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PRICE BEHAVIOR UNDER ALTERNATIVE FORMS OF PRICE EXPECTATIONS

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Price is the causal mechanism through which economists traditionally have studied the interaction of market demand and supply for a commodity. Price elasticities reflect changes in demand and supply for the commodity in response to movements in its price while cross-elasticities measure the effects of such price movements on the demand and supply of other commodities. Price ratios indicate the relative scarcities of commodities. Shifts in preference, technological innovations, excesses and scarcities in both final outputs and factor inputs, whether temporary or permanent eventually manifest themselves through price.

Economists, recognizing that the superposition of changes in a multiplicity of economic variables will influence price, have formulated models of market behavior that explicitly consider the net effect of such changes. This purpose is usually accomplished by including additive random terms in the equations which are thought to characterize behavior in the market. These "catch-all" concepts are considered to be a primary determinant of the price that will prevail at every moment of time.

Economists have also emphasized producers' expectations as determinants of price behavior. More precisely put, the way producers form their expectations of what price will prevail in the market, when their goods are put on sale, substantially influences price itself. The topical

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literature may be exemplified by the cobweb theorem described by Ezekiel<sup>(1)</sup>, the adaptive expectations of Nerlove<sup>(2)(3)</sup>, the rational expectations suggested by Muth<sup>(4)</sup>, and the implicit expectations of Mills<sup>(5)</sup>. Most of the scholarly work on this subject has involved a synthesis of behavioral relationships which assumedly hold in the market and a historical analysis of the price behavior, which the alternative formulations of expectations and random terms imply.

Along with the above-mentioned developments, variance has come to be used frequently as a measure of the degree of uncertainty in economic systems that contain random processes. The minimization of uncertainty (variance) is not considered to be the ultimate goal of the rational man. Conceptually, it is thought of as a cost that is borne by those involved in economic activity, in return for which they receive some form of economic compensation. A good example of this phenomenon at work is to be found in Markowitz<sup>(6)</sup> where uncertainty is shown to be counterbalanced by returns on investments.

Rather than departing from these well-trodden paths, the current effort hopefully will broaden them by analyzing the components of the variance of price. In the models to be considered, a price process  $\{p_t\}$  is assumed to be evolving through time. It is composed of fluctuations of different lengths of time. The relative importance of these fluctuations in shaping the over-all process may show how well a particular model leads to the price behavior commonly observed. This elaboration will extend the traditional synthesis mentioned above by means of the theory of stationary time series, or more precisely, the spectral representation of such series.\* Questions to be answered are,

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\*For a discussion of stationary time series and their spectral representations see Ref. 7, Chap. 6.

for example: What is the relative importance of fluctuations of different lengths of time in the price process? How will reducing the production period affect price? How influential are price expectations on price?

Attention will be restricted to situations wherein temporary shifts in economic phenomena are dominant over permanent changes in the economic structure. This focus precludes any change in the equilibrium price, and, therefore, simplicity dictates that deviation from equilibrium only be considered.\*

Economists commonly develop models of price expectations based on discrete time. For purposes of continuity and comparison with past work on this subject, the current formulations will also be based on discrete time; however, it should be noted the production process is often a pipeline into which raw materials are continually flowing and from which output is constantly emanating. The work of this paper does not pertain to such a process, but rather to the batch type of process like that in agriculture.

Consider a commodity whose demand is unaffected by the prices of other commodities and whose supply is independent of the production of other goods. Let  $p_t$  be a discrete function of time and assume that demands and supply are linear functions of price so that

$$(1a) \quad D_t = -ap_t$$

$$(1b) \quad S_t = bp_t^e + \sum_{\tau=0}^{\infty} c_{\tau} \epsilon_{t-\tau}$$

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\* Muth (8) has described circumstances in which permanent changes in the economic structure manifest themselves through the stochastic process which influences supply. This seems to be an unrealistic oversimplification, since permanent changes should more appropriately be introduced through changes in the slopes of the demand and supply curves and the equilibrium price.

The quantity  $p_t^e$  is the price at the beginning of the production process that producers expect to prevail at time  $t$ , when their goods go on sale.  $\{\epsilon_t\}$  is a discrete stationary stochastic process in the strict sense, whose realizations occur at unit intervals.  $c_\tau$  is a weighting function that accounts for the influence of the realizations of  $\{\epsilon_t\}$  in periods following their occurrence. Assume that the mean and variance of  $\{\epsilon_t\}$  are  $\{0, \sigma^2\}$  respectively. Letting demand equal supply yields

$$(2) \quad ap_t + bp_t^e = - \sum_{\tau=0}^{\infty} c_\tau \epsilon_{t-\tau} .$$

This is the relationship around which the investigation will focus.

If the second moments of  $\{p_t\}$ ,

$$(3) \quad E(p_t p_{t+v} | v) \quad v = 0, \pm 1, \pm 2, \dots, \pm \infty,$$

are finite and dependent on  $v$  alone, the process is said to be wide-sense stationary and possesses an autocovariance function,

$$(4) \quad R_v = E(p_t p_{t+v} | v) \quad v = 0, \pm 1, \pm 2, \dots, \pm \infty.$$

This means that the relationship between prices at different times is dependent only on the time lag between them and in no way on historical time. Such phenomena are characteristic of the short run, in which permanent changes occur slowly and have little effect on price. Note that  $R_v$  is symmetric around  $v$  equal to zero.

It is known that the autocovariance function of a stationary process possesses a spectral representation

$$(5) \quad f(\omega) = \frac{1}{2\pi} \sum_{\nu=-\infty}^{\infty} R_{\nu} e^{-i\omega\nu} \quad 0 \leq \omega \leq \pi ;$$

and conversely,

$$(6) \quad R_{\nu} = \int_0^{\pi} f(\omega) \cos \omega \nu d\omega \quad \nu = 0, \pm 1, \pm 2, \dots \pm \infty.*$$

Observe that  $R_0$  is the variance of  $\{p_t\}$  and is given by

$$(7) \quad R_0 = \int_0^{\pi} f(\omega) d\omega .$$

$f(\omega)d\omega$  is a spectral decomposition of variance into additive components whose sum equals the variance of the process. Therefore,

$$\frac{f(\omega_0)d\omega}{\int_0^{\pi} f(\omega)d\omega}$$

is the relative contribution to the variance made by an infinitesimal band of frequencies around  $\omega_0$ . Economists customarily speak of the lengths of fluctuations with respect to time. Frequency is simply the inverse of time, so that fluctuations taking long periods of time to work themselves out will correspond to low frequencies, whereas short fluctuations are identified with high frequencies. This gives us a method for determining the relative importance of fluctuations of different lengths of time in the variance of  $\{p_t\}$ . If we think of variance as a measure of uncertainty we may consider the spectral density function as the distribution of uncertainty among fluctuations of different lengths of time.

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\*See Ref. 7, pp. 161-166. It is assumed that the spectral density function is everywhere in the interval  $\{0, \pi\}$  continuous. This implies that  $\{p_t\}$  does not contain any strictly periodic components.

It may be asked in what sense this paper discusses the variance of price. More precisely put, is the distribution of variance under scrutiny that which derives from the historical past of the process, or is it a cross-sectional view of the components of variance at any moment of time? These two concepts coincide for stationary processes so that, if a small band of frequency contributes  $x$  to the variance during the historical process, it will also contribute  $x$  at any moment of time.

#### Case 1

As a simple example, consider a market situation wherein the length of the production process is short compared to unity, and/or inconsequential holding costs permit the storage of large quantities of the commodity, which can be instantaneously offered for sale in response to favorable prices. This is the case of perfect price knowledge. Assume that producers react to current prices, and that the random disturbances make themselves felt only at the moment of their realizations. Then

$$(8) \quad p_t^e = p_t, \text{ and}$$

$$(9) \quad c_\tau = \delta_\tau,$$

where  $\delta_\tau$  is the Dirac delta function.

$$(10) \quad \delta_\tau = \begin{cases} 1 & \tau = 0 \\ 0 & \tau \neq 0. \end{cases}$$

Therefore, price may be expressed as

$$(11) \quad p_t = \frac{-\epsilon_t}{a + b},$$

with mean zero and autocovariance

$$(12) \quad R_v = \frac{\sigma^2}{(a+b)^2} \delta_v \quad v = 0, \pm 1, \pm 2, \dots \pm \infty.$$

The corresponding spectral density function (s.d.f.) is

$$(13) \quad f(\omega) = \frac{\sigma^2}{\pi(a+b)^2} \quad 0 \leq \omega \leq \pi.$$

Equation (13) implies that frequencies in the interval  $\{0, \pi\}$  uniformly contribute to the variance. It should be noted that, since  $\{\epsilon_t\}$  is a discrete process whose realizations occur at unit intervals, the most rapid fluctuations that can exist must have a period of two units of time. This means that  $\pi$  corresponds to a period of two units of time. Were  $\{\epsilon_t\}$  to be continuous, all frequencies from zero to infinity would uniformly contribute to the variance.\* Specification of a non-zero interval of unity is, therefore, a convenient way of restricting the frequency range to one of relevance. This case is of course the natural outcome of a process whose realizations are completely unrelated to each other, as shown by Eq. (12), and therefore favors no particular band of frequencies.

### Case 2

In reality, many disturbances affect demand and supply long after the times of their realizations. Bad weather, for example, lessens the productivity of soil even during succeeding periods of more favorable conditions. Reductions in income may influence demand for a particular commodity long after the economy has begun to recover. To illustrate

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\*See ibid., p. 166.

a situation in which the entire past history of the supply disturbances makes itself felt, consider a geometric weighting scheme,

$$(14) \quad c_{\tau} = (1-\alpha) \alpha^{\tau} \quad 0 < \alpha < 1 ,$$

such that

$$(15) \quad p_t = \frac{-(1-\alpha) \sum_{\tau=0}^{\infty} \alpha^{\tau} \epsilon_{t-\tau}}{a+b}$$

Here the influence of past disturbances declines as time evolves. The corresponding spectral density function is

$$(16) \quad f(\omega) = \left[ \frac{(1-\alpha)^2 \sigma^2}{\pi(a+b)^2 (1-2\alpha \cos \omega + \alpha^2)} \right] \quad 0 \leq \omega \leq \pi .$$

### Case 3A

In general, the length of the production process is such that producers can only guess what the prevailing price will be when the goods, which are about to be processed, are put on sale. If the disturbances are not autocorrelated, it would seem rational for producers to assume that the equilibrium price will prevail. Then

$$(17a) \quad p_t = \frac{-\epsilon_t}{a} ;$$

$$(17b) \quad f(\omega) = \frac{\sigma^2}{\pi a^2} \quad 0 \leq \omega \leq \pi .$$

The s.d.f.'s of Cases 1 and 3A are shown in Fig. 1 for purposes of comparison. Observe that using the mean as the expected price results in a uniformly higher variance (Case 3A) than would exist if producers were able to respond immediately to price movements.

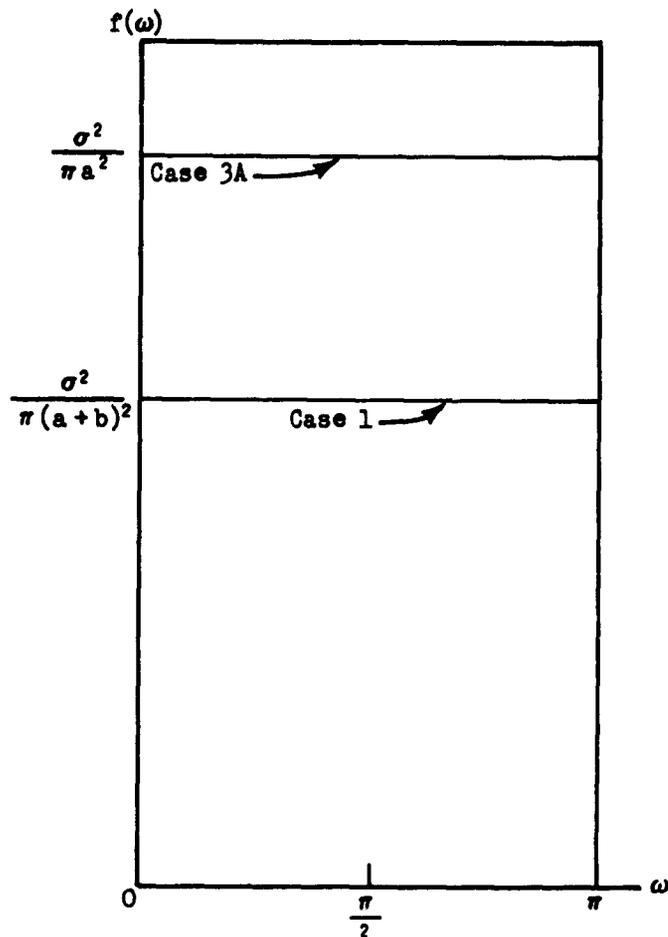


Fig. 1 -- Spectral Density Functions of Cases 1 and 3A

Case 3B

Were producers to use the equilibrium price as their estimate of future price, in the presence of geometrically weighted disturbances, price would be given by

$$(18a) \quad P_t = \frac{-(1-\alpha) \sum_{\tau=0}^{\infty} \alpha^{\tau} \epsilon_{t-\tau}}{a}$$

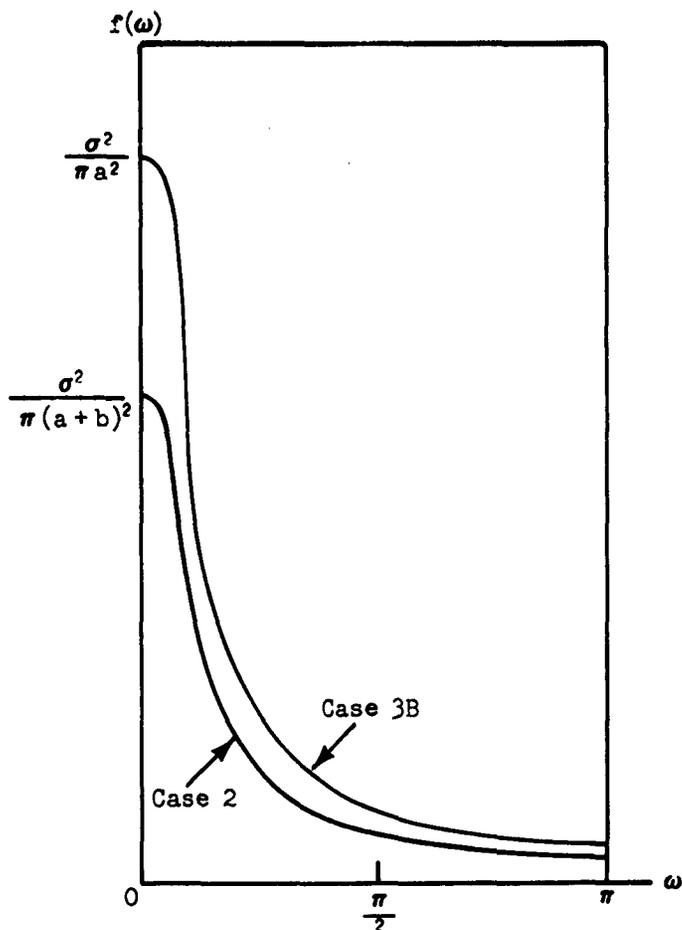


Fig. 2 -- Spectral Density Functions of Cases 2 and 3B

The resultant s.d.f. is

$$(18b) \quad f(\omega) = \left[ \frac{(1-\alpha)^2 \sigma^2}{\pi a^2 (1-2\alpha \cos \omega t + \alpha^2)} \right] \quad 0 \leq \omega \leq \pi,$$

and is shown in Fig. 2 along with the s.d.f. of Case 2.

Observe that in both cases the curves of variance shape downward to the right. This means that these price processes will be characterized by long-term drifts in price around equilibrium and that the changes in

price in successive periods will be small compared to long-term changes. The two curves contrast, as in Fig. 1, in that the curve of perfect knowledge is everywhere lower than the curve resulting from the use of the equilibrium price. This difference becomes smaller in absolute terms at higher frequencies, but the relative relationship is constant throughout.

It is of course somewhat unrealistic to think that producers would continue to intend to produce the equilibrium quantity period after period in spite of this long-term drift in price. It is true that sharply rising cost curves around equilibrium may restrict response to price movements somewhat. However this seems to be unlikely behavior for producers since no cognizance is taken of long term movements which would allow them to adjust capacity appropriately.

It will nevertheless be helpful to use these two curves for comparative purposes in what follows.

The s.d.f.'s associated with perfect knowledge are effectively the distributions of uncertainty which must exist regardless of the form of price expectations. The s.d.f.'s for equilibrium behavior represent the distribution of uncertainty if suppliers pay no attention to price. The aim of producers is to reduce uncertainty as close as possible to the lower bound. We shall see what distributions result from the alternative models considered.

Another observation about Fig. 2 should be made in passing. The larger is  $\alpha$  (i.e., the more influential is the past in the present), the faster are the declines of the s.d.f.'s. This means that the more relevant is the past in the present, the more apparent will be the long-term fluctuations in price.

The fact that  $f(\omega)$  is greater than zero over a range of frequencies does not mean that corresponding fluctuations will be observable in  $\{p_t\}$ . The summation of equally weighted fluctuations of different lengths of time will not generate a particular appearance for  $\{p_t\}$ . Fluctuations of a given length will, however, be discernible to the extent that  $f(\omega)$ , at that frequency, dominates  $f(\omega)$  for the remaining frequencies in the immediate vicinity. In Cases 2 and 3B, one should therefore expect long-term drifts in price with no fixed period.

We see that uncertainty may be categorized with respect to time. Producers would naturally like to reduce short- as well as long-run uncertainty to a minimum. The presence of autocorrelated phenomena of the sort described give rise to long-term drifts in price. If producers were to assume that the equilibrium price would prevail in the future, they would be sorely disappointed. In reality successive periods of above- and below-equilibrium prices would occur. It seems only natural that producers would observe this behavior and try to take advantage of it. It is generally taken for granted that economic phenomena are autocorrelated, and, therefore, it is this situation with which the economist is mainly preoccupied. Attention is now turned to the effects on price resulting from price expectations formed in several alternative ways suggested in the literature.

#### Case 4A

Consider a situation in which disturbances are not autocorrelated and the price at the beginning of the production process is assumed by producers to prevail at the time of sale. This is the essence of the "cobweb" theorem which Ezekiel<sup>(1)</sup> has attributed to Ricci<sup>(9)</sup>, Schultz<sup>(10)</sup>

and Tinbergen. (11) Then

$$(19) \quad ap_t + bp_{t-\tau_1} = -\epsilon_t,$$

where  $\tau_1$  is the production time. This results in

$$(20) \quad p_t = -\frac{1}{a} \sum_{n=0}^{\infty} \left(-\frac{b}{a}\right)^n \epsilon_{t-n\tau_1}.$$

Note that if  $b$  is greater than  $a$ , fluctuations in price will increase in magnitude as time evolves. This means that the process is dependent on historical time and that the variances is heteroscedastic. Therefore, the process is non-stationary. If  $b$  is less than  $a$ ,  $\{p_t\}$  is stationary and it is this situation to which attention is confined.

The corresponding spectral density function is

$$(21) \quad f(\omega) = \frac{\sigma^2}{\pi(a^2 + 2ab \cos \omega\tau_1 + b^2)} \quad 0 \leq \omega \leq \pi;$$

it is shown in Fig. 3 along with the spectra of Cases 1 and 3A.

In Fig. 3, the common practice of setting  $\tau_1$  equal to the interval between successive disturbances has been followed. Notice that the variance attributable to long- and medium-term fluctuations has been reduced as compared to Case 3A, but that the variance of the extremely short fluctuations has been increased. This implies that price will change radically in the short run, precisely the result which is most undesirable.

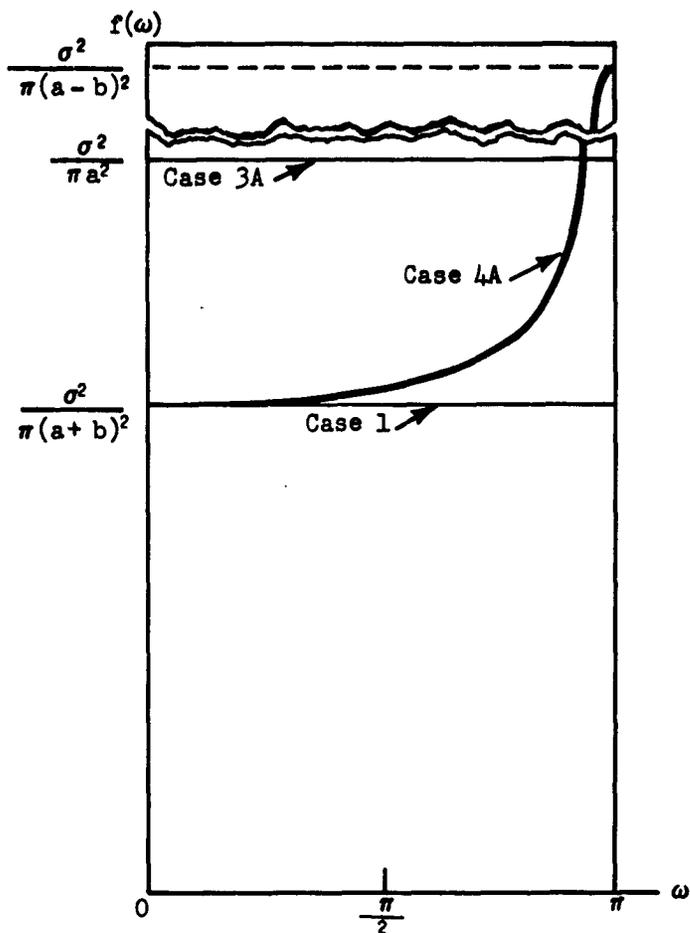


Fig. 3 -- Spectral Density Functions of Cases 1, 3A and 4A

#### Case 4B

Turning now to the situation in which disturbances are autocorrelated results in

$$(22a) \quad ap_t + bp_{t-\tau_1} = -(1-\alpha) \sum_{\tau=0}^{\infty} \alpha^\tau \epsilon_{t-\tau}$$

$$(22b) \quad p_t = -\frac{(1-\alpha)}{a} \sum_{n=0}^{\infty} \left(-\frac{b}{a}\right)^n \sum_{\tau=0}^{\infty} \alpha^{\tau-n\tau_1} \epsilon_{t-n\tau_1-\tau}$$

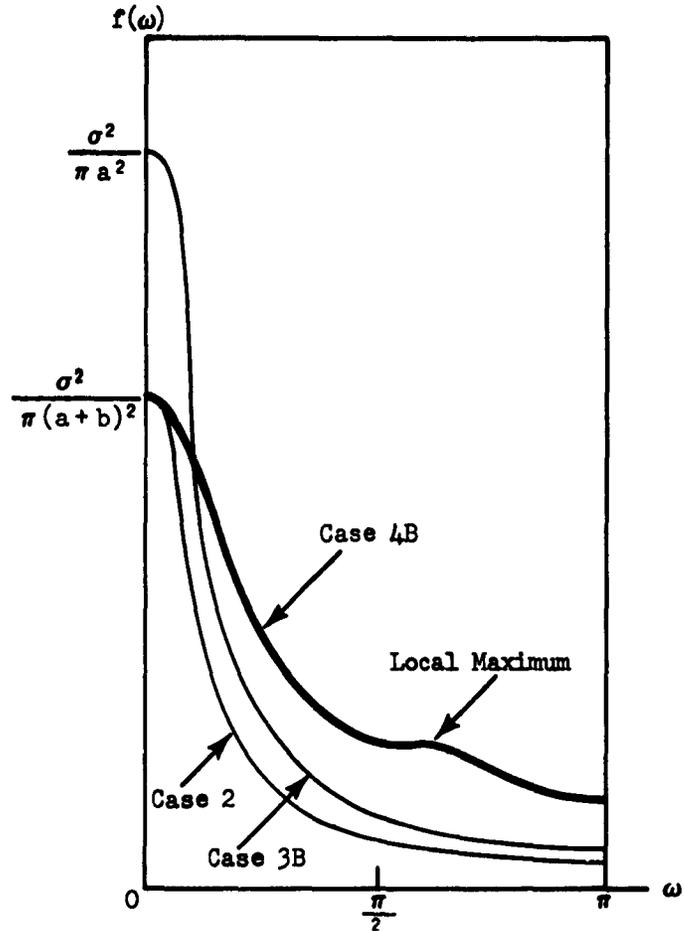


Fig. 4 -- Spectral Density Functions of Cases 2, 3B and 4B

$$(22c) \quad f(\omega) = \left[ \frac{(1-\alpha)^2 \sigma^2}{\pi(a^2 + 2ab \cos \omega \tau_1 + b^2)(1 - 2\alpha \cos \omega + \alpha^2)} \right] \quad 0 \leq \omega \leq \pi.$$

For  $\tau_1$  equal to unity, a local maximum occurs in  $f(\omega)$  (see Fig. 4) at

$$(22d) \quad \omega = \cos^{-1} \left[ \frac{\alpha(1+\alpha^2) + ab(a^2+b^2)}{2(\alpha^2 - a^2 b^2)} \right] \quad 0 \leq \omega \leq \pi.$$

Notice that the s.d.f. of Case 4B slopes down to the right except for the single "bump" at the local maximum. To the extent that a concentration

of variance exists around the mid-band frequency in this bump,  $\{p(t)\}$  will exhibit a corresponding periodicity. However, this periodicity will be somewhat obscured by the presence of long-term drift. Note also that, since  $\alpha^2$  is usually less than  $a^2 b^2$  this maximum will occur at a frequency greater than  $\pi/2$  implying a period of less than 4 time units. Observe also that the extremely short fluctuations of Case 3A are nowhere in evidence in Case 3B.

The professional literature abounds with justifiable criticisms of the use of the cobweb theorem in the presence of unrelated disturbances. However this last illustration suggests greater plausibility for the cobweb theorem in the presence of autocorrelated disturbances.

#### Case 5A

It seems reasonable to suppose that producers try to profit from past errors in their expectations when theorizing about future price. The "adaptive" expectations model outlined by Nerlove<sup>(2)(3)</sup> is designed to account for such behavior. It is assumed that the change in expectations is proportional to the most recent error in prediction, so that

$$(23a) \quad p_t^e - p_{t-1}^e = (1-\beta)(p_{t-1} - p_{t-1}^e).$$

$$(23b) \quad p_t^e = (1-\beta) \sum_{\tau=0}^{\infty} \beta^\tau p_{t-\tau-1} \quad 0 < \beta < 1.$$

Assuming that disturbances are uncorrelated results in

$$(24a) \quad f(\omega) = \frac{\sigma^2(1-2\beta\cos\omega+\beta^2)}{\pi\{[b(1-\beta)-a\beta]^2 + 2a[b(1-\beta)-a\beta]\cos\omega+a^2\}} \quad 0 \leq \omega \leq \pi.$$

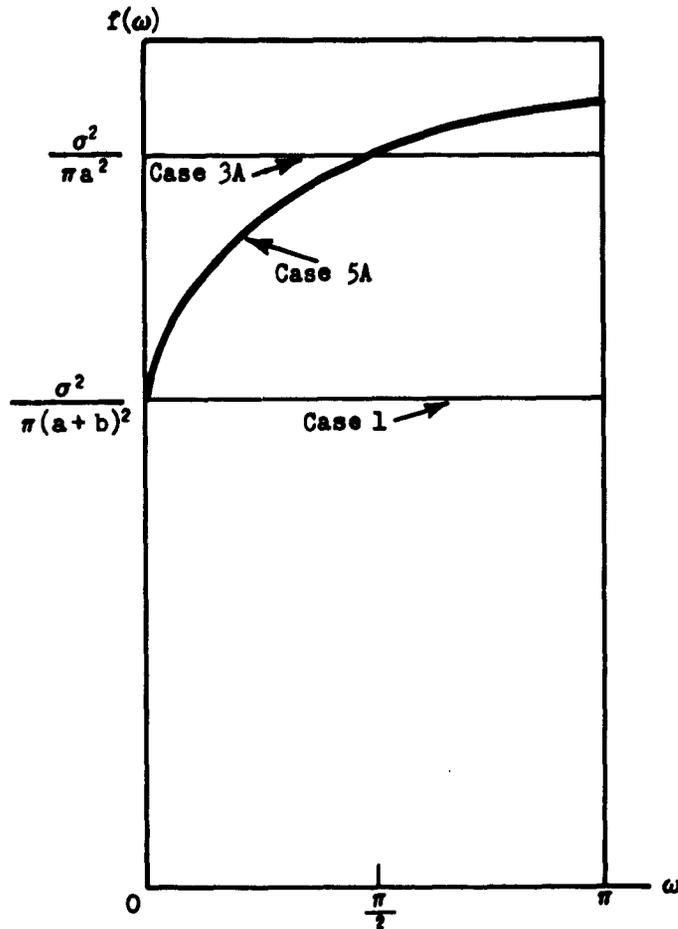


Fig. 5 -- Spectral Density Functions of Cases 1, 3A and 5A

The stability condition that

$$(24b) \quad -\frac{(1+\beta)}{(1-\beta)} < \frac{b}{a} < 1$$

is required in order for (24a) to exist.\*

Figure 5 shows the corresponding  $f(\omega)$  along with those of Cases 1 and 3A. One observes that the variance at low frequencies has been

\* See Ref. 2, p. 233.

reduced, compared to Case 3A, whereas the variance at high frequencies has been increased. It can be shown that the smaller is  $\beta$  (i.e., the more important is the adjustment in Eq. (23a)), the greater will be this distortion in the s.d.f. The limiting form occurs when  $\beta$  equals zero and the adaptive formulation coincides with the cobweb theorem.

Mills<sup>(12)</sup> has criticized the adaptive expectations formulation on the grounds that the most rational estimate of future price is the equilibrium price. He therefore concludes that rational producers cannot be expected to form their estimates of future prices by using the adaptive expectations formulation, since it will inevitably lead them to wrong results. The example of Case 5A agrees with this criticism in that the redistribution of variance seems to be the disadvantage of producers.

In a reply to Mills, Nerlove<sup>(13)</sup> admits the validity of this argument for the case in which the disturbances are not autocorrelated; however, he justifiably points out that disturbances in economies are usually related and it is to such cases that adaptive expectations are pertinent. With this in mind, consider the effect on variance of assuming adaptive expectations in the presence of geometrically autocorrelated phenomena.

#### Case 5B

Price is given by

$$(25) \quad ap_t + b(1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} p_{t-\tau-1} = -(1-\alpha) \sum_{\tau=0}^{\infty} \alpha^{\tau} \epsilon_{t-\tau},$$

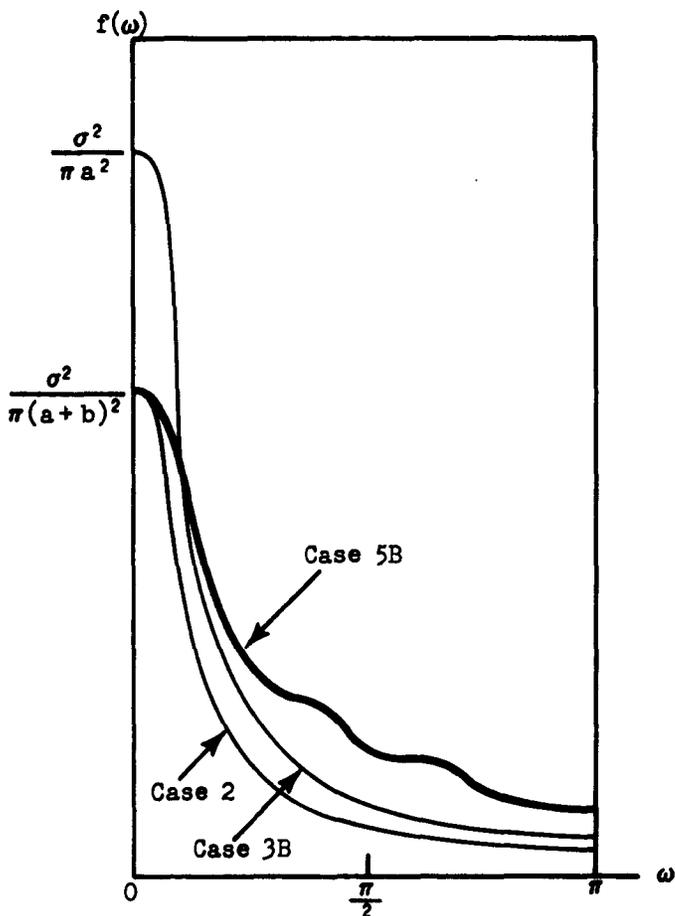


Fig. 6 -- Spectral Density Functions of Cases 2, 3B and 5B

which yields

$$(26) \quad f(\omega) = \frac{\{[\gamma(1-\alpha)^2 \sigma^2 / (1-2\alpha \cos \omega + \alpha^2)] [1-2\beta \cos \omega + \beta^2]\}}{\pi \{[b(1-\beta) - a\beta]^2 + 2a[b(1-\beta) - a\beta] \cos \omega + a^2\}} \quad 0 \leq \omega \leq \pi .$$

Figure 6 compares the spectra for this case with those of Cases 2 and 3B.

Observe that the curve of variance for adaptive expectations is everywhere greater than the curve associated with perfect price knowledge

(Case 2), and diverges from it as frequency increases. This implies that the use of adaptive expectations in place of perfect price knowledge results in an increase in variance primarily for short-term movements. Bear in mind, however, that this increase is less than in the case of the cobweb theorem.

Comparing the adaptive expectations curve with the variance that would result if producers assumed the equilibrium price would prevail (Case 3B) shows us that the former curve is below the latter for low frequencies and above it at high frequencies, just as was true for the cobweb theorem. It can be shown that, at most, two local maxima may exist in  $f(\omega)$  for Case 5B. As mentioned before, these "bumps" may sometimes be noticeable concentrations of variance and may then be somewhat observable in  $\{p_t\}$ . It should be noted in passing that the s.d.f. for the cobweb theorem will lie above the s.d.f. for the adaptive expectations in the range of short-run fluctuations. This means that less short-run uncertainty is to be associated with the former than with the latter.

Before continuing to an analysis of the variance associated with the rational expectations of Muth<sup>(4)</sup>, it seems appropriate to discuss how well the assumption of rationality holds for producers in the adaptive expectations formulation when disturbances are autocorrelated as assumed here. We have seen that the Nerlove model causes a reduction in long-term variance and an increase in short-term variance relative to the variance associated with assuming the equilibrium price will prevail. Here it is not possible to speak of a reduction in variance as being a decline in uncertainty, because what has actually taken place is a redistribution of relative uncertainty toward short-term movements.

Such a redistribution forces producers to seek access to sources of short-term financing to help them over temporary adversities. Similarly, the ease with which producers can change their level of production is a determinant of how this increased short-run variance will affect them. Were sellers to behave as in Case 3B, long periods of profit accumulation would be followed by extended periods of unfavorable prices. This, of course, would not occur with any fixed timing, and producers could not be certain as to how long either behavior would continue. This price behavior will exist somewhat, even with the use of adaptive expectations. However long fluctuations will be less accentuated than if the equilibrium price is used.

It may appear reasonable to assume that sellers would try to take advantage of this drift in price even at the risk of creating some short-run uncertainty. Remember, however, that the short-term variance is small compared to the over-all variance, and that a small increase in absolute terms may be very significant relative to the already existing short-run uncertainty.

#### Case 6

Muth<sup>(4)</sup> argues that if one assumes rational behavior on the part of producers, their informed guess of future price will be the same as that suggested by economic theory. He considers price to be a weighted linear combination of the disturbances affecting supply. It is assumed that expected price is a linear combination of the corresponding disturbances up to the moment that expectations are formed, and that these disturbances are weighted just as those in price are; that is,

$$(27a) \quad p_t = \sum_{\tau=0}^{\infty} w_{\tau} \epsilon_{t-\tau} ;$$

$$(27b) \quad p_t^e = \sum_{\tau=0}^{\infty} w_{\tau} \epsilon_{t-\tau-1} .$$

Muth acknowledges the fact that, in the presence of uncorrelated disturbances, the equilibrium price would be the rational expectation.

Therefore, attention is confined to the autocorrelated case.

Here

$$(28a) \quad a \sum_{\tau=0}^{\infty} w_{\tau} \epsilon_{t-\tau} + b \sum_{\tau=0}^{\infty} w_{\tau} \epsilon_{t-\tau-1} = -(1-\alpha) \sum_{\tau=0}^{\infty} \alpha^{\tau} \epsilon_{t-\tau} ;$$

$$(28b) \quad w_0 = - \frac{(1-\alpha)}{a} ; \text{ and}$$

$$(28c) \quad w_{\tau} = - \frac{(1-\alpha)\alpha^{\tau}}{a+b} \quad \tau = 1, 2, \dots \infty .$$

Therefore,

$$(29) \quad p_t = - \frac{(1-\alpha)}{a} \epsilon_t - \frac{1-\alpha}{(a+b)} \sum_{\tau=1}^{\infty} \alpha^{\tau} \epsilon_{t-\tau} ,$$

and

$$(30) \quad f(\omega) = \left\{ \frac{(1-\alpha)^2 [(a+b)^2 - 2(a+b)b\alpha \cos \omega + b^2 \alpha^2]}{\pi a^2 (a+b)^2 (1 - 2\alpha \cos \omega + \alpha^2)} \right\} \quad 0 \leq \omega \leq \pi .$$

Figure 7 shows an s.d.f. which clearly contrasts with those of the alternative models considered. For Case 6,  $f(\omega)$  begins at a point

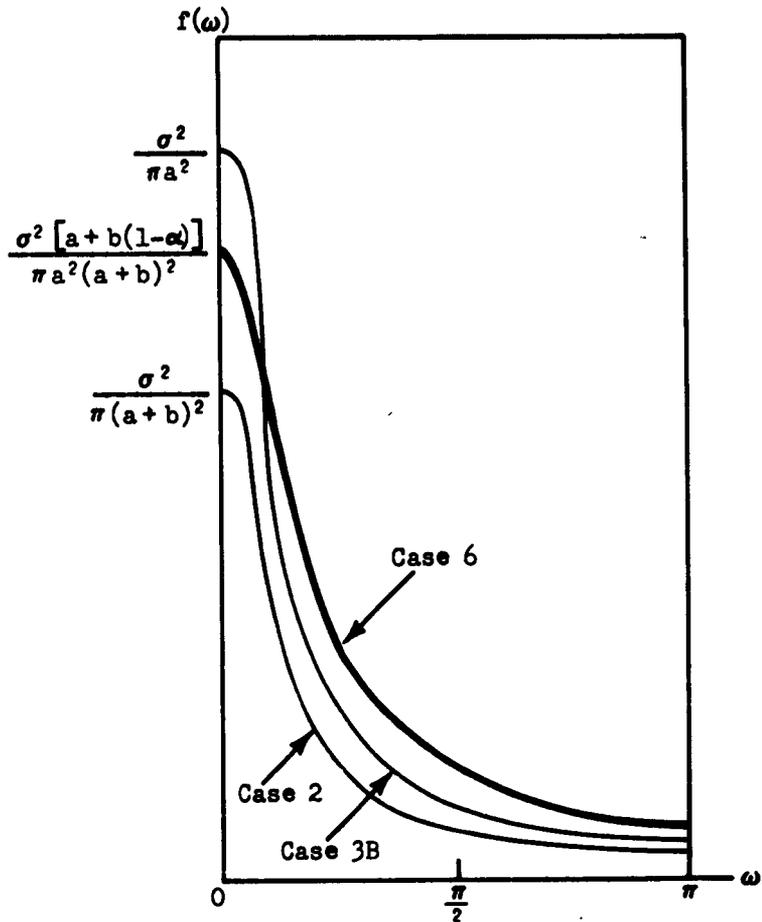


Fig. 7 -- Spectral Density Functions of Cases 2, 3B and 6

above the curve of perfect price knowledge, whereas the other s.d.f.'s mentioned begin at exactly the same point

$$\frac{\sigma^2}{\pi(a+b)^2}$$

This means that the variance attributable to long-term fluctuations is greater in the rational expectations model than in the other two considered. As previously mentioned, however, one must also consider short-run variance. A comparison of the current model under scrutiny

with the former two shows that the rational expectations hypothesis yields an s.d.f. that is lower than the other two in the upper frequency range. Note, however, that this model nevertheless has a greater s.d.f. in the upper frequency range than does the equilibrium price model.

One may therefore conclude that the rational expectations hypothesis is more effective in keeping down uncertainty in the range of short-run fluctuations, whereas the adaptive expectations yields less uncertainty in the range of long-run fluctuations. It should be mentioned in passing that Muth<sup>(8)</sup> has shown that, if the disturbances are the sum of permanent and transitory effects, adaptive expectations coincide with rational expectations.

#### CONCLUSIONS AND OBSERVATIONS

Alternative forms of price expectations have been described here for the purpose of examining the distribution of price variance which they imply both in the presence of uncorrelated and autocorrelated disturbances. It has been shown that the cobweb theorem, the adaptive expectations formulation, and the rational expectations hypothesis all cause a distribution of variance favoring short-run uncertainty as contrasted with the distribution of variance that would result if the equilibrium price were expected to prevail.

The cobweb theorem gave rise to extremely short-run periodicities in the presence of uncorrelated disturbances. For autocorrelated disturbances, the rational expectations hypothesis yielded a higher variance than did the cobweb theorem and the adaptive expectations formulations for lower frequencies, but had a lower variance at high frequencies. All three models showed smaller variances at low frequencies than resulted in the case of equilibrium price.

As stated before, it seems reasonable to suppose that the equilibrium price expectation is the most rational in the case of uncorrelated disturbances. Indeed no price patterns would exist even if producers had a perfect knowledge of price, and, therefore, rational behavior dictates expectations which do not induce autocorrelated behavior such as is shown in Cases 4A and 5A.

Turning to the case of autocorrelated disturbances uncovers a much more involved problem. Unlike classical prediction problems, the form of producers' expectations influences the behavior of price. Therefore, one must decide what sort of expectations result in behavior favorable to producers in order to decide what is rational behavior. The degree to which a particular distribution of uncertainty, over that which inherently must exist, is tenable is dependent on the particular characteristics of the market considered. To assume that producers always try to produce the same quantity is to imply that they do not attempt to take advantage of long-term drift in price. This conclusion is of course contrary to fact for most industries. One would expect producers to plan their long-run future capacity on the basis of long-run price movements. Therefore, they will be interested in keeping long-run uncertainty due to expectations down, especially since the built-in uncertainty to begin with, is primarily associated with long-term movements.

One concludes on the basis of the analysis presented that the form price expectations take must be partly associated with the sorts of uncertainty that producers are willing to live with. The fact that the rational expectations hypothesis yields the smallest variance for short-run fluctuations of the three models considered is certainly a point in its favor; however, this advantage must be weighed against

the greater variance it induces for long-term drift, as compared with adaptive expectations.

Probably the most serious short-coming of this paper has been the glaring omission of any explicit mention of inventories. For this the writer humbly apologizes. The intent of this paper has been to describe, with the aid of spectral descriptions, the resultant uncertainty of alternative forms of price expectations in the presence of uncorrelated and autocorrelated disturbances. Simplicity was needed in order not to obscure the meaning of the results obtained. Hence the omission of inventories. Hopefully, future work will enable the writer to remedy this shortcoming and consider the results of including inventories.

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