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EFFICIENCY ENHANCEMENT BY PHASE FOCUSING AND COLLECTOR DEPRESSION

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FOREWORD

This report was prepared by the Electron Physics Laboratory, Department of Electrical Engineering, The University of Michigan, Ann Arbor, Michigan, on Air Force Contract No. AF 30(602)-2834 under Project No. 5573 of Task No. 557303 entitled "The Design and Construction of a High-Power UHF Crestatron." The secondary report number is Technical Report No. 55 under Project 05228.

The author gratefully acknowledges discussions with Dr. J. G. Meeker and the work of Chong Rhee in carrying out the numerous calculations for these investigations.

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ABSTRACT

A theoretical study of the efficiency enhancement possible through collector segmentation and depression is made using information obtained from the nonlinear interaction theory for traveling-wave-like devices. The velocity characteristics of the spent beam are used directly to calculate the improvement factors and the optimum collector segment potentials as functions of various operating parameters.

Specific calculations are made for both maximum gain and maximum power output operating conditions. Also efficiency improvement through phase focusing is discussed and experimental data are given and correlated with the theoretical predictions. It is found that improvement factors of approximately 1.5 to 2 are predicted, and similar results are obtained experimentally.

Results are also given for the backward-wave oscillator, for which the improvement factor is higher due to the low interaction efficiency and the corresponding low velocity spread in the spent beam.

PUBLICATION REVIEW

This report has been reviewed and is approved.

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EFFICIENCY ENHANCEMENT BY PHASE FOCUSING AND COLLECTOR DEPRESSION*

INTRODUCTION

Nonlinear calculations\(^1,2,3\) of the electron-wave interaction process in TWA's, klystrons and BWO's have permitted the calculation of interaction efficiencies along with information about the energy state of the spent beam. These calculations have been primarily concerned with the r-f interaction process and have not considered the effects of collector depression on the overall efficiency of the device. In this report the effect of collector depression on device efficiency is studied theoretically and calculations are presented for the TWA and the BWO under a variety of operating conditions. The conversion process in a TWA and a BWO is one of converting the electron beam kinetic energy to r-f energy. It is well known that the efficiency of conversion is limited by the fact that the electrons slow down as they give up energy to the r-f wave so that eventually they drop out of near synchronism with the wave and the interaction ceases. It is logical to suppose that the wave velocity could be slowed to maintain the beam-wave synchronism and that a resulting increased efficiency would be obtained. Recently a detailed theoretical study of this problem was made and some successful experiments were carried out\(^4\). The interaction efficiency of a broadband, high-power helix-type TWA operating in the S-band regime was increased by a factor of 2.5 to 3 times. Interaction efficiencies of near 50 percent have been obtained on this amplifier.

The phase focusing process discussed above relates to the fundamental interaction process, whereas collector depression affects the "plate-circuit" efficiency. Much of the power usually dissipated in the collector may be conserved by operating the collector at a lower potential than the r-f circuit**. In view of the wide velocity spread in the spent beam of a TWA it is usually necessary to segment the collector and optimize the voltage on each segment. Collector depression is most effective in devices with low interaction efficiencies since the velocity spread is small. As the interaction efficiency is increased the efficiency improvement factor obtained with collector depression decreases (for a fixed number of segments) because of the increased velocity spread. The efficiency enhancement obtainable with collector depression is determined in this report for TWA's with uniform velocity circuits and for TWA's with tapered velocity circuits. The effect on backward-wave oscillator performance is also considered.

THEORETICAL DEVELOPMENT

The nonlinear calculations of beam-wave interaction are made for representative electron charge groups passing through the interaction region, where they are acted upon by both r-f circuit and space-charge fields. The total number of these charge groups constitutes the entering d-c beam charge. The velocity was defined in terms of the normalized Lagrangian dependent variables as follows:

\[
\left( \frac{u_j}{u_0} \right)^2 = [1 + 2Cu(y, \Phi_0)]^2, \tag{1}
\]

where

\[u_j \triangleq \text{the total velocity of the } j\text{th charge group, and}\]
\[u(y, \Phi_0) \triangleq \text{the normalized r-f velocity for the } j\text{th group.}\]

The total velocity for a particle is simply related to its energy in electron volts or joules as:

\[
\frac{mu^2}{2} - eV_0 = -eV = \varepsilon, \tag{2}
\]

where \(V\) is the equivalent r-f potential for a particle and is positive for energy given up (deceleration) by an electron and negative when energy is absorbed.
by the electron (acceleration). For an electron with the average velocity \( u_0 \), Eq. 2 simplifies to

\[
\frac{m u_0^2}{2} - e V_0 = 0, \tag{3}
\]

which simply states the equivalence of the electron kinetic and potential energies. Combining Eqs. 1, 2 and 3 yields

\[
\left( \frac{u_0}{u} \right)^2 = [1 + 2 C u (y, \Phi_{0d})]^2 = 1 - \frac{V}{V_0}, \tag{4}
\]

where the index \( j \) again indicates the particular electron charge group in question.

Thus it is clear that a knowledge of \((1 + 2 C u_j)\) at the output of a TWA, a klystron, or a BWO permits calculation of the plate circuit efficiency for a multi-segment collector system. It will be assumed that "m" electron charge groups are injected into the interaction region and that the beam transmission efficiency is 100 percent.

Electrons are injected at the input to the interaction region over one cycle of the r-f wave and hence it is convenient to define various characteristic energies per cycle of the beam. These are

- \( \varepsilon_{d-c} \) = d-c beam input energy to the interaction region,
- \( \varepsilon_i \) = r-f input energy to the circuit,
- \( \varepsilon_d \) = r-f energy dissipated on the circuit,
- \( \varepsilon_c \) = d-c energy given to the collector, and
- \( \varepsilon_o \) = r-f energy at the circuit output.

Also several characteristic efficiencies can be defined as follows:

- \( \eta_e \) = electronic conversion efficiency, and
- \( \eta_o \) = overall efficiency.

The above efficiencies may now be written in terms of the previously defined energies. Assume that "m" charge groups are injected into the interaction region. The electronic efficiency is then

\[
\eta_e = \frac{\varepsilon_o - \varepsilon_i + \varepsilon_d}{\varepsilon_{d-c}} = 1 - \frac{\varepsilon_c}{\varepsilon_{d-c}}, \tag{5}
\]

and the overall efficiency

\[
\eta_o = \frac{\varepsilon_o - \varepsilon_i}{\varepsilon_{d-c}}. \tag{6}
\]

The r-f energy given up by the beam may be related to the electron r-f potentials as follows.

\[
\varepsilon_o - \varepsilon_i + \varepsilon_d = \sum_{j=1}^{m} V_j, \tag{7}
\]

where generally (for high gain) \( \varepsilon_i << \varepsilon_o \).

In the case of the conventional TWA with the collector operated at the r-f structure potential the above indicated energies and efficiencies are

\[
\varepsilon_{0-c} = m V_o, \tag{8a}
\]

\[
\varepsilon_i = \sum_{j=1}^{m} (V_o - V_j), \tag{8b}
\]

\[
\sum_{j=1}^{m} (V_o - V_j) \]

\[
\eta_o = 1 - \frac{1}{m} \sum_{j=1}^{m} \left( 1 - \frac{V_j}{V_o} \right) = \frac{1}{m} \sum_{j=1}^{m} \left( \frac{V_j}{V_o} \right), \tag{8c}
\]

\[
\eta_o = \frac{\varepsilon_o}{m V_o}, \tag{8d}
\]

if \( \varepsilon_i << \varepsilon_o \).

The electronic efficiency given by Eq. 8c may also be written as

\[
\eta_e = \frac{1}{m} \sum_{j=1}^{m} \left( 1 - [1 + 2 C u (y, \Phi_{0d})]^2 \right). \tag{9}
\]

The efficiency expressions for multi-segment collector tubes may be developed systematically from the above relations if it is assumed that specific fractions of the output beam are collected at given potentials with one segment of the collector at the structure potential. Suppose now that the collector is segmented and that the segment operating d-c potentials are such that \( V_o > V_{c1} > V_{c2} > V_{c3} \ldots > V_{cm} \), that is, there are \( r + 1 \) segments. A schematic collector configuration illustrating the potential arrangement is shown in Figure 1. The energy given up to the individual collector segments is given by
The overall interaction efficiency for the \( r + 1 \) segment collector device is
\[
\eta_{r+r+1} = \frac{\varepsilon_a - \varepsilon_f}{\varepsilon_{d-c, r+1}},
\]
and thus
\[
\frac{\eta_{r+r+1}}{\eta} = \frac{\varepsilon_a - \varepsilon_f}{\varepsilon_{d-c, r+1}} = \frac{\eta_{0, r+1}}{\eta_0}.
\]

The electronic efficiency for the \( r + 1 \) segment collector device may be written in terms of the equivalent r-f potentials as
\[
\eta_{0, r+1} = \frac{\sum_{i=1}^{m} \left( \frac{V_i}{V_o} \right)}{P_1 \left( \frac{V_{c-r}^{r}}{V_o} \right) + P_2 \left( \frac{V_{c-1}^{r}}{V_o} \right) + \ldots + P_r}.
\]

Equation 18 may also be written in terms of particle velocities as
\[
\eta_{e, r+1} = \frac{\sum_{i=1}^{m} \left( 1 - [1 + 2Cu(y, \Theta_{c0})]^2 \right) \varepsilon_{d-c, r+1}}{P_1 \left( \frac{V_{c-r}^{r}}{V_o} \right) + P_2 \left( \frac{V_{c-1}^{r}}{V_o} \right) + \ldots + P_r}.
\]

In terms of the r-f energy delivered to the circuit the overall efficiency is written as
\[
\eta_{0, r+1} = \frac{2C \left( A_{\text{max}}^2 - A_o^2 \right) \left( 1 - \frac{d\theta(y)}{dy} \right)}{(1 + Cb) \left[ \frac{P_1 \left( \frac{V_{c-r}^{r}}{V_o} \right)}{m} + \frac{P_2 \left( \frac{V_{c-1}^{r}}{V_o} \right)}{m} + \ldots + \frac{P_r}{m} \right]}
\]

Equations 15 through 20 may now be used to calculate the "partial" and "overall" efficiencies for
depressed collector devices for an arbitrary number of collector segments but with the last segment at the r-f structure potential.

**GRAPHICAL EVALUATION SCHEME**

Recently H. Krage* suggested to the author that a graphical method could be utilized to determine the optimum collector segment voltages. The graphical method also uses the velocity characteristics of the spent beam and is useful to obtain approximate results and calibrate one’s intuition.

Figure 2 serves to illustrate the graphical procedure for a two-stage collector. The velocity squared, \((1 + 2Cu)^2\), is plotted vs. the “charge group” number.

The power conserved by using a two-stage collector is seen from the graph to be the sum of the areas of the two rectangles under the velocity curve. For any given case the segment voltages are chosen to maximize the total power conserved.

**RESULTS OF CALCULATIONS**

It is easily seen from Eqs. 17–20 that the efficiency enhancement achievable through depressed collector operation is directly related to the velocity spread in the spent beam. As pointed out previously, the greater the velocity spread in the output beam the greater the number of collector segments needed to achieve a given overall efficiency. The amount of velocity spread existing in the beam is directly dependent upon the electronic interaction efficiency.

The above theory may be used to calculate improvement factors for any operating conditions. The most significant information is to be gleaned from the calculations on devices operating at maximum gain conditions and at maximum saturation output (maximum efficiency). A comparison of these two sets of results is most interesting. The results have been calculated for various cases as taken from the efficiency curves computed with the aid of a one-dimensional nonlinear theory. These are shown in Figures 3 and 4 for 1-, 2-, and 3-stage collectors along with the efficiency for a device without collector depression. In Figure 3 it is apparent that the efficiency improvement factor decreases as \(\omega_n/\omega\) increases, as is to be expected since increased coulomb repulsion forces lead to greater velocity spread in the electron beam. The improvement factor is also seen to decrease, in Figure 4, with an increase in the gain parameter \(C\). This also is reasonable since an enhanced interaction achieved through tighter coupling between the beam and the circuit should naturally lead to greater velocity spreads.

Close scrutiny of these results indicates that in general if one operates at a condition for maximum small-signal gain to achieve linearity of operation at the expense of efficiency, the same “overall” efficiency may be achieved as in the maximum-power-output (saturation-efficiency) case by utilizing one additional collector segment. In a particular instance technological problems may define an upper limit on the number of collector segments permitted.

In order to achieve these maximum efficiency enhancement factors the various collector segments must be operated at the optimum potentials. Typical calculated efficiency improvement factors as a function of the various segment potentials are shown in Figures 5 and 6 for specific operating points.

Examination of the various cases indicates that the optimum collector segment voltages are approximately as given in the following table.

Figure 3. Efficiency with Collector Depression vs. QC

Figure 4. Efficiency with Collector Depression vs. C

Figure 5. Efficiency Improvement Factor vs. Collector Segment Potential. (C = 0.1, B = 1.0, QC = 0.125, b = 0.65, d = 0, y = 6.8)

Figure 6. Efficiency Improvement Factor vs. Collector Segment Potential. (C = 0.1, B = 1.0, QC = 0.125, b = 1.5, d = 0, y = 7.2)

TABLE 1

<table>
<thead>
<tr>
<th>No. of Stages</th>
<th>V_0/V_0</th>
<th>V_0/V_0</th>
<th>V_0/V_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>
The theoretical development and calculations are independent of $V_0$ and thus apply to relativistic beams. Specific calculations have been made for relativistic beams as a function of the relativity factor and large efficiency improvement factors were found.

**SEGMENTED COLLECTORS IN PHASE-FOCUSED TUBES**

Recent theoretical studies with supporting experiments have indicated that the efficiency of linear beam devices can be improved by utilizing phase-focusing techniques. One wonders whether further improvement can be achieved in such tubes by collector segmentation and depression. Since tubes with high interaction efficiencies are characterized by a high degree of velocity spread in the beam, it would be expected that the efficiency improvement factor through the use of a depressed collector would be less than that in the case of lower-efficiency tubes. Calculations similar to those presented early in this report have been made on a variable-pitch tube and its uniform-pitch equivalent and are presented in Figure 7.

The efficiency is seen to be improved by a factor of 1.36 as a result of phase focusing through pitch variation of the helix and the efficiency of the variable-pitch tube is seen to be always greater than its uniform-pitch counterpart, independent of the degree of collector subdivision.

The use of a 3-stage depressed collector on the uniform-pitch tube gives an efficiency improvement factor of 2.2, whereas the improvement factor for a 3-stage collector on the variable-pitch tube is only 1.7. However, the overall efficiency is higher by a factor of 1.05.

**DEPRESSED COLLECTORS ON BWO'S**

All of the previous work has been directed towards efficiency improvement in forward-wave amplifiers. Since the interaction efficiency of the O-type BWO is so low it is a likely candidate for depressed collector operation.

The general theory was used to calculate efficiency improvement factors for a large-signal BWO which has an electronic interaction efficiency of 9 percent. The overall efficiency vs. the degree of collector segmentation is shown in Figure 8. The very large im-

![Figure 7. Efficiency Improvement for Variable Phase Velocity Tubes. ($C = 0.1, d = 0, QC = 0$)](image1)

![Figure 8. Efficiency Improvement by Collector Depression in a BWO](image2)
Improvement is due to the fact that the conversion efficiency is low and hence the velocity spread in the beam is low.

**EXPERIMENTAL RESULTS**

A simple one segment depressed collector was constructed for a low-gain power amplifier operating in the S-band frequency regime. For this particular tube the limit of collector depression was 35 percent of the helix voltage at full drive power. The improvement in efficiency obtained is illustrated in Table II.

**TABLE II**

<table>
<thead>
<tr>
<th>Electronic Efficiency</th>
<th>Overall Efficiency with 35 Percent Depression</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>23.3</td>
</tr>
<tr>
<td>18</td>
<td>27.8</td>
</tr>
<tr>
<td>21</td>
<td>32.4</td>
</tr>
<tr>
<td>24</td>
<td>37.1</td>
</tr>
<tr>
<td>27</td>
<td>41.5</td>
</tr>
</tbody>
</table>

This TWA is characterized by \( C \approx 0.1 \) and \( QC \approx 0.15 \). The theoretical improvement factor for collector depression when it is operated at maximum power output is 1.53, which compares favorably with the experimental improvement factor of 1.54.

A high-power helix-type traveling-wave amplifier was constructed with a variable phase velocity structure in which the helix pitch was varied approximately cubically over the last 15 percent of its length. The phase velocity was reduced to 50 percent of its initial value by this means. Recent experimental data taken on this amplifier are shown in Figure 9 for a wide range of frequencies. The efficiency improvement factor due to phase focusing varies between 1.3 and 2.0 depending upon operating conditions. The greatest significance of these results is the fact that efficiency improvement has been achieved over an extremely wide bandwidth and that efficiencies of 5 times \( C \) have been obtained. The collector on this tube was operated at the circuit potential. If single-stage collector depression is used the improvement factor is estimated to be approximately 1.25.

In many cases TWA's are operated with power outputs below the saturation output in order to obtain linearity in the phase shift characteristics. Calculations have indicated that much of the effi-

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![Figure 9](image-url)  
*Figure 9. Maximum Efficiency vs. Frequency Before and After Velocity Tapering. (a) Variable Pitch: Voltage and Attenuator Position Optimized for Maximum Power Output. (b) Uniform Pitch: Voltage and Attenuator Position Optimized for Maximum Power Output.*
ciency can be recovered by utilizing a multi-segment depressed collector. However, one usually has to use an additional collector segment to obtain the same overall efficiency.

**CONCLUSIONS**

The nonlinear calculations of the state of the spent beam in a TWA and BWO have been used in collector depression studies. A theory has been developed to calculate the efficiency improvement factors for collector depression from the velocity spread data on the spent beam. In addition the optimum collector segment potentials are obtained from the calculations. Efficiency improvement factors of 1.5 have been predicted and these have been substantiated by experiment.

The degree of improvement obtainable is directly a function of the amount of velocity spread in the beam. However, it is shown that a small improvement can still be achieved by collector depression even in phase-focused amplifiers.
LIST OF REFERENCES


A theoretical study of the efficiency enhancement possible through collector segmentation and depression is made using information obtained from the nonlinear interaction theory for traveling-wave-like devices. Specific calculations are made for both maximum gain and maximum power output operating conditions. Also efficiency improvement through phase focusing is discussed. Improvement factors of approximately 1.5 to 2 are predicted and verified experimentally. Results are also given for the backward-wave oscillator.

1. Theoretical Development
2. Segmented Collectors in Phase-Focused Tubes
3. Depressed Collectors on BWO's
4. Experimental Results
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