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TRANSLATION

CHOICE OF SAFETY FACTOR AND COMPUTATION IN DESIGNING ELECTROMAGNETIC MECHANISMS OF A REQUIRED DEPENDABILITY

By

Ya. A. Rips

FOREIGN TECHNOLOGY DIVISION

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BY: Ya. A. Rips

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TRANSLATION SERVICES BRANCH
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

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CHOICE OF SAFETY FACTOR AND COMPUTATION IN DESIGNING ELECTROMAGNETIC
MECHANISMS OF A REQUIRED DEPENDABILITY

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The dependability of a device based on a determined principle of action with fixed conditions and a given time of operation, is determined by the peculiarities of its design, the quality of the materials used in it, and the technology of its production. Therefore, along with the problem of computing the dependability of a given device there is a great deal of significance in a certain sense in the reverse problem, designing a device of a given dependability, and this article is devoted to a consideration of this problem, as applied to electromagnetic mechanisms.

As is known quantitatively the dependability of a device is determined by the probability of its faultless working under determined operational conditions during an established length of time. Experimentally the dependability is approximately measured by a relative number of devices, average as to probability which function faultlessly under given conditions during a stated time $t$. In this arrangement the number of functioning devices should be referred to their whole number put out by production. In other words, if in an interval of time from zero to $t$ there worked faultlessly $R(t)\%$ of manufactured devices, and consequently there got out of order $Q(t)\%$ \[
= \int [1 - R(t)] 100\% \]
\[
of the devices, then the dependability in the interval of time under consideration approximately equals $R(t)$.

The device is capable of faultless working in the case where not one of its properties or operating characteristics remains within previously determined limits. The probability of a given occurrence for the $p$th of $r$ characteristics of the device can be analytically determined by the probability
of the fulfillment of some criterion of dependability of the characteristics of the device during the course of time \( t \) \[ R_d(t) = P(A_d > B_d) \] (1)

where \( A_d \) and \( B_d \) are the properly chosen parameters of the device which characterize, respectively, the position of the characteristic being researched and the allowable limit of its displacement.

If the occurrences which characterize the fulfillment of the criteria of dependability for each of \( r \) characteristics of the device are independent of each other, then the dependability of the device as a whole can be determined through the dependability of the separate characteristics in the following fashion:

\[ R(t) = \prod_{i=1}^{r} R_{d_i}(t) \] (2)

The working of an electromagnetic mechanism is generally determined by several characteristics. Keeping in mind the identity of the method of computing the dependability of any of the characteristics of an electromagnetic mechanism, it is to the point to limit oneself to the determination of one of them, for example, the dependability of the operation \( R_1(t) \). The dependability of an electromagnetic mechanism is computed with the aid of expression (2). Just in the same way for a given value of dependability of a mechanism as a whole one may select the corresponding dependability of its characteristics.

As criteria for the dependability of the operation of an electromagnetic mechanism let us consider the fulfillment of the relationship \( f_s > f_n \). In accordance with this the analysis given below relates to devices with relay action usually used in control circuits and automation in communications, and is not extended to electromagnetic mechanisms (measuring mechanisms, etc.) for which the criteria of dependability of operation are different.
In computing the value $R_3(t)$ the parameters $A_p$ and $B_p$ are considered as
with laws of distribution corresponding to the moment of time $t$ \([1, 2] \). 
In the given case $A_p$ and $B_p$ represent, respectively, the electromagnetic $F_e$
and the mechanical counteraction $F_m$ of the force of the electromagnetic
mechanism in the first critical point (in operation) at the moment of time
t. In most cases one may consider with sufficient precision that the dis-
tribution $F_e$ and $F_m$ correspond to the normal law:

$$
\Phi(F_e) = \frac{1}{\sqrt{2\pi\sigma_e^2}} e^{-\frac{(F_e - \mu_e)^2}{2\sigma_e^2}}
$$

$$
\Phi(F_m) = \frac{1}{\sqrt{2\pi\sigma_m^2}} e^{-\frac{(F_m - \mu_m)^2}{2\sigma_m^2}}
$$

where $\sigma_e$ and $\sigma_m$ are the average quadratic deviations; the lines above the
symbol signify mathematical expectation (average value) of
random magnitude.

For such an affirmation there exist theoretical bases. The random val-
ues $F_e$ and $F_m$ prove to be functions of a great number of comparatively small
(i. e., having small average quadratic deviations) independent random argu-
ments $x_1, \ldots, x_k$, the dimensions of the parts, parameters of the original
materials, etc. In this situation the values of the deviations $\Delta F_e$ and $\Delta F_m$
from the average values $\mu_e$ and $\mu_m$ are determined through the deviation of
the arguments $\Delta x_1, \ldots, \Delta x_k$ from their average values $\bar{x}_1, \ldots, \bar{x}_k$ by the
device of the sum. Under given circumstances on the basis of Lyapunov's
theorem known from the theory of probabilities, one may affirm that the laws
of distribution of the random values $F_e$ and $F_m$ will be practically normal.

The existing statistical data confirm the indicated proposition. This
can be seen from the experimental and theoretical distributions of electro-
mechanical and mechanical forces obtained for 100 specimens of the relay RKM.
does not depend on the parameters of the device. Therefore the designing of electromagnetic mechanisms of a given dependability should be done from the condition of assuring a given value $R_{10}^*$, taking into consideration only the gradual failures and equal to:

$$R_{10}^* = \frac{R_{10}}{R(y)}.$$  

Let us designate by $Y$ the expression standing in the formula (4) under the sign of Laplace's function:

$$Y = \frac{1}{\sqrt{\left(\frac{\sigma_y}{\mu_y}\right)^2 + \left(\frac{\sigma_t}{\mu_t}\right)^2}}.$$  

From the equality (4) it is seen that the value $R_y(t)$ is uniquely connected with the value $Y$ and the choice of the first is fully determined by the latter. In Fig. 2 there is shown a graph of the dependence of $Y$ on the value $R_{10}^*$. Since $R_{10}^*$ is close to unity for convenience along the $Y$ axis there are plotted the values $-\ln Q = -\ln(1 - R_{10}^*)$. By making use of this graph in accordance with the given value of $R_{10}^*$ one can find the corresponding value of $Y$, and vice versa.

An analysis of the expression (6) shows that $Y$ is determined by three parameters: the safety factor $\sigma_t$ and the relative average quadratic deviations $\sigma_y^2$ and $\sigma_t^2$ of the random values $Y$ and $Y_0$, while the given value $Y$ can be assured at different combinations of them. These
parameters prove to be functions of the time of the operation of the device. Their initial values $\bar{x}_0$, $\bar{y}_0$, and $\bar{z}_0$ with $t = 0$ determine the value $Y(0) = y$, corresponding to the initial production dependability $R_1(0)$, which is measured by the probability that the device put out will be in working order $[3]$. The values $\delta_{x0}$ and $\delta_{y0}$ are determined by the quality of the design and materials used, and also by the peculiarities of the technology of the production. The degree of the dependence of the values $\delta_{x0}$, $\delta_{y0}$, and $\bar{x}_0$ on the time of working characterises the stability of the device with relation to disturbing factors. In the process of $\bar{x}_0$ decreases and $\delta_{x0}$ and $\delta_{y0}$ increase in comparison with their initial values, as a result of which the dependability of the work drops $[2]$. The essence of all the measures for raising the quality and the operational dependability amounts to a lowering of the values $\delta_{x0}$ and $\delta_{y0}$ and reducing the change in the values $\delta_{x0}$, $\delta_{y0}$, and $\bar{x}_0$ in operation. As depends on how successfully the given problem is solved one chooses one or the other initial value of the safety factor $\bar{x}_0$, by which one also assures the desired level of dependability. The choice of the value $\bar{x}_0$ is done from the condition that in the course of the period of the service $T$ of the device its dependability will be equal to the given $R_{10}$. In this situation the necessary value $\bar{x}_0$ in proportion as $\delta_{x0}$ and $\delta_{y0}$ are greater and the changes $\delta_{x0}$, $\delta_{y0}$ and $\bar{x}_0$ are greater in the process of operation. In practice up to the present time the value $\bar{x}_0$ has been determined by the experimental method with tested devices al-
ready created or else through experience in operation of analogous apparatuses.

Increase in the initial value of the safety factor $\delta_0$ increases the dependability of a given device, but at the same time increases its weight and cost and the expenditure of material. On the other hand, with a given safety factor $\delta_0$, the dependability increases with the reduction of $\delta'_{30}$ and $\delta'_{40}$, and with a slower change of $\delta'_{3t}$, $\delta'_{4t}$, and $\delta_t$. In this case also there is an increase in the cost and the labor involved in the device since it is necessary to use more high-quality material, increase the precision in the making of the parts, etc.

From what has been explained it is clear that the problem of creating a device with a given dependability is not unique and has a multitude of decisions corresponding to determined combinations of values of $\delta'_{30}$, $\delta'_{40}$, and $\delta_0$ and determined form of the functions $\delta'_{3t}$, $\delta'_{4t}$, and $\delta_t$. The choice of the optimum combination of the indicated values in solving a concrete problem demands consideration of various supplementary conditions. The combination mentioned above can be found by joint determination of all the parameters, and this should be done taking into consideration a series of technical-economic factors.

This question merits special investigation. In this article the problem of designing electromagnetic mechanisms of a given dependability is considered as the choosing of the respective initial values for the safety factor with completely determined values for the remaining parameters.

As is known one distinguishes the working $s$, the production (rated) $s_p$, and the operational $s_\tau$ of the safety factor which are connected by the ratios

$$s = s_p s_\tau$$

or

$$s = s_p^2$$

(7)

The electromagnetic mechanism should be designed with the initial working safety factor $\delta_0$. The production safety factor is determined by the values $\delta'_{30}$ and $\delta'_{40}$, depending on the design and production factors. The value
of the operational safety factor depends on the sensitivity of the device to disturbing action and is considered as change in the values $G_{t}, G_{Mt}$, and $a_{t}$ in the course of a period of service.

As a result of the change in $G_{t}, G_{Mt}$, and $a_{t}$ in the working of the device there is a change in the value $R_{i}(t)$ with relation to the component of the initial production dependability $R_{i}(0)$. However, whatever the character of this change might be it is necessary that the value $R_{i}(T)$ in the course of the period of service of the device $T$ be equal to the previously selected value $R_{i}^{*}(0)$. By taking this into consideration one can draw the conclusion that the condition for rational designing of the devices is that the component of the initial production dependability $R_{i}(0)$, calculated by the formula (4) with $G_{t} = G_{t}, G_{Mt} = G_{Mt},$ and $a_{t} = a_{t},$ should be numerically equal to $R_{i}^{*}(T)$ computed with $G_{t} = G_{Mt}, G_{Mt} = G_{Mt},$ and $a_{t} = a_{t}$ (Fig. 3):

$$R_{i}(0) = R_{i}^{*}(T)$$

This follows from the fact that in rational designing the supplementary possibilities included in the electromagnetic device, as a result of the introduction of the operational safety factor, at the end of the period of service should be completely exhausted. Therefore the level of dependability determined by the value $R_{i}(0)$ is maintained and should be selected as equal to the prescribed value $R_{i}^{*}(0)$ and the known values $G_{t}$ and $G_{Mt}$ without taking into account the dependencies $W_{t}, G_{t},$ and $G_{Mt}$ in time.

The considerations expressed here have a general character and are applicable to devices of any kind.

The designing of electromagnetic mechanisms is done by known methods.
In doing this in accordance with the technical conditions of the designing problem one chooses a kinematic scheme and design of the mechanism, determines the characteristics of the final element, and builds the specific mechanical feature of the device. Afterwards one determines the magnitude of the counteracting mechanical force \((\bar{F}_{m0})_{kp}\) at the critical point, and after design of the magnetic system from the condition \((\bar{F}_{m0})_{kp} = (\bar{F}_{c0})_{kp}\) one computes the o. d. (operational dependability) \(I_{op}\).

The following step in the computation is the working out of the working tractional characteristic and design of the winding whereby one should take into consideration the need for creating an electromagnetic mechanism of a given dependability \(R(T)\), which is determined by the proper choice of the initial safety factor \(\varepsilon_0\). The necessary value of \(\varepsilon_0\) in the case under consideration is attained by an increase in the o. d. of the winding. However, this leads to an increase in the amount of copper in the winding and the power required by it.

Let us assume that the desired value of the component of dependability \(R(T)\) in the course of the period of service is equal to \(R_{10}\). Then in accordance with the equalities (4), (6), and (8) we will get

\[
R_{10}^{-1} = \frac{1}{2} \left| 1 + \Phi(y) \right|
\]

where

\[
y = \frac{\tau_{sp} - 1}{\sqrt{\left(\frac{\tau_{sp}}{\tau_{sp}}\right)^2 + \left(\frac{\tau_{sp}}{\tau_{sp}}\right)^2}}
\]

The magnitude \(y\) can be determined in accordance with the given value \(R_{10}\) with the aid of the graph presented in Fig. 2. The value \(\tau_{sp}\) as a function of \(y\) is found by the following relationship:

\[
\tau_{sp} = \frac{1 + \sqrt{1 - \left(1 - \frac{\tau_{sp}}{\tau_{sp}}\right)^2 \left[1 - \Phi\left(\frac{\tau_{sp}}{\tau_{sp}}\right)\right]}}{1 - \Phi\left(\frac{\tau_{sp}}{\tau_{sp}}\right)}
\]

In accordance with the expression (10) in Fig. 4 there are presented the de-
pendencies $\bar{\varepsilon}_{np}$ on the magnitude $\frac{\delta_{2x}}{F_{30}}$ for different values of $n = \left(\frac{\delta_{10}}{F_{10}}\right) / \left(\frac{\delta_{30}}{F_{30}}\right)$.

Having computed the last two expressions earlier by the stated method \([3,7]\), in accordance with the given graphs one can determine the value of the necessary production safety factor $\bar{\varepsilon}_{np}$.

The method of computing the required value of the factor $\bar{\varepsilon}_{3k}$ for given conditions of operation is presented in \([2,2]\). Having determined $\bar{\varepsilon}_{np}$ and $\bar{\varepsilon}_{3k}$ by the expression (7) we find the initial working safety factor $\bar{\varepsilon}_0$.

The value obtained $\bar{\varepsilon}_0$ represents the safety factor according to the tractional forces at the first critical point. If the magnetic circuit of the electromagnetic mechanism is not saturated then the safety factor by ampere turns $\bar{\varepsilon}_0$ will equal $\bar{\varepsilon}_0 = \sqrt{\bar{\varepsilon}_0}$, and the working o. d.

$$/w_p = \sqrt{\bar{\varepsilon}_0}/w_{op}. \quad (11)$$

By knowing the o. d. one can construct the working tractional characteristic and make the computation of the parameters of the winding: diameter of the wire $d$, number of turns $w$, resistance of the winding $r$, and the power lost in the winding $P$.

In designing the electromagnetic mechanisms of a given dependability a definite interest is afforded by the dependence of the power $P$, the volume $V_M$ and the cost of the copper $C$, the diameter of the wire, and the number of turns on the value of the initial working safety factor $\bar{\varepsilon}_0$. They enable one to compare and evaluate different versions of mechanisms from the point of view of economy. Such dependences which show change in these values.
referred to their magnitudes \( P_0, V_{\text{M0}}, C_0, d_0, \) and \( m_0 \) with \( \bar{m} = 1 \) presented in Fig. 5.

See Page 11a

For the purpose of obtaining much generalization the curves presented in Fig. 5 are given in conventional units while as a unit of measurement \( \bar{m}_0 \), there is taken the value

Fig. 6

See Page 11a

\[
\frac{\text{d}^2 \phi_0}{\text{d}^2 \psi_0} = \lambda_0
\]

where \( l_k \) is the length of the coil;

\( D \) is the inside diameter of the coil;

\( k_y \) is the embedding factor;

\( \mu_c \) is the heat emission factor;

\( \theta \) is the allowable temperature in the heating of the coil;

\( \rho \) is the specific resistance of the wire;

\( k \) is the ratio of the diameters of the wire with insulation and without insulation.

The units of change of magnitude \( \frac{P}{P_0}, \frac{V}{V_{\text{M0}}}, \frac{C}{C_0}, \frac{d}{d_0}, \) and \( \frac{m}{m_0} \) are easily expressed by \( \lambda \) and the values in the table. In constructing the graph \( \frac{C}{C_0} = f(d) \) there was used the curve of the dependence of the cost of a unit of weight of active wire material of the brand PEL on its diameter, which is shown in Fig. 6.

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The method explained makes possible computation in projecting such an im-
portant factor as to what the dependability is in the working of the device, enables one to choose rationally the safety factor, and evaluate the effect of the parameters of the device on its dependability. All these elements are essential in the designing of dependable devices with minimum cost and expenditure of materials. It is clear that the procedure given can be used in computing not only electromagnetic mechanisms but also other devices.

Supplement. By way of example to illustrate the points involved let us consider the choice of the safety factor and the designing of an electromagnetic relay on DC with a given operational dependability.

Let the given value $R_{10}$ in the course of the period of the service be equal to 0.9999. This signifies that excluding sudden breakdowns in the course of the period of the service there is allowed failure in operation on the average of one relay among 10,000 in operation. It is necessary to design the apparatus in such a way that this condition will be satisfied.

As a result of the designing of the relay by the ordinary procedure one determines the value $I_{up}$ as equal, let us assume, to 100. Further by the method expounded in (3) one computes the values $\delta_{so}$ and $\delta_{mg}$, which we will take, respectively, as equal to 0.1 and 0.05.

Then one determines $Q=1=R^*_{10} = 0.0001$ and $(-lnQ)=9.2$. In accordance with this value with the aid of the graph given in Fig.2 we will find the value $y=3.72$, then the value $\delta_{so}/\delta_{so}$ = 0.372 and for $n = \frac{0.05}{0.1} = 0.5$ by the curves presented in Fig. 4. We determine the value $\delta_{np}=1.64$. Besides, let $\delta_{nk}=1.8 \frac{1}{2}$. Then $\delta_{o}=1.64 \cdot 1.8 = 2.96$ and $\delta_{f}=\sqrt{2.96}$ $= 1.72$. In accordance with the formula (11) the necessary o. d. will be equal to $I_{mp}=1.72 \cdot 100 = 172$.

For evaluation of the supplementary expenditure due to which there is assured the necessary safety factor $\delta_{o}$ and the value $R^*(T)$ is raised from
0.5 to 0.9999 let us find the unit of measurement $\bar{a}_0$. In doing this we take

$$D = 0.5 \text{ cm} ; \ t_k = 3 \text{ cm} ; \ \theta = 50^\circ \text{C} ; \ \mu = 12 \cdot 10^{-4} \text{ w/cm}^2 \cdot \text{ C} ; \ k_x = 0.9 ; \ k = 12; \ \rho = 0.0175 \cdot 10^{-4} \text{ ohm} \cdot \text{ cm}. \ \text{Then} \ A = 15.1 \ \text{and the value} \ \bar{a}_0 \ \text{in these units is} \ \frac{2.96}{15.1} = 0.196. \ \text{Afterwards by the curves presented in Fig. 5 and with the aid of the table we will find that} \ \begin{align*}
\frac{P}{P_0} &= 1.2 \left( \frac{A}{1+A} \right) - 1.13; \\
\frac{V_q}{V_{\text{wq}}} &= 0.33 \left( \frac{A^2}{1+A} \right) - 3.22; \\
\frac{C}{C_0} &= 0.31 \frac{A^b}{\sqrt{1+A}} - 2.3.
\end{align*}

\textbf{References}


4. Sotakov, B. S., Elements of Automatic and Telemechanical Equipment, Gosenergoizdat (State publishing office for power-engineering literature), 1950.

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