L} \), a unit vector from the station to the satellite in inertial coordinates is

\[
\mathbf{L} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}
\]

Declination = \( \sin^{-1}(z) \)

Right ascension = \( \tan^{-1}\left(\frac{y}{x}\right) \)

Hour angle = LST - RA

Some camera stations require the hour angle and declination of each flash. Since the ephemeris is stepped in time by an amount not equal to the time interval between flashes, but by a multiple of the interval between flash sequences, the time derivatives of \( x \), \( y \), and \( z \) are computed and multiplied by the time between flash intervals to obtain the new coordinates of each flash. With these new coordinates, the right ascension and declination are then computed as noted above.
The time derivatives are
\[ \dot{x} = r \left[ \begin{array}{c} \frac{1}{2} \cos \lambda + \\
(1 - \sin^2 i \sin^2 u)^{1/2} \cos \lambda \\
(1 - \sin^2 i \sin^2 u)^{1/2} \sin \lambda \\
(1 - \sin^2 i \sin^2 u)^{1/2} \cos \lambda \\
(1 - \sin^2 i \sin^2 u)^{1/2} \sin \lambda \end{array} \right] + \\
\left[ \begin{array}{c} \dot{r} [-\sin i \cos \lambda \sin^2 u + \sin^2 i \sin u \cos u \sin \lambda]
/(1 - \sin^2 i \sin^2 u)^{1/2} \cos \lambda \\
(1 - \sin^2 i \sin^2 u)^{1/2} \sin \lambda 
/(\cos^2 u \cos^2 i \sin^2 u) (\cos i \sin \lambda + \sin i \sin u \cos u \sin \lambda)
+ \Omega - \omega_s \end{array} \right] + \\
\dot{y} = -r \left[ \begin{array}{c} \frac{1}{2} \sin \lambda \\
-\sin i \cos \lambda \sin^2 u + \sin^2 i \sin u \cos u \sin \lambda \\
(1 - \sin^2 i \sin^2 u)^{1/2} \cos \lambda \\
(1 - \sin^2 i \sin^2 u)^{1/2} \sin \lambda \end{array} \right] + \\
\left[ \begin{array}{c} \dot{r} [-\sin i \cos \lambda \sin^2 u + \sin^2 i \sin u \cos u \sin \lambda]
/(1 - \sin^2 i \sin^2 u)^{1/2} \sin \lambda \\
(1 - \sin^2 i \sin^2 u)^{1/2} \cos \lambda 
/(\cos^2 u \cos^2 i \sin^2 u) (\cos i \sin \lambda - \sin i \sin u \cos u \sin \lambda) + \\
\Omega - \omega_s \end{array} \right] + \\
\dot{z} = \dot{r} \sin i \sin u + \dot{r} \cos i \sin u \cos u + r \sin i \cos u \sin u \cos u \sin \lambda \]

\[ \text{12) Image size of the flash} \]

Image diameter (in microns) = \( a_1 q + a_2 q^2 + a_3 q^3 \)

where \( a_1, a_2, a_3 \) are functions of the plate emulsion. For 103F emulsion developed 8 minutes \( a_1 = 7.468, a_2 = -0.112237, a_3 = .0008352 \)

\[ q = \left( \frac{D}{s} \right) \left( T \: B \right)^{1/2} \: e^{-0.46 \: \Delta m / \sin h} \]

\( D = \) aperture of camera

\( s = \) distance from satellite to observer = \( |TS| \)

\( T = \) transmission of lens

\( B = 9000 \) if \( \theta \leq 45^\circ \)

\( 150(105 - \theta_1) \) if \( \theta > 45^\circ \)

\( \Delta m = \) atmospheric extinction at the zenith for the station; when an actual measurable value is unavailable, \( \Delta m = 1.25 \) (the factor for moderate haze).

\( h = \) elevation angle
13) Right ascension, $\alpha_m$, and declination, $\delta_m$, of the moon are obtained from a lunar ephemeris listed in memory. $\alpha_m$ and $\delta_m$ are given for 0 hours U.T of each day of the year. A six point interpolation yields the desired quantities at any time, $t$. For purposes of this program, the vector from the center of the Earth to the Moon is taken as being the same as the vector from the observer to the Moon. The angle between the station-satellite vector and the station-moon vector, $\mathbf{TM}$, is:

$$\text{Moon Angle} = \cos^{-1}\left(\mathbf{TS} \cdot \mathbf{TM}\right)$$

Moon Phase, $P_m$, is determined from

$$P_m = R.A. - R.A.\oplus$$

if $P_m = 0^\circ \pm 45^\circ$, Phase new

$$P_m = 180^\circ \pm 45^\circ$$, Phase full

Otherwise, Phase quarter

14) Observation Selection. From a series of possible observations, one observation (the least valuable one) is discarded. If a station has made $n$ good observations at azimuths $A_1, A_2, A_3, \ldots A_n$ and it is now possible to make an observation at azimuth $B$, a "value" is assigned to azimuth $B$ in the following fashion:

$$A = \frac{1}{n} \sum_{i=1}^{n} A_i$$

and

$$p = \frac{1}{n} \sum_{i=1}^{n} (A_i - \bar{A})^2$$
take as the "value" of the observation

\[ V(B) = e^{-p} \sin^2 \left( \frac{\bar{A} - B}{2} \right) \]

Note that \( 0 \leq V(B) \leq 1 \), that \( V(B) \) is zero if \( \bar{A} \approx B \), and that it also is small if \( p \) is large (that is, there is a large "scatter" in the azimuths already observed). The values \( n, p, \) and \( \bar{A} \) are "updated" as additional good observations are made.
INPUT AND OUTPUT FORMATS

A. For Determining Acquisition Data and Selecting Observations

The OGP program uses logical tapes 8 and 11 for input. Logical tapes 9, 10, 12 and 14 are used as intermediate binary tapes.

The acquisition data and selected observations are available as printed output on logical tape 5 with data select zero. With data select 4 the teletype messages to APL, NASA and the 1381st Geodetic Survey Squadron are punched from tape 5. (Teletype messages are also available as printed output on logical tape 6 with data select zero). Control cards necessary for this computation are input cards 1 through 6 (Table I) and four station cards for each observing site involved (Table II). Note that in the preparation of station cards (Table II) a station may be designated as a "Share Station."

B. For Ephemeris Computation Only

If so desired, the ephemeris portion only is available in the form of latitude, longitude and time. In this instance the first six control cards must be followed by four blank cards.
<table>
<thead>
<tr>
<th>Card No.</th>
<th>Card Column</th>
<th>Format</th>
<th>Contents</th>
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<tr>
<td>1</td>
<td>1-6</td>
<td>12A6</td>
<td>Year, month, day of this run</td>
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<tr>
<td></td>
<td>7-72</td>
<td></td>
<td>Job heading</td>
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<tr>
<td>2</td>
<td>1-12</td>
<td>F12.9</td>
<td>EPOCHT (days and decimals of day)</td>
</tr>
<tr>
<td></td>
<td>13-16</td>
<td>I4</td>
<td>IYEAR (year of epoch)</td>
</tr>
<tr>
<td></td>
<td>17-22</td>
<td>F6.3</td>
<td>CONANG (1/2 light angle in deg.)</td>
</tr>
<tr>
<td></td>
<td>23-28</td>
<td>F6.3</td>
<td>SUNTST (Test for sun elev. in deg.)</td>
</tr>
<tr>
<td></td>
<td>35-40</td>
<td>F6.3</td>
<td>DELTAE (ΔE in deg.)</td>
</tr>
<tr>
<td></td>
<td>42-56</td>
<td>F15.15</td>
<td>QJ2 (J₂)</td>
</tr>
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<td>57-71</td>
<td>F15.15</td>
<td>QJ3 (J₃)</td>
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<td>3</td>
<td>1-10</td>
<td>F10.6</td>
<td>WO (ω₀ in deg.)</td>
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<td>12-21</td>
<td>F10.6</td>
<td>RAO (Ω₀ in deg.)</td>
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<td>23-32</td>
<td>F10.5</td>
<td>AXIS (semi-major axis in nautical miles)</td>
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<td>34-43</td>
<td>F10.9</td>
<td>ECCNO (eccentricity)</td>
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<td>45-54</td>
<td>F10.8</td>
<td>XINC (inclination in degrees)</td>
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<td>56-61</td>
<td>F6.3</td>
<td>QMASS (mass of satellite in kilograms)</td>
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<td></td>
<td>63-72</td>
<td>F10.4</td>
<td>AREA (cross sectional area of satellite in square centimeters)</td>
</tr>
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<td>4</td>
<td>1-5</td>
<td>I5</td>
<td>NSTA (the number of stations to be considered)</td>
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<td>5</td>
<td>1-10</td>
<td>I10</td>
<td>IRM (the maximum number of flash sequences to be allowed per revolution)</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>I10</td>
<td>ILM (the maximum number of flash sequences to be allowed per load)</td>
</tr>
<tr>
<td></td>
<td>23-34</td>
<td>F12.9</td>
<td>GDT (integer by which the interval between clock pulses is multiplied to determine time interval at which acquisition date is computed; the elements are integrated at one half this interval)</td>
</tr>
<tr>
<td>Column</td>
<td>Decade</td>
<td>Field</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td>40-49</td>
<td>I10</td>
<td>INF</td>
<td>Initial number of flash sequences executed to date</td>
</tr>
<tr>
<td>50-59</td>
<td>I10</td>
<td>INFL</td>
<td>Initial number of injections executed to date</td>
</tr>
<tr>
<td>60-71</td>
<td>F12.9</td>
<td>TIMP</td>
<td>Epoch of an even flash time</td>
</tr>
<tr>
<td>72</td>
<td>I1</td>
<td></td>
<td>Blank if refined projections - &quot;one&quot; if long range (an identifier for message to NASA)</td>
</tr>
<tr>
<td>1-12</td>
<td>F12.9</td>
<td>DDTT</td>
<td>Time in seconds between flashes in a sequence</td>
</tr>
<tr>
<td>13-24</td>
<td>F12.9</td>
<td>TSTOP</td>
<td>Stop time in days</td>
</tr>
<tr>
<td>25-28</td>
<td>F4.1</td>
<td></td>
<td>Minimum angle between station-satellite vector and station new moon vector in degrees</td>
</tr>
<tr>
<td>29-32</td>
<td>F4.1</td>
<td></td>
<td>Minimum angle between station-satellite vector and station quarter moon vector in degrees</td>
</tr>
<tr>
<td>33-36</td>
<td>F4.1</td>
<td></td>
<td>Minimum angle between station-satellite vector and station full moon vector in degrees</td>
</tr>
<tr>
<td>39-50</td>
<td>F12.9</td>
<td>ELTIM</td>
<td>Time in days at which acquisition data computations begin</td>
</tr>
<tr>
<td>53-57</td>
<td>F5.2</td>
<td></td>
<td>Minimum image size in microns</td>
</tr>
<tr>
<td>60</td>
<td>I1</td>
<td></td>
<td>Blank for acquisition data; 1 for ephemeris computation only</td>
</tr>
</tbody>
</table>
TABLE II
STATION CARDS

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Card Column</th>
<th>Format</th>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-24</td>
<td>4A6</td>
<td>Station number and name (number in col. 1-4 with leading zeros)</td>
</tr>
<tr>
<td></td>
<td>25-34</td>
<td>F10.5</td>
<td>XLAT (station latitude in degrees)</td>
</tr>
<tr>
<td></td>
<td>35-44</td>
<td>F10.5</td>
<td>XLONG (Station west longitude in degrees)</td>
</tr>
<tr>
<td></td>
<td>45-54</td>
<td>F10.2</td>
<td>HEIGHT (height of station above MSL in feet)</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>I1</td>
<td>&quot;1&quot; if this is MOTS station</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>I1</td>
<td>&quot;1&quot; if this is camera station</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;2&quot; if this is range station</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;3&quot; if this is range rate station</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>I1</td>
<td>&quot;1&quot; if this station can only share flashes</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>F10.7</td>
<td>DIAM (camera aperture in mm)</td>
</tr>
<tr>
<td></td>
<td>21-30</td>
<td>F10.7</td>
<td>TRANS (lens transmission factor)</td>
</tr>
<tr>
<td></td>
<td>31-40</td>
<td>F10.7</td>
<td>QM (atmospheric extinction at observer zenith)</td>
</tr>
<tr>
<td></td>
<td>41-50</td>
<td>F10.7</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>51-60</td>
<td>F10.7</td>
<td>A2</td>
</tr>
<tr>
<td></td>
<td>61-70</td>
<td>F10.7</td>
<td>A3</td>
</tr>
<tr>
<td>3</td>
<td>1-15</td>
<td>F15.10</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>16-30</td>
<td>F15.10</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>Lowest elevation angle (degrees) attainable as a function of azimuth</td>
</tr>
<tr>
<td>1-3</td>
<td>(I3)</td>
<td></td>
<td>Elevation Angle 0°-20° Azm (0°N)</td>
</tr>
<tr>
<td>4-6</td>
<td>(I3)</td>
<td>&quot;</td>
<td>&quot; 20-40</td>
</tr>
<tr>
<td>7-9</td>
<td>(I3)</td>
<td>&quot;</td>
<td>&quot; 40-60</td>
</tr>
<tr>
<td>10-12</td>
<td>(I3)</td>
<td>&quot;</td>
<td>&quot; 60-80</td>
</tr>
<tr>
<td>13-15</td>
<td>(I3)</td>
<td>&quot;</td>
<td>&quot; 80-100</td>
</tr>
<tr>
<td>16-18</td>
<td>(I3)</td>
<td>&quot;</td>
<td>&quot; 100-120</td>
</tr>
<tr>
<td>19-21</td>
<td>(I3)</td>
<td>&quot;</td>
<td>&quot; 120-140</td>
</tr>
<tr>
<td>Elevation Range</td>
<td>Elevation Angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>----------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22-24</td>
<td>(I3) 140-160</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25-27</td>
<td>(I3) 160-180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28-30</td>
<td>(I3) 180-200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31-33</td>
<td>(I3) 200-220</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34-36</td>
<td>(I3) 220-240</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37-39</td>
<td>(I3) 240-260</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40-42</td>
<td>(I3) 260-280</td>
<td></td>
<td></td>
</tr>
<tr>
<td>43-45</td>
<td>(I3) 280-300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>46-48</td>
<td>(I3) 300-320</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49-51</td>
<td>(I3) 320-340</td>
<td></td>
<td></td>
</tr>
<tr>
<td>52-54</td>
<td>(I3) 340-0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
GLOSSARY OF SYMBOLS USED

- \( A \) Cross sectional area of satellite in square centimeters
- \( C_{14} \) (longitude of perigee of sun-longitude of sun at January 0.0 for year of interest)
- \( C_{15} \) \( \dot{2} \) (eccentricity of earth's orbit)
- \( C_{16} \) \( \tan^2 \) (mean obliquity of ecliptic/2)
- \( C_{17} \) \( \tan \) (mean obliquity of ecliptic)
- \( J \) \( 1623 \times 10^{-6} \)
- \( H \) \( -6 \times 10^{-6} \)
- \( K \) \( 9 \times 10^{-6} \)
- \( \ell_0 \) longitude of sun at any time
- \( m \) mass of satellite in kilograms
- \( n \) Mean motion of satellite \( (GM)^{1/2}/a^{3/2} \)
- \( R_e \) radius of earth at perigee
- \( R_{eh} \) radius of earth at sub-satellite position
- \( V \) Satellite velocity
- \( \dot{\theta}_1 \) \( 0^\circ.98564724 \)
- \( \dot{\theta}_2 \) \( 360^\circ.98564724 \)
- \( \mu \) GM
- \( \phi_s \) Satellite latitude
- \( \lambda_E \) East longitude
Program Constants and Conversion Factors

READ INPUT CARDS AND STORE OUTPUT TITLES
EPOCHT = Epoch of Elements in Days
IYEAR = Year of Epoch
CONANG = 1/2 Cone Angle of Strobe Light
SUNTST = Maximum Sun Elevation Angle
DELTAE = ΔE
QJ2 = J2
QJ3 = J3
WO = ω₀
RAO = Ω₀
AXIS = a₀
ECCNO = e₀
XINC = i₀
QMASS = Mass of Satellite
AREA = Area of Satellite
NSTA = No. of Stations
NREV = No. of Revolutions to be Considered
DM1 = M₀
IRM = Max. No. of Flashes/Rev.
ILM = Max. No. of Flashes/Load
INFL = Initial No. of Flash Loads
TIMP = Epoch of an Even Clock Pulse
IP = Flag to Define Long Range or Refined Predictions
DDTT = Time Between Clock Pulses
TSTOP = Final Epoch
QLU1 = Min. Angle Between Satellite & New Moon
QLU2 = Min. Angle Between Satellite & Qtr. Moon
Q = Min. Angle Between Satellite & Full Moon
ELTIM = Time Look Angles Start
QIMSZ = Min. Image Size

Call CONGET for Annual Constants

To P.2
From P. 1

- Is $\Delta E = 0$?
  - Yes: $\Delta E = 10^\circ$
  - No

- Is $\text{CONANG} = 0$?
  - Yes: $\text{CONANG} = 85^\circ$
  - No

- Is $\text{SUNTST} = 0$?
  - Yes: $\text{SUNTST} = 10^\circ$
  - No

- Is $\text{HTST} = 0$?
  - Yes: $\text{HTST} = 15^\circ$
  - No

Write Input on Output Tape

To P. 3
Write on Output Tape:
"PROGRAM GIVEN MORE STATIONS THAN IT CAN HANDLE - WILL CONTINUE WITH FIRST 100"

Initialize Parameters Controlling Flow of Logic and Set Revolution Counter to 1
Flag 1 = 0 NCT = 0
Flag 2 = 0 M = 1
Flag 3 = 0 IFIRST = 1
TIME = EPOCHT
DT = TTIMP/86400
DDT = DDTT/86400
INFR = INF
JAR = INF
JAJ = INF
IELTI = 1
CRAR = 0
IR = 0
IL = 0
ITWO(J) = 0
J = 1, NSTA

Compute True Anomaly and Argument of Latitude
Write on Output Tape:
Beginning Revolution No. X

Go To 10001
Call Subroutine APS to Write Header for Message to APL on Tape 6

Compute Time of 1st Flash of All Sequences Scheduled for This Load Time and Write Them on Tape 6 as Message to APL

Call Subroutine Apend to Write End of Message to APL

Call Subroutine MH to Write Header to NASA on Tape 6

Write LHA & Dec. of All Flashes of Each Sequence Scheduled for This Load Time on Tape 6 as Message to NASA

Call Subroutine AFHED to Write Header for Message to APCS on Tape 6

Write Az. & Elev. Z to Center Flash of Each Sequence Scheduled for This Load Time on Tape 6 as Message to APCS

Call Subroutine AFEND to Write End of Message to APCS on Tape 6

Write on Output Tape 5 "OBSERVATIONS SELECTED FOR THIS LOAD" Time, Station No., Revolution No., \( \phi \) Sat., \( \lambda \) Sat., Range, Azimuth Elev. Angle

Rewind Tapes 9, 10, 12 and Set Up Restart Elements and Parameters

- \( A = ARS \)
- \( A = ARS \)
- \( AZI = AZRS \)
- \( RA = RARS \)
- \( U = URS \)
- \( ECCN = ECRS \)
- \( XINC = XIRS \)
- \( W = WRS \)
- \( DMI = DMRS \)
- \( INF = INFR \)
- \( E = ERS \)
- \( TIME = TRS \)
- \( XSINE = SINERS \)
- \( CRAR = RCRAR \)
- \( FLAG2 = 0 \)
- \( TLP = TIME \)
- \( IL = 0 \)
- \( H = HRS \)
- \( IR = 0 \)
- \( Jo = VRS \)
- \( FLAG1 = 0 \)
- \( M = MRS \)
- \( NCT = 0 \)

Is \( IFLAG4 = 3 \) ?

Yes

Call Subroutine SAO to Write Message to SAO on Tape 6

Write Total Flash Sequences and Total Loads on Tape 5

Call Subroutine Flex to Edit Tape 6 and Write TTY Messages on Tape 5

No

Go To 10001
Have All Stations Been Considered for This Time

Is $\text{NCT} - \text{NSTA} = 0$

Yes $\rightarrow \text{NCT} = 0$

No $\rightarrow \text{Go To 77}$

91

Is $\text{OLDSIN} < 1$

Yes $\rightarrow \text{Go To 80}$

No $\rightarrow \text{Go To 80}$

Is $\text{SINUZ} < 1$

Yes $\rightarrow \text{Go To 80}$

No $\rightarrow$

$E' = E - 2\pi$
$U' = U - 2\pi$

Compute New Perigee Height and Call Subroutine TLU for New Atmospheric Density and Scale Height
NIR = IR

Go To 425

Is NIR_0

No

Is (IR - IRM)_0

No

T1 = 9999999999

DO 206 L = 1, NIR

Is TR (L)_0

No

Is (UR(L) - T1)_0

Yes

T1 = VR(L)

LL = L

206

CONTINUE

TR (LL) = -1

Yes

Do 211 L = 1, NIR

Is TR (L)_0

Yes

IL = IL + 1

TL(IL) = TR(L)

SL(IL) = SR(L)

VL(IL) = VR(L)

211

CONTINUE

425

SET RESTART ELEMENTS

IR = 0   XIRS = OXIRS
ILP = IL  WRS = OWRS
ARS = OARS  URS = OURS
RARS = ORARS  VRS = ours
ECRS = OECRS  MRS = IMAS
TRS = OTRS  DMRS = ODMRS
RCRAR = ORCRAR  ERS = OERS
SINERS = OSINERS
HRS = OHRS
AZRS = OAZRS
319

M = M + 1

Have All Revolutions Been Made

Is (NREV - M) < 0

Yes

Set IFLAG4 = 3
Write on Output Tape
5 Restart Elements

Go To 273

No

Is TIME - TSTOP < 0

Yes

No

Is IFLAG4 = 3

Yes

No

Is IFLAG4 = 3

Yes

TIME = TIME + DT
STIME = TIME

Is IFLAG4 = 3

Yes

No

Is (TIME - ELTIM) < 0

Yes

IELTI = 2
CRAR = CRAR + GDT
TIME = STIME + DDT * CRAR
DT = DDT * GDT

No

CRAR = CRAR + GDT
TIME = STIME + CRAR * DDT
DT = DDT * GDT
Compute Total Perturbative Effect Of Bulge and Drag on the Elements at This Time

Set Flags Telling That a New Time Has Been Computed

\[
\begin{align*}
TNEW1 &= 0 \\
TNEW3 &= 0 \\
TNEW4 &= 0 \\
OLDSIN &= \sin(u) \\
SINUZ &= \sin(u-u_0)
\end{align*}
\]

Is IELTI = 1

Yes: Go To 80

No

Is IFRIST = 1

Yes: Go To 91

NO

COMPUTE:
- Greenwich Sidereal Time
- Longitude of Sun
- Right Asc. of Sun
- Dec. of Sun
- Satellite Latitude
- Satellite Longitude
- Distance from Center of Earth to Sat.
Call Subroutine AMESS
To Get Magnetic Dip and Dec. of Sat.

Is DIP < 0
Yes
Flash South Light

No
Flash North Light

Is This North Flash
Yes

Is This Time On An Odd Clock Pulse
Yes

No
Is This Time On An Even Clock Pulse
Yes

No
Go To 91

Do 40432
J = 1, NSTA

IONE (J) = ITWO (J)
ITWO (J) = 0

40432
CONTINUE
Consider Next Station
Compute:
Sidereal Time at Station
Hour Angle Sun
Topocentric

Is
This An
Optical Site

Yes
Compute Elevation
Angle of Sun (ELEVS)

No

Is
(ELEVS - SUNTST) ≤ 0

Yes
Compute Elevation Angle to Satellite

Call Subroutine
LUNY to Get RA
and δ of Moon

Compute Angle
Between Sta. Sat.
Vector & Center
Light Cone

Call Subroutine
PLATIM to Get
Image Size

Call Subroutine
LU to Get
Moon Phase

Go To 90

Compute Azimuth to Sat.
& Test to See if View is
Obstructed, Compute Sat.
Illumination Angle

Call Subroutine
VAL to Assign Value
to this Observation

Compute Topocentric Hour
Angle & Declination
of Each Flash in the
Sequence

Write Output Tape 5:
STATION
TIME
SAT. ELEV. ANGLE
SAT. AZIMUTH
RANGE
SAT. LATITUDE
SAT. LONGITUDE
IMAGE SIZE
LIGHT ANGLE
MOON ANGLE
SUN ELEV.
MAG. DIP & DEL
STA. TYPE

Go To 90

32
APPENDIX I

Three teletype messages are prepared directly from magnetic tape.

Number One is to the Applied Physics Laboratory in Silver Springs, Maryland. It contains the following information:

1. Day number since January 0
2. Year number since 1900
3. Number of last injection into satellite memory + 1
4. The proper light to flash at any scheduled time
5. Time of the first flash in each sequence in milliseconds.

The format of the message is as follows:

ANNA IB OPTICAL --- XXX --- XX - XXXXXX

where a - days
   b - year
   c - injection number

The times given are in milliseconds from 0 hours of the day noted in the message.

A zero in d denotes a North flash and a 1 denotes a South flash.

Number Two is to NASA Goddard Space Flight Center, Greenbelt, Maryland. Contents and format are as follows:

f g b

 XXXXX / XXXXXX AFCRL X2 PREDICTIONS

The times given in the message are in milliseconds from 0 hours of the day noted in the message.

A zero in d denotes a North flash and a 1 denotes a South flash.
where

\[ f \] = satellite identification number

\[ g \] = year, day and month

\[ h \] = type of look angle (R for refined, \( P \) for long range)

\[ i \] = station number

\[ j \] = sequence number this day

\[ k \] = day number since Jan. 0

\[ l \] = hour

\[ m \] = time of day

\[ n \] = seconds to nearest 1/10 second

\[ o \] = sign of Local Hour Angle (1 indicates negative, 0 indicates positive)

\[ p \] = Local Hour Angle to nearest 1/10 degree

\[ q \] = sign of declination (1 indicates negative, 0 indicates positive)

\[ r \] = declination to nearest 1/10 degree

Number Three is to the 1381st Missile Survey Squadron, Orlando, Florida. Contents and format as follows:

STA. NO. DAY HR MIN SEC PASS NO. SEQ. NO. AZ(0°N) ELEV (day)

XXXX XXX XX XX XX. XXX XXXXXX XXXXX XXX. XX XXX. XX

34
REFERENCES


2. Baker and Makemson (1960); An Introduction to Astrodynamics; Academic Press


5. Brown, D. C. (1962); Determination of Expected Image Diameters; unpublished report

6. Shea, M. A. (May 1962); Geomagnetic Field Calculations; Physics Department, University of New Hampshire, Durham, N. H.
APPENDIX II

Geomagnetic Field Calculations For a 5 Degree Polynomial

If one assumes that the earth's magnetic field at the point $r, \theta, \lambda$ (radial distance, co-latitude, and east longitude) arises from purely internal sources, then the components of the field $(X, Y, Z)$ are given by:

$$X = \sum_{n=1}^{N} \sum_{m=0}^{n} \frac{d}{d\theta} P^m_n \left[ g^m_n \cos m\phi + h^m_n \sin m\phi \right] \left(\frac{R_e}{r}\right)^{n+2}$$

$$Y = -\frac{1}{\sin \theta} \left[ \sum_{n=1}^{N} \sum_{m=0}^{n} P^m_n \left[ -m g^m_n \sin m\phi + m h^m_n \cos m\phi \right] \left(\frac{R_e}{r}\right)^{n+2} \right]$$

$$Z = \sum_{n=1}^{N} \sum_{m=0}^{n} P^m_n \left[ -(n+1) g^m_n \cos m\phi - (n+1) h^m_n \sin m\phi \right] \left(\frac{R_e}{r}\right)^{n+2}$$

where $P^m_n$ are the Schmidt orthogonal functions (using the associated Legendre functions; see Chapman and Bartels, Geomagnetism - Vol. II; pages 606-615); and $g^m_n$ and $h^m_n$ are the Schmidt coefficients of the geomagnetic field; $R_e$ is the earth's radius in kilometers.

The total magnetic field, $F$, is then given by:

$$F = (X^2 + Y^2 + Z^2)^{1/2}$$

where $F$ is calculated in gauss. To convert to gamma, the relationship $1 \text{ gauss} = 10^5 \text{ gamma}$ is used.
Flow Chart for Geomagnetic Field Computations

Input to Subroutine
Geodetic Latitude
Geodetic Longitude
r, (Focus to Sat. Dist.)

Calculate the Schmidt Orthogonal Functions for Degree 5

Determine x, y, and z Components of Field

Determine Horizontal, Vertical and Total Field Components

Compute DIP and DEC

Return
Contains an analysis and description of a computer program written for the Philco 2000. The program computes acquisition data to a magnetically stabilized satellite that carries a flashing light on each pole. The satellite motion is described by an osculating ellipse and consideration is given to the orientation of the light, whether or not the observer is in darkness, the expected image size of the strobe light on a photographic plate, and the relative position of the moon. In addition, the program selects the most geometrically "valuable" observation to make and automatically prepares three teletype messages that are sent to the observing sites.

1. Kahler, H.R., Nixon, W.T., Moroney, R.M.

AF Cambridge Research Laboratories, Bedford, Mass., Geophysical Research Directorate
AFCRL - 63 - 445 Unclassified Report