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LONG LINE EFFECT IN CONNECTION WITH CAVITY RESONANCES

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ABSTRACT

When resonant cavities are separated from relatively small obstacles by long waveguides, either multiple resonances or resonant frequency shifts may occur. This long line effect is explored for both transmission and reflection cavities and is found to be much more serious in the former than in the latter case. It can usually be eliminated by matching procedures.
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I. Introduction

When cavities are separated from relatively small discontinuities by long transmission lines, one can, to a first order, expect both cavity resonances and resonances of the system formed by the long lines and the discontinuities adjacent to them. The latter are closely spaced and in part presumably fall into or near the cavity resonant region. Such situations can be expected to "pull" the actual cavity resonant frequencies to a larger or lesser extent, depending on the detailed parameters involved. These "long line" effects are explored below for both transmission and reflection cavities.

Work in this connection was first stimulated by the unexpected appearance of multiple, relatively closely spaced, transmissions of a transmission cavity located between two long waveguides. The subsequent difficulty of deciding on "the" cavity resonance and the disparity between the measured "cavity resonant frequencies" of this cavity as the measurement was repeated several times, with what seemed to be very slight variations, necessitated a detailed analysis.

A simple microwave configuration which might give rise to the long line effect is shown in Fig. 1, typical patterns of transmitted power in Fig. 7 and a convenient network representation for the physical structure in Fig. 4.

![Configuration which might give rise to long line effect.](image)

It is in this representation, and a comparable one in the reflection cavity case, which is analyzed below.

This effect has presumably found its way into many measurements involving long waveguides and cavities, notably measurements at cryogenic temperatures, which usually necessitate transmission lines which are many wavelengths long. If not detected, eliminated, or understood, it may well either cause undue experimental difficulty or unnecessarily large errors. Fortunately, the cure, when it is practical at all, lies not in long calculations or elaborate calibration schemes, but in relatively simple laboratory precautions, i.e., in matching.

II. Cascaded Transmission Lines and Ideal Transformers

Since the representations of the physical structures of interest are based on ideal transformers and transmission lines, a few preliminary remarks concerning these elements
and their relation to each other are in order. The scattering transfer matrix of a two-port is known to be very useful in connection with work involving cascaded network elements. The definition of this matrix, $\mathbf{r}$, as employed here, is

$$
\begin{pmatrix}
  a_1 \\
  b_1 \\
  a_2 \\
  b_2
\end{pmatrix} =
\begin{pmatrix}
  r_{11} & r_{12} \\
  r_{21} & r_{22}
\end{pmatrix}
\begin{pmatrix}
  b_1 \\
  a_1 \\
  b_2 \\
  a_2
\end{pmatrix},
$$

(1)

where $a$ and $b$ are the complex amplitudes of the incident and reflected waves and where subscripts 1 and 2 refer to ports 1 and 2 of the two-port. It is readily seen that the scattering transfer matrix of a transmission line of length $l$ is

$$
\tau =
\begin{pmatrix}
  e^{jkl} & 0 \\
  0 & e^{-jkl}
\end{pmatrix},
$$

(2)

that of an ideal transformer with turns ratio $n:1$, as shown in Fig. 2(a), can be shown to be

$$
\tau = \frac{n^2 + 1}{2n} \begin{pmatrix} 1 & N \\ N & 1 \end{pmatrix}, \quad N \equiv \frac{n^2 - 1}{n^2 + 1}
$$

(3a)

and that of an ideal transformer with turns ratio $1:n$, as shown in Fig. 2(b), then follows as

$$
\tau = \frac{n^2 + 1}{2n} \begin{pmatrix} 1 & -N \\ -N & 1 \end{pmatrix}.
$$

(3b)

Here $n^2$ is, of course, the insertion VSWR of the transformer and $N$ the associated reflection coefficient magnitude i.e., $|S_{11}|$. With $n$ restricted to the range $1 < n < \infty$, it follows that $0 < N < 1$. 

![Fig. 2 Ideal transformer and its dual.](image-url)
The two transformers in Fig. 2 are "dual" to each other in the sense that they may be employed to represent the same structure at "dual planes", i.e., $T_1$ and $T_2$ are respectively located $\lambda_0/4$ away from $T_1'$ and $T_2'$. As a consequence, a variety of representations involving transmission lines and transformers, some convenient, some not, are possible for a given lossless structure. For example, when loss is neglected, a transmission cavity can be represented to a good approximation as in Fig. 3(a), or when $n_2:1$, the dual of transformer $1:n_2'$ is employed, as in Fig. 3(b). Assuming the transmission line length $\ell$ in Fig. 3(a) to be almost equal to the corresponding physical cavity length, as it usually is, the length $\ell' = (\ell + \lambda_0/4)$ in the representation of Fig. 3(b) though proper, may lead to confusion and error. Also, quite apart from physical waveguide lengths, networks with transformers disposed as in Fig. 3(a) resonate when $\ell = n\lambda_0/2$, while those with transformers disposed as in Fig. 3(b) resonate when $\ell' = (n\lambda_0/2 + \lambda_0/4)$, so that the former are seen to be more in accord with the usual notions of resonance.

Fig. 3 Alternative representation of a transmission cavity.

Representations of the type shown in Fig. 3(b), i.e., with all transformers faced in the same direction, have been used, for example, in microwave filter synthesis. For the present purpose, however, it is felt that representations such as that in Fig. 4, which use transformers in the sense of Fig. 3(a) are preferable.

III. Transmission Cavity, Long Line and Small Obstacle

Let the transmission properties of a lossless system constituted of a transmission cavity which is separated from a small discontinuity by a long transmission line be considered. The corresponding equivalent circuit is shown in Fig. 4. (Note the order of transformers: 1, 3, 2). Assuming that waveguides $\ell_1$ and $\ell_2$ are of the same cross-section, a single value of $\chi$ is associated with both. Since the scattering coefficient $S_{21}$,
Fig. 4 Representation of transmission cavity, long line and lossless discontinuity.

which is given by

$$S_{21} = \frac{b^2}{a_1} \left| a_2 = 0 \right.$$  \hspace{0.5cm} (4)

is of interest in connection with the transmission properties of the system, it is worth noting from eq. (1) that

$$S_{21} = \frac{1}{\tau_{11}}.$$  \hspace{0.5cm} (5)

To find the system resonances (transmissions) one need therefore only examine the minima of $|\tau_{11}|$, as will be done below.

Starting with the matrices of eqs (2) and (3), one finds on cascading that $\tau_{11}$ for the network of Fig. 4 is

$$\tau_{11} = \frac{(n_1^2 + 1)(n_2^2 + 1)(n_3^2 + 1)}{8n_1n_2n_3} \left[ e^{j\kappa(\ell_1 + \ell_2)} e^{-N_1N_3} - N_2N_3 e^{j\kappa(\ell_1 - \ell_2)} - j\kappa(\ell_1 - \ell_2) \right] - j\kappa(\ell_1 + \ell_2) \right] \right].$$  \hspace{0.5cm} (6)

This expression is still perfectly general in that no assumptions have been made; it refers to any three ideal transformers with turns ratios in the indicated directions, separated by transmission lines of arbitrary length. On making the very good assumption that reasonably near resonance the transformers have constant values, so that only $\kappa\ell_1$ and $\kappa\ell_2$ are frequency dependent, one sees that the first factor can be discarded and only the minima of a function $F(\kappa)$, which is proportional to $|\tau_{11}|$, needs to be examined. Factoring out $\exp\left[j\kappa(\ell_1 + \ell_2)\right]$ from eq. (6), $F(\kappa)$ is taken as
This function can be visualized as a planar "celestial" system as follows: It gives the variation of the distance from a point (the origin), which is fixed in space with respect to a "sun" (located at +1), to a "satellite" of a "moon" of a "planet" of the same sun.

Equation (7) is invariant to the interchange of indexes 1 and 2, so that in the case of interest it is not material whether the cavity lies to the left or the right of the long line. However, let us assume for definiteness that transformers 1 and 3 are associated with the cavity, so that $N_1$ and $N_3$ are both reasonably close to unity and $l_1$ is short. The length $l_2$ then represents the long line and is large, and $N_2$ represents the discontinuity.

The motion of the satellite is somewhat easier to picture when eq. (7) is reordered as

$$F(K) = \left| 1 - N_1 N_3 e^{-j2\kappa l_1} - N_2 N_3 e^{-j2\kappa l_2} + N_1 e^{-j2\kappa (l_1 + l_2)} e^{j2\kappa l_2} \right|. \quad (7)$$

In this equation one recognizes $N_2 \left[ N_1 \exp(-j2\kappa l_1) - N_3 \right]$ as a vector with an $\kappa l_1$ dependent magnitude which rotates essentially at $\exp(-j2\kappa l_2)$, i.e., very quickly. It is at a minimum at the resonant frequency of the cavity, $2\kappa l_1 = 2\pi$. When $N_2 = 0$, i.e., when there is no obstacle, eq. (8) shows $F(K)$ to be the distance from an origin to a circle of radius $N_1 N_3$ centered at the point 1. The frequency associated with that point on this circle located closest to the origin is the resonant frequency of the cavity itself. However, when $N_2 \neq 0$, $F(K)$ moves quickly through a series of maxima and minima. Each such minimum corresponds to a maximum value of $|S_{21}|$; the frequency associated with it is consequently taken as a system resonance point.

Figure 5 shows $F(K)$ for a particular set of network parameters, though not explicitly, as the distance from the origin to the satellite path (dashed spiral). The ratio $l_2/l_1$ has been taken as 36 in this case. The cavity irises have insertion VSWR's of 19 and 39 respectively; the discontinuity has an insertion VSWR of 3. For convenience, but quite arbitrarily, the phase of $2\kappa_0 l_2$ has been taken as $2\pi$ at cavity resonance so that the spiral is symmetric about the line $0 - 1$. In the absence of the discontinuity, $F(K)$ would be the distance from the origin to the path of the planet. The circles $C_1$, $C_2$, and $C_3$ represent limits of sensitivity of the detection system so that transmissions corresponding to any portion of path lying within such a circle can be seen by the experimenter. For example, assuming $C_1$ (lowest sensitivity), cavity resonance would not be visible in the absence of the obstacle ($N_2 = 0$), but resonances become barely visible at
Fig. 5 Typical long line effect for a transmission cavity: $l_2/l_1 = 6$. 

$\frac{l_2}{l_1} = 36$
$N_1 = .9$
$N_3 = .95$
$N_2 = .5$
$2\kappa_0 l_2 = 2n\pi$
two different frequencies when the obstacle is present. Assuming sensitivity limits $C_2$ and $C_3$, respectively, five and seven frequency-wise roughly equispaced separate transmissions would be observable.

It can be seen that in general the cavity resonant frequency cannot be expected to coincide with one of the system transmission frequencies. The important consequence of this fact is that the cavity resonant frequency cannot be determined, except very approximately, in the presence of additional obstacles separated from the cavity by a long transmission line. When more than one obstacle is present, multiple transmissions can of course also be expected.

When the "long" line is substantially shorter than in the example of Fig. 5, the multiple resonances may no longer be observable, but a substantial difference may exist between the cavity resonant frequency and the system resonant frequency. This is demonstrated in Fig. 6 which differs from that of Fig. 5 only in that $l_2/l_1$ now equals six and in that $2\kappa_0 l_2$ is no longer an even multiple of $\pi$. The figure shows clearly that the satellite moves much more slowly, as it should, that no multiple resonances lie within sensitivity circles $C_1$, $C_2$ and $C_3$ and that symmetry no longer exists about the $0-1$ line. However, it is most important that the system resonant point now lies at a frequency corresponding to $1.014\kappa_0$, and since there is only a single resonant point, it may inadvertently be mistaken for the cavity resonant point.

The "long line effect" has been subjected to qualitative, though not detailed, experimental verification. The power transmitted through circuits of the type under discussion was displayed on an oscilloscope against a frequency scale. A swept source was employed. Ratios of $l_2/l_1$ up to about 47 and a variety of discontinuities were used. Transmissions were predictably spaced and their average amplitudes responded especially to changes in $N_2$. The character of the transmissions near $\kappa_0$ changed rapidly with any small change in $l_2$ or $N_2$. Sketches of typical patterns are shown in Fig. 7.

IV. Reflection Cavity, Long Line and Small Obstacle

One might expect a long line effect, similar to the one discussed above, in connection with a reflection cavity $(n_2, l_2)$ when it is separated from a discontinuity $n_1$ by a long transmission line $l_1$ as shown in Fig. 8. Here $n_2$ is quite large while $n_1$ is
Fig. 6 Typical long line effect for a transmission cavity: $\frac{L_2}{L_1} = 36$. 

$\frac{L_2}{L_1} = 6$

$N_1 = .9$

$N_3 = .95$

$N_2 = .5$

$2\kappa_0 L_2 = 2n\pi + \frac{\pi}{6}$
Fig. 7 Typical transmission patterns in long line effect.

Fig. 8 Representation of discontinuity, long line and reflection cavity.
much smaller. In view of the lossless idealization of the model, one must seek a criterion of resonance other than the normal absorption which takes place in a real dissipative cavity. Such a criterion can be based on the rate of change with \( \kappa \) of \( \theta \), the phase of the input reflection coefficient \( \Gamma \). While expressions for \( d\theta/d\kappa \) are easily derived they do not lead to readily interpretable results unless \( d\theta/d\kappa \) is laboriously computed and then plotted against \( \kappa \). A step by step graphical analysis of the situation, however, based on the "tangent" relation \(2\) for lossless two-ports does give insight, as will be shown, into the long line effect associated with reflection cavities.

With reference to Fig. 7, the electrical length of the short-circuited transmission line \( l_2 \) is \( \kappa l_2 \). As frequency \( (\kappa) \) is varied, it effectively acts as a moving shorting plunger at plane \( T_b \) so that, with \( n_2 \) assumed constant over the frequency range involved, a nodal shift \( \theta_2 \) governed by

\[
\tan \theta_2 = -\frac{n_2}{n_1} \tan(\kappa l_2)
\]

(9)

takes place to the left of transformer \( n_2 \). The frequency dependence of \( \theta_2 \) is not related to \( \kappa l_1 \); as \( \kappa \) increases \( \theta_2 \) decreases, so that the associated minimum moves towards the right. The effective short-circuited length of transmission line to the right of plane \( T_a \) then is \( \kappa l_1 - \theta_2 \) so that the nodal shift \( \theta_1 \) to the left of transformer \( n_1 \) is given by

\[
\tan \theta_1 = \frac{-l_1}{n_1} \tan (\kappa l_1 - \theta_2).
\]

(10)

Note that \( \theta_1 \) is not \( \theta \), the phase of \( \Gamma \), but is simply relate to it by \( \theta = 2\theta_1 + \pi \).

Fig. 9 shows the various phases plotted qualitatively against \( \kappa l_2 \) in units of \( \pi \) radians. The ordinate has been compressed by a factor of eight, \( l_1/l_2 \) has for convenience been taken (as small) as seven and \( -\theta_1 \), rather than \( \theta_1 \), has been plotted to conserve space and add perspective. The points of maximum (positive) slope of the \( -\theta_1 \) versus \( \kappa l_2 \) plot indicate the resonances. It can be seen that the cavity resonances which are located at \( \kappa l_2 = \pi, 2\pi \) are sharp, quite distinct from the "long line" resonances and are influenced by these elements only very slightly. The oscilloscope display of absorptions one would expect to see in this case, but with the normal losses present, is shown below the phase plots. It is understood, of course, that a microwave system could never approach the broad bandedness implied by the example of Fig. 9; nevertheless when \( l_1/l_2 \) is substantially larger than the value used here, the phase behavior in a reasonable band near cavity resonance is of the type shown.
Fig. 9 Typical long line effect for reflection cavity.
The actual resonances over a frequency band of various combinations of reflection cavities, long lines and obstacles were observed on an oscilloscope in the laboratory. They were, as expected, distinctly of the two types (cavity and long line) shown in Fig. 9. Interactions of the two types of resonances in the sense of very small resonant frequency and absorption changes were apparent only when the resonances were very close to each other. These effects were much less marked than those observed in connection with transmission cavities.

V. Conclusions

It has been shown theoretically and verified experimentally that relatively small obstacles separated from either transmission or reflection cavities can give rise to many resonances in the neighborhood of the actual cavity resonance itself. In the reflection cavity case, cavity resonances tend to remain distinct from long line resonances although a small measure of frequency "pulling" does occur when a long line resonance is very close to a cavity resonance. Matching out of the power source, directional couplers, bends, etc., separated from the cavity by a long transmission line, may consequently be indicated when the actual resonant frequency of the cavity is of importance. When no long line resonances are located very near the cavity resonant point, however, such precautions are probably unnecessary in this case.

In the transmission cavity case the long line effect is potentially much more serious. The actual resonance of the cavity is no longer distinguishable and completely misleading large resonances (transmissions) of the system may occur. When multiple resonances are observable near cavity resonances, though one may predominate in amplitude, the predominant resonance is in all probability still quite far from the actual cavity resonant frequency; when no multiple resonances are explicitly observed the possibility of some frequency pulling, even with shorter "long lines", still exists. These effects can be especially troublesome when, for example, frequency adjustments are made by use of a detection system which is subsequently replaced by some other termination since, presumably, the effect may be present, and in different forms, with either of these terminations, unless they are well matched. If these effects are to be avoided, all precautions including the use of well tuned isolators (at the far end from the cavity of any long lines present) should be seriously considered.

While this long line effect was noted and considered in connection with a cryogenic system in which long waveguides are required to reach into a dewar, it no doubt occurs also in a variety of entirely different situations in which cavities and long transmission lines must be used.
References
