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SOME ENTRAINMENT PROPERTIES OF A TURBULENT
AXI-SYMMETRIC JET

By
George Naim Paris

Research Report No. 39

January 15, 1963



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**SOME ENTRAINMENT PROPERTIES OF A TURBULENT
AXI-SYMMETRIC JET**

**By
George Naim Faris**

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The Aerophysics Department
Mississippi State University**

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LIST OF SYMBOLS

D	Nozzle diameter, ft
K	Kinematic momentum, ft^4/sec^2
M	Momentum flux, lbs
n	Exponent related to downstream distance, $1 \leq n \leq 2$
Q	Quantity of flow, ft^3/sec
r	Distance measured in radial direction from jet centerline, ft
R	Distance measured in radial direction from jet centerline to the edge of the jet, ft
u	Velocity within the turbulent area in the x-direction, ft/sec
U	Maximum jet velocity, ft/sec
w	Cole's wake function
ω	Distance in radial direction from core edge, ft
W	Distance in radial direction from core edge to edge of jet, ft
x	Distance in downstream direction, ft
y	Distance measured perpendicular to surface, ft
β	Wake function
δ	Distance in y-direction to edge of boundary layer, ft
ρ	Density of air, $\text{lb-sec}^2/\text{ft}^4$
θ	Angle, degrees

Subscripts

c	Denotes core
e	Denotes exit of nozzle ($x = 0$)
o	Denotes centerline of jet
T	Denotes total

INTRODUCTION

The jet mixing phenomena is frequently encountered in many engineering problems such as jet pumps, ejectors, and combustion chambers. The turbulent entrainment process of the jet forms the basis for many of its applications. For example, the basis of a jet pump, or an ejector, lies in the property of a moving jet of primary fluid to entrain a secondary fluid and move it downstream. Jet pumps, a primary example of turbulent mixing, have been investigated as a power source for boundary layer control studies in aircraft (Ref. 1). Even with the low efficiency usually associated with jet pumps, their use is found profitable because of their small size, simplicity, and light weight. Ejectors, utilizing the same entrainment principle, have found application in aircraft and ground effect machines for thrust augmentation purposes (Ref. 2). For these and other reasons the diffusion and entrainment properties of free jets have been the subject of extensive experimental and theoretical analysis.

Turbulent jet mixing of incompressible fluids was theoretically analyzed by Tollmien (Ref. 3) in 1926 by the use of Prandtl's mixing length theory. In 1935 Kuethe (Ref. 4) developed an approximate method for computing the profiles near the exit of a round jet discharging air into a medium at rest. From experimental results, Reichardt (Ref. 5), in 1941, introduced the constant exchange coefficient concept over the mixing zone. In 1947 Liepmann and Laufer (Ref. 6), using hot wire anemometer techniques for measuring turbulent fluctuations, showed that neither the mixing length nor the exchange coefficient is constant across the mixing region. Albertson, et al (Ref. 7), in 1950, by presuming that the viscous effects have no influence on the mixing region concluded that the diffusion characteristics

of the flow are dynamically similar under all conditions. Consequently, they used the Gaussian normal probability function to describe the velocity profiles throughout the mixing region.

In the present investigation, experiments were performed to verify the extension of Cole's Wake Law for two-dimensional boundary layers (Ref. 8) to the case of axi-symmetric free jet flows.

APPARATUS AND METHOD

The apparatus used in this investigation is shown schematically in Figure (1).

The primary air flow was furnished by a two-horsepower portable electric blower. A settling chamber, in which aluminum honeycomb was placed, was connected to the blower and the fluid was finally released into the atmosphere through a cylindrical nozzle one inch in diameter. A powerstat, together with a constant voltage transformer, was utilized to regulate the speed of the jet. The exit velocity of the fluid was maintained constant by preserving a constant differential static pressure between two pressure taps located on the circumferences of the settling chamber and the cylindrical nozzle respectively. A Kollsman helicopter airspeed indicator was used to measure velocities within the jet by means of static and total head probes mounted on a three-directional rotary table. The brass probes were connected to the airspeed indicator by plastic tubing as shown in Figure (2). In addition, an aural stethoscope was used to differentiate between laminar and turbulent flow regions.

The issuing jet was checked for symmetry by measuring the velocity profiles across the jet, Figure (3). Measurements of the profiles were taken at a differential static pressure equal to six inches of water which corresponded to a centerline velocity of 176 ft/sec at the nozzle exit. Velocities less than 16 ft/sec were not recorded due to possible inaccuracies in the airspeed indicator. From the measured velocity profiles the values for the total volume rate of flow and the total kinematic momentum were graphically determined.

DISCUSSION

The Structure of the Jet

The primary flow, except in the turbulent boundary layer at the walls of the nozzle, emerges as a laminar jet with uniform velocity which, beginning at the rim of the nozzle, becomes increasingly turbulent downstream as the mixing region of primary and entrained flow spreads inward. This turbulent region, surrounding the laminar core, continues to expand until it merges at the jet centerline at some distance downstream. In the present investigation this distance was 4.3 nozzle diameters, which compares favorably with Keuthe's 4.44 nozzle diameters (Ref. 4), and Davies' et. al. 4.35 nozzle diameters (Ref. 9). In addition to spreading inward, the turbulent region spreads outward as shown in Figure (4). The surrounding fluid is accelerated and the primary flow is decelerated although the total kinematic momentum of primary and entrained fluid remains constant, Figure (5). The total volume rate of flow, however, increases in the downstream direction (Ref. 10).

A New Jet Profile Description

In order to obtain an accurate description of this entrainment process, and extension has been made of an existing entrainment theory now available in the study of turbulent boundary layers.

Immediately at the exit of the nozzle, the turbulent portion of the flow can be approximately described by Cole's two-dimensional Wake Law. Cole's wake function, $w(\frac{y}{s})$, can be approximated by the form suggested by Newman (Ref. 11) as:

$$w(\frac{y}{s}) = 1 + \sin \pi (\frac{y}{s} - \frac{1}{2}) \quad (1)$$

which, for two-dimensional wake profiles, may also be written as:

$$\frac{u}{U_0} = \frac{1}{2} \left[1 - \cos \frac{\pi Y}{S} \right] \quad (2)$$

or
$$\frac{u}{U_0} = 1 + \beta \quad (3)$$

where
$$\beta = -\frac{1}{2} \left[1 + \cos \frac{\pi Y}{S} \right]$$

It has been recently suggested by Cornish (Ref. 12) that for axis-symmetric jet flows, β becomes a function of $(1 - \frac{\omega}{W})^2$ where $\frac{\omega}{W} = 1 - \frac{Y}{S}$. With this transformation, Cole's two-dimensional Wake Law is extended to describe velocity profiles for jet flows when written in the following form:

$$\frac{u}{U_0} = \frac{1}{2} \left[1 - \cos \pi \left(1 - \frac{\omega}{W} \right)^2 \right] \quad (4)$$

Neither Cole's two-dimensional Wake Law nor Cornish's axis-symmetric form can describe the profiles in the core region since the flow is neither purely two-dimensional nor fully developed jet flow, Figure (6). Therefore, the following form has been suggested by Cornish to describe these intermediate profiles:

$$\frac{u}{U_0} = \frac{1}{2} \left[1 - \cos \pi \left(1 - \frac{\omega}{W} \right)^n \right]; \quad 1 \leq n \leq 2 \quad (5)$$

This relation becomes Cole's two-dimensional Wake Law when $n = 1$ and will describe the fully developed jet profiles when $n = 2$. Profiles within the core region are described with intermediate values of n . Because of the experimental difficulty in measuring velocities near the periphery of

the jet, equation (5) is written in the following form:

$$\frac{u}{U_0} = \frac{1}{2} \left[1 - \cos \pi \left\{ 1 - \left(1 - (.5)^{1/n} \right) \frac{\omega}{\omega_{u/U_0} = .5} \right\}^n \right]; \quad 1 \leq n \leq 2 \quad (6)$$

For $n = 1$, equation (6) will reduce to Cole's two-dimensional Wake Law; however, as n varies between 1 and 2, equation (6) describes the flow within the core region. Velocity profiles have been measured experimentally within the core region and have been compared to Cornish's theory in Figure (7). The value of n for each profile varies with distance downstream of the nozzle. This variation of n as determined experimentally is shown in Figure (8).

Beyond the core region, where profile similarity is preserved, equation (6) will describe the fully developed jet profiles when $n = 2$ as shown in Figure (9). Equation (6) with $n = 2$ has been compared with profiles developed by Schlichting and Tollmien in Figure (10).

Summation of Profile Behavior

From the preceding analysis the behavior of the profiles can be deduced. In the mixing region immediately downstream of the nozzle, the turbulent portions of the velocity profiles are nearly two-dimensional. As the distance from the nozzle is increased, these profiles gradually change until at the end of the laminar core the turbulent profiles become fully developed axi-symmetric jet flow.

CALCULATION OF ENTRAINMENT PROPERTIES

The Quantity of the Flow

At any distance, x , downstream the total volume rate of flow can be expressed as:

$$Q_T = \int_0^{2\pi} \int_0^R u r dr d\theta \quad (7)$$

The boundary conditions are:

$$\text{Core Region; } 0 \leq r \leq r_c; \quad u = U_0 = U_e = \text{const.} \quad (8)$$

$$\text{Wake Region; } r_c \leq r \leq R; \quad U_e \geq u \geq 0 \quad (9)$$

$$R - r_c = W \quad \text{and} \quad r - r_c = w$$

Substituting equation (5) in equation (7) and integrating, using the series expansion for the cosine function, the following general equation for the volume rate of flow is obtained:

$$Q_T = \pi r_c^2 U_e + \pi W^2 U_0 \left[\frac{1}{2} - \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma \pi^{2\gamma}}{(2\gamma)!(2n\gamma+1)} \right. \\ \left. + \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma \pi^{2\gamma}}{(2\gamma)!(2n\gamma+2)} \right] + \pi r_c W U_0 \left[1 - \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma \pi^{2\gamma}}{(2r)!(2n\gamma+1)} \right]$$

Equation (10) is compared with the data in Figure (11). In the profile similarity region where $r_c = 0$ and $n = 2$, equation (10) will reduce to

$$Q_T = 0.394 U_0 R^2 \quad (11)$$

The Momentum Flux

The momentum flux at any distance, X , downstream may be expressed as:

$$M_T = \int_0^{2\pi} \int_0^R \rho u^2 r dr d\theta \quad (12)$$

For an incompressible flow, ρ is constant, and the kinematic momentum, K , is:

$$K_T = \frac{M_T}{\rho} \quad (13)$$

Thus:

$$K_T = \int_0^{2\pi} \int_0^R u^2 r dr d\theta \quad (14)$$

Substituting equation (5) in equation (14) and integrating, using the series expansion for the cosine function, the following general equation for the kinematic momentum is obtained:

$$\begin{aligned} K_T = & \pi U_0^2 r_c^2 + \frac{1}{2} \pi U_0^2 W^2 \left[\frac{3}{4} - 2 \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma \pi^{2\gamma}}{(2\gamma)!(2n\gamma+1)} \right. \\ & + \frac{1}{2} \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma (2\pi)^{2\gamma}}{(2\gamma)!(2n\gamma+1)} - \frac{1}{2} \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma (2\pi)^{2\gamma}}{(2\gamma)!(2n\gamma+2)} + 2 \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma (\pi)^{2\gamma}}{(2\gamma)!(2n\gamma+2)} \left. \right] \\ & + \frac{1}{2} \pi U_0^2 W r_c \left[\frac{3}{2} - 2 \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma \pi^{2\gamma}}{(2\gamma)!(2n\gamma+1)} + \frac{1}{2} \sum_{\gamma=0}^{\infty} \frac{(-1)^\gamma (2\pi)^{2\gamma}}{(2\gamma)!(2n\gamma+1)} \right] \quad (15) \end{aligned}$$

In the profile similarity region where $r_c = 0$ and $n = 2$, equation (15) will reduce to:

$$K_T = 0.193 U_0^2 R^2 \quad (16)$$

Thus:

$$U_0 = \frac{2.276}{R} \sqrt{K_T} \quad (17)$$

In the present investigation the radius of the jet increased linearly beyond the core region as shown in Figure (12). This linear variation can be described as:

$$R = .0558 + .29X; \quad X \geq X_c \quad (18)$$

Substitution of equations (18) and (17) in equation (11) yields:

$$Q_T = (.05 + .26X) \sqrt{K_T}; \quad X \geq X_c \quad (19)$$

Similarly, the substitution of equation (18) in equation (17) yields the following relation for the centerline velocity

$$U_0 = \frac{2.276 \sqrt{K_T}}{.0558 + .29X}; \quad X \geq X_c \quad (20)$$

Using the experimental value for the kinematic momentum, $K_T = 188 \frac{Ft^4}{Sec^2}$, equations (19) and (20) will reduce respectively to:

$$Q_T = 0.685 + 3.5X; \quad X \geq X_c \quad (21)$$

and

$$U_0 = \frac{31.2}{.0558 + .29X} ; \quad x \geq x_c \quad (22)$$

Equations (21) and (22) are compared with measured values for the volume rate of flow and the centerline velocity in Figures (13) and (14) respectively.

In Figure (15) these relations are compared with measured data for the velocity profiles, jet radius, and core radius.

CONCLUSION

The turbulent portions of the velocity profiles of a subsonic axisymmetric free jet discharging fluid into the atmosphere can be described at any distance downstream by Cornish's extension of Cole's two-dimensional wake law.

$$\frac{u}{U_0} = \frac{1}{2} \left[1 - \cos \pi \left(1 - \frac{w}{W} \right)^n \right] \quad 1 \leq n \leq 2$$

Beyond the core region, the profiles can be described by the following special form of the general relation:

$$\frac{u}{U_0} = \frac{1}{2} \left[1 - \cos \pi \left(1 - \frac{r}{R} \right)^2 \right]$$

The total volume rate of flow beyond the core region can be derived from the profile description and expressed as:

$$Q_T = 0.394 U_0 R^2$$

Also, the total kinematic momentum downstream from the core region can be expressed as:

$$K_T = 0.193 U_0^2 R^2$$

The preceding relations are applicable to the general case of axisymmetric jets. To fully determine the characteristics of any specific jet, a further relation between Q_T , U_0 , or R and X for that jet is required.

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REFERENCES

1. Wagner, F. G. Proposal for a Program in Jet Pump Research for Application to BLC System Design. (Convair-San Diego, Report ZA-263), 1957.
2. Johnson, J. K., Jr., Shumpert, P. K. and Sutton, J. F. Steady Flow Ejector Research Program. (Lockheed-Georgia Company, Report ER-5332), 1961.
3. Tollmien, W. Calculation of Turbulent Expansion Processes. (N.A.C.A. TM 1085), (Sept. 1945), Cited in Pai, S.I., Fluid Dynamics of Jets (New York, D. Van Nostrand, 1954), P. 97.
4. Keuthe, A. M. Investigations of the Turbulent Mixing Regions Formed by Jets. (Journal of Applied Mechanics, Vol. 2, No. 3), (Sept. 1935).
5. Reichardt, H. Über eine neue Theorie der freien Turbulenz. (ZAMM Bd. 21, Nr. 5), (Oct. 1941), Cited in Pai, S.I., Fluid Dynamics of Jets, P. 97.
6. Liepmann, H. W., and Laufer, J. Investigations of Free Turbulent Mixing. (N.A.C.A. TM1257), (Aug. 1947).
7. Albertson, M. L., Dai, Y. B., Jensen, R. A., and Rouse, H. Diffusion of Submerged Jets. (Transactions of the American Society of Civil Engineers, Vol. 115), 1950, pp. 639-697.
8. Coles, D. The Law of the Wake in the Turbulent Boundary Layers. (Journal of Fluid Mechanics, Vol. 1), (London, 1956).
9. Davies, P.O.A.L., Barratt, M. J., and Fisher, M. J. Turbulence in the Mixing Region of a Round Jet. (Aeronautical Research Council, A.R.C. 23728), (April 1962).
10. Schlichting, H. Boundary Layer Theory. New York, McGraw-Hill, 1955. 1st ed.
11. Newman, B. G. Personal Communication, (1956), Cited in Cornish, J. J., III, A Universal Description of Turbulent Boundary Layer Profiles with or without Transpiration. (Mississippi State University, Aerophysics Department, Research Report No. 29), (June 1960), P. 25.
12. Cornish, J. J., III. Unpublished Report, Aerophysics Department, Mississippi State University, (1962).

Figure 1

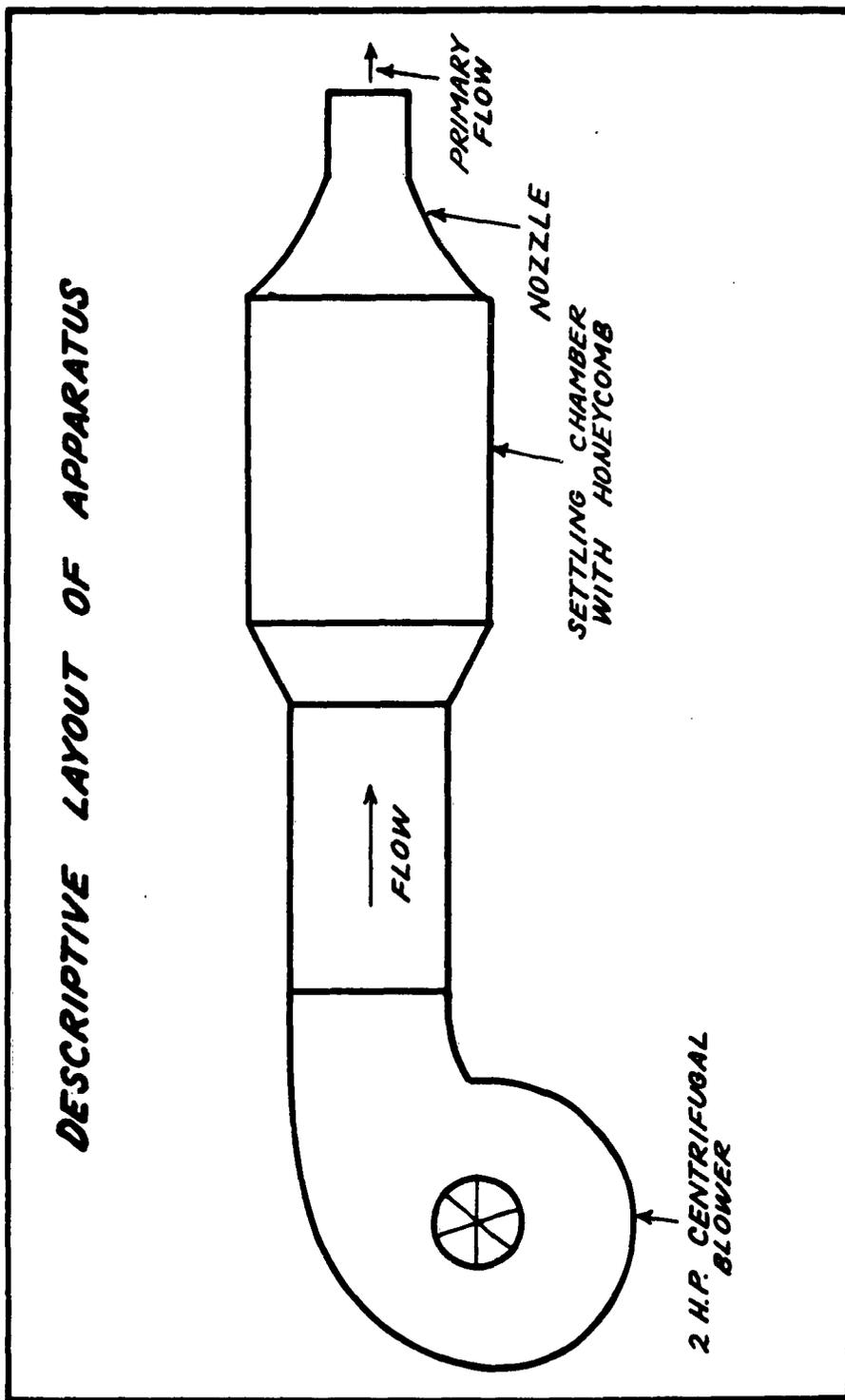
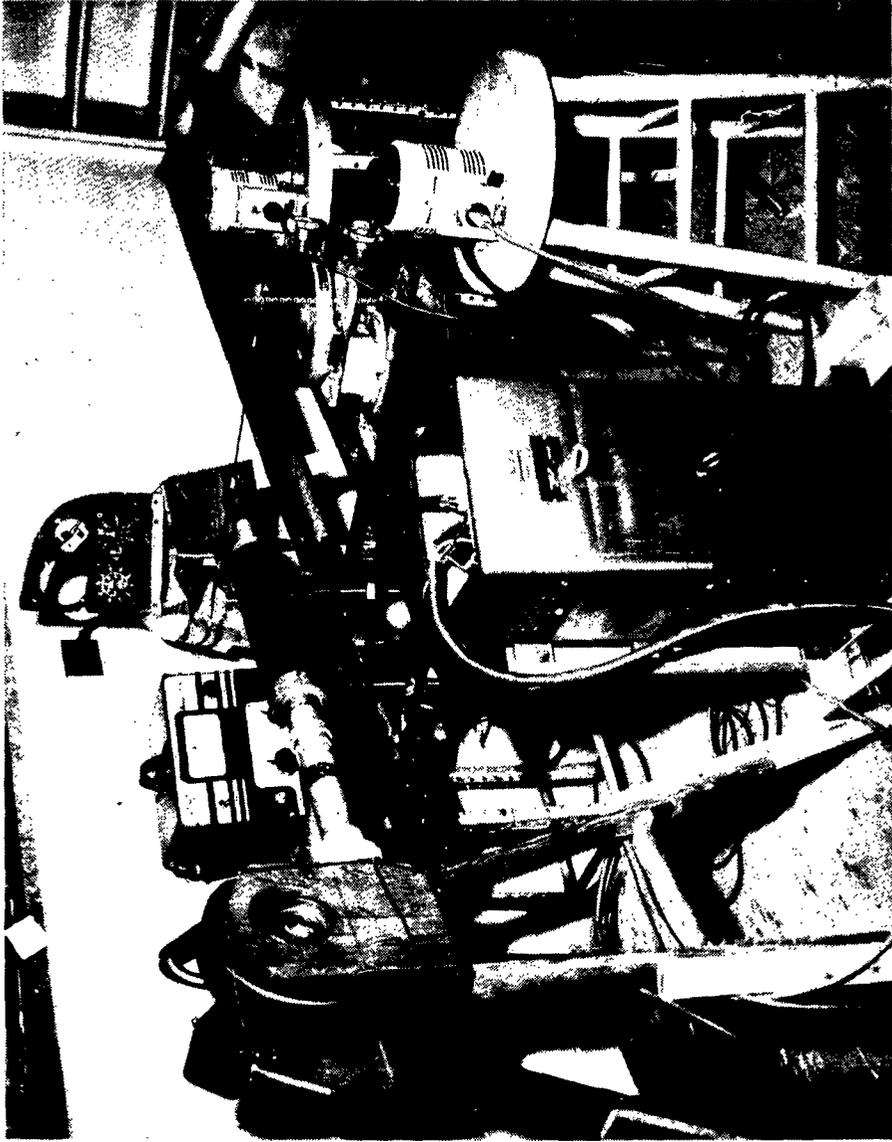


Figure 2



Experimental Apparatus

Figure 3

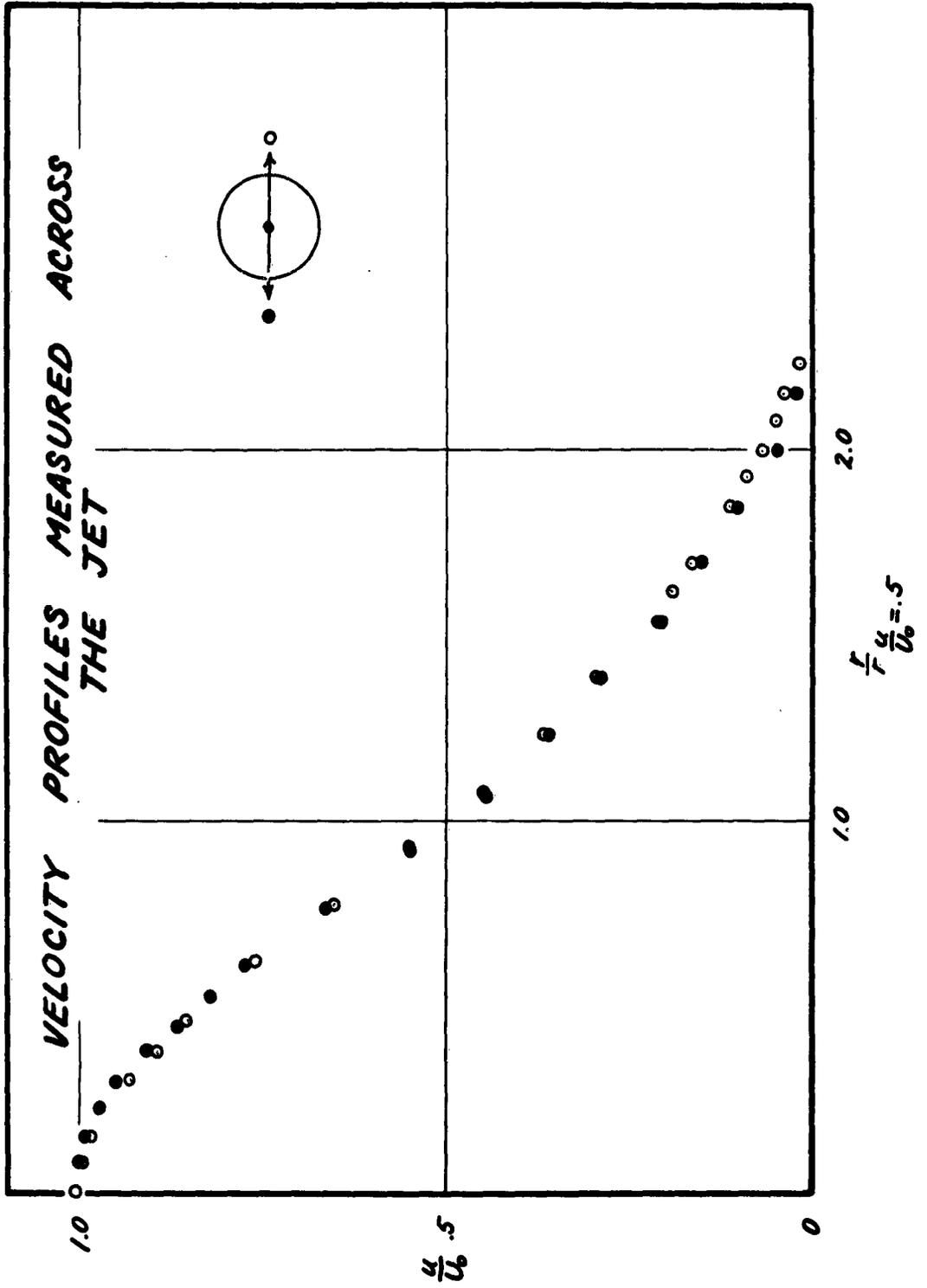


Figure 4

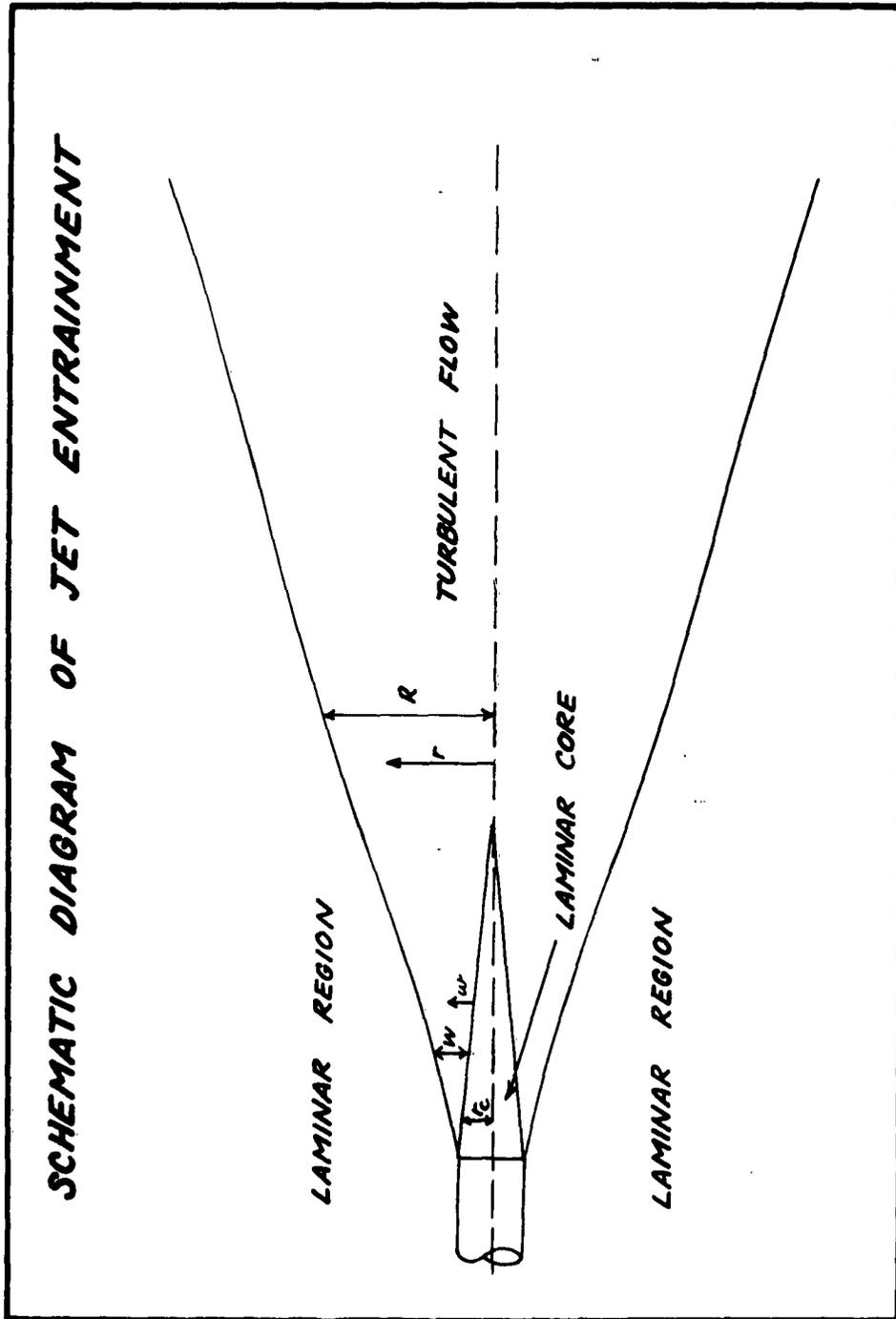


Figure 5

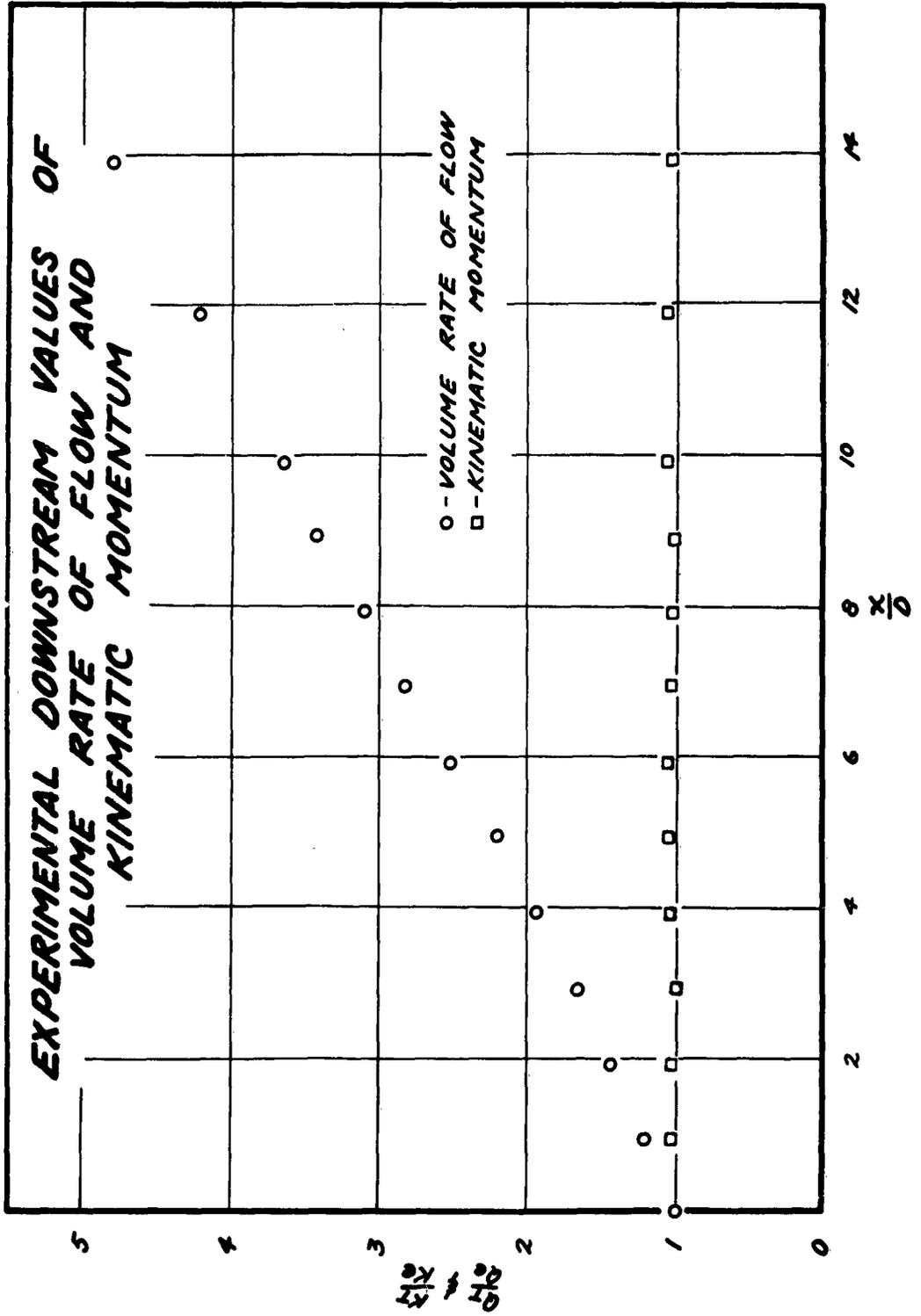


Figure 6

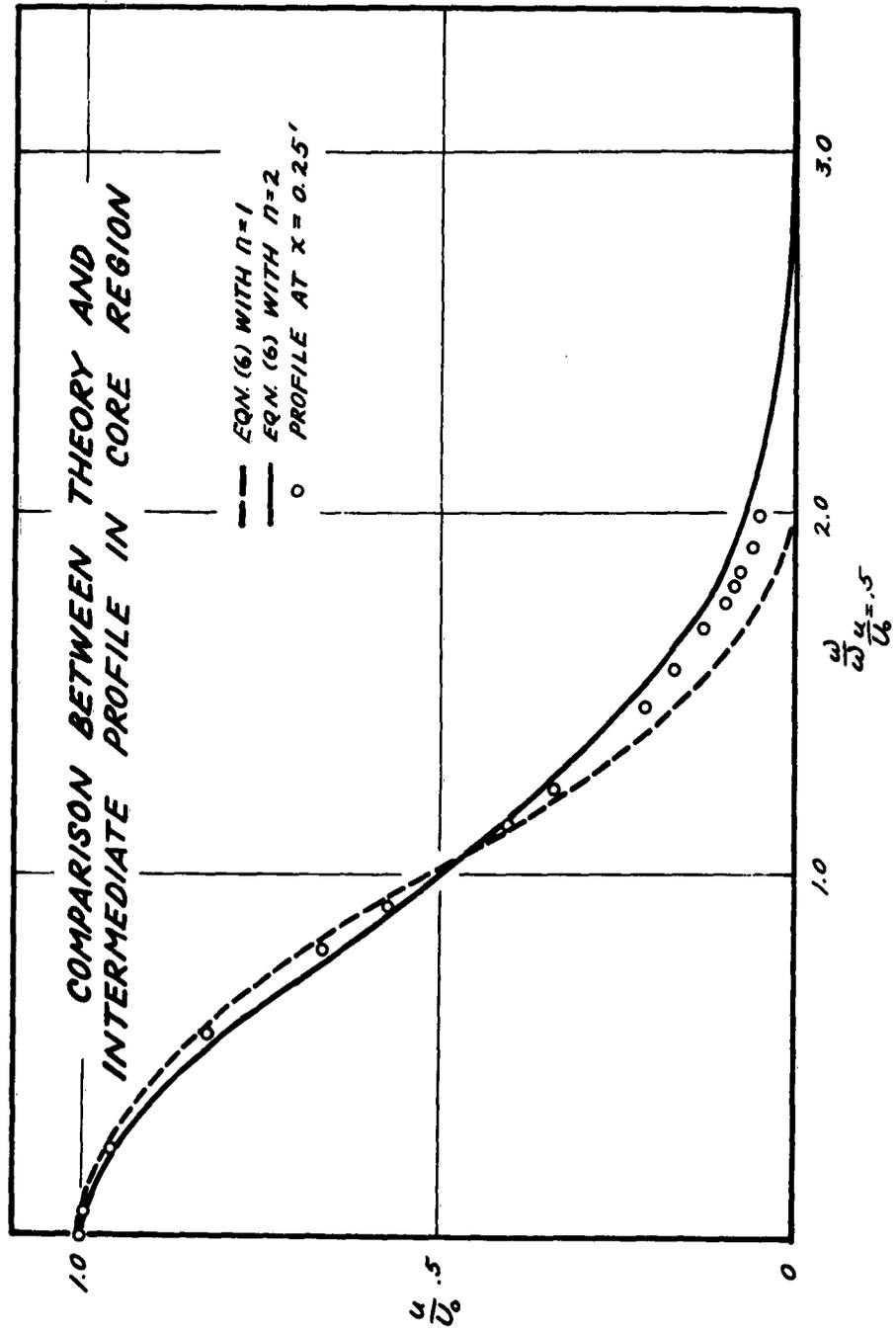


Figure 7

A COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL PROFILES IN CORE REGION

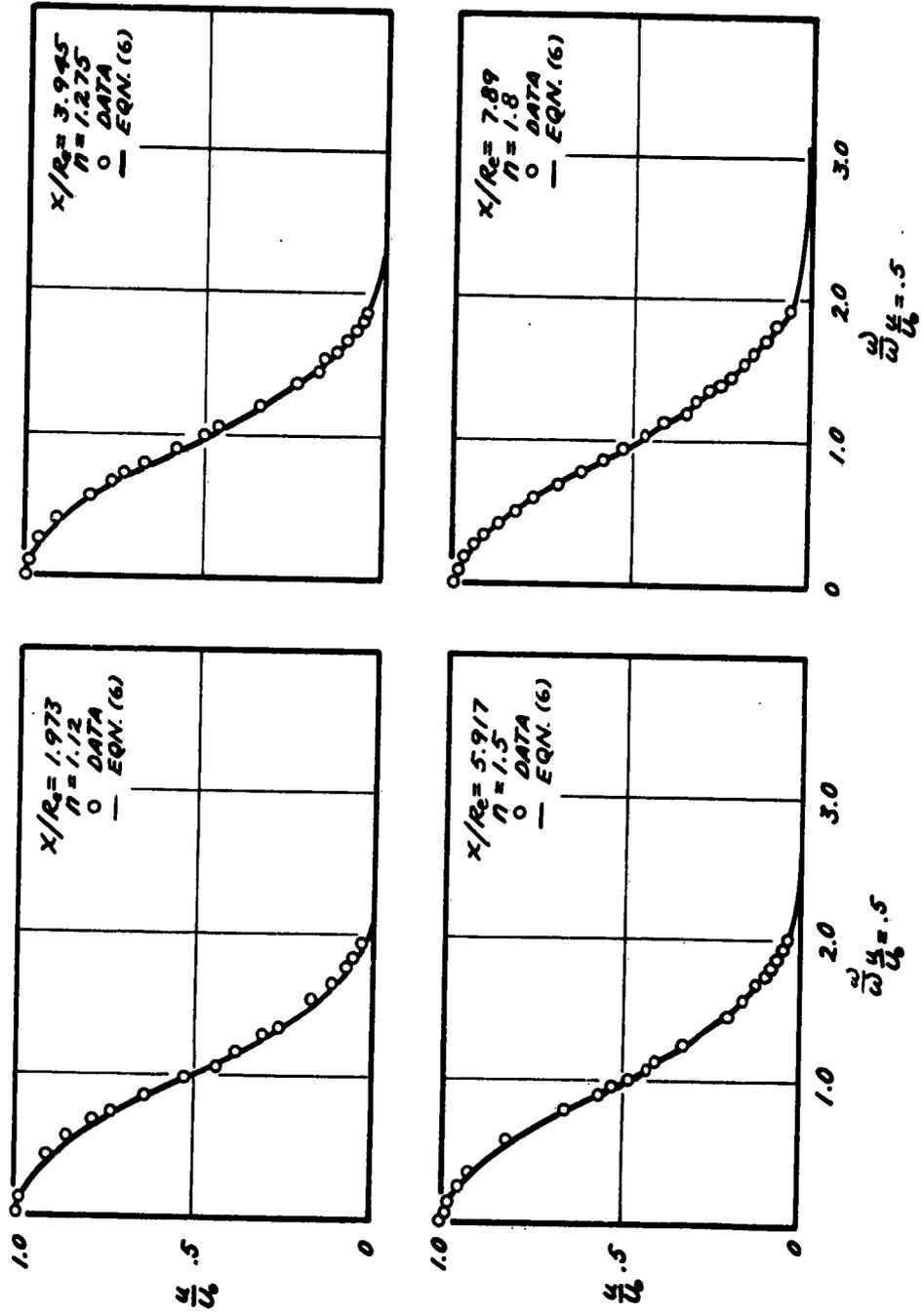


Figure 8

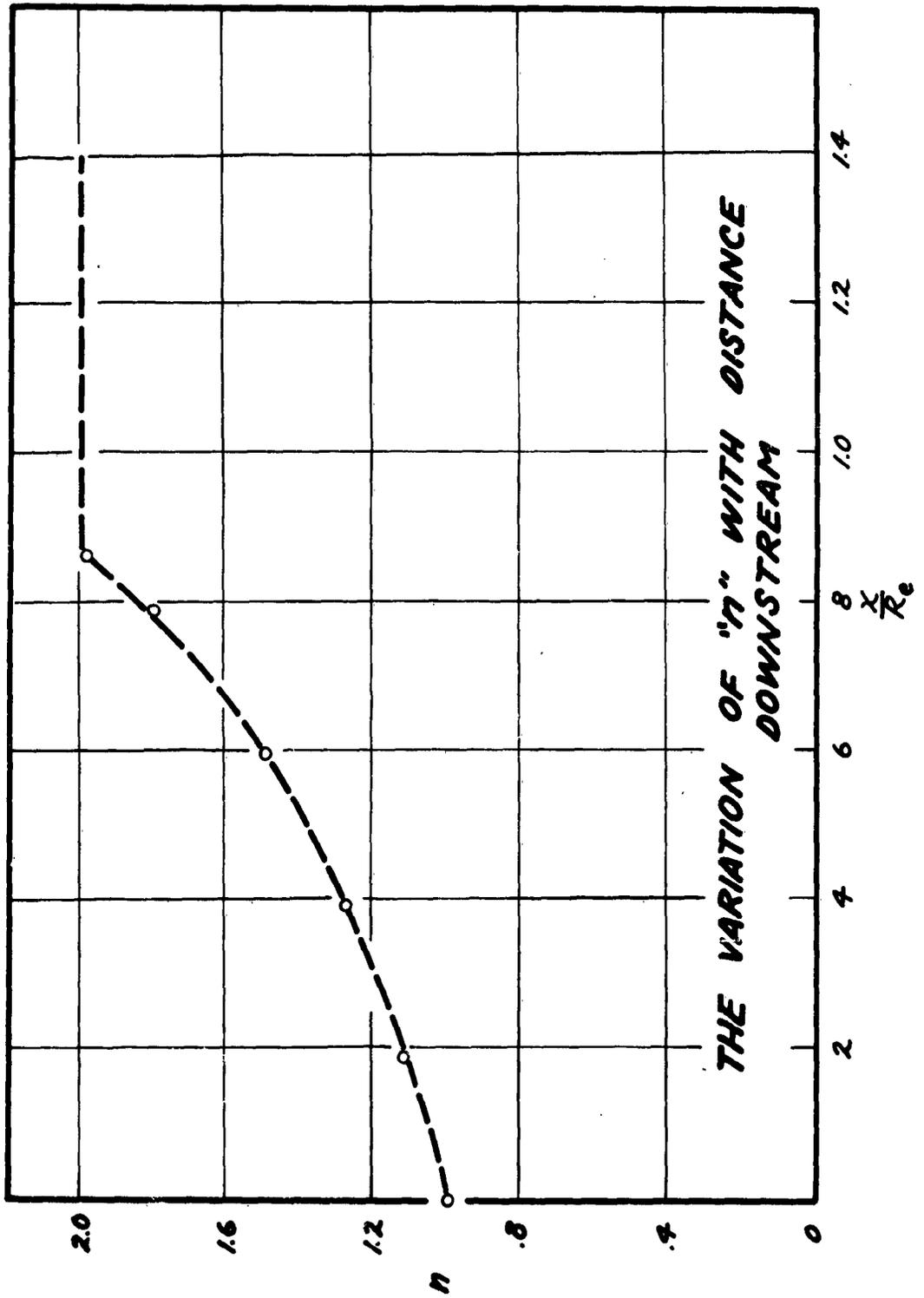


Figure 9

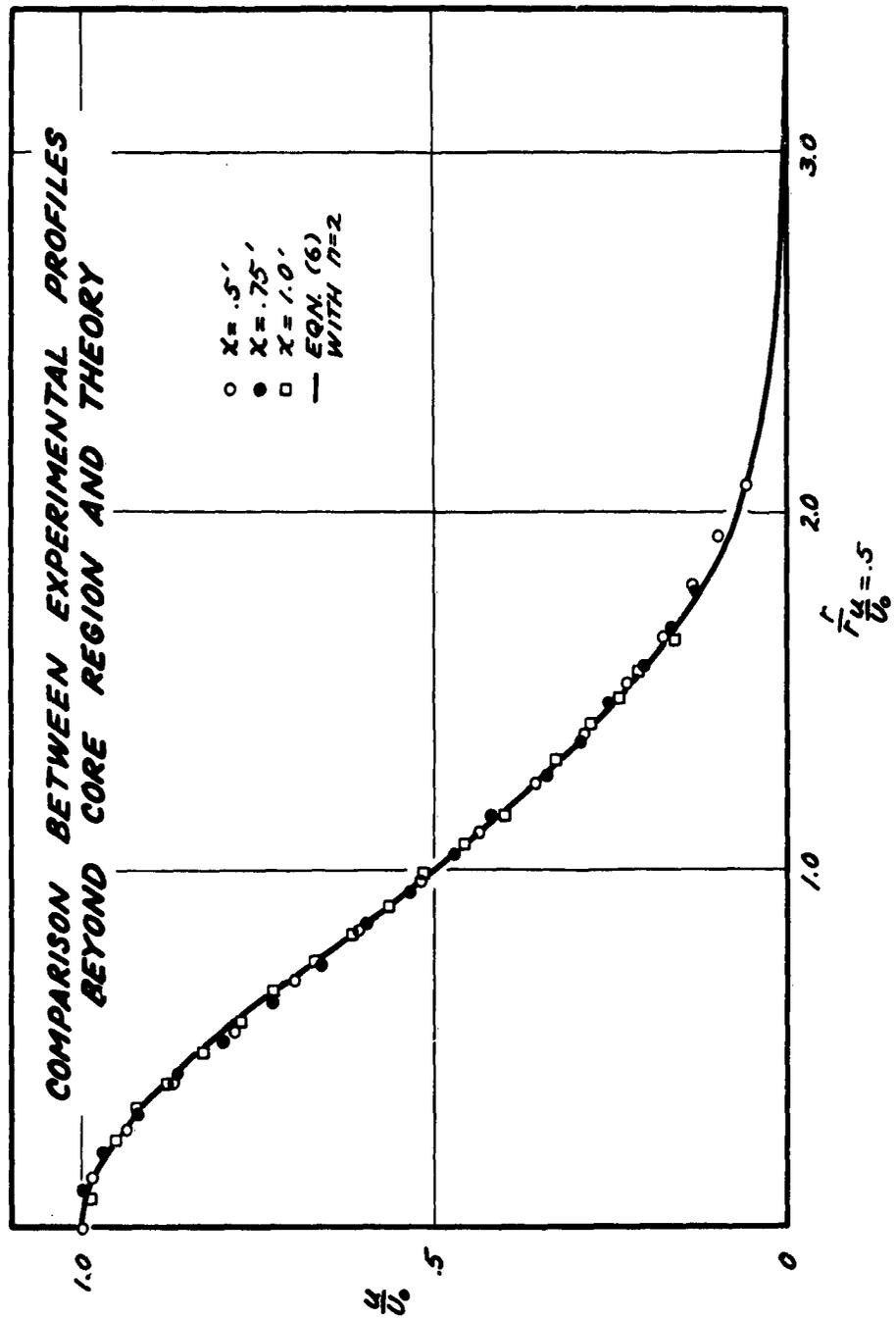


Figure 10

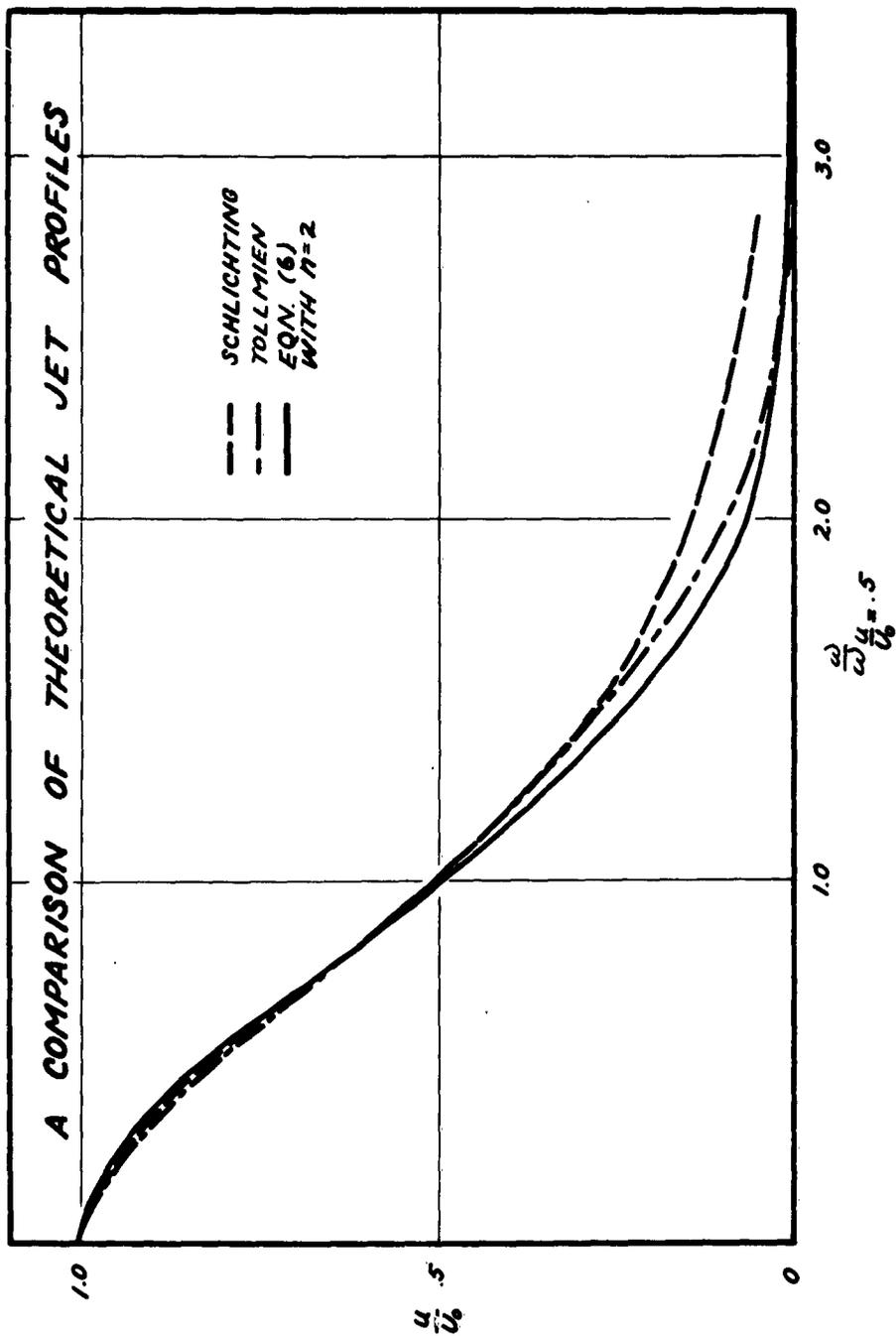


Figure 11

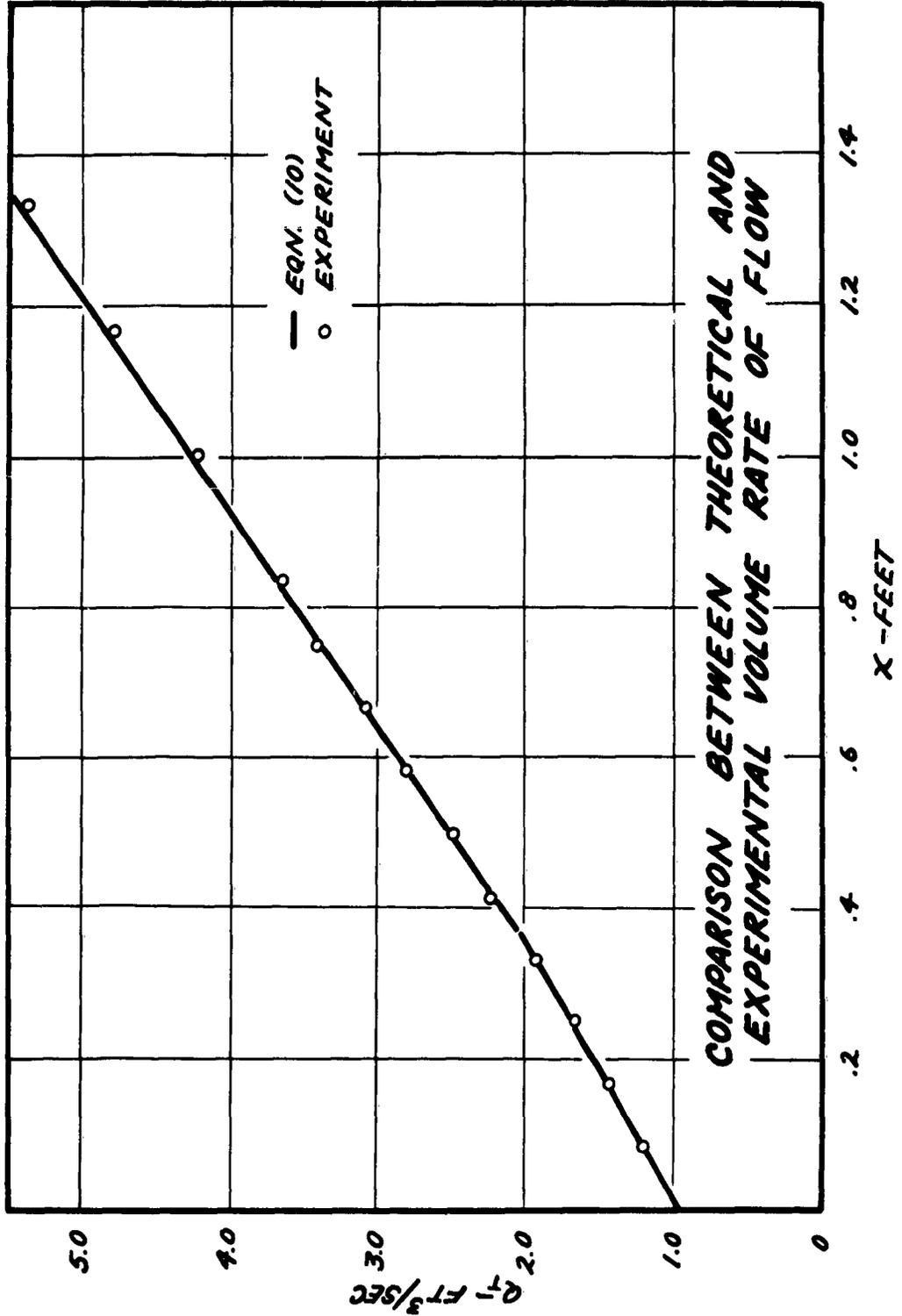


Figure 12

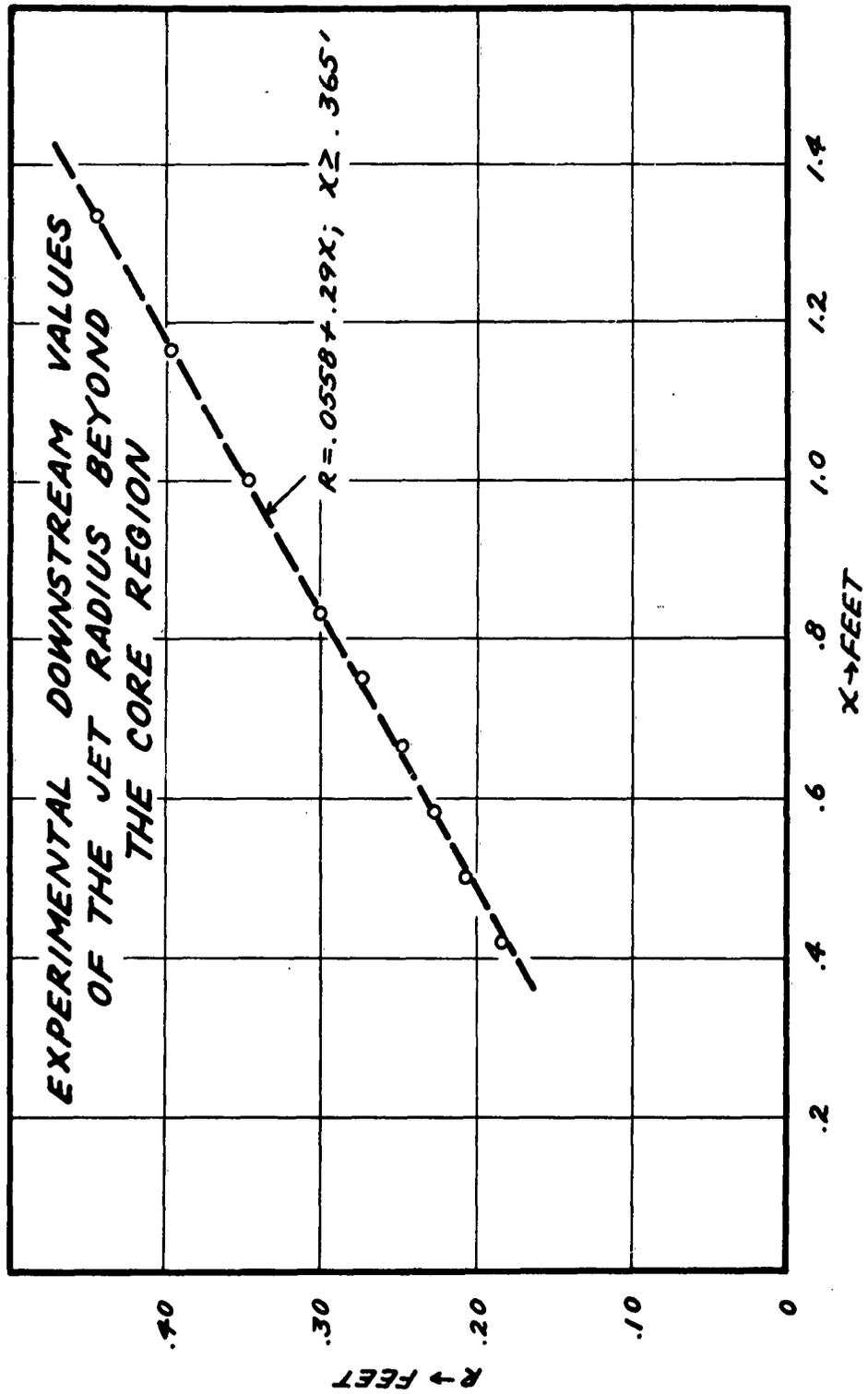


Figure 13

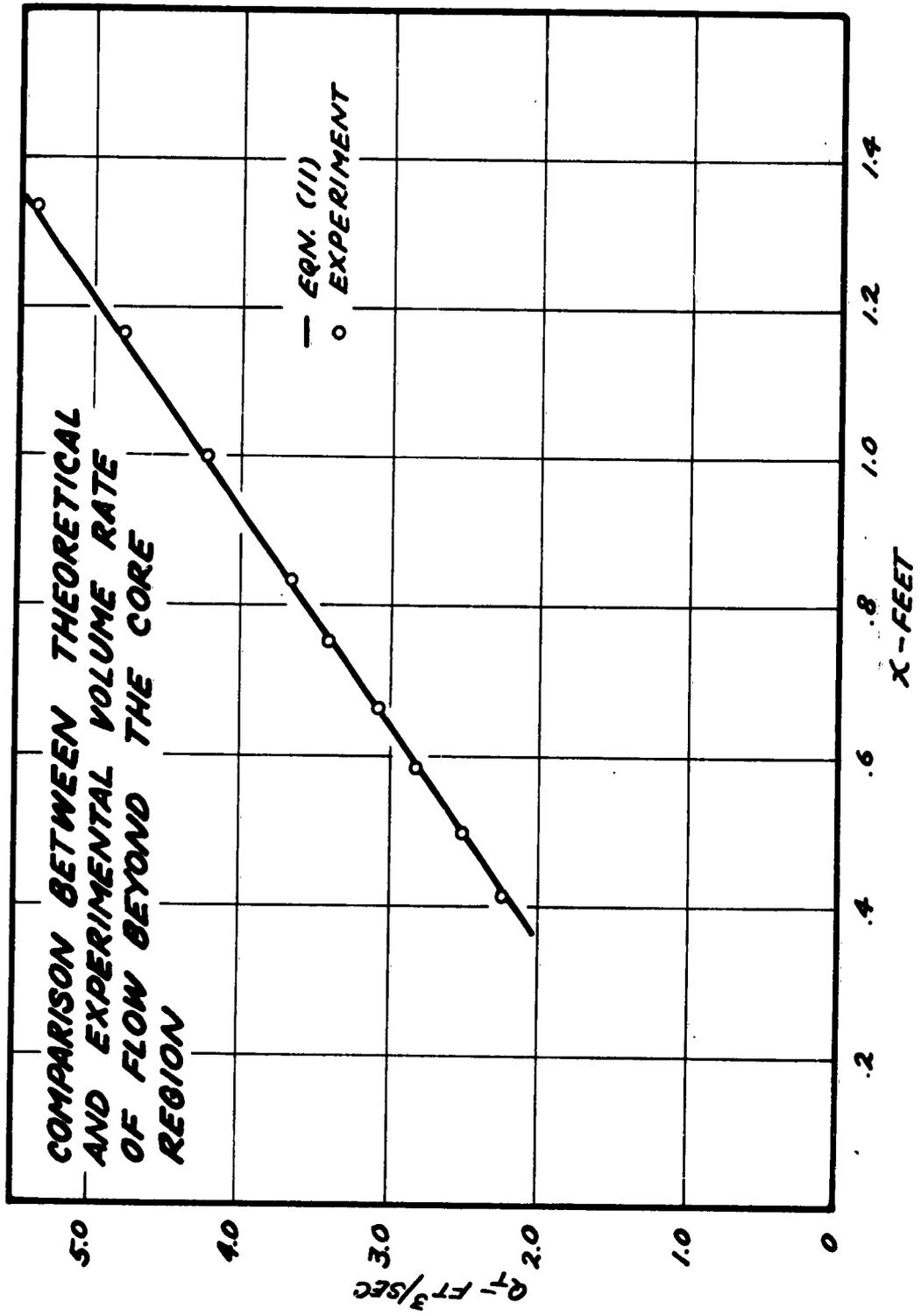


Figure 14

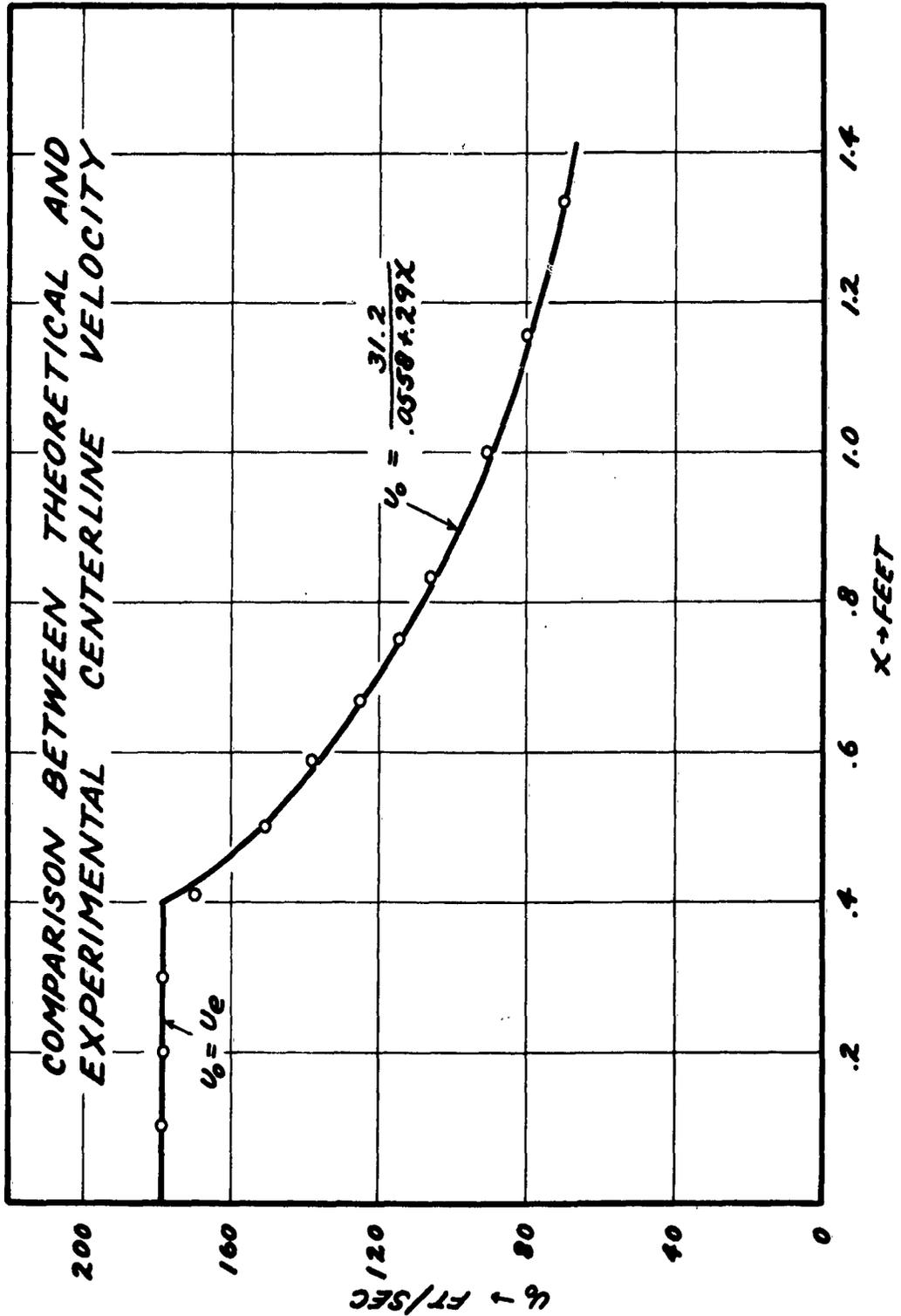


Figure 15

