INVESTIGATION OF GLASS-METAL COMPOSITE MATERIALS

P. A. Lockwood

Contract NOrd 15764
Fourth Annual Progress Report
Covering Period March 15, 1958 to June 15, 1959
(Also includes Thirteenth Quarterly Progress Report)

ALL DISTRIBUTION OF THIS REPORT IS CONTROLLED.
QUALIFIED USAV USERS SHOULD REQUEST THROUGH
THE CHIEF, BUREAU OF NAVAL WEAPONS, WASHINGTON D.C.

Qualified requesters may obtain copies of this report from ASTIA.

Owens-Corning FIBERGLAS Corporation
Basic and Applied Research Center
Newark, Ohio
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>SUMMARY</td>
<td>2</td>
</tr>
<tr>
<td>DISCUSSION</td>
<td>9</td>
</tr>
<tr>
<td>Development of Glass-Metal Composites For Elevated Temperature Service</td>
<td>9</td>
</tr>
<tr>
<td>Research Into Strength of Metal-Coated Glass Fibers and Methods of Improving the Coated Fiber Strength</td>
<td>11</td>
</tr>
<tr>
<td>Research Into Composites</td>
<td>15</td>
</tr>
<tr>
<td>Theoretical Studies of Glass-Metal Composites</td>
<td>19</td>
</tr>
<tr>
<td>FUTURE PLANS</td>
<td>21</td>
</tr>
<tr>
<td>APPENDIX A - Tables and Figures</td>
<td>22</td>
</tr>
<tr>
<td>APPENDIX B - Treatise by Dr. E. Saibel, &quot;A Derivation of the Stress-Strain Relationship of Composite Bars&quot;</td>
<td>43</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Unsintered Fused Silica-Copper Composites. Tensile Strength</td>
<td>23</td>
</tr>
<tr>
<td>II</td>
<td>Unsintered Fused Silica-Aluminum-Nickel Composites. Tensile Strength</td>
<td>24</td>
</tr>
<tr>
<td>III</td>
<td>Exploration of Variables Controlling Tensile Strength and Coating Smoothness of Aluminum Coated &quot;E&quot; Glass Fibers</td>
<td>25</td>
</tr>
<tr>
<td>IV</td>
<td>Tensile Strength Comparisons</td>
<td>41</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thermal Expansion of Single Fibers - Lead Alloy (1% Zn, 1-1/2% Cd) Coated &quot;E&quot; Glass</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>Thermal Expansion of Single Fibers - Zinc Coated &quot;E&quot; Glass</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>Thermal Expansion of Single Fibers - Aluminum Alloy (5% Zn, 1% Cd) Coated &quot;E&quot; Glass</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>Thermal Expansion of Coated &quot;E&quot; Glass Single Fibers after Repeated Heating</td>
<td>33</td>
</tr>
<tr>
<td>5</td>
<td>Tensile Stress-Strain Curves</td>
<td>34</td>
</tr>
<tr>
<td>6</td>
<td>Tensile Stress-Strain Curves</td>
<td>35</td>
</tr>
<tr>
<td>7</td>
<td>Stress-Strain Differences with Heat Treatment</td>
<td>36</td>
</tr>
<tr>
<td>8</td>
<td>Tensile Stress-Strain vs Fiber Breaks</td>
<td>37</td>
</tr>
<tr>
<td>9</td>
<td>Tensile Stress-Strain vs Fiber Breaks</td>
<td>39</td>
</tr>
<tr>
<td>10</td>
<td>Thermal Expansion of Composites</td>
<td>42</td>
</tr>
</tbody>
</table>
INVESTIGATION OF GLASS-METAL COMPOSITE MATERIALS

INTRODUCTION

This report covers research effort expended on Naval Ordnance Contract NOord 15764 in the period March 15, 1958, to June 15, 1959. It comprises work reported in the Tenth, Eleventh, and Twelfth Quarterly Progress Reports and work performed in the period December 15, 1958, to June 15, 1959, inclusive of the thirteenth quarter for which a separate report will not be submitted. On January 21, 1959, the research facilities of the Owens-Corning Fiberglas Corporation at Newark, Ohio, were disrupted by a flood. Approximately three months were required to return the Laboratory to normal operating condition and to replace the considerable number of test specimens that were lost. By permission of Mr. George B. Butters, Contracting Officer, the reporting period for the Fourth Annual Progress Report was extended to June 15, 1959.

The major effort was directed to improving the characteristics of glass-metal composites by determining the variables affecting their behavior. One investigation was concerned with the factors controlling the strength of single filaments in the operation of forming and coating them with molten metals. In another investigation the physical properties of composite materials were examined in relation to the theories which have been evolved to explain or predict their behavior.

The work represents the combined effort of Messrs. J. I. Aber, R. E. Evans, P. A. Lockwood, E. E. Mattern, N. L. Leedy and Dr. H. B.
Whitehurst and Dr. J. W. Michener. Consulting work was done by Dr. Edward Saibel of Rensselaer Polytechnic Institute and Dr. T. S. Shevlin of The Ohio State University.

Physical property measurements of the glass-reinforced metal test bars were performed at The Ohio State Experimental Station under the direction of Dr. T. S. Shevlin and at the Owens-Corning Fiberglas Testing Laboratories.

Invaluable assistance in the preparation of the text of this report was given by Mr. J. A. Grant of the Research Laboratories administrative staff of Owens-Corning Fiberglas.
SUMMARY

Work at the start of the fourth year of research on glass-metal composites was mainly directed to creating composites of acceptable strength-to-weight ratio suitable for service in the 1,500 - 2000°F range.

Direct high temperature analogues of glass-aluminum composites could not be made by the techniques successful for glass-aluminum. Neither "E" glass nor fused silica fibers could be coated in the fiber-forming operation or vacuum injection cast with metals melting above aluminum. The alloys used, mainly those of copper, oxidized deeply and too rapidly and the fibers were excessively embrittled.

An adaptation of powder metallurgy techniques comprising mixing fibers with powdered metals, hot pressing into green compacts and sintering was explored with partial success. Samples of nickel, copper, stainless steel, chromium, and brass with "E" glass or fused silica fibers, both bare and aluminum coated, were carried through the green compact stage. Sintering was not satisfactorily accomplished due to equipment limitations. To overcome the difficulty, major expenditures for capital equipment were required and for that reason work on high temperature composites was temporarily discontinued.

A study was initiated to discover the underlying mechanisms responsible for the physical properties exhibited by glass-aluminum composites. Work was divided into two sections. In one, the factors controlling the strength of metal-coated fibers were investigated; in the other, the
properties of composites were examined to determine the interactions of glass and metal in producing the final composite properties.

To facilitate tensile testing of single fibers a multihead tester was developed capable of pulling eight fibers simultaneously and providing a trace record of stress versus time for each. The device provided the requisite sensitivity and greatly accelerated testing work.

Quality of metal coatings was found not to be a factor affecting fiber strengths. It had been assumed that thickness, smoothness and completeness of coatings correlated with higher fiber tensile strengths. In one series of experiments using three glasses varying from excellent to poor in coatability with two aluminum alloys, the coated fiber tensile strength was a constant percentage of the virgin fiber strength of each glass independent of coating quality. For 1100 aluminum the coated fiber tensile strength was 20 - 24 per cent of virgin fiber strength and for a 94 per cent aluminum - 5 per cent zinc - 1 per cent cadmium alloy, 17 - 23 per cent. In another series of experiments five parameters in the operation of fiber forming and metal coating believed to control coating quality and strength were studied. It was found that the parameters studied were not the only controlling ones; some other variable influenced coating thickness and tensile strength to a greater extent. Strengths were not reproducible in reruns. A set of parameters was established after much labor which would give reproducible and relatively even coatings of metal on the fibers. This was accomplished with a combination of relatively high pulling speed and careful metering of
metal flow. Tensile strength of fibers so produced was relatively low, averaging 107,000 ± 8,000 psi.

The strength of aluminum-coated glass fiber was found to be affected by humidity the same as bare glass fiber. Strength was 20 per cent greater in 5 per cent relative humidity versus 60 per cent relative humidity atmosphere. Lead-coated fibers were not so affected.

Experiments were performed on the thermal expansion of metal-coated fibers and glass-metal composites. Efforts were made to correlate the results with the treatise by R. B. Wiley on "Thermal Expansion of Glass-Metal Composites."

The experiments were not decisive and correlation was therefore not possible. More work under finer control is needed in this area.

The properties of glass reinforced aluminum composites were compared with those of sintered aluminum powder (SAP) with the object of determining if the mechanisms by which the matrix metal of each are reinforced are the same or different. The results indicated that glass-aluminum composites had properties which were similar or superior to those of the SAP materials but that the mechanisms were not the same. The tensile strength of glass-aluminum composites is achieved at interparticle spacings considerably larger than the theoretical minimum required by SAP materials theory. This became apparent only at elevated temperatures. The difference was enhanced with increasing glass content and was taken to indicate that the factor determining the minimum tensile strength of

---

glass-aluminum composites is dependent on the presence of the glass in filamentous form. Since the comparison is made on the basis of the observation that the tensile strength of SAP materials is dependent on interparticle spacing of the oxide particles, no conclusion can be drawn regarding the mechanism operating in the fiber reinforced materials. Two other basic differences in behavior between the two materials were explored. In tests at 900°F indications were that the curves obtained by plotting the logarithm of stress-to-rupture against time-to-failure would be considerably flatter for glass-aluminum composites than for SAP, as would be expected if ultimate failure depended on fiber fracture rather than creep or flow of the metal. Permanent strain-hardening could not be induced in glass-aluminum composites as compared to SAP materials in which strain-hardening is one of the strengthening mechanisms operating.

The stress-strain diagrams of glass-metal composites were examined with the object of learning or explaining the role of each component in the strengthening mechanism. Matrix metals used were lead, zinc, and aluminum alloys; five of the nine aluminum alloys were variously heat treated. Marked differences were observed but none of such a nature as to explain behavior of the individual components. Stress-strain diagrams were of four general types:

1. Initial yield followed by a straight line segment
2. Continuously changing slope
3. Typical of a metal
4. Continuously but irregularly changing slope
Moduli computed from the curves varied widely at initial stressing. In a few cases moduli up to 25 million psi were indicated for composites which should not go over ten or eleven million psi. Moduli of three to seven million psi were computed for the straight line portion of 37 percent of the curves (Type 1). If it is assumed that the modulus of the materials is an additive function of the area of the components, the modulus of the composites in the strained condition would be in the neighborhood of 2.2 million psi.

Breaks were of two types: one with a tangent modulus of more than one million psi and one with a tangent modulus of less than one half million psi. These results indicate that initial strength of the matrix does not appear to be the primary factor in determining strength of the composites and that shape of the stress-strain curves is affected more by heat treatment than by any other variable investigated.

Due to the large disparity in tensile strengths between vacuum injection cast glass-metal composites and handbook data for the matrix metal in the form of commercial wrought alloy bars, the effect of vacuum injection casting on strength was investigated.

Vacuum cast bars of the commercial alloy as received and diluted with one part 1100 aluminum to three of alloy were made in the same manner as glass-metal composites but omitting the glass fibers. The results of room temperature tests were not as decisive as desired due to loss of the undiluted commercial alloy bars which were not replaced. Compared on a cast basis, the indications were that dilution of the matrix alloy lowered its strength relatively little. The vacuum injection cast
glass-aluminum composites behaved similarly to the cast diluted alloy bars with the exception of reduced elongation. The major difference between cast and wrought alloy would appear to be one of basic structure. As previously reported the properties of wrought bars fade around 400°F whereas vacuum injection cast glass-aluminum composites continue strong to 700 - 900°F.

Attempts were made to relate the rate of fiber breakage in composites under stress to shapes of the stress-strain curves. Results showed that the fiber breaks in glass-lead bars could account for some of the change in the shape of the stress-strain curve. Results with glass-aluminum bars were not as positive. The rate of fiber breakage increased exponentially just prior to failure of the sample and bore no relation to shape of the curve. Work in this direction was discontinued.

Dr. Edward Saibel has composed a model system for glass-metal composites designed to explain and predict the properties of composites. His treatise, attached to this report as Appendix B, is similar to Dr. H. B. Whitehurst's treatise on the principles of fibrous reinforcements but takes plasticity of the matrix into account. Experimental data fit both theories with some unexplained deviations.

---

It was obvious that composites with strength in the range 1500 - 2000°F would require a glass of higher temperature endurance than "E" glass. For this reason a method of drawing fibers from fused silica rods was developed and placed into laboratory scale operation. Attempts were made to coat the fused silica fibers with various metals at forming. Aluminum was applied successfully but metals with melting points above aluminum (copper alloys) were not; oxide developed rapidly preventing the metal from contacting the fiber. In those few cases where continuous coating of high temperature alloys was achieved the fibers were brittle and difficult to handle. A modification of the aluminum coating technique will be necessary if metals which melt above 1500°F are to be applied directly to fibers in the forming operation.

A few trials were made at vacuum injection casting molten metal around the fibers, but copper and its alloys oxidize so rapidly that the method was impractical. For this reason adaptations of conventional powder metallurgy techniques were developed in which fibers coated with metal powders were hot pressed in dies to form green compacts. A slurry composed of metal particles suspended in a viscous liquid which served as a heat fugitive binder was applied to the fibers in the forming operation. The particles adhered well enough to permit placing the fibers in pressing dies. Dipping the fibers in ethylene glycol and rolling them in
metal powders was found to be an easier method for effecting the combination.

Hot pressed compacts of copper, nickel, stainless steels, nickel-chromes and brass reinforced with fused silica fibers were produced. One bar was made with zirconium powder. Compacts containing over 50 per cent glass could not be made satisfactorily and the best glass percentages were relatively low, generally less than 10 per cent. With the exception of copper the bars all showed low green compact strength. This is not unusual for powdered metal compacts, but it does make handling more difficult. The use of 1100 aluminum precoats on the fibers resulted in denser, more easily handled compacts and was especially helpful for nickel compacts. Strengths of green compacts of copper (Table I) and nickel (Table II) were not exceptionally high but considering the low per cent of theoretical density achieved the results were quite favorable.

Sintering was not satisfactorily accomplished due to inadequacy of available furnaces in maintaining suitable protective atmospheres. Differential thermal contraction between fiber and metal leading to severe internal stressing as the compacts cooled over a long temperature range was another problem encountered. The delamination observed was believed to be in part due to that cause and in part to oxidation. Possibly the differential shrinkage problem could be overcome by placing the bars under light to moderate pressure during the sintering and cooling steps.

At this time the necessary equipment for the high temperature work was not available at the Owens-Corning Fiberglas Corporation Research
Laboratories. For this reason it was recommended that the Navy Bureau of Ordnance consider another contractor or subcontractor having adequate facilities for investigating these high temperature composites. In the meantime the Owens-Corning Fiberglas Corporation Research facilities would be used to investigate the mechanism by which glass fibers and metal matrixes interact to produce the properties exhibited by the present composite materials.

Research Into Strength of Metal-Coated Glass Fibers and Methods of Improving the Coated Fiber Strength

For this phase of the work a faster, more sensitive method of measuring the tensile strength of single fibers was required. A multi-head tester was developed and constructed which was capable of testing eight single fibers simultaneously and recording the results automatically.\(^1\) With an auxiliary jig (fork) for mounting fibers in the tester, several hundred tests per day were quite possible. Tensile strengths were computed from the recorded loading at break and the fiber diameter as measured with a filar micrometer.

The first experiments were designed to check the correlation, if any, between tensile strength and quality of the coating on the fibers. Assumptions had been that smooth, uniform, thin coatings yielded higher strengths. Glasses R178, "E" and "C" arranged in order of decreasing coatability were coated with 1100 aluminum and the standard coating alloy (94 per cent Al -

5 per cent Zn - 1 per cent Cd). Within relatively narrow limits the coated fiber strength was a constant percentage of the virgin fiber strength for all three glasses, 20 - 24 per cent for 1100 aluminum and 17 - 23 per cent for the standard alloy.

These results indicated coating quality was not a controlling factor in coated fiber tensile strengths.

More recent work was done to determine what factors in the fiber forming and coating operation affect coating quality and fiber strength. The variables involved were:

1. Fiber forming temperature
2. Fiber pulling speed
3. Distance of metal coater from fiber-forming tip
4. Fiber diameter
5. Flow rate of metal onto the fiber

The results of the first tests were not very encouraging and it was decided to rerun the tests using statistical methods of changing these variables. (See Table III)

Before this was done it was decided to try improving the fiber forming and coating method to eliminate possible outside variations. This work involved forming as uniform a fiber as possible and producing as smooth a coating as possible. After a considerable expenditure of time, it was found that by using very high pulling speeds, such as 10,000 feet per minute and over, and controlling the flow of the metal by metering procedures that the above objectives could be accomplished. It was found, though, that the relative strength of these coated fibers was low
compared to other experimental results. The fiber strengths were 107,000 psi with a normal variation of ± 8,000 psi as compared to a normal overall average of 125,000 psi.

The experimentation changing the five parameters mentioned earlier was now reapplied on a statistical basis except that the flow of the metal was metered as best possible. The results of these experiments are shown in Table I and as can be seen other unrecognized variables are still influencing the experiments. This is best indicated by the fact that when a series of conditions were repeated in the experiment that the variations in results were as wide or wider than when changing the variables. There appeared to be no correlation between the variables and tensile strength, between the variables and fiber coating uniformity, and between fiber coating uniformity and tensile strength.

During this work it was found that fibers which were allowed to stand overnight showed a reduction in strength compared to fibers which were tested within a few hours of the time they were formed. This fact had also been noted in the Basic Physics Research Department and was accredited to the room humidity. A series of tests were designed in which coated fibers were tested in the virgin condition and metal-coated condition in atmospheres of 60 per cent and 5 per cent relative humidity. The virgin fibers showed an approximately 20 per cent increase in strength when tested in the low humidity condition. This was found to be true with aluminum-coated fibers also but not with lead-coated fibers. The results of this test were rather surprising in that it had always been assumed, based on microscopic examination, that the coating quality of aluminum-
coated fibers was superior to that of lead-coated fibers. No other explanation than coating discontinuity or porosity would appear to explain the differences between the two materials. This conclusion was further emphasized on applying wax coatings over the metal coatings. Although the wax did not eliminate change in tensile strength with changing humidity, the effect was greatly reduced. Aluminum wires tested as controls showed no change in strength with varying humidity.

Another more surprising indication was that several times forks would appear with exceptional strength of over 250,000 psi against an overall average of about 125,000 psi for coated fibers. These would seldom appear as single fibers on one fork, but rather as whole forks; and repeated checking into fiber diameter and the equipment failed to show any malfunction. All work to trace the reason for these exceptional strengths has ended in failure and at the present time no methods have been devised which will seemingly lead to an explanation.

A series of thermal expansion tests on metal-coated fibers was started and results of these tests are recorded in Figures 1, 2, 3, and 4. The reason for this work was an attempt to correlate experimental results with theory set forth in the treatise on "Thermal Expansion of Glass-Metal Composites" by R. B. Wiley. Correlation was not possible either because the changes were smaller than experimental error or because the treatise did not cover enough of the variables involved.

---

Research Into Composites

As previously reported, work was done to check the possibility that the reinforcing mechanisms operating in glass-metal composites might be the same as in dispersed oxide particle composites of which sintered aluminum powder (SAP) composites are typical. In summation the properties of glass-reinforced aluminum and SAP composites were found to be similar in many ways but due to the differences in interparticle spacing on which SAP theory is based it was concluded that the fibers did not act in the same way to modify the properties of the aluminum. Particle spacing in glass fiber reinforced aluminum is many times the minimum normally used in sintered aluminum powders and for similar properties there are differences of several hundredfold. This behavior was evident only at elevated temperatures and was intensified with increasing glass fiber content indicating dependence on presence of the glass in filament form. Additionally, the two materials differ in stress-rupture and cold working properties. Stress-to-rupture versus time-to-failure curves for glass-aluminum composites look to be flatter than those for SAP indicating again dependence on fibers rather than discrete particles and metal flow. Permanent strain-hardening could not be induced in glass-aluminum composites but is known to operate in SAP materials as one of the strengthening mechanisms. Cold working glass-aluminum composites either by extrusion or rolling is difficult at best; severe cracking is encountered with glass

---

contents much over 10 per cent when these operations are done in the normal manner. In general no increase in strength after cold working and annealing was noted and for those samples that could be rolled with great difficulty the strength actually decreased. This behavior indicated a distinctly different mechanism of reinforcement for glass-fiber metal composites.

A better understanding of the nature of glass-metal composites was the primary reason for studying changes in stress-strain curves caused by varying the matrix metal. Correlation of rate of fiber breakage in composites under stress with shape of the stress-strain curves was attempted by recording the sound of the breaking fibers. The objective in both cases was to determine effect of the fibers on the matrix materials. Preliminary results have been reported.\(^1\)

Results with glass-fiber lead composites indicated that the fiber was the factor controlling the shape of the stress-strain curve. As the rate of fiber breakage increased, the shape of the curve changed at approximately the same rate.

While all the stress-strain curves are not shown in this report, three of them illustrate the variations caused by changing the matrix metal. Figure 5 presents a comparison of stress-strain curves for glass-fiber lead, glass-fiber aluminum, and glass-fiber zinc composites. The composites were prepared by vacuum injection casting and contain

---

approximately 20 per cent glass fibers by volume. Tests were conducted at room temperature.

Figure 6 shows the relationships between three aluminum matrix composites vacuum injection cast with 20 per cent "E" glass fibers by volume. The first curve is for 1100 aluminum. The second curve is for 2014 aluminum solutioned and aged after casting. The third curve is for 4032 aluminum also with solution and aging heat treatment after casting. These three curves show the difference that alloys can make in composites.

Figure 7 presents four curves showing the effects of various types of heat treatment on a composite of 2014 aluminum vacuum injection cast with 20 per cent "E" glass fibers by volume. The curves represent the as cast, annealed, solutioned, and solutioned and aged conditions of heat treatment. These are average curves typical of results obtained and do not represent any one composite.

The work on the stress-strain sound recording of aluminum composites proceeded very satisfactorily using the modified sound recording system previously described.1 Figures 8 and 9 present typical stress-strain curves with representative fiber breakage rate curves for aluminum composite materials. Figure 8 presents a glass-aluminum composite in which the matrix material is 1100 aluminum and Figure 9 presents a composite in which the matrix material is 2014 aluminum after solutioning and aging. As can be seen from these curves, the shape of the stress-strain curve cannot be readily predicted from the change in the rate of fiber breakage.

In fact, due to the almost entirely exponential shape of the fiber breakage curve, the only thing that can be discerned is that when one is checking the sound of fiber breakage during a stressing operation approximately five to ten seconds' warning is given as to the time at which the composite will break. Work with the stress-strain sound experiment, therefore, has been discontinued at the present time.

Due to the wide difference in the ultimate tensile strengths found experimentally for glass-aluminum composites and the ultimate strengths as published for the commercial alloys, it was decided to see what effect the casting system used in producing composites might have on the strength of the commercial alloys. Rods of the various alloys were vacuum cast in the same manner as for composites but with fiber omitted. Also, due to the fact that glass fibers normally are coated with 1100 aluminum, bars were made of the commercial alloy diluted with 25 per cent 1100 aluminum. This percentage is about the same dilution of the matrix alloy as in typical vacuum injection cast composites. The experiment turned out not to be as decisive as desired due to loss of the undiluted alloy bars before testing which were not replaced after the flood. Tensile strengths of composites are compared with strengths of diluted alloy bars and handbook data in Table IV. Any conclusions drawn from these data may be labeled speculative and based on slim evidence. However, the data tend to show that dilution of the matrix with casting alloys lowers its strength relatively little. The composites have strengths like the diluted alloy bars indicating the addition of fibers did not weaken them. It is inferred that the main difference between cast and wrought commercial alloy bars.
arises from differences in basic metal structure in the two conditions. This difference is erased at temperatures above 400°F for commercial alloys versus composites which continue strong to 700 - 900°F.

To check R. B. Wiley's treatise "Thermal Expansion of Glass-Metal Composite Materials," \(^1\) checks were made on the thermal expansion of glass-metal composites. In particular lead, zinc, and aluminum composites were studied. The test apparatus did not function properly for the higher temperatures required in testing the glass-aluminum composites, but curves were obtained for the lead and zinc materials and data on 2014 aluminum-glass composites were available from earlier trials. These are shown in Figure 10. But after re-examination of the experimental apparatus, it is felt that while the curves are valid, they were not under fine enough control to either prove or disprove the work done by R. B. Wiley. Future work in this area is still planned.

**Theoretical Studies of Glass-Metal Composites**

In the 10th Quarterly Progress Report, Dr. H. B. Whitehurst presented a paper in which he used glass reinforced plastic composites as a model to describe glass-metal composites. The main consideration in this treatise is that both the composite components are deformed elastically under stress. Dr. E. Saibel of Rensselaer Polytechnic Institute has since followed this same line of reasoning but has utilized known experimental and

---

theoretical work on metals to devise a set of experimental curves for glass-aluminum composites allowing for the plastic deformation of metals under stress. This treatise is found in Appendix B. Both of these theoretical works seem to fit fairly closely to experimental data, but there are deviations which cannot be explained exactly. Whether it is due to the assumptions that had to be made or whether it is due to an experimental error or by coincidence that the curves happened to follow those of the theories is not understood. A conclusive test to discern the facts has not been found at the present time.
FUTURE PLANS

Work in the next period will follow the same pattern of trying to discern what the effects of the glass and metal are in producing the unique set of properties found for the composite materials. Most of this work will be an extension of the present work and will be divided into two classes.

One will be concerned with attempts to prove validity and applicability of any of the three theories advanced, i.e.; R. B. Wiley's treatise, Dr. H. B. Whitehurst's theory, and Dr. E. Saibel's theory.

The other area of interest is concerned with prosecuting studies on both aluminum-coated fibers and composites at high temperatures. Specifically, fiber strengths and stress-strain behavior of various composites at elevated temperatures will be examined. Stress-strain behavior of composites with low percentages of glass is of particular interest.
APPENDIX A

TABLES AND FIGURES
TABLE I
UNSINTERED FUSED SILICA-COPPER COMPOSITES
TENSILE STRENGTH

Fused Silica Fibers. 0.0005 - 0.001 Inch Diameter.
3% by Wt. Oriented Parallel
Hot Pressed at 900°F & 16,6 tons/sq.in. Not Sintered.

<table>
<thead>
<tr>
<th>Temperature °F</th>
<th>Tensile Strength Psi</th>
<th>Elongation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room</td>
<td>46,900</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td>32,000</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>4,420</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>14,050</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>10,500</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>22,500</td>
<td>1.56</td>
</tr>
<tr>
<td>1100</td>
<td>4,043</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>3,800</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>1,900</td>
<td>—</td>
</tr>
<tr>
<td>1500</td>
<td>484</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>402</td>
<td>6.25</td>
</tr>
<tr>
<td></td>
<td>402</td>
<td>—</td>
</tr>
<tr>
<td>Control</td>
<td>No Glass Fibers</td>
<td></td>
</tr>
<tr>
<td>Room</td>
<td>27,500</td>
<td>0.78</td>
</tr>
</tbody>
</table>
TABLE II
UNSINTERED FUSED SILICA-ALUMINUM-NICKEL COMPOSITES
TENSILE STRENGTH

Fused Silica Fibers. 0.00075 - 0.0012 Inch Diameter.
Precoated with 1100 Aluminum. Oriented Parallel.
Hot Pressed at 1100°F & 14 tons/sq.in. Unsintered.
Tested at Room Temperature

<table>
<thead>
<tr>
<th>Composition - % by Wt.</th>
<th>Tensile Strength</th>
<th>Average Tensile Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>Al Ni</td>
<td>(Psi)</td>
</tr>
<tr>
<td>5</td>
<td>5 90</td>
<td>3323</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4075</td>
</tr>
<tr>
<td>10</td>
<td>10 80</td>
<td>6947</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9542</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6491</td>
</tr>
<tr>
<td>*16.66</td>
<td>16.66 66.66</td>
<td>10637</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11412</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8113</td>
</tr>
</tbody>
</table>

*Specific Gravity: Theoretical 4.99
Actual 4.42
89.1% of Theoretical
**TABLE III**

**EXPLORATION OF VARIABLES**

**CONTROLLING TENSILE STRENGTH AND COATING SMOOTHNESS**

**OF ALUMINUM COATED "F" GLASS FIBERS**

| Single Fiber Data on 24 or More "F" Glass Fibers | Coated With 1100 Aluminum Metal |

<table>
<thead>
<tr>
<th>Bushing Pulling Temp. *°F</th>
<th>Dist. of Coater From Tip—In.</th>
<th>Pulling Speed—Ft./Min</th>
<th>Coater % Metal Head In Coater In.</th>
<th>Bare Fiber Dia. In. x 10^-5</th>
<th>Coated Fiber Dia. Min. In. x 10^-5</th>
<th>Coated Fiber Dia. Max. In. x 10^-5</th>
<th>Ave. Coated Fiber Dia. In. x 10^-5</th>
<th>Ave. Coated Fiber Thickness 3 Grams</th>
<th>Fiber Weight (Timed Runs)</th>
<th>Ave. Bare Coating</th>
<th>Ave. Coated Coating</th>
<th>Fiber Weight Applied x 10^3 psi</th>
<th>Tensile % Metal Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>2400</td>
<td>7,000</td>
<td>1.5</td>
<td>100</td>
<td>30</td>
<td>36</td>
<td>36</td>
<td>3</td>
<td>0.254</td>
<td>0.504</td>
<td>46</td>
<td>156</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>4,300</td>
<td>1.5</td>
<td>100</td>
<td>38</td>
<td>48</td>
<td>60</td>
<td>54</td>
<td>0.271</td>
<td>0.512</td>
<td>45</td>
<td>140</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>2,700</td>
<td>1.5</td>
<td>100</td>
<td>53</td>
<td>74</td>
<td>89</td>
<td>82</td>
<td>0.281</td>
<td>0.512</td>
<td>45</td>
<td>140</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>2,890</td>
<td>1.5</td>
<td>100</td>
<td>58</td>
<td>64</td>
<td>78</td>
<td>71</td>
<td>0.357</td>
<td>0.633</td>
<td>43</td>
<td>125</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>2,160</td>
<td>1.5</td>
<td>100</td>
<td>64</td>
<td>70</td>
<td>95</td>
<td>82</td>
<td>0.357</td>
<td>0.577</td>
<td>38</td>
<td>124</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>1,260</td>
<td>1.5</td>
<td>100</td>
<td>83</td>
<td>96</td>
<td>109</td>
<td>103</td>
<td>0.350</td>
<td>0.450</td>
<td>22</td>
<td>94</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>1,260</td>
<td>1.5</td>
<td>100</td>
<td>88</td>
<td>100</td>
<td>111</td>
<td>106</td>
<td>0.387</td>
<td>0.570</td>
<td>26</td>
<td>85</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>51.0</td>
<td>1.5</td>
<td>100</td>
<td>136</td>
<td>177</td>
<td>177</td>
<td>177</td>
<td>0.440</td>
<td>0.701</td>
<td>37</td>
<td>73</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>7,000</td>
<td>1.5</td>
<td>100</td>
<td>32</td>
<td>36</td>
<td>62</td>
<td>49</td>
<td>0.297</td>
<td>0.778</td>
<td>62</td>
<td>165</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>2,890</td>
<td>1.5</td>
<td>100</td>
<td>59</td>
<td>63</td>
<td>105</td>
<td>84</td>
<td>0.389</td>
<td>0.761</td>
<td>49</td>
<td>97</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>1,260</td>
<td>1.5</td>
<td>100</td>
<td>99</td>
<td>102</td>
<td>132</td>
<td>117</td>
<td>0.493</td>
<td>0.780</td>
<td>37</td>
<td>83</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>2,890</td>
<td>1.5</td>
<td>50</td>
<td>53</td>
<td>60</td>
<td>80</td>
<td>70</td>
<td>0.324</td>
<td>0.787</td>
<td>59</td>
<td>130</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>1,260</td>
<td>1.5</td>
<td>50</td>
<td>93</td>
<td>127</td>
<td>151</td>
<td>139</td>
<td>0.479</td>
<td>0.652</td>
<td>27</td>
<td>84</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>7,000</td>
<td>1.5</td>
<td>50</td>
<td>33</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>0.296</td>
<td>--</td>
<td>--</td>
<td>97</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>1,260</td>
<td>1.5</td>
<td>12</td>
<td>89</td>
<td>160</td>
<td>123</td>
<td>111</td>
<td>0.394</td>
<td>0.555</td>
<td>25</td>
<td>--</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>2,890</td>
<td>1.5</td>
<td>12</td>
<td>55</td>
<td>66</td>
<td>101</td>
<td>84</td>
<td>0.362</td>
<td>0.554</td>
<td>35</td>
<td>117</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>7,000</td>
<td>1.5</td>
<td>12</td>
<td>34</td>
<td>37</td>
<td>55</td>
<td>46</td>
<td>0.301</td>
<td>--</td>
<td>--</td>
<td>121</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>2,500</td>
<td>2.5</td>
<td>100</td>
<td>59</td>
<td>63</td>
<td>122</td>
<td>93</td>
<td>0.344</td>
<td>0.594</td>
<td>42</td>
<td>119</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>2.5</td>
<td>100</td>
<td>45</td>
<td>67</td>
<td>85</td>
<td>76</td>
<td>0.433</td>
<td>1.216</td>
<td>67</td>
<td>126</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>2550</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>51</td>
<td>61</td>
<td>77</td>
<td>69</td>
<td>0.424</td>
<td>0.680</td>
<td>37</td>
<td>123</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Bushing Temp. °F</td>
<td>Pulling Speed Ft/Min</td>
<td>Dist. of Coater</td>
<td>% Metal Head In</td>
<td>Bare Fiber Dia. In. x 10^-5</td>
<td>Coated Fiber Dia. Min. In. x 10^-5</td>
<td>Coated Fiber Dia. Max. In. x 10^-5</td>
<td>Ave. Coated Fiber Dia. In. x 10^-5</td>
<td>Ave. Thickness Coating In. x 10^-5</td>
<td>Fiber Weight (Timed Runs) Grams</td>
<td>% Metal Applid</td>
<td>Tensile Strength x 10^3 psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------</td>
<td>----------------</td>
<td>----------------</td>
<td>------------------</td>
<td>--------------------------------</td>
<td>---------------------------------</td>
<td>-------------------------------</td>
<td>--------------------------------</td>
<td>-----------------------------</td>
<td>----------------</td>
<td>-----------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>5,500</td>
<td>2.5</td>
<td>100</td>
<td>38</td>
<td>58</td>
<td>63</td>
<td>61</td>
<td>11</td>
<td>0.326</td>
<td>0.427</td>
<td>35</td>
<td>175</td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>42</td>
<td>56</td>
<td>69</td>
<td>63</td>
<td>9</td>
<td>0.242</td>
<td>0.726</td>
<td>60</td>
<td>145</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,500</td>
<td>1.5</td>
<td>100</td>
<td>63</td>
<td>81</td>
<td>92</td>
<td>86</td>
<td>11</td>
<td>0.421</td>
<td>0.815</td>
<td>48</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>1.5</td>
<td>100</td>
<td>47</td>
<td>56</td>
<td>51</td>
<td>61</td>
<td>6</td>
<td>0.353</td>
<td>0.690</td>
<td>42</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>1.5</td>
<td>54</td>
<td>120</td>
<td>87</td>
<td>21</td>
<td>0.350</td>
<td>1.160</td>
<td>70</td>
<td>148</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>2,550</td>
<td>1.5</td>
<td>100</td>
<td>63</td>
<td>51</td>
<td>99</td>
<td>75</td>
<td>6</td>
<td>0.329</td>
<td>0.700</td>
<td>53</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,000</td>
<td>2.5</td>
<td>100</td>
<td>61</td>
<td>68</td>
<td>157</td>
<td>126</td>
<td>4</td>
<td>0.387</td>
<td>1.490</td>
<td>74</td>
<td>114</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>2.0</td>
<td>100</td>
<td>61</td>
<td>44</td>
<td>64</td>
<td>41</td>
<td>4</td>
<td>0.297</td>
<td>0.765</td>
<td>61</td>
<td>97</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>39</td>
<td>52</td>
<td>75</td>
<td>64</td>
<td>4</td>
<td>0.368</td>
<td>0.697</td>
<td>44</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>5,500</td>
<td>2.0</td>
<td>100</td>
<td>39</td>
<td>61</td>
<td>68</td>
<td>65</td>
<td>12</td>
<td>0.337</td>
<td>0.896</td>
<td>52</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>7,000</td>
<td>2.0</td>
<td>100</td>
<td>33</td>
<td>41</td>
<td>63</td>
<td>52</td>
<td>9</td>
<td>0.341</td>
<td>0.838</td>
<td>59</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>2,500</td>
<td>2.5</td>
<td>100</td>
<td>58</td>
<td>68</td>
<td>97</td>
<td>83</td>
<td>12</td>
<td>0.348</td>
<td>0.681</td>
<td>49</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>1,000</td>
<td>2.0</td>
<td>100</td>
<td>94</td>
<td>106</td>
<td>132</td>
<td>119</td>
<td>12</td>
<td>0.333</td>
<td>0.453</td>
<td>27</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>53</td>
<td>67</td>
<td>104</td>
<td>86</td>
<td>17</td>
<td>0.460</td>
<td>0.712</td>
<td>35</td>
<td>163</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>2,500</td>
<td>2.0</td>
<td>100</td>
<td>53</td>
<td>57</td>
<td>92</td>
<td>75</td>
<td>11</td>
<td>0.316</td>
<td>0.448</td>
<td>30</td>
<td>114</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>48</td>
<td>52</td>
<td>111</td>
<td>82</td>
<td>17</td>
<td>0.372</td>
<td>0.769</td>
<td>52</td>
<td>152</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,500</td>
<td>1.5</td>
<td>100</td>
<td>66</td>
<td>68</td>
<td>98</td>
<td>83</td>
<td>8</td>
<td>0.435</td>
<td>0.650</td>
<td>33</td>
<td>126</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>5,500</td>
<td>2.5</td>
<td>100</td>
<td>42</td>
<td>64</td>
<td>53</td>
<td>57</td>
<td>6</td>
<td>0.366</td>
<td>0.771</td>
<td>53</td>
<td>135</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>2.5</td>
<td>100</td>
<td>56</td>
<td>57</td>
<td>50</td>
<td>44</td>
<td>9</td>
<td>0.337</td>
<td>0.533</td>
<td>37</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,500</td>
<td>1.5</td>
<td>100</td>
<td>63</td>
<td>76</td>
<td>124</td>
<td>100</td>
<td>16</td>
<td>0.422</td>
<td>1.237</td>
<td>66</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>1.5</td>
<td>100</td>
<td>42</td>
<td>47</td>
<td>68</td>
<td>59</td>
<td>8</td>
<td>0.411</td>
<td>0.890</td>
<td>54</td>
<td>146</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,500</td>
<td>1.5</td>
<td>100</td>
<td>52</td>
<td>65</td>
<td>114</td>
<td>90</td>
<td>14</td>
<td>0.392</td>
<td>0.646</td>
<td>39</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>1.0</td>
<td>100</td>
<td>44</td>
<td>56</td>
<td>64</td>
<td>60</td>
<td>8</td>
<td>0.320</td>
<td>0.404</td>
<td>21</td>
<td>131</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>3.0</td>
<td>100</td>
<td>47</td>
<td>60</td>
<td>61</td>
<td>60</td>
<td>6</td>
<td>0.367</td>
<td>0.637</td>
<td>42</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>5,500</td>
<td>2.5</td>
<td>100</td>
<td>39</td>
<td>47</td>
<td>106</td>
<td>77</td>
<td>18</td>
<td>0.336</td>
<td>1.668</td>
<td>80</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>Pulling Temp.</td>
<td>Pulling Speed</td>
<td>Pulling From</td>
<td>Head In Forming</td>
<td>Dist. of Coater</td>
<td>% Metal Head In Coater</td>
<td>Bare Fiber Dia. x 10^-5</td>
<td>Coated Fiber Dia. x 10^-5</td>
<td>Coated Fiber Dia. x 10^-5</td>
<td>Coated Fiber Dia. x 10^-5</td>
<td>Ave. Coated Fiber Dia. x 10^-5</td>
<td>Ave. Fiber Thickness</td>
<td>Fiber Weight (Timed Runs) Grams</td>
<td>% Metal Applied</td>
</tr>
<tr>
<td>--------------</td>
<td>---------------</td>
<td>--------------</td>
<td>----------------</td>
<td>----------------</td>
<td>------------------------</td>
<td>--------------------------</td>
<td>--------------------------</td>
<td>--------------------------</td>
<td>--------------------------</td>
<td>----------------------------</td>
<td>----------------------</td>
<td>-------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>47</td>
<td>58</td>
<td>93</td>
<td>78</td>
<td>15</td>
<td>0.364</td>
<td>0.752</td>
<td>61</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>2,500</td>
<td>2.5</td>
<td>100</td>
<td>54</td>
<td>59</td>
<td>87</td>
<td>73</td>
<td>9</td>
<td>0.393</td>
<td>1.166</td>
<td>74</td>
<td>135</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,500</td>
<td>2.5</td>
<td>100</td>
<td>64</td>
<td>65</td>
<td>76</td>
<td>70</td>
<td>3</td>
<td>0.404</td>
<td>0.679</td>
<td>49</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>5,500</td>
<td>1.5</td>
<td>100</td>
<td>35</td>
<td>39</td>
<td>83</td>
<td>61</td>
<td>12</td>
<td>0.288</td>
<td>1.016</td>
<td>72</td>
<td>173</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>47</td>
<td>58</td>
<td>59</td>
<td>58</td>
<td>5</td>
<td>0.357</td>
<td>0.527</td>
<td>32</td>
<td>127</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>2,500</td>
<td>1.5</td>
<td>100</td>
<td>57</td>
<td>61</td>
<td>116</td>
<td>88</td>
<td>15</td>
<td>0.337</td>
<td>0.748</td>
<td>55</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>2.5</td>
<td>100</td>
<td>41</td>
<td>47</td>
<td>72</td>
<td>59</td>
<td>10</td>
<td>0.356</td>
<td>0.776</td>
<td>54</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>1.5</td>
<td>100</td>
<td>41</td>
<td>47</td>
<td>54</td>
<td>52</td>
<td>5</td>
<td>0.369</td>
<td>0.562</td>
<td>34</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>5,500</td>
<td>1.5</td>
<td>100</td>
<td>41</td>
<td>47</td>
<td>54</td>
<td>52</td>
<td>5</td>
<td>0.346</td>
<td>0.576</td>
<td>65</td>
<td>127</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>1.5</td>
<td>100</td>
<td>41</td>
<td>47</td>
<td>54</td>
<td>52</td>
<td>5</td>
<td>0.378</td>
<td>0.700</td>
<td>46</td>
<td>191</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>41</td>
<td>47</td>
<td>54</td>
<td>52</td>
<td>5</td>
<td>0.378</td>
<td>0.700</td>
<td>46</td>
<td>191</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>2.5</td>
<td>100</td>
<td>48</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>10</td>
<td>0.330</td>
<td>0.776</td>
<td>57</td>
<td>162</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>5,500</td>
<td>2.5</td>
<td>100</td>
<td>44</td>
<td>49</td>
<td>58</td>
<td>53</td>
<td>5</td>
<td>0.430</td>
<td>0.610</td>
<td>29</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>2.5</td>
<td>100</td>
<td>48</td>
<td>65</td>
<td>70</td>
<td>68</td>
<td>10</td>
<td>0.384</td>
<td>0.715</td>
<td>16</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,500</td>
<td>2.5</td>
<td>100</td>
<td>63</td>
<td>75</td>
<td>82</td>
<td>78</td>
<td>7</td>
<td>0.412</td>
<td>0.670</td>
<td>24</td>
<td>134</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,500</td>
<td>1.5</td>
<td>100</td>
<td>64</td>
<td>69</td>
<td>109</td>
<td>89</td>
<td>12</td>
<td>0.407</td>
<td>0.736</td>
<td>45</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>2475</td>
<td>2,500</td>
<td>1.5</td>
<td>100</td>
<td>42</td>
<td>48</td>
<td>72</td>
<td>60</td>
<td>9</td>
<td>0.384</td>
<td>0.888</td>
<td>51</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>2450</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>50</td>
<td>51</td>
<td>81</td>
<td>66</td>
<td>8</td>
<td>0.381</td>
<td>0.761</td>
<td>59</td>
<td>129</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>2,500</td>
<td>1.5</td>
<td>100</td>
<td>54</td>
<td>59</td>
<td>106</td>
<td>83</td>
<td>14</td>
<td>0.311</td>
<td>0.661</td>
<td>53</td>
<td>127</td>
<td></td>
</tr>
<tr>
<td>2425</td>
<td>2,500</td>
<td>2.5</td>
<td>100</td>
<td>59</td>
<td>66</td>
<td>67</td>
<td>66</td>
<td>3</td>
<td>0.339</td>
<td>0.611</td>
<td>45</td>
<td>110</td>
<td></td>
</tr>
</tbody>
</table>
TABLE III (Continued)

Reruns of Earlier Data
Six Fibers or More Tested

<table>
<thead>
<tr>
<th>Bushing Temp. °F</th>
<th>Pulling Speed Ft./Min</th>
<th>Dist. of Coater Head In Tip--In.</th>
<th>Bare Fiber Dia. x 10^{-5}</th>
<th>Coated Fiber Dia. x 10^{-5}</th>
<th>Coated Fiber Dia. x 10^{-5}</th>
<th>Ave. Coated Fiber Thickness (Timed Runs) Grams</th>
<th>Fiber Weight % Metal Applied</th>
<th>Tensile Strength x 10^3 psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>2300</td>
<td>2,700</td>
<td>1.0</td>
<td>100</td>
<td>41</td>
<td>51</td>
<td>73</td>
<td>62</td>
<td>11</td>
</tr>
<tr>
<td>2400</td>
<td>4,000</td>
<td>1.0</td>
<td>100</td>
<td>40</td>
<td>52</td>
<td>64</td>
<td>58</td>
<td>9</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>1.0</td>
<td>100</td>
<td>34</td>
<td>46</td>
<td>70</td>
<td>58</td>
<td>12</td>
</tr>
<tr>
<td>2300</td>
<td>2,700</td>
<td>1.5</td>
<td>100</td>
<td>43</td>
<td>46</td>
<td>85</td>
<td>66</td>
<td>11</td>
</tr>
<tr>
<td>2400</td>
<td>4,000</td>
<td>1.5</td>
<td>100</td>
<td>38</td>
<td>56</td>
<td>65</td>
<td>61</td>
<td>11</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>1.5</td>
<td>100</td>
<td>39</td>
<td>41</td>
<td>56</td>
<td>48</td>
<td>11</td>
</tr>
<tr>
<td>NO RUN - Glass would not coat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>4,000</td>
<td>2.0</td>
<td>100</td>
<td>40</td>
<td>49</td>
<td>59</td>
<td>54</td>
<td>7</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>2.0</td>
<td>100</td>
<td>37</td>
<td>41</td>
<td>60</td>
<td>50</td>
<td>7</td>
</tr>
<tr>
<td>NO RUN - Glass would not coat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>4,000</td>
<td>2.5</td>
<td>100</td>
<td>40</td>
<td>43</td>
<td>91</td>
<td>67</td>
<td>12</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>2.5</td>
<td>100</td>
<td>38</td>
<td>42</td>
<td>77</td>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td>2300</td>
<td>2,700</td>
<td>3.0</td>
<td>100</td>
<td>42</td>
<td>50</td>
<td>94</td>
<td>72</td>
<td>15</td>
</tr>
<tr>
<td>2400</td>
<td>4,000</td>
<td>3.0</td>
<td>100</td>
<td>39</td>
<td>51</td>
<td>85</td>
<td>68</td>
<td>14</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>3.0</td>
<td>100</td>
<td>36</td>
<td>45</td>
<td>57</td>
<td>51</td>
<td>13</td>
</tr>
<tr>
<td>NO RUN - Glass would not coat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO RUN - Glass would not coat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>4.0</td>
<td>100</td>
<td>46</td>
<td>56</td>
<td>64</td>
<td>60</td>
<td>7</td>
</tr>
<tr>
<td>Bushing</td>
<td>Pulling Speed</td>
<td>Temp</td>
<td>% Metal</td>
<td>Dist. of Coater</td>
<td>Bare Dia.</td>
<td>Coated Dia.</td>
<td>Ave. Fiber Diameter</td>
<td>Ave. Fiber Thickness</td>
</tr>
<tr>
<td>---------</td>
<td>---------------</td>
<td>------</td>
<td>---------</td>
<td>----------------</td>
<td>-----------</td>
<td>-------------</td>
<td>---------------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>1.0</td>
<td>100</td>
<td>4.0</td>
<td>4.1</td>
<td>63</td>
<td>53</td>
<td>6</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>1.0</td>
<td>100</td>
<td>4.2</td>
<td>4.4</td>
<td>63</td>
<td>54</td>
<td>6</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>1.0</td>
<td>100</td>
<td>4.9</td>
<td>4.9</td>
<td>68</td>
<td>58</td>
<td>8</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>1.0</td>
<td>100</td>
<td>4.2</td>
<td>4.6</td>
<td>68</td>
<td>57</td>
<td>5</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>1.0</td>
<td>100</td>
<td>4.0</td>
<td>4.2</td>
<td>68</td>
<td>55</td>
<td>8</td>
</tr>
<tr>
<td>2500</td>
<td>7,000</td>
<td>1.0</td>
<td>100</td>
<td>3.7</td>
<td>4.2</td>
<td>72</td>
<td>57</td>
<td>10</td>
</tr>
</tbody>
</table>

* One small cone on coater lip
** Double cone on coater lip
Figure 1 - Thermal Expansion of Single Fibers.
Lead Alloy (1% Zn, 1-1/2% Cd) Coated "E" Glass
Figure 2 - Thermal Expansion of Single Fibers - Zinc Coated "F" Glass
Figure 3 - Thermal Expansion of Single Fibers - Aluminum Alloy (5% Zn, 1% Cd) Coated on Glass
Figure 1 - Thermal Expansion of Coated Pyrex Glass Single Fibers after Repeated Heating
Figure 5 - Tensile Stress-Strain Curves

20% "E" Glass Fibers by Volume
Matrix = Pure Metals
Orientation = Parallel
(Tested at Room Temperature)
Figure 6 - Tensile Stress-Strain Curves

20% "E" Glass Fibers by Volume
Matrix - Aluminum Alloys
Orientation - Parallel
(Tested at Room Temperature)
Figure 7 - Stress-Strain Differences with Heat Treatment

20% "E" Glass Fibers by Volume
Matrix - 20\% Aluminum
Orientation - Parallel
(Tested at Room Temperature)
Figure 8a - Tensile Stress-Strain vs Fiber Breaks
Figure 8b - Tensile Stress-Strain vs Fiber Breaks

1100 Al Coated
20% "E" Glass Fibers by Volume
Orientation - Parallel
Matrix - 1100 Al
Annealed
1100 Al Coated
20% "E" Glass Fibers by Volume
Orientation Parallel
Matrix 201b Al
As Cast

Figure 9a - Tensile Stress-Strain vs Fiber Breaks
Figure 9b - Tensile Stress-Strain vs Fiber Breaks

1100 Al Coated
20% "E" Glass Fibers by Volume
Orientation - Parallel
Matrix - 2014 Al
Solutioned and Aged

Stress-Strain
Fiber Breaks

Strain, in./in. x 10^3

Figure 9b - Tensile Stress-Strain vs Fiber Breaks
TABLE IV
TENSILE STRENGTH COMPARISONS
Composites - 20% by Volume "E" Glass
Parallel Oriented 1100 Aluminum Coated
with Aluminum Matrixes
Tested at Room Temperature

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Ultimate Tensile Strength</th>
<th>Elongation %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>psi x 10^-3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>As Cast</td>
<td>An-</td>
</tr>
<tr>
<td>1100 Aluminum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite</td>
<td>14.5</td>
<td>14.1</td>
</tr>
<tr>
<td>Laboratory Casting*</td>
<td>9.1</td>
<td>5.2</td>
</tr>
<tr>
<td>Commercial (Wrought)**</td>
<td>18.0</td>
<td>13.0</td>
</tr>
<tr>
<td>2014 Aluminum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite</td>
<td>23.2</td>
<td>18.5</td>
</tr>
<tr>
<td>Laboratory Casting*</td>
<td>14.7</td>
<td>--</td>
</tr>
<tr>
<td>Commercial (Wrought)**</td>
<td>--</td>
<td>25.0</td>
</tr>
<tr>
<td>4032 Aluminum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite</td>
<td>21.3</td>
<td>--</td>
</tr>
<tr>
<td>Laboratory Casting*</td>
<td>17.9</td>
<td>--</td>
</tr>
<tr>
<td>Commercial (Wrought)**</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>5056 Aluminum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite</td>
<td>18.8</td>
<td>17.2</td>
</tr>
<tr>
<td>Laboratory Casting*</td>
<td>19.4</td>
<td>16.3</td>
</tr>
<tr>
<td>Commercial (Wrought)**</td>
<td>60.0</td>
<td>42.0</td>
</tr>
<tr>
<td>3051A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite</td>
<td>22.7</td>
<td>17.6</td>
</tr>
<tr>
<td>Laboratory Casting*</td>
<td>23.5</td>
<td>--</td>
</tr>
<tr>
<td>Commercial (Casting)**</td>
<td>26.0</td>
<td>--</td>
</tr>
</tbody>
</table>

*The laboratory control castings contain 75% of the matrix alloy and 25% 1100 Aluminum, which is the ratio of the matrix alloy and the fiber coating alloy in composites.

**Handbook data.
APPENDIX B

TREATISE BY DR. E. SAIBEL

A Derivation of the Stress-Strain Relationship of Composite Bars
A DERIVATION OF THE STRESS-STRAIN RELATIONSHIP OF COMPOSITE BARS.

by

Edward Saibel

I. G. Tadjbakhsh

May 28, 1959
SUMMARY

A method has been developed by means of which stress-strain relationship of glass reinforced composite materials may be predicted from the stress-strain curves of the individual materials. The method is based on the fundamentals of the theory of elasticity and takes into account the non-linear behavior of materials. Flow curves have been determined for ordinary room temperature and at elevated temperatures.

Nomenclature

With the exception of symbols defined in the body of the paper the following nomenclature is used.

\( \sigma \) = uniaxial stress
\( a \) = a universal constant taken as 4300 psi (see reference 1)
\( E \) = uniaxial strain
\( \sigma_u \) = ultimate strength of metal
\( \beta \) = a constant dependent upon \( \sigma_u \) and \( \sigma \) (equation 3)
\( E_o \) = initial modulus of elasticity of metal i.e. at zero strain
\( E_m \) = instantaneous modulus of metal
\( E_g \) = modulus of elasticity of glass
\( T \) = temperature in degree Fahrenheit
\( \nu \) = Poisson's ratio
Introduction

The fundamental problem is the behavior of a single glass fiber surrounded by a thin layer of metal coating under the application of an axial load. A composite bar with a high concentration of such glass fibers embedded in it, may then be assumed to act principally in the same manner that a single fiber does. In the following, first the separate behavior of the component parts will be examined and then the combined problem will be investigated.

Behavior of Metal

The coating metal being ordinarily of aluminum compounds essentially exhibits a nonlinear stress-strain relationship when subjected to loads. This nonlinear behavior plays an important part in describing the behavior of composite bars especially at lower ranges of temperature where the strength of metal becomes comparable with that of glass. We will take for the stress-strain relationship of the metal the following relationship (reference 1) which has been derived from phenomenological considerations of polycrystalline metals.

\[ \sigma = \sigma_0 \log \frac{\lambda_1 + \lambda_2 \varepsilon_0}{\lambda_2 + \lambda_4 \varepsilon_0} \tag{1} \]

where \( \sigma \) is the stress, \( \varepsilon \) is strain and \( \sigma_0 \) is a universal constant and

\[ \lambda_1 = \sqrt{1+\beta^2} + (1+\beta) \]
\[ \lambda_2 = \sqrt{1+\beta^2} - (1+\beta) \]
\[ \lambda_3 = \sqrt{1+\beta^2} + (1-\beta) \]
\[ \lambda_4 = \sqrt{1+\beta^2} - (1-\beta) \tag{2} \]
\[ \lambda_4 = \sqrt{1 + \beta^2} - (1 - \beta) \]

\[ \kappa = \frac{E_0}{\sqrt{1 + \beta^2}} \]

\( \beta \) being a constant described below.

Equation (1) is an isothermal relationship. For any given temperature of application of stress, the constant \( \beta \) is related to the ultimate true strength of the metal at that temperature and is to be found from the relationship

\[ \frac{\lambda_1}{\lambda_3} = \frac{\sigma_t}{\sigma_u} \quad (3) \]

If in addition the initial modulus of elasticity of the metal \( E'_0 \) is known, all the required constants in equation (1) can be determined.

The instantaneous modulus \( E_m(\varepsilon) = d\sigma/d\varepsilon \) is given by

\[ E_m = \frac{4E_0(1+\beta^2)\varepsilon_k \varepsilon}{(\lambda_1 e^{\kappa e} + \lambda_3)(\lambda_3 e^{\kappa e} + \lambda_4)} \quad (4) \]

Figure 1 illustrates all the characteristics of equation (1).
For low temperatures (75°F - 200°F), $\sigma_0$ is rather large and from

(3) \[ \beta \gg 1 \quad \text{In this case the proceeding equations can be somewhat simplified and take the following form} \]

\[
\sigma = \sigma_0 \log \frac{2\beta + 1}{2/\beta + \exp(-\frac{E_m}{\sigma_0})} 
\]

(5) \[
E_m = E_0 \frac{2/\beta}{2/\beta + \exp(-\frac{E_m}{\sigma_0})} 
\]

(7) \[
\beta = \frac{1}{2} \exp \left( \frac{E_m}{\sigma_0} \right) 
\]
However, when temperatures are rather high (200-700°F) $\beta$ becomes comparable with unity and one must use the more accurate equations (3)
and (4). For even higher temperatures these equations cease to be valid and creep must be taken into account.

Behavior of Glass

Glass is an almost perfect example of an elastic material. Its stress-strain relationship up to the point of breaking can be assumed to be

$$\sigma = E \varepsilon$$

(8)

or incrementally

$$d\sigma = E \, d\varepsilon$$

(9)

Furthermore its behavior changes very little with variation of temperature, for example its modulus of elasticity $E_g$ changes by less than one percent in a rise of 250°F.

Stress-Strain Relationship for Composite Fiber

In many problems of mechanics a simple and straightforward approach often provides an adequate answer for the phenomenon under consideration along with the promise of being mathematically less cumbersome. A higher level of exactitude is warranted if experiment points to an inadequacy of the simpler approach. In the absence of sufficient experimental evidence, three different approaches have been adopted for this problem. In the first the effect of the difference of Poisson's ratio of the two media has been
neglected, in the second this effect has been taken into account while simplifying the problem by assuming that the metal coating is very thin. And in the third approach the complete problem has been considered. The final justification and limitation of any one of these approaches is of course dictated by experimental evidence.

A. Approach Number One

Consider the arrangement shown in Figure 2. The two media occupy the regions bounded by concentric circles of radii \( R_1 \) and \( R_2 \).

Let

\[
A_m = \text{area of metal}
\]

\[
A_g = \text{area of glass}
\]

\[
P = \text{load on fiber}
\]

\[
\sigma_m = \text{uniform stress in metal, equation (1) or (5)}
\]

\[
\sigma_g = \text{uniform stress in glass, equation (8)}
\]

\[
\varepsilon_m = \text{axial strain in metal}
\]

\[
\varepsilon_g = \text{axial strain in glass}
\]

Then assuming the condition of continuity in the form

\[
\varepsilon_g = \varepsilon_m = \varepsilon
\]  

(10)

and neglecting the reduction of the areas due to Poisson ratio effect
we have

\[ P = A_m \sigma_m + A_G \sigma_G \]  

(11)

or if we introduce

\[ x_m = \frac{A_m}{A_m + A_G} \]

\[ x_G = \frac{A_G}{A_m + A_G} \]

\[ \sigma = \frac{P}{A_m + A_G} = \text{observed stress} \]

we obtain with the aid of equations (1) and (8)

\[ \sigma = x_m \left[ \sigma_0 \left( \lambda_0 \frac{\lambda_1 + \lambda_2 e^{-k_1}}{\lambda_3 + \lambda_4 e^{k_2}} \right) \right] + x_G E_G \varepsilon \]  

(12)

This equation should yield a good approximation in all cases in which the difference in Poisson's ratio of the materials is not too great. Its range of applicability should include single fibers and also composite bars which have a uniform distribution of glass fibers throughout their cross-sections.
B. Approach Number Two

Assuming that at a certain instant of loading, a hydrostatic pressure \( p \) acts on the glass core, any further increase in load \( P \) will bring about a change in this pressure \( p \). Then for the glass core we can write an incremental form of generalized Hooke's Law

\[
d \varepsilon_x = \frac{1}{E_y} \left[ d \sigma_y - \nu_y (d \sigma_y + d \sigma_z) \right]
\]  
(13)

\[
d \varepsilon_y = \frac{1}{E_y} \left[ d \sigma_y - \nu_y (d \sigma_y + d \sigma_z) \right]
\]  
(14)

but

\[
d \sigma_y = d \sigma_y = -d \rho
\]
Therefore

\[ d \varepsilon_y = \frac{1}{E_y} \left[ d \sigma_y + 2 \gamma_y d \rho \right] \]  

(19)

\[ d \varepsilon_g = -\frac{1}{E_g} \left[ (1-\gamma_g) d \rho + \gamma_g d \sigma_g \right] \]  

(16)

And for the metal when

\[ R_i >> t \]

we can write in a similar manner (Fig. 4)

\[ d \varepsilon_m = \frac{1}{E_m} \left[ d \sigma_m - \nu_m (d \sigma_m + d \sigma_m) \right] \]  

(17)

\[ d \varepsilon_m = \frac{1}{E_m} \left[ d \sigma_m - \nu_m (d \sigma_m + d \sigma_m) \right] \]  

(18)

but approximately

\[ \sigma \sigma_m = \frac{R_i}{t} d \rho \]

\[ \sigma \sigma_m = -\frac{i}{t} d \rho \]
Therefore

\[ d \varepsilon_{x_m} = \frac{1}{E_m} \left[ d \sigma_{x_m} - \nu_m (\frac{R_i}{t} - \frac{1}{\varepsilon}) d\sigma \right] \] (19)

\[ d \varepsilon_{x_m} = -\frac{1}{E_m} \left[ (\frac{1}{\varepsilon} + \nu_m \frac{R_i}{t}) d\sigma + \nu_m d\sigma_{x_m} \right] \] (20)

Assuming that no slip and no separation occurs at the interface of the two media, we have as the requirements of continuity

\[ d \varepsilon_{x_m} = d \varepsilon_{x_g} \] (21)

\[ d \varepsilon_{x_m} = d \varepsilon_{x_g} = d\varepsilon \] (22)

The first of these implies

\[ \frac{1}{E_g} \left[ (1 - \nu_g) d\sigma + \nu_g d\sigma_{x_g} \right] = \frac{1}{E_m} \left[ (\frac{1}{\varepsilon} + \nu_m \frac{R_i}{t}) d\sigma + \nu_m d\sigma_{x_m} \right] \]

from which

\[ d\sigma = \frac{1}{K} \left( \nu_g E_m d\sigma_{x_g} - \nu_m E_g d\sigma_{x_m} \right) \] (23)

where

\[ K = E_g \left( \frac{1}{\varepsilon} + \nu_m \frac{R_i}{t} \right) - E_m \left( 1 - \nu_g \right) \] (24)
Substituting for \( dp \) in (16) and (20) we have

\[
E_g \, d\varepsilon = d\sigma_{g} + \frac{2R}{K} \left( \nu g E_m \, d\sigma_m - \nu g E_g \, d\sigma_{m} \right)
\]

(25)

\[
E_m \, d\varepsilon = d\sigma_{m} - \frac{1}{K}\left( \frac{R}{t} - \frac{1}{2} \right) \left( \nu g E_m \, d\sigma_m - \nu g E_g \, d\sigma_{m} \right)
\]

(26)

where use has been made of (22). Inverting these we get

\[
d\sigma_{g} = (A\, E_m + E_g) \, d\varepsilon
\]

(27)

\[
d\sigma_{m} = (-B\, E_m + C\, E_m) \, d\varepsilon
\]

(28)

where

\[
A = \frac{3R^2}{E_m} \div \left( \frac{R}{t} - \frac{1}{2} \right)
\]

\[
B = \frac{1 - \frac{1}{2} - \frac{2R}{E_g}}{\nu g E_g} \div \left( \frac{R}{t} - \frac{1}{2} \right)
\]

(29)

\[
C = \frac{3}{E_m} + \frac{1}{E_g} \left( \frac{1}{t} + \frac{R}{t} \right) \div \left( \frac{R}{t} - \frac{1}{2} \right)
\]

Let now

\[
dP = \text{increment of applied load}
\]

\[
d\sigma = \frac{dP}{A_m + A_g} = \text{increment in observed stress}
\]
Then equilibrium requires that

\[ d\sigma = x_m \, d\sigma_{m} + x_g \, d\sigma_{g}, \]  

or with the aid of (27) and (28)

\[ d\sigma = \left[ -B \, x_m \, E_m^\varepsilon + (C \, x_m + \Lambda x_g) \, E_m + x_g \, E_g \right] d\varepsilon \]  

where

\[ E_m = 4E_0 (1+\beta^2) \varepsilon^{k_2} \left( \lambda_1 \varepsilon^{k_4} + \lambda_3 \right) \left( \lambda_2 \varepsilon^{k_4} + \lambda_3 \right) \]

For stress we have

\[ \sigma = \int d\sigma \]

After carrying out the integration we obtain

\[ \sigma = -x_m \, B \, E_0 \, \varepsilon \left\{ \frac{\sqrt{1+\beta^2}}{\beta} \left( \frac{2\varepsilon^{k_4} + b}{a \varepsilon^{k_4} + c \varepsilon^{k_4} + b} \right) - 7 \right\} + \]

\[ \left[ \frac{\lambda_2 \varepsilon^{k_4} + \lambda_3}{\lambda_4 \varepsilon^{k_4} + \lambda_3} \right] + \left( (C \, x_m + \Lambda x_g) \, E_m \right) \left[ \frac{\lambda_1 + \lambda_2 \varepsilon^{k_4}}{\lambda_2 + \lambda_4 \varepsilon^{k_4}} \right] \]

\[ + x_g \, E_g \, E \]

where

\[ a = 2(1+\sqrt{1+\beta^4}) \quad b = 4\beta^4 \quad c = 2(1-\sqrt{1+\beta^4}) \]

\[ T = (1-\sqrt{1-\beta^4} + \beta^4) / (1+\beta^4) \]
Thus within the framework of assumptions made, equation (32) represents the flow curve of a single glass fiber. As it stands this equation is not applicable for reinforced composite bars. For such bars, especially when the glass content is very low, metal coating around each single glass core is very thick and assumptions made regarding hoop and radial stresses no longer hold. For very high glass content one can consider the entire bar as a single fiber. In that case an equivalent \((R_i/t)\) may be computed from

\[(\frac{t}{R_i})^2 + 2(\frac{t}{R_i}) - \frac{X_m}{X_g} = 0\]

or

\[\frac{t}{R_i} = \sqrt{1 + \frac{X_m}{X_g}} - 1\]  \hspace{1cm} (34)

C. Approach Number Three

N. I. Muskhelishvili (Reference 2) in his Mathematical Theory of Elasticity considers the problem of extension of a composite bar possessing rotational symmetry. If both bodies have linear stress-strain relationship, he gives the following formula for the magnitude of applied load, referring to figure (2)

\[P = S_g \cdot \ell\]  \hspace{1cm} (35)

where

\[S_g = \pi R_i^2 E_1 + \pi (R_i^2 - R_i^1) E_f\]  \hspace{1cm} (36)

\[K_{33} = \frac{4\pi (1 - \nu^2)(R_i^1 - R_i^2)R_i^2}{Y_i(R_i^1 - R_i^2) + Y_2 R_i^2 + 2\mu R_i^2} \hspace{1cm} (37)\]

\[Y = \frac{2(1 - \nu - \nu^2)}{E}, \hspace{0.5cm} \mu = \frac{1 - \nu}{E}\]
It is significant to notice that \( k_{23} \) is always a positive constant which adds to the rigidity of extension of the bar irrespective of the sign of \( (\nu_1 - \nu_2) \). In our notation equation (35) reads

\[
\sigma = \left[ \frac{D}{F + \frac{G}{E_m}} + x_m E_m + x_g E_g \right] \varepsilon
\]

where

\[
D = 4 (\nu_m - \nu_j)^2 x_g
\]

\[
F = \frac{E}{E_g} (1 - \nu_j^2 - \nu_j^2)\varepsilon
\]

\[
G = 2 \left[ (1 - \nu_m - \nu_m^2) \frac{x_g}{x_m} + (1 + \nu_m) \frac{1}{x_m} \right]
\]

Writing equation (38) in the incremental form

\[
d\sigma = \left[ \frac{D}{F + \frac{G}{E_m}} + x_m E_m + x_g E_g \right] d\varepsilon
\]

and integrating it in order to take into account the variation of \( E_m \) with strain we get after considerable mathematical manipulation

\[
\sigma = \frac{H}{F - B} \log \left| \frac{(e - \gamma)(1 - B)}{(e - \gamma^2)} \right| + x_m \alpha \log \left| \frac{\lambda_1 + \lambda_2 e^{\frac{ke}{\lambda_2}}}{\lambda_3 + \lambda_4 e^{-\frac{ke}{\lambda_4}}} \right|
\]

\[
+ x_g E_g \varepsilon
\]
where

\[
H = \frac{D}{2k \, G^* (1+\sqrt{\beta^2})}, \quad G^* = \frac{G}{\sqrt{E_0 (1+\beta^2)}}
\]

\[
(p, q) = \frac{1}{4(1+\sqrt{\beta^2}) G^*} \left[ -(F+q^2 G^*) \pm \sqrt{(F+q^2 G^*)^2 + 16\beta^2 G^*^2} \right]
\]

The first term of equation (41) represents a correction to equation (12).

**Conclusion**

On the basis of theoretical analysis and numerical calculations a few observations can be made regarding the stress-strain relationships derived above. Equation (12) provides a relatively simple means of predicting flow curves for single fibers as well as for composite bars, and in all cases where the difference in Poisson's ratio of the two materials may be neglected. The use of equation (32) is to be confined to single fibers and composite bars of more than 70% glass of uniform distribution. Equation (41) is more cumbersome to evaluate numerically but is applicable to single fibers as well as to composite bars of uniform glass distribution. In the case of bars with 20% glass it has been found that equations (12) and (41) almost coincide. The difference is very small.

It must be pointed out that the three methods of approach explained above are really various degrees of approximation to the actual problem, starting with elementary considerations and ending with more accurate methods in which the interaction of the two media has been considered.
Examples

For purposes of illustration and comparison the following numerical examples have been worked out for equations (12), (32) and (41).

General Data

\( T = \text{temperature} = 500^\circ F \)
\( \sigma_u = \text{ultimate strength of } 2a-H12 \text{ aluminum} = 4300 \text{ psi} \)
\( E_o = \text{initial modulus of metal} = 7.8 \times 10^6 \text{ psi} \)
\( \nu_m = 0.33 \)
\( E_g = \text{modulus of glass} = 11.3 \times 10^6 \text{ psi} \)
\( \nu_g = 0.2 \)
\( C = \text{a universal constant} = 4300 \text{ psi} \) (see reference 1)

Then from (2) and (3)

\( \beta = 1.1752 \)
\( \lambda_1 = 3.718 \)
\( \lambda_2 = -0.632 \)
\( \lambda_3 = 1.368 \)
\( \lambda_4 = 1.718 \)
\( k = 2.332 \times 10^3 \)

and from (33)

\( a = 5.086 \)
\( b = 5.524 \)
\( c = 1.086 \)
\( T = 0.349 \)
I. Single glass fiber (30% metal, 70% glass)

\[ R_i = 2.5 \times 10^{-4} \text{ inches} \]
\[ t = 0.5 \times 10^{-4} \text{ inches} \]
\[ X_j = 0.7 \]
\[ X_m = 0.3 \]

from (29)
\[ A = 0.269 \]
\[ B = 0.141 \times 10^{-6} \]
\[ C = 5.4 \]
\[ CX_m + AX_j = 1.81 \]

II. Composite bar (80% metal, 20% glass)

\[ X_m = 0.8 \]
\[ X_j = 0.2 \]

\[ t/R_i = -1 + \sqrt{1 + \frac{X_m}{X_j}} = 1.296 \]
\[ R_i/t = 0.809 \]
\[ A = 3.883 \]
\[ B = 2.063 \times 10^{-6} \text{ in}^4/\text{lb} \]
\[ C = 25.492 \]
\[ CX_m + AX_j = 21.17 \]

and
\[ D = 0.01322 \]
\[ F = 1.274 \times 10^{-7} \text{ in}^2/\text{lb} \]
\[ G = 2.56 \]
\[ G^* = 0.3446 \times 10^{-7} \text{ in.}^2/\text{lb.} \]

\[ \frac{D}{2kG^2(1+\sqrt{k})^2} = 0.0324 \times 10^3 \text{ psi} \]

\[ \psi = 0.111 \]

\[ \delta = -1.922 \]
### Numerical Data for Stress-Strain Curves

**T=500 °C, 25-H12 Aluminum, "E" Glass**

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\sigma_f$</th>
<th>$\sigma_m$</th>
<th>$\sigma_{30% \text{ metal}}$</th>
<th>$\sigma_{80% \text{ metal}}$</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.2 \times 10^{-3}$</td>
<td>$2.26 \times 10^3$</td>
<td>$1.34 \times 10^3$</td>
<td>$1.98 \times 10^3$</td>
<td>$3.65 \times 10^3$</td>
<td>$1.52 \times 10^3$</td>
</tr>
<tr>
<td>0.4</td>
<td>4.52</td>
<td>2.309</td>
<td>3.857</td>
<td>6.175</td>
<td>2.031</td>
</tr>
<tr>
<td>0.6</td>
<td>6.78</td>
<td>2.988</td>
<td>5.642</td>
<td>9.454</td>
<td>3.746</td>
</tr>
<tr>
<td>0.8</td>
<td>9.04</td>
<td>3.453</td>
<td>7.364</td>
<td>11.842</td>
<td>4.57</td>
</tr>
<tr>
<td>1.0</td>
<td>11.3</td>
<td>3.762</td>
<td>9.038</td>
<td>13.962</td>
<td>5.27</td>
</tr>
<tr>
<td>1.4</td>
<td>15.82</td>
<td>4.085</td>
<td>12.3</td>
<td>17.701</td>
<td>6.432</td>
</tr>
<tr>
<td>1.8</td>
<td>20.34</td>
<td>4.214</td>
<td>15.502</td>
<td>21.098</td>
<td>7.439</td>
</tr>
<tr>
<td>2.0</td>
<td>22.6</td>
<td>4.244</td>
<td>17.1</td>
<td>22.735</td>
<td>7.92</td>
</tr>
<tr>
<td>2.5</td>
<td>28.25</td>
<td>4.287</td>
<td>21.06</td>
<td>26.764</td>
<td>9.03</td>
</tr>
<tr>
<td>3.0</td>
<td>33.9</td>
<td>4.296</td>
<td>25.02</td>
<td>30.736</td>
<td>10.22</td>
</tr>
<tr>
<td>3.5</td>
<td>39.55</td>
<td>4.3</td>
<td>28.98</td>
<td>34.698</td>
<td>11.35</td>
</tr>
<tr>
<td>4.0</td>
<td>45.2</td>
<td>4.3</td>
<td>32.93</td>
<td>38.653</td>
<td>12.48</td>
</tr>
<tr>
<td>5</td>
<td>56.5</td>
<td>4.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>67.8</td>
<td>4.3</td>
<td>48.75</td>
<td>54.473</td>
<td>17.0</td>
</tr>
<tr>
<td>7</td>
<td>79.1</td>
<td>4.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>113</td>
<td>4.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>169.5</td>
<td>4.3</td>
<td>119.94</td>
<td>125.663</td>
<td>37.34</td>
</tr>
<tr>
<td>20</td>
<td>226</td>
<td>4.3</td>
<td>165.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>282.5</td>
<td>4.3</td>
<td></td>
<td>59.94</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>339</td>
<td>4.3</td>
<td>238.29</td>
<td>238.29</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>452</td>
<td>4.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
References
