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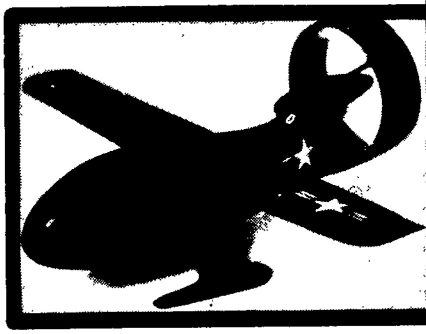
MEAN VELOCITY DISTRIBUTION IN THE OUTER LAYER OF A TURBULENT BOUNDARY LAYER

By

Tatsuya Matsui

Research Report No. 42 January 19, 1963



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Tatsuya Matsui

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By Tatsuya Matsui*

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ABSTRACT

The intermittent-eddy-viscosity hypothesis is proposed for the outer layer in a turbulent boundary layer flow, and it assumes an alternate appearance of zero eddy viscosity in the non-turbulent region and of a constant eddy viscosity in the turbulent region. The equation of motion is simplified by assuming the velocity defect law and that the velocity defect is small compared with the free stream velocity. With these assumptions the distributions of mean velocity and shear stress in the outer layer of a turbulent boundary layer on a flat plate for constant pressure are obtained using the intermittency factor measured by Klebanoff. The calculated results show good agreement with those measured.

INTRODUCTION

A turbulent boundary layer is divided into two parts, that is, the wall region and the outer region, the boundary between them being at about fifteen per cent of the boundary layer thickness from the surface. It is a physical characteristic of the wall region that the shear stress is almost constant. The flow adjacent to the wall is, however, laminar, whereas the flow is entirely turbulent in the layer further than $yU_t/\nu=30$, where the mean velocity distribution is well described with the so-called logarithmic wall law. Thus the wall region or the constant shear layer is subdivided into three layers, the laminar sublayer, the transition region and the turbulent region governed by the logarithmic law.

The physical features in the outer region are that the mean velocity distribution has similarity if it is plotted in the form of velocity defect, which is the difference from the free stream velocity, instead of velocity itself. The velocity defect law in the logarithmic form can describe the velocity profile well into the wall region, but it can describe the velocity in only a small part of the outer region. The power law for the velocity profile is purely empirical, though it describes the profile fairly well for its simplicity. The parabolic distribution proposed by Hama[1] describes the profile all over the outer region. The wake function proposed by Coles[2] was derived empirically as an expression for the deviation of the velocity from the logarithmic wall law. As the velocity distribution by Coles, however, does not go smoothly into the free stream velocity, an outer wall function has been proposed by Cornish[3] to improve this defect of the Law of Wake. These laws are useful for practical purposes, although they seem to lack physical foundations.

Assuming that the eddy viscosity is constant in the outer region, the velocity profile in the form of a co-error function was obtained analytically by Townsend[4] and by Hinze[5]. The profile shows appreciable deviations from the measured values at the outer edge of the boundary layer. The reason for the discrepancy may be due to the characteristics of flow changing intermittently from turbulent to non-turbulent flow in the outermost layer. A velocity profile was derived by the writer[6] taking the intermittency into account, but it too did not agree well with the measured results.

The outer region ranges over about eighty-five per cent of the turbulent boundary layer, and it is not affected directly by the shear at the wall, whereas the mean flow in the wall region is governed by the wall stress. It is a characteristic of the layer that the eddy viscosity is nearly constant within a portion about forty-five per cent of the total thickness from the wall and the intermittency is observed outside of this region.

In the present paper the mean velocity distribution in the outer region is derived analytically taking the characteristics of the flow into account and is compared with experimental results.

In Chapter 1, the Intermittent-Mean-Velocity Hypothesis is proposed, and in connection with this treatment, a new mean velocity distribution near the center of a turbulent pipe flow is derived by making use of the mixing length according to Kikuradse[7].

In Chapter 2, the Intermittent-Eddy-Viscosity Hypothesis is proposed, and the mean velocity distribution is derived in the form of velocity defect by making use of the intermittency factor according to Klebanoff[8].

The wake function by Coles and the outer wall function by Cornish are compared with the results calculated by the present forumla for velocity distribution. The shape factor, eddy viscosity and mixing length are discussed.

NOTATION

A, B	Constants in logarithmic velocity defect law (Equation 2-27)
a, b, c	Constants in the expression of mixing length in pipe flow (Equation 1-10)
C	Constant in the logarithmic wall law (Equation 1-14)
d	$d^2 = (U_o \delta/\varepsilon_o)(d\delta/dx)$, constant in Equation 2-13
H	Shape factor
l	Mixing length
R	Pipe radius
$R_\varepsilon, R_{\varepsilon_o}$	Eddy Reynolds number based on ε and ε_o, respectively
U	Mean velocity in x-direction
U_o	Free stream velocity or velocity at pipe center
U_t	Mean velocity in the turbulent region of intermittent, outer layer
U_τ	Friction velocity, $U_\tau = \sqrt{\tau_o/\rho}$,
V	Mean velocity in y-direction
u, v	Turbulent velocity fluctuations in x- and y-directions, respectively
$\overline{uv}$	Turbulent shear stress per unit mass
x	Distance measured along surface
y	Distance normal to surface measured from surface
α	Constant in the expression of eddy viscosity by Clauser
γ	Intermittency factor
δ	Boundary layer thickness
δ^*	Displacement thickness
ε	Mean eddy viscosity in outer layer, $\varepsilon = \gamma \varepsilon_o$
ε_o	Eddy viscosity in the turbulent region of outer layer
ζ	$\zeta = 5(y/\delta - 0.78)$
θ	Momentum thickness
κ	Constant in the logarithmic wall law, Equation 1-14

λ	Constant in the expression of the center velocity in pipe flow
ν	Kinematic viscosity
ξ	$\xi = y/\delta$
ρ	Fluid density
τ	Shear stress
τ_0	Shear stress at wall

CHAPTER 1

THE INTERMITTENT-MEAN-VELOCITY HYPOTHESIS

1-1. Introduction

It was shown by Schubauer[9] that the turbulent energy distribution in a boundary layer divided by the intermittency factor coincides closely with the distribution in pipe flow. According to Schubauer, such close agreement could not occur unless the average turbulent energy was distributed through the turbulent regions of a boundary layer just as it is in the fully turbulent section of a pipe flow.

This fact suggests that the mean velocity distribution in the turbulent region in a boundary layer may coincide with that in a fully turbulent pipe flow, because the turbulent energy is taken from the mean-flow kinetic energy through the turbulent shear stress in that layer. From careful study of the oscilloscope records of velocity fluctuation, it was noticed by Klebanoff[8] that "the trace has somewhat of a square-wave appearance in the intermittent region, and that the non-turbulent regions seem to be at a constant level corresponding to that of the free stream, while the turbulent regions are seen to be centered about some lower level. The difference between the velocity of the outside potential flow and that existing in the turbulent regions seemed to depend on how far past the measuring position the instantaneous edge of the layer extended at the particular instant".

If the mean velocity level in the turbulent region may be assumed to be expressed with the extrapolation to the outer layer of the logarithmic wall law in the deeper, non-intermittent turbulent region, which is valid in the fully turbulent pipe flow, the average mean-velocity distribution in the intermittently turbulent, outer layer will be described by the average of the free stream velocity and that of the logarithmic wall law, that is,

$$U = (1-\gamma)U_o + \gamma U_t \ , \qquad (1-1)$$

where U , U_o and U_t denote the average mean velocity, the free stream velocity and the mean velocity in the turbulent region according to the logarithmic law, respectively, and γ denotes the intermittency factor. The expression can be written alternately in the form of velocity defect, which is known to be appropriate to describe the mean velocity distribution in the outer region,

$$U_o - U = \gamma(U_o - U_t) \ , \qquad (1-2)$$

The mean velocity distribution was calculated by the writer[6] making use of the following empirical formulas by Klebanoff for the velocity defect and for the intermittency factor,

$$\frac{U_o - U_t}{U_\tau} = -5.75 \log_{10}\frac{y}{\delta} + 2.3 \ , \qquad (1-3)$$

$$\left.\begin{aligned} \gamma &= \frac{1}{2}(1 - \operatorname{erf}\zeta) \ , \\ \zeta &= 5\left(\frac{y}{\delta} - 0.78\right) , \end{aligned}\right\} \qquad (1-4)$$

$$\frac{U_o - U}{U_\tau} = \gamma\left(-5.75 \log_{10}\frac{y}{\delta} + 2.3\right) , \qquad (1-5)$$

where U_τ denotes the friction velocity $\sqrt{\tau_o/\rho}$, τ_o , the wall shear stress, ρ , density, y , the distance from the wall, and δ , the boundary layer thickness. The velocity distribution calculated by Equation (1-5) shows poor agreement with the experimental results as shown in Fig. 1.

Now it is recalled that the turbulent intensity in a boundary layer divided by γ coincides with that in pipe flow. Then the equation for $(U_0 - U_t)/U_\tau$ should be presumably that for pipe flow instead of for boundary layer flow, that is

$$\frac{U_0 - U_t}{U_\tau} = -5.62 \log_{10} \frac{y}{\delta} + 0.8 , \qquad (1\text{-}6)$$

which was derived by Hinze[10] based on the measurements by Laufer[11]. In this case, U_0 denotes the maximum velocity at the center of pipe. The use of this formula does not give good results either. It has been noticed, however, that for $y/\delta > 0.15$ the logarithmic velocity distribution deviates from the actual velocity distribution. A correction function for this deviation has been empirically introduced by Millikan[12], Hinze[13], and Cornish[3].

In the present chapter, a new formula is derived analytically for the mean velocity distribution in a fully turbulent pipe flow, and the mean velocity distribution in a boundary layer will be calculated using the new formula.

1-2. The mean velocity distribution in a fully turbulent pipe flow

According to the momentum transfer theory, the turbulent shear stress can be expressed as

$$\tau = \rho l^2 \frac{dU}{dy} \frac{dU}{dy} , \qquad (1\text{-}7)$$

where τ and l denote shearing stress and mixing length, respectively. As the shearing stress in a pipe flow is distributed linearly, it is written as

$$\tau = \tau_0 (1 - y/R) , \qquad (1\text{-}8)$$

where R is the radius of a pipe. According to Nikuradse[7], the mixing length in a pipe flow is represented by the expression

$$\frac{l}{R} = 0.14 - 0.08\left(1-\frac{y}{R}\right)^2 - 0.06\left(1-\frac{y}{R}\right)^4, \qquad (1\text{-}9)$$

Now the mixing length is assumed as

$$\frac{l}{R} = a^2 - b^2\left(1-\frac{y}{R}\right)^2 - c^2\left(1-\frac{y}{R}\right)^4, \qquad (1\text{-}10)$$

where the numerical constants, a , b and c are left to be determined later.

(i) Near the wall, or $y/R << 1$

In this case, the mixing length is known to be proportional to the distance from the wall,

$$l = \kappa y \,, \qquad (1\text{-}11)$$

where κ is a constant. In order to satisfy this condition, the following relations has to hold between the three constants.

$$a^2 - b^2 = c^2 \,, \qquad (1\text{-}12)$$

$$4a^2 - 2b^2 = \kappa \,, \qquad (1\text{-}13)$$

It is well known that the logarithmic wall law,

$$\frac{U}{U_\tau} = \frac{1}{\kappa} \ln \frac{yU_\tau}{\nu} + C \qquad (1\text{-}14)$$

is derived from equations (1-7), (1-8) and (1-11), where ν is the kinematic viscosity and C is a constant. This equation cannot give the value of the maximum velocity at the center when we put $y = R$, because it has been derived for the flow near the wall. When the deviation from the actual value is denoted by λ, the maximum velocity U_0 is given by the equation

$$\frac{U_0}{U_\tau} = \frac{1}{\kappa} \ln \frac{R U_\tau}{\nu} + C + \lambda \, . \qquad (1\text{-}15)$$

Then we obtain the velocity defect from equations (1-14) and (1-15) as follows:

$$\frac{U_0 - U}{U_\tau} = -\frac{1}{\kappa} \ln \frac{y}{R} + \lambda \, , \quad \text{for } y/R \ll 1 . \qquad (1\text{-}16)$$

(ii) Near the center of a pipe, or $(1 - y/R) \ll 1$.

In this case the mixing length is represented approximately by the following expression

$$\frac{l}{R} = a^2 - b^2 \left(1 - \frac{y}{R}\right)^2 . \qquad (1\text{-}17)$$

From equations (1-7), (1-8) and (1-17) the differential equation for the mean velocity results in the form

$$\frac{dU}{dy} = \frac{\sqrt{1 - y/R}}{a^2 - b^2 (1 - y/R)^2} \cdot \frac{U_\tau}{R} \, , \qquad (1\text{-}18)$$

from which the velocity defect is obtained by integration.

$$\int_{U}^{U_0} \frac{dU}{U_\tau} = \int_{y}^{R} \frac{\sqrt{1-y/R}}{a^2 - b^2(1-y/R)^2}\, d\left(\frac{y}{R}\right). \tag{1-19}$$

The result is

$$\frac{U_0 - U}{U_\tau} = \frac{1}{b\sqrt{ab}}\left[\tanh^{-1}\sqrt{\frac{b}{a}\left(1-\frac{y}{R}\right)} - \tan^{-1}\sqrt{\frac{b}{a}\left(1-\frac{y}{R}\right)}\,\right], \tag{1-20}$$

$$\text{for } (1-y/R) << 1 .$$

When $x << 1$,

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots ,$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots ,$$

therefore,

$$\tanh^{-1} x - \tan^{-1} x = \frac{2}{3} x^3 \left(1 + \frac{3}{7} x^4 + \cdots\right).$$

As the argument of $\tanh^{-1}$ and $\tan^{-1}$ in Equation (1-20) is smaller than unity, the velocity defect can be expressed approximately by the equation

$$\frac{U_0 - U}{U_\tau} = \frac{1}{3a^2}\left(1-\frac{y}{R}\right)^{3/2}\left[1 + \frac{3b^2}{7a^2}\left(1-\frac{y}{R}\right)^2\right]. \tag{1-21}$$

Using another expansion of $\tanh^{-1} x$,

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x} = -\frac{1}{2} \ln (1-2x+\cdots) ,$$

another approximate formula of the velocity defect can be derived.

$$\frac{U_0 - U}{U_\tau} = -\frac{1}{2b\sqrt{ab}} \left\{ \ln\left[1-2\sqrt{\frac{b}{a}\left(1-\frac{y}{R}\right)}\right] + 2\sqrt{\frac{b}{a}\left(1-\frac{y}{R}\right)} \right\} . \qquad (1\text{-}22)$$

The four numerical constants a, b, c and λ have to be determined so that Equation (1-16) for $y/R << 1$ may be smoothly joined to Equation (1-20) for $(1-y/R) << 1$ at some point.

By equating right-hand sides of these equations and their derivatives, respectively, we have

$$-\frac{1}{\kappa} \ln \frac{y}{R} + \lambda = \frac{1}{b\sqrt{ab}} \left[\tanh^{-1} \sqrt{\frac{b}{a}\left(1-\frac{y}{R}\right)} - \tan^{-1} \sqrt{\frac{b}{a}\left(1-\frac{y}{R}\right)} \right] , \qquad (1\text{-}23)$$

and

$$\frac{1}{\kappa} \cdot \frac{1}{\frac{y}{R}} = \frac{\sqrt{1-\left(\frac{y}{R}\right)^2}}{a^2 - b^2\sqrt{1-\left(\frac{y}{R}\right)^2}} . \qquad (1\text{-}24)$$

Now we have four equations (1-12), (1-13), (1-23) and (1-24) for the four unknown constants, provided that the value of κ is taken to be 0.40 as usual. It is seen from Equation (1-24) that the value of b/a changes rapidly between $y/R = 0.30$ and 0.31 when κ is substituted by Equation (1-13). If the point of $y/R = 0.3027$ is chosen as the joining point, we obtain the values of constants as follows:

$$\left.\begin{aligned} a^2 &= 0.14 , \\ b^2 &= 0.08 , \\ c^2 &= 0.06 , \\ \text{and} \qquad & \\ \lambda &= 0.1785 . \end{aligned}\right\} \qquad (1\text{-}25)$$

Putting these values into equations (1-10), (1-16), (1-20), (1-21) and (1-22), the results come as follows:

$$\frac{l}{R} = 0.14 - 0.08\left(1-\frac{y}{R}\right)^2 - 0.06\left(1-\frac{y}{R}\right)^4 , \qquad (1\text{-}26)$$

$$\frac{U_0 - U}{U_\tau} = -5.75 \log\frac{y}{R} + 0.1785 \qquad (1\text{-}27)$$

$$\text{for } \; y/R < 0.30 ,$$

$$\frac{U_0 - U}{U_\tau} = 10.868\left[\tanh^{-1}\left\{0.869\sqrt{1-\frac{y}{R}}\right\} - \tan^{-1}\left\{0.869\sqrt{1-\frac{y}{R}}\right\}\right] \qquad (1\text{-}28)$$

$$\text{for } \; y/R > 0.30 ,$$

and approximately

$$\frac{U_0 - U}{U_\tau} = 4.76\left(1-\frac{y}{R}\right)^{3/2}\left[1+0.244\left(1-\frac{y}{R}\right)^{2}\right], \qquad (1\text{-}29)$$

or

$$\frac{U_0 - U}{U_\tau} = -5.43\left\{\ln\left[1-1.74\sqrt{1-\frac{y}{R}}\right] + 1.74\sqrt{1-\frac{y}{R}}\right\}. \qquad (1\text{-}30)$$

The expression for mixing length is the same as that of Nikuradse. The value of λ in the equation for velocity distribution near the wall is smaller than that of Hinze. The classical formula, which was empirically deduced by Darcy[14],

$$\frac{U_0 - U}{U_\tau} = 5.08\left(1-\frac{y}{R}\right)^{3/2}, \qquad (1\text{-}31)$$

being the first approximation, Equation (1-29) maybe regarded as a second approximation. Equation (1-30) is akin to the universal velocity distribution deduced from Karman's similarity law.

$$\frac{U_0 - U}{U_\tau} = -\frac{1}{\kappa}\left\{\ln\left[1-\sqrt{1-\frac{y}{R}}\right] + \sqrt{1-\frac{y}{R}}\right\}. \qquad (1\text{-}32)$$

In Fig. 2, Equations (1-27) and (1-28) are plotted to be compared with the experimental results taken from the date of channel flow[11] and pipe flow[15] by Laufer and of pipe flow by Komatsu[16]. It also contains the velocity distribution by the Law of Wake deduced by Coles, which is modified with the outer wall function by Cornish. The experimental results seem to depend appreciably on Reynolds Number, and there is a slight difference between the mean velocity in channel flow and that in pipe flow. The velocity distribution by Cornish coincides fairly well with the channel flow, while the present distribution curve passes through the midst of all measured points.

1-3. The mean velocity distribution in the outer layer of a turbulent boundary layer

As mentioned in 1-1, if the velocity in the turbulent part of the intermittently turbulent region is assumed to be expressed by the formula for a fully turbulent pipe flow, Equation (1-28), the mean velocity distribution in the outer layer of a turbulent boundary layer may be calculated by equations (1-2) and (1-28). That is,

$$\frac{U_0-U}{U_\tau}=10.868\,\gamma\left[\tanh^{-1}\left\{0.869\sqrt{1-\frac{y}{R}}\right\}-\tan^{-1}\left\{0.869\sqrt{1-\frac{y}{R}}\right\}\right], \quad (1\text{-}33)$$

where γ is the intermittency factor given by Equation (1-4). It can be seen in Fig. 1 that the velocity distribution was not improved by Equation (1-33).

Now it appears that the intermittent-mean-velocity hypothesis is not valid in the outer region of turbulent boundary layer. In the following chapter, another hypothesis will be proposed.

CHAPTER 2

THE INTERMITTENT-EDDY-VISCOSITY HYPOTHESIS

2-1. Introduction

In the preceding chapter, the shearing stress was assumed to be given by Equation (1-7) according to the momentum transfer theory, and to be distributed linearly as shown in Equation (1-8). The assumption is made for a pipe flow, not for a boundary layer flow. Besides, the concept of the intermittent-mean-velocity hypothesis is not based on any other sound foundation from the dynamical point of view than from the phenomenological one. Now we shall start from the fundamental equations of motion for the boundary layer flow. Concerning the expression of shearing stress in terms of mean velocity, there are several ways as found in textbooks. The first of them is the mixing length theory, including the momentum transfer theory and the vorticity transfer theory. The second is the mechanical similarity rule by Karman. Prandtl's new assumption for the eddy viscosity is related to free turbulent flow. The constant eddy viscosity concept by Boussinesq comes last, though it is the oldest one, because it is newly applied by Townsend and Hinze. According to the experimental results by Klebanoff, the eddy viscosity in the turbulent boundary layer is not constant, but it has a maximum value near the layer of $y/\delta = 0.3$, and it changes slightly between $y/\delta = 0.2$ and 0.5; whereas outside of the region it falls rapidly to zero. The assumption of constant eddy viscosity may be permissible only for the region of $y/\delta < 0.5$. In the region where the eddy viscosity changes rapidly to zero, the intermittency of flow pattern can be observed. It is shown by Hinze that the eddy viscosity divided by the intermittency factor is nearly constant throughout the outer region, in the same way as it is shown by Schubauer that the turbulent energy distribution divided by the intermittency factor shows a fairly good agreement with in pipe flow. This factor shows a fairly good agreement with in pipe flow. This fact seems to suggest the intermittent-eddy-viscosity hypothesis proposed in the following section.

2-2. The intermittent-eddy-viscosity hypothesis

The shearing stress in turbulent flow is given in terms of the eddy viscosity as follows:

$$\tau = \rho \varepsilon \frac{\partial U}{\partial y} , \qquad (2\text{-}1)$$

where ε denotes the average eddy viscosity in a layer in the outer region, and not having a constant value, but being a function of y/δ. According to the suggestion mentioned before, it is assumed that the eddy viscosity has a constant value ε_0 in the turbulent region, while it is zero in the non-turbulent region, and the eddy viscosity of ε_0 and of zero appears intermittently at a specified point. This is the intermittent-eddy-viscosity hypothesis and it is written as

$$\varepsilon = \gamma \varepsilon_0 , \qquad (2\text{-}2)$$

where γ is the intermittency factor. The same concept as this is suggested by Hinze. The shearing stress in the outer layer is now

$$\tau = \rho \gamma \varepsilon_0 \frac{\partial U}{\partial y} . \qquad (2\text{-}3)$$

In a fully turbulent flow, the shearing stress is mainly due to the turbulent fluctuation and is expressed as

$$\tau = -\rho\, \overline{uv}\ . \tag{2-4}$$

where u and v denote the turbulent velocity fluctuations in x- and y-direction, respectively. From equations (2-3) and (2-4) we have

$$-\overline{uv} = \gamma \varepsilon_0 \frac{\partial U}{\partial y}\ . \tag{2-5}$$

We can verify this relation by making use of the experimental results of $\overline{uv}$, γ and U , which were measured independently. The turbulent shear stress is plotted against the velocity gradient in Fig. 3. According to the constant eddy viscosity concept, the shear stress should be linear to the velocity gradient itself. The relations between the shear stress and the velocity gradient is shown with small crosses in Fig. 3. It can be seen in the figure that the assumption of constant eddy viscosity is valid between $y/\delta = 0.2$ and 0.5 , but that the assumption should not be applied beyond $y/\delta = 0.5$. When the shear stress is plotted against the velocity gradient multiplied by the intermittency factor γ , instead of the velocity gradient itself, which are shown with small circles in the same figure, it appears that the linearity between them is well confirmed all over the outer region. Thus the intermittent-eddy-viscosity hypothesis seems to be valid in the outer layer of turbulent boundary layer flow.

In Fig. 3, the shear stress is given in non-dimensional form divided by the square of friction velocity, the velocity is given in non-dimensional form of velocity defect divided by friction velocity, and the distance from the surface is divided by the boundary layer thickness. Therefore, the slope of the straight line gives the eddy viscosity in non-dimensional form, that is, $\varepsilon_o / \delta U_\tau$, and the value is 0.062 for this case.

2-3. <u>The distribution of mean velocity and shear stress in the outer layer of turbulent boundary layer</u>

The equation of motion is given for the mean velocity in turbulent boundary layer with zero pressure gradient as follows:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{1}{\rho}\frac{\partial \tau}{\partial y}\ , \tag{2-6}$$

or in terms of velocity defect,

$$U_o\left(1-\frac{U_o-U}{U_o}\right)\frac{\partial}{\partial x}(U_o-U) + V\frac{\partial}{\partial y}(U_o-U) = -\frac{1}{\rho}\frac{\partial \tau}{\partial y}\ . \tag{2-7}$$

In the outer layer we can assume that

$$\frac{U_o-U}{U_o} << 1\ ,$$

and by the order-of-magnitude procedure we arrive at the approximate equation of (2-7),

$$U_o\frac{\partial}{\partial x}(U_o-U) = -\frac{1}{\rho}\frac{\partial \tau}{\partial y}\ . \tag{2-8}$$

According to the intermittent-eddy-viscosity hypothesis the shear stress is

$$\tau = -\rho \gamma \varepsilon_0 \frac{\partial}{\partial y}(U_0 - U) \; . \tag{2-9}$$

Assuming the similarity of velocity defect, we put

$$\left.\begin{aligned} \frac{U_0 - U}{U_\tau} &= f(\xi) \; , \\ \xi &= y/\delta \; , \end{aligned}\right\} \tag{2-10}$$

where $f(\xi)$ is the velocity distribution function. If we keep in mind that

$$\begin{aligned} \left(\frac{\partial}{\partial x}\right)_y &= \left(\frac{\partial}{\partial x}\right)_\xi + \left(\frac{\partial}{\partial \xi}\right)_x \left(\frac{\partial \xi}{\partial \delta}\right)_y \frac{d\delta}{dx} \\ &= \left(\frac{\partial}{\partial x}\right)_\xi - \frac{\xi}{\delta}\frac{d\delta}{dx}\left(\frac{\partial}{\partial \xi}\right)_x \; , \end{aligned}$$

and

$$\left(\frac{\partial}{\partial y}\right)_x = \frac{1}{\delta}\left(\frac{\partial}{\partial \xi}\right)_x \; ,$$

we obtain the equation of motion

$$\frac{U_0}{U_\tau}\frac{dU_\tau}{dx} f - \frac{U_0}{\delta}\frac{d\delta}{dx}\xi f' = \frac{\varepsilon_0}{\delta^2}\frac{d}{d\xi}(\gamma f') \; , \tag{2-11}$$

where the prime means differentiation with respect to ξ . Except near the edge of boundary layer, f and $\xi f'$ are of the same order of magnitude. As shown in the Appendix,

$$-\frac{1}{U_\tau}\frac{dU_\tau}{dx} << \frac{1}{\delta}\frac{d\delta}{dx} .$$

Hence, it is permissible to neglect the first term on the left-hand side of Equation (2-11), which then becomes

$$\frac{d}{d\xi}(\gamma f') = -\frac{U_o\delta}{\varepsilon_o}\frac{d\delta}{dx}\xi f' . \tag{2-12}$$

Since the left-hand side of this equation is a function of ξ alone, the equation can be correct only if

$$\frac{U_o\delta}{\varepsilon_o}\frac{d\delta}{dx} = d^2 , \tag{2-13}$$

that is, a constant, independent of x . Integrating Equation (2-12), we have

$$\ln\frac{\gamma f'}{[\gamma f']_{\xi=0}} = -d^2\int_0^{\xi}\frac{\xi}{\gamma}d\xi .$$

From equations (2-9) and (2-10)

$$\tau = -\rho\gamma\varepsilon_o\frac{U_\tau}{\delta}\frac{df}{d\xi} \tag{2-14}$$

and since $\tau_0 = \rho U_\tau^2$ and $\gamma = 1$ when $\xi = 0$,

$$[\gamma f]_{\xi=0} = -\frac{U_\tau \delta}{\varepsilon_0}\,.$$

Hence

$$\ln(-\gamma f') = \ln\left(\frac{U_\tau \delta}{\varepsilon_0}\right) - d^2 \int_0^\xi \frac{\xi}{\gamma}\, d\xi\,, \tag{2-15}$$

or

$$\frac{df}{d\xi} = -\frac{U_\tau \delta}{\varepsilon_0} \cdot \frac{1}{\gamma} \exp\left[-d^2 \int_0^\xi \frac{\xi}{\gamma}\, d\xi\right]. \tag{2-16}$$

Integrating this equation from infinity to ξ, we obtain the velocity distribution function as follows:

$$f(\xi) = \frac{U_\tau \delta}{\varepsilon_0} \int_\xi^\infty \frac{1}{\gamma} \exp\left[-d^2 \int_0^\xi \frac{\xi}{\gamma}\, d\xi\right], \tag{2-17}$$

since f tends to zero when ξ goes to infinity. For the shear stress distribution we have from equations (2-14) and (2-16)

$$-\frac{\overline{uv}}{U_\tau^2} = \exp\left[-d^2 \int_0^\xi \frac{\xi}{\gamma}\, d\xi\right]. \tag{2-18}$$

In order to determine the values of two constants, d^2 and $U_\tau \delta / \varepsilon_0$ using Equation (2-15), $\log(-\gamma f')$ is plotted against $\int_0^\xi (\xi/\gamma)\, d\xi$ in Fig. 4, when the measured values of γ and f by Klebanoff were used. It can be seen that the plotted points are on a straight line.

It suggests validity of the relation given by (2-15). Values of the two constants obtained are:

$$\left.\begin{aligned} &\frac{U_\tau \delta}{\varepsilon_o} = 16.19 \;, \quad \text{or} \quad \frac{\varepsilon_o}{U_\tau \delta} = 0.062 \;, \\ &\text{and} \\ &d^2 = 4.50 \;. \end{aligned}\right\} \qquad (2\text{-}19)$$

This value for non-dimensional eddy viscosity, $\varepsilon_o / U_\tau \delta$, is in good agreement with that obtained before from the shear stress in Fig. 3. The mean velocity distribution of Equation (2-17) with the constants of (2-19) is shown in Fig. 1. The calculated values show an excellent agreement with those measured by Klebanoff. The velocity distribution by the law of wake modified with the outer wall function by Cornish is also shown in the same figure. This distribution will be discussed in the next section.

The shear stress distribution calculated by Equation (2-18) with the value of d^2 of (2-19) is shown in Fig. 5, and it is also in good agreement with that measured by Klebanoff.

2-4. The wake function and the outer wall function

Coles[2] has proposed that the mean velocity distribution may be written in the form

$$\frac{U}{U_\tau} = f\left(\frac{yU_\tau}{\nu}\right) + \frac{\Pi(x)}{\kappa} w\left(\frac{y}{\delta}\right) , \qquad (2\text{-}20)$$

which is now known as the law of wake, and $w(y/\delta)$ is called the wake function. $\Pi(x)$ denotes the profile parameter. κ is an empirical constant in the logarithmic law, that is

$$f\left(\frac{yU_\tau}{\nu}\right) = \frac{1}{\kappa} \ln \frac{yU_\tau}{\nu} + C \quad , \qquad \text{(2-21)}$$

where C is another empirical constant. Coles used the numerical values of $\kappa = 0.40$ and $C = 5.10$. The profile parameter $\Pi(x)$ is given the value of 0.55 for a flow at constant pressure. Then the velocity profile is written as

$$\frac{U}{U_\tau} = \frac{1}{0.40} \ln \frac{yU_\tau}{\nu} + 5.10 + \frac{0.55}{0.40}\, w\left(\frac{y}{\delta}\right) ,$$

or in the form of velocity defect

$$\frac{U_0 - U}{U_\tau} = -\frac{1}{0.40} \ln \frac{y}{\delta} + \frac{0.55}{0.40}\left[2 - w\left(\frac{y}{\delta}\right)\right] ,$$

since the wake function is normalized so that it has the value $w(1) = 2$. From this equation we have

$$w\left(\frac{y}{\delta}\right) = 2 - \frac{0.40}{0.55}\left(\frac{U_0 - U}{U_\tau} + \frac{1}{0.40} \ln \frac{y}{\delta}\right) . \qquad \text{(2-22)}$$

By using Equation (2-17) for the velocity defect, the value of $w(y/\delta)$ can be calculated. The result is shown in Fig. 6 with that of Coles. The wake function by Coles is seen to be smaller than the present result. Besides, the calculated wake function has the maximum at about $y/\delta = 0.9$; whereas Coles' wake function has no maximum value. This leads to the non-zero slope at the edge of boundary layer, as was pointed out by Cornish.

The outer wall function has been proposed by Cornish[3] to improve Coles' description of the velocity profile, that is,

$$\frac{U}{U_\tau} = \frac{1}{\kappa} \ln \frac{yU_\tau}{\nu} + C + \frac{\Pi}{\kappa} w\left(\frac{y}{\delta}\right) + \omega\left(\frac{y}{\delta}\right), \qquad (2\text{-}23)$$

where $\omega(y/\delta)$ is the outer wall function. The velocity defect is written as

$$\frac{U_0 - U}{U_\tau} = -\frac{1}{\kappa} \ln \frac{y}{\delta} + \frac{\Pi}{\kappa}\left[2 - w\left(\frac{y}{\delta}\right)\right] - \omega\left(\frac{y}{\delta}\right),$$

since $\omega(1) = 0$. With $\kappa = 0.40$ and $\Pi = 0.55$, this equation becomes

$$\frac{U_0 - U}{U_\tau} = -\frac{1}{0.40} \ln \frac{y}{\delta} + \frac{0.55}{0.40}\left[2 - w\left(\frac{y}{\delta}\right)\right] - \omega\left(\frac{y}{\delta}\right),$$

from which we have

$$\omega\left(\frac{y}{\delta}\right) = -\frac{1}{0.40} \ln \frac{y}{\delta} + \frac{0.55}{0.40}\left[2 - w\left(\frac{y}{\delta}\right)\right] - \frac{U_0 - U}{U_\tau}. \qquad (2\text{-}24)$$

By using the values of Coles for the wake function and Equation (2-17) for the velocity defect, the outer wall function can be calculated, and it is shown in Fig. 7. The difference between the outer wall function of Cornish and the present result is very small except between $y/\delta = 0.6$ and 0.8. It should be noted, however, that the values of constants used by Cornish were $\kappa = 0.412$ and $2\Pi/\kappa = 2.5$, so that the value of Π was 0.515. With these values the discrepancy will be seen to become larger.

2-5. The shape factor

The shape factor is defined as the ratio of displacement thickness and momentum thickness of boundary layer. Velocity defect is similar if it is divided by friction velocity, not by free stream velocity. Accordingly, the shape factor is a function of friction velocity. The relation between them is written as follows:

$$H = \frac{\delta^*}{\theta} = \frac{1}{1 - \frac{I_2}{I_1} \cdot \frac{U_\tau}{U_0}} , \tag{2-25}$$

where

$$I_1 = \int_0^1 \frac{U_0 - U}{U_\tau} d\xi , \quad I_2 = \int_0^1 \left(\frac{U_0 - U}{U_\tau}\right)^2 d\xi ,$$

and

$$\left.\frac{\delta^*}{\delta} = I_1 \frac{U_\tau}{U_0} , \quad \frac{\theta}{\delta} = I_1 \frac{U_\tau}{U_0} - I_2 \frac{U_\tau^2}{U_0^2} \right\} \tag{2-26}$$

In order to estimate the values of I_1 and I_2, we use the logarithmic law

$$\frac{U_0 - U}{U_\tau} = -A \ln \xi + B \tag{2-27}$$

with $A = 2.5$ and $B = 2.3$ from $\xi = 0$ to $\xi = 0.1$ and the present formula of equation (2-17) with (2-19) from $\xi = 0.1$ and $\xi = 1$, and we obtain $I_1 = 3.675$, $I_2 = 24.50$, and $I_2/I_1 = 6.67$. With this value of I_2/I_1, H is plotted against U_τ/U_0 in Fig. 8. This result is seen to be in a better agreement with the measured values than that of Hama with the value of $I_2/I_1 = 6.1$. It may be noted that if the logarithmic formula (2-27) is used throughout the boundary layer, the ratio is

$$\frac{I_2}{I_1} = \frac{2A^2 + 2AB + B^2}{A + B} \quad ,$$

and the value $I_2/I_1 = 6.1$ is obtained with $A = 2.5$ and $B = 2.3$. If we use Equation (2-17) only, we obtain $I_2/I_1 = 5.87$. With Cornish's outer wall function, almost the same result as mentioned above is obtained as shown in Fig. 8.

2-6. <u>Eddy viscosity and mixing length</u>

Clauser noticed that the constant pressure turbulent profile dropped so abruptly at the wall as to appear to extrapolate to a non-zero velocity at the wall, and that this characteristic shape of the turbulent profile came from the circumstances that the laminar sub-layer next to the wall and the flow adjacent to it had a lower viscosity than the eddy viscosity prevailing in the main body of the turbulent flow. He attempted to simulate a turbulent boundary layer profile to a laminar profile with a slip velocity on the wall and with an appropriate eddy viscosity, instead of a laminar viscosity.

If the present formula of Equation (2-17) is extrapolated to the wall, the velocity at the wall or the slip velocity U_s is obtained,

$$\frac{U_0 - U_s}{U_\tau} = \sqrt{\frac{\pi}{2d^2}}\,\frac{U_\tau \delta}{\varepsilon_0} \quad , \qquad (2\text{-}28)$$

which is calculated to have a constant value of 9.54 with (2-19).

As shown in preceding sections, eddy viscosity cannot be regarded as constant in a turbulent boundary layer, but it is nearly constant in the turbulent part in the outer layer. The eddy viscosity in the turbulent part is written from Equation (2-5) as follows:

$$\varepsilon_0 = -\frac{\overline{uv}}{\gamma \frac{\partial U}{\partial y}} ,$$

and $\varepsilon_0 / U_\tau \delta$ is 0.062 as mentioned before. From $U_\tau = 1.85$ ft/sec. and $\delta = 3.0$ in. of Klebanoff's data, we have $\varepsilon_0 = 0.0287$ ft^2/sec.

Since $U_\tau \delta = U_0 \delta^* / I_1$, from Equation (2-26) and $I_1 = 3.675$, we have another expression of ε_0,

$$\varepsilon_0 = 0.0168\, U_0 \delta^* \tag{2-29}$$

From this, we have the eddy Reynolds number

$$R_{\varepsilon_0} = \frac{U_0 \delta^*}{\varepsilon_0} = 59.5 \tag{2-30}$$

The eddy viscosity by Clauser is

$$\varepsilon = \alpha U_0 \delta^*$$

and α is universally 0.018. Therefore, the eddy Reynolds number is $U_0 \delta^* / \varepsilon = 56$. It is noted that the present results coincide rather remarkably with those of Clauser in spite of the difference in definition of ε_0 and ε.

In definition of ε_o given by Equation (2-3), the transfer of mean flow momentum is assumed implicitly to be due to the small eddy or gradient type of transfer, the large eddy of convective type of transfer being neglected. From the constancy of ε_o shown before, it may be assumed that the turbulent part in the outer layer is composed of small eddies of the same size and of the same strength. In other words, the motion of a fluid element may be regarded as random walk of flight velocity v and of flight length ℓ. Then the eddy viscosity is written as follows:

$$\varepsilon_o = \frac{1}{2} v \ell$$

where the flight length ℓ corresponds to the mixing length. According to the measurement by Klebanoff, the y-component of turbulent intensity v/U_o divided by the intermittency factor has roughly a constant value between 0.035 and 0.043. If we take the value $v/U_o = 0.04$, we have from Equation (2-29)

$$\frac{\ell}{\delta^*} = 0.84$$

With Equation (2-26) and the value of $I_1 = 3.765$ and $U_\tau/U_o = 0.037$, the mixing length is obtained in terms of the boundary layer thickness, that is

$$\frac{\ell}{\delta^*} = 0.117 \qquad (2\text{-}31)$$

It can be seen that the mixing length is small compared with the boundary layer thickness. This result is consistent with the assumption of gradient type of transfer of the mean flow momentum. The effect of large eddies is taken into account through the intermittency factor.

CONCLUSION

A new formula for the mean velocity distribution of a fully turbulent pipe flow, which was derived with reference to the mixing length measured by Nikuradse, agrees well with experimental results. The mean velocity profile in the outer layer of a turbulent boundary layer was calculated by the intermittent-mean-velocity hypothesis, using the new formula for pipe flow, and the result did not agree with the experiments.

It has been shown by study of measured results by Klebanoff that the eddy viscosity in turbulent region of outer layer is nearly constant and that in non-turbulent region is zero, so that the turbulent shear stress is expressed in terms of the average eddy viscosity which is the product of the constant eddy viscosity and the int -mittency factor. It is suggested to call this concept the intermittent-eddy-viscosity hypothesis. The velocity profile and the shear stress distribution in the outer layer deduced from this concept are in good agreement with the experimental results.

Coles' wake function should be modified, and for this purpose Cornish's outer wall function gives a little larger values than those calculated, though the difference is very small.

The relation between the shape factor and the friction velocity is well described with the present velocity profile.

The eddy Reynolds number, $U_0 \delta^* / \varepsilon_0$, has the value of 60, and the mixing length has a constant value of order of one-tenth of boundary layer thickness, so that the gradient type transfer of mean flow momentum can be assumed.

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APPENDIX

Assuming that the velocity is a function of $U_\tau y/\nu$,

$$\frac{U}{U_\tau} = F\left(\frac{U_\tau y}{\nu}\right) \tag{1}$$

and substituting δ for y, we obtain the freestream velocity U_o,

$$\frac{U_o}{U_\tau} = F\left(\frac{U_\tau \delta}{\nu}\right) \tag{2}$$

Differentiation of Equation (2) with respect to x yields

$$-\frac{1}{\delta}\frac{d\delta}{dx} = \left[\frac{1}{F'}\frac{\nu}{U_\tau \delta}\left(\frac{U_o}{U_\tau}\right)^2 + \frac{U_o}{U_\tau}\right]\frac{d}{dx}\left(\frac{U_\tau}{U_o}\right), \tag{3}$$

where F' denotes the derivative of F with respect to $U_\tau \delta/\nu$. Since the local skin friction coefficient $c_f = 2(U_\tau/U_o)^2$, U_o/U_τ becomes very large at large Reynolds numbers. If the velocity is assumed to have the logarithmic form

$$\frac{U}{U_\tau} = F\left(\frac{U_\tau y}{\nu}\right) = A \ln \frac{U_\tau y}{\nu} + B \tag{4}$$

we have the following relation by differentiation

$$\frac{1}{F'}\frac{\nu}{U_\tau \delta} = A \tag{5}$$

Strictly speaking, this relation cannot be valid in the outer layer. However, the order-of-magnitude relation will remain unchanged. Therefore, $\nu / F' U_\tau \delta$, may be of the order of unity, because A has the value of about 2.5. Accordingly, the second term in the bracket on the right-hand side of Equation (3) can be neglected. In other words, it is very small in comparison with the left-hand side.

$$-\frac{U_0}{U_\tau}\frac{d}{dx}\left(\frac{U_\tau}{U_0}\right) << \frac{1}{\delta}\frac{d\delta}{dx} \quad . \tag{6}$$

Figure 1

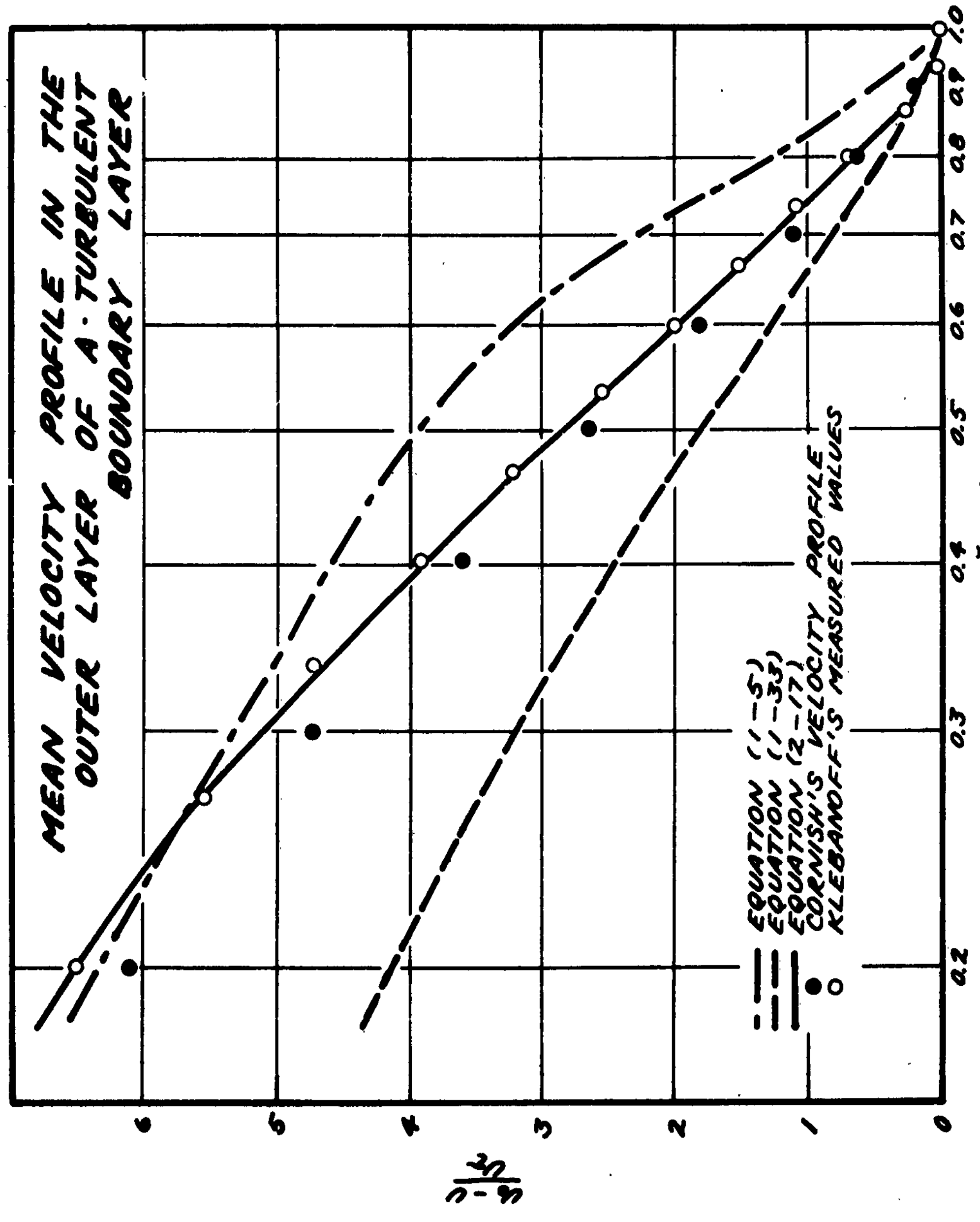
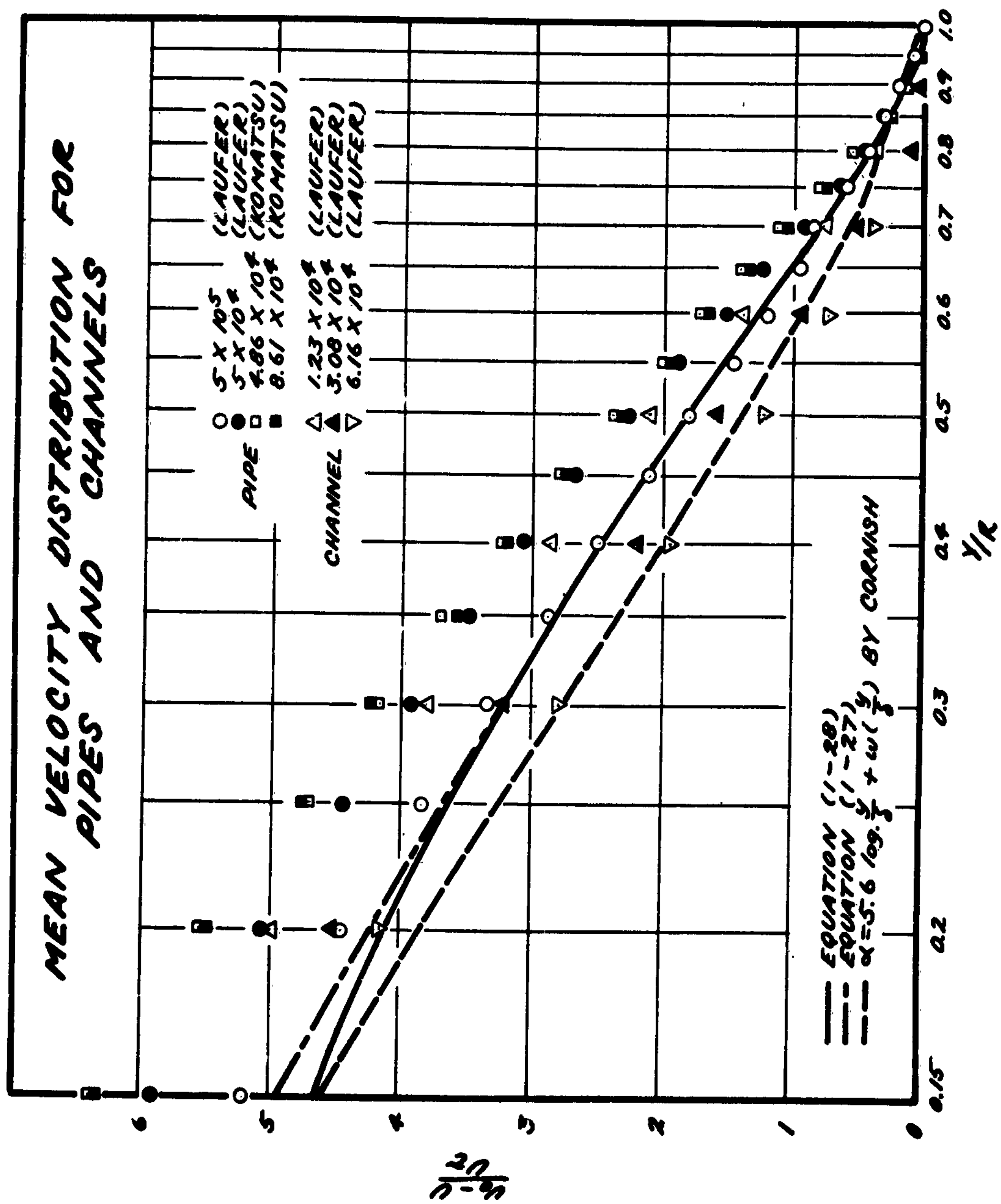
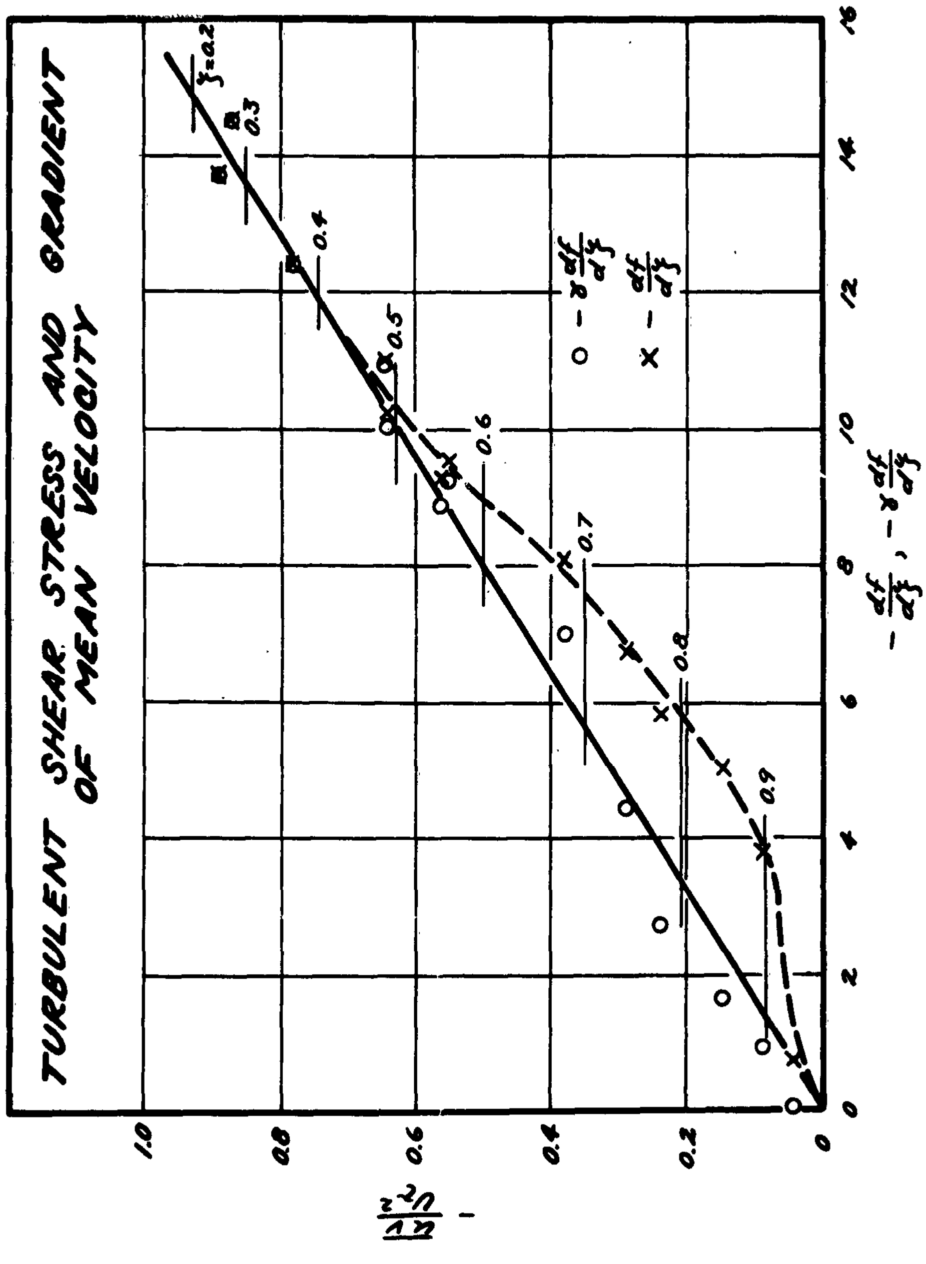

MEAN VELOCITY PROFILE IN THE OUTER LAYER OF A TURBULENT BOUNDARY LAYER
EQUATION (1-5)
EQUATION (1-33)
EQUATION (2-17)
CORNISH'S VELOCITY PROFILE
KLEBANOFF'S MEASURED VALUES
$\xi = y/\delta$
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\frac{u_\delta - u}{u_\tau}$
0
1
2
3
4
5
6

Figure 2



MEAN VELOCITY DISTRIBUTION FOR PIPES AND CHANNELS
PIPE
5 X 10^5 (LAUFER)
5 X 10^4 (LAUFER)
4.86 X 10^4 (KOMATSU)
8.61 X 10^4 (KOMATSU)
CHANNEL
1.23 X 10^4 (LAUFER)
3.08 X 10^4 (LAUFER)
6.16 X 10^4 (LAUFER)
EQUATION (1-28)
EQUATION (1-27)
α=5.6 log y/δ + ω(y/δ) BY CORNISH
y/R
0.15
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
1
2
3
4
5
6

Figure 3



TURBULENT SHEAR STRESS AND GRADIENT
OF MEAN VELOCITY
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
12
14
16
ξ=0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Figure 4

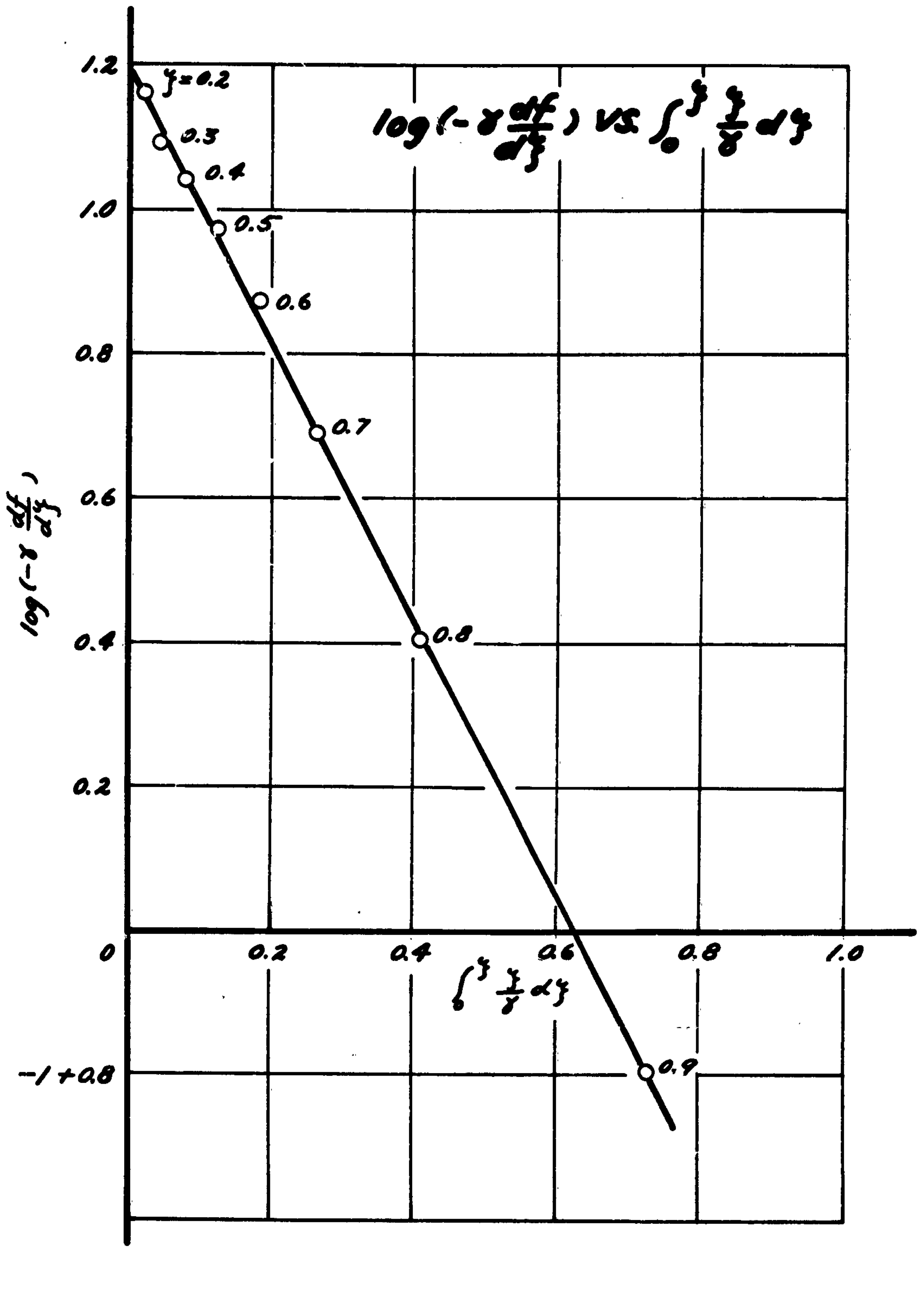
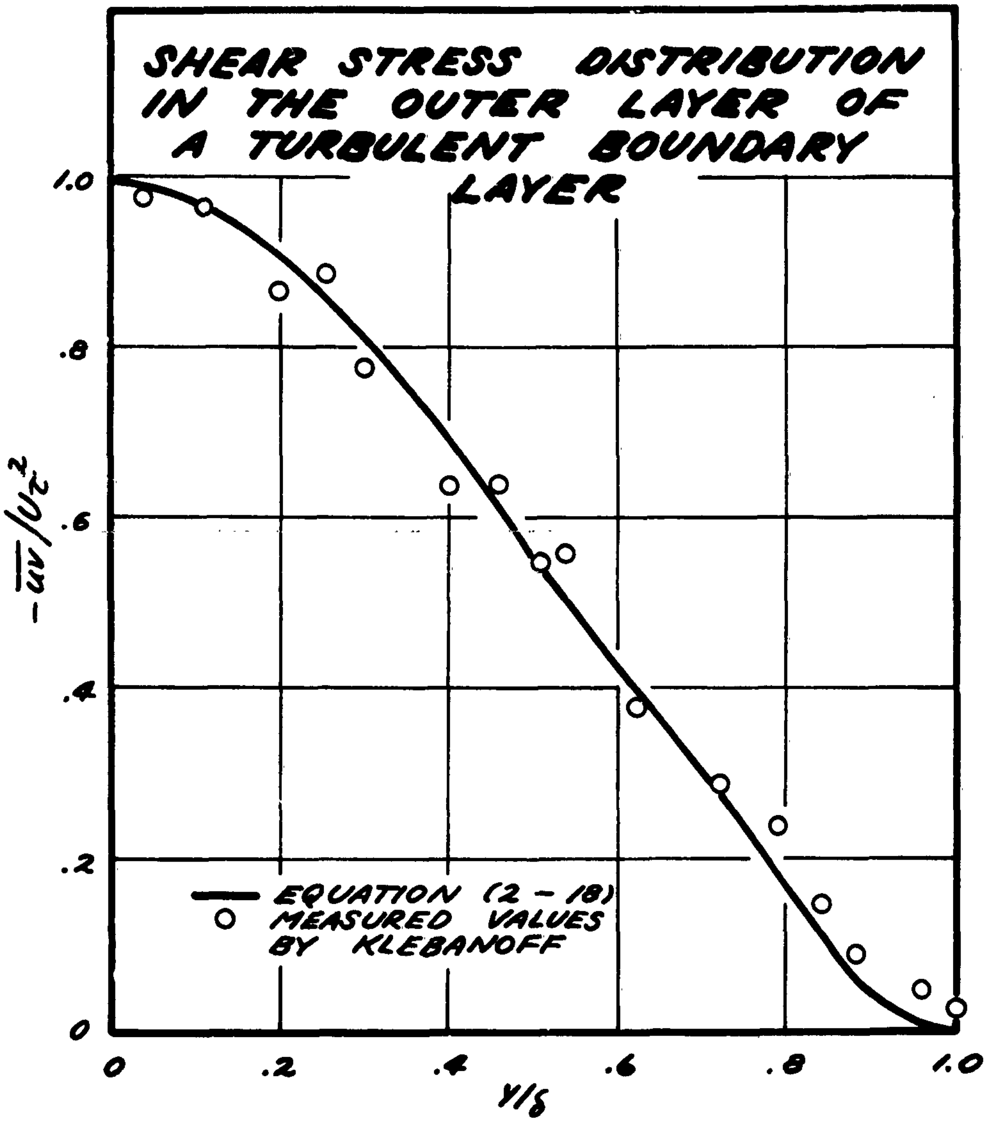
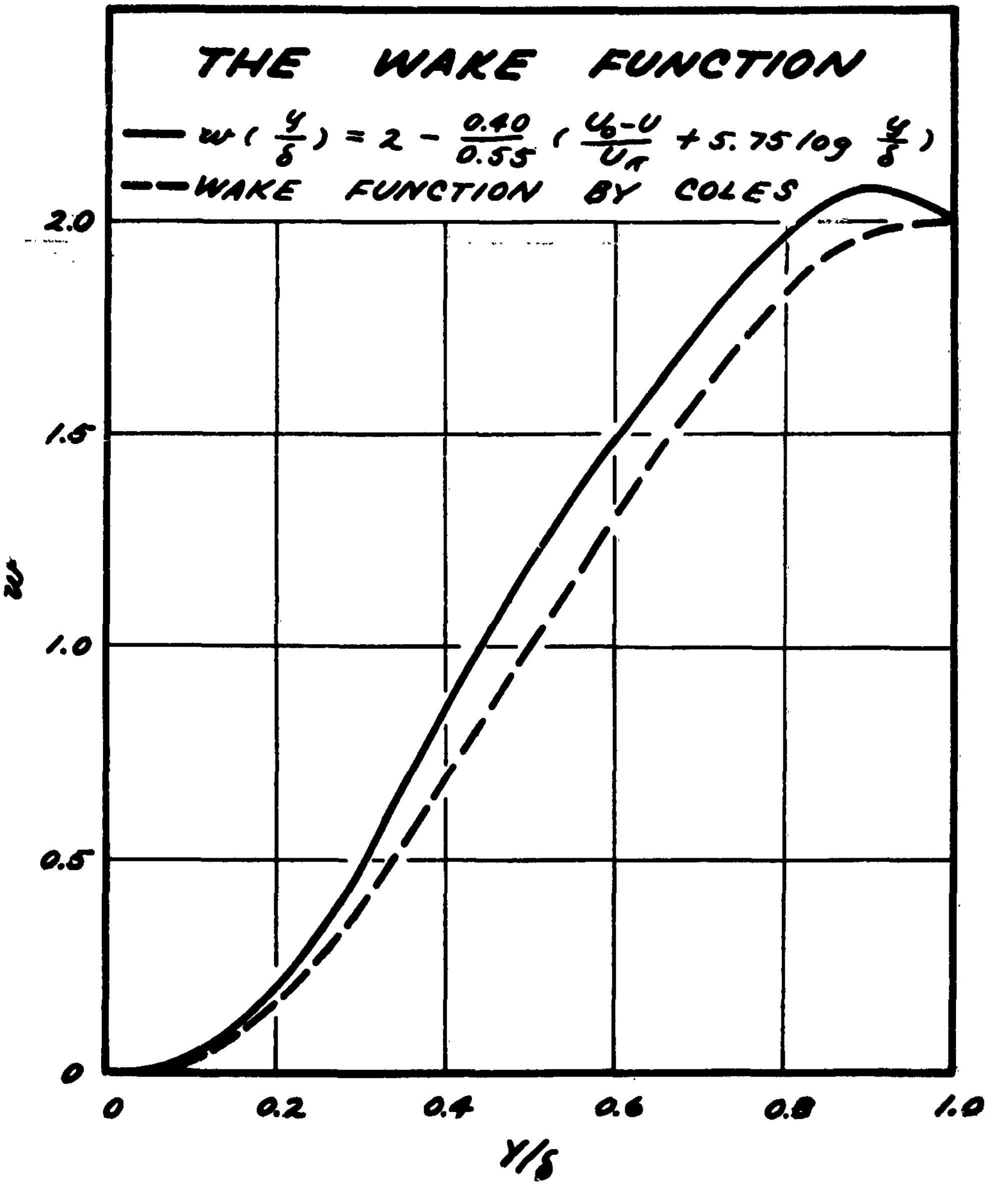

log(- γ dφ/dζ) vs ∫₀^ζ ζ/γ dζ
ζ = 0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.2
1.0
0.8
0.6
0.4
0.2
0
-1 + 0.8
0.2
0.4
0.6
0.8
1.0
log(- γ dφ/dζ)
∫₀^ζ ζ/γ dζ

Figure 5



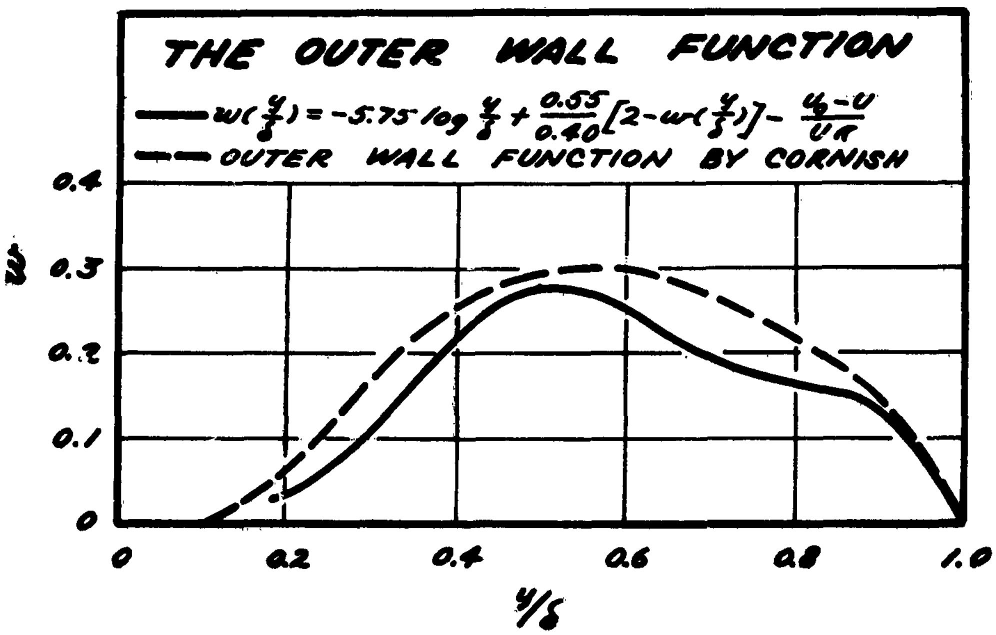
SHEAR STRESS DISTRIBUTION IN THE OUTER LAYER OF A TURBULENT BOUNDARY LAYER
1.0
.8
.6
.4
.2
0
$-\overline{uv}/U_\tau^2$
0
.2
.4
.6
.8
1.0
y/δ
EQUATION (2 – 18)
MEASURED VALUES BY KLEBANOFF

Figure 6



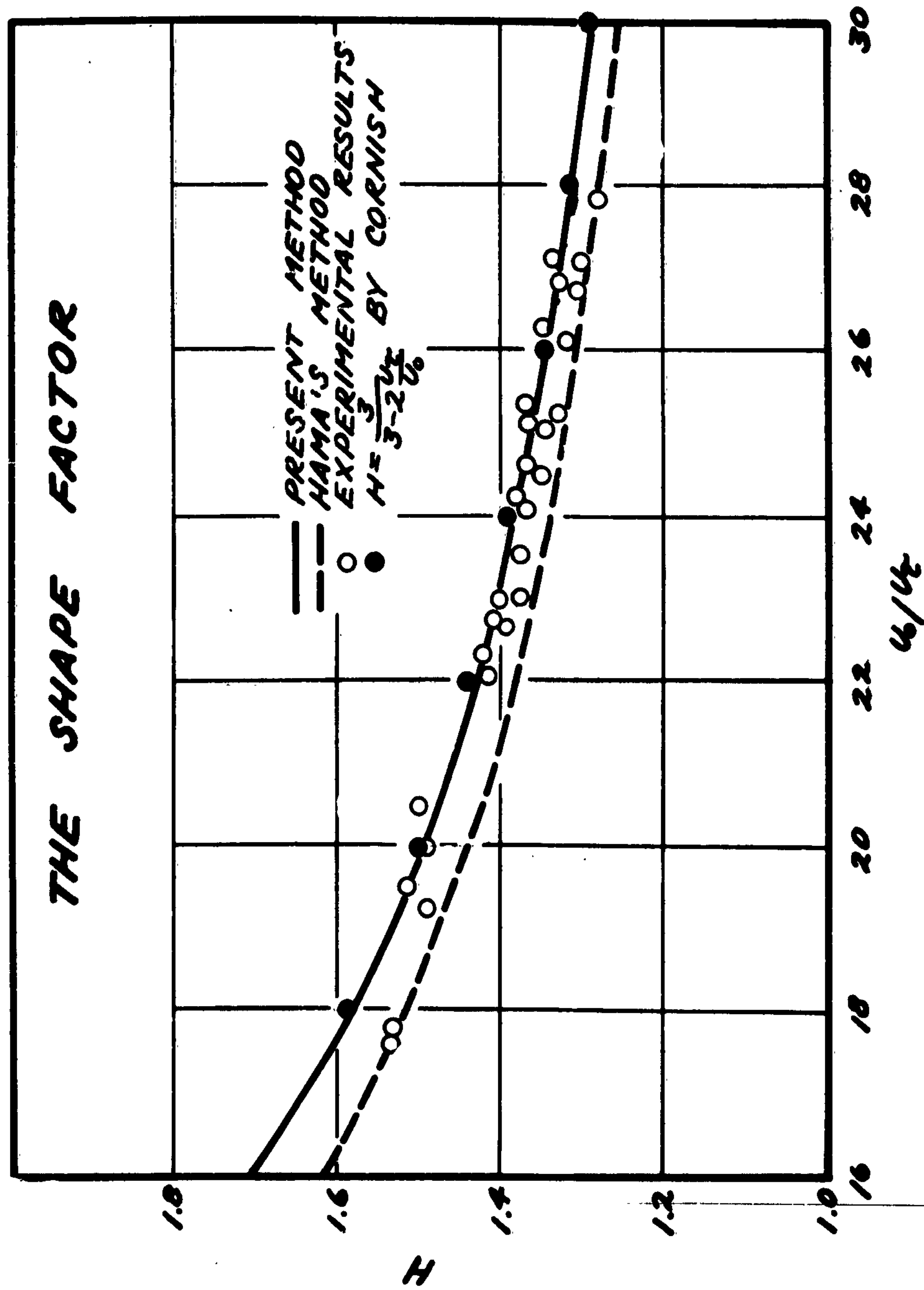
THE WAKE FUNCTION
— $w(\frac{y}{\delta}) = 2 - \frac{0.40}{0.55}(\frac{U_0-U}{U_\tau} + 5.75 \log \frac{y}{\delta})$
--- WAKE FUNCTION BY COLES
2.0
1.5
1.0
0.5
0
w
0
0.2
0.4
0.6
0.8
1.0
Y/δ

Figure 7



THE OUTER WALL FUNCTION
$w(\frac{y}{\delta}) = -5.75 \log \frac{y}{\delta} + \frac{0.55}{0.40}[2 - w(\frac{y}{\delta})] - \frac{u_0 - u}{u_\tau}$
OUTER WALL FUNCTION BY CORNISH
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1.0
y/δ

Figure 8


THE SHAPE FACTOR
PRESENT METHOD
HAMA'S METHOD
EXPERIMENTAL RESULTS BY CORNISH
H
1.0
1.2
1.4
1.6
1.8
16
18
20
22
24
26
28
30