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# A PRELIMINARY DISCUSSION OF POSSIBLE APPLICATION OF FLUID-FILM BEARINGS TO AUXILIARY POWER A-C GENERATORS IN SPACE VEHICLES

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## 1. Introduction

The present report contains a preliminary, general discussion is given of fluid-film bearings for auxiliary power, AC-generators in space vehicles as a possible alternative to conventional, rolling-element-type bearings, the use of the latter being precluded due to relatively high temperature conditions and environmental factors. The generators, rated at 30 to 100 KVA at 24,000 rpm, are to be turbine-driven, self-contained units, having no external seals between rotating and stationary elements. The sealed containers are to withstand external pressures of 0 to 1 atmosphere and the ambient generator temperature is given as 600°F.

Substances specified as possible working fluids for the closed-cycle turbine are: mercury, sulfur, rubidium and aluminum bromide. These fluids are ~~to be~~ considered as lubricants for the journal and thrust bearings of the generators and the feasibility is discussed regarding ~~of~~ their use in the liquid and/or in the gaseous state, within a very roughly defined but limited temperature - pressure regime. forms the first and very important topic of this discussion. We stipulate that the maximum metal and fluid temperatures within the bearings should not exceed 800°F and assume that the fluid can be supplied as a liquid and/or as a gas at a supply pressure not exceeding 3 atmospheres absolute. Here and elsewhere we also discuss likely effects of condensation (liquid droplets in gas-lubricated bearings) and of evaporation (vapor bubbles in liquid-lubricated bearings), particularly with reference to appreciable temperature and pressure variations in the vicinity of the dew point.

We consider the effects of the ambient environment and the limitations imposed by it on the successful operation of certain types of bearings. This is followed by an inquiry into the feasibility of operating the generators during launching of the vehicle. A representative value of acceleration for a large vehicle of 5g is assumed [ /9 ] and examples are worked out for the load carrying capacity of typical liquid and gas-lubricated journal bearings. The bearing load arising from the gyroscopic couple in a vehicle which tumbles in space is also considered, while in another section attention is given to severe vibrations which may be encountered during launching and initial flight.

The steady-state operation of the bearings is discussed, particularly with reference to weightlessness of the rotating mass and the limitations imposed on bearing design by stability considerations when the journals rotate at high speeds in a nearly concentric position. Possible advantages and disadvantages of liquid and gaseous lubricants in a weightless environment are considered with regard to scavenging and circulation.

In developing the numerous and frequently overlapping topics of this report, we progressively eliminate from further consideration those types of fluid-film bearings which are unsuitable from the lubricant, environment and operational point of view, thereby narrowing down the field to those bearing types which appear to be most promising.



NOTE: Numerals in brackets refer to Bibliography

Whereas no definite recommendations are made for specific bearing and journal materials, general requirements are briefly discussed with reference to high temperature operation, particularly from the point of view of metal expansion, stability, creep and compatability of materials at starting and stopping.

The appendices contain such information as is available with reference to fluid properties and methods of estimating the latter (Appendix 1). A brief, general outline of the theory of fluid-film bearings is given (Appendix 2) and is supplemented by examples of problems for which pertinent theoretical solutions and/or experimental data exist.

The application of fluid-film bearings discussed in this report is poorly defined and the variables are as numerous as their effects are unpredictable. Both, the analytical and the experimental aspects of such an investigation involve extremely difficult problems in the domains of fluid flow, heat transfer and thermodynamics, as well as of metallurgy, chemistry, theory of elasticity, and the broad field of dynamics of continuous media. Theoretical solutions, experimental data and reliable design information for fluid-film bearings, in which the geometry and the properties of both lubricants and bearing materials, are subject to appreciable changes at elevated, non-uniform temperatures, are almost non-existent, even for steady-state operation.

This report, therefore, does not pretend to go beyond discussing some of the more obvious problems which may be encountered with exotic lubricants at elevated temperatures, in an unusual environment.

2. The Lubricating Media

The desirability to lubricate the bearing with a single phase fluid is an obvious one. A realistic and reliable design, based on predictable fluid characteristics, would be limited to either a liquid or a gaseous lubricant. Whereas a few vapor bubbles in a liquid film, or some minute liquid droplets suspended in a gas film are not likely to impair seriously the performance of a bearing, the operating characteristics may become quite erratic and unpredictable if rapid evaporation or condensation is allowed to occur.

In a high-speed liquid bearing, operating in an essentially concentric position (under conditions of weightlessness), the alternate formation and collapse of vapor locks may give rise to unsteady motion and, possibly, to the total destruction of the bearing. Similar remarks apply also to a gas bearing if rapid condensation is precipitated due to appreciable temperature and pressure changes near the dew point of the substance. It is not advisable, therefore, that the operating pressures and temperatures for either liquid or gas-lubricated bearings, be they self-acting or externally-pressurized, should lie close to the dew point of the lubricant. For this reason also (among many other, equally important considerations) it is essential to insure that the ambient and/or supply pressures and temperatures of the lubricant be maintained as closely as is possible within the specified design limits.

No extensive information is available with regard to properties of suggested lubricants, particularly the viscosity. The only exception, possibly, is mercury. In Appendix 1, therefore, formulas are given for reasonably reliable estimates of the viscosity of dilute gases and for very rough* estimates of the viscosity of liquids. Figure A1-1 and the appended table give the vapor pressures of the suggested lubricants as a function of temperature [29]. The curve for aluminum bromide has been extrapolated, since no data is available for pressures above 1 atmosphere.

Since the normal melting points of all four lubricants fall below 208°F (and are not likely to change very appreciably for an increase in pressure of 2 to 3 atmospheres) it becomes immediately apparent that, in the region bounded approximately by a temperature range between 600°F and 800°F and a pressure range between 45 psia and 1 psia, rubidium will be always available in liquid and aluminum bromide in gaseous form. In this case, the choice between a gas-lubricated and a liquid-lubricated bearing, self-acting or externally-pressurized**, is determined by the nature of the fluid.

With regard to mercury, the choice is dual. If the bearing temperatures can be maintained below approximately 700°F and the ambient

*Errors in excess of 50% may be incurred.

**At 24000 rpm the hydrodynamically-induced load carrying capacity may be appreciable.

pressure in the order of 15 psia, a liquid mercury bearing can be operated. A mercury vapor bearing suggests operating temperatures closer to 800°F. For mercury, the vapor pressure curve "divides", so to speak, the suggested temperature-pressure range and although mercury, both liquid and gaseous, is an attractive lubricant from the point of view of viscosity, the problems of condensation and/or evaporation must be considered in the light of previous remarks. Finally, we note from Figure A1-1, that in the temperature-pressure range specified in the foregoing, sulfur will be available mainly in the liquid phase, particularly if the ambient pressure is of the order $> 10$ psia.

So far we have not discussed the important property, the viscosity of the lubricant. With the exception of mercury, viscosity data for the remaining substances are not readily available for states in which they may be considered as suitable lubricants. For the purpose of comparison with viscosities of well known substances at room temperatures, we note that the viscosity of water at 70°F is approximately $1.45 \times 10^{-7}$ lb-sec/in$^2$ and that of air is approximately $2.65 \times 10^{-9}$ lb-sec/in$^2$. At low pressures the viscosity is essentially a function of temperature only.

Figure A1-2 shows the viscosity of liquid mercury. At 600°F the viscosity of liquid mercury is of the same order as that of water at room temperatures and decreases but little with increasing temperature, a desirable characteristic. Figure A1-3 shows the viscosity of mercury vapor, the dashed line representing extrapolated values obtained by means of Sutherlands formula, equation (A1-5). We note that at 700°F gaseous mercury is approximately 3 times as viscous as air at 70°F. The same figure also shows approximate values of viscosity of gaseous aluminum bromide, calculated by means of formula (A1-1), Appendix 1. At 700°F one may expect the viscosity of $Al\ Br_3$ to be approximately 10 times that of the viscosity of air at 70°F.

Liquid sulfur has a most unusual viscosity characteristic, Figure A1-4. In the neighborhood of 300°F it has a viscosity of the order 10 times that of water at room temperature, but at 400°F its viscosity is approximately 20,000 times as high as the viscosity of water at room temperature. No information is available for liquid sulfur in the range of 600° to 800°F. In view of the unusual behavior of this element, caused by changes in molecular structure, no attempt was made to estimate the viscosity of liquid sulfur in the aforementioned temperature range. Suffice it to say, that unless the viscosity of liquid sulfur decreases at least by a factor of $10^3$ between 400°F and 600°F, the "pitch-like" quality of this substance appears to preclude its successful application as a liquid lubricant in a high-speed bearing. On the other hand, its useful range as a gaseous lubricant is associated with temperatures which are either too high, or too close to the dew point.

No viscosity data are available for rubidium, but the properties of this metal are likely to resemble those of cadmium, potassium and sodium. The viscosities of these metals [ 28 ] at representative points of their liquid-phase temperature range are tabulated below.

| Metal | T (°F) | μ lb-sec/in$^2$ |
|---|---|---|
| Cadmium (Cd) | 752 | $3.14 \times 10^{-7}$ |
| | 932 | $2.67 \times 10^{-7}$ |
| | 1112 | $2.24 \times 10^{-7}$ |
| Potassium (K) | 153.3 | $.746 \times 10^{-7}$ |
| | 482 | $.374 \times 10^{-7}$ |
| | 1292 | $.197 \times 10^{-7}$ |
| Sodium (Na) | 219.7 | $.994 \times 10^{-7}$ |
| | 482 | $.552 \times 10^{-7}$ |
| | 1292 | $.264 \times 10^{-7}$ |

The viscosities of other liquid metals are of the same order of magnitude ($3.07 \times 10^{-7}$ - $1.73 \times 10^{-7}$ for lead between 826°-1551°F), so that one may expect the viscosity of liquid rubidium to be comparable with that of water at 70°F and, generally, with the viscosities of other liquid metals.

Summarizing, we conclude from the preceding paragraphs that, in the specified temperature-pressure range, rubidium may be considered as a liquid lubricant, aluminum bromide as a gaseous lubricant, and that with mercury either alternative is feasible, provided that care is exercised to avoid an operating condition in which either rapid evaporation, or condensation take place. It appears that sulphur does not hold out much promise as a lubricating medium in the considered temperature-pressure range.

3. <u>The Environment</u>

In the course of preliminary discussions with Jack and Heintz Inc., one of the suggestions made was that the entire alternator assembly, including the bearings, be placed in the turbine condenser, in an environment corresponding to ambient conditions of approximately 1 psia and 600°F. If we bear in mind that in the absence of gravitational forces the bearings operate in an essentially concentric position, when the pressure throughout the film of a self-acting bearing varies but little from the ambient pressure, and that no gravity sump exists for liquid lubricants, we unavoidably impose additional limitations on the already restricted choice of bearing and lubricant. Referring to Figure A1-1, we note that aluminum bromide is available in gaseous form at very low pressures in the 600°F-800°F temperature range. The load capacity of a self-acting gas bearing, however, is dictated by the ambient pressure level, so that only externally-pressurized bearings can be considered in this case. Rubidium, on the other hand, is still a liquid at low pressures in this temperature range and could be considered for both self-acting and externally-pressurized bearings. In the absence of gravity, scavenging of liquid lubricants is a problem, since no natural gravity sump exists and the forces acting on a liquid in a weightless environment are mainly due to surface tension and centrifugal effects. From a point of view of lubrication, flooding has no adverse effect on the bearing performance, but removal of the lubricant-condensate from the condenser, which contains the stator and a rotor revolving at 24,000 rpm, is certainly one of the most important design problems to be considered.

Mercury, fed to the bearing as a liquid, may also tend to evaporate within the clearance and give rise to an undesirable condition of "mixed-phase" lubrication. Liquid mercury, for example, supplied at 700°F and 30 psia, will begin to vaporize when the pressure in the film drops to approximately 20 psia and continue to do so at an ever increasing rate as the fluid enters extensive regions within the bearing clearance, in which the pressure is only slightly above the 1 psia condenser level. On the other hand, mercury may offer an attractive solution from the point of view of lubricant scavenging. If the ambient pressure and temperature (and those in the liquid-film) could be sufficiently well controlled, so that the substance remains predominantly liquid within the film, but is caused to evaporate upon leaving the clearance, the removal of a vapor from the ambient surroundings of the rotor to a separate condenser may offer a considerable advantage over scavenging of a liquid in the absence of gravity.

It may be remarked at this point that there is really no clear cut distinction between self-acting and externally-pressurized bearings, particularly in the absence of gravitational forces. A liquid lubricant is normally supplied at some points on the unloaded side of a bearing, under pressure which is generally but an insignificant fraction of the maximum pressure in the film. The very same bearing becomes an "externally-pressurized" bearing under no load (weightless) conditions. The journal becomes slightly displaced to an eccentric equilibrium position and the resultant of pressure forces due to the combined effects of eccentricity and of external sources is zero.

Figure 1a shows a schematic diagram of the system when the bearings and the rotor are placed in the condenser. A more preferable arrangement, one which would afford a better control of the ambient conditions, is shown in the schematic diagram of Figure 1b. From the point of view of lubricant removal and recirculation under conditions of weightlessness, a gaseous lubricant, or a lubricant which would evaporate very rapidly in the ambient surroundings, seem to offer a definite advantage over a liquid. Placing the bearings in a surrounding other than the condenser, and a reasonable degree of latitude in controlling the ambient pressure and temperature, appear to be desirable features to be considered in future design.

The question of stability is discussed elsewhere in this report. With reference to high-speed operation of fluid-film bearings in a nearly concentric position, we only wish to remark at this point that this condition is precisely that in which an instability, generally referred to as "Whirl", is most likely to occur. One must resort to special types of "anti-whirl" bearings, or induce artificially a high degree of eccentricity in order to insure stable operation at high rotational speeds.

Summarizing the contents of this section, we conclude that means must be provided for efficient lubricant scavenging, and that in the absence of gravity a gas may be superior to a liquid from this point of view. Placing the bearings in a temperature and pressure controlled ambient environment, other than the condenser, appears to be preferable. The use of mercury as a liquid lubricant which evaporates upon leaving the bearing clearance may offer a possible solution to avoid the accumulation of liquid in the surroundings of a high-speed rotor, provided that the ambient temperatures and pressures in the operating range of a liquid mercury bearing can be adequately controlled. Finally, remedies must be found to counteract the tendency of fluid-film bearings to become unstable at high rotational speeds and at small eccentricity ratios.

4. Bearing Loads

The most difficult problem posed by future design considerations are the severe bearing loads encountered during launching and caused by both vehicle acceleration and vibrations. The effects of vibrations will be discussed in the following section. Here we shall only consider the load due to a constant acceleration, assuming a value of 5g to be representative for a large vehicle. A ratio of approximately 1.15 lb/KVA is specified for the alternators in the 30 to 100 KVA range (a weight of 35 lb for a 6" x 10" rotor of a 30 KVA generator and a weight of 110 lb for a 9" x 16" rotor of a 100 KVA generator). Since the vehicle may tumble in space, we also estimate the maximum allowable rps of tumbling which would result in a 5g bearing load.

It is assumed here that the generators are required to supply power throughout the launching phase and are started during the count-down period. It is appropriate to remark at this point that if the generators are to be started in space, and if tumbling is controlled, it is possible to operate, at least in principle, all types of fluid bearings considered so far, be they liquid or gas-lubricated, self-acting or externally-pressurized. (In the latter case, a gas-lubricated bearing may have the advantage from the point of view of lubricant removal from the ambient surroundings of the generator.)

We shall demonstrate that no externally-pressurized bearing of reasonable dimensions is likely to operate successfully if the vehicle acceleration is 5g and the supply pressure is limited to 3 atmospheres absolute. As an example we take a rotor with two 3" x 3" journal bearings. The configuration is shown in Figure 2. The bearing has 3 pressure pads, separated by venting grooves,* each pad extending over an arc of approximately 120°. The pads have very shallow, circumferential grooves, fed by supply nozzles, or orifices. The journal is shown in a position corresponding to a displacement along the load line, although at high rotational speed the attitude angle would be greater than zero and an additional load carrying capacity would be induced in a manner analogous to that of a purely self-acting bearing. For simplicity also, we shall assume a linear pressure drop from the edge of the groove to the periphery of the pad, and a uniform pressure plateau extending over the region bounded by the groove. It will be taken for granted that the clearance, the eccentricity and the dimensions of supply restrictors allow us to maintain the required pressure levels in individual pads.

*Grooved and groove-separated pad-bearings of this type, as well as self-acting journal step-bearings (Rayleigh type), are inherently more stable and less likely to "whirl" at high rotational speeds. [6, 11, 14, 17, 18]

The pad dimensions are approximately 3" x 3" and those of the region bounded by the grooves 1-1/2" x 1-1/2". For a supply pressure $P_s = 45$ psia and an ambient pressure Pa = 10 psia, we assume that a pressure $P_r = 22.5$ psia ($P_r = 0.5\ P_s$) is maintained in the grooves of pads 2 and 3, and that the corresponding pressure for pad 1 is $P_r = 37.5$ psia ($P_r = 0.835\ P_s$). These magnitudes are equivalent to pressure differentials, $\Delta P_r$, of 12.5 psi and 27.5 psi above the ambient pressure.

The assumed pressure distribution is that of a "truncated pyramid" (See Figure 2) and we may calculate the resultant force for a single pad, F, as follows

$$F = \frac{1}{3}\left(A + A_r + \sqrt{AA_r}\right)\Delta P_r = A_e\,\Delta P_r \tag{1}$$

in which A and $A_r$ are the areas of the pad and of the region bounded by the groove, and

$$A_e = \frac{1}{3}\left(A + A_r + \sqrt{AA_r}\right) \tag{2}$$

The average pressure difference acting on the pad area, A, is:

$$\overline{\Delta P} = \frac{F}{A} = \frac{A_e\,\Delta P_r}{A} \tag{3}$$

The effective force, acting at the center of each pad, is the average pressure times the projected area of A, normal to the radius from the bearing center to the center of the pad, $A'$. The effective force, $F'$, is:

$$F' = A'\,\overline{\Delta P} = A'\,\frac{A_e}{A}\,\Delta P_r \tag{4}$$

We have:

$$A = 3 \times 3 = 9 \text{ in}^2$$

$$A_r = 1.5 \times 1.5 = 2.25 \text{ in}^2$$

and from equation (2):

$$A_e = 5.25 \text{ in}^2$$

Approximately also

$$A' = 2 \times (1.5 \sin 60°) \times 3 = 7.8 \text{ in}^2$$

so that

$$F' = 4.55 \ \Delta P_r$$

The magnitude of the effective force for pad 2 and 3 is:

$$F_{2,3}' = (4.55)\ (12.5) = 57 \text{ lb}$$

and that for pad 1 is:

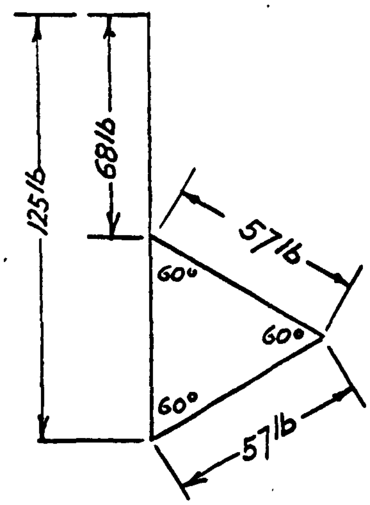
$$F_1' = (4.55)\ (27.5) = 125 \text{ lb}$$

The total load carrying capacity for this bearing is

$$L = F_1' - 2F_2' \cos 60°$$

$$= 125 - 57 = 68 \text{ lb}$$

In this case, the load carrying capacity of two 3" x 3", externally-pressurized journal bearings is only 135 lb, whereas the load, due to the mass of a rotor accelerated to 5g, is 175 lb for the smallest, and 550 lb for the largest rotor in the proposed 30 KVA - 100 KVA generator series. Since the effective areas vary as




the square of the linear bearing dimensions, and since neither clearance nor eccentricity can be excessively large, the limitations imposed by a low supply pressure and a high acceleration during launching, coupled with high-speed and temperature considerations, preclude a successful application of externally-pressurized bearings.

We next turn our attention to load carrying capacities, likely to be developed in self-acting, liquid and gas-lubricated bearings. We shall consider plain journal bearings, but we shall also bear in mind that plain journal bearings are generally unstable if operated at high speeds and at low eccentricity ratios. Stability is often achieved at the expense of the load carrying capacity. For the present, however, the plain journal bearing will suffice for the purpose of assessing the order of magnitude of load carrying capacities. We shall make use of computer results presented in two papers by A. A. Raimondi [4,5], although numerous other sources of information, both theoretical and experimental, are available. A brief review and a summary of basic theory of fluid film bearings is also presented in Appendix 2.

The performance of a liquid-film bearing is usually specified in terms of dimensionless parameters:

$$S = \left(\frac{R}{C}\right)^2 \frac{\mu N}{\bar{P}} = \text{Sommerfeld number}$$

$$\varepsilon = \frac{e}{C} = \text{Eccentricity ratio}$$

$$\frac{L}{D} = \text{Length to diameter ratio}$$

and, for partial bearings,

$$\beta = \text{Angle subtended by bearing arc}$$

For gas bearings data are more conveniently presented in terms of:

$$\lambda = 2\pi \left(\frac{R}{C}\right)^2 \frac{\mu N}{Pa} = \text{Compressibility number*}$$

and $$\frac{\bar{P}}{Pa} = \frac{W}{LDPa} = \text{Load ratio}$$

because the gas bearing performance depends on the ambient pressure level.

In the foregoing

$C$ = bearing clearance, radial
$e$ = eccentricity
$D = 2R$ = journal diameter

*This is Raimondis expression for $\lambda$. Most authors use $= \frac{6\mu\omega}{Pa}\left(\frac{R}{C}\right)^2$.

L = bearing length
$\bar{P}$ = unit load (load/projected bearing area)
N = journal speed (rps)
$\mu$ = viscosity
Pa = ambient pressure (absolute)

Similar quantities appear in the parameters which characterize the performance of slider bearings.

We shall consider a 3" x 3" full journal bearing, L/D = 1; R/C = 1000; $\varepsilon$ = 0.6*; N = 400 rps (24,000 rpm). For the liquid lubricant we take mercury at 650°F, $\mu \approx 1.3 \times 10^{-7}$ lb-sec/in$^2$ (the viscosity of liquid rubidium is expected to have the same order of magnitude) and for the gaseous lubricant we consider aluminum bromide at the same temperature, with an estimated $\mu = 23 \times 10^{-9}$ lb-sec/in$^2$. An ambient pressure of Pa = 15 psia is assumed.

Example 1 - Liquid mercury

From reference [ ], for a cavitated full journal bearing we have:

$$\text{at } \varepsilon = 0.6;\ S = \left(\frac{R}{C}\right)^2 \frac{\mu N}{\bar{P}} = 0.14$$

$$\bar{P} = \left(\frac{R}{C}\right)^2 \frac{\mu N}{S} = \frac{(10^6)(1.3 \times 10^{-7})(4 \times 10^2)}{1.4 \times 10^{-1}} = 370 \text{ psi}$$

The total projected bearing area (2 bearings) is 18 in$^2$. The load carrying capacity is:

$$W = 370 \times 18 = 6650 \text{ lb}$$

*For high speed, high temperature gas bearings, operation at high eccentricity ratios may become unsafe.

This is more than sufficient to withstand a 5g-loading for the largest rotor mass. Unlike gas-lubricated bearings, the liquid-lubricated bearing may be operated, at least for short periods of time, at very large eccentricity ratios (the "boundary lubrication" region), without running the risk of severe damage, or total destruction.

Since the temperature rise is of interest in the case of the more viscous, liquid lubricants, it can be shown [32], with the assumption that the energy dissipated in the film is available for increasing the temperature of the lubricant, that:

$$\Delta\tau = \frac{J\Upsilon c\Delta T}{P} = \frac{8\pi}{2 - \frac{Q_s}{Q}} \cdot \frac{\left(\frac{R}{C} f\right)}{\left(\frac{Q}{RCNL}\right)} \tag{5}$$

in which, in addition to symbols previously defined, the quantities in equation (5) are as follows:

$\Delta T$ = temperature rise of lubricant
$\Upsilon$ = weight density of lubricant
$c$ = specific heat of lubricant
$Q$ = the volume time-rate of lubricant entering the film
$Q_s$ = the volume time-rate at which the lubricant leaves the sides of the bearing
$f$ = coefficient of friction
$J$ = mechanical equivalent of heat (9336 in-lb/Btu)

In example one, for the case of mercury, we have:

$$\Upsilon = 0.491 \frac{\text{lb}}{\text{in}^3} \; ; \; c = 0.0324 \frac{\text{Btu}}{\text{lb-°F}}$$

For S = 0.14, reference [ 4 ] gives:

$$\Delta\tau = 18$$

hence:

$$\Delta T = \frac{\Delta\tau\bar{P}}{J\gamma c} = \frac{(1.8 \times 10^{1})(3.7 \times 10^{2})}{(9.34 \times 10^{3})(4.91 \times 10^{-1})(3.24 \times 10^{-2})} = 45°F$$

For gases, the increase in temperature would be substantially less and tend to increase the viscosity and, therefore, the load carrying capacity of the bearing.

Example 2 - Gaseous Aluminum Bromide

From reference [ 5 ], we have for a full journal bearing:

$$\text{at } \varepsilon = 0.6; \quad \lambda = 2\pi\left(\frac{R}{C}\right)^2 \frac{\mu N}{Pa} = \frac{(6.28)(10^{6})(2.3 \times 10^{-8})(4 \times 10^{2})}{1.5 \times 10^{1}} = 3.85$$

$$\frac{\bar{P}}{Pa} = 1.4 \text{ or } \bar{P} = 21 \text{ psi}$$

so that for a total projected area of two 3" x 3" bearings, we obtain a load carrying capacity of

$$W = 21 \times 18 = 380 \text{ lb}$$

which will support a 75 lb rotor mass, accelerated to 5g.

Aluminum bromide appears to be a very viscous gas, that is if one is to place any reliance on the calculated viscosity values, Appendix 1.

We shall, consequently, carry out a similar calculation for mercury vapor at 750°F to which there corresponds a value of $\mu \approx 8.3 \times 10^{-9}$ lb-sec/in$^2$. The reason for selecting this rather high temperature is that at Pa = 15 psia mercury condenses at approximately 675°F. Since peak pressures in the converging part of the film are likely to attain values twice as large, or more, as that of the ambient pressure, condensation may occur if the temperature is too low. At the same time, the ambient pressure can only be lowered at the expense of the load carrying capacity.

Example 3 - Mercury Vapor

In this case we have

$$\text{at } \varepsilon = 0.6; \quad \lambda = 2\pi \left(\frac{R}{C}\right)^2 \frac{\mu N}{Pa} = (3.85)\ \frac{(8.3 \times 10^{-9})}{(23 \times 10^{-9})} = 1.39$$

$$\frac{\bar{P}}{Pa} = 1.12 \text{ or } \bar{P} = 16.8 \text{ psi}$$

and a corresponding load carrying capacity of

$$W = 16.5 \times 18 \approx 300 \text{ lb}$$

which is sufficient to support a 60 lb rotor mass when the acceleration is 5g. The maximum pressure attained in the film [5] is approximately

$$P_{max} = 1.43 \text{ Pa} = 21.5 \text{ psia}$$

so that no condensation will occur (see Figure A1-1).

It has previously been stated that plain journal bearings are unstable if operated at high speeds and at small eccentricity ratios. We do not wish to anticipate the contents of the next section, but it must be realized that "anti-whirl" types of journal bearings may have load carrying capacities of the order of only 50 to 75 percent of plain journal bearings. Whereas in the case of liquid-lubricated bearings there appears to be more than enough to spare with regard to load carrying capacity, the corresponding magnitudes which can be realized with gas-lubricated bearings are marginal. It may well be that the latter are suitable only as means of support for small, light rotors.

One is limited by both bearing size and bearing clearance. On the other side, the appearance of typical $\lambda$ vs. $\bar{P}/P_a$ (parameters $\varepsilon$ and L/D) clearly indicates their asymptotic behavior (a fact supported by theoretical considerations [1,2,5]). For values of

$$\lambda = 2\pi \left(\frac{R}{C}\right)^2 \frac{\mu N}{P_a} \gtrsim 1$$

the load carrying capacity of most self-acting gas bearings cannot be increased by any appreciable amount, except by raising the ambient pressure, $P_a$. Thus, even if it were feasible to operate in a high and, possibly, non-uniform temperature environment with larger clearance ratios, R/c (e.g. to decrease the clearance) no benefit would ensue by increasing $\lambda$ beyond the value of approximately 1. Similar remarks apply with regards to the possible use of high-viscosity gases.

One must also consider gyroscopic effects due to the angular velocity of possible tumbling motion of the vehicle in space. Assuming that in the worst possible case the vehicle tumbles at $N_T$ rps in a plane passing through the spin axis of a cylindrical rotor, weight W ($I = WD^2/8g$), revolving at $N_S$ rps, we can estimate $(N_T)_{max}$ for a limiting bearing load $F_{max} = nW$ and a bearing span, L. Taking this oversimplified approach for the purpose of estimating the severity of gyroscopic loads, the maximum gyroscopic moment is:

$$4\pi^2 \, I N_S \, (N_t)_{max} = FL \qquad (6)$$

or

$$\frac{4\pi^2 \, D^2 \, W N_S (N_t)_{max}}{8g} = nWL \qquad ($$

in which

$$n = \frac{F_{max}}{W}$$

so that

$$(N_t)_{max} = \frac{2nLg}{(\pi D)^2 \, N_S} \qquad (7)$$

Taking $n = 5$; $N_S = 400$ rps; $L = 12$ in; $D = 7.5$ in, we have, approximately

$$(N_t)_{max} \approx \frac{1}{5} \text{ rps} = 12 \text{ rpm}$$

Summarizing briefly the contents of this section, we conclude that high bearing loads, accompanying the launching of a vehicle, could safely be carried on liquid-mercury and liquid-rubidium-lubricated bearings, with an additional advantage gained at starting, but with a possible disadvantage of liquid lubricants in a weightless environment. Self-acting gas-lubricated bearings may also be designed to withstand considerable accelerations, particularly for less massive rotors. Smaller frictional losses would contribute to cooler ambient surroundings of the generator, while the removal of a gaseous lubricant in a weightless environment from the ambient bearing and rotor surroundings may be easier than that of a liquid. Finally, the load carrying capacity of externally-pressurized bearings at a maximum supply pressure of 3 atmospheres is insufficient if not supplemented by an appreciable, self-induced (hydrodynamically) pressure field. External pressurization, however, will undoubtedly play an important role in starting a generator equipped with self-acting, gas-lubricated bearings.

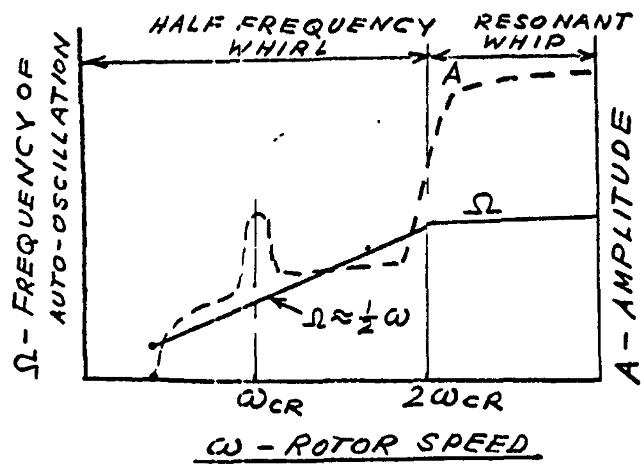
5. Instability and Critical Speeds

The bearings, the rotor and the fluid film constitute a complex, distributed parameter system, the response of which is governed by the mass and by the elastic properties of its components and the quasi-elastic and dissipative characteristics of the film. The formulation of the problems involves the equations of motion of all moving members, as well as the fluid-dynamic equations. This results in a set of partial and generally non-linear differential equations and boundary conditions which are to be satisfied. It is necessary to make simplifying assumptions and consider only special cases. Only a limited number of computer and linearized solutions exist for very simple systems and bearing configurations. Good experimental data is equally difficult to obtain. Precise manufacture of test bearings, alignment and isolation of equipment represent a formidable task, but even more difficult is the problem of accurate and reliable measurement of eccentricity, pressure and temperature in the bearing clearance. Vibration measurements of a rotating and a translating shaft, with amplitudes of the order of a fraction of the clearance, have taxed the ingenuity of many investigators.

A type of instability invariably associated with gas, as well as liquid-lubricated journal bearings, particularly when operated at high speeds and at low eccentricities (vertical or lightly loaded shafts), is referred to in literature as "half-frequency whirl". The name derives from the fact that the angular velocity of the line of centers is approximately one half the speed of rotation of the shaft about its axis. The amplitude of the whirling motion may grow with the speed of rotation when the threshold speed has been reached, so that the danger of total bearing destruction arises if the amplitude becomes as large as the clearance. This situation is particularly dangerous in the case of plain, gas-lubricated journal bearings, which possess neither sufficient damping (low viscosity), nor any protection that a liquid film may provide to the surfaces. Since perfect symmetry, balancing and alignment are never realized, not even in symmetrical, two-bearing-rotors, "angular" modes of whirl are frequently observed and, generally, at a lower threshold speed than that of the "translatory" mode.

The scope and nature of this report do not permit us to go into details. Suffice it to say that no very reliable and theoretically well founded criteria exist for predicting the threshold frequency of whirl in liquid-lubricated bearings. This is even more true in the case of gas-lubricated bearings*. The subject is an extremely controversial one and this type of instability is frequently confused with other types of vibrations and auto-oscillations associated with rotors mounted in fluid-film bearings. For example, when the rotational speed of the shaft is approximately <u>twice</u> that of its "first critical" speed, another type of self-excited vibration may occur. This is known as "resonant whip" and is frequently confused with "half-frequency whirl". The frequency of this vibration is equal to approximately the "first critical" and <u>it persists</u>

*The reader is referred to the following, typical references on the subject which are listed in the bibliography: [9,10,12,7] and [6,14,17,18,13] at approximately this frequency when the shaft speed is increased above twice the value of the "first critical". Large and destructive amplitudes may build up. The latter phenomenon usually occurs in elastic shafts and may be absent in the case of short, rigid rotors.




A criterion, due to Boeker and Sternlicht [10,11,13], and several variants thereof, have been used in predicting the threshold frequencies of half-speed whirl in liquid and gas-lubricated journal bearings possessing a high degree of symmetry.

The criterion requires that:

$$M\omega^2 \left(\frac{1}{K} + \frac{1}{K_2}\right) < 4 \tag{8}$$

in which M is the rotor mass, $\omega$ is the angular speed of rotation, K is the rotor stiffness and $K_2$ is defined as:

$$K_2 = \frac{\partial F_r}{\partial r} = \text{"radial film stiffness"}$$

where $F_r$ is the component of the resultant film-force, F, along the line of centers.

The difficulty arises from the fact that the meaning and functional dependence of $K_2$ vary from author to author, and that all claim good correlation with experimental data, regardless of whether the particular bearing is externally-pressurized, self-acting, or both, or whether the lubricant is compressible, or incompressible. Fisher, Cherubim and Fuller [ 14 ], for example, assume for $K_2$ a value obtained from static load-deflection characteristics, assuming that the components of the film force, F, are the same as if the load were applied quasi-statically, with the journal center following the eccentricity locus. They support this argument by stating that a value of $K_2$ derived in this manner is in good agreement with values of $K_2$ determined on the basis of (a) frequency of rotor vibrating in bearings when subjected to impact and (b) the observation of the "first critical" speed. The agreement between calculated and experimental results is of the order of 6-9%, which is excellent in view of the complexity of the problem.

A new approach to $K_2$ was introduced recently by the staff of the General Electric Co. Bearing Center, under the leadership of Dr. Sternlicht [ 13 ]. They now consider that the fluid force, F, (and, therefore, $K_2$) depends not only on the eccentric position of the journal at a given instant of time, but also on the velocity. Neither the magnitude, not the direction of the velocity are known. The authors, therefore, prescribe different values and directions of the velocity and solve the dynamic Reynold's equation (Appendix 2) by digital computer techniques. In order to avoid difficulties in obtaining results for compressible fluids, the term $\partial\rho/\partial t$ is neglected in the fluid-dynamic equations, although this step cannot be justified. Tests conducted with vertical rotors and gas-lubricated bearings show very good agreement between predicted and empirical results [136]. Although this writer is rather skeptical with regard to the simple criterion in which the most important term, $K_2$, is given so many different interpretations, we give Elwell's [136] summary of the half-frequency whirl theory for rigid, lightly loaded rotors.

In the case of the "translatory" mode of whirl

$$M\omega^2 \times \left(\frac{1}{K} + \frac{1}{K_2}\right) < 4 \tag{8}$$

and if the rotor is very rigid, $1/K \approx 0$. The threshold frequency of the "translatory" mode of whirl then becomes:

$$\omega_T^{\,2} = \frac{4K_2}{M} \quad \text{(translatory whirl)} \tag{9}$$

For a symmetrical, two-bearing rotor $K_2' = 2K_2$ applies.

For the case of the "angular" mode of whirl of a symmetrical, two-bearing rotor, we have:

$$\frac{\ell^2}{2} K_a + 2K_3 > \left(\frac{\omega}{2}\right)^2 \left(I_t - 2I_p\right) \tag{10}$$

in which

$$K_3 = \frac{L^2}{K_a} \tag{11}$$

so that with additional, simplifying assumption with regard to effects of misalignment

$$\omega_a^2 = \frac{2}{3}\left(\frac{3\ell^2 - L^2}{I_t - 2I_p}\right) K_a \text{ (Angular whirl)} \tag{12}$$

In the foregoing equations:

$\ell$ = Bearing span, center to center
$L$ = Bearing length
$I_t, I_p$ = Transverse and polar moments of intertia of rotor

and $K_a$ is the translatory radial stiffness at one bearing corresponding to the "angular" threshold speed of whirl, $\omega_a$.

The predictions based on equations (9) and (12) will be as good as the K-values. Nevertheless, satisfactory results have been obtained with [ 13 ] or without [ 14 ] introducing refinements in the determination of $K_2$ or $K_a$. Since the critical speed value varies here as the square root of either K, the error is always halved. This may be, indeed, a blessing in disguise.

In reference [13a] the authors also consider the "resonant frequencies" of a symmetrical, rigid rotor. The center of gravity does not coincide with the geometrical axis of the rotor, but is displaced by an amount equal to $\delta$ in the plane of symmetry perpendicular to the axis. It is assumed here that the center of the journal moves in a circular orbit, radius $e_o < C$, around the bearing center and that the unbalance rotates with the same angular velocity as the journal. The "resonant frequencies" for specific values of $\delta$, $e_o$ and $\omega$ are given by the expression

$$\omega_{res} = \frac{F}{me_o\omega\left[1 - \left(\frac{\delta}{e_o}\right)^2\right]^{1/2}} \tag{13}$$

where F is the fluid-force exerted on the journal, and values of its dimensionless equivalent

$$f = \frac{2\pi}{\mu\omega LD\left(\frac{R}{C}\right)^2} F$$

for this type of motion are tabulated in reference [13a] for various ratios of $\varepsilon_o = e_o/C$. Since an unbalance is a rotating load, one may apply these results in analogous situations in the case of an electrical unbalance.

An analytical investigation of whirl in gas-lubricated bearings is presently conducted at The Franklin Institute by Mr. V. Castelli. The dimensionless parameters which must be maximized, or minimized in order to insure stability, are:

$$\left.\begin{aligned} \lambda &= \frac{6\mu\omega}{P_a}\left(\frac{R}{C}\right)^2 \text{ (minimized)} \\ B &= \frac{2\lambda}{\omega}\sqrt{\left(\frac{R}{C}\right)\frac{P_a}{M}} \text{ (maximized)} \\ \frac{\bar{P}}{P_a} &= \frac{W}{P_a LD} \text{ (maximized)} \\ \frac{L}{D} &= \frac{L}{2R} \text{ (minimized)} \end{aligned}\right\} \tag{14}$$

The parameter $\lambda$ tells us, that for a prescribed angular velocity, gas viscosity and bearing diameter, the smallest possible clearance should be used and that a high ambient pressure is desirable. The parameters B and $\bar{P}/P_a$ indicate that stability favors non-massive, but heavily loaded rotors. Finally, long bearings are more likely to be unstable than short bearings. The reader will be aware, of course, that one must refer to dimensionless parameters as a whole, and not to dimensional quantities individually. The preceding remarks are merely intended to give some kind of a physical interpretation to possible effects of speed, weight, size, etc. Requirements applicable to liquid-lubricated bearings are almost identical, except that the ambient pressure affects only the degree of film cavitation, but does not otherwise affect the load carrying capacity of the bearing. In the limiting case of $\lambda \rightarrow 0$, the compressible and incompressible solutions give the same results, but note that for the same $\lambda$ an increase in the viscosity by a factor of $10^2$, for example, is equivalent to a corresponding increase of the clearance by a factor of 10. This, and remarks made in the last section, show that in the case of liquid-lubricated bearings neither load, nor stability requirements impose as severe limitations on the bearing clearance ratio as in the case of gas-lubricated bearing. This is an important consideration, since high temperatures and temperature gradients call for a bearing with the greatest possible clearance. Moreover, the values of $\lambda$ are likely to be so high that no further benefit could be derived from an additional increase (by increasing R/C), because of the asymptotic behavior at the load when $\lambda \rightarrow \infty$.

On the other hand, a large R/C ratio is most important from the point of view of stability, but because of limitations imposed by high temperatures and temperature gradients we cannot take advantage of stabilizing the bearing by reducing the clearance.

It is difficult to predict the response of the rotor mass when the entire generator and the bearing supports are subjected to random disturbances, or steady vibrations within a wide frequency band. A condition of resonance at some frequency of excitation is to be expected and much will depend on the method of isolation of the generator. If at the time of launching, or during flight, the mode and frequency of excitation transmitted to the bearings is such that a resonant condition is induced in the quasi-elastic rotor and fluid-film system, one can only rely on the dissipative properties of the film to limit the amplitudes of relative bearing and journal displacement to a reasonable fraction of the clearance. Liquids, because of their higher viscosities, are certainly superior from this point of view. The "first natural" of the rotor will probably be below the 400 cps, operating speed, so that one may not have to worry about frequencies of excitation in the $10^3$ to $10^4$ cps band width, provided the bearings can be designed for the rotor to "ride over the critical" and for whirl-free operation at the rated speed. We do not wish to minimize the adverse effect of shock and vibrations, but it would be futile at this stage to speculate with regard to the response of an unknown, distributed parameter, non-linear system to unspecified inputs.

Summarizing the content of this section, we conclude that the most important aspect of a successful bearing design is to insure stable and whirl-free operation. All other requirements may be met, but if the bearing is inherently unstable when operated at high speeds in a nearly concentric position, it will be useless half way to Venus, though it may support a 50g load when the vehicle starts on its way. We have pointed out that plain journal-bearings are the worst offenders from this point of view and that the problem of instability of rotors in fluid-film bearings has not been completely solved. One is not as helpless, however, as it may seem, since bearings have been designed and operated well in excess of 100,000 DN. In the next section, therefore, we shall consider again briefly the physical mechanism of half-frequency-whirl and the requirements for constructing anti-whirl bearings. We shall then consider one type of anti-whirl bearing, known to have been used successfully in many high-speed applications. An approximate estimate of the load carrying capacity of such a bearing will also be made, for both a liquid and a gas.

## 6. Anti-Whirl Bearings and Their Load Capacity

Returning to the subject of "half-frequency whirl", the following simple illustration of the phenomenon is given. Referring to Figure 5, consider a very long bearing, completely filled with a liquid. If the shaft is vertical, or unloaded, the bearing and the journal are concentric. If a load, W, is applied as shown, the Sommerfeld bearing causes the journal center to be displaced at right angles to the load line. If there is no load, but the journal is accidentally displaced by a small distance, $e_o$, from 0 to 0', the pressure forces in the converging half of the film will tend to drive the journal in an orbit around the bearing center, because in this position there is a moment $-We_o$ acting on the journal. For a small displacement and laminar flow conditions, the velocity profile will be nearly linear, Figure 5. The quantity of liquid transferred at the point of maximum clearance to the lower bearing half must equal the quantity of liquid entering the upper bearing half at the point of minimum clearance. To satisfy the conservation condition we must have:

$$\frac{1}{2} R\omega (C + e_o) - \frac{1}{2} R\omega (C - e_o) = 2R\Omega e_o \qquad (15)$$

Hence

$$\Omega = \frac{1}{2} \omega \qquad (16)$$

showing that the whirl frequency, $\Omega$, is one half of $\omega$, the angular speed of rotation of the journal. This simple approach gives a qualitative explanation of the phenomenon, but with loaded, finite journals and compressible lubricants the analysis of the whirl problem is much more complex.

Nevertheless, it becomes apparent that if the journal of a plain cylindrical bearing is accidentally displaced from the concentric position, one may expect a large component of the resultant fluid-force at right angles to the displacement, which tends to drive the journal into an orbit, or into a path which spirals out. The properties of an anti-whirl bearing must be such as to return the journal to the equilibrium position along the shortest possible path. This can be achieved by introducing discontinuities into the film, that is by dividing the cylindrical bearing surface into several regions, the resultant pressure force at each region tending to restore the journal to the equilibrium position. The "balkanization" of the bearing surface can be accomplished in several ways. The simplest method is to introduce several axial grooves in the bearing, so that each sector acts as a partial bearing. A more elaborate method is a pivoted-slider construction, similar to that of the usual thrust bearing, except that the sliders are curved around the journal. This type of bearing is claimed to be very stable [ 11 ] and possesses self-aligning properties. Another very efficient method is the adaptation of the Rayleigh step to the journal bearing [ 6,15 ]. It is this type of bearing which we intend to discuss. In reference [ 6 ], the author describes the successful application of the Rayleigh step to gas-lubricated journal and thrust bearings of generators and motors. This writer had the opportunity to inspect some of the units in Germany. One AC generator, rated at 60 KW at 1500 rpm, had a 485 lb rotor supported on two air-lubricated, 11.2" x 8.0" bearings, having a clearance ratio as small as C/R = 0.00072

(R/C = 1400). No high temperatures or large deflections had to be considered in this case. A smaller, rotor, when mounted in plain journal bearings, began to whirl at 3600 rpm, but operated perfectly up to 30,000 rpm when mounted in a type of bearing shown in Figure 3. The eccentricity locus of a multiple-pad journal bearing has the character of (II), Figure 4b, while a plain, finite journal has a locus which corresponds to the schematic (I). The multiple-pad bearings, unlike plain journal bearings, develop an appreciable force which tends to oppose the displacement of the journal along the line of centers, while the transverse component remains small. This is the desired anti-whirl property, but it can only be achieved at the expense of the load carrying capacity. We shall, consequently, estimate the load carrying capacity of the Rayleigh-step journal-bearing, Figure 3, and compare the results with those obtained with mercury and mercury vapor in Examples 1 and 3 for the case of the plain 3" x 3" journal bearing.

Example 4 - Liquid Mercury Anti-Whirl Bearing

Consider the 3" x 3" Rayleigh-step journal-bearing in Figure 3. Design charts for certain optimum proportions for this type of bearing have been supplied to the writer through the courtesy of Dr. H. Drescher, Göttingen, Germany, and approximate values of parameters used in this example have been obtained with their aid. The parameters are:

$\frac{\bar{P}h^2}{\mu UZ}$ = Dimensionless load

$\frac{B}{Z}$ = Ratio of length to width of slider

$\frac{b}{B}$ = Step length ratio

$\frac{z}{Z}$ = Step pocket width ratio

$\frac{h}{d}$ = Ratio of clearance and of step depth

in which $\bar{P}$ is unit load based on the projected bearing area, $\mu$ is the lubricant viscosity, U is the surface velocity of the slider, and the meaning of all other symbols is indicated in Figure 3.

The local clearance in a journal is given by

$$h = C\,(1 + \varepsilon \cos\theta) \tag{17}$$

where $\varepsilon$ is the eccentricity ratio, e/C, and $\theta$ is measured from the line of centers and $h = h_{max}$. The local clearance is not constant, but for the purpose of obtaining approximate values of the load capacity, we shall assume a clearance value corresponding to the midpoint of each of the six individual steps (at 1/2 B). The center of pressure will be taken at approximately 5° from the step, opposite to the direction of rotation of the journal. The eccentricity locus will be assumed to be nearly parallel to the load line. We neglect the curvature and assume that the total force contributions of each "plain" slider, $F_{1,2...6}$, act at the centers of pressure. We take, in addition to data in Example 1:

$$B = Z = 1.5";$$

$$\frac{B}{Z} = 1; \ \frac{b}{B} = \frac{z}{Z} = 0.75;$$

For a maximum allowable eccentricity ratio of $\varepsilon = 0.6$, we obtain:

$$h_1 = 0.60 \times 10^{-3} \text{ in}$$
$$h_{2,6} = 1.05 \times 10^{-3} \text{ in}$$
$$h_{3,5} = 1.95 \times 10^{-3} \text{ in}$$
$$h_4 = 2.40 \times 10^{-3} \text{ in.}$$

An optimum ratio for slider 1 is:

$$\frac{h_1}{d} = 0.8; \ d = 0.75 \times 10^{-3} \text{ in}$$

For this value of d, we then obtain the remaining five h/d ratios. The dimensionless loads of all pads can now be determined. There are two bearings and two rows of pads for each bearing (Figure 3). The contributions to the load carrying capacity (from each pair of pads) of two bearings are:

$$2F_1 = 1530 \text{ lb}$$
$$2F_{2,6} = 452 \text{ lb}$$
$$2F_{3,5} = 84 \text{ lb}$$
$$2F_4 = 45 \text{ lb}$$

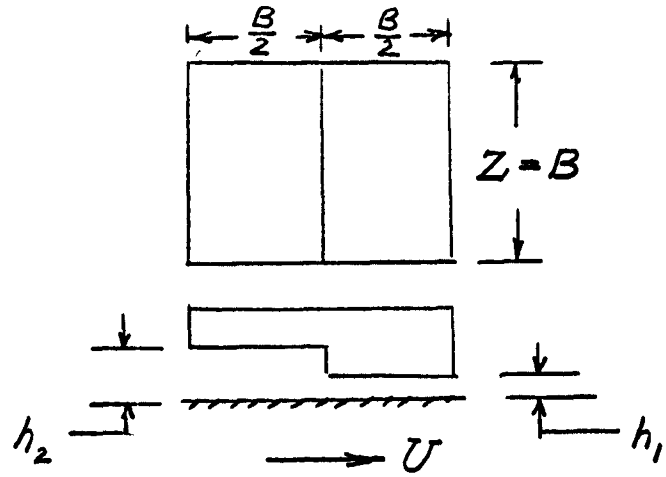
A vector diagram for these forces is shown in Figure 4a. The resultant, R is approximately:

$$R = 1800 \text{ lb}$$

It is reasonable to assume that this is the order of magnitude of load which this bearing can support. Note that this is only 40% of the load which a plain, liquid-mercury-lubricated journal bearing can support under identical conditions (See Example 1). Nevertheless, a 110 lb rotor (100 KVA generator) requires a load capacity of only 550 lb when accelerated to 5g, whereas the available capacity is more than 3 times as large and, therefore, more than sufficient. We note that slider 1 provides approximately 5/6 of the total load carrying capacity. Since a gas-lubricated Rayleigh type journal bearing will behave similarly, we shall limit our considerations to this slider alone when the lubricant is mercury vapor.

Example 5 - Mercury Vapor Anti-Whirl-Bearing

The only available* design data are due to Gross [ 3 ] and are limited to sliders having a slenderness ratio $B/Z = 1$, a step length ratio $b/B = 0.5$ and ratios $h_1/h_2 = 2$ and $h_1/h_2 = 3$ (See diagram below).




*Useful data for the infinitely long step bearing (e.g. very small B/Z ratios) is contained in a paper by K. C. Kochi, reference [ 16 ].

Unlike the Drescher step bearing, there are no pockets in this slider ($z/Z = 0$). The dimensionless load

$$\bar{P} = \frac{W}{P_a BZ}$$

is given as a function of the speed-compressibility parameter, defined here as:

$$\Lambda = \frac{6\mu UB}{P_a h_2^2}$$

We have for $h_1/h_2 = 2$ and $h_2 = 0.6 \times 10^{-3}$ in, $h_1 = 2 \times 0.6 \times 10^{-3} = 1.2 \times 10^{-3}$ *in*, which corresponds to

$$d = h_2 = 0.6 \times 10^{-3} \text{ in}$$

If we retain the two-row pad construction of Example 4, then for a pad

$$B = Z = 1.5''$$

and with $\mu = 8.3 \times 10^{-9}$ lb-sec/in$^2$; $P_a = 15$ psia;

$$\Lambda = \frac{(6)(8.3 \times 10^{-9})(3.14 \times 4 \times 10^2 \times 3)(1.5)}{(15)(0.36 \times 10^{-6})} = 52$$

the dimensionless load is:

$$\bar{P} = 0.45$$

For two bearings there are 4 pads corresponding to position 1 (e.g., approximately normall to the load line) and these contribute:

$$2F_1 = (4)(15)(1.5)(1.5)(0.45) = 58 \text{ lb}$$

as compared with

$$2F_1 = 1530 \text{ lb}$$

for liquid mercury and a for a similar pad arrangement in Example 4.

For the plain, mercury-vapor-lubricated journal bearing, a load carrying capacity of 300 lb was estimated in Example 3. If we were to select more favorable slider proportion, dispensing with the two-row arrangement and introducing a pocket construction in order to reduce side leakage, we could hardly expect to carry more than 100 lb with two 3" x 3" bearings at $\varepsilon = 0.6$, which is not sufficient to support even the smallest, 35 lb rotor at an acceleration of 5g. Nevertheless, it may be possible to lengthen the bearings and to increase the diameters of the journals. It may well be that the bearing will be stable with three instead of with six sliders along the periphery. Aluminum bromide is apparently 3 times as viscous as mercury vapor and, all other things being equal, would increase $\Lambda$, by a factor of 3. For a ratio of $h_1/h_2 = 3$, we would then obtain a dimensionless load value for the same slider of 0.95, as compared with 0.45 in the previous case, an increase of over 50%. The mounting of the lighter and medium size rotors of the 30 KVA - 100 KVA generator series in whirl-free, gas-lubricated bearings is, therefore, feasible but more difficult to achieve than with liquid-lubricated bearings.

Summarizing the content of this section, we note that the price one must pay for achieving stability is high, in as much as the load carrying capacity will be reduced. This represents no problem with liquid-lubricated bearings, because there is a sufficient surplus and reserve to withstand g-loads considerably higher than 5. We do not exclude the possibility of using gas-lubricated bearings, although their load carrying capacity appears to be marginal. The choice is limited, because one cannot reduce the clearance under the stipulated operating conditions, nor allow the eccentricity to become too large. Moreover, with higher values of the bearing number, $\Lambda$, one enters an area of diminishing returns, because the load approaches an asymptotic value. The highest possible ambient pressure and bearing size compatible with other requirements are recommended in the case of gas-lubricated bearings. Despite the apparent disadvantages, gas bearings may provide the answer to many problems, such as scavenging in a weightless environment, dissipation of energy in the form of heat, etc. The reader should not dismiss the possibility of their use and he should also be aware that the approximate calculations in this report are limited to particular bearing dimensions and configurations and are, therefore, by no means conclusive.

7. Bearing Materials - General Requirements

The selection of bearing materials poses a formidable problem. Certain materials may combine a number of desirable properties, but may be deficient in other equally important characteristics. We begin with discussing the requirements and the conditions which must be met in order to insure a bearing life of approximately 10,000 hours of steady-state operation. This part of the discussion will best be served by a very approximate calculation of clearance reduction due to the combined effects of temperature, centrifugal forces, and creep. The reader will realize that the necessary oversimplification of the problem restricts the usefulness of this calculation, but that a rough estimate of clearance reduction may be obtained in this manner. A full treatment of the problem, if one were to account for stresses and deformations produced by temperature gradients, boundary constraints, the distributed mass of the rotating journal and creep, while taking simultaneously into consideration the temperature dependence of Young's modulus, Poisson's ratio etc., is beyond the scope of this report..

For simplicity, we postulate a linear and unconstrained expansion of the bearing and of the journal, proportional to their "average" temperatures. In calculating the growth and maximum stress of a high-speed, solid journal, we assume a decrease in the value of the elastic modulus from $30 \times 10^6$ psi to $25 \times 10^6$ psi and an increase of Poisson's ratio from 0.3 to 0.33 at 800°F. In estimating the creep rate at that temperature, the maximum stress value is taken. The example is worked for a 3 inch journal, rotating at 24,000 rpm, and a bearing with an initial clearance ratio $R_o/C_o = 1000$ at $T_o = 70°F$. The coefficients of linear expansion and the creep rate assumed in this example are relatively low, but well within the capabilities of alloys employed in high temperature applications.

Example 4

Let:

$R_o$ = Radius of journal at temperature $T_o$

$C_o$ = Initial clearance (radial) at temperature $T_o$

$C_1$ = Operating clearance (radial)

$\delta = \delta_T + \delta_\omega + \delta_c$ = Total clearance reduction, due to thermal expansion, centrifugal forces and creep respectively.

$\alpha$ = Coefficient of expansion

$\Delta T$ = "Average" temperature rise above $T_o$

$p$ = Direct stress

$E$ = Young's modulus

$\sigma$ = Poisson's ratio

$\rho$ = Mass density of journal material

$\omega$ = Angular velocity of journal

The subscripts "j" and "b" refer to "journal" and "bearing", and "r" and "t" denote the "radial" and the "tangential" directions of the direct stresses.

(a) To determine the clearance reduction due to thermal expansion, have:

$$\delta_T = \alpha_j R\Delta T_j - \alpha_b (R_o + C_o)\Delta T_b \qquad (18)$$

Assuming $\alpha = \alpha_j = \alpha_b$ as a minimum requirement when $\Delta T_j > \Delta T_b$:

$$\delta_T = \alpha R_o \Delta T_j \left(1 - \frac{R_o + C}{R_o} \cdot \frac{\Delta T_b}{\Delta T_j}\right) \approx \alpha R_o \Delta T_j \left(1 - \frac{\Delta T_b}{\Delta T_j}\right) \qquad (19)$$

Assuming that we can select an alloy which has a relatively low coefficient of expansion, $\alpha = 2 \times 10^{-6}$ in/in/°F, and that the difference between the "average" journal and bearing temperatures does not exceed 100°F, we take:

$$\alpha = 2 \times 10^{-6} \text{ in/in/°F}$$
$$R_o = 1.5 \text{ in}$$
$$\Delta T_j = 1/2\ (800 + 700) - 70 = 680\text{°F}$$
$$\Delta T_b = 1/2\ (700 + 600) - 70 = 580\text{°F}$$

$$1 - (\Delta T_b/\Delta T_j) = 1 - 0.85 = 0.15$$

so that:

$$\delta_T = (2 \times 10^{-6})(1.5)(680)(0.15) = 0.305 \times 10^{-3} \text{ in}$$

For a ratio $R_o/C_o = 1000$, $\delta_T$ represents approximately 1/5 of the initial radial clearance, $C_o$.

(b) To determine the clearance reduction due to centrifugal forces acting on a solid journal [30], we have:

$$\delta_\omega = \frac{\omega^2 \rho R_o^3}{4E}(1 - \sigma) \qquad (20)$$

We take:

$$\omega = 2\pi N = 6.28 \times 400 = 2512 \text{ Rad/sec}$$
$$\rho = \gamma/g = 0.283/386 = 0.735 \times 10^{-3} \text{ lb-sec}^2/\text{in}^4$$
$$\sigma = 0.33$$
$$E = 25 \times 10^{-6} \text{ lb/in}^2$$

and obtain:

$$\delta_\omega = (6.3 \times 10^6)(0.735 \times 10^{-3})(3.38)(0.67)(10^{-8}) = 0.105 \times 10^{-3} \text{ in}$$

For a ratio $R_o/C_c = 1000$, $\delta_\omega$ represents approximately 1/10 of the initial clearance, $C_o$.

(c) To determine the clearance reduction due to creep, we first calculate the maximum stress [ 30 ] for a solid journal (center of journal):

$$(p_r)_{max} = \frac{\omega^2 \rho R_o^{\ 2}}{8} \cdot \frac{3 - 2\sigma}{1 - \sigma} = (p_t)_{max} \tag{21}$$

$$(p)_{max} = (0.125)(6.3 \times 10^6)(0.735 \times 10^{-3})(2.25)(3.5) = 4.550 \text{ lb/in}^2$$

At a temperature of 800°F and a stress of approximately 5000 psi, plain carbon steels have creep rates of the order of $0.1\%/10^4$ hrs. For the same conditions, creep rates of alloy steels used in high-temperature [31] applications (for example, 19-9 W-$M_o$, Timken 16-25-6, Inconel X or Hastelloy B) are at least 100 times smaller. If we select an alloy which has a creep rate of 0.01% per 10,000 hours at a temperature of 800°F and a stress of 5000 psi,

$$\delta_c = 1.5 \times 10^{-4} = 0.15 \times 10^{-3} \text{ in}$$

For a ratio $R_o/C_o = 1000$, $\delta_c$ represents approximately 1/10 of the initial radial clearance, $C_o$.

We thus have for the total clearance reduction:

$$\delta = \delta_T + \delta_\omega + \delta_c = 0.56 \times 10^{-3} \text{ in}$$

which represents a 37% reduction of the initial radial clearance $C_o = 1.5 \times 10^{-3}$ in, and an operating radial clearance $C_1 = 0.94 \times 10^{-3}$ in.

If the journals are hollow and efficiently cooled, so that temperature gradients can be minimized, thermal stresses, as well as stresses caused by centrifugal forces, can be greatly reduced. This would tend to reduce the clearance loss and enhance the dimensional stability of the journal over its entire length. Assuming the bearing to be cooler than the journal, it may be advisable to select a bearing material having a larger value of the coefficient of expansion than the journal material. The nature of the bearing support must be such that the bearing sleeve is not unduly restrained from expanding and some provision for the self-alignment of bearings with respect to the journals must be made*. The initial dimensions of the bearing and of the journal must be determined on the basis of operating clearance requirements with regard to eccentric operation and with due allowance made for thermal distortion of bearing

*For example, mounting the bearings in metal diaphragms which resist lateral deflections but permit the axial alignment of the bearing and of the journal. Gimballing and pivoted journal bearing construction represent two of many other alternatives.

surfaces. There is not much latitude in the selection of the clearance ratio, particularly in the case of self-acting, gas-lubricated bearings, both from the load carrying capacity and stability point of view, so that the absence of appreciable temperature gradients and effective temperature control of the shaft and of the bearings are essential.

We next turn our attention to the problem of starting and stopping. In an experimental model, both starting and stopping may be required at frequent intervals, but in a space vehicle the problem may be reduced to a single start prior to launching. In the case of liquid lubricants, no particular problem exists unless a "dry" start is contemplated. The frictional compatability of materials need not be considered if provision is made to lift the rotor before starting by means of an external (with respect to the bearing) pressure source, so that no metal-to-metal contact occurs when the journal begins to rotate. This is quite important in starting "hot", self-acting, gas-lubricated bearings and may be accomplished by simply supplying the gas under pressure through a number of small orifices, located in the lower bearing half. (There are other possibilities of avoiding journal and bearing contact, such as resting the rotor on stops which protrude above the bearing surface only by a small fraction of the total clearance, and which possess good dry friction characteristics.)

If a "dry" start of a gas-bearing is contemplated, the bearing and journal materials must be compatible. Literature abounds with data on the compatibility of numerous materials, and tests have been conducted for a wide range of temperature and load conditions and for a variety of lubricants, surface coating and ambient environments. The choice of materials and of its surface treatment will vary with each particular application and it is not proposed to make specific recommendations, particularly since frictional compatability of materials does not seem to represent a major problem for a limited number of starts and stops. We shall summarize, however, the remarks of Drescher[6], which may serve as a guide in the selection of materials if a "dry" start is unavoidable. The asperities and surface irregularities are subject to both elastic and plastic deformation. The degree to which elastic deformation takes place before some of the asperities deform plastically, depends on the elasticity and hardness of the materials. Such factors as surface finish, the grain structure of the material, and many others are also important. Although no functional relationship is known, a combination of large hardness and a low elastic modulus is favorable. Since both the journal and the bearing materials are compressed, the effect is additive; but since plastic deformation will occur first in the asperities of the softer material, it is the hardness of the latter which is decisive. The dimensionless parameter which is to be maximized (assuming that the bearing hardness, $H_b$, is less than the hardness of the journal, $H_j$) is:

$$H_b \left( \frac{1}{E_b} + \frac{1}{E_j} \right)$$

in which $E_b$ and $E_j$ are the elastic moduli of the bearing and of the journal materials. Thus, in addition to properties discussed in the preceding paragraphs, materials which are relatively hard and elastic at elevated temperatures provide suitable combinations within limits dictated by other requirements.

Summarizing the contents of this section, the most important prerequisite from the material point of view is the ability to maintain dimensional tolerances. A low coefficient of thermal expansion and a low creep rate are desirable. Unless the bearing size is considerably larger than 3 inches, no high strength is called for. The materials must be reasonably compatible, able to retain a good surface finish and hardness at temperatures of the order of 800°F. For a limited number of "dry" starts and stops, the susceptibility of materials to spalling or galling is to be considered, rather than overall "good wear" characteristics. Chemical activity between the materials and the lubricating medium must, of course, be reduced to a minimum. It is definitely recommended that the journals (or the entire shaft) be hollow and that every effort be made to avoid appreciable temperature gradients. High thermal conductivity of materials is desirable as well as efficient cooling, especially in regions where heat is conducted to the bearings from the turbine, or from the generator.

8. Conclusions and Recommendations

The recipients of this report are undoubtedly aware of the variety and complexity of problems which must be solved before one can even contemplate the successful operation of generators under the postulated conditions. It is natural for every specialist to stress the difficulties to be anticipated in his area of responsibility and to minimize or oversimplify other equally important problems. Nor does this writer claim to be immune to this tendency. Nevertheless, we believe that a turbo-generator must be designed "radially outward", that is consideration must first be given to the method of supporting the rotor. No amount of sophistication in alternator and turbine design can possibly supply the answer to one simple question, namely "Will the rotor revolve?" If no other benefit accrues from this discussion, we hope at least that the prospective contractors, engineers, and designers will have realized that bearings, the proverbial "rectangles with two diagonals", require particularly careful attention.

We emphasize once more that the discussion presented in this report (and the conclusions which may be drawn from it) is neither exhaustive, nor final. The manner in which the problem is started is such that for each topic discussed there will be those in whose opinion other aspects of bearing operation are more important and have been omitted in this report. There will be those who disagree with everything that has been stated, those who offer an innumerable number of alternative solutions to each and every suggestion, and those who are critical of either oversimplification, or overstatement of the problem.

The author has followed a procedure of summarizing the contents of each section before entering the next topic of discussion. Here we make an attempt to review the overall aspects of this complex, fluid-film bearing problem, to arrive at certain conclusions and to make a number of recommendations.

With regard to liquid lubricants, both rubidium and mercury offer definite possibilities in the specified temperature-pressure range. Rubidium, since it has a very low vapor pressure in the 600°-800°F temperature range (Figure A1-1) and a viscosity of the order of water at room temperature, appears to offer a particularly good solution for a liquid-lubricated bearing, even if the bearing is to be operated in the ambient surroundings of the condenser. Liquid mercury would require some degree of temperature and pressure control if rapid vaporization and cavitation within the film is to be avoided. The limited viscosity data of sulfur indicate that the liquid might still retain the "pitch-like" quality in the operating temperature range and is probably unsuitable for high speed operation. The corrosive tendency of this substance, particularly in the presence of residual amounts of oxygen in the system, may have an adverse effect on the bearings.

With regard to gaseous lubricants we can again eliminate sulfur, which is readily available as a gas, but only at relatively high temperatures and low pressures. We have no experimental viscosity data of aluminum bromide. Calculations, based on equations contained in Appendix 1, indicate that this gas may be 10 times as viscous in the operating temperature range as air at room temperature. We do not know what its chemical and other characteristics are and how the substance would affect the generator performance as a whole, but aluminum bromide gives an indication of being a superior, gaseous lubricant. Mercury vapor requires an operating temperature from 700° to 800°F, rather than from 600° to 700°F, e.g., at the high extreme of the contemplated temperature range.

The weightless environment poses the problem of lubricant scavenging if the lubricant is a liquid. It is realized that the medium is condensed before being returned to the "boiler", so that the problem of circulation will exist at one point of the closed loop, or another. We assume that the generator cannot operate when wholly, or even partly submerged in a liquid and we take it for granted that a purely gaseous environment in a separate (from the condenser) generator housing is desirable. It is also assumed that the substance can be supplied and withdrawn from the ambient surroundings of the bearings and of the generator in gaseous form and that this represents a lesser problem than the scavenging of a liquid lubricant. If the latter consideration does not apply, we recommend unreservedly that liquid-lubricated bearings be used, since they are capable of withstanding much higher g-loads than gas bearings, have better damping characteristics, can be safely operated at high eccentricity ratios and with larger clearances, and will not be immediately destroyed on first metal-to-metal contact between the journal and the bearing. On the other hand, the gas bearing may offer the advantages of easier scavenging and circulation and of appreciably less energy dissipation in the form of heat that must be removed from the ambient surroundings of the generator. With the exception of rubidium, liquid films will tend to evaporate and cavitate if the bearing is operated in a nearly concentric position and in a low-pressure condenser environment. A separate bearing and generator housing, one in which the ambient temperature and pressure can be controlled to a reasonable degree, is recommended.

The two extremes of operating conditions require the same bearing to perform equally well when subjected to high accelerations and vibrations during launching and when the rotor becomes weightless. It has been pointed out that a supply pressure of 3 atmospheres absolute is too low for an externally-pressurized bearing of reasonable dimensions, particularly in the case of larger rotors. External pressurization is a convenient method of rotor support at starting with self-acting gas bearings. A hybrid design, in which the self-generated load carrying capacity of a gas bearing is augmented by external pressurization, is feasible.

We have discussed the problem of whirl, an instability which is invariably present in plain and lightly loaded, high-speed journal-bearings, and we have demonstrated that the load carrying capacity of anti-whirl bearings is much less than the capacity of plain bearings of comparable dimensions. Despite this reduction, the liquid-lubricated, anti-whirl bearings have ample reserve of load capacity, so that no problem exists in this area during launching. In the case of gas-lubricated bearings, even the plain journal bearings may have a capacity which is marginal. If an anti-whirl gas-bearing is to be employed, an increase · size, combined with a sophisticated design, in which the best possible choice of clearance ratio, pad configuration and utilization of the maximum available area are incorporated, will provide, we believe, sufficient capacity to withstand a load of the order of 5g.

Finally, with regard to bearing materials, we have no reason to think that the limitations are imposed by considerations of either strength, or creep. What is most important in this application is a low coefficient of thermal expansion and minimization of temperature gradients. The journals, and possibly the whole shaft, should definitely be hollow and adequately cooled. The bearing shell must not be unduly constrained and provision must be made for self-alignment. The bearing materials must have a high degree of dimensional stability, and every attempt should be made to insure minimum distortion, reduction, or increase of the nominal, operating clearance. These considerations are particularly important in the case of gas-lubricated bearings, which generally require much smaller clearances than liquid-lubricated bearings and in which metal-to-metal contact at high speeds results in total destruction. The bearing surfaces may also be subjected to the corrosive influence of the lubricating media. It is well known that this problem exists with sulfur, particularly in the presence of residual oxygen and moisture. The corrosive influence of mercury in the considered temperature range does not appear to be very serious, especially with steels containing molybdenum and and titanium as alloying elements, or with titanium inhibited mercury (33). No information is available on possible chemical effects of Rubiduim and Aluminum Bromide on bearing materials in the considered temperature range. The selection of particular materials, surface coatings, methods of mounting and manufacture cannot be discussed until the problem is defined in more specific terms.

## 9. Bibliography

1. Theory and Design Data for Continuous-Film Self-Acting Journal Bearings of Finite Length, Elrod H.G. and Malanoski S.B., The Franklin Institute, Report No. I-A2049-13, Contract Nonr-2342(00), Task NR 061-113, Nov. 1960

2. Refinements of the Theory of the Infinitely Long, Self-Acting Gas-Lubricated Journal Bearing, Elrod H.G. and Burgdorfer A., presented at First International Symposium on Gas-Lubricated Bearings, Washington, D.C., 1959

3. Numerical Analysis of Gas Lubricating Films, Gross W.A., presented at First International Symposium on Gas Lubricated Bearings, Washington, D.C., 1959

4. A Solution for the Finite Journal Bearing and Its Application to Analysis and Design III, Raimondi A.A. and Boyd J., Westinghouse Research Laboratories, Research Report 8-0525-R7, 1957

5. A Numerical Solution for the Gas Lubricated Full Journal Bearing of Finite Length, Raimondi A.A., ASLE Paper No. 60 LC-14, 1960

6. Special Characteristics of Air-Lubricated Hydrodynamic Bearings with Reference to Practical Applications, Drescher H., presented at First International Symposium on Gas Lubricated Bearings, Washington D.C., 1959 (also Private Communication from H. Drescher)

7. Perturbation Analysis of the Stability of Self-Acting, Gas-Lubricated Journal Bearings, Castelli V. and Elrod H.G., The Franklin Institute, Report No. I-A2049-11, Contract Nonr 2342(00), Task NR 097-343, Feb. 1960

8. Private Communication from V. Castelli

9. Varieties of Shaft Disturbances due to Fluid Films in Journal Bearings, Newkirk B.L., Trans. ASME, Vol. 78, 1956

10. Investigation of Translatory Fluid Whirl in Vertical Machines, Boeker G.F. and Sternlicht B., Trans. ASME, Vol. 78, 1956

11. Gas-Lubricated Bearings - A Critical Survey, Boeker G.F., Fuller D.D. and Kayan C.F., WADC Technical Report 58-495, 1958

12. A Theory of Oil Whip, Hori Y., Journal of Applied Mechanics, Vol. 26, No. 2, 1959

13. (a) Dynamic Stability Aspects of Cylindrical Journal Bearings Using Compressible and Incompressible Fluids, Sternlicht B., Poritsky H., and Arwas E., General Electric Company, Report for ONR, Contract Nonr 2844(00), Task No. NR 097-348, 1959

(b) Gas Bearing Stability Study - Vertical Rotor Investigation, Elwell R.C., Hooker R.J., and Sternlicht B., General Electric Company, Report for ONR, Contract Nonr 2844(00), Task No. 097-348, 1960

14. Some Instabilities and Operating Characteristics of High-Speed Gas-Lubricated Journal Bearings, Fischer G.K., Cherubim J.L., and Fuller D.D., ASME Paper No. 58-A-231, 1958

15. The Stepped Shape Film Applied to a Journal Bearing, Archibald F.R., Journal of The Franklin Institute, Jan. 1952

16. Characteristics of Self-Lubricated Stepped Thrust Pad of Infinite Width with Compressible Lubricant, Kochi K.C., ASME Paper No. 58-A-194, 1958

17. Gas Lubricated Self-Acting Journal Bearings, Ford G.W., Harris D.M., and Pantall D., Atomic Energy Research Establishment, Report AERE ED/R1662, Harwell (England), 1955

18. Review of Research on Gas Bearings in the U.K. Atomic Energy Authority, Whitley S., presented at First International Symposium on Gas Lubricated Bearings, Washington, D.C., 1959

19. Lubrication Reference Manual for Missile and Space Vehicle Propulsion at Temperatures Above 700°F, Macks F., WACD Technical Report 58-638, Vol. I, Part I, 1959

20. A Study of Stability of Externally Pressurized Gas Bearings, Licht L. and Elrod H.G., Journal of Applied Mechanics, Vol. 27, No. 2, 1960

21. An Analytical and Experimental Study of the Stability of Externally-Pressurized Gas-Lubricated Thrust Bearings, Licht L. and Elrod H.G., The Franklin Institute, Report No. I-A2049-12, Contract Nonr 2342(00), Task NR 061-113, 1961

22. Self-Excited Vibrations of an Air-Lubricated Thrust Bearing, Licht L., Fuller D.D., Sternlicht B., Trans. ASME, Vol. 80, 1958

23. Air-Hammer Instability in Pressurized Journal Gas-Bearings, Licht L., ASME Paper No. 60-WA-10, 1960 (to be published Trans. ASME)

24. Axial Relative Motion of a Circular Step Bearing, Licht L., Trans. ASME, Journal of Basic Engineering, Vol. 81, No. 2, 1959

25. Extension of the Conducting Sheet Analogy to Externally-Pressurized Gas Bearings, Licht L., ASME Paper No. 60-LUB-1, 1960 (to be published Trans. ASME)

26. Notes on Transport Phenomena, Bird R.B., Stewart W.E., and Lightfoot E.N., Wiley and Sons Inc., New York, 1958

27. Molecular Theory of Gases and Liquids, Hirschfelder J.O., Curtiss C.F., and Bird B.R., Wiley and Sons Inc., New York, 1954

28. The Reactor Handbook, Vol. 2, Atomic Energy Commission, AECD-3646, U.S. Gov't. Printing Office, Washington, D.C., 1955

29. International Critical Tables of Numerical Data of Physics, Chemistry, and Technology, McGraw-Hill Book Co., 1926

30. A Treatise on the Mathematical Theory of Elasticity, Love A.E.H., Dover Publications, New York, 1944

31. Metals Handbook, American Society of Metals, 1949

32. The Theory of Film Lubrication, Boswall R.O; Longmans, Green and Co. Ltd., New York, 1928

33. A Lunar Power Plant, Armstrong R. H., Carter J. C., Hummel H. H., Janicke M. J. and Marchaterre J. F., Argonne National Laboratory, Report ANL-6261, December 1960.

## APPENDIX 1

### Estimation of Viscosities

For monatomic gases, the viscosity may be calculated from the following expression, based on theories due to Chapman and Enskog [26,27]:

$$\mu = 2.6693 \times 10^{-5} \frac{\sqrt{MT}}{\sigma^2 \Omega_\mu} \tag{A1-1}$$

$$\sigma \doteq \left( \frac{2.3\ \tilde{V}_m^{(sol)}}{1.2615} \right)^{1/3} \tag{A1-2}$$

$$\Omega_\mu = f\left(\frac{KT}{\varepsilon}\right) \tag{A1-3}$$

in which $\mu$ is the viscosity in gm $cm^{-1}$ $sec^{-1}$ (poise), $\sigma$ is the "collision diameter" of the molecule in Angstroms ($10^{-8}$ cm), M is the molecular weight, T the absolute temperature in °K, K is the Boltzmann constant, $\varepsilon$ is the maximum energy of attraction between a pair of molecules and $\Omega_\mu$ is a slowly varying function of the dimensionless temperature $KT/\varepsilon$ (see tabulation, reference [26], pg. 13). The $\tilde{V}_m^{(sol)}$ term denotes the volume of one mole of the substance in the solid state and at the melting point in $cm^3$ $mol^{-1}$ and the ratio $\varepsilon/K$ may be taken as

$$\varepsilon/K = 1.92\ T_m \tag{A1-4}$$

where $T_m$ is the absolute temperature at the melting point in °K.

Similar expressions for $\sigma$ are given in reference [ 26 ] when the boiling and/or critical point of the substance is the reference datum.

Although this formula is derived for dilute, monatomic gases, it gives remarkably good results for polyatomic gases. It has also been extended by Curtiss and Hirschfelder to include multi-component gas mixtures. Another useful relation [27,29] in the estimation of the viscosity of gases, due to Sutherland, is

$$\frac{\mu}{\mu_o} = \frac{T_o + C}{T + C} \left(\frac{T}{T_o}\right)^{3/2} \tag{A1-5}$$

where $\mu_o$ is the viscosity at the temperature (absolute) $T_o$. If the viscosity is known for two or more values of the temperature, the constant C can be determined.

Several methods have been developed for the computation of viscosities of dense gases and are given on pp. 14-16 in reference [ ].

Rough estimates of the viscosity of liquids may be obtained from Trouton's Rule (based on Eyring's theory) [ ]:

$$\mu = 3.98 \times 10^{-3} \ (\rho/M) \ e^{3.8 \ T_b/T}$$

in which $\rho$ is the mass density gm cm$^{-3}$, M the molecular weight, $T_b$ is the temperature at the normal boiling point in °K, and T the liquid temperature in °K [ 26 ].

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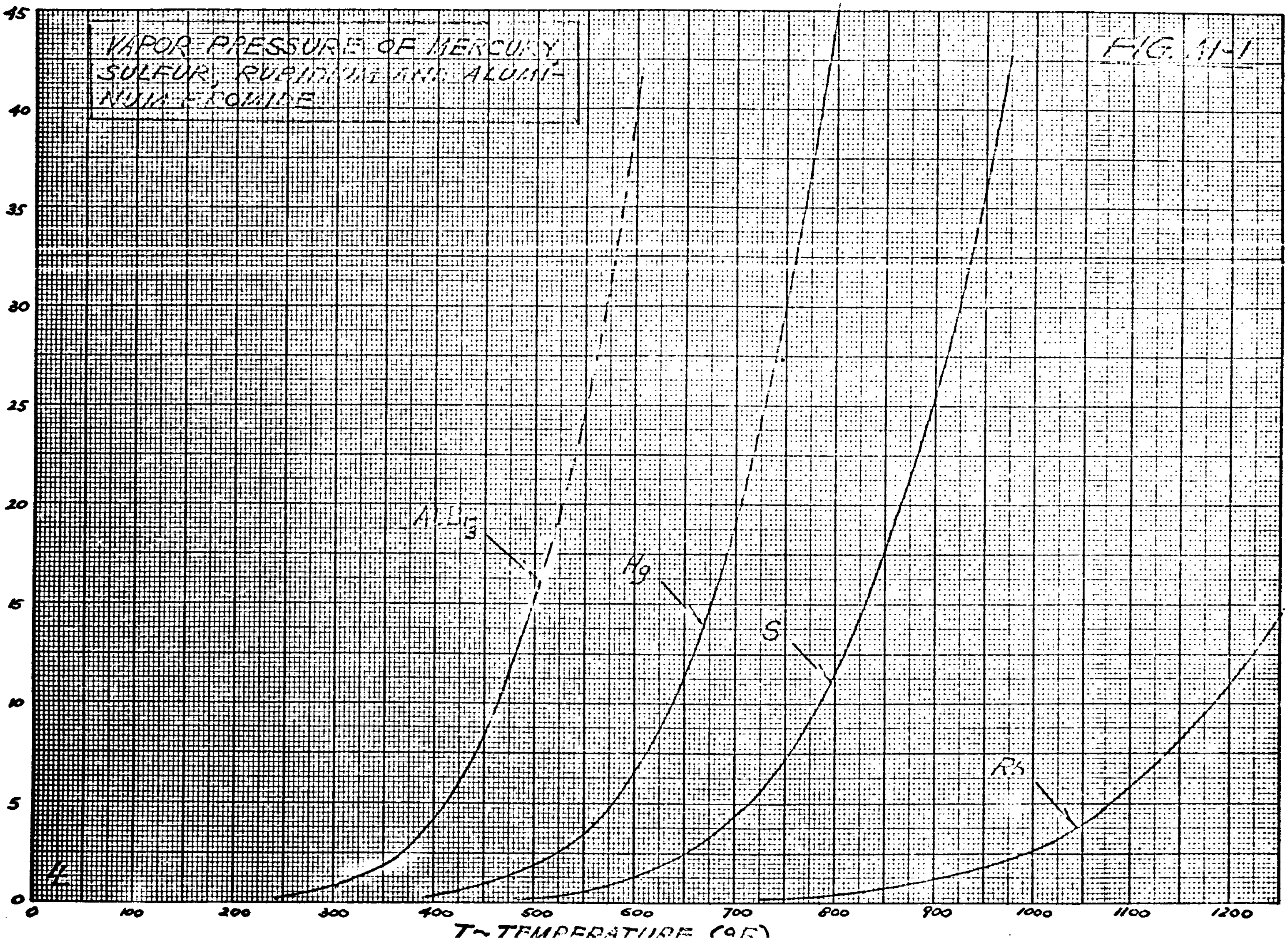

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TITLE VAPOR PRESSURES OF LUBRICANTS

## TABLE A-I

**Hg**

| T °F | P psia |
|---|---|
| 402.3 | 0.40 |
| 456.4 | 0.98 |
| 480.1 | 1.4 |
| 504.9 | 2.0 |
| 557.9 | 4.0 |
| 541.2 | 6.0 |
| 616.5 | 8.0 |
| 637.0 | 10.0 |
| 676.1 | 15.0 |
| 706.0 | 20.0 |
| 730.1 | 25.0 |
| 750.6 | 30.0 |
| 768.5 | 35.0 |
| 784.4 | 40.0 |
| 798.9 | 45.0 |
| 812.1 | 50.0 |

**S**

| T °F | P psia |
|---|---|
| 392 | 0.04 |
| 482 | 0.23 |
| 572 | 0.93 |
| 608 | 1.47 |
| 644 | 2.28 |
| 680 | 3.46 |
| 716 | 5.07 |
| 752 | 7.27 |
| 788 | 10.2 |
| 832 | 14.1 |
| 860 | 18.3 |
| 896 | 24.3 |
| 932 | 31.9 |
| 963 | 41.2 |
| 1004 | 52.6 |
| 1040 | 66.3 |

**Rb**

| T °F | P psia |
|---|---|
| 732 | 0.19 |
| 858 | 0.77 |
| 957 | 1.93 |
| 1148 | 7.74 |
| 1254 | 14.7 |

**$AlBr_3$**

| T °F | P psia | |
|---|---|---|
| 244 | 0.19 | |
| 303 | 0.77 | |
| 349 | 1.93 | |
| 441 | 7.74 | |
| 494 | 14.7 | |
| 550 | 23.3 | * |
| 600 | 41.1 | * |

* Extrapolated.



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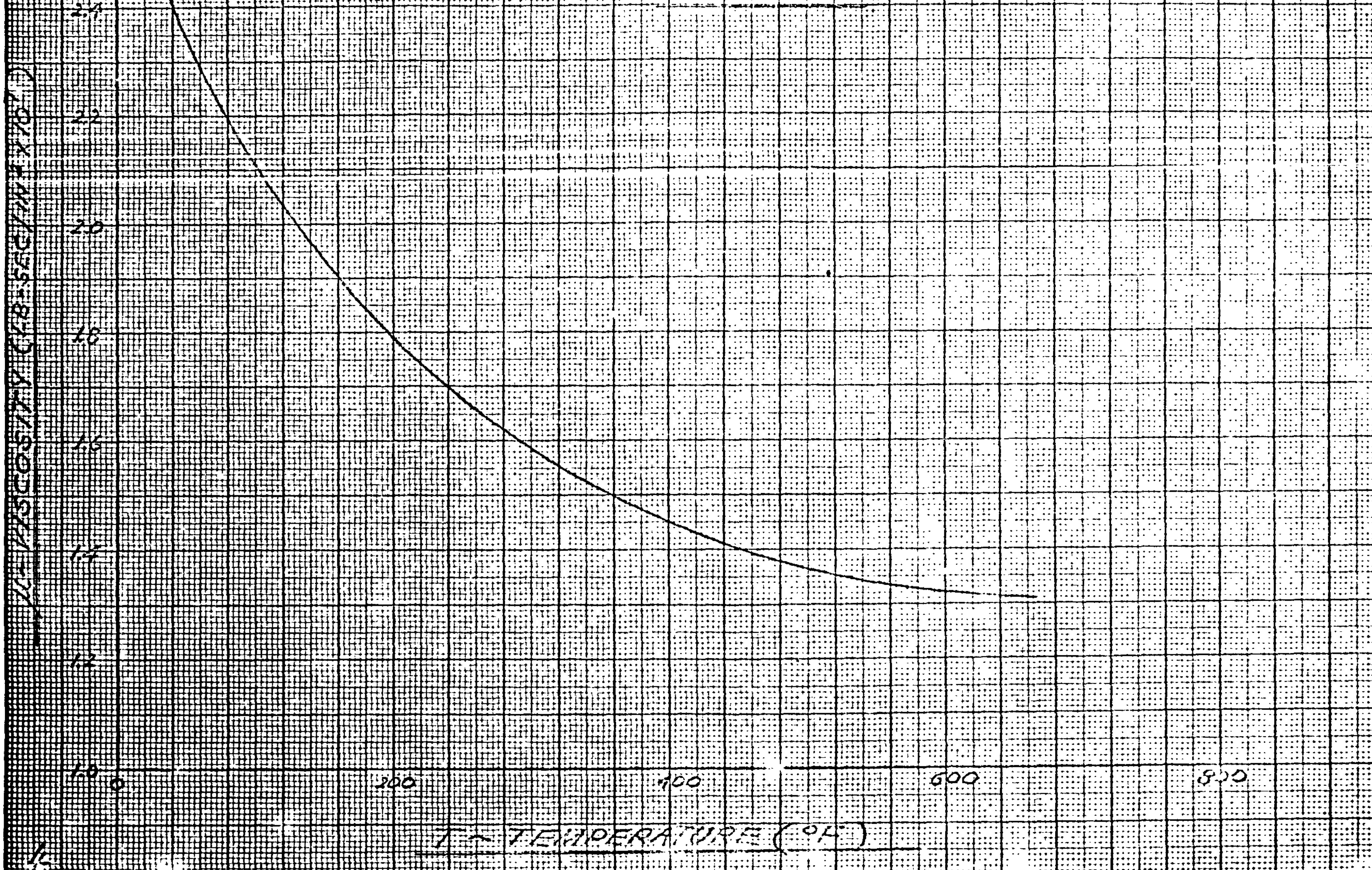
VISCOSITY OF LIQUID MERCURY

FIG. AI-2



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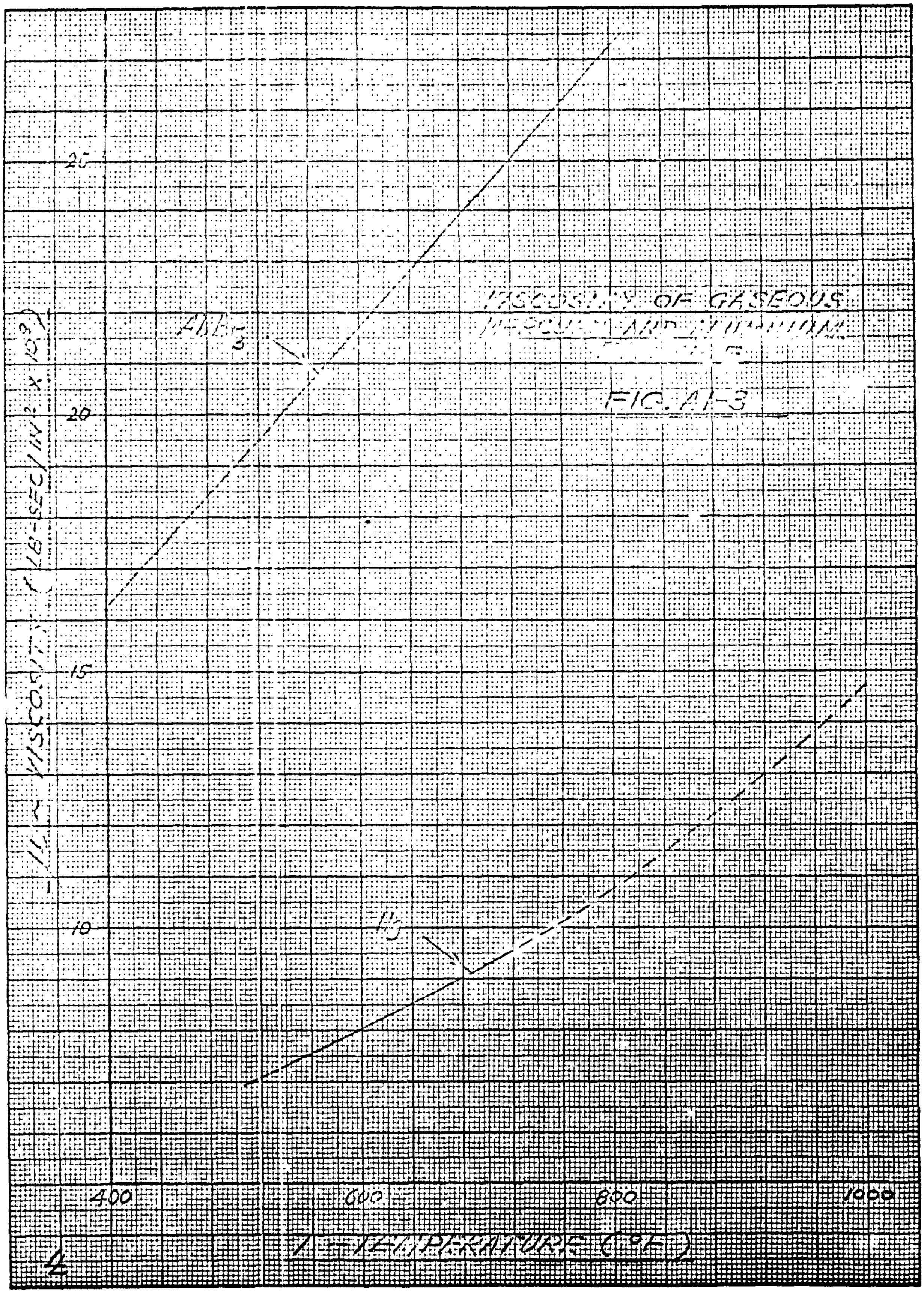

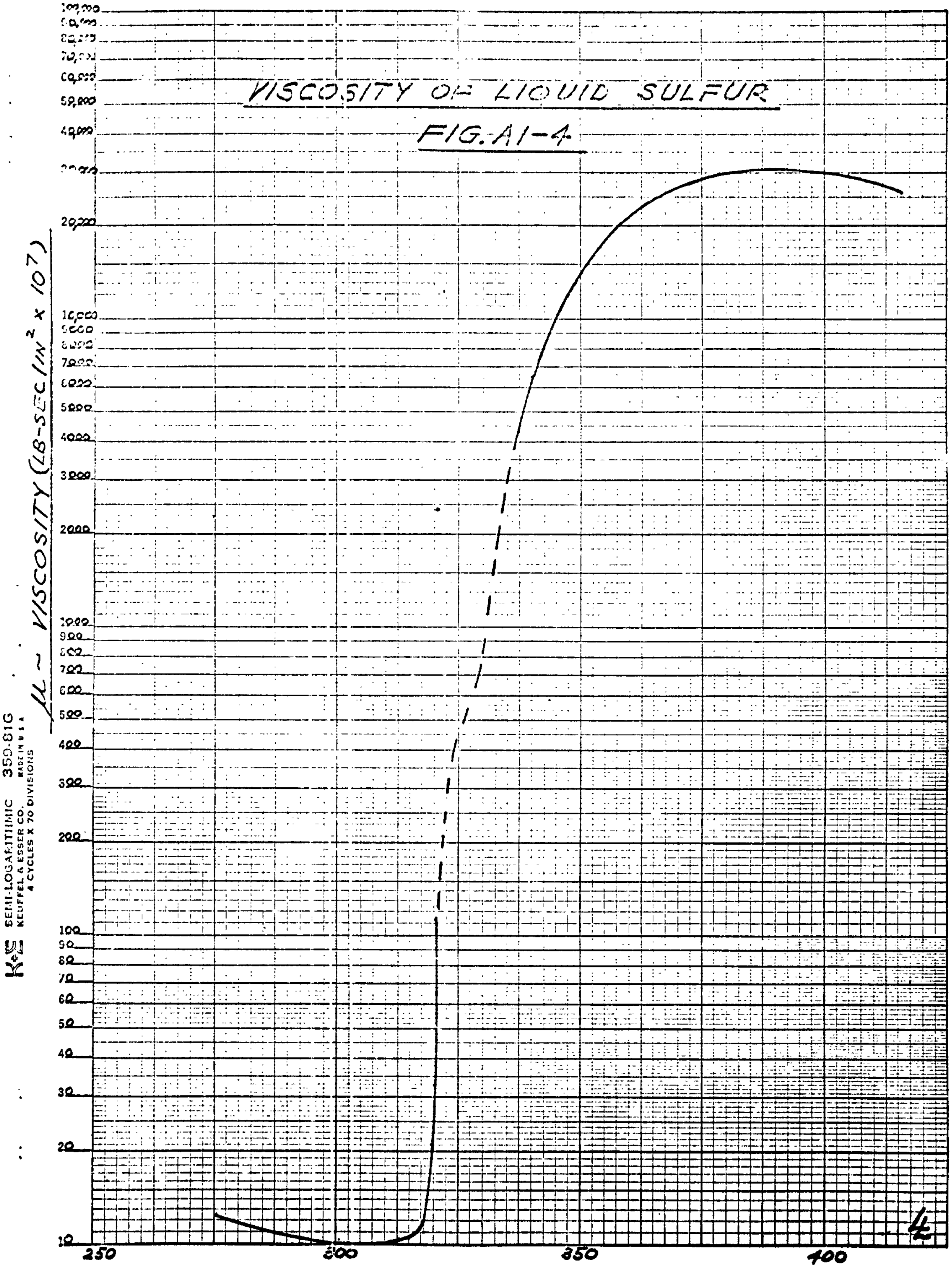
VISCOSITY OF LIQUID SULFUR
FIG. A1-4
μ ~ VISCOSITY (LB-SEC/IN² x 10⁷)
250
300
350
400
10
20
30
40
50
60
70
80
90
100
200
300
400
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600
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TITLE VISCOSITIES OF LUBRICANTS

# TABLE A-II

## LIQUID Hg

| T<br>°F | $\mu$<br>$\frac{lb\text{-}sec}{in^2} \times 10^5$ |
|---|---|
| 32 | 2.44 |
| 68 | 2.25 |
| 122 | 2.02 |
| 212 | 1.75 |
| 302 | 1.58 |
| 392 | 1.46 |
| 482 | 1.39 |
| 572 | 1.33 |
| 662 | 1.31 |

## LIQUID S

| T<br>°F | $\mu$<br>$\frac{lb\text{-}sec}{in^2} \times 10^5$ |
|---|---|
| 253.4 | 15.86 |
| 275.9 | 12.56 |
| 301.1 | 10.28 |
| 313.3 | 10.43 |
| 317.8 | 11.00 |
| 318.6 | 13.75 |
| 319.1 | 20.95 |
| 320.0 | 35.10 |
| 320.5 | 112.1 |
| 329.0 | 725.0 |
| 339.8 | 6,525 |
| 363.2 | 23,200 |
| 374.9 | 28,565 |
| 392.0 | 31,175 |
| 410.0 | 29,725 |
| 422.6 | 27,695 |
| 428.0 | 26,970 |

## GASEOUS Hg

| T<br>°F | $\mu$<br>$\frac{lb\text{-}sec}{in^2} \times 10^9$ | |
|---|---|---|
| 523 | 7.16 | |
| 595 | 7.94 | |
| 696 | 9.24 | |
| 716 | 9.48 | |
| 850 | 11.7 | * |
| 1000 | 14.7 | * |

## GASEOUS $AlBr_3$ (†)

| T<br>°F | $\mu$<br>$\frac{lb\text{-}sec}{in^2} \times 10^9$ |
|---|---|
| 392 | 16.0 |
| 500 | 19.1 |
| 608 | 21.9 |
| 716 | 24.9 |
| 806 | 27.4 |

(*) SUTHERLAND'S FORMULA

(†) CALCULATED FROM EQUATION (A1-1), APPENDIX I.

# Appendix 2

## Review of Pertinent Theory of Fluid-Film Bearings

The purpose of this appendix is to provide a limited outline of the type of problem involved in the analysis of fluid-film bearings. The complexity of the analysis is commensurate with and reflects the difficulties encountered in obtaining experimental data. The first expression is the "dynamic lubrication equation", an extension of the more familiar form of Reynolds equation to non-steady cases.

For an arbitrary motion of two surfaces, j and b, constituting the boundaries of an established laminar film, the equation may be written in a form given in the derivation of V. Castelli [8]:



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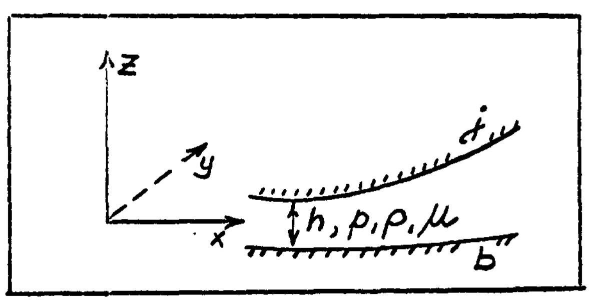
$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\mu}\frac{\partial}{\partial x}p\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\mu}\frac{\partial}{\partial y}p\right)=$$

$$6\Big\{2h\frac{\partial \rho}{\partial t}+2\rho(w_j-w_b)$$

$$-\rho\left[(u_j-u_b)\frac{\partial h}{\partial x}+(v_j-v_b)\frac{\partial h}{\partial y}\right]$$

$$+h\left[\frac{\partial}{\partial x}\rho(u_j+u_b)+\frac{\partial}{\partial y}\rho(v_j+v_b)\right]\Big\} \qquad (A2\text{-}1)$$

in which $u, v, w$ are the velocity components of the bounding surfaces, $j$ and $b$, at points $x, y, z$ in a cartesian coordinate system. The density, $\rho$, and the viscosity, $\mu$, are variable.




The dynamic lubrication equation must be compatible with the equations of motion of the elastic bodies $j$ and $b$ and satisfy the condition $p = p_a$ at the peripheries of the boundaries. In the case of externally-pressurized bearings, the statement of boundary conditions must also account for the mass flow entering (or leaving) the sources of supply.

At this point the reader will have realized the degree of complexity involved in the solution of the problem of whirl for example.

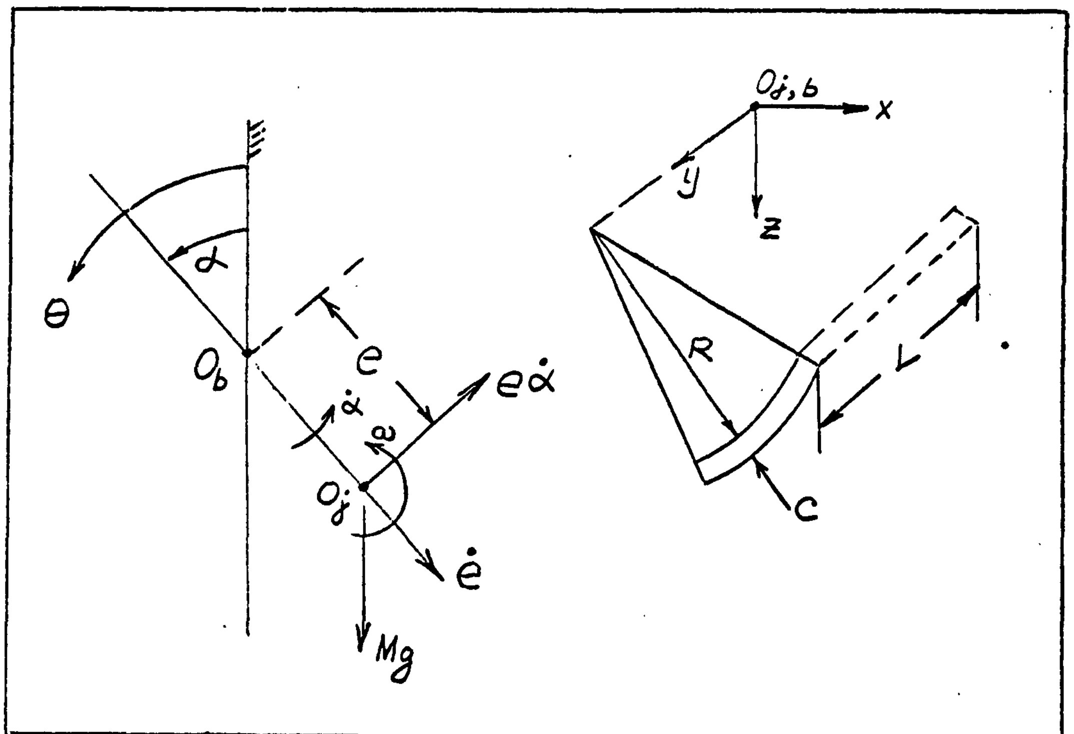
Let us limit the discussion to the case of a perfectly rigid and symmetrical rotor and a stationary bearing. Perfect alignment is postulated at all times, so that the only motion permitted is that of the journal axis in the xy plane and the uniform rotation of the journal about its own axis.

If we consider the isothermal case [1,2,8] of a perfect gas, $p/\rho$ = constant, $\mu$ = constant and $U_j = R\omega$, equation (A2-1) becomes:

$$\frac{\partial}{\partial\theta}\left(PH^3\frac{\partial P}{\partial\theta}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial\eta}\left(PH^3\frac{\partial P}{\partial\eta}\right)=$$
$$\lambda\left(\frac{\partial}{\partial\tau}PH+\frac{\partial}{\partial\theta}PH\right) \qquad \text{(A2-2)}$$

in which

$$\theta = x/R\ ;\ \eta = y/L\ ;\ \tau=\tfrac{1}{2}\omega t$$
$$\lambda = \frac{6\mu\omega}{p_a}\left(\frac{R}{C}\right)^2\ ;\ P = p/p_a$$
$$H=\frac{h}{C}=1+\epsilon\cos(\theta-\alpha)\ ;\ \epsilon = e/C$$



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If $M$ is the combined mass of the rotor and the shaft, then the equations of motion* are:

$$\ddot{X} = \frac{R}{C}\frac{P_a}{M}\int_0^{2\pi} P\sin\theta\, d\theta$$
$$\ddot{Z} = \frac{R}{C}\frac{P_a}{M}\int_0^{2\pi} P\cos\theta\, d\theta + \frac{g}{C} \qquad \text{(A2-3)}$$

in which

$$X = \frac{x}{C}\ ;\quad Z = \frac{z}{C}\ ;\quad X^2 + Z^2 = \epsilon^2$$

so that:

$$\alpha = \tan^{-1}\frac{X}{Z} \quad \text{and also} \quad H = 1 + X\sin\theta + Z\cos\theta$$

while velocities and accelerations of the $(XZ)$ and $(\epsilon\alpha)$ systems are related as follows:

$$\dot{X} = \dot{\varepsilon}\sin\alpha + \varepsilon\dot{\alpha}\cos\alpha$$
$$\dot{Z} = \dot{\varepsilon}\cos\alpha - \varepsilon\dot{\alpha}\sin\alpha$$
$$\ddot{X} = (\ddot{\varepsilon} - \varepsilon\dot{\alpha}^2)\sin\alpha + (\varepsilon\ddot{\alpha} + 2\dot{\varepsilon}\dot{\alpha})\cos\alpha \qquad \text{(A2-4)}$$
$$\ddot{Z} = (\ddot{\varepsilon} - \varepsilon\dot{\alpha}^2)\cos\alpha - (\varepsilon\ddot{\alpha} + 2\dot{\varepsilon}\dot{\alpha})\sin\alpha$$

---

* Friction forces are negligibly small in comparison with pressure forces acting on the journal.

The case of the very long bearing ($R/L \to 0$) is being investigated at the Franklin Institute by Mr. R. Castelli. High-speed digital computer techniques are being applied to solutions of representative cases, corresponding to values of dimensionless parameters referred to in Section 5. Preliminary results give every indication that a concentrically operated, plain journal gas-bearing will become unstable (whirl) at very moderate rotational speeds, unless the clearance is reduced to give an $R/c$ ratio far in excess of the permissible maximum under the contemplated operating conditions of the alternators.

A perturbation stability analysis for certain types of externally-pressurized gas-bearings has been performed by the writer [22,23,24]. A distributed parameter approach was used for the case of a circular thrust bearing [20] for which experimental data is now also available [21]. The problem is of considerable import-



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ance also in this case, since external pressurization may be required when a self-acting gas bearing is started, or in hybrid design.

In order to insure that no "air hammer" occurs in pocket and grooved types of externally-pressurized bearings, the following physical quantities should be minimized or maximized:

Minimized:

1. Depth of grooves and pockets (Small total volume)
2. Pressure difference across the supply restrictors
3. The supported mass

Maximized

1. Cross-sectional area of supply nozzles or orifices
2. Length of annular flow passage
3. Area ratio of the annular and the pocket (or groove circumscribed) regions.



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Returning now to equation (A2-2), the steady-state case corresponds to $\frac{\partial}{\partial \tau}(PH) = 0$.

Following the notation of Elrod [1], the steady-state equation for a continuous-film journal-bearing, lubricated with a perfect gas for which the viscosity depends on temperature only; may be written as follows:

$$\frac{\partial}{\partial \theta}\left(H^3 \Pi \frac{\partial \Pi}{\partial \theta}\right) + \frac{\partial}{\partial \xi}\left(H^3 \Pi \frac{\partial \Pi}{\partial \xi}\right) = \Lambda \frac{\partial}{\partial \theta}(H\Pi) \qquad \text{(A2-5)}$$

$$\theta = x/R \; ; \; \xi = y/R \; ; \; \Pi = p/p_a$$

$$H = \frac{h}{c} = 1 + \epsilon \cos \theta$$

$$\Lambda = \frac{6 \mu \omega}{p_a}\left(\frac{R}{c}\right)^2$$

Elrod has shown that at the point of maximum pressure in the film [1,2]:

$$T_{max} - T_w \leq \frac{U_w^2}{8(k/\mu)_w} \qquad \text{(A2-6)}$$

where $T_w$ and $U_w$ are the temperature and the velocity of the moving wall; $(k/\mu)_w$ is the ratio of thermal conductivity and viscosity at the wall, at the point of $p_{max}$. It follows, that with the surfaces maintained at uniform temperatures, the isothermal film is a reasonable assumption. He also derived an important "mass-content" relation for continuous-film, finite journal bearings:

$$\left.\begin{aligned} &\oint h^3 \chi \, d\theta = 0 \\ &\chi \equiv \int_{p_a}^{p} \frac{\rho}{\mu} \, dp \end{aligned}\right\} \qquad (A2\text{-}7)$$

which, for the case of a perfect gas of constant viscosity, gives an explicit relation between the integral on one hand, and the ambient pressure and eccentricity ratio on the other.

$$\oint h^3 p^2 = 2\pi C^3 p_a^2 \left(1 + \tfrac{3}{2}\epsilon^2\right) \qquad (A2\text{-}8)$$

Replacing in equation (A2-5)

$$\Pi \quad \text{by} \quad \Pi^* = p^*/p_a = \left(1 + \frac{2\mu_a}{\beta_{T,a}} \chi\right)^{1/2}$$

$$\Lambda \quad \text{by} \quad \Lambda^* = \frac{6\mu_a \omega}{\beta_{T,a}} \left(\frac{R}{C}\right)^2$$

in which

$$\beta_{T,a} = \left[\frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T\right]_a$$

is the isothermal bulk modulus at $(p_a, \rho_a)$, Elrod then shows that the same type of equation describes the lubrication by fluids other than gases, namely fluids belonging to a class for which

$$\left.\begin{array}{l}\rho = A(T) + B(T)\,p \\ \mu = \mu(T)\end{array}\right\} \qquad \text{(A2-7)}$$

so that performance data obtained for isothermal, perfect-gas lubricating films apply to <u>all</u> continuous film journal bearings, provided the lubricant properties are as in (A2-7) and $p_a$ is replaced by $\beta_{T,a}$.

Although it is more convenient to represent data for incompressible fluids in terms of the Sommerfeld number

$$S = \frac{\mu N}{(W/LD)} \cdot \left(\frac{R}{C}\right)^2$$

than in terms of

$$\Lambda = \frac{6\mu\omega}{p_a}\left(\frac{R}{C}\right)^2 \text{ and } \bar{P} = W/p_a LD ,$$



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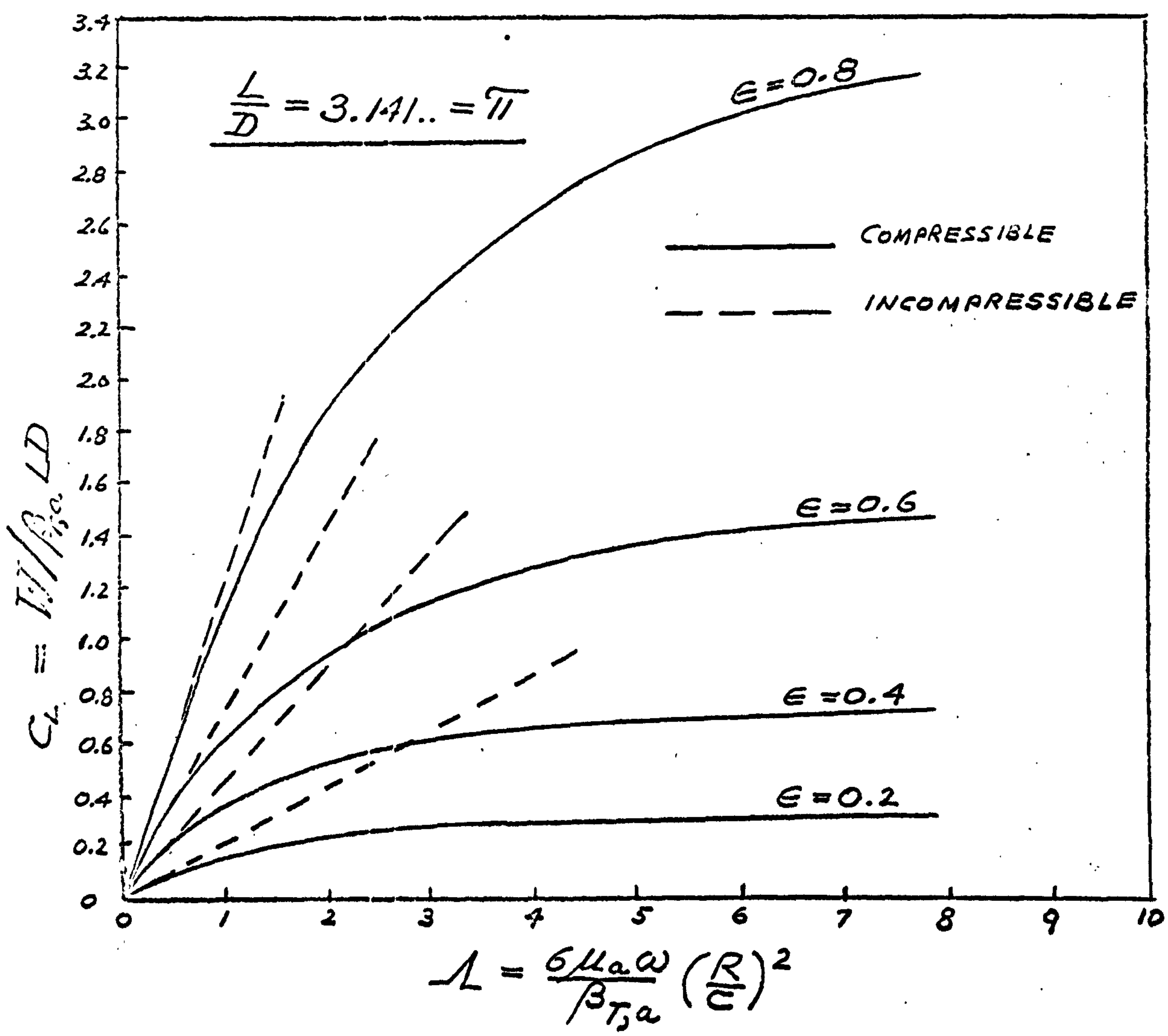
it is instructive to compare compressible and incompressible solutions when expressed in terms of the same dimensionless parameters, especially the limiting behavior when $\Lambda \to 0$, $\Lambda \to \infty$, $\epsilon \to 0$. A roughly sketched chart of this type is shown below (see reference [1]).



The chart shows that for $\Lambda \to 0$, the load carrying capacities of liquid and gas-lubricated bearings tend toward the same value. When $\Lambda \to \infty$, the load carrying capacity of liquid-lubricated bearings becomes very large. On contrast, the load carrying capacity of gas-lubricated bearings has an upper bound as $\Lambda \to \infty$, so that for a given speed and lubricant and a safe $\epsilon_{max}$ no further benefit may be derived by means of large $R/C$ ratios, (eg. reducing $C$, for example). One must then increase the ambient pressure, $p_a$, and the projected area $LD$.

Although both liquid and gas-lubricated journal bearings can easily be designed, they become invariably unstable when operated at high speeds in a nearly concentric position (whirl). It is much more difficult to design anti-whirl, gas-lubricated journal bearings with a high enough capacity to withstand the g-loads during launching. Limited data exist for plain slider bearings and plain step bearings. In attempting to "wrap" such sliders around the journal [15], the attitude of the journal center, the effect of curvature and the location of the center of pressure have to be estimated. A considerable amount of sound intuition and experience is involved.

Before concluding this brief survey, we wish to point out that the solution of the lubrication equation represents no difficulty for externally-pressurized, parallel-surface ($h$ = constant) thrust bearings, regardless of whether the fluid is a liquid, or a gas. If the flow is laminar and isothermal, and if - in the case of a gas - the method of feeding insures a low Mach number throughout the field of flow, we can write the lubrication equation in the form [25]:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (A2-8)$$

in which

$$\phi = p$$

if the fluid is incompressible, or

$$\phi = ap^2 + b$$

for a compressible fluid, with the costants "a" and "b" adjusted to give identical boundary conditions in either case.

Equation (A2-8) is, of course, that of Laplace. "Difficult" configurations represent no problem, since solutions can be obtained by means of a simple, conducting-sheet electric analog.



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## SCHEMATIC DIAGRAMS SHOWING POSSIBLE LOCATIONS OF ROTOR AND BEARINGS IN CLOSED SYSTEM

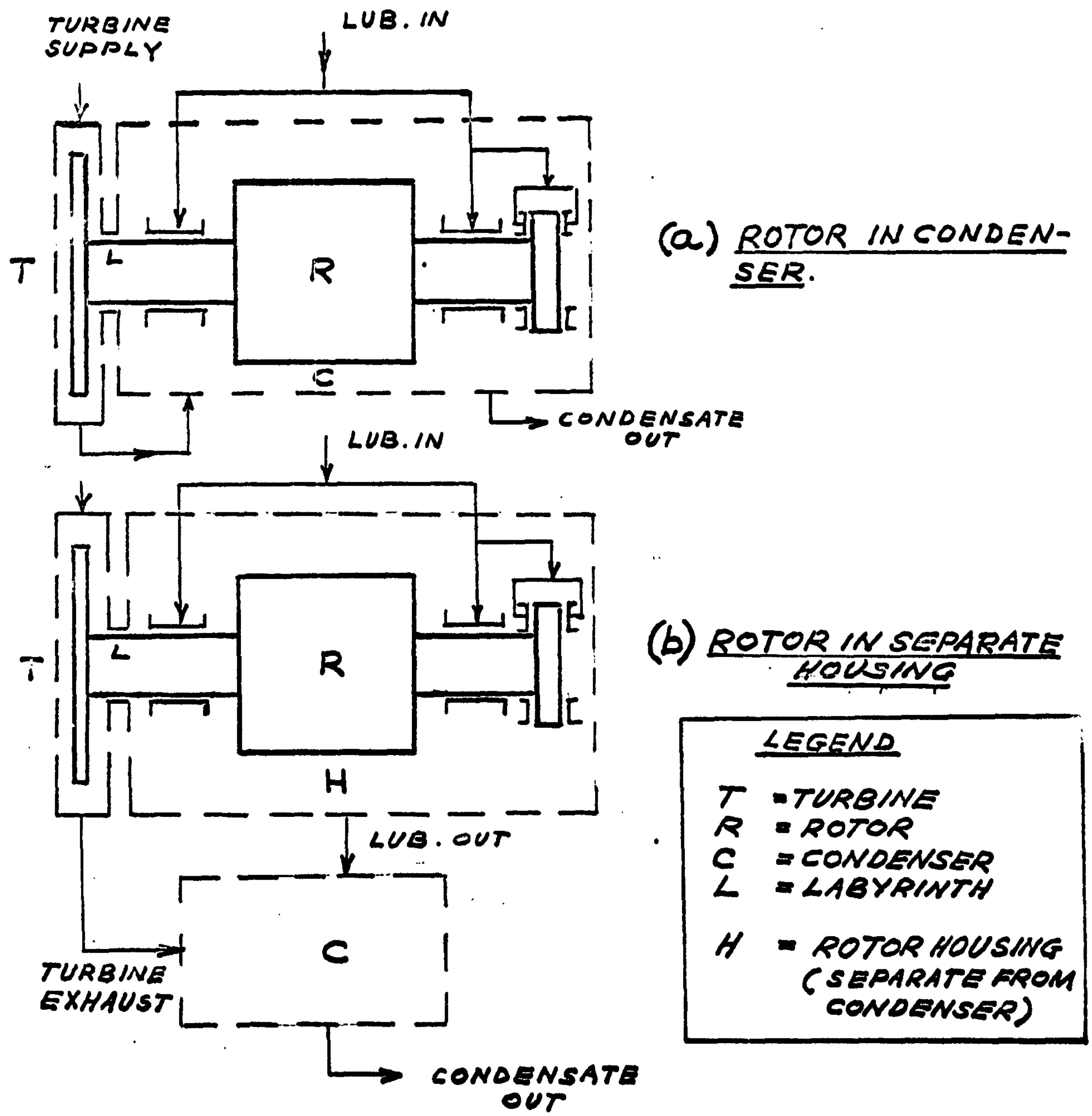



FIG. 1

EXTERNALLY-PRESSURIZED GROOVED JOURNAL BEARING.

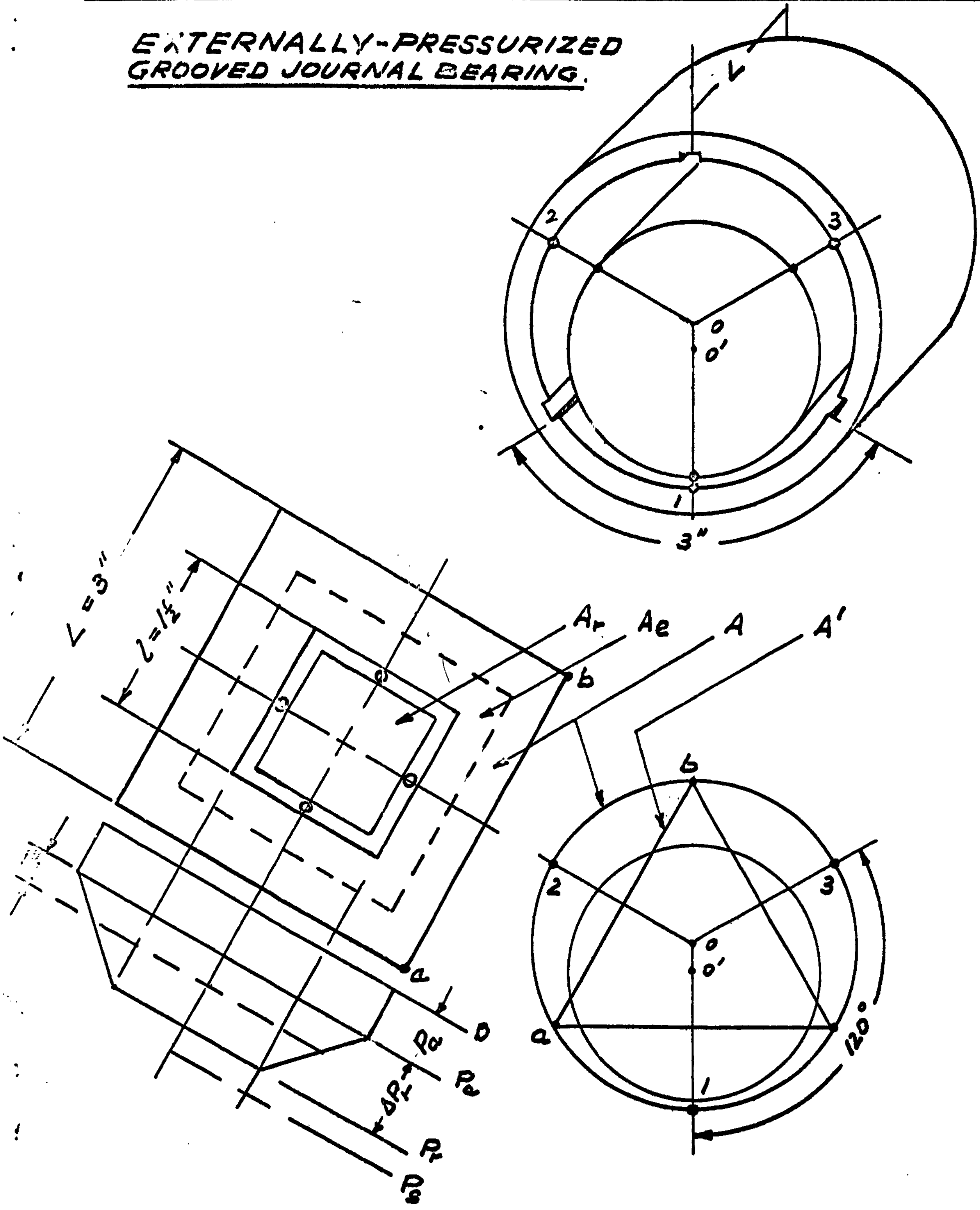
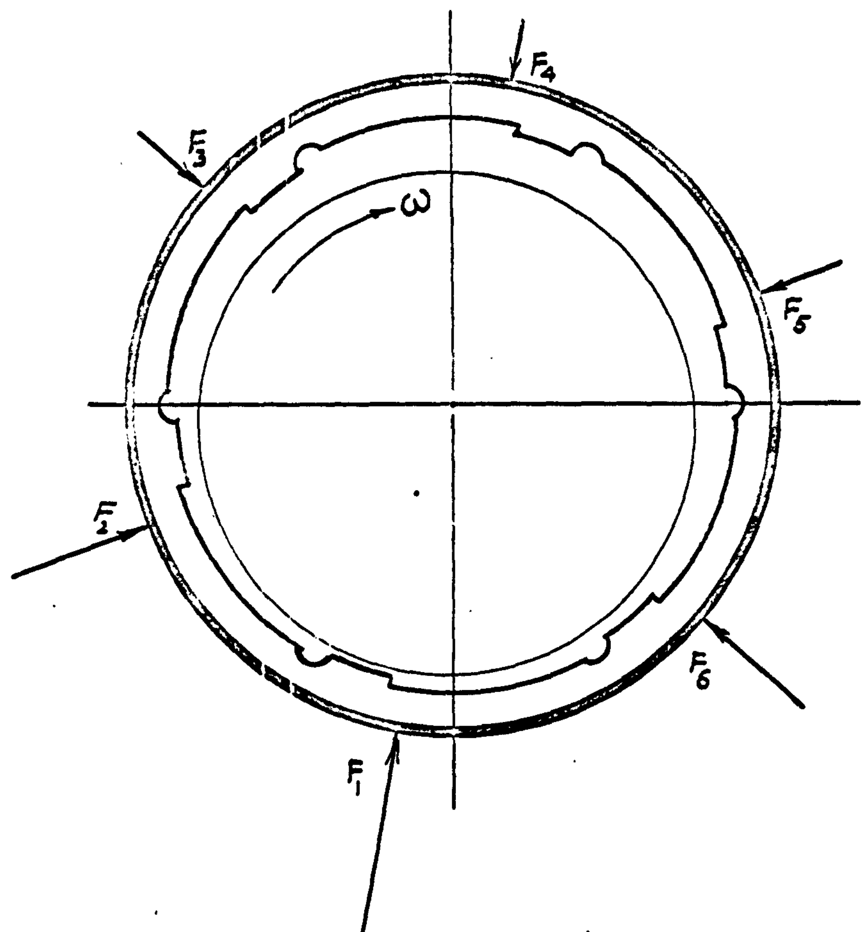



FIG. 2

TITLE RAYLEIGH-STEP JOURNAL BEARING



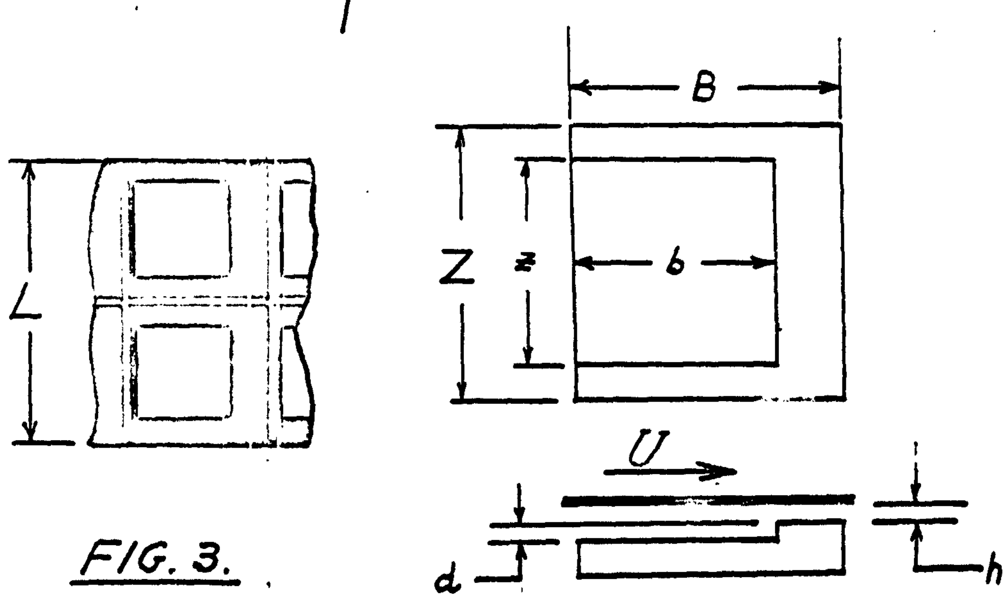




FIG. 3.



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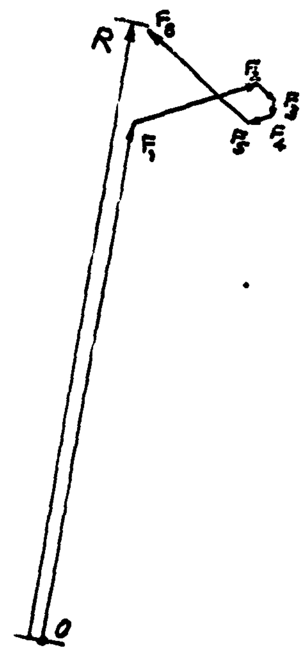
(a) FORCE VECTOR DIAGRAM FOR RAYLEIGH STEP JOURNAL BEARING IN FIG.3.




(b) SCHEMATIC DIAGRAM OF TYPICAL ECCENTRICITY LOCI
(I) PLAIN JOURNAL BEARING
(II) RAYLEIGH STEP JOURNAL BEARING

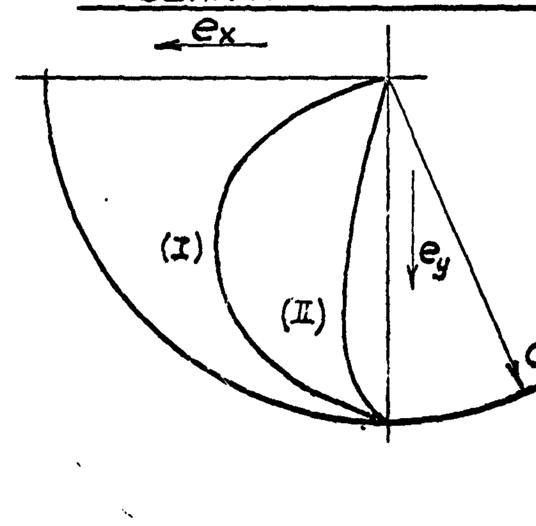



FIG. 4

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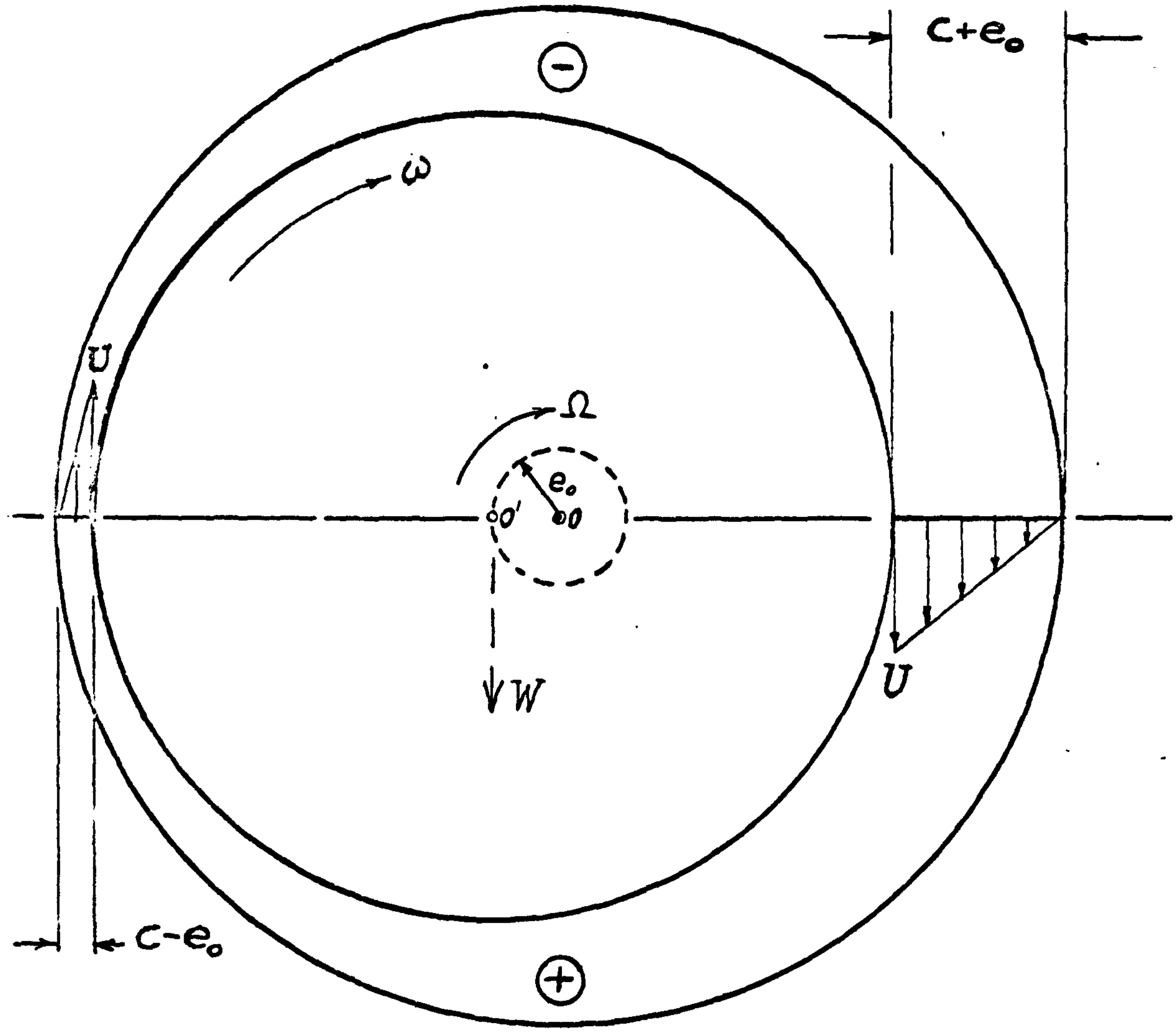


FIG. 5.



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