NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
A communications system (see Fig. 1) consisting of active elements $T_i$ (converters, amplifiers) and passive elements $\mathcal{M}_i$ (transmission lines and paths, interstage networks, filter networks, etc.) is considered. The transfer function of the system is $\gamma$ and its overall insertion loss due to the passive elements is:

$$G = \prod_{i=1}^{m+1} s_i$$

where $s_i$ is the power-insertion loss introduced by a passive element $\mathcal{M}_i$ and $m$ is the number of elements; $K_i \geq 1$ is the power-amplification of the $i$-th active element. Under the assumption that the noise introduced by each passive element is $\sigma_i^2$ and that of each active element referred to its input is $\sigma_i^2$, the noise power at the output of the system can be written as:

$$P_o = (P_i + \sigma^2) \frac{K_1 K_2 \cdots K_m}{\epsilon_1 \epsilon_2 \cdots \epsilon_{m+1}} + (\sigma_1^2 + \sigma_2^2) \frac{K_1 K_2 \cdots K_m}{\epsilon_1 \epsilon_2 \cdots \epsilon_{m+1}} + \frac{K_1 K_2 \cdots K_m}{\epsilon_1 \epsilon_2 \cdots \epsilon_{m+1}} + \frac{K_1 K_2 \cdots K_m}{\epsilon_1 \epsilon_2 \cdots \epsilon_{m+1}} + \frac{K_1 K_2 \cdots K_m}{\epsilon_1 \epsilon_2 \cdots \epsilon_{m+1}} + \sigma^2$$

where $P_i$ is the noise power at the input of the system. It is seen from Eq. (4) that for $K_i = 1$ (where $i = 1, 2, \ldots, m$) the noise figure due to the design parameters is a minimum. The system of Fig. 1a can be simply represented by that of Fig. 1c under these conditions. The expression for the output noise is differentiated with respect to $K_i$ and $m$ and it is found that
Dependence of the noise figure can be expressed as:

\[ N_{\text{min}} = 1 + \frac{1}{P_a} \left| e^{\frac{(\sigma_n^2 + \sigma_T^2)}{P_a}} \ln \left( \frac{G}{K_{\text{max}}} \right) - \frac{\sigma_n^2}{\gamma} + \sigma_T^2 \right| \]  

where \( K_{\text{max}} = \frac{P_a}{(P_a + \sigma_T^2)} \), where \( P_a \) is the permissible output power. For the case when \( P_a = \sigma_T \), \( K_{\text{max}} = 1 \) and \( \gamma = 1 \), it is found that the minimum noise figure is achieved when the transmission system is divided into equal sections in which the insertion loss of the passive elements and the gain of the active elements are equal to \( e = 2.71 \); the number of active plus passive pairs is \( m = \ln G \). The noise figure for \( \gamma = 1 \) and \( K_{\text{max}} = 1 \) for any \( m \) is given by:

\[ N = 1 + \frac{1}{P_a} \left( \frac{P_a = \sigma_T}{\gamma = 1} \right) (\sigma_n^2 + \sigma_T^2)^{m+1/m} \]  

This expression permits evaluation of the deviation of the noise figure from the optimum value. There are 6 figures.

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