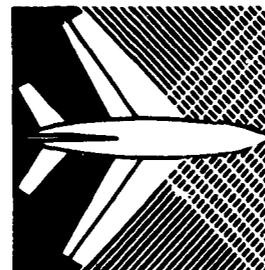
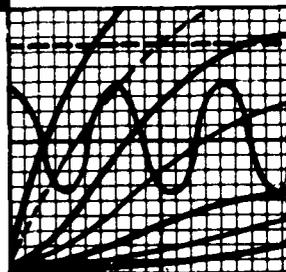
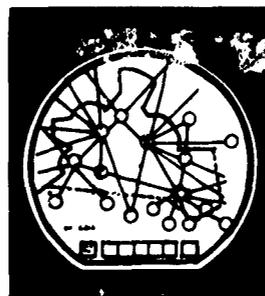


6332



461671

ASTIA

4761-10-F (II)
Task No. 116-8R
(Portion of Phase 1)

STUDY AND ANALYSIS OF SELECTED LONG-DISTANCE NAVIGATION TECHNIQUES

Volume II

Final Report

JAMES O'DAY
IRVIN SATTINGER
ROBERT SCOTT
JOSEPH SULLIVAN

ASTIA

APR 18 1963

Prepared by

NAVIGATION AND GUIDANCE LABORATORY
Institute of Science and Technology
THE UNIVERSITY OF MICHIGAN

March 1963

Federal Aviation Agency,
Systems Research and Development Service,
Research Division, Contract ARDS-436



4761 - 10 - F (II)
Task No. 116-8R
(Portion of Phase 1)

STUDY AND ANALYSIS OF SELECTED LONG-DISTANCE NAVIGATION TECHNIQUES

Volume II

Final Report

JAMES O'DAY
IRVIN SATTINGER
ROBERT SCOTT
JOSEPH SULLIVAN

March 1963

Prepared by
Navigation and Guidance Laboratory
Institute of Science and Technology
THE UNIVERSITY OF MICHIGAN
Ann Arbor, Michigan

Federal Aviation Agency
Systems Research and Development Service,
Research Division, Contract ARDS-436

NOTICES

Sponsorship. The work reported herein was conducted by the Institute of Science and Technology for the Federal Aviation Agency, Systems Research and Development Service, Contract ARDS-436. Contracts and grants to The University of Michigan for the support of sponsored research by the Institute of Science and Technology are administered through the Office of the Vice-President for Research.

Note. This report reflects the views of the contractor, who is responsible for the facts and the accuracy of the data presented herein, and does not necessarily reflect the official views or policy of the Federal Aviation Agency.

Distribution. Initial distribution is indicated at the end of this document.

PREFACE

This report is a final summary of the investigations carried out during the period September 21, 1961, to July 31, 1962, by the Institute of Science and Technology of The University of Michigan for the Systems Research and Development Service, Federal Aviation Agency, under Contract No. ARDS-436, for the purpose of studying and analyzing four selected long-range navigation techniques usable by non-military ocean-crossing aircraft. The purpose of this report is to permit comparisons with other systems not considered in this study, and evaluation of systems considered in relation to desired track separation. The FAA Project Manager has been Mr. Nathaniel Braverman.

The Navigation and Guidance Laboratory of the Institute conducts work encompassing two general areas: (1) analytical and experimental research in navigation and guidance systems, techniques, sensors, and investigation of associated underlying physical and mathematical phenomena; and (2) supporting efforts in areas of work intimately associated with navigation and guidance. The Navigation and Guidance Laboratory, besides retaining its own staff of research personnel, is free to consult with and invite participation of members of the University faculty.

CONTRIBUTORS

Although it is impossible to acknowledge all of those who participated in this report, the authors wish to express especial thanks to the following colleagues:

Anderson, William	Kuipers, J
Conlon, Howard	Minty, George
Crabtree, Raymond	Porter, W. A.
Fedchenko, Robert	Puig, Hiram
Filkins, Lyle	Richardson, Wyman
Friberg, H. B.	Smith, Newbern
Jebe, Emil	Tirrill, Lloyd
Kazda, Louis	Vukovich, Joseph
	Young, Wendell

The authors also wish to acknowledge the valuable cooperation and assistance received from the staff of the System Research and Development Service during this study. We are particularly indebted to Mr. Nathaniel Braverman, Chief of the Technical Research Branch, for his technical direction, guidance and counsel. Mr. Charles Santora's contributions to the program, particularly in respect to very-low-frequency techniques, were very helpful. In his capacity as technical consultant to the Systems Research and Development Service, Mr. Nathan Marchand of Marchand Electronics Laboratory provided valuable assistance in the search for technical information and in the critical analysis of various navigation systems.

CONTENTS

Notices	ii
Preface	iii
List of Contributors	iv
List of Figures	vii
List of Tables	x
Abstract	1
1. Introduction	1
1.1. Nature of Report	1
1.2. The Accuracy Problem	2
1.3. Statistical Description of Accuracy	4
1.4. Estimated Costs	7
2. Summary of Major Conclusions and Recommendations	9
2.1. Heading Reference Systems	9
2.2. Very-Low-Frequency Systems	10
2.3. Inertial Systems	10
2.4. Satellite Systems	11
3. Heading References	12
3.1. Introduction	12
3.2. Hybrid Systems	13
3.3. The Slaved Gyromagnetic Compass	16
3.4. Free Gyros	23
3.5. Heading Reference Derived from Celestial Information	31
3.6. North-Seeking Gyros	43
3.7. Variation Correction	45
4. Very-Low-Frequency Radio Navigation Systems	53
4.1. Introduction	53
4.2. Accuracy Evaluation	55
4.3. Lane Identification and Tracking	87
4.4. Transmitter Requirements	93
4.5. Problems of Introduction and Use	94
4.6. System Configurations Other Than Hyperbolic-Hyperbolic	96
4.7. Summary and Conclusions	102
5. Inertial Navigation Techniques	105
5.1. Introduction	105
5.2. Accuracy of Inertial Systems	107
5.3. Other Characteristics of Inertial Systems	122
5.4. Reliability	134
5.5. Economic Considerations	136
5.6. Timely Availability and Future Projections	137
6. Use of Satellites for Long Distance Navigation	140
6.1. Introduction	140
6.2. TRANSIT Navigation Satellite System	141
6.3. Angle-Measuring Systems	161
6.4. Range-Measuring Systems	175
6.5. Conclusions and Recommendations	181

Appendix A: Radiation of Very-Low-Frequency Energy from Aircraft Antennas	191
Appendix B: Derivation of the d_{rms} Error of Hyperbolic-Hyperbolic, Hyperbolic-Elliptic, and Rho-Rho Navigation Systems	193
B.1. Hyperbolic-Hyperbolic Systems	193
B.2. Rho-Rho Systems	196
B.3. Hyperbolic-Elliptic Systems	198
Appendix C: Correlation Coefficient Between Time Differences for a Three-Station Fix	204
Appendix D: System Accuracy Contours for a Three-Station Hyperbolic System on a Spherical Earth	206
D.1. d_{rms} Error on a Spherical Earth	206
D.2. Explanation of Accuracy Contours	242
D.3. Computation of Accuracy Contours with an Analog Computer	244
D.4. Analog Computer Circuit	247
D.5. Conclusions	254
Appendix E: Description of the OMEGA and DELRAC Very-Low-Frequency Hyperbolic Radio Navigation Systems.	255
E.1. OMEGA	255
E.2. The Airborne OMEGA Receiver	259
E.3. DELRAC	260
References	265
Bibliography	269
Distribution List	271

FIGURES

1. Cross-Track Error Limits 13

2. Presently Acceptable Cross-Track Error Distributions 14

3. Parallel-Track Separation Standard 14

4. Future Cross-Track Error Distributions 16

5. Compass Headings 18

6. Free-Gyro Mode of Operation 24

7. Heading Errors Vs. Actual Heading for Several Bank Angles (ϕ) 25

8. The Basic Astrocompass 33

9. Kollsman Periscopic Sextant 34

10. Kollsman Photoelectric Sextant (KS-85) 35

11. Kollsman Sky Compass 37

12. Kollsman Automatic Astrocompass Control and Indicator Panels 39

13. Kollsman Automatic Astrocompass Amplifier and Computer Components 40

14. Kollsman Automatic Astrocompass and Astrotracker 41

15. Difference Between Actual Flight Path and Great Circle 47

16. No Change in Magnetic Variation, 8° Meridian Convergence
Between 30° and 40° Longitude 47

17. No Meridian Convergence, Magnetic Variation Increasing Westbound 48

18. No Meridian Convergence, Magnetic Variation Decreasing Westbound 49

19. Change in Error Variation 51

20. Correlation Function of the Variation Error 51

21. Geometry for One LOP. 56

22. Fix Geometry 57

23. Path Geometry for Computation of Diurnal Shift 65

24. Typical Diurnal Shift 65

25. Time-Difference Diurnal Variation 67

26-33. d_{rms} -Error Isogram for a Hyperbolic Navigation System on a
Spherical Earth 78-86

34. Probability of Correct Lane Identification with a Single Lane Reading 89

35. Interferometer 98

36. Hypothetical Inertial-System Position-Error Propagation 116

37. Hybrid System Block Diagram 117

38. Weighting Functions 119

39. Hybrid-System Error Vs. Time 119

40. Navigation Error Normal Alignment and Erection 128

41. Navigation Error Fast "Scramble" Erection and Alignment 128

42. Hybrid System Position Indicated Geometry 131

43. Performance Comparison Chart 133

44. Geometry of TRANSIT Position—Determination Method 143

45. Distribution of Fix-Renewal Intervals with TRANSIT System for
Single Coverage Areas 154

46. Distribution of Fix-Renewal Intervals with TRANSIT System for Double
Coverage Areas 154

47. Allowable Fix-Renewal Interval of Combined TRANSIT and Inertial
Systems 156

48. Allowable Fix-Renewal Interval of Combined TRANSIT and Doppler
Systems 157

49. Geometry of Satellite Observation 167

50. Effect of Satellite Parallax in Reducing Error in Position Fix 168

51. Geometry of Position Fix 168

52. Position Fix for Moving Aircraft 169

53. Distribution of Coverage-Gap Intervals for Various Numbers of
Satellite Passing Over Aircraft 171

54. Allowable Fix-Renewal Interval of Angle-Measurement Satellite
System Combined with Inertial System 173

55. Hyperbolic-System Error Parallelogram 193

56. LOP Error Diagram 194

57. Rho-Rho System Error Diagram 197

58. Hyperbolic-Elliptic System Geometry 199

59. Hyperbolic-Elliptic System Error Parallogram 200

60. Hyperbolic-Elliptic System Error Diagram 200

61-95. d_{rms} -Error Isogram for a Hyperbolic Navigation System on a
Spherical Earth 209-243

96. Intersection of LOP's 1 and 2 Which Determine Position of Point P 244

97. Triangle S_2MS_1 Located on a Spherical Earth 245

98. Triangle MPS_1 of Figure 91 245

99. Computer Block Diagram for Solving and Plotting Constant d_{rms}
(K_1) Error Contours 245

100. Computer Circuit for Generating Y_1 249

101. Computer Circuit for Generating X_1 250

102. Computer Circuit for Generating $2\phi_1$ 251

103. Computer Circuit for Solving

$$\frac{d_{rms}^2}{C^2 \sigma^2} = \frac{1}{4 \sin^2 \theta} \left[\frac{1}{\sin^2 \phi_1} + \frac{1}{\sin^2 \phi_2} + \frac{2\rho \cos \theta}{\sin \phi_1 \sin \phi_2} \right] \dots \dots \dots 252$$

104. Analog High-Gain Amplifier with a Servo-Jet Potentiometer in the Feedback Path for Division	253
105. OMEGA Transmission Sequence	255
106. OMEGA Master Station Block Diagram	256
107. OMEGA Slave-Station Synchronizer Block Diagram	257
108. OMEGA Navigation Receiver Block Diagram	258
109. DELRAC Transmission Sequence	262
110. Simplified Block Diagram of DELRAC Receiver	263

TABLES

I. Probabilities Combined in Circles of Varying Radius	6
II. Relations Among Statistics Used to Describe the Error Distribution in the Circular Normal (Two-Dimensional) Distribution Case	7
III. Observed Drift Vs. Time	29
IV. Summary of Drift-Rate Signs	30
V. Factors Affecting Flight Path	50
VI. Phase Velocity at 10.2 kc (V/C) Ratios	63
VII. Bandwidth Requirements for a VLF Navigation System Operating at 10 kc Without Externally Derived Rate Information	69
VIII. Typical Single-Axis (x Axis) Error Propagation	110
IX. Accuracy Criteria	111
X. Typical Error-Source Summary of a Velocity-Damped Inertial System for a 60-Minute Flight East at 200 Knots	114
XI. System Errors Vs. Time	120
XII. Flight Times	121
XIII. Inertial Navigation Systems	138
XIV. Distribution of Fix-Renewal Intervals for Single Coverage of TRANSIT Satellites	153
XV. Coverage Gaps with 3440-Nautical-Mile-Altitude Satellites	171
XVI. Gap in Double Coverage	172
XVII. Estimates of Range-Measuring System	179
XVIII. Comparison of Navigation Satellite Systems	189

STUDY AND ANALYSIS OF SELECTED LONG-DISTANCE NAVIGATION TECHNIQUES

Volume II Final Report

ABSTRACT

The state of the art and the development potential of heading references, VLF (very-low-frequency) radio systems, inertial techniques, and satellite systems have been considered for their applicability to long-range ocean-crossing nonmilitary aircraft from 1965 to 1975. The navigation systems discussed here are by no means the only competitors for position and course determination over transoceanic and high-altitude transcontinental regions; our data and information should permit further comparison with other systems.

We conclude that magnetically slaved and good free gyroscopes will continue to be the principal heading references for commercial aircraft; we recommend improvement programs in magnetic compasses and the use of nonfloated friction-averaging gyros.

We do not expect that VLF systems will have been sufficiently operated to be acceptable for commercial aviation before 1975. However, their ability to cover large areas and inherent accuracy make them attractive if certain propagation and instrumentation problems can be solved.

In their present form inertial systems are competitive with doppler navigation systems, at least in accuracy. The choice of an inertial system for commercial flight depends on cost, reliability, and convenience. Product improvement and the recently lowered cost of inertial platforms may make these systems attractive in the near future, particularly for higher speed aircraft.

The present configuration of TRANSIT, the only satellite reference system scheduled for implementation, exhibits time gaps which would be serious for aircraft use. The single-fix accuracy appears to be adequate if additional satellites are orbited to provide more frequent fixes.

1

INTRODUCTION

1.1. NATURE OF REPORT

This is the final report on contract ARDS-436, Study and Analysis of Selected Long Distance Navigation Techniques. It consists of a review of the state of the art and development potential for each of the several techniques studied. These are (1) improved heading references, (2) very-low-frequency ground-based radio navigation techniques, (3) inertial navigation techniques and

(4) navigation using earth satellites for nonmilitary ocean-crossing aircraft. An explicit presentation of the work program is contained in the statement of work of the contract, appended to Volume I.

The above techniques have been considered for their applicability to the high long-range traffic over ocean routes expected in the period 1965 to 1975. Principal attention was directed to routes terminating in the United States and its possessions, although some consideration was given to world-wide coverage for those systems which provide it easily. In an effort to describe the operational environment, information regarding current and future routes, aircraft types and schedules was collected.

Sources of information for work on this contract were many. While detailed references are given in following sections, it seems worthwhile to note here the variety of sources—military development agencies, several military operational units, many manufacturers and developers, several airlines and airline associations, and several military testing agencies. One major effect in connection with using such a variety of sources was the disparity of information regarding similar equipment or systems. For systems in the development state it is difficult enough to arrive at precise cost and performance estimates. For systems which exist only as proposals—or gleams in the engineer's eye—the cost and performance estimates must be made carefully and then viewed suspiciously.

This report is divided into two volumes. Volume I includes this introductory section, which discusses the approach to the problem; and a summary section, which presents conclusions and recommendations relative to each of the four areas of study. Appendix A comprises the statement of work of the contract under which this study has been conducted; Appendix B is a letter summary of the report's major conclusions and may be useful to the reader as a quick survey.

Volume II discusses the problem in more detail. After an introduction and a brief summary its four main sections take up heading references, VLF radio systems, inertial systems, and satellite-reference systems. Five appendixes discuss VLF systems in still more detail.

1.2. THE ACCURACY PROBLEM

Taking accuracy as a measure of performance of a navigation system, we find that various "kinds" of accuracy exist. These are not comparable although they are often treated as such in side-by-side comparisons of competitive systems. Common usage of "accuracy" may include any one of the following.

- (a) The fundamental accuracy limit for a particular system is determined by the physical limitations inherent in the method, or by our knowledge of the underlying physical

constants. For example, radar is limited by (among other factors) the knowledge of the propagation velocity of electromagnetic waves.

- (b) Ideal performance today is the accuracy attained by existing research and development systems under ideally controlled laboratory conditions. It also comprises predicted system performance based upon present-day component accuracy under laboratory conditions.
- (c) Ideal performance in the foreseeable future is the same as (b) except that an extrapolation is made to some future date. The prediction of improved performance is (or should be) based on normal research and development progress; breakthroughs cannot be programmed, and it should not be assumed that they will occur.
- (d) Operational accuracy is the accuracy of the production system operated, calibrated, and maintained by airline personnel in the field rather than by the design engineers. Operational accuracy is sometimes estimated by subjecting ideal performance results to some degradation factor.
- (e) Special operating condition performance refers to accuracy under unfavorable conditions which may further degrade the accuracy from (d). Included in this category are short warmup times, temporary power failures, high latitudes, etc. In some cases, special conditions may deny use of the system entirely.

Most statements on accuracy are given without due regard to the reliability of the system. Malfunctions or large inaccuracies for which specific causes are suspected are customarily removed from the statistical analysis of errors. Thus, quoted accuracy, even when based on field experiments, may not be a valid measure of system performance.

Most navigation systems require initial settings or calibration. These may take the form of null and scale adjustments, starting point or transmitter locations, and/or north reference direction. Conditions may limit the precision with which these operations can be performed. The result is that any system of a specified type—no matter how accurately it performs when correctly calibrated—may exhibit inadequate accuracy under normal operating conditions. We conclude that "system accuracy" must be interpreted in its broadest sense to include the effect of ground equipment, information regarding initial conditions, setup procedures, auxiliary data sources, and vehicle behavior.

The relation between ideal performance and field performance also deserves comment. It does not suffice to multiply ideal performance by an arbitrary degradation factor. To do so would be to ignore the reliability factor, which may favor a simple but crude system over a "more

accurate" system that is likely to be complex and delicate. Furthermore, the "more accurate" system will require complicated calibration and alignment operations as well as highly specialized maintenance procedures. When these are improperly performed under normal operating conditions, the basically "more accurate" system may do poorly in comparison with a crude and simple system. In other words, a comparison of the laboratory performance of two systems is insufficient to predict their comparative effectiveness under operational conditions.

In looking for civil applications deriving from military developments, we must realize that the civil operator has different performance requirements. In a military navigation mission, the problem might be to navigate with a precision of 100 yards 70% of the time, so that a bomb might be placed effectively. The military operator is willing (although perhaps not happy) to see a complete failure the other 30% of the time. Complete failure may mean missing the target by ten miles, or even the loss of the aircraft. While either of these undesirable results may be acceptable to the military, they cannot be tolerated by the commercial operator. In contrast, he would place much more stress on the reliability of the equipment—and thus the overall "performance" of a system—rather than its occasional spectacular accuracy.

It is sometimes a characteristic of very complex systems that "when they are good, they are very very good—and when they are bad, they are horrid." Important criteria, then, for civil navigation aids are their reliability and fail-safe characteristics. These criteria favor certain types of hybrid and redundant systems yielding consistent, even though mediocre, performance.

In this report we have attempted to report operational accuracy under normal flight conditions. In many instances, of course, data indicating such accuracy are not available, in which case notation is made as to the meaning of the "accuracy" information given.

1.3. STATISTICAL DESCRIPTION OF ACCURACY

In recognizing these general concepts of accuracy we must add that statistical terminology for describing the accuracy is, unfortunately, not standardized. First, the technical distinction between accuracy and precision is mixed in many presentations. For clarity we state here that precision merely means the repeatability of a measurement expressed in suitable terms. On the other hand, an accurate system is capable of measuring the true value of a quantity, or we say it is unbiased. The five common usages of accuracy listed above would ordinarily combine the assessment of bias and repeatability into one single accuracy figure.

In navigation terms we say a system is accurate if the average of a long series of fix determinations in the neighborhood of a target destination is this target's location. Such a statement,

however, says nothing about dispersion of the individual fixes, which is described by a precision measure.

The navigation problem increases the terminology confusion because of the increased number of parameters and the variety of available measures for describing the two-dimensional error structure. Thus, we find that the assessment of accuracy may include one or more of the following:

- (a) d_{rms} error
- (b) variance
- (c) standard deviation or standard error
- (d) moments of the radial error
- (e) root mean square
- (f) mean square error
- (g) CEP, the circular probable error

In the univariate (one-dimensional) case the first moment is taken about the origin and other moments are taken about the mean. The bivariate or two-dimensional problem again calls attention to moments about the origin when the radial error is considered.

The purpose of this report is to permit comparisons with other systems not considered in this study and evaluation of systems considered in relation to desired track separations.

Specifications of accuracy presented in this report are given in terms of the d_{rms} error, a parameter often used for this purpose. The interpretation of this statistical method of specifying accuracy (actually, precision) is briefly summarized in this section.

Consider a position-measurement process in which measurement errors along each of two orthogonal axes have a normal distribution and are uncorrelated. If the standard deviations of the errors along the two axes are equal ($\sigma_x = \sigma_y$), a circle with its center at the mean values of the position coordinates is the locus of points having constant probability density. It can be shown that the probability is 0.632 that a single position measurement will fall within such a circle having a radius of $d_{\text{rms}} = \sqrt{\sigma_x^2 + \sigma_y^2}$. For a circle of radius $2 d_{\text{rms}}$ the probability increases to 0.982.

In the general case of position determination the errors in x and y may be correlated, and σ_x is not equal to σ_y . For the bivariate (two-dimensional) normal distribution it is possible to make a transformation (consisting of a rotation of axes) which will remove the correlation of the errors. The distribution may then be described by two new variances along the two new orthogonal axes. In this general situation, an ellipse represents the locus of points having con-

stant probability density. The parameter d_{rms} , as defined above, may still be used to represent the spread of individual position determinations. This quantity d_{rms} describes the radius of a circle that is arbitrarily drawn, since the shape of the distribution in plan view is elliptical. Within this circle, however, the probability of obtaining a single position determination lies within a certain narrow range even though the ratio of the two standard deviations σ_y/σ_x (taken as the smaller over the larger) may be unknown in the range zero to one. Table I shows the range of probabilities for several circles. It seems most natural, perhaps, to employ the d_{rms} statistic for the uncorrelated case with $\sigma_x = \sigma_y$, but the maximum error is about 8% for the examples in Table I if $\sigma_x > \sigma_y$. The specification of positional error by a d_{rms} circle tends to simplify calculations, and it may be noted that such specification for a circle of $1.4 d_{rms}$ radius or larger is always conservative, in the sense that the errors are considered to be greater than they actually are. If improved values of the probabilities are desired, they can be read from available tables when the ratio of the standard deviation is known for the uncorrelated situation.

TABLE I. PROBABILITIES CONTAINED IN CIRCLES
OF VARYING RADIUS

Radius of Circle	Probability for $\sigma_x = \sigma_y$	Probability for $\sigma_y = 0$
1 d_{rms}	0.632	0.683
2 d_{rms}	0.982	0.954
3 d_{rms}	0.999	0.997

It is worth pointing out that most of the statistics used to describe the radial error distribution or the position determination distribution are inter-related; when one is known, others are readily obtained. This derivation is most easily made when $\sigma_x = \sigma_y$ in the uncorrelated case. Since this case is of interest for a variety of navigation problems, some of the inter-relations are given here. For the noncircular distribution case it appears preferable to consult suitable tables for describing the two-dimensional error situation.

In the navigation problem we take d as the radial error or the straight line deviation of the position determination from the true position. Hence, $d^2 = x^2 + y^2$ from the geometry of the situation. Thus, we find that d_{rms} may be described as the square root of the average value of d^2 . Other statistics that may be considered are the variance of d , the standard deviation of d , and the CEP. Taking $\sigma_x = \sigma_y = \sigma$ as indicated above, we show the relations among various statistics in Table II.

TABLE II. RELATIONS AMONG STATISTICS USED TO DESCRIBE THE ERROR DISTRIBUTION IN THE CIRCULAR NORMAL (TWO-DIMENSIONAL) DISTRIBUTION CASE

Name of Statistic	Value in Terms of $\sigma_x = \sigma_y = \sigma$	Equivalent d_{rms} Units (σ)
d_{rms}	$\sqrt{2} \sigma$	1.000
Mode of d	σ	0.707
Median of d	$\sqrt{1.386} \sigma$	0.833
Mean of d	$(\pi/2)^{1/2} \sigma$	0.866
CEP	$\sqrt{1.386} \sigma$	0.833
Standard Deviation of d	$\left(\frac{4-\pi}{2}\right)^{1/2} \sigma$	0.463
Variance of d	$\left(\frac{4-\pi}{2}\right) \sigma^2$	0.3035

In view of the frequent application of the d_{rms} statistic in the navigation literature and its general usefulness in the two-dimensional error problem, we have tried to present this statistic consistently when summarizing system or component capabilities. When other statistics are used (e.g., those indicated in Table II), we have tried to make clear their proper interpretation and their relation to the d_{rms} error.

1.4. ESTIMATED COSTS

For systems under development the problems of estimating costs are similar to those of estimating accuracy. In fact, even for systems in existence the acquisition cost can vary widely. A good example is the recent purchase by TWA of a dual doppler radar, supplemented by Edo Loran for 12 Boeing 707 aircraft. The doppler equipment included dual transmitter-receivers, dual sensor tracker units, dual control units, a single antenna, two computers, two computer controllers, and two indicators. Purchase cost of the dual DRA-12-A doppler radar navigation system is quoted by Bendix as approximately \$25,000. TWA's estimate of the cost of the program of installing 12 systems was \$1,800,000—or \$150,000 per aircraft. The apparent discrepancy is not hard to explain, but is worth some discussion to indicate the elements involved. In addition to the \$25,000 for the basic equipment, spares are needed. For an airline like TWA, which operates over long routes, appropriate spares may be more than 50% of the initial cost. In the case of this installation program, the 707's had to be modified by Boeing to accept the doppler antenna, the compass system had to undergo a very careful (and expensive) calibration,

and the installation of the equipment required the aircraft to be pulled from normal service for a period of time—thus causing a loss of income. The point of this example is that there may be many factors which must be considered in the cost of a particular system to the airline. These factors make a straightforward comparison difficult. While it has not been possible to consider all of them, many of them are discussed in connection with the various systems.

In general, cost figures presented in this report are best estimates for purchase of a small quantity of a developed item. In certain instances additional estimates are given for abnormal installation costs and developmental costs.

2

SUMMARY OF MAJOR CONCLUSIONS AND RECOMMENDATIONS**2.1. HEADING REFERENCE SYSTEMS**

Magnetic compasses will continue to be a principal source of heading information aboard aircraft for many years. In connection with new aircraft development programs, we recommend early consideration of magnetic compass installation. Developments such as the miniaturized magnetic azimuth detector and an improved compass amplifier promise to be useful in future high speed aircraft.

The accuracy potentially available in a nonfloated friction-averaging gyroscope as a heading reference would be appropriate in combination with a doppler radar navigation system. In addition, such gyroscopes have nearly the same warmup and maintenance characteristics of current directional gyros and should not cause any serious introduction problems.

The calibration of aircraft magnetic compasses by "electrical" rather than physical swinging would probably be useful for commercial airlines, which depend heavily on magnetic compasses.

As improved free gyros become available, corrections for the earth's rotation must be made more precisely. Present compass controllers require improvement. One convenience would be separate knobs for latitude correction and drift rate correction.

Automatic celestial trackers would be both convenient and more precise than present hand-operated sextants, although these advantages would come with a decrease in reliability. In addition, their availability aboard an operational aircraft would permit a more precise check of free-gyro heading references.

"Inertial quality" free gyros, in spite of their present advanced performance and decreased cost, are not recommended because they are appreciably better than required for the commercial operation. If this better performance came at no increase in cost or trouble, they would be welcome; but this is not now the case.

North-seeking gyros are in operation and exhibit performance somewhat better than magnetically slaved gyros, but their complexity and the consequent possibility of errors resulting from undetermined failure make them unacceptable as the sole heading reference aboard an aircraft.

Radio-celestial systems offer the possibility of periodic heading checks in all weather. However, the equipment is large, relatively complex, and expensive; for most future aircraft the advantage of being able to see through clouds does not seem to be worth the price.

Operational performance data on free gyros are sparse because data have not been taken in response to a real need, and errors in the instrumentation and recording of the data are not readily identifiable. As new gyros become commercially available, carefully designed test programs will be a great aid in evaluating their potential.

2.2. VERY-LOW-FREQUENCY SYSTEMS

Range-measuring, azimuth-measuring, and hyperbolic-elliptical VLF systems have been proposed in the past, but the only extensive experimental effort toward a system has been with the hyperbolic system, OMEGA. In this study the proposed DELRAC system, basically very similar to OMEGA, has also been considered. Analysis of OMEGA development has shown that the accuracy required for commercial over-ocean navigation can be provided by a station configuration with station baselines from 3000 to 5000 nautical miles long under normal propagation conditions. Propagation anomalies may cause the error to be greater than allowable, although current programs aimed at understanding these anomalies have a good chance of reducing the errors to an acceptable value. From the standpoint of commercial aviation the most serious problem is that lane resolution for hyperbolic systems in this frequency region has not been satisfactorily determined. Only experimental readout equipment exists, and this would not be satisfactory for commercial operation. Although the initial cost of the system is high, repetitive costs are not large and the system is close to being economically feasible for commercial aviation.

Several problems make the introduction of operational VLF systems unlikely between 1965 to 1975. The propagation research necessary to insure acceptable operation under all states of the ionosphere and in all areas may require several years. The ambiguity resolution problem has not been conclusively solved and will require considerable experimentation. The airborne computer required for coordinate conversion is not available and may require some time to develop. Finally, the requirement for stations on foreign soil will almost surely raise diplomatic problems.

2.3. INERTIAL SYSTEMS

We have concluded that inertial navigation systems are competitive with doppler navigation systems in accuracy and performance, but that these qualities do not overshadow considerations of cost for the present aircraft. An airline would have to be convinced of the increased convenience, adequate reliability, and moderate costs before inertial systems would find wide acceptance for subsonic jets.

The recent trends in lowered cost make the inertial system—either a pure inertial system or possibly a doppler-inertial system—most attractive for future supersonic aircraft. Commercial operational experience with today's aircraft to determine the usefulness of inertial systems would be desirable. We recommend that an experimental evaluation of a current inertial system be conducted, with aircraft flying over land rather than ocean so that good data on the accuracy of the experiment can be obtained.

Many of the objections to inertial systems, such as long warmup times, difficult alignment procedures, poor reliability, and initial and maintenance costs, have been at least partially resolved. Large-scale production of at least one platform indicates that more operational problems will be reduced in the near future.

2.4. SATELLITE SYSTEMS

Navigating aircraft by means of satellites appears to be economically and technically feasible. Such systems would be capable of handling any volume of traffic, and would provide world-wide all-weather coverage. If used with random, rather than synchronized, orbits, they would provide a position-fixing capability which is adequate only when combined with dead-reckoning methods. With this combination it would be possible to navigate the aircraft with an error whose d_{rms} value remained in the 2- to 6-nautical-mile range. For certain methods the user's equipment would not be excessively large or expensive. Overall system reliability is open to some question pending an actual demonstration. For all systems substantial installations of ground-based and satellite equipment would be required to make the method available for navigation purposes.

Satellite navigation could be available for introduction and use only during the latter part of the 1965-1975 period; TRANSIT is the system which would be available earliest. With respect to fix renewal interval, the TRANSIT system, even if expanded to eight satellites from the presently planned four, would not meet the requirement for a fix renewal every 500 nautical miles for aircraft at speeds higher than Mach 1. The other methods considered, which are not under active development, would meet this requirement except for occasional coverage gaps.

If research and development beyond currently active programs is undertaken, we recommend that these should initially be confined to relatively small scale efforts directed toward the critical factors which are likely to affect proposed systems.

3 HEADING REFERENCES

3.1. INTRODUCTION

Aircraft operating and maintenance personnel desire heading reference equipment which requires no maintenance, calibration, variation compensation, or corrections during flight. Ordinarily they settle for something less than perfect.

Apparently, any accuracy better than $1/4^{\circ}$ standard deviation is useful only in an inertial navigation system. The $1/4^{\circ}$ figure has been selected for several reasons. First, it is difficult to set a free-gyro reference more precisely. Second, the currently available commercial doppler radar has an inherent cross-track error due to the doppler radar alone on the order of $1/4^{\circ}$ standard deviation. Third, use of a more precise heading reference introduces some difficult maintenance, installation, and calibration problems. As for reliability, there should be a positive indication of any continuous error greater than perhaps one degree, although random errors of this amount would be acceptable if they correlated over not more than a few minutes.

The combination of accuracy, reliability, and convenience is not currently available in any single instrument. Consequently, it is common practice to carry two basic heading references which can be compared, and a third which can be used to check which of the other two is correct in case of a discrepancy.

Accuracy data for heading references originate from so many sources and are obtained by such a large variety of methods that they are difficult to interpret. For example, a particular gyro used commercially has been quoted by the manufacturer as exhibiting a maximum error of 0.5° per hour. Airlines using this gyro will remove it from service if the drift rate exceeds 3° per hour, a common service figure. This startling difference arises for several reasons: the manufacturer's tests may not be conducted in a typical operational environment; airline personnel may not be as adept at setting in required corrections for latitude; the airline may not provide the maintenance required for precise operation; or the airline may be quite satisfied with the combination of poorer performance and reduced maintenance.

The performance ratings from a series of tests are not very meaningful unless the conditions of the tests are known. Johnsville Naval Air Development Center has done an extremely good job in designing bench tests which simulate flight operation in Navy aircraft. The Center has found, for example, that performance of a free gyro in a 15° and a 3° test on a Scorsby table and a bench test are required to represent flight performance. The manufacturer may have conducted only a 1° or 3° Scorsby test in arriving at his quoted accuracy.

The difference between two such tests is often greater for very good equipment than for cheaper and more rugged equipment.

In spite of the uncertainties of available data, certain conclusions have been drawn. The summary (Section 2.1) presents certain conclusions and recommendations, first, with regard to useful developments; composite systems incorporating these developments are discussed in Section 3.2. Then slaved gyromagnetic compasses, free gyros, celestial heading references, north-seeking gyros, and variation correction are discussed.

3.2. HYBRID SYSTEMS

Airline operations require a high degree of reliability of the heading-reference system. Rather than a normal error distribution allowing some probability of very large errors, the airline operator desires a truncated distribution which will essentially never exhibit errors larger than some acceptable, though undesirable, amount.

Consider the situation shown in Figure 1. A flight from origin to destination is defined as acceptable if the cross-track error is no larger than "a" 99% of the time. 1% of the time a larger error is acceptable, as long as it does not exceed "b." The values of a and b are not defined, and are intended only to represent the nature of the problem. The probability of exceeding b must be extremely small, and such an event would be considered a catastrophe.

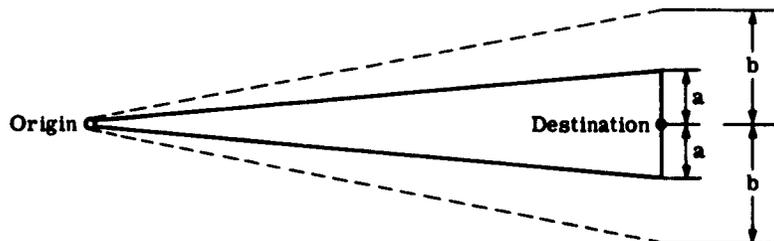


FIGURE 1. CROSS-TRACK ERROR LIMITS

It is difficult to measure, calculate, or define this latter probability. About all that can be said is that it must be very small; whether it is one part in 10^6 or one in 10^7 is neither important nor possible to determine. Figure 2 shows several error distributions which satisfy this need, and, as long as they all are truncated at $\pm'b$, there is not much difference in their operational value. For purposes of air traffic control separation standards are defined. The separation is chosen on the basis that two aircraft will have an extremely small probability of occupying the same track. Again, it is difficult to state an exact probability; the desire is that "never" shall two aircraft collide. In this study we may define some value "c" which is

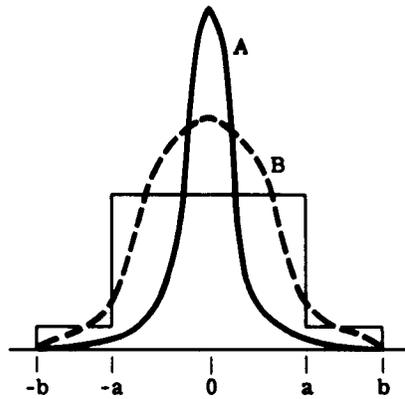


FIGURE 2. PRESENTLY ACCEPTABLE CROSS-TRACK ERROR DISTRIBUTION

half of the parallel track separation distance. Thus one may draw another distribution curve on which "c" is shown as the maximum allowable error (Figure 3).

Compliance with the curve of Figure 3 can be achieved either by having a heading reference so good that the cross-track error never accumulates to a value greater than c, or by providing intermittent fixes which essentially reset the heading reference. In the latter case the poorer the heading reference the more frequent the fix renewal. The former case is simply a limiting form of the latter with no renewals, and "b" of Figure 2 must be less than or equal to "c" of Figure 3. Clearly this condition must exist, for example, if one desired to use pure doppler radar without intermediate navigation fixes over the Atlantic.

Simple systems which fulfill the requirements of Figures 2 and 3 can be designed. If the error distribution is normal, a very small standard deviation would be required to have a

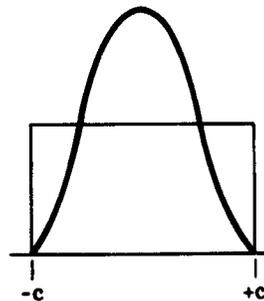


FIGURE 3. PARALLEL-TRACK SEPARATION STANDARD

probability of not exceeding "b" or "c" of, say, 10^{-6} (see curve A of Figure 2). In addition, account must be taken of the probability of complete failure, so that such a system would have to be most reliable.

However, hybrid systems can be designed which yield the desired performance of curve B and are for practical purposes "just as good" as A (Figure 2). Systems currently used by transoceanic aircraft are all of this type. It is common, for example, to carry two magnetically slaved gyros of good accuracy and moderate reliability, and to back them up with a simple magnetic compass of extreme reliability but somewhat less accuracy. For polar flights two free gyros are supplemented and occasionally corrected by celestial observations.

It seems clear that heading-reference systems of the future will be hybrid in character. The combination of systems used will depend on the characteristics of the particular airline: whether it flies principally polar or middle latitudes; whether it has a professional navigator in the cockpit; which system is most economical in terms of cost, fuel and time savings; what separation standards must be maintained over the routes flown; what are the aircraft's capabilities. The possible combinations of simple systems which can be used to generate a hybrid are many. There are magnetic compasses with various accuracies and reliabilities; very complex earth-rate direction references, celestial equipment ranging from hand-operated to fully automatic, free gyros, external-fix corrections, and radio-direction finding. The use of two components which depend upon different sources of information, or different power supplies, is often advisable. For example, the USAF B-58 has an earth-rate-direction reference from the doppler-inertial system, a conventional magnetic reference, and an automatic celestial input. Such a system is designed to have a very small standard deviation, similar to curve A of Figure 2, and with proper care can be very reliable.

When considering the problem of reducing track separation on over-ocean routes, we must make the value of "c" smaller. If the entire cross-track error results from the heading reference, then the value of "b" must be made smaller. This situation arises only if no intermediate fixes are taken. It is difficult to do this in a practical system without having some effect on the value of "a," too, but this should not be mistaken for the primary need (Figure 4). As demonstrated by a reduction of "b" and "c," desired performance can be achieved only if the better available components are properly combined. A better free gyro, for example, may only be useful if it can be corrected by an automatic astro-compass or a position-fixing system.

Although many improved developments are noted in the following pages, the reader should keep in mind that no one of these is a panacea for the over-ocean traffic separation problem, and that only judicious design can lead to a real operational improvement. In particular,

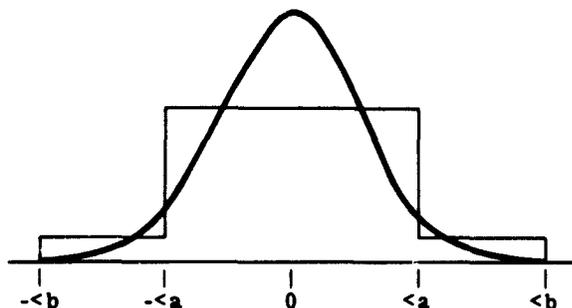


FIGURE 4. FUTURE CROSS-TRACK ERROR DISTRIBUTION

adopting a new and more accurate device may often yield a less reliable system. Here, again, careful system design is necessary to achieve desired performance.

3.3. THE SLAVED GYROMAGNETIC COMPASS

3.3.1. GENERAL DESCRIPTION AND USE. The state of the art of this class of equipment is represented essentially by product-improved versions of the Sperry C-11 compass. Under Air Force sponsorship, Sperry has conducted a program to improve both the magnetic and the inertial characteristics of the slaved gyromagnetic compass. The material in this section was extracted principally from reports of the Sperry work [1].

The slaved gyromagnetic compass consists basically of a directional gyro indicator electrically connected to a magnetic azimuth detector. In other words, it is a gyro-stabilized magnetic compass in which the gyro is aligned with the earth's magnetic meridian by means of a magnetic azimuth detector [2].

The stabilizing element of the compass system is an electrically driven gyro rotor which spins about a horizontal axis. The axis of rotation is maintained in the horizontal plane by a leveling system. Any lateral drift of the rotor spin axis in this horizontal plane is corrected by the action of the magnetic azimuth detector when operated in the slaved mode. The system can also be operated as a free gyro. Errors in the free-gyro mode are discussed in Section 3.4 of this report.

Before discussing the accuracy of a slaved magnetic compass system, we should review the common magnetic deviations.

Magnetic deviation may be defined as the angular difference between compass heading and magnetic heading. Deviation is dependent upon the magnetic latitude and also upon the individ-

ual vehicle—its trim and loading, whether it is pitching or rolling, the heading, and the location of its compass. The total deviation on any compass heading (CH) is the algebraic sum of the various types of deviation. The total deviation d for a vehicle in a horizontal attitude is: $d = A + B \sin CH + C \cos CH + D \sin 2 CH + E \cos 2 CH$, where A, B, C, D, E are coefficients representing the types of deviations which contribute to the total [3].

Coefficient A is the coefficient of constant deviation. This error is caused by the misplacement of the lubber line of the compass.

Coefficients B and C are the coefficients of semicircular deviation. They are caused by subpermanent magnetism induced in the hard iron of the aircraft.

Coefficient B is caused by the magnetic fields along the longitudinal axis of the aircraft. Its maximum effect is apparent on headings of east and west. The deviation is proportional to the sine of the compass heading.

Coefficient C is caused by the magnetic fields along the lateral axis of the aircraft. The maximum effect is apparent on north and south headings. The deviation is proportional to the cosine of the compass heading.

The errors represented by coefficients B and C are called one-cycle errors.

Coefficient D represents quadrantal deviation which are proportional to the sine of twice the compass heading. These deviations are caused by induced magnetism in horizontal soft iron which is symmetrical with respect to the compass.¹

Coefficient E represents quadrantal deviations which are proportional to the cosine of twice the compass heading. These deviations are caused by induced magnetism in horizontal soft iron which is asymmetrical with respect to the compass.

Deviations represented by Coefficients D and E are called two-cycle errors. These are not ordinarily compensated for in aircraft compass systems because of their small value but are included in the deviation card.

These coefficients and their total deviations are shown in Figure 5, which was taken from Reference 3.

3.3.2. ACCURACY STUDY—SLAVED MODE. The errors in a slaved gyromagnetic compass system can be divided into two major classes, static and dynamic. Static errors are

¹"Soft-iron" is defined ideally as that material which has zero coercive force and zero remnant induction. Actually, the term is applied to materials with very little coercive force and remnant induction, which causes hysteresis loops of small area. (See the Bibliography, N. S. Spencer and G. F. Kucera.)

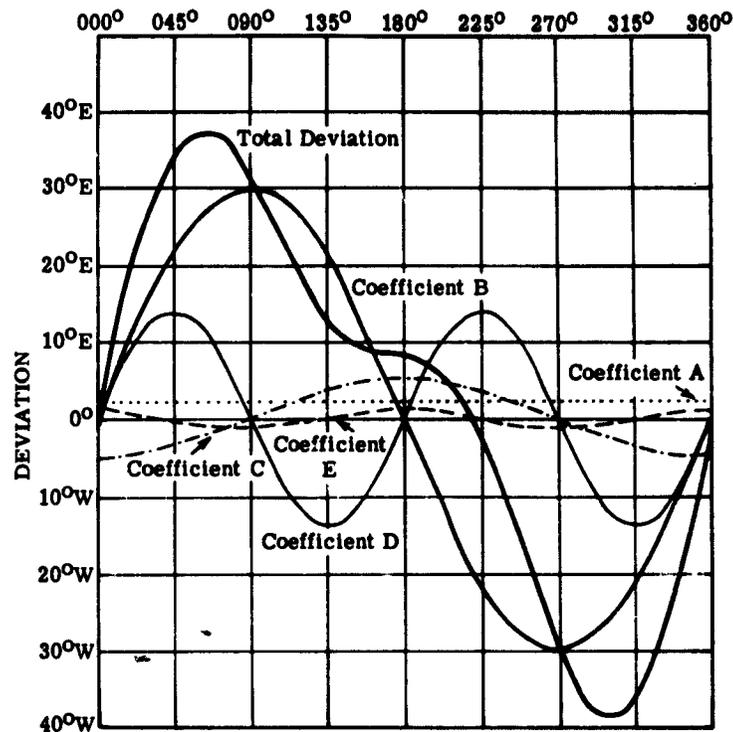


FIGURE 5. COMPASS HEADINGS

associated with ground swinging; both static and dynamic errors are associated with air swinging [1].²

Static errors are produced by disturbing fields external to the compass system. These fields, mainly due to ferromagnetic bodies in the aircraft, cause the magnetic azimuth detector to transmit erroneous heading signals. The ferromagnetic bodies can produce either a fixed field (hard-iron effect) or an induced field (soft-iron effect). Disturbing fields due to d-c cables can also be produced. All these disturbing fields can be treated by two methods; they can be eliminated or they can be neutralized. The elimination of these fields is usually out of the question in a completed aircraft. Compensation for their effect is the only practical solution.

Conventional "precision" ground swinging and compensation procedures using a digital readout periscopic sextant as performed by TWA results in static heading errors on the order of 0.1° on a calibrated heading. Time required for swinging is four or five hours. Although

²Bowditch defines swinging as "placing the vehicle on various headings to determine compass deviation."

not used commercially, an "electrical ground swing" with Sperry's MC-1 Compass Calibrator has been used on military aircraft for some time [4, 5]. Careful use of this equipment is reported by the Air Force to result in similar static-heading errors (0.1°) and has been reported to require 1/4 to 1/2 the time and effort required for physical swinging. The cost to the Air Force for 100 MC-1's was somewhere between \$10,000 and \$20,000 each. This equipment is used by most of the major airframe manufacturers (United Aircraft, Lockheed, Douglas, Boeing), but to our knowledge no airline currently uses it. While certain advantages over conventional ground swinging are claimed (for example, the continuous measurement of the true magnetic field may be expected to eliminate time-varying errors during a swing), there is the disadvantage that two-cycle errors caused by horizontal soft iron will not be corrected. Though this disadvantage may not be important in current aircraft, it is likely to be serious in future supersonic transports.

A precision gyro compass of the type represented by the Autonetics MABLE and ABLE developments could be used as a heading reference during a compass swing. In its present form this equipment might need some modification to be adapted to the compass swinging job, and its advantages over an electrical compass calibrator are not obvious.

The high performance of present and future aircraft that necessitates accurate heading information, aggravates, at the same time, the dynamic-error problems. An aircraft in flight will experience accelerations causing the magnetic azimuth detector to sense a direction other than that of the horizontal component of the earth's field. The gyro will be subject to real and apparent precession. The airframe will distort. Eddy currents will produce disturbing fields, and the system's slaving response may not be suited to the immediate situation. Insofar as these errors are random, their effect decreases with distance. They are discussed in the following paragraphs.

3.3.3. ACCELERATION EFFECTS. The magnetic azimuth detector is pendulously suspended in a dampening fluid. When subjected only to the acceleration due to gravity, it will detect the horizontal component of the earth's magnetic field. However, in flight, accelerations having components in the horizontal plane will cause the sensing unit to be inclined to the horizontal. A horizontal acceleration in the direction of the longitudinal axis of the aircraft (in an east-west heading) of one g with a duration of 20 seconds will produce an error of only 0.87° in a typical system. An acceleration of only 0.1 g, however, with a duration of three minutes will produce an error of 6° .

A lateral acceleration (parallel to the horizontal projection of the transverse axis) will cause the sensing element to hangoff from the gravity vertical. The resulting error will be

most pronounced on north-south headings. Encountered in flight are two accelerations of this type — Coriolis and turning (including the "north turning error").

The Coriolis error is directly proportional to speed, sine of the latitude, tangent of the dip angle, and the cosine of the heading. The error for a 500-knot aircraft heading due north at a latitude of 15° would amount to only 0.15° . At 75° latitude the error would be about 1.4° [1].

In a coordinated turn the hangoff of the sensing element will be equal to the angle of bank. The angle of bank for a given rate of turn depends upon the aircraft's speed. A "standard rate" turn (3° per second) at 100 knots will result in a bank angle of approximately 15° . At 500 knots the bank angle would be approximately 53° . The extent to which this hangoff will introduce errors depends upon the type of slaving and the parameters of the turn. A slow turn maintained for several minutes could result in a large error. For example, a one-twelfth standard rate turn held for six minutes would result in an error of about 9° to 12° . During such a situation the gyro not only receives false information but also is subjected to a false vertical. The leveling system then precesses the spin axis into a plane that is not horizontal. The component of this precession appears as an error in azimuth.

A special case of the turning error is called the north-turning error. When an aircraft is on a northerly heading (in the northern hemisphere), in an area where there is a vertical component of the earth's magnetic field, and rolls into a coordinated turn in either direction, the magnetic sensing element will respond to the vertical component in such a manner as to indicate a turn in the opposite direction. If steering information is now being obtained from the compass system, a correction will be made that will increase the turn, which will continue until the total turn exceeds the desired turn; this will be recognized only after returning to level flight. At this time a turn in the opposite direction will be initiated, and an error will generate in the opposite direction. Thus, the aircraft's attempts to maintain a constant heading will result in an oscillatory path.

On a southerly heading the error produced by the vertical component is in such a direction that the indicated turn emphasizes the actual turn, and the result is a dampening of any oscillation. In the southern hemisphere the situation is reversed.

Under certain conditions an acceleration error can be produced by oscillatory yaw, especially in high-performance aircraft. In a coordinated dutch-roll maneuver the dynamic vertical will coincide with the aircraft's z axis and will average the g vertical. An error produced by this hangoff will average zero. However, in an uncoordinated dutch-roll (typical of high performance aircraft) the dynamic vertical will not average the g vertical, and a hangoff error

will be produced. This error can be on the order of a few tenths of a degree; whether or not it is important depends largely upon the characteristics of the particular aircraft.

Turbulence in flight will produce transient accelerations. These accelerations, causing the magnetic sensing element to swing in a damped oscillatory manner and being of short duration, will not influence the gyro heading.

We have now examined all the acceleration effects. The next dynamic error to be considered is caused by earth rate and meridian convergence.

Since the direction of the spin axis of a free gyro tends to remain fixed in space, the rotation of the earth will cause an observer on the earth to sense an apparent drift. This apparent drift is called earth rate. Meridian convergence, on the other hand, is the apparent drift that results from the gyro's being transported over the surface of the earth. The meridian convergence is a function of the sine of the heading angle. This means that on an easterly heading it will add to the earth's rate, whereas on westerly heading it will subtract.

The magnitude of the error due to the earth rate effect in a slaved system depends upon the geographic latitude and the slaving rate. With a slaving rate of one degree per minute per degree at the latitude of New York this error would amount to 0.16° . At a latitude of 65° this error would amount to 0.23° . It should be noted that most equipment exhibits a slaving rate of 1° to 2° /minute. The error due to meridian convergence is a function of the aircraft's velocity, the slaving rate, the tangent of the magnetic dip angle, and the sine of the magnetic heading. The maximum error due to meridian convergence in an area where the magnetic dip is 80° is about 0.26° for a slaving rate of one degree per minute per degree and a 500-knot aircraft.

3.3.4. GIMBAL ERROR. Errors due to the geometry of a two-gimbal system can result when an aircraft departs from a level attitude. These errors develop as a result of changes in both pitch and roll. For small bank angles the error is for practical purposes a two-cycle error. For a bank angle of 30° this error could amount to a maximum of 4° . For bank angles of only 5° the two-cycle error would amount to 0.2° . In every case, however, the error disappears when the aircraft returns to straight and level flight and for that reason is not of much importance in commercial missions.

3.3.5. AIRCRAFT DISTORTION. When an aircraft is at rest on the ground, the weight of the structure is supported by the undercarriage. When the aircraft is airborne, the same weight is supported by the lift of the wings. This change in the loading of the structure can cause a noticeable distortion of the wing tips with respect to the axis of the aircraft. If the magnetic azimuth detector is located in the wing tip, it is possible that an index error will occur when the aircraft is airborne. This error can be classified as constant, since the wing

remains in nearly the same position throughout flight. When necessary, it can be corrected by airborne swinging of the compass.

3.3.6. EDDY CURRENTS. It is possible that a magnetic field can be produced by eddy currents in the conductors of a moving aircraft. However, since there are numerous conductors and the orientation is somewhat random, the total field due to the eddy currents is very likely to be zero.

Direct current flowing to lights and control motors in the aircraft may produce a magnetic field causing errors in the magnetic azimuth detector output. How great this effect is, of course, depends on the magnitude of the current and the proximity of the conductor to the magnetic detector. Some currents may never appear until the aircraft is airborne, thus defeating the value of a careful ground swing. A one-ampere current flowing in a wire one foot below a fluxvalve will cause approximately a one degree error in heading at the worst point of the one cycle error. If the horizontal component of the field is 0.18 oersted (the value at New York). In the design of the aircraft care should be taken to locate conductors away from the magnetic detector, and in swinging the aircraft tests should be made both with and without current flowing in conductors near the magnetic azimuth detector.

3.3.7. CAPABILITIES OF PRESENT MAGNETICALLY SLAVED GYRO SYSTEMS. A large number of manufacturers produce slaved gyro systems in a variety of qualities and prices.

As a rule, the operational accuracy of a slaved system is quoted as 0.75° or 1° rms and is essentially independent of cost. (See Section 1.2 for discussion of this accuracy's meaning.) The reason for this uniformity is that in the magnetic mode all the systems are essentially the same unless special conditions, such as radical maneuvers or attitudes, are required. It will be of more interest to compare the qualities of the associated gyroscopes. This will be done in Section 3.4.

3.3.8. MAGNETIC REFERENCES FOR SLAVING. For many years compasses have been an afterthought in the design of aircraft. The exigencies of proper structural and aerodynamic design have, perhaps quite properly, been foremost in the aeronautical engineer's mind. The magnetic detector was then installed in the place with the smallest residual field, provided there was room there.

Test engineers at Johnsville Naval Air Development Center were using an R-4-D for experimental magnetic compass work and discovered an eight-foot piece of steel in a wing, along with 1000 steel screws. To clean up the aircraft so that good compass performance could be

obtained required a large amount of effort. Modern aircraft, even though they are large, have space limitations created by aerodynamic design which make compass installation difficult.

Sperry Phoenix has had a development program for a subminiature magnetic azimuth detector for present and future high-speed aircraft. Prime consideration was given to minimum size, wide environmental capabilities, and compatibility with existing compass systems. A breadboard model was 1.5 inches wide and 1.25 inches high—of great importance when considering installation in thin vertical stabilizers. Performance was similar to that of the J-2 compass system. Although the development was discontinued for lack of an immediate application, Sperry engineers believe that potential performance is better than the J-2 system. The development work to date may be of use for future aircraft capable of Mach 2 and Mach 3 speeds.

Of perhaps more importance to the magnetic performance of detectors is the nature of the slaving amplifier. Sperry improvement of the AGC in this amplifier permits achieving $1/4^{\circ}$ to $1/2^{\circ}$ standard deviation accuracies down to field strengths of about 0.02 oersteds horizontal component. (Current compasses exhibit such accuracy down to only 0.18 oersteds). The elliptical area surrounding the magnetic pole defined by a 0.020-oersted horizontal component contour (which for this Sperry development represents the area in which magnetic heading is unusable) is quite small—about one hour's flight time over the long axis of the ellipse at Mach 1.

Although there are still problems of a rapid change of variation at high latitudes, and a somewhat larger standard deviation of variation, the Sperry development introduces a possibility of fair magnetic performance on polar routes. Even if the primary heading reference for polar routes continues to be a free gyro, this improved magnetic performance would provide a useful backup.

3.4. FREE GYROS

3.4.1. INTRODUCTION. Gyroscopic instruments may be divided into two classes: free gyros and non-free gyros. A free gyro may be defined as a displacement gyro whose spin axis orientation is not slaved to coincide with some externally defined reference. For example, the orientation of the spin vector of an ideal free gyro would remain fixed in space. The use of the term "free gyro" when discussing heading references is somewhat misleading. The block diagram of Figure 6 illustrates what is commonly meant by free-gyro mode of operation when discussed in the context of heading-reference systems. Operation in the free-gyro mode merely means that the gyro is no longer slaved to the external reference. However, since the function of the heading-reference system is to provide a measure of aircraft heading in earth

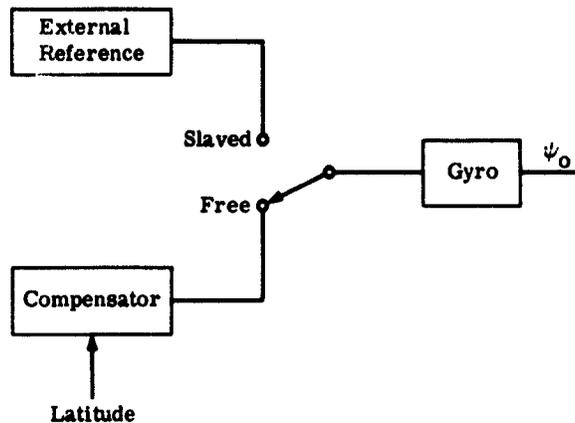


FIGURE 6. FREE-GYRO MODE OF OPERATION

coordinates, compensation must be provided for the fact that the coordinates of the earth are rotating with respect to the orientation of the spin vector of the free gyro.

It might be helpful to regard the sensitive element (which in principle tries to maintain its spin vector orientation fixed in space) simply as an input-output device. Its output is angular orientation and its derivatives; its input is torque. In an ideal gyroscopic heading-reference system this input torque would be zero except for the compensation torques intentionally provided in order to maintain a reference in the desired coordinates.

3.4.2. SOURCES OF ERROR. The sources of the errors which will limit the performance of a free gyro functioning as a heading reference may be placed in three groups: external reference error, compensation error, and errors relating to practical gyro design.

External reference error. It is clear that the accuracy in the output measure of heading provided by a free gyro cannot exceed the orientation accuracy given to the spin axis by the external reference. It perhaps should be re-emphasized that a free gyro is merely trying to function as a memory and retain its initial orientation.

Compensation error. As was discussed earlier, a free gyro used as a heading reference often includes compensation for earth rate, since the desired measure of aircraft heading is defined in earth coordinates which are rotating with respect to inertial space. Since the compensation torque required by the free gyro is a function of the latitude of aircraft position, the total error attributed to this compensation requirement will include the input error of aircraft latitude and the instrumentation error of the compensating device.

Errors relating to practical gyro design. Any practical gyro design will be limited in performance by the residual torques affecting the sensitive element. The sources of drift, which are well known to the designers of gyros, are often categorized as follows: (1) anisotropic drift, which is proportional to g^2 and relates to the geometry of the gimbal structure and its bending properties; (2) unbalanced drift, which is proportional to g and is attributed simply to that residual unbalance which is impossible or impractical to remove in a given design; (3) certain spurious torques, which act on the gyros and cause them to precess or drift from their prescribed orientation with respect to inertial space—this is perhaps the most important single problem in the design of inertial systems. Often included in these torques are the effects of non-viscous friction or "stiction."

Geometric errors. Another error in heading measure, which troubles not only free gyros but slaved gyros as well, is related to the gimbaling geometry. This geometrical effect, often referred to as intercardinal tilt error, is characteristic particularly of two-degrees-of-freedom gyros. Since this error relates to the geometry of the gimbaling, it can be analytically expressed in Figure 7 as follows.

Directional gyro, intercardinal tilt error:

$$\tan \psi_G = \frac{\sin \psi \cos \phi - \cos \psi \sin \theta \sin \phi}{\cos \psi \cos \theta}$$

$$\text{Heading error} = \psi - \psi_G$$

where ψ_G = heading as measured by directional gyro; ψ = actual heading

θ = aircraft elevation angle; ϕ = aircraft bank angle

Gyro spin axis is oriented to point north and maintained parallel to earth tangent plane.

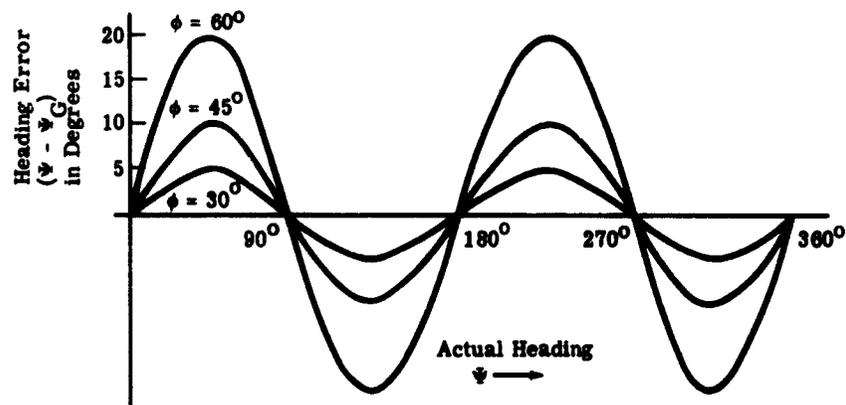


FIGURE 7. HEADING ERRORS VS ACTUAL HEADING FOR SEVERAL BANK ANGLES (φ)

3.4.3. TWO-GYRO PLATFORMS. Intercardinal tilt error can be eliminated by the use of a two-gyro platform. Ordinarily such a platform has an all-attitude capability and yields a heading reference for a maneuvering aircraft which is not degraded as discussed in the last paragraph. The utility of such a platform for commercial aircraft is doubtful, since minor deviations from true heading can be accepted during turns and such aircraft are not in acrobatic service.

3.4.4. INERTIAL NAVIGATION SYSTEMS. An inertial navigation system of the type discussed in Section 5 constitutes a precise heading reference. In addition, in combination with a doppler radar it offers the possibility of airborne gyrocompassing as an external reference for a free gyro. Detailed discussions of the properties of such a platform appear in Section 5.2, and it will only be noted here that it would not be economical to have such an instrument for heading reference alone.

3.4.5. EQUIPMENT DEVELOPMENT PROGRAMS. Gyroscopes may be acquired today with drift rate performance ranging from more than 10° /hour-maximum to 0.001° /hour-rms. For the accuracy required by commercial aircraft the latter is not of great interest. One interesting technique is that of friction averaging by rotating or oscillating the gimbal bearings in a gyroscope. Several manufacturers (Kearfott, Bell) have used this technique, and it is known commercially by several names. Sperry Gyroscope Company has developed the Roto-Race bearing, the principle of the C-11 gyro. A recent improvement of the Roto-Race is described in a final report on the development of a non-floated "inertial quality" gyroscope [6]. Using the same "Roto-Race" principle, and extremely careful construction, Sperry built a directional gyro with random drift rates of about 0.03° /hour-rms. Since this was a developmental unit, cost information is not firm. The manufacturer estimates that it can be produced for \$6000, which puts it between the C-11 Roto-Race gyro and a floated gyro.

For much equipment a significant price reduction follows a large production order, a saving which occurs because tooling must be set up only once, and, if the production run is large enough, automation may be used extensively. Such a gain does not appear to be as large a factor in high-accuracy gyroscopes as in, say, automobiles. What makes a gyroscope good is largely hand work and testing; and these, being primarily labor costs, do not change much with mass production. Consequently, for some time the costs \$9000, \$6000, \$3000, and \$1000 are likely to be realistic for the floated (0.01° /hour-rms), advanced friction averaging (0.03° /

hour-rms) C-11 or equivalent (0.25 or 0.5°/hour-maximum), and the more conventional slaved system gyro (4°/hour-maximum), respectively.³

In fact, these costs are all small enough that it would probably be worthwhile for an airline to buy one which gave a real advantage. At this writing it would appear that this unit would be the 0.25°/hour instrument as represented by the Kearfott friction-averaging gyro or the Sperry Roto-Race gyro. There is every reason to believe that performance as demonstrated by the recent Sperry development will be realized in a commercially available gyro, which is no more difficult to maintain than present instruments. On the other hand, the floated gyro, while capable of somewhat better performance, brings with it disadvantages of longer warmup, alignment, and drift rate correction times, and more critical maintenance.

A major improvement in gyro performance brings with it a need for better control of the latitude-dependent earth-rate parameter. At 45° latitude, an error of 1° in setting in the latitude correction results in an error of 0.13°/hour—a value which is quite acceptable with a 1°/hour gyro but unacceptable for a 0.1°/hour unit. Consequently, one can expect that a somewhat more expensive controller will be used with a better gyro. A 0.01°/hour gyro would deserve latitude settings accurate to a tenth of a degree or better, but such information would not be easy to use even if it could be obtained.

With today's directional gyros it is common to bias out the drift rate by applying a false latitude correction. Tables are prepared for the flight personnel to permit them to correct for drift rates observed in flight in this manner. One development, which the Army has sponsored, incorporates two controls on the gyro drift rate—one to set the latitude rate, and the other to bias out the drift rate. This arrangement has been judged convenient by those who used it, and may be highly desirable when the gyros are more stable.

3.4.6. EXTERNAL REFERENCES. As has been noted, the free gyro is not, by itself, capable of determining true heading—it is only a memory. Therefore, one must have a means for inserting true heading at the beginning of a flight, and if the drift rate of the gyro is such as to produce a serious cross-track error on a long flight, one should have a method of correcting this error during flight.

³It is conventional for the poorer gyros to be rated in terms of maximum error; the more precise ones, in terms of rms error. This convention evidently results from the military specifications on magnetically slaved gyroscopes, which insist on no worse than a particular performance, and from the use of higher quality gyros on platforms where errors expressed in terms of standard deviation are useful in error propagation equations. Since the errors of the poorer gyros are probably not normally distributed, it is not easy to quote all of these types in the same terms.

Several such methods exist. The magnetically slaved gyrocompass has already been discussed; slaving provides a continuous correction to the gyro in latitudes where the horizontal component of the magnetic field is large enough (Section 3.3.8). The earth-rate-direction reference, or airborne gyrocompass, will be discussed in Section 3.6. This system derives heading from the earth's rotational rate, and it may be said that it effectively provides continuous correction for a free gyro.

True heading can be inserted at the beginning of a flight in several ways. A known direction at the airport can be observed visually (by the equivalent of a gunsight) and this direction then transferred to the free gyro. Alternatively a ground-based gyrocompass could be mechanically coupled to the aircraft for insertion of true heading. Of these, the gunsight method appears to be about as accurate as the other, and considerably simpler.

Two other methods will be mentioned. The free-gyro heading can be corrected intermittently or continuously by referring to some celestial body and using the appropriate auxiliary data. Alternatively, the effective track of an aircraft, which may be directly related to the heading if drift angle is known, can be determined by a sequence of geodetic fixes. Again, this method is usually an intermittent technique, although rates of travel through a radio field, for example, could make it continuous. The celestial method has been investigated and is reported in Section 3.4.7.

3.4.7. ANALYSIS OF OPERATIONAL USE OF A FREE GYRO HEADING REFERENCE SYSTEM

3.4.7.1. Summary. This section describes some analyses of records obtained from the Office of the Chief of Navigation, Pan-American Airways. These records contained copies of the original "Free Gyro Logs" maintained by Pan-Am navigators on North Atlantic flights during 1961. The "Logs" covered about 150 trips for some 25 aircraft in the year 1961. The number of trips per aircraft varied from one to fifteen, and numbers of eastbound and westbound trips are also widely out of balance.

Some "Logs" were found to be incomplete or otherwise unusable. In a number of cases a single trip was broken up into several flights or legs for analysis because of intermediate landings, a gap in the record, or a change of mode of operation. No information was given in these records about maintenance, servicing, adjustments, or changes of the gyro units in the aircraft.

Each aircraft, either a Douglas DC-8 or Boeing 707, had two Sperry C-11 systems installed. The navigational procedure used the master gyro of the system as the primary gyro. About every 20 or 30 minutes the navigator aboard made a celestial fix and compared the

heading so obtained with the primary gyro heading indication. The "Free Gyro Logs" contain the necessary tabulations of these comparisons and a plot against time of the differences between these two heading indications. Resets of the free gyro were made by the navigator whenever the gyro drift was considered excessive. No records were kept on the performance of the secondary gyro in each aircraft.

In approaching the analysis of these records of operational performance of a free-gyro heading-reference system, our interest has been to obtain information to answer the following questions:

- (a) What is the average drift rate with time of the primary gyro in these aircraft trips?
- (b) How does this average drift rate behave from trip to trip?
- (c) What is the nature and magnitude of the residual variation from the line of average drift rate for each flight?

In order to obtain information on these three points, we made a regression analysis of observed drift versus time. As an example, consider the hypothetical series of Table III.

TABLE III. OBSERVED DRIFT VS. TIME

Greenwich Mean Time	Difference* Grid Heading — #1 Compass (°)	Elapsed Time (min)
1800	0	0
1820	-1	20
1850	-3	50
1910	-2	70
1930	-4	90
2000	-7	120
2020	-9	140
2040	-11	160
2100	-8	180
2130	-10	210

*Resets if any have been removed.

The elements in the central column were labeled as Y_{t_j} and in the right hand column as T_j . A linear regression of y on T seemed a reasonable assumption for most flights. Many flights, of course, showed large deviations from the assumed linear regression; perhaps this indicates some nonlinearity in the gyro behavior, but this matter has not been pursued. The available data may contain much additional useful information, but time has not permitted extracting it.

As a model for the linear regression, we may write

$$Y_{t_j} = a + \beta_t T_j + \epsilon_j$$

The least squares estimate b_t of β_t then measures the average drift rate for a single flight. Both the sign and magnitude of the drift rate has been studied. For assessing the residual variation, the statistic $s_{y.t}^2$ is an unbiased estimator of the variance of the ϵ_j 's which are considered to have mean zero and variance $\sigma_{y.t}^2$.

With regard to sign of the drift rate, only four out of seventy-two flights showed a zero drift rate. Only one such flight was observed for each of four of seven aircraft summarized. Thus, the excellent performance cannot be associated with equipment or a second officer. For the other sixty-eight flights, the signs are summarized in this 2×2 contingency table with marginal totals as shown in Table IV.

TABLE IV. SUMMARY OF DRIFT-RATE SIGNS

Sign of Drift Rate	Flight Bound		
	West	East	
Plus	11	17	28
Minus	23	17	40
	34	34	68

This small table is a composite of the seven tables which could be prepared for each aircraft. Eastbound flights appear to be balanced as to sign, but westbound flights show a preponderance of negative drift. Aircraft were not uniform in this preponderance: some showed more positives, and some more negatives. Standard statistical analysis of the results in the table indicates that the observed results or results more extreme would occur by chance only about once in ten trials.⁴

Considering next the magnitude of the average drift rate, we found that the westbound flights showed an average value for b_t of $\bar{b}_t = -0.935^0$ per hour while the eastbound flights gave $\bar{b}_t = +0.0114^0$ per hour. The latter is very small, the former rather large. Unfortunately, the variation of the average drift rate from flight to flight about these averaged values was

⁴For explanation of the above techniques, refer to any standard text in statistical methods (e.g., G. W. Snedecor, *Statistical Methods*, 5th Ed., ISU Press, Ames, Iowa, 1956; or W. Dixon and F. Massey, Jr., *Introduction to Statistical Analysis*, 2nd Ed., McGraw-Hill, New York, 1957).

⁵For chi square for contingency tables, see Snedecor.

found to be large; standard deviation of the average drift rate within aircraft was 2.33° per hour for eastbound flights and 1.64° per hour for westbound flights.

Analyses of the residual variation now available are based on only 36 flights, but it is believed that the information is indicative of the general situation. Interpretation of the variance $s_{y,t}^2$ described above, and its square root, the standard deviation $s_{y,t}$ is interesting. It would appear that these quantities must contain at least two components of variation. One component is due to the errors in the celestial fixes taken to determine the status of the free gyro indication; the second is due to the supposedly random drift variation of the gyro from its average drift rate on a particular flight. Perhaps this second component can be divided further into subcomponents.

Of the 36 flights summarized at this date three showed extremely large values of $s_{y,t}^2$ (i.e., $s_{y,t}^2 > 14$). Averaging then over the 33 flights included, we found that the $\bar{s}_{y,t}^2 = 1.8424$ from which $\bar{s}_{y,t} = 1.36^{\circ}$. Separation into east- and westbound flights gave the following results:

Eastbound: 15 flights; $s_{y,t}^2 = 1.7696$

Westbound: 18 flights; $s_{y,t}^2 = 1.8836$

We conclude that the residual variation is homogeneous with respect to direction of flight. For judging these results of the regression analyses it is of interest to add at this point a report that the overall average correlation between y_t and T was found to be $r = 0.839$.

The numerical results above may be summarized as follows. No actual data are available to us for separating the residual variation into its components. We may hazard the opinion that our estimate $\bar{s}_{y,t}^2 = 1.8424$ comprises about equal contributions from the celestial fix error and from gyro random drift sources. Since the points are plotted about every 20 or 30 minutes, we may say that in such a period there is about a one-degree contribution to the residual variation from each of the two sources. Returning to the average drift rate, we have found the overall average drift rate to be small for eastbound flights, but it was about -1° /hour for westbound flights. Both flight directions showed large variations among aircraft from the average aircraft value. The standard deviations were 2.3° /hour eastbound and 1.6° /hour westbound for the drift rates about their respective average values.

3.5. HEADING REFERENCE DERIVED FROM CELESTIAL INFORMATION

3.5.1. DESCRIPTION OF METHOD. The determination of direction by visual or optical celestial means is as old as history. It still provides the most accurate and stable heading reference. For aircraft use, however, there are limitations in accuracy because of instrumentation errors, position uncertainties, and time errors. In addition, cloud cover can make the system unusable.

A spherical triangle on a celestial sphere is called a celestial triangle. The navigation triangle is the celestial triangle formed by arcs of a celestial meridian, an hour circle, and a vertical circle. The terrestrial counterpart is also called a navigation triangle, being formed by arcs of two meridians and a great circle connecting the two places on earth, one on each meridian. It is formed by the assumed position of the observer and the geographical subpoint of the celestial body (the place having the body in its zenith). This subpoint is of particular interest in determining direction by celestial means. At any particular time from any particular place on the surface of the terrestrial sphere there is only one true azimuth to the subpoint of the body. This true azimuth can be obtained from tables, such as H. O. 249. By measuring the relative bearing from the aircraft to the celestial body and knowing the true azimuth of the celestial body, the aircraft heading can be determined.

For years navigators requiring true heading information from celestial observations have relied on the common astrocompass, which is essentially a celestial compass. By sighting known celestial bodies the true heading of the aircraft can be obtained. This device has been used primarily in the polar regions as an occasional check on gyro heading and is not suitable for a continuous heading reference. It is mentioned here because it represents a basic tool for determining celestial heading. Its accuracy is quite limited, perhaps 3° or 4° standard deviation. The astrocompass is shown in Figure 8.

3.5.2. PRESENT EQUIPMENT AND TECHNIQUES

3.5.2.1. The Periscopic Sextant. A more modern device is the periscopic sextant. In addition to measuring celestial altitudes the periscopic sextant can be used to determine true heading. Thus the need for the rather crude astrocompass is eliminated. Of course, this also eliminates a need for an astrodome and the attendant danger associated with pressurized aircraft. The Kollsman periscopic sextant is shown in Figure 9.

True heading can be obtained with this sextant by either the true-bearing or the relative-bearing method. For the true-bearing method (1) set the true azimuth of the celestial body in the azimuth dial window, (2) bring the body into collimation, and (3) read the true heading under the vertical crosshair on the azimuth scale. If precomputation techniques are being used, a true-heading reading is obtained every time an observation for an LOP is made. For the relative-bearing method (1) bring the body into collimation, (2) turn the azimuth crank until 0 degrees is under the vertical crosshair, (3) read the relative bearing of the body in the azimuth dial window, and (4) solve for the true azimuth of the body and determine the true heading from this.

The accuracy of the true heading with the use of the periscopic sextant depends on several factors: (1) the accuracy of one's assumed position (this determines the accuracy to which the

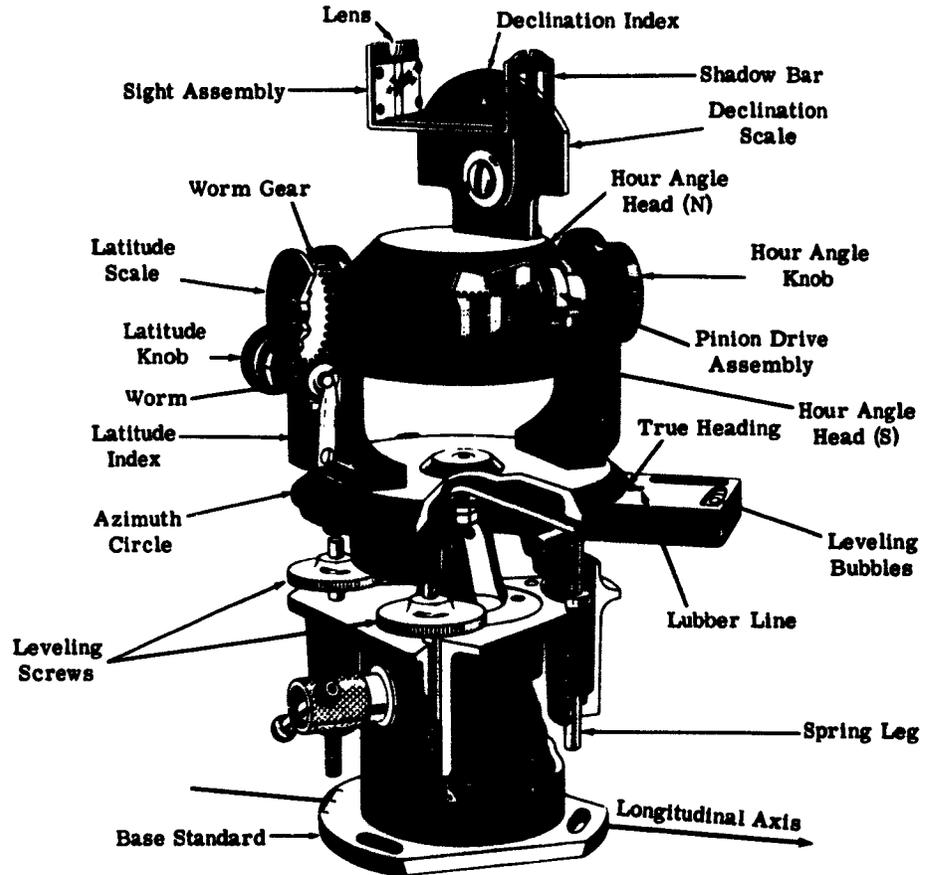


FIGURE 8. THE BASIC ASTROCOMPASS



FIGURE 9. KOLLSMAN PERISCOPIC SEXTANT

true azimuth of a celestial body can be determined); (2) the skill of the operator; (3) most important, the altitude of the body. For star altitudes up to 45° the accuracy is very nearly constant. From 45° to 70° the accuracy degrades slightly. Above 70° of altitude there is a heavy degradation in azimuth accuracy and, of course, at 90° it is indeterminate.

While the azimuth dial on the periscopic sextant used by most commercial carriers can be set to one tenth of a degree, the statistical study of field data obtained during the period of this contract has indicated that the accuracy under operational conditions has a standard deviation of about 1.0° (Section 3.4.7). Since this is likely to be a random error, the overall accuracy of the heading information can be better than this with multiple observations.

The Kollsman periscopic sextant is priced from \$2600 to \$3000.

3.5.2.2. A Photoelectric Sextant. Primarily with strategic bombers, considerable improvement has been made during the last few years in celestial equipment for aircraft. The Kollsman photoelectric sextant KS-85 (AVN-1), which has been used successfully in military

aircraft for some time, is now declassified and available for commercial airlines. Although no commercial airline has used this equipment to date, Pan-American World Airlines is making arrangements to test the Kollsman KS-85 in December 1962. The equipment is shown in Figure 10.



FIGURE 10. KOLLSMAN PHOTOELECTRIC SEXTANT
(KS-85)

There are several distinct advantages to this type of equipment for over-ocean flights. According to the manufacturer, the photoelectric sextant is less susceptible to errors resulting from oscillatory roll and yaw of jet aircraft than the standard periscopic sextant. In addition, the entire operation can be conducted from the copilot's seat. The entire system weighs 47 pounds and consists of three units: a tracker, a central amplifier, and a computer indicator. The computer indicator, measuring $5\frac{3}{4} \times 4\frac{3}{4} \times 7\frac{3}{16}$ inches, is the only unit that has to be accessible in flight. Precomputed star information is set into the computer-indicator and, when the tracker unit locks on the desired star, true-heading information is displayed in the center window to a least count of 0.1° . The rms accuracy of true heading is stated as 0.3° for star altitudes of -5° to 45° . The accuracy degrades slightly from 45° to 70° of star altitude, and heavily above 70° of altitude. Of course, in addition to obtaining a true heading, the altitude of the tracked star is continually up-dated (with a dynamic rms error of 4.0 minutes), enabling the operator to plot an LOP by conventional celestial plotting techniques.

The manufacturer's brochure lists the following information. At night stars can be tracked down to -5° , star and planet sensitivity is +4.5 to -4.0 magnitude. The photoelectric sextant is capable of moon tracking to $1/4$ of full moon. The sensitivity control—"Star Brightness"—settings of BRT, MED, and DIM on the control panel aids in the acquisition of the desired star in the presence of dimmer stars. During the day the sun can be tracked down to -5° , the moon down to $1/4$ of full moon. Venus and Jupiter can be tracked when favorably located. The manufacturer also states that in twilight a 0.0 magnitude star can be tracked from 20,000 feet altitude when the sun is just trackable.

The mechanical limitations of the vertical platform are given as $\pm 7^\circ$ for both roll and pitch (based on accuracy); the system is not damaged by unlimited tilt.

The mechanical limitations of the celestial tracker are elevation -5° to 90° (relative to the aircraft) and azimuth $\pm 1\frac{1}{2}$ revolutions with automatic unwinding.

The celestial tracker employs a rectangular search. The width of this search pattern is 7° total, and the height is 3° total. If the required celestial body lies anywhere within this pyramid, it will be acquired. This feature makes the training of personnel comparatively simple, especially when precomputed star information is to be used.

At the end of the present program with the military, the commercial price of the photoelectric sextant is quoted as approximately \$30,000. This price is a manufacturer's estimate and does not include aircraft modifications, etc.

3.5.2.3. The Sky Compass. A second piece of modern equipment is the Kollsman Sky Compass (Figure 11), an instrument designed to yield true headings under conditions where

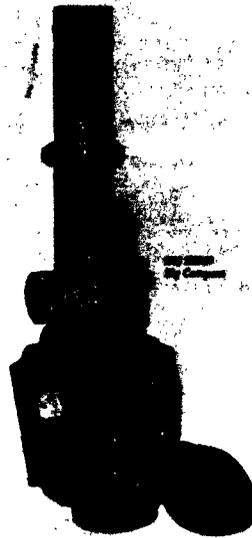


FIGURE 11. KOLLSMAN SKY COMPASS

the magnetic compass is impossible to use—in other words, at comparatively high latitudes and in the regions of the magnetic poles. This equipment analyzes the indirect or polarized rays of the sun reflected from the zenith when the sun is below the horizon or completely obscured by a partially overcast sky.

This technique is not new; the first instrument to use this principle was made by Dr. A. H. Pfund of Johns Hopkins University. In 1948, at the instigation of the Bureau of Aeronautics, the Bureau of Standard developed an adaptation of the Pfund Instrument. The Kollsman Compass involves the same principles as the Pfund Instrument, but is a new design and the first production unit of this type.

As mentioned before, this instrument performs well in polar region twilight. This may be a real advantage for high speed aircraft on east to west flights where the twilight period may last for most of the flight.

This equipment fits in the same mount as the Kollsman Periscopic Sextant. In order to obtain true heading with the sky compass it is first necessary to set in the precomputed

azimuth of the sun on the azimuth counter. The second step is to obtain the proper light match or dark match on a reticle and then read the true heading as projected on the azimuth scale. Accuracy in azimuth readings of the sun is quoted by the manufacturer as better than $1/2^\circ$ standard deviation.

The Kollsman Sky Compass (Figure 11) is not being used by any air carrier at present, although approximately 150 units are in use by the Navy and Air Force. The manufacturer's estimate of the cost is between \$1800 and \$2000.

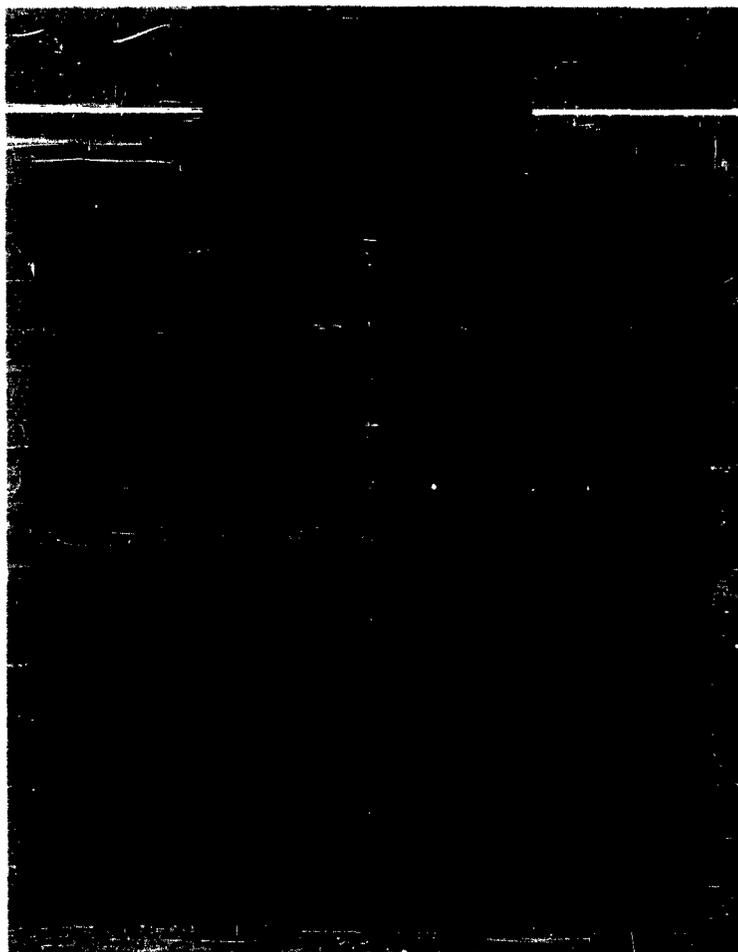
3.5.2.4. The Automatic Astrocompass. The final piece of equipment to be considered in this section is the Kollsman Automatic Astrocompass (Kollsman type KS-50). This has been operational in the B-52 for several years and represents the most accurate celestial-heading device studied during this program (Figures 12, 13, 14).

The automatic astrocompass is an electromechanical airborne device which produces a true-heading reference accurate to an rms error of six minutes of arc, according to the manufacturer's handbook. It has the ability to lock onto and continuously track a selected celestial body producing continuous heading as an electrical servo output and as a visual reference displayed on a counter. In addition, the system computes and displays true azimuth, celestial altitude, altitude intercept and heading correction (i.e., the difference between magnetically generated heading information and the astro heading). The inputs to the system are the aircraft's geographic position, the sidereal position of the selected celestial body, the position of the sidereal coordinate system in relation to that of the geographic (the Greenwich hour angle), and an estimated heading of the aircraft at the present time (best available true heading).

The automatic astrocompass uses these data to compute the line of sight to the celestial body by the altitude and relative bearing of the aircraft. This positions the tracking device so that it looks in the general area of the selected body. When the tracker locates the selected body, it will lock on and continually track, evaluating the observed information against the computed information and transmitting corrected true heading.

The system comprises twenty separate units divided into three groups. One group consists of the control and indicator units, which must be available to the operator. The second group contains those units which may be remotely located in any convenient portion of the aircraft structure—the power supply, amplifier, and mechanical computer.

The third group is the tracker, which must be located on the top of the aircraft and close to the longitudinal axis. The fore and aft alignment of tracker must be parallel to the longitudinal axis of the aircraft, and the vertical position should correspond within a few degrees to the normal vertical of the aircraft in flight attitude.



**FIGURE 12. KOLLSMAN AUTOMATIC ASTROCOMPASS CONTROL
AND INDICATOR PANELS**



**FIGURE 13. KOLLSMAN AUTOMATIC ASTROCOMPASS AMPLIFIER
AND COMPUTER COMPONENTS**

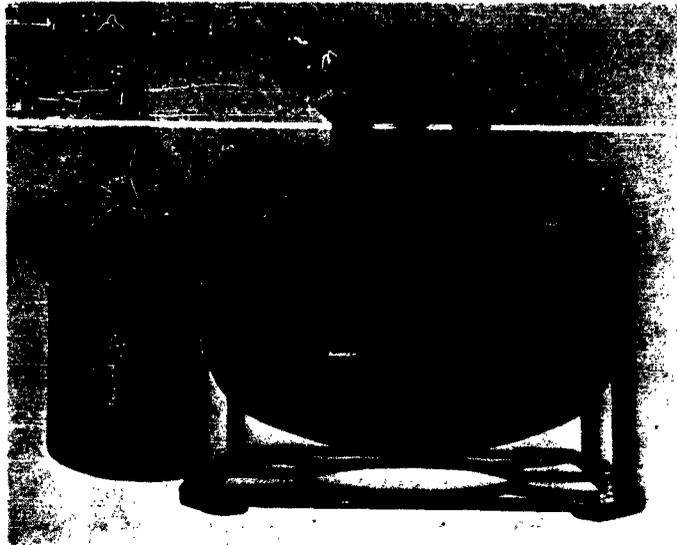


FIGURE 14. KOLLSMAN AUTOMATIC ASTROCOMPASS AND ASTROTRACKER

The three groups are shown in Figures 12, 13, and 14. Not shown in Figure 12 is that the star data display panel actually consists of three identical panels, enabling the operator to preset the sidereal hour angle and declination for three different celestial bodies. This equipment is somewhat bulky. It weighs close to 150 pounds and is estimated by the manufacturer to cost about \$65,000, including installation.

The principal advantage of a celestial heading reference is its basic accuracy. We have discussed here a number of instruments which can provide true heading from a celestial source, ranging from the inexpensive astrocompass, which requires a fair amount of training and is not very accurate, to the sophisticated and expensive automatic astrocompass, which is more accurate than necessary for a commercial carrier and probably too complex. There is a possible tradeoff in the case of the \$30,000 photoelectric sextant, since its accuracy is apparently much better than that of the manual periscopic sextant. Particularly in conjunction with high quality gyro heading references ($1/4^{\circ}$ /hour or better), this photoelectric sextant may provide a real advantage.

3.5.3. RADIO-CELESTIAL-HEADING DETERMINATION. For radio-celestial determination of aircraft heading two elements are required: first, a body in space capable of emitting a radio signal of sufficient strength to be tracked; and second, a tracking device aboard the aircraft capable of measuring the azimuth relative to a reference axis in the aircraft. In addition, it is expected that this information would serve only to correct a free gyro. The radio technique has the obvious advantage of being essentially an all-weather device and, hence, of particular value to low altitude aircraft, which must operate below the clouds for long periods of time.

Since the only natural celestial bodies which radiate enough energy in the radio spectrum to be useful are the sun and the moon, it is assumed that adding several satellites would be necessary. Such a system, used for position determination, is discussed in Section 6.3. In that section it is pointed out that satellites in relatively high orbits enjoy certain advantages, not the least of which is having orbits predicted accurately some weeks or months in advance. Consequently, tables could be made available to flight personnel so that heading could be determined in essentially the same way as with the more conventional optical instruments. The radio sextant would be similar to the Collins AN/SRN-4, but the antenna could be smaller. With an 18-inch parabolic receiving antenna the sun could be tracked, thus providing a backup to the basic satellite system.

System accuracy dictates that bodies be tracked only when they are below 45° altitude, but in a high-speed aircraft the astrodome could probably not protrude far enough to scan down to the horizon. Thus a very limited part of the sky would be available.

A pointing accuracy of 1 minute of arc (rms) has been estimated by the developer. Achieving this sort of accuracy with the total system would require that there be no other errors. But there will inevitably be some error in position of the aircraft, and it is interesting to investigate its contribution. For example, if the aircraft has a position error of 10 nautical miles, then the azimuth error varies from 10 to 13 minutes of arc as the satellite proceeds from the horizon to 45° . A second critical value is the time reference used. Since the satellite is traveling at one mile per second, or about four times the earth's rotation rate, time errors become four times more important.

The coverage analysis found in Section 6.3.2.2 for the angle-measuring system applies to radio azimuth determination, with one important exception—a satellite directly overhead will provide no heading reference and, as has been indicated, a degraded reference when its altitude is above 45° . Consequently, observation time is reduced although some time would be available at the beginning and end of every pass. It is unreasonable to assume continuous

(24-hour) heading information from this source (see Section 6.3.2.2 for a discussion of coverage vs. number of satellites in orbit).

The size, weight, and cost of this system would be identical with that of the altitude measuring system. The numbers are repeated here for convenience. The complete cost of a navigation system installed in an aircraft has been estimated at \$20,000 to \$30,000. This includes the antenna, receiver, and tracking mechanism; but it excludes the vertical reference system. Without the vertical reference system the weight is estimated at 300 pounds.

In spite of its basic accuracy and all-weather capabilities, this system does not appear very attractive as the only heading reference for high speed commercial aircraft because of its bulk and installation problems. If, however, the full satellite navigation system were used aboard an aircraft, this would offer a convenient and useful way for obtaining azimuth corrections for a free-gyro heading reference.

3.6. NORTH-SEEKING GYROS

Gyrocompasses have been used principally aboard ships, where vehicle velocities and sudden accelerations are small and latitudes of operation are generally below 70° . Under these conditions errors of $1/2^\circ$ are common, and maximum errors of 3° occur. One of the most significant errors, the speed error, arises because the vessel has an uncorrected velocity component in a north-south direction. The speed error causes an error in the apparent rotational vector of the earth, resulting in a heading error at the compass readout. For example, a ship at latitude 30° N steaming on true course 045° at a speed of 20 knots has an error of 1.04° . Any reasonable estimate of speed permits correction adequate for ship navigation purposes [7].

With aircraft, using airspeed as an input, one can see that large errors would be common. Consequently, gyrocompasses are of little use unless a good value of aircraft velocity is available. However, the advent of doppler radars has brought the capability to derive precise velocity information for aircraft, and the gyrocompass has become much more attractive [8].

Several airborne gyrocompasses have been built and tested. The ID-551/APN-105 was constructed under Air Force contract [9]. The earth rate directional reference was built by AC Spark Plug's Boston Facility (formerly the Dynatrol Corporation), and the doppler was the LFE APN-105. This particular model was tested briefly in the final stage of the contract, but the hurried experimental procedures were inadequate for proper evaluation. In bench tests with simulated doppler-derived velocities, errors were within tolerances ($1/4^\circ$ standard deviation). In airborne tests, however, no standards were available except the gyromagnetic compasses aboard the test aircraft, so that no useful accuracy information was obtained.

Another AC Spark Plug platform has been used with a GPL radar in the Airborne Long Range Input to the SAGE System. This combined the AJN-10 inertial platform with the APN-144 doppler radar, and production units have been installed in the EH-121 aircraft at Idlewild Airport. The true-heading error is said to be 0.17° standard deviation, but the APN-144 has a quoted standard deviation of the drift angle of 0.25° . One may only assume that the heading reference is basically better than the doppler, but that the overall system performance must be the rms sum of the two, or approximately 0.33° standard deviation. The U. S. Navy Bureau of Ships has contracted with the Sperry Gyroscope Division of Sperry Rand Corporation to build several models of a combination gyrocompass, gyromagnetic compass, and free gyro. With a precise velocity input (as from a doppler radar) the gyrocompass mode of operation is expected to exhibit a standard deviation of $1/4^\circ$ at moderate latitudes. The equipment resulting from this contract is expected to be delivered in late 1962 and will be evaluated at Johnsville Naval Air Development Center.

The only one of these systems for which production cost information is available is the GPL-AC Spark Plug combination. Cost to the Air Force, as provided by GPL, was

Doppler	\$ 16,000
Navigation computer	22,000
AC inertial platform	<u>70,000</u>
Total	\$108,000

The main advantages of such a system are the high accuracy and the ability to use inertially derived velocity during periods when the doppler is not operating. Further, if a doppler-inertial system is already aboard the aircraft, little additional cost or complexity is involved in adding the airborne gyrocompassing feature. A number of disadvantages are apparent: alignment time is 30 to 40 minutes on top of 10 to 15 minutes gyro heating time, accuracy at very high latitudes is poor, continuous power is required (loss of power, or large transients, results in loss of alignment), and system accuracy depends on the accuracy of the doppler velocity and position information. Doppler malfunctions (not complete failures) can cause large undetectable errors. Finally, for complexity much greater than a magnetic system, performance is not much better.

Not enough operational information has been collected to draw any conclusions on system reliability. It can be observed, however, that the mean time between failure must be appreciably lower than that of either the doppler radar or the inertial platform, since both of these must be operating in order to produce the heading reference. It is doubtful that at the present state of development the gyrocompass offers any improvement for the commercial operator.

3.7. VARIATION CORRECTION

Non-time-varying uncertainties in the definition of the magnetic field are a serious source of heading errors. The Coast and Geodetic Survey has indicated that isogonic lines in the United States have a probable error of 0.5° and, over less investigated areas of the earth, 0.8° .⁶ Certain regions which have not been surveyed magnetically for many years are considerably worse than this. Perhaps conveniently, the areas of poorest magnetic information are also the least traveled by commercial aircraft—the Indian Ocean, for example. At any rate, for the North Atlantic the 0.8° figure is applicable. Translated into a 2σ value, this indicates a 95% probability of being within $\pm 2.4^\circ$ [10-18].

In addition to the uncertainty of the non-time-varying error, a number of time-varying errors are possible. Since the source of most of these is external to the earth, the source magnitude is independent of latitude. Consequently, such errors increase appreciably at high latitudes, where the horizontal component of the earth's field is small.

Diurnal changes have a magnitude of about 0.1° at middle latitudes and 0.2° at near polar latitudes. During periods of high sunspot activity these values are doubled.

Magnetic storms cause compass system oscillations with periods of several hours and amplitudes of several degrees at middle latitudes. At high latitudes the amplitude may again be doubled. For example, a recent storm which caused a maximum error of 3° at Tucson, Arizona, caused a 7° error at Sitka, Alaska. Unfortunately, the direction of the error cannot be predicted from one point to another some distance away, although the approximate magnitude can. Magnetic storms can be predicted under certain circumstances—notably a tendency to recur in twenty-eight days, the rotational period of the sun with respect to the earth—but this predictability is not consistent enough to be of much value to aircraft operators.

Data going into a world isogonic chart, as published every five years by the Coast and Geodetic Survey, are a strange mixture of information derived over many years. Some information as old as 1900, but updated by a knowledge of annual change, from nearby points, is still used. Since World War II, a fairly consistent program of airborne magnetometry has been performed by the United States Coast and Geodetic Survey, Air Force, and Hydrographic Office. While errors in airborne magnetometry may be a little larger than those in data obtained by specially constructed ships, the rate of data acquisition is so much greater that it is now the principal source for over-ocean surveys.

Users sometimes ask why the Coast and Geodetic Survey does not publish isogonic charts every year, since new data are available continuously. The answer is that the probable error

⁶Personal correspondence with Coast and Geodetic Survey.

of the data is so much greater than the annual change that annual publication would be of little value.

One may conclude from this discussion that magnetic compasses, after all corrections are applied, will still not yield a very precise indication of true heading; and instantaneous accuracies of better than about 1.0° standard deviation at middle latitudes are not foreseen at this time.

Magnetic variation in flight is allowed for by applying a bias correction to the indicated magnetic heading. This correction may be obtained from a plot of variation contours on a map of the region being flown over, from tables prepared for particular flight routes, or by reading from a variation cam, which is a physical memory of world-wide variation. Variation contours on a map require some interpolation and careful application of the rules for determining compass heading from true heading; but, if properly handled, they are as accurate as any other method. When a select family of routes is flown, a table of variation may be prepared. TWA, for example, has prepared tables of magnetic heading to fly from any integral latitude at a 10° -longitude line to any integral latitude within 5° at the adjacent 10° -longitude line. In this case the average variation for the path segment is applied over the entire segment (see below). In the case of cam correction, the entire variation table is stored on a cam positioned by a latitude-longitude input, and the appropriate variation is applied continuously. This method is convenient, but it does require a continuous generation of latitude and longitude—and this is not often convenient for commercial operators.

Two types of error in position occur because of the practice of using an average value of variation over a route segment rather than applying variation continuously. If the field varies linearly over the path, an actual path followed tends to be an arc to the right or left of the great circle path, which arrives at the correct destination. If the field is nonlinear, there will be a position error at the destination. This error can be calculated, but it is ordinarily quite small. The case of a linear field is shown in Figure 15 [19].

From 30° W to 40° W	
True Course =	260° (measured at meridian nearest halfway)
Average Magnetic Variation =	27° W
Magnetic Course =	287°
Deviation =	None
Magnetic Heading =	287 (no wind)

As can be seen from Figure 15, the actual flight path does not necessarily follow the great circle route. An understanding of this can be obtained by examining the individual factors.

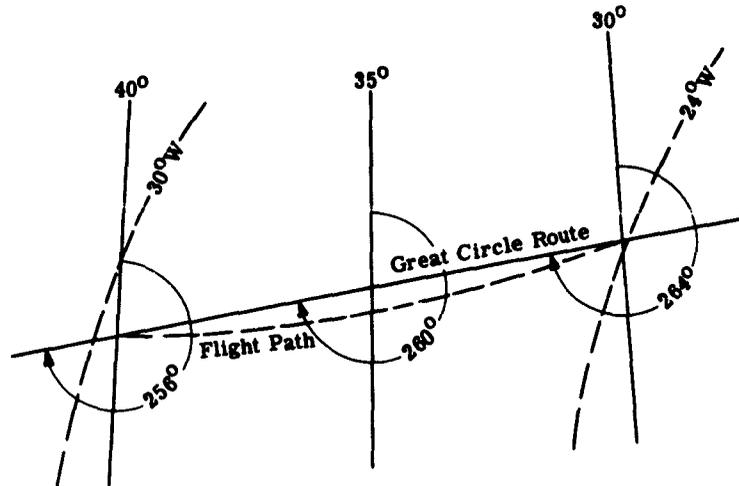


FIGURE 15. DIFFERENCE BETWEEN ACTUAL FLIGHT PATH AND GREAT CIRCLE

Assume no magnetic variation, but a total meridian convergence of 8° in 10° of longitude at approximately 55° N latitude (Figure 16, Reference 12).

At midpoint—True Course = 260°

At beginning—Great Circle Course = 264°
 Course Flown = $\frac{260^\circ}{4^\circ}$
 Course discrepancy = 4°

At end—Great Circle Course = 256°
 Course Flown = $\frac{260^\circ}{-4^\circ}$
 Course discrepancy = -4°

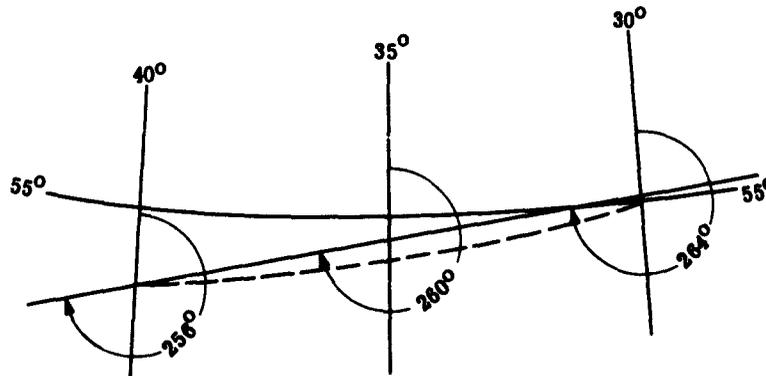


FIGURE 16. NO CHANGE IN MAGNETIC VARIATION, 8° MERIDIAN CONVERGENCE BETWEEN 30° and 40° LONGITUDE

The resulting flight path is shown as a dashed line to the south of the great circle route. For a flight of 350 nautical miles, the cross-track discrepancy at the midpoint would be 6.1 nautical miles.

A similar situation in the southern hemisphere would result in the flight path being to the north of the great circle route.

Now assume no meridian convergence, but an increase in magnetic variation of 6° when westbound (Figure 17).

$$\begin{aligned} \text{At midpoint—TC} &= 260^\circ \\ \text{Average Variation} &= \frac{27^\circ \text{ W}}{287^\circ} \\ \text{MC} &= \end{aligned}$$

$$\begin{aligned} \text{At beginning—MC} &= 287^\circ \\ \text{Variation at } 30^\circ &= \frac{24^\circ \text{ W}}{263^\circ} \\ \text{TC} &= \end{aligned}$$

$$\begin{aligned} \text{At end—MC} &= 287^\circ \\ \text{Variation at } 40^\circ &= \frac{30^\circ \text{ W}}{257^\circ} \\ \text{TC} &= \end{aligned}$$

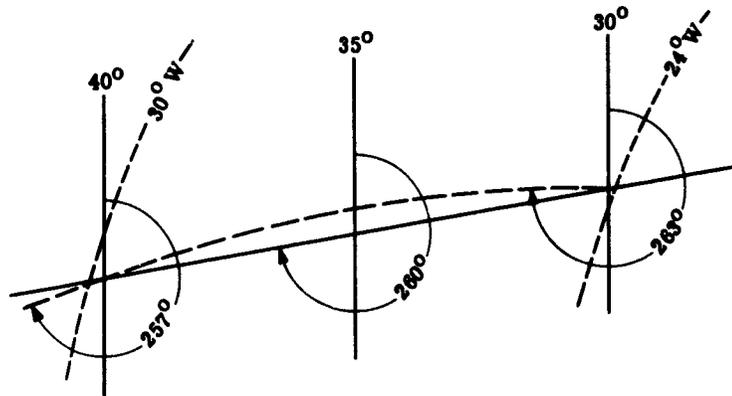


FIGURE 17. NO MERIDIAN CONVERGENCE, MAGNETIC VARIATION INCREASING WESTBOUND.

The resulting flight path is shown as the dashed line to the north of the great circle route. For a flight of 350 nautical miles, the cross-track discrepancy would be 4.5 nautical miles.

If the magnetic variation decreases 12° when westbound, assuming no meridian convergence, the following situation exists (Figure 18).

$$\begin{aligned} \text{At midpoint—TC} &= 260^\circ \\ \text{Average Variation} &= \frac{20^\circ \text{ W}}{280^\circ} \\ \text{MC} &= \end{aligned}$$

$$\begin{aligned} \text{At beginning—MC} &= 280^\circ \\ \text{Variation} &= \frac{26^\circ \text{ W}}{254^\circ} \\ \text{TC} &= \end{aligned}$$

$$\begin{aligned} \text{At end—MC} &= 280^\circ \\ \text{Variation} &= \frac{14^\circ \text{ W}}{266^\circ} \\ \text{TC} &= \end{aligned}$$

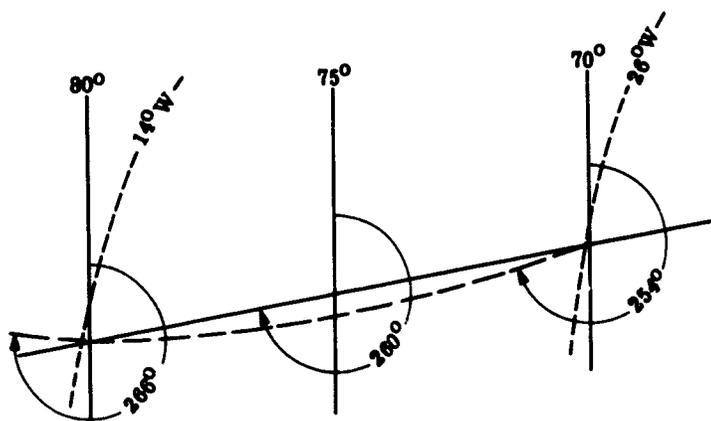


FIGURE 18. NO MERIDIAN CONVERGENCE, MAGNETIC VARIATION DECREASING WESTBOUND.

The resulting flight path is shown as the dashed line to the south of the great circle route. For a flight of 350 nautical miles, the cross-track error would be 9.2 nautical miles.

In the same manner it can be shown that easterly variations increasing numerically to the west will move the flight path to the south, while easterly variation decreasing to the west will move it to the north.

These facts are summarized in Table V.

The combinations of meridian convergence and changing variation will increase or decrease the effect of either one alone. In the vicinity of James Bay, Canada, the cross-track discrepancy is increased. It is fortunate that over most of the North Atlantic the two effects nearly cancel.

TABLE V. FACTORS AFFECTING FLIGHT PATH

Movement of Flight Path	Meridian Convergence	Factors			
		Westerly Variation Increasing Westerly	Westerly Variation Decreasing Westerly	Easterly Variation Increasing Westerly	Easterly Variation Decreasing Westerly
Northern Hemisphere	South	North	South	South	North
Southern Hemisphere	North	North	South	South	North

3.7.1. REDUCTION IN CROSS-TRACK ERROR RESULTING FROM RANDOM NATURE OF THE ERROR IN VARIATION. The heading error in an ocean crossing with a magnetic compass is a function of the precision of deviation compensation, readout errors, and the incomplete knowledge of variation correction over the route. The method of determining variation over the ocean is such that a residual random error of approximately 1° standard deviation exists. This means that on any single determination of heading, there is about a 2/3 chance that it is within 1°.

Neglecting all other errors (readout, deviation, etc.), if the magnetic field were in error by this amount over the entire path, one would correctly expect a 1° standard deviation error at the end of a flight. If the magnetic field variation error is random, one can expect a smaller error than this over long flights [20, 21].

Let us suppose that the transoceanic flight distance R can be broken up (magnetically) into n segments of equal length r = R/n such that the error in measured or computed variation is constant over each segment, and then changes randomly to a new constant value for the next segment, etc. Such a situation is shown in Figure 19. While this is somewhat artificial, it is a fairly reasonable limiting situation. In effect we may then represent the correlation function of the variation error as in Figure 20.

Now consider an aircraft traveling the entire path. In the first interval of length r the error in compass variation is assumed constant, so that at the end of that interval the plane will have a cross-course error equal to rε, where ε is the error in radians.

Similarly, in the second and third intervals the cross-course error will be rε₂ and rε₃ respectively. Since ε is a random variable, the cross-course error at the end of the flight can be computed from the sum of the error components

$$S_E^2 = r^2 \epsilon_1^2 + r^2 \epsilon_2^2 + \dots + r^2 \epsilon_n^2 \tag{1}$$

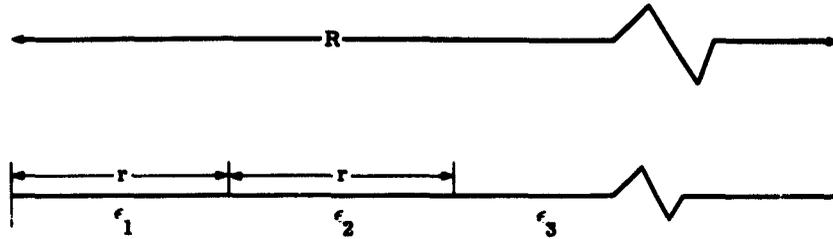


FIGURE 19. CHANGE IN ERROR VARIATION



FIGURE 20. CORRELATION FUNCTION OF THE VARIATION ERROR

In terms of standard deviations, where σ is in radians we may write an expression of similar form for the entire flight:

$$R^2 \sigma_E^2 = nr^2 \sigma_\xi^2$$

and

$$R\sigma_E = \sqrt{n} r \sigma_\xi$$

Since $R = nr$,

$$R\sigma_E = \frac{R}{\sqrt{n}} \sigma_\xi \text{ or } \sigma_E = \frac{\sigma_\xi}{\sqrt{n}} \tag{2}$$

Thus, the standard deviation at the end of a long flight is $1/\sqrt{n}$ times the standard deviation of the individual segments.

No precise value of the distance in which the correlation goes to zero over ocean regions is available, but some data taken on land indicate that 100 miles would not be unreasonable for the non-time-varying error. In this case, for a 2000-mile trip the expected error would be approximately $\frac{2000\sigma}{\sqrt{20}}$, where σ is the standard deviation of the variation in the field in radians. One may draw the conclusion, then, that magnetic heading may be effectively accurate to $\sigma/4$ or $1/4^\circ$ for a flight across the Atlantic.

The case considered here used a rather artificial correlation function. It is probable that one would use a correlation distance much smaller than 100 miles. Consequently, the actual cross-course error caused by the standard deviation of variation at the end of a 2000-mile flight may be negligible. Of course, the error assumed here was without bias. It is anticipated that if any real bias existed in the field, it would be discovered by repeated flights, so that it could be allowed for subsequently. It is concluded, then, that cross-track errors in such long flights are likely to derive from sources other than the poorly mapped variation field.

The exception to this conclusion is that errors caused by magnetic storms exhibit different properties. The distance over which such errors are correlated is likely to be more like 500 miles—an appreciable fraction of the total flight path. In addition, such errors are likely to exhibit a larger standard deviation. Consequently, during such storms one might expect terminal cross-track errors two or three times as large as on magnetically quiet days. Even though the existence of a magnetic storm may be known to the pilot, there is a real need for an auxiliary heading reference, preferably one using a different basic source of information. (See discussion of hybrid systems in Section 3.2.2.)

VERY-LOW-FREQUENCY RADIO NAVIGATION SYSTEMS**4.1. INTRODUCTION**

4.1.1. HISTORY. The systems considered here are ground-based radio frequency systems operating at frequencies below 50 kc and at ranges up to at least 3000 nautical miles. During World War II the LORAN-A hyperbolic navigation system gained a reputation for reliably providing relatively accurate fixes at long range. Following the widespread acceptance of LORAN-A, a need for still greater range and accuracy was evident. Low-frequency LORAN operating at 180 kc was tested with the hope that this would provide the answer. Because of propagation variations and dispersion, the pulse measuring techniques of LORAN precluded obtaining the accuracy that was desired. It was evident, however, that achievement of high accuracy at long range would require the use of still lower frequencies. Consequently, the RADUX hyperbolic system was devised by Dr. J. A. Pierce of the Cruft Laboratory at Harvard. The RADUX system was to operate at 40 kc, using phase comparison measurements rather than the pulse measurements of LORAN-A. It was found, however, that the cyclic ambiguities of the phase measurements could not be resolved at 40 kc because of path dispersion. About the same time Dr. Pierce and others were conducting experiments in VLF propagation at about 16 kc and found remarkable stability in the propagation medium. Many experimenters then began investigating the VLF (i.e., 3 to 30 kc) frequency region for the use of both navigation techniques and standard frequency broadcast. The encouragement given by the propagation experiments over very long ranges led Dr. Pierce and the U. S. Navy to develop the OMEGA system for world-wide navigation. Another result was the proposal of the DELRAC system by Decca Navigator Company, Ltd.

Although the development of OMEGA was based primarily on the need for world-wide surface navigation, the possible application of such a system to transoceanic air navigation is interesting. It is the purpose of this report to investigate this latter application with particular attention to the needs of the air carrier industry.

4.1.2. SYSTEM CONFIGURATIONS. Navigation systems designed to provide very long range navigation in the VLF region may take several forms, which differ primarily in the fundamental physical measurements that are made to obtain a navigation fix or "present position." The more important examples of the types of systems that have been considered are

- (a) Theta-theta (based upon azimuth measurements)
- (b) Rho-rho (based upon range measurements)
- (c) Hyperbolic-hyperbolic (based upon range-difference measurements)
- (d) Hyperbolic-elliptic (based upon range, range-difference, or range-sum measurements)

All of these systems determine position by two separate measurements, each of which determines a line of position; the intersection of two of these lines is the position.

Rho-rho and theta-theta systems have several advantages over pure hyperbolic systems. Coordinate conversion is less complicated, corrections for propagation variations are more easily applied, and only two ground stations are required for a fix. When transmitters and antennas are very expensive, as they will be for long-range very-low-frequency stations, the last advantage may represent a large reduction in investment. Hyperbolic-elliptic systems present a problem in coordinate conversion similar to that of hyperbolic-hyperbolic systems, although again only two stations are required to determine position. One possible advantage of hyperbolic-elliptic over rho-rho systems is the high accuracy of the hyperbolic measurement and the good line-of-position crossing angle, which is always 90° for hyperbolic-elliptic systems. Both rho-rho and hyperbolic-elliptic systems, however, require the use of a highly accurate and precise local time standard or clock for the measurement of one or more absolute distances. Clocks, usually in the form of oscillators to provide sufficient stability and accuracy, have not been available in the past; for this reason, most effort has been devoted to hyperbolic-hyperbolic systems.

All of the systems might be used in the normal manner, with the transmitters on the ground and the receiver in the aircraft, or in an "inverse" manner, with the transmitter in the aircraft and the receivers on the ground. In Appendix A, however, it is shown that the amount of power that can be radiated from an aircraft at very low frequencies is extremely small. Only one type of antenna was considered in the analysis, but the results given are probably better than those for most other types with the possible exception of a long trailing wire. It is assumed, however, that a trailing wire antenna several thousand feet long is not an acceptable type for air carrier operation. Not considered in Appendix A is the low radiating efficiency of aircraft antennas, which, although more difficult to evaluate analytically, will further reduce the possible radiated power. This is one of the reasons we may restrict our considerations to configurations in which the systems are operated with transmitters on the ground and the receiver in the aircraft.

The evaluation of the four systems will be made for commercial aircraft on over-ocean and over-land routes up to 3000 nautical miles in length, with required accuracies of 2 to 6 nautical miles d_{rms} radial error. The evaluation will also include considerations relevant to operation in aircraft with speeds up to Mach 3 and to the operational availability of the system from 1965 to 1975.

4.1.3. SYSTEMS CONSIDERED. The evaluation given here will be limited primarily to existing systems. The OMEGA system, the only VLF system actually in existence today, pro-

vides to a large extent the basis of this evaluation. The DELRAC system is a proposal of Decca Navigator, Ltd., and is essentially a Decca system time-shared and translated to a lower frequency. It is interesting to note that the OMEGA and DELRAC systems are so similar that most of the analyses to be presented for the OMEGA system will also apply to DELRAC; the two systems differ mainly in instrumentation methods. Together, they represent the present technology and state-of-the-art of long-range VLF hyperbolic systems.

Since proposals for systems other than hyperbolic have been made in the past and will undoubtedly be made in the future, a brief analysis of rho-rho, theta-theta, and hyperbolic-elliptic systems will be given later, in Section 4.6.

4.2. ACCURACY EVALUATION

4.2.1. ACCURACY SPECIFICATION

4.2.1.1. Description of Hyperbolic System Measurements. All hyperbolic navigation systems, including OMEGA and DELRAC, permit the derivation of position by determining the difference in distances between the receiver and a pair of transmitting stations. For short baseline systems the earth is commonly considered flat; then the loci of such distance differences form a hyperbola. Two such measurements will define two hyperbolas on the earth, and their intersection gives the receiver's position. The actual measurements are always the differences in the time of arrival of two radio signals; the distance difference is obtained by a knowledge of the physics of wave propagation. VLF systems make the time-difference measurement by comparing the phase of the two signals and relating this phase difference to a time difference. Any error in the system can be related, then, to an equivalent error in the time-difference measurement. Figure 21 shows the geometry of such a system for one line of position (hyperbola). If the master station transmits a wave of the form⁷

$$E_m = \cos(\omega t + a) \quad (3)$$

the signal received at the slave *s* is then

$$E_s = \cos(\omega t + a + \psi_{ms}) \quad (4)$$

The signal is then retransmitted by the slave and appears at the receiver as

$$E_{R_s} = \cos(\omega t + a + \psi_{ms} + \delta + \psi_{sr}) \quad (5)$$

⁷Since we are interested only in the phase of the signals, the amplitude is taken as one for simplicity of expression.

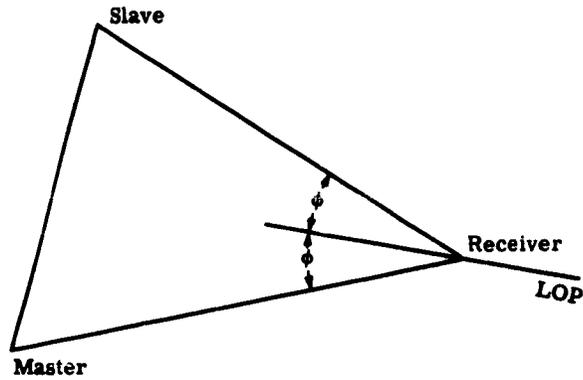


FIGURE 21. GEOMETRY FOR ONE LOP

The signal transmitted directly from the master station to the receiver appears as

$$E_{R_m} = \cos (\omega t + \alpha + \psi_{mr}) \tag{6}$$

where $\omega = 2\pi f$ and f = the radio frequency of the signal

- ψ_{ms} = phase delay over path from master to slave
- ψ_{sr} = phase delay over path from slave to receiver
- ψ_{mr} = phase delay over path from master to receiver
- δ = synchronizing error of the slave station

The phase difference between the two received signals at the receiver is then

$$\Delta\psi = \psi_{ms} + \psi_{sr} + \delta - \psi_{mr} + \epsilon \tag{7}$$

where ϵ is the instrumental error of the receiver. The phase difference $\Delta\psi$ can be related to a time difference by

$$\Delta T = \frac{\Delta\psi}{\omega} \cdot 10^6 \mu\text{sec} \tag{8}$$

The cyclic nature of $\Delta\psi$ gives rise to ambiguities which are spaced along the baseline at distance intervals of

$$L = \frac{\lambda}{2} = \frac{c}{2f} \tag{9}$$

where λ is the wavelength of the radio frequency signal (f) and c is the velocity of propagation. Each of these intervals is called a lane in most phase or cycle-measuring systems.

4.2.1.2. d_{rms} Measure of Systems Error. The position accuracy of hyperbolic systems has been statistically analyzed by several authors [22, 23]. A commonly used accuracy index and the one used here is the specification of a circle whose radial dimension is one or more times the d_{rms} error. The interpretation and usefulness of the d_{rms} statistic is discussed in Section 1.3.

The d_{rms} error is used to show a fictitious circle within which the probability of finding any one measurement is within a certain narrow range. The specification of position error by this circle is always conservative, and the calculations are usually simple. The d_{rms} error is derived in Appendix B and is given in nautical miles by

$$d_{rms} = \frac{0.08076}{\sin \theta} \sqrt{\frac{\sigma_{\Delta T_1}^2}{\sin^2 \phi_1} + \frac{\sigma_{\Delta T_2}^2}{\sin^2 \phi_2} + \frac{2\sigma_{\Delta T_1} \sigma_{\Delta T_2} \rho_{\Delta T_1, \Delta T_2} \cos \theta}{\sin \phi_1 \sin \phi_2}} \quad (10)$$

where $\rho_{\Delta T_1, \Delta T_2}$ = correlation coefficient between ΔT_1 and ΔT_2 and
 $\sigma_{\Delta T_1}$ and $\sigma_{\Delta T_2}$ are the standard deviations of the time difference measurements in μsec .
 θ , ϕ_1 , and ϕ_2 are shown in Figure 22.
 θ = angle of cut between position lines (for three stations $\theta = \phi_1 + \phi_2$)
 2ϕ = angle at receiving point between one pair of transmitters

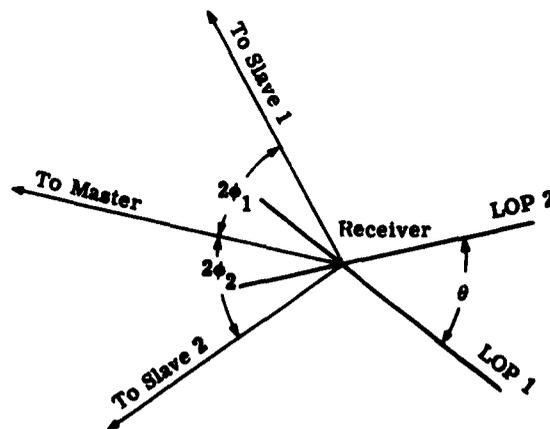


FIGURE 22. FIX GEOMETRY

4.2.1.3. Hyperbolic System Errors. From the preceding, it appears that a quantitative investigation of errors may be reduced to an investigation of the errors of the time difference measurements ΔT_1 and ΔT_2 . For evaluation of the d_{rms} error, the errors in the time difference measurements will be assumed to have a zero mean and a variance $\sigma_{\Delta T}^2$. Means other than zero will be caused by diurnal propagation variations, incorrect assumptions of phase velocity, and anomalous propagation conditions. These causes will be considered in sections dealing with these subjects. Although it is not necessary that errors in ΔT be normally distributed, it is reasonable to assume that they are so distributed because such errors are the sum of several error sources and many of the error sources themselves have been found to be nearly normal [24].

In considering random errors, then, we shall assume errors in ΔT to be normal with zero mean or expectation and of variance $\sigma_{\Delta T}^2$. Combining Equations 7 and 8,

$$\Delta T = t_{\text{ms}} + t_{\text{sr}} + t_{\delta} - t_{\text{mr}} - t_{\epsilon} \quad (11)$$

If all the terms of ΔT are independent (uncorrelated), then

$$\sigma_{\Delta T}^2 = \sqrt{\sigma_{t_{\text{ms}}}^2 + \sigma_{t_{\text{sr}}}^2 + \sigma_{t_{\text{mr}}}^2 + \sigma_{t_{\delta}}^2 + \sigma_{t_{\epsilon}}^2} \quad (12)$$

Both t_{δ} and t_{ϵ} are instrumentation errors and are independent of each other and all other terms. The remaining terms represent propagation times, and some correlation between these terms might be expected. However, Norton [24] cites evidence that the correlation between propagation times, when the transmitters are separated by a distance (S) greater than 40λ wavelengths, is negligible; that is,

$$\rho \approx 0 \text{ when } S > 40\lambda \quad (13)$$

At the OMEGA operating frequency of 10.2 kc

$$40\lambda = 634 \text{ nautical miles} \quad (14)$$

a distance which is small compared to the station separations or baseline distances with which we are concerned. Experiments with the OMEGA system at 10.2 kc give evidence that ρ is approximately 0.5 when $S = 100$ miles.⁸ With baselines several thousand miles long, it appears reasonable to assume that the terms in Equation 12 are independent.

The correlation-coefficient $\rho_{\Delta T_1, \Delta T_2}$ in Equation 10 is a measure of the degree to which a change in ΔT_1 will be accompanied by a change in ΔT_2 . If ΔT_1 and ΔT_2 were independent, then

⁸Private communication with Dr. J. A. Pierce, of the Cruft Laboratory, Harvard University.

$\rho_{\Delta T_1, \Delta T_2}$ would be 0. Since the master-receiver path is common to both measurements, any change in t_{mr} will affect both ΔT_1 and ΔT_2 . If the correlation between the individual paths is assumed equal to zero, we still have time-difference correlation remaining because of the common master-to-receiver path. Appendix C shows that over a large portion of the service area the value of ρ might be expected to equal about 0.3. From an analysis of an experimental L.F. LORAN system (180 kc) Crichlow found that $\rho_{\Delta T_1, \Delta T_2} = 0.309$ [22, 25]. Since Norton's data agree with the results of Appendix C, we shall assume in this report that

$$\rho_{\Delta T_1, \Delta T_2} \approx 0.3 \quad (15)$$

4.2.2. PROPAGATION TIME

4.2.2.1. Random Errors in Propagation Time. An analysis of errors in the phase delay over the transmission paths requires, of course, an investigation of propagation characteristics. Much of the attention that has been given VLF systems has been devoted properly to the propagation problem. The justification for systems operating in this frequency range has been the high stability of phase velocity over long distances with little signal attenuation. Although a great deal of material is available on the theory of very-low-frequency propagation, we will rely here largely on the experimental data that has been gathered by researchers investigating navigation applications and time-standard capabilities.

Until recently, VLF stations were not frequency-stabilized well enough for accurate observations of phase over long periods of time. As more and more stations become stabilized (the Navy is engaged in such a program) and atomic time standards become available to more investigators, the amount of experimental data collected will greatly increase. At present, even though several active measurement programs are underway, the data are so new that some investigators are not yet prepared to release them. Nevertheless, enough information is available to provide a reasonable assessment of the problems, and areas where limiting bounds cannot be established will be indicated.

For convenience, we shall treat the propagation time t as a random variable with mean μ_t and variance σ_t^2 . The standard deviation measures the precision or the ability of the system to give consistent position information at a fixed location. Much of the experimental data gathered to date provide an evaluation only of phase stability (σ_t or precision) over specific paths. In 1957 J. A. Pierce published measurements of the phase of a trans-Atlantic signal when compared with a stable local oscillator [26]. The signal was transmitted by GBR at 16 kc from Rugby, England, to Cambridge, Massachusetts—a distance of 2800 nautical miles. Pierce found a standard deviation during night and day of about 2 μ sec, with maximum devia-

tions from the mean of about $\pm 5 \mu\text{sec}$. The daytime value of the phase delay was lower than the night value because of the lowering of the ionosphere. This diurnal variation will be discussed later as part of the predictability problem. Casselman, Heritage, and Tibbals have published similar data on the round trip time of a 12.2-kc signal over a 2270-nautical mile path from Hawaii to San Diego [27]. The measurements, taken every six minutes over a six-day period in January 1958, gave a standard deviation of $5 \mu\text{sec}$ in the daytime and $4 \mu\text{sec}$ at night. Measurements were also made over a 24-hour period at frequencies from 10.2 to 15.2 kc. It was found that the nighttime variations were much lower at the lower frequencies, with peak to peak variations of only $8 \mu\text{sec}$, at 10.2 kc, and approximately $50 \mu\text{sec}$ at 15.2 kc. If path reciprocity is assumed, the one-way transmissions have standard deviations equal to one-half the above values. The standard deviations just given were for six-minute readings of the phase delay. Although the diurnal variations observed were similar to those of Pierce, the daytime values were not so constant.

Other observers have determined standard deviations of propagation time over paths up to 3000 nautical miles long in the very-low-frequency region, and Norton has used these data to obtain an "ionospheric roughness parameter" with which he computes the standard deviation of phase of the received signal [24]. Norton's model might be used to gain some insight into the functional relation between the standard deviation of phase and transmission distance. This model indicates that the phase variation should vary as the square root of the number of ray hops, or approximately as the square root of distance. However, the model is based on the assumption that propagation follows a single ray path. Watt and Plush point out in their investigation of VLF for standard frequency broadcasts that at long distances several rays would probably contribute to the field at the receiver so that the standard deviation would be reduced [28]. Since the number of contributing rays increases with distance, the standard deviation of phase would be expected to increase at a slower rate than the square root of distance.

Not enough experimental data are yet available for conclusive study of the variation of the standard deviation with range. For example, the National Bureau of Standards has been measuring the phase of station GBR at 16 kc in Boulder over a 4000-nautical mile path. Chilton³ gives values over this path of

$$\sigma = 2.6 \mu\text{sec in daylight}$$

$$\sigma = 5.1 \mu\text{sec at night}$$

³Personal communication with Charles J. Chilton, National Bureau of Standards, Boulder Laboratories.

These of Chilton compare with the values of Pierce over a 2800-nautical mile path of

$$\sigma = 1.4 \mu\text{sec in daylight}$$

$$\sigma = 2.8 \mu\text{sec at night}$$

These values are in a ratio of about 1.8, although the square root of the distance ratio is about 1.19. On the other hand, the data of Casselman, et al., do substantiate the conclusion of Norton that at about 10 kc the day and night standard deviations are nearly the same [24]. One of the problems encountered here is that because of the few data available and because of possible differences in averaging techniques it is difficult to make comparisons.

Much of the data taken so far on the standard deviation of propagation time have been taken over paths in the temperate zone. It would be interesting to examine data from both arctic and auroral paths; the National Bureau of Standards is taking data over an arctic path, but results are not yet available. The GBR-to-Boulder path for which data are available just touches the auroral zone, and Chilton points out that a standard deviation based on data taken over this path for a six-month period and including some magnetic disturbances and meteor showers gives a value of about $\sigma = 8.36 \mu\text{sec}$, although these data include night values at 16 kc which according to Casselman, et al., have a higher variation than at 10 kc.

The Naval Electronics Laboratory has also taken data on actual phase difference measurements using an OMEGA receiver.¹⁰ Data were taken at the Naval Electronics Laboratory in San Diego on the received phase difference between transmitters in Hawaii and Forestport, New York. Both transmitters were synchronized with the Balboa, Canal Zone, Station. Data taken from August 6 to September 2, 1960, gave a maximum standard deviation of about $7 \mu\text{sec}$. The transmission paths involved here range from 2000 to 4000 nautical miles. Since the Hawaii and Forestport stations were both synchronized on Balboa, four transmission paths are involved. Since the data at 10 kc for the Hawaii-San Diego path gave standard deviations of 2 to $2.5 \mu\text{sec}$, we might assume $\sigma = 3 \mu\text{sec}$ for each of these longer paths. If

$$\sigma_t = 3 \mu\text{sec} \quad (16)$$

and

$$\sigma_{\Delta T} = \sqrt{4\sigma_t^2} \quad (17)$$

$$\sigma_{\Delta T} = 6 \mu\text{sec} \quad (18)$$

¹⁰ Personal communication with Mr. E. R. Swanson of Naval Electronics Laboratory.

This value is in reasonable agreement with the measurements made at the Forestport and Hawaii stations.

Dr. J. A. Pierce reports that, on the basis of recent data taken on the OMEGA system in service tests, with much of the data based on receivers operating in the Caribbean region, the standard deviation of the time of propagation over a single path is $2 \mu\text{sec}$, and the standard deviation of the time-difference measurements is 5 to 6 μsec .¹¹ These measurements were all made at 10.2 kc.

On the basis of available experimental evidence it will be assumed throughout the rest of this report that the standard deviation of the time of travel over a "quiet" ionospheric path at 10.2 kc is about 3 μsec . A "quiet" path is one undisturbed by anomalous conditions, such as meteor showers, sudden ionospheric disturbances (SID's), etc. These effects will be discussed later in this report. Further experimentation, particularly over arctic paths, may indicate that this figure is somewhat optimistic. It is likely, however, that investigations will indicate that the value of 8.36 μsec given by Chilton is close to the upper bound for 10.2 kc.

4.2.2.2. Diurnal Variation and Phase Velocity. The accuracy with which we can predict the mean-time-difference readings at any given location depends largely on the accuracy with which we can predict the mean of the time of propagation t between two points. To evaluate the accuracy with which we can predict the mean readings, we must investigate the behavior of the mean of the propagation time, or the phase velocity upon which t depends. The often mentioned diurnal variation is a function of the change in velocity from night to day when the ionosphere lowers. The phase velocity can also be expected to vary with other parameters; for example, ground conductivity and orientation of the path in relation to the direction of the earth's magnetic field in the ionosphere. Although a great deal of theory directly related to our subject has been developed, the computations required to evaluate all of the pertinent phenomena are laborious and would still contain uncertainties because the models are incomplete. In addition, few experimental data have been gathered in this area.

Two problems have faced investigators here. To make a direct measure of propagation time over a path has been impossible because a sufficiently accurate synchronization of "clock" at both ends of the paths has been unavailable. The OMEGA experiments, on the other hand, have provided accurate time difference measurements, but much of the data has been taken at sea where accurate receiver location information has not been available.

¹¹Personal communication with Dr. J. A. Pierce of Cruft Laboratory, Harvard University.

Several investigators have attempted to use a combination of experimental data and theoretical models. Most such attempts have suffered from uncertainties in the models. Westfall, for example, has found that by using an approximation of Wait's for the phase velocity at long distances, he could use experimental data on the diurnal shift to calculate the ionospheric heights [29, 30]. He obtained a result of $h = 59.8$ km for the daytime height and 72.3 km at night. Using these heights and the same model for velocity, we obtain the following ratios between phase velocity and the free-space velocity at 10.2 kc,

$$\frac{v_{\text{day}}}{c} = 1.0076$$

$$\frac{v_{\text{night}}}{c} = 0.999498$$

where v is the phase velocity, and c is the velocity of light.

Wait, however, points out that the model used by Westfall was very approximate and suggests the use of a better model, which results in a deduced ionospheric height change of 18 to 20 km at 10.2 kc [31]. The most recent theoretical development of the mode theory by Wait has been compared to the OMEGA data taken by NEL, which agree well with the velocities computed by Wait [32, 33].

$$\frac{v_{\text{day}}}{c} = 1.0027$$

$$\frac{v_{\text{night}}}{c} = 0.9996$$

Pierce has used the data taken in the OMEGA service tests, in which receivers on ships were accurately located by LORAN (a precise 2-Mc phase-comparison position-determination system) to derive values of the phase velocity.¹² Pierce has observed that the velocity during daylight hours is not constant, nor are the values the same for land and sea. The values he derived are shown in Table VI. These are the ones now being used in the OMEGA program

TABLE VI. PHASE VELOCITY
AT 10.2 kc (V/C) RATIOS

	Land	Sea
Early or Late Daylight	1.00229	1.00256
Noon	1.00261	1.00310
Night	0.99945	0.99927

¹²Personal communication with Dr. J. A. Pierce of Cruft Laboratory, Harvard University.

and probably represent the best data to date. These figures agree well with the last over-sea values given by Wait.

Just how reliable these values are is difficult to determine. Dr. Pierce states that the standard deviation of velocity ratio may be given approximately by

$$\sigma_{v/c} = \frac{0.003}{f_{\text{kc}}}$$

based on his OMEGA data. This would give

$$\sigma_{v/c} = 3 \times 10^{-4} \text{ at } 10 \text{ kc}$$

At a range of 4000 nautical miles from the station, the resulting standard deviation of the computation of time of propagation would be

$$\sigma_{\mu_t} = 7.5 \text{ } \mu\text{sec}$$

It should be noted, however, that errors in the mean of the propagation time from the master to the slave station are readily removable by system calibration so that σ_{μ_t} above is applicable only to the transmitter-to-aircraft paths.

At long ranges the value of the phase velocity is very nearly independent of range. At short ranges the velocity varies because of interference between the various skywave modes and the ground wave. The same interference causes partial cancellation of the field at short ranges. Hence, the transmitter stations should be placed so that the user is always at least 1000 miles from each station.

The change of velocity from night to day values as the ionosphere lowers results in a daily change in propagation time, known as the diurnal shift or diurnal variation. The stability of the diurnal shift is dependent on daily changes in the difference between night and day values of velocity; the change in the diurnal variation is small. Pierce reports a change in the height of the GBR diurnal variation at Cambridge of only $\pm 1 \text{ } \mu\text{sec}$ throughout the entire year. The Naval Electronics Laboratory has taken data on the Forestport, New York, and Balboa, Canal Zone, stations; the data show that the diurnal shift in the time difference is caused by the combination of the shifts in each of the three transmission paths involved. Over each of the consecutive ten-day periods during which data were taken, the variation in the diurnal shift was much less than the standard deviation of each reading, which was about $7 \text{ } \mu\text{sec}$, as mentioned before. The data available to the writer do not, however, permit an actual determination of the stability other than that it is much better than $7 \text{ } \mu\text{sec}$. Between two of the ten-day periods a shift in the diurnal variation was observed; this was about $9.4 \text{ } \mu\text{sec}$ between the mean of one ten-day period and

the mean of the following. This change has not yet been explained, although Dr. Pierce thinks that it represents equipment error.

Dr. Pierce has been able to compute diurnal shifts that agree well with experimental data over the Balboa-to-Corpus Christi path by the following model. In Figure 23 a receiver is located at R and a transmitter at T, although it makes no difference if they are interchanged. We are assuming here that the transmitter is at the eastern end of the path. Control points are selected 777 km from each end of the path, and it is assumed that the path-transmission time changes linearly as the ionosphere between these points changes height. The phase of the signal at the receiver then has a trapezoidal shape as a function of time as shown in Figure 24.

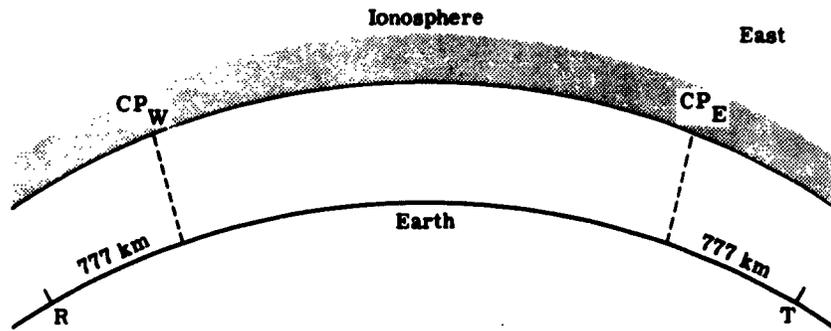


FIGURE 23. PATH GEOMETRY FOR COMPUTATION OF DIURNAL SHIFT

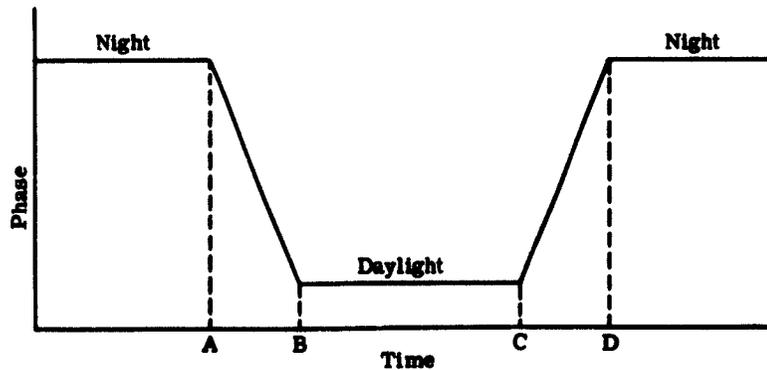


FIGURE 24. TYPICAL DIURNAL SHIFT

The break points on the curve (A, B, C, D) can be defined as

A is the beginning of sunrise at the eastern control point and occurs when

$$\cos \chi = -0.16 \quad (19)$$

where χ = the sun's zenith angle.

B is the end of sunrise at the western control point and occurs when

$$\cos \chi = +0.04 \quad (20)$$

C is the beginning of sunset at the eastern control point and occurs when

$$\cos \chi = 0.0 \quad (21)$$

D is the end of sunset at the western control point and occurs when

$$\cos \chi = -0.25 \quad (22)$$

The definitions of sunrise and sunset given here do not coincide exactly with the astronomical definitions. This method has worked well for Dr. Pierce, although the corners of the trapezoid are rounded off in actual patterns.

The time duration of the change from night to daylight and daylight to night defined here will be neither necessarily equal nor of constant length over a given path because of the seasonal change in the time of sunrise and sunset. At moderate latitudes the height of the shift is constant. However, there are times of the year when the GBR-Boulder path is never in complete darkness. The effect of this, as observed by the National Bureau of Standards at Boulder, is that the diurnal shift over this path is triangular and never reaches the full night value during the summer months.

The diurnal variation in a time-difference reading is the combination of the variations over the individual paths and at times may go negative when path times are subtracted in a hyperbolic system. The maximum shift that may occur is about 120 μ sec for a system where all paths are about 5000 nautical miles long. If the receiver is near the bisector of the angle formed by the station baselines, the diurnal shift is zero, although there are peaks up to 120 μ sec high during the sunrise and sunset hours. The time-difference diurnal variation over one day would then look something like that shown in Figure 25.

The operational problem of dealing with the diurnal shift might be reduced by removing it from the master-to-slave station paths, thus reducing the paths that are affected from five to three. This could most easily be done by the use of ultrastable oscillators at the slave stations which would be locked to the master transmission with a 24-hour averaging time. Although the

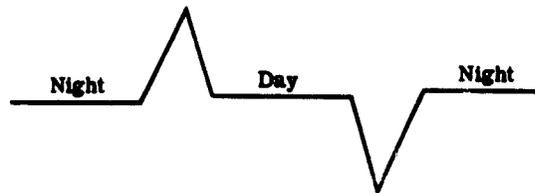


FIGURE 25. TIME-DIFFERENCE DIURNAL VARIATION

experimental OMEGA chain now in use does not do this, it is expected that it will eventually operate in this manner.

The diurnal shift over a path 5000 miles long in an east-west direction may require nearly five hours for completion. Since many commercial flights take less time, the system must provide the required accuracy even during the diurnal shift. Corrections require knowledge of both present time and receiver position. The length of time required to complete a diurnal shift on north-south paths is much shorter, but the slope, or change in time difference per unit time, is high, so that an accurate time correction is also needed for these paths. For example, the data taken by NEL at Corpus Christi, Texas, on the Forestport, New York - Balboa, Canal Zone, station pair represent a time difference where all paths are predominantly north-south. In the Corpus Christi data the total diurnal change is about 50 μsec in the time-difference readings. The shift from day to night values at sunrise occur with a maximum rate of about 10 $\mu\text{sec}/\text{hour}$. The rate is not constant, however, and the total change of 50 μsec requires about 7 hours. On the other hand, sunrise strikes several of the stations at nearly the same time and the total change takes place in about two hours at a maximum rate of 50 $\mu\text{sec}/\text{hour}$. After the transition from night to day, an overshoot of 13 μsec was observed; some of this can be explained as a result of the superposition of individual paths, but Blackband of the Royal Aircraft Establishment has noted a similar overshoot of about 5 μsec at 16 kc on a single path from Rugby to Malta [34].

4.2.2.3. Anomalous Propagation. Sudden ionospheric disturbances (SID) cause sudden shifts in both the time of propagation and the time differences. The intensity of SID's varies, but the effect of intense disturbances is a shift in the time difference of about 40 μsec . The peak deviation is usually reached in about 15 minutes with a recovery time of about 1.5 hours. These numbers result from observation at Lima, Peru, on the Hawaii-Balboa pair, but are typical of

SID's that have been observed by several investigators.¹⁸ Although SID's with sufficient magnitude to disturb VLF systems are infrequent, they do sometimes occur several times a day and may actually overlap on occasions. Periods have been observed in the OMEGA data where the duration of a disturbance caused by several SID's lasted two to three hours.

Pierce noted an effect from magnetic storms on his GBR data [26]. This effect was a phase jitter at night with a phase uncertainty of about 5 μ sec. It was not noted during the day. Black-band reports a more serious disturbance on the GBR signal as received at Malta [34]. The data in this area are not sufficient to draw any quantitative conclusions, and an assessment of the errors due to magnetic storms will have to await more experimentation.

Chilton, of the National Bureau of Standards, has also observed anomalous phase effects from meteor shower ionization [35]. During the Perseid shower of August, 1960, the phase of the GBR signal at Boulder was perturbed from the monthly average by as much as 14 μ sec during the dawn hours.

4.2.3. INSTRUMENTATION ERRORS

4.2.3.1. Bandwidth and Noise Consideration. Intimately related to instrumentation error is the noise response of the detection scheme. The OMEGA system and presumably other VLF systems use a linear servo detector. The predetection bandwidth is determined by the IF response of the receiver, and the postdetection bandwidth is determined by the servo characteristics. Ideally we would analyze such a system by defining the noise environment and then evaluating the effect of the noise on the system.

VLF systems operate with an extremely narrow bandwidth in an environment of impulsive atmospheric noise. Receiver noise is seldom important because of the high atmospheric noise level.

An ideal analysis is beyond the scope of this evaluation. The analysis of atmospheric noise is difficult, and the spectrum of such impulsive non-stationary noise after passage through an extremely sharp filter cannot be conveniently determined [36]. In addition, a detailed knowledge of the receiver circuits, which is not available, would be required. Furthermore, the present airborne receiver may not be representative of future designs. It is possible, however, to perform a simplified analysis and reach some conclusions about the range of error. The same analysis will be used as a basis for estimating transmitter requirements.

The minimum bandwidth requirements will be determined for receiver operation in typical aircraft of speeds up to Mach 3.

¹⁸Personal communication with J. A. Brogden of NRL.

If changes in the propagation medium are neglected, an OMEGA receiver in a fixed location would receive a signal of zero spectral width. If the aircraft is moving with a constant radial velocity (measured in the direction of the transmitter), the signal will be displaced in frequency by a doppler shift, but will still have no bandwidth. If, however, the radial velocity is changing, the signal does exhibit a finite bandwidth. It is possible to supply externally derived estimates of radial velocity to servo detectors and perform the final integration on the difference between the actual and estimated signal. In such a case the final effective bandwidth required is only that bandwidth determined by the velocity error.

Without external velocity or rate information the receiver postdetection bandwidth must include the doppler shift from the maximum radial velocity either to or from the transmitter. During aircraft maneuvers the bandwidth is not zero because of the transient effects on the signal frequency. The effect of a maneuver is analyzed here by assuming that the aircraft is orbiting in a circular pattern far from the transmitter. The changing radial velocity then modulates the frequency of the received signal in a sinusoidal manner, and the bandwidth can be found by classical FM analysis.

Table VII gives the bandwidth required by a system operating without external rate information. The bandwidth is given for an aircraft flying at a constant radial velocity, and for an aircraft flying at the same velocity in a circular orbit. The assumed turning rate is a 2g, or "panic," turn.

TABLE VII. BANDWIDTH REQUIREMENTS FOR A VLF NAVIGATION SYSTEM OPERATING AT 10 kc WITHOUT EXTERNALLY DERIVED RATE INFORMATION

Aircraft Types	Maximum Ground Speed in Knots	Bandwidth Required for Constant Velocity	Bandwidth Required for 2g Turn
Piston	300	0.00526 cps	0.035 cps
Subsonic Jet	686	0.0121 cps	0.0458 cps
Supersonic Jet	1740	0.0305 cps	0.0786 cps

Propagation effects will also contribute to signal frequency shift as either an increase or a decrease in received signal frequency. A diurnal shift of 50 μ sec/hour is equivalent to a bandwidth at 10 kc of

$$BW = 0.000278 \text{ cps} \quad (23)$$

Similarly, an SID of 60 μ sec in 15 minutes requires a bandwidth of

$$BW = 0.001334 \text{ cps} \quad (24)$$

Transmitting station frequency stability might also be considered, but with stability of at least one part in 10^9 the shift is not significant compared to the shifts given above. For the total postdetection bandwidth we can now add the contributions given above. For a supersonic jet the bandwidth is then

Maneuver Transients	0.078600 cps
SID	0.001334 cps
Diurnal Variation	0.000278 cps
Total Bandwidth	0.080212 cps

The phase measurement made by the receiver will be the phase of the signal plus noise compared to the phase of a local reference. If we assume a Gaussian distribution of the noise in the integrator bandwidth (over such a narrow bandwidth this is a good assumption), error in the phase measurement will be [28, 24]

$$\sigma_{\phi} = \frac{1}{\sqrt{2}} \frac{n}{c} \quad (25)$$

where c and n are the rms carrier and noise voltages. If we assume that the equivalent time measurement has a standard deviation of $3 \mu\text{sec}$ at 10 kc , then

$$\sigma_t = \sigma_{\phi} / \omega \quad (26)$$

where $\omega = 2\pi f$ and f = the radio frequency and for

$$\sigma_t = 3 \mu\text{sec} \quad (27)$$

$$\frac{c}{n} = 11.50 \text{ db}$$

This is the carrier-to-noise ratio required in the effective receiver bandwidth. If the receiver bandwidth is the value given above for a supersonic aircraft, and the noise-bandwidth relationship of thermal noise is used,

$$c/n = 11.50 \text{ db in a bandwidth of } 0.08 \text{ cps} \quad (28)$$

$$c/n = -29.47 \text{ db in a bandwidth of } 1 \text{ kc} \quad (29)$$

Pierce gives a value of 75 db above $1 \mu\text{v/m}$ as the value of "Kansas" noise, exceeded only 5% of the time in North America. "Kansas" noise is the noise in a 1-kc bandwidth at 10 kc [26]. Casselman and Tibbals measured a noise of $300 \mu\text{v/m}$ at San Diego at 12 kc in a 100-cps bandwidth [37]. This is 59.5 db above $1 \mu\text{v/m}$ in a 1-kc bandwidth. Crichlow gives a value of about 60 db above $1 \mu\text{v/m}$ at 10 kc in a 1-kc bandwidth for the median noise in a summer noise grade of 4 between 0800-1200 hours local time [38]. This is the highest noise value Crichlow gives

except for a few small areas of the world. The correction from Reference 21 required to give the noise exceeded only 10% of the time is 10 db. The Crichlow noise value, exceeded only 10% of the time, is then 70 db above $1 \mu\text{v/m}$ in a 1-kc bandwidth at 10 kc. This value is in excellent agreement with the measured values given by Pierce and Casselman and Tibbals and will be used here.

Watt and Plush give the following expression for the VLF field strength, which has been derived from a mode of propagation model of Wait [28]. When $d > 2000$ km

$$E = K + P_r - 10 \log_{10} f(\text{kc}) - 10 \log_{10} [a \sin (d/a)] - \alpha d/1000 \quad (30)$$

where E = vertical field strength in db above $1 \mu\text{v/m}$

P_r = radiated power in db above 1 kw

f_{kc} = signal frequency in kc

a = radius of the earth ($\sim 6,400$ km)

d = distance from the source in km

$K = 94.8$ for night paths when the height of the ionosphere is 90 km ($K = 97.5$ when $h = 70$ km)

α = attenuation rate per 1000 km

Watt and Plush give an extrapolated value of $\alpha = 4.5$ db/1000 km for 10 kc, but on the basis of theoretical work by Wait and experimental measurements by Taylor a better value would seem to be [35, 39]

$$\alpha = 3 \text{ db/1000 km} \quad (31)$$

Using this value ($\alpha = 3$) and 70 db noise in a 1-kc bandwidth, we finally find the required radiated power for a standard deviation of time measurement of $3 \mu\text{sec}$ in a supersonic aircraft without rate information

$$P_r = 8.1 \text{ kw, } d = 3000 \text{ nautical miles, } \sigma_t = 3 \mu\text{sec} \quad (32)$$

$$P_r = 138 \text{ kw, } d = 5000 \text{ nautical miles, } \sigma_t = 3 \mu\text{sec} \quad (33)$$

Presently, however, the airborne OMEGA receiver designed and tested by NRL does not have a postdetection bandwidth as wide as used in these calculations. The present receiver is designed to use external rate information and is required to track errors only in the external rate of up to 250 miles/hour (wind error, etc.). From information from NRL the bandwidth of the NRL-OMEGA receiver is calculated to be about 0.013 cps, with a rise time for a step in-

put of about 75 seconds.¹⁴ Using this bandwidth the powers required would be lowered by 7.9 db, or the power required for operation to 5000 nautical miles would be

$$P_r = 22 \text{ kw} \quad (34)$$

for the present NRL-OMEGA receiver.

The stations now in use have radiated powers of 5 kw at Halku, 2 kw at Balboa, and 250 watts at Forestport. On a recent flight test the receiver operated satisfactorily at over 5000 miles from the Balboa Station [40-45]. Because of the impulsive, non-stationary nature of atmospherics, the simplified noise analysis given here is pessimistic. Receivers using some form of correlation detection with long integrating periods can discriminate against such noise more effectively than against thermal noise of the same rms value. It should be noted, however, that noise caused trouble during a hail storm. Information has not yet been published specifying the noise environment encountered during the NRL flights. The noise environment may have been more favorable than the noise level used in the analysis given above. Thus the above analysis and the NRL tests indicate that an increase in transmitter power by a factor of 8 would be required to accommodate a supersonic transport in which the receiver was not rate-aided.

4.2.3.2 Transmitter Synchronization Errors. In the present operation of the OMEGA chain, the slave transmitters are synchronized with the signal received from the master station. If the phase of the transmitted signal at the slave station is not in phase with the received signal but is of a constant phase difference, it is added to the phase readings at the receivers. Since this phase difference is constant, it can be either removed from the readings at the receiver or removed by a phase adjustment at the slave transmitter. A random variation in the phase synchronization, however, will cause a random error in the received time differences. Such an error was labeled t_δ in Equation 11; the standard deviation of this error is σ_{t_δ} . Casselman, Heritage and Tibbals report that a continuous measurement of station synchronization shows that the phase of the signal transmitted by the slave relative to the phase of the received master signal varies by less than $\pm 1 \mu\text{sec}$ [27]. The measurement was made with a monitor receiver having an accuracy of at least $0.5 \mu\text{sec}$ [37]. It might be assumed that $1 \mu\text{sec}$ represents the 3σ error since this value is never exceeded. This accuracy of synchronization is evidently reliable even with low master signal strengths. No increase in error was noted when the master station in Hawaii radiated only 50 mw to the slave station 2270 nautical miles away at San Diego. Therefore, it will be assumed in this system evaluation that

$$\sigma_{t_\delta} = 0.3 \mu\text{sec} \quad (35)$$

¹⁴ Personal communication from A. F. Thornhill of NRL.

If the slave station were operated from an ultrastable oscillator locked to the master transmissions with a 24-hour averaging period, the synchronizing error would be a function of oscillator stability and frequency synchronization. Such a change has already been suggested to reduce the problem of diurnal variation (see Section 4.2.2.1 and References 40-45).

Pierce has shown that the standard deviation of the frequency of a clock (oscillator) set by a VLF path across the Atlantic on 16 kc is about 2 parts in 10^{11} . This result was obtained after a 24-hour-averaging-period and gives a standard deviation of time measurement of about $1.6 \mu\text{sec}$ [26, 39]. This procedure assumes a local oscillator with a frequency drift of only 1 part in 10^{11} for several days. The AUTOMICHRON used by Dr. Pierce approaches this requirement, but with questionable reliability. It appears possible to achieve this stability with excellent reliability by using several AUTOMICHRONS with their outputs automatically averaged.¹⁶ Reder, Winkler, and Bickort, who are working on long-range clock synchronization for USASRDL, have reported results from an experiment by Dr. Pierce using the 10.2-kc signal from Haiku, Hawaii, as received at Cambridge [46]. The results Dr. Pierce received were $\sigma_t \approx 3.2 \mu\text{sec}$ over the path described. A value for σ_{t_0} for "24-hour-average" synchronization of

$$\sigma_{t_0} \approx 4 \mu\text{sec} \quad (36)$$

will be used in this report. The precision of Dr. Pierce's results probably does not warrant fractional values and so the next higher integer has been chosen.

4.2.4. SUMMARY OF ACCURACY AND CONSIDERATION OF ROUTES AND SERVICE AREAS

4.2.4.1. Error Summary. The factors that contribute to random errors in the time-difference measurement discussed in this report are

- (a) Random propagation errors over a "quiet path"
- (b) Receiver instrumentation error
- (c) Slave station synchronization error

Errors that will be summarized later are

- (a) Error in assumption of phase velocity
- (b) Anomalous propagation
- (c) Computation of diurnal shift

Random errors have been assigned these values:

- (a) Propagation errors, quiet path $\sigma_t = 3 \mu\text{sec}$; disturbed path $\sigma_t = 8.36 \mu\text{sec}$

¹⁶Personal communication with Dr. J. A. Pierce of Cruft Laboratories, Harvard University.

(b) Receiver instrumentation $\sigma_{t_e} = 3 \mu\text{sec}$

(c) Synchronizing, phase locked $\sigma_{t_\delta} = 0.3 \mu\text{sec}$; "24-hour-averaging" $\sigma_{t_\delta} = 4 \mu\text{sec}$

Since it is reasonable to assume that these random errors are independent, the variances (σ^2) of the individual errors can be added to obtain a variance of the total error from these sources. If a "phase locked" slave station is used, there are three paths involved in each time-difference measurement, while if "24-hour-averaging" of an ultrastable oscillator is used, only two paths are involved.

If a "phase locked" slave system is used, the standard deviation of the time difference will then be (compare Equations 11 and 12)

$$\sigma_{\Delta T} = [3(3)^2 + 3^2 + (0.3)^2]^{1/2} \quad (37)$$

$$\sigma_{\Delta T} \cong 6 \mu\text{sec} \quad (38)$$

If a 24-hour-averaging slave system is used, the standard deviation of the time differences will then be

$$\sigma_{\Delta T} = [2(3)^2 + (3)^2 + (4)^2]^{1/2} \quad (39)$$

$$\sigma_{\Delta T} \cong 6.3 \mu\text{sec}$$

Since it is reasonable to assume that future OMEGA stations will operate in the 24-hour-averaging-mode, the result of Equation 39 will be used. However, operation in the same manner with an assumed disturbed path gives

$$\sigma_{\Delta T} = [2(8.36)^2 + (3)^2 + (4)^2]^{1/2} \quad (40)$$

$$\sigma_{\Delta T} \cong 12.8 \mu\text{sec}$$

It might be noted that if the disturbed-path figure for σ_t is assumed, it is advantageous to use the 24-hour-averaging system because the total error of a phase locked system with "disturbed paths" would be

$$\sigma_{\Delta T} = 14.8 \mu\text{sec} \quad (41)$$

A reasonable estimate of the standard deviation of the time-difference measurement from random-error sources is in the range $6 \leq \sigma_{\Delta T} \leq 12 \mu\text{sec}$.

$$\sigma_{\Delta T} = 6 - 12 \mu\text{sec} \quad (42)$$

Errors in the assumption of phase velocity might be treated as random errors if an airborne computer were continually computing coordinates. On the other hand, if a chart is used to determine position, the chart could be corrected for overland paths. Either a chart or a computer might be corrected periodically for changes in propagation velocities as they are evaluated by monitor receivers at strategic locations. If we assume that the value of the standard deviation of our knowledge of phase velocity is that given by Pierce

$$\sigma_t = 7.5 \text{ } \mu\text{sec} \quad (43)$$

then the resultant contribution to the standard deviation of the time difference in hyperbolic system operation is

$$\sigma_{\Delta T} = (2 \sigma_t^2)^{1/2} \quad (44)$$

$$\sigma_{\Delta T} = 10.6 \text{ } \mu\text{sec} \quad (45)$$

Only two paths are thus affected (master to receiver and slave to receiver) because the master-to-slave path can be calibrated. We shall assume that the error in the time-difference measurement due to the error in the assumption of the phase velocity is a random variable, although its value would in all probability decrease with experience. The amount of land in a path is not a serious problem, because if we use Pierce's values for phase velocity, a difference in a path of 500 nautical miles in the amount of land would only change the path-propagation time by 1.5 μsec .

If the diurnal variation is to be computed on board the aircraft, it will require a knowledge of time and position. The ratio between the error in the knowledge of time (T) and the resulting error in the time difference (ΔT) will be the same as the slope of the diurnal shift. The example cited earlier of a fast diurnal shift was 50 $\mu\text{sec}/\text{hour}$, or;

$$\Delta T/T = 0.84 \text{ } \mu\text{sec}/\text{minute} \quad (46)$$

In computing the diurnal shift a time error of 5 minutes would then result in an error of 4.2 μsec in the time-difference computation.

The error resulting from an error in assumed position will be the algebraic sum of the errors in each path. The error over any path will be

$$\epsilon_T = \frac{d}{RT} DV \quad (47)$$

where d = radial error in position relative to transmitter

RT = actual distance from receiver to transmitter

DV = height of the diurnal variation at receiver location in μsec

However, DV is the difference in time of propagation from night to day or

$$DV = \frac{RT}{V_n} - \frac{RT}{V_d} \quad (48)$$

where V_n = night velocity

V_d = daytime velocity

$$DV = \frac{RT}{C} \left(\frac{1}{V_n} - \frac{1}{V_d} \right) \quad (49)$$

$$DV \approx \frac{RT}{C} \left(\frac{V_d}{C} - \frac{V_n}{C} \right) \quad (50)$$

Combining Equations 50 and 47 gives

$$\epsilon_T = \frac{d}{C} \left(\frac{V_d}{C} - \frac{V_n}{C} \right) \quad (51)$$

Using Pierce's values for V_d and V_n over sea water we find that the maximum value of the error over a single path is

$$\epsilon_T = \frac{d}{C} 0.00383 \quad (52)$$

or

$$\epsilon_{T/d} = 0.0236 \text{ } \mu\text{sec/nautical mile} \quad (53)$$

Sometimes the errors over the master and slave paths may add, so that the maximum error in the time difference would be

$$\frac{\epsilon \Delta T}{d} = 0.0472 \text{ } \mu\text{sec/nautical mile} \quad (54)$$

Therefore an error in assumed position of 100 nautical miles would result in a maximum time-difference error of 4.72 μsec .

Present knowledge of ionospheric physics does not permit reliable prediction of SID's. Furthermore, the experience of NRL with their airborne OMEGA receiver in flight tests has indicated that a navigator may not recognize the occurrence of an SID if the receiver output is not continuously recorded. Since serious SID's affect a large portion of the service area, although not necessarily to the same extent, they might be reported to the users by ground monitors if reliable communications could be maintained. The same might be true of ionospheric shower

effects; since such effects often last several days, it might be possible to determine corrective measures. How successful such an approach might be is not yet evident.

4.3.4.2. Routes and Service Areas. The evaluation of position error resulting from the time-difference errors that have been discussed depends on an evaluation of the d_{rms} error given by Equation 10. Appendix D discusses the computation of Equation 10 over a spherical earth and presents the results of computations for several master and slave station configurations. The results of Appendix D will be used here to evaluate the probable d_{rms} error over regions of interest.

One of the primary goals, as interpreted here, is the evaluation of VLF systems, especially hyperbolic systems, for use over routes up to 3000 nautical miles in length. The most heavily traveled 3000-mile over-ocean route is the North Atlantic route between New York and London; this route will be used for illustrative purposes.

Figures 26, 27, 28, and 29 show system configurations with baseline lengths of 3000, 4000, 5000, and 5000 nautical miles and baseline angles of 120° , 120° , 90° , 120° , respectively. The contours shown on the plots are for constant values of K where

$$K = \frac{d_{rms}}{c\sigma_{\Delta T}} \quad (55)$$

where c is the velocity of light. If d_{rms} error in nautical miles is desired, and $\sigma_{\Delta T}$ is in μsec , then

$$d_{rms} = 0.1618K\sigma_{\Delta T} \quad (56)$$

K is a multiplying factor of the computed error resulting from the hyperbolic geometry. The plots are azimuthal equidistant projections, and the distance scale is constant in a radial direction from the origin or master station. The azimuth to any point can be measured at the origin or master station. Angles measured at any other point will be in error, as will distances measured other than radially from the master station.

The 3000-mile route from New York to London has been plotted on Figures 26, 27, and 28, along with the boundaries of a rectangle 1000 nautical miles wide centered on the route. The lines forming the area plotted are great-circle lines on the globe. The rectangles are plotted with the stations located at the following points.

Figure	Master Station	Slave Station 1	Slave Station 2
26	Thule	Florida	North Turkey
27	Thule	Central America	Eastern Libya
28	Puerto Rico	Aleutian Range	Turkey

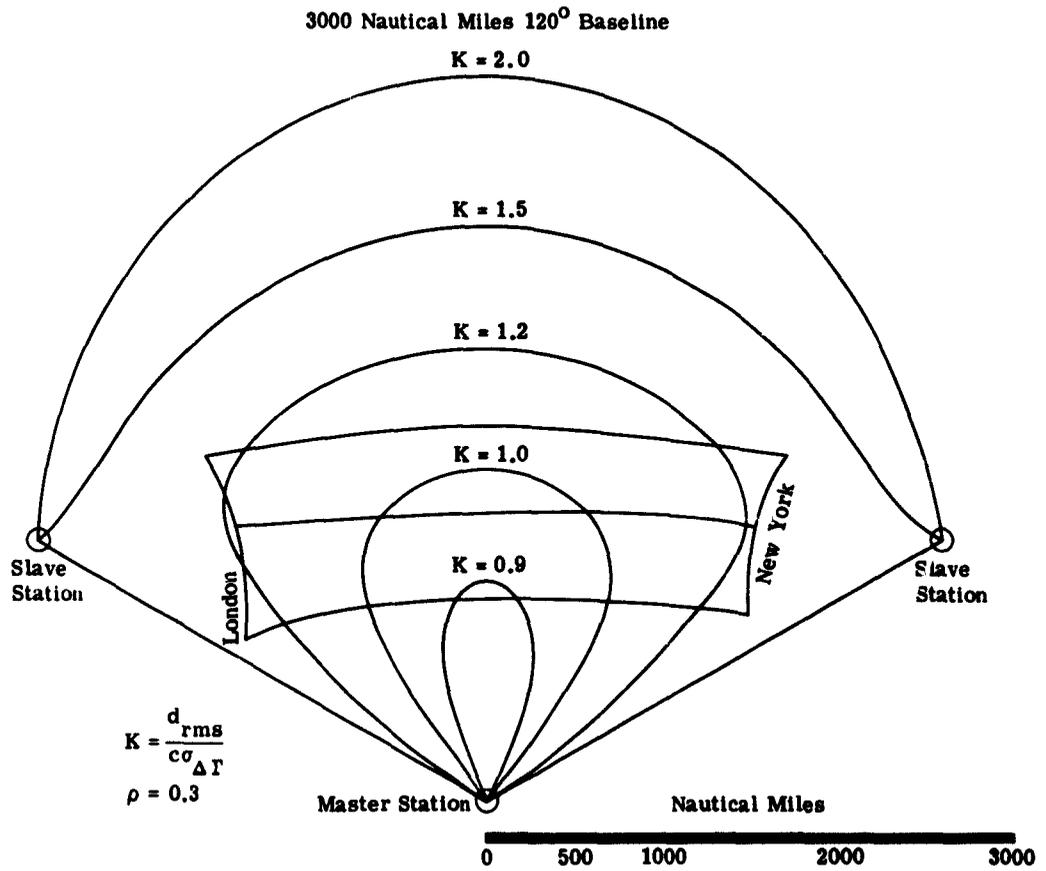


FIGURE 26. d_{rms} - ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

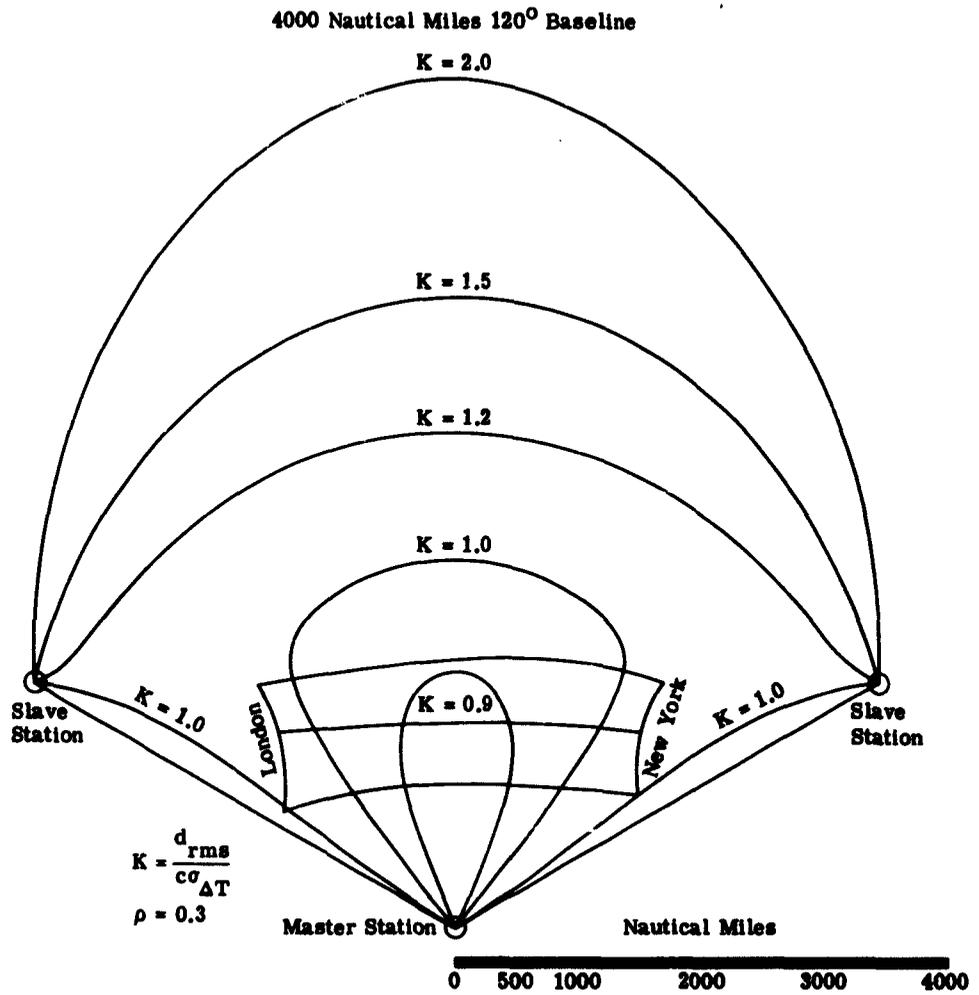


FIGURE 27. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

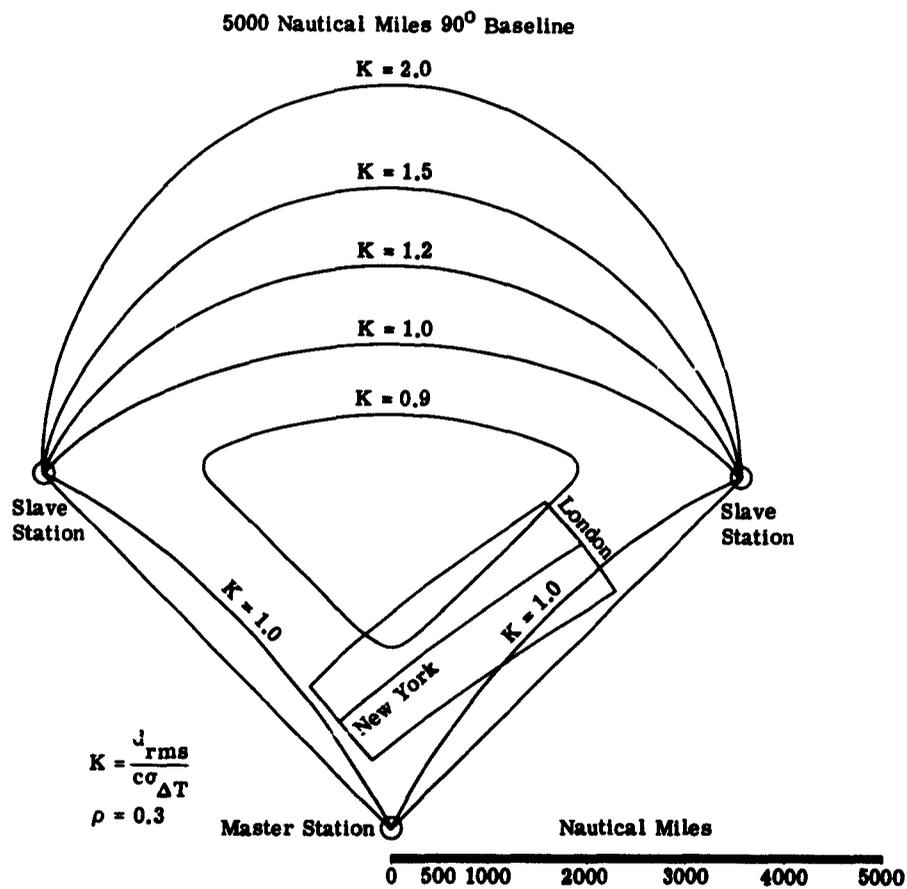


FIGURE 28. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

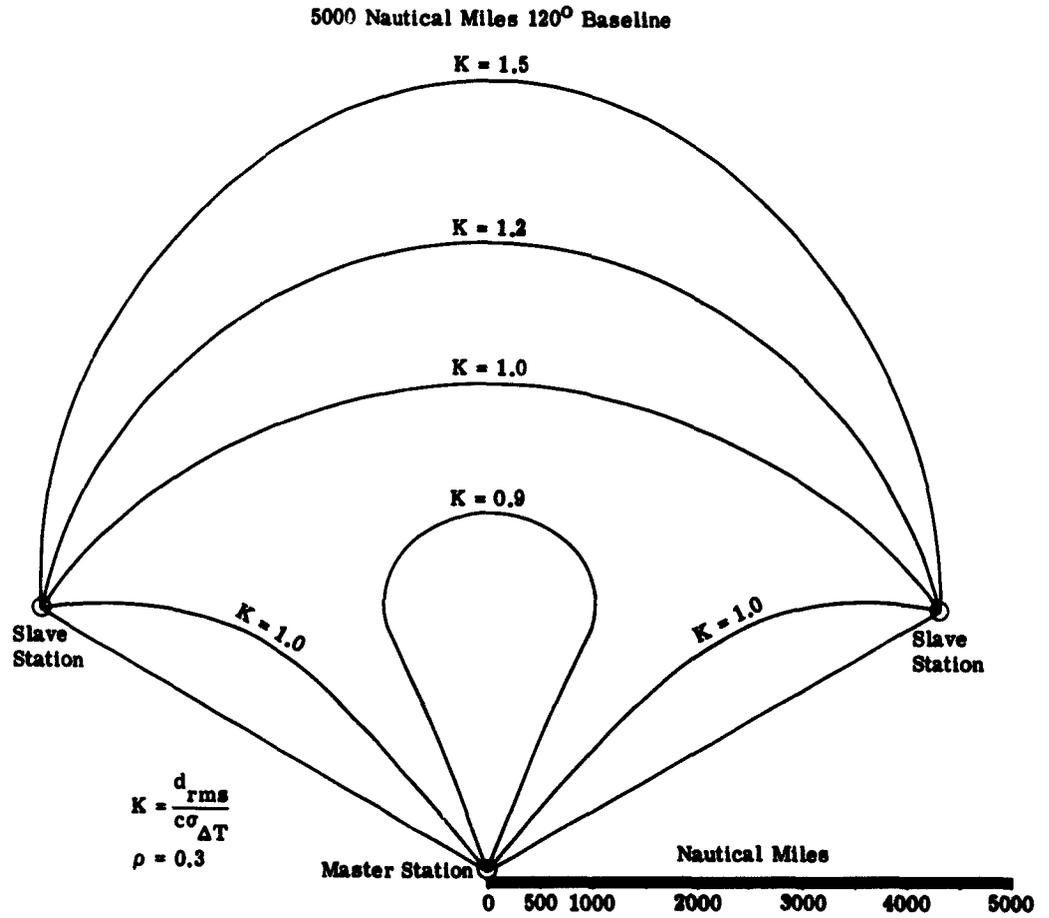


FIGURE 29. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

The reason for considering the 1000- by 3000-mile rectangle for the New York-London route was to show the coverage over a sizable area of the North Atlantic; 1000 miles is the airspace width that commercial carriers might be expected to occupy [47].

The North Atlantic rectangle is largely contained within the $K = 1.0$ contour by the 4000-mile system, and within the $K = 0.9$ contour by the 5000-mile system. In both examples, the stations are all at least 1000 miles from the rectangle. The minimum possible value of K would be realized only by a system of four stations forming baselines that intersect in right angles. At the point of intersection, θ , ϕ , and ϕ_2 are each 90° and

$$K = 0.707 \quad (57)$$

The system configurations described above provide coverage over large areas with little degradation in accuracy because of geometry.

Using the larger value of random propagation error given in Equation 42 and treating the phase velocity uncertainty given in Equation 45 as a random error, we have as the total random error in the time differences

$$\sigma_{\Delta T}^2 = 12^2 + 10.6^2 \quad (58)$$

$$\sigma_{\Delta T} = 16 \text{ } \mu\text{sec} \quad (59)$$

For the 4000-mile, 120° system (Figure 27) we then can cover the North Atlantic routes with an accuracy of

$$d_{\text{rms}} = 0.1618 \times 16 = 2.59 \text{ nautical miles} \quad (60)$$

Using the 5000-mile 90° system, as shown in Figure 28, we find the error to be

$$d_{\text{rms}} = 0.1618 \times 0.9 \times 16 = 2.33 \text{ nautical miles} \quad (61)$$

It is also interesting to examine the total coverage that might be included by the $K = 1$ contour on each of these plots. For this reason, the information has been plotted on a globe for each of the selected configurations given by Figures 26, 27, 28, and 29. Photographs of the globe with each configuration are shown in Figures 30, 31, 32, and 33.

It should be noted that 5000-mile baseline systems give coverage over a large area with little change in the value of the d_{rms} error. Figure 33 shows a 5000-mile baseline, 120° system with the master station located at Quinhagak, Alaska, and slave stations on the Galapagos Islands and in Turkey. The coverage now is extended to West Europe, the Mediterranean area, Central America and the Caribbean, most of the United States, all of Canada, and polar routes.

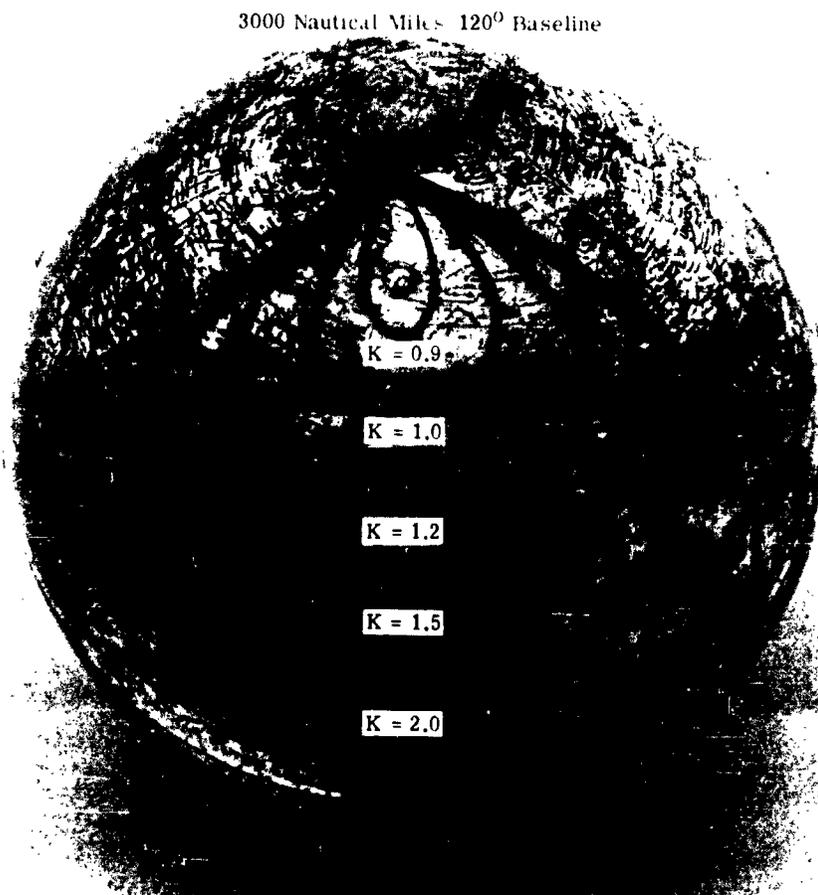


FIGURE 30. d_{rms} - ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM

4000 Nautical Miles 126° Baseline

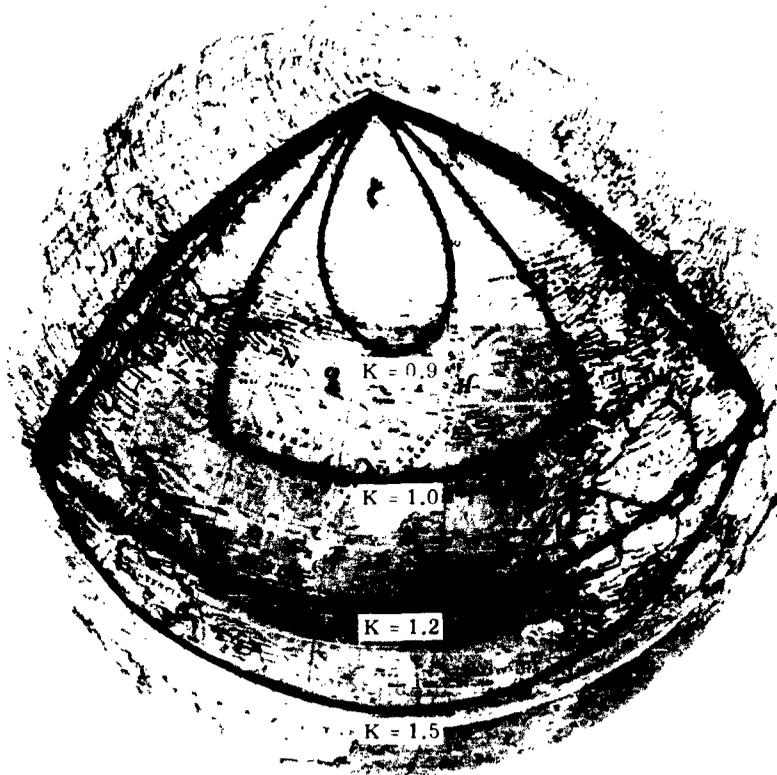


FIGURE 31. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM

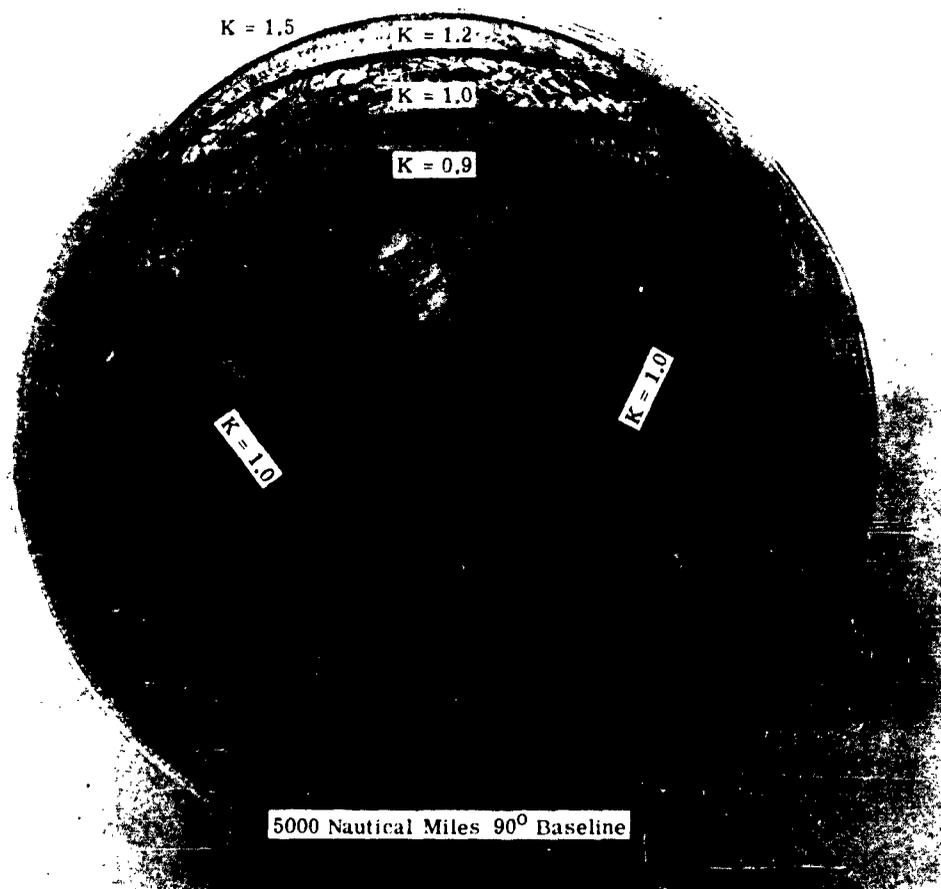


FIGURE 32. d_{rms} - ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM

5000 Nautical Miles 120° Baseline

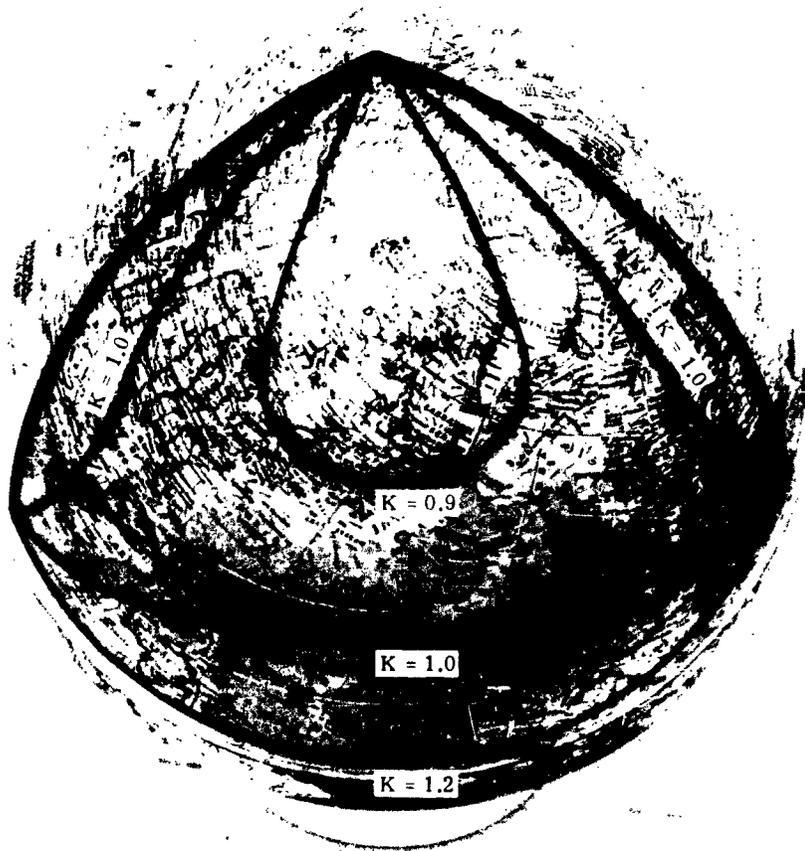


FIGURE 33. d_{rms} - ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM

Accuracy over the North Atlantic route is nearly as good as that provided by a short baseline system intended to service only that area.

Because of the great coverage illustrated by Figure 33, the United States Navy is considering a global system with four to six stations located near the equator, and one as close to each pole as possible. The earth would then be covered by eight service areas similar to that shown in Figures 28 and 32. The North Atlantic and a large area of the Pacific might be covered with the four stations shown in Figure 33, with one more in the Pacific.

The accuracy indicated here is better than required for over-ocean commercial navigation. If a d_{rms} error of six nautical miles is allowed, then from

$$K = \frac{d_{\text{rms}}}{c\sigma_{\Delta T}} \quad (62)$$

we obtain

$$\sigma_{\Delta T} = \frac{d_{\text{rms}}}{Kc} = \frac{6}{0.1618} = 37 \mu\text{sec} \quad (63)$$

If we assume a random error of $\sigma_{\Delta T} = 16$, we may allow an additional random component of

$$\text{additional allowable } \sigma_{\Delta T} = \sqrt{37^2 - 16^2} = 33.2 \mu\text{sec} \quad (64)$$

4.3. LANE IDENTIFICATION AND TRACKING

4.3.1. LANE IDENTIFICATION. All phase measuring systems suffer from ambiguities between regions over which the phase difference changes by one cycle. The regions of ambiguity are often called lanes, and the width of each lane LW is

$$LW = \frac{\lambda}{2 \sin \phi} \quad (65)$$

where λ is the wavelength of the transmitted signal and 2ϕ is the angle subtended by the slave stations at the receiver. At 10 kc the lane width is 15,000 meters on the baseline.

Before a user can establish his position, some method must be provided for resolving the ambiguities. The method commonly used in currently operational phase comparison navigation systems is to provide a second, but less accurate, measure of position. The less accurate or "coarse" information must then have either no ambiguities, or ambiguity regions of such width that they may be resolved by dead reckoning or auxiliary navigation aids.

The accuracy of the coarse measurement required to resolve the lane ambiguities reliably has often been given as half a lanewidth. The reliability of resolution depends, however, on the

lanewidth, the accuracy and precision of both the coarse and the fine measurements, and the decision process used. It has usually been assumed that the fine measurement is substantially more accurate and precise than the coarse measurement. Thus, the coarse measurement is used to define the lane, and the fine measurement used to find position within the lane. In other words, the fine reading can be considered the fractional portion of the total reading; and the coarse reading, the integral portion.

If the integral portion of a single coarse measurement is used as the integral portion of the final measurement, the probability that the lanes have been correctly resolved is simply the probability that the integral part of the coarse reading is correct. If, in addition, the errors in the coarse measurement are assumed to be normally distributed with mean 0 and standard deviation σ_c given in percent of a lanewidth, then the probability of correct lane identification is

$$P = 2\Phi\left(\frac{100}{\sigma_c}\right) - 2\sigma \left[0.3989 - \phi(100/\sigma_c)\right] \quad (66)$$

where

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad (67)$$

$$\Phi(x) = \int_0^x \phi(t) dt \quad (68)$$

The probability of correct lane identification by such a scheme has been computed as a function of σ_c and is shown by the dashed line of Figure 34.

However, the scheme described above does not take advantage of the information contained in the fractional part of the coarse measurement. One common method which does make use of the entire coarse measurement is the selection of that integer which, together with the fine measurement, would make the final result differ from the entire coarse measurement by less than one-half lane. If the fine measurement were made by an error-free system, the lane identification would be correct whenever the coarse measurement erred by less than one-half lane. This requirement has been the basis for the frequent statement that the accuracy of the coarse system must be better than one-half lane. If the fine system has errors that are normally distributed with mean 0 and a standard deviation of σ_f given in percent of a lanewidth, then for correct lane identification the error in the coarse measurement minus the error in the fine measurement must be less than one-half lanewidth, or

$$-\frac{LW}{2} < \epsilon_c - \epsilon_f < \frac{LW}{2}$$

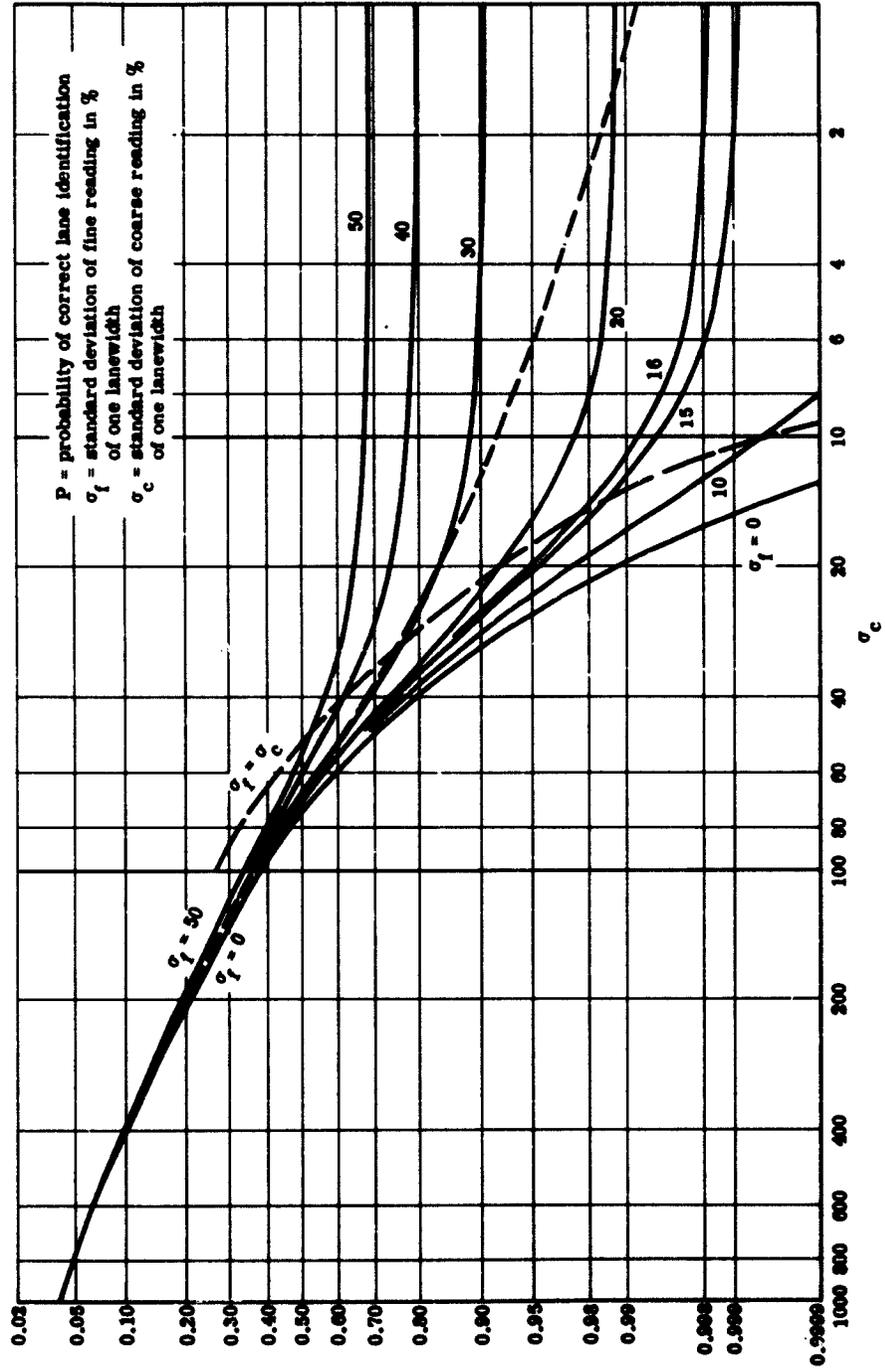


FIGURE 34. PROBABILITY OF CORRECT LANE IDENTIFICATION WITH A SINGLE LANE READING

where ϵ_c = the error in the coarse measurement in lanes

ϵ_f = the error fine measurement in lanes

The probability of correct lane identification from a single set of measurements with this scheme is shown in Figure 34 by the solid lines.

Also shown in Figure 34 is the line along which $\sigma_c = \sigma_f$. To the right of this line the probabilities shown by the solid lines are of little interest, because in this region the coarse system is more accurate than the fine system and the justification for a fine system is questionable. Equation 63 has shown that the accuracy requirement of a 6-nautical mile d_{rms} error might be satisfied by $\sigma_f = 37 \mu\text{sec}$. At 10.2 kc one lanewidth is $98 \mu\text{sec}$ or $\sigma_f = 38\%$. If the coarse system provides an accuracy of $\sigma_c \leq 38\%$ at 10.2 kc, again the need for the fine system is questionable. We might restrict our attention, then, to the region to the left of $\sigma_c = 38\%$ and below the line along which $\sigma_c = \sigma_f$. If the expected value of σ_f is 16% (see Equation 58), the maximum probability of correct lane identification from a single lane reading is 0.76. It should be noted that in the region of interest more reliable resolution results from the first scheme described for choosing the lane number (where no attention is given to the fractional part of the coarse measurement) when $\sigma_f > 30\%$. Although both schemes have been used in the past, the difference in reliability provided by the two is not great unless σ_f is a small fraction of a lanewidth.

This analysis has been limited to the probability of correct lane identification with a single measurement, under the assumption that the mean errors of both the coarse and the fine measurements are 0. The probability of correct lane identification would be greatly increased if several consecutive coarse or lane readings were averaged, provided that the correlation between the errors of the measurements was small over the averaging period.

There are two methods of making the coarse measurement. Pulse systems, such as LORAN-A and LORAN-C, measure time difference on pulse-leading edges with no ambiguity. Continuous-wave systems usually make a second time-difference measurement at a second but lower frequency with larger regions of ambiguity. The pulse method is impossible at very low frequencies. The antennas that must be used to radiate any appreciable power have such narrow bandwidths that the pulse rise times are too long. The antenna of Navy station NBA in Balboa, Canal Zone, is used as the master station antenna in the current OMEGA experiments and is typical of antennas that must be used for long range propagation at VLF. It consists of six towers, each 600 feet high, spaced to provide a flat top 1200 feet by 2400 feet. The Q of the antenna at 18 kc is 700, and the pulse rise time is about 15 msec. Time measurements on the envelope of pulses from such an antenna suffer from errors caused by dispersion of the propagation medium and from inability to define a specific point on a slowly rising pulse. Stone, et al., have obtained time measurements on the NBA station with a probably error of 500 μsec , at

Washington, D. C. Stone thinks that the error may eventually be reduced to 100 μ sec, but the technique shows no hope of ever providing reliable lane resolution at VLF [48].

All VLF proposals have been based on the lower frequency technique. The "coarse" frequencies that have been tried range from 200 to 4000 cps and are obtained in the receiver as a difference frequency. In both the original OMEGA concept and DELRAC two individual carriers are transmitted and mixed in the receiver. In the RADUX experiments one carrier was transmitted with low-frequency modulation (200 cps) and the receiver effectively obtained the coarse signal as the difference between the carrier and sidebands.

The multiple frequency schemes are subject to lane identification error resulting from a phenomenon that might be called "time or phase multiplication." When two sinusoidal signals of the form

$$\sin(\omega_1 t + \phi) \quad (69)$$

and

$$\sin(\omega_2 t + \theta) \quad (70)$$

are combined to obtain the difference frequency, the result is of the form

$$\sin[(\omega_1 - \omega_2)t + \phi + \theta] \quad (71)$$

Any change in the phase of one of the inputs results in an equal phase change in the difference frequency, and differential time delay in the input signals is multiplied by the ratio of input to output frequency

$$\frac{\partial t}{\partial(\omega_1 - \omega_2)} = \frac{\partial t}{\partial \omega_1} \frac{\omega_1}{\omega_1 - \omega_2} \text{ or } \frac{\partial t}{\partial \omega_2} \frac{\omega_2}{\omega_1 - \omega_2} \quad (72)$$

Thus, for a 10-kc system with lanewidths of 100 μ sec, a variation of 5 μ sec in the relative phase of the two signals would cause a variation of 50 μ sec in the phase of a 1-kc-difference frequency.

The great sensitivity of lane resolution to dispersive propagation is the reason Decca Navigator, Ltd., has proposed resolution in steps for the DELRAC system. In 1959 Casselman, et al., published the results of an experiment conducted to investigate the two-frequency lane resolution capabilities of OMEGA [27]. Two frequencies spaced 1 kc apart were transmitted from Hawaii to San Diego in a manner very similar to that which would be used in an operational system. The phase of the difference frequency (1 kc) was recorded over several 24-hour periods, and the frequencies used ranged from 10.2 to 18.2 kc. In general the data indicated that lane resolution would be possible during the daylight hours at frequencies below 14.2 kc. During hours of dark-

ness, however, reliable lane resolution would not be possible at any of the frequencies used. Apparently the error-correlation time is so long that averaging would not help much. Lane resolution with a difference frequency of $f/3$ might be possible where f is the basic frequency—say, 10 kc. Here the "phase multiplication" problem is reduced by a factor of 3, and, even with less correlation between the phase velocities for the two frequencies, lane resolution is likely to be possible. On this matter both the OMEGA and the Decca people have come to the same conclusion. It should also be pointed out that the reduction of ambiguities in steps of three requires several steps before dead reckoning can make the position information completely unambiguous; this technique has not been conclusively demonstrated experimentally.

4.3.2. TRACKING. The present experimental OMEGA system does not provide lane resolution. Present position is only obtained through continuous tracking of "fine" position data. The opinion of people working on the OMEGA development at NEL and NRL is that continuous tracking is possible and, therefore, the added complexity and size of lane resolution receivers is not worth the benefits these might provide. Their opinion, however, is based on surface vehicle experience with continuous recordings of the receiver output. In an aircraft the problem is more difficult. The NRL receiver used in recent flight tests has no lane identification capability and the NRL test report suggests the development of a two-frequency OMEGA with one step of lane resolution [40-45]. The proposed width of the second lane is about 40 miles. If this resolution is possible, the 20-mile accuracy required to remove further ambiguities might be possible with auxiliary navigation aids or dead reckoning. The present 10.2-kc OMEGA lane-width is 7.9 nautical miles on the baseline; to remove ambiguities, position knowledge must be good to about four miles on the baseline.

Any navigation system good enough to resolve the ambiguities of the present OMEGA system (without a coupled lane identification system) will almost meet the accuracy requirements of this study. The use of a strip chart recorder for readout is suggested by NRL because this would allow correction of lane counts that have been lost. The use of the strip chart would also permit rapid recognition of anomalous propagation (SID's) which might go unnoticed on a counter-type display.

The use of a continuous chart record of output with manual signal evaluation may not be a desirable operating procedure for modern air carrier operation in view of the trend to reduce crews. If an instantaneous readout is necessary, such as that provided by a counter-type display, or if the readings occasionally must be sent automatically over data links for air traffic control use, the problem of lane identification is serious. A modern jet can fly across one OMEGA lane in about 41 seconds; a Mach 3 supersonic jet, in about 16 seconds. Loss of signal for 20 seconds

or 8 seconds, respectively, might result in an improper lane count. This loss might be caused by local atmospheric noise of extremely high amplitude or incorrect rate information during maneuvers, if the system is rate-aided.

4.4. TRANSMITTER REQUIREMENTS

Transmitter power requirements were discussed in Section 2.3.1. With the exception of the Forestport Station, transmitters now used for OMEGA (92 kw at Balboa and 5 kw at Haiku) were sufficient for the NRL flight test out to at least 5000 miles, except during a hailstorm. The estimated requirement computed in Section 4.2.3.1 was

$$P_r = 22 \text{ kw (See Equation 34)} \quad (73)$$

Some discrepancy would be expected because of the assumptions made in the noise analysis.

If external rate information is not available, the power required to serve supersonic jets (Mach 3) was estimated to be (Equation 33)

$$P_r = 138 \text{ kw} \quad (74)$$

Although a 3000-nautical-mile-baseline system would require much less power, the benefits of a 5000-mile system may warrant considering it.

No cost analysis of transmitting stations capable of radiating these powers at low frequencies has been made for this study; the problem necessitates a fairly comprehensive design analysis before reliable estimates can be made.

The low radiating efficiency of VLF antennas makes it economical to use a mediocre antenna and a large transmitter (i.e., in the range of several kw to several hundred kw the transmitter power is cheaper than antenna efficiency). Voltage effects (insulator breakdown, corona, etc.), which limit the amount of power that can be fed into a given antenna, also limit the possible reduction in antenna size and efficiency.

The NBA transmitter at Balboa consists of six towers, each 600 feet high, spaced to provide a rectangular flat top 2400 feet long by 1200 feet wide. The maximum power radiated by this antenna at 10.2 kc is about 2 kw. The slave station at Forestport, on the other hand, is a single vertical 1200 feet high, limited by voltage breakdown consideration to 250 watts of radiated power. During severe icing conditions the radiated power has to be dropped to 15 watts.

M. L. Tibbals of NEL has given an estimate for an OMEGA station of ten million dollars. Nine million is for an antenna with a flat top supported in the center by a 1200-foot tower and

supported on the edges by five towers, each 800 feet high and spaced 1500 feet from the center.¹⁶ The remaining million dollars is for the transmitter. This antenna description sounds like half of the Cutler, Maine, VLF communication station. The cost of each Cutler antenna was considerably above this estimate. Dr. Pierce has suggested the use of 10-kw stations and antennas with 1500-foot center towers and an umbrella supported by 500-foot towers. Dr. Pierce estimates that the efficiency would be 10%, and that the cost would be five to eight million dollars each.

Northrup Ventura Division of Northrup Corporation has recently announced the development of a short circular antenna called the "Directional Discontinuity Ring Radiator" (DDRR). They have suggested using it for radiating vertically polarized energy from antennas which must be shorter than one quarter wavelength. Upon inquiry Northrup indicated that a DDRR could be constructed for operation at 10 kc; the antenna would be 5000 feet in diameter and 500 feet high, and would have an efficiency of 65%.¹⁷ Initial cost estimates of Northrup Ventura run from twenty to thirty-five million dollars, and this is admittedly conservative. The efficiency, however, is high. Northrup is reportedly working on an initial design and estimate for an antenna less efficient (about 10%) at a lower cost. The results of the investigation are not yet available.

4.5. PROBLEMS OF INTRODUCTION AND USE

This section will discuss some of the problems of introducing a VLF hyperbolic navigation system into the world of modern transoceanic air carrier operations; it will also discuss several miscellaneous aspects of VLF systems that have not been covered above. Because of the trend toward smaller crews in airline operations a navigation system that might require a navigator or increase the crew work load would be inappropriate unless the payoff were quite high.

The present method of obtaining a VLF fix is almost identical to that of standard LORAN. The output of the receiver is two time differences, which may be displayed on a dial, counter, or strip recorder. The time differences are then plotted on a chart on which lines of constant time difference have been printed. If an accuracy of 2 to 6 nautical miles is required, the navigator must be able to plot with an accuracy of better than 0.03° to 0.1° latitude. Such plotting is certainly not impossible, but it forces the navigator to use larger-scale maps than used now, when the trend is toward small-scale maps for navigation. The correction for the diurnal shift now has to be computed by the operator, and the strip chart may have to be inspected for lost lanes and anomalous propagation. These procedures are nearly as complicated as present celestial techniques, and are incompatible with the concept of reducing crew duties.

¹⁶ Personal communication with M. L. Tibbals of Naval Electronics Laboratory.

¹⁷ Personal communication with Stanley Boyle, Northrup Ventura Division, Northrup Corporation.

The requirement, then, is for a navigation computer capable of transforming the hyperbolic system coordinates into some geographic coordinate system. A computer for use in conjunction with a hyperbolic system receiver might also perform a second, almost equally important function. The NRL aircraft receiver requires externally supplied rate information with a radial velocity error less than 250 miles per hour.

Convenient operation and rate accuracy requirements both indicate the need for deriving the rate information automatically, particularly during maneuvers. If standard airspeed indicators and heading references or doppler systems are to be used, a computer too is required.

Aircraft computers for solving these problems, particularly the coordinate conversion problem, are not yet available. Several groups have worked on the coordinate conversion problem, but no solutions offering sufficient accuracy are yet in sight.

Attempts to analyze the reliability and maintenance aspects of projected systems have met with only mediocre success. The MTBF of the present NRL airborne receiver is estimated to be about 300 to 500 hours. Receiver maintenance would present no particular problems to the airlines. A small amount of technician training might be required, but no unique problems would be encountered. Unless computers are used for coordinate conversion and correction, crew training would be roughly equivalent to that required for LORAN-A. Transmitting systems should be reliable, although the Navy time-standard station NBA, which is also serving as the OMEGA master station, is down for maintenance several days a year. Because of the high costs and the massiveness of antennas and antenna-loading circuits, reliability by duplication seems unlikely. One of the characteristics of massive VLF antenna systems, furthermore, is the long time required for repair when trouble does develop.

Cost estimates are only rough. An attempt at predicting receiver costs on the basis of comparing available commercial aviation equipment was unsuccessful because no equipment of similar complexity and circuitry could be found. Mr. Brodgen and Mr. Thornhill of NRL estimate the cost of production model OMEGA aircraft receivers at about \$5000. The Decca receiver costs about this, but contains no servo components. The currently available LORAN-C receiver, on the other hand, contains similar servo circuitry, but is considerably more complex and sells for about \$28,000. A reasonable guess might be \$8000 to \$10,000 for a production model of the airborne OMEGA receiver. This figure might be reduced, and reliability might be increased, if the servos were replaced with solid-state devices; this change is being investigated for LORAN-C receivers. Nevertheless, \$8000 to \$10,000 might not include the cost of lane identification circuits, and definitely would not include a computer, installation, and maintenance and repair equipment and supplies. The total cost to the airlines per installation would be many

times this. Transmitter stations have already been estimated at from ten to thirty million dollars. If systems are planned only for smaller service areas, the cost of the stations might be materially reduced.

A formidable problem to be faced before the adoption of a VLF navigation system might be the international agreement. Historically, all such endeavors have met with strong political pressures, and adopting systems dependent upon international cooperation and participation has been extremely slow.

4.6. SYSTEM CONFIGURATIONS OTHER THAN HYPERBOLIC-HYPERBOLIC

4.6.1. THETA-THETA SYSTEMS. Theta-theta, or azimuth, systems permit the determination of a fix by measuring the angles from the aircraft to two known locations. The intersection of the lines (LOP) thus defined is unique. (The second solution at the antipodal point is beyond the range of interest here.) The angles at either the aircraft or the ground may be measured, at either location, in two ways: the direction of arrival of a radio wave may be determined, or interferometer techniques may be used. An example of the measurement of angle of arrival is the loop direction finder; the Adcock direction finder, an example of the interferometer technique, can be used with the directive antenna array at the transmitter or receiver. Examples of both methods are the Adcock direction finder and the CONSOLAN system.

Appendix A shows that the maximum power that may be radiated from an aircraft antenna is less than 115×10^{-6} watts at 40 kc. Using Equation 30 to find the field strength at 1000 nautical miles with $\alpha_{40\text{kc}} = 7$ db/km, we find

$$E < -6.39 \text{ db above } 1 \mu\text{v/m} \quad (75)$$

With an atmospheric noise level of 70 db above $1 \mu\text{v/m}$ (Section 4.2.3.1), the signal-to-noise ratio in a 1-kc bandwidth is then

$$\frac{S}{N} < -76.39 \text{ db (1-kc bandwidth)} \quad (76)$$

At lower frequencies the antenna efficiency and voltage limited power are lower and, even though the path loss is slightly less, the signal-to-noise ratio is lower. Signal-to-noise ratios of this magnitude are extremely low for operation with the integration times allowable.

Long-range applications must be confined to systems with the transmitter on the ground. Three schemes are then possible: (1) a loop-type direction finder might take bearings on at least two transmitters; (2) an interferometer with spaced antennas might be used on the aircraft; and (3) an "inverse" interferometer might be used with spaced transmitting antennas and the measurements taken in the aircraft—this is the CONSOLE or CONSOLAN technique.

Loop antenna direction finders, or angle-of-arrival-type equipment can be exemplified by two modern aircraft ADF systems. They are the military ARN-59 and the commercial Collins Radio model DF 201 ADF system. Both sets represent the state of the art in aircraft DF equipment, and no major improvements in accuracy are expected in the foreseeable future. Both sets also have an instrumental accuracy of $\pm 2^\circ$. This accuracy is at low frequencies and does not include propagation effects or the effects of the aircraft itself.

An error of six nautical miles in a 1000-mile range is the least stringent combination of range and accuracy that we may consider acceptable for this study. If the line of position crossing angles are 90° (an ideal case)

$$d_{\text{rms}} = \left(r_1^2 \sigma_{\theta_1}^2 + r_2^2 \sigma_{\theta_2}^2 \right)^{1/2} \quad (77)$$

where r_1 and r_2 are the distances to the transmitters

σ_{θ_1} and σ_{θ_2} are the standard deviations of the bearing error

If both stations are at the same range and both measurements are of the same accuracy

$$r_1 = r_2 = r \quad (78)$$

and

$$\sigma_{\theta_1} = \sigma_{\theta_2} = \sigma_\theta \quad (79)$$

then

$$d_{\text{rms}} = \sqrt{2} r \sigma_\theta \quad (80)$$

If the allowable d_{rms} error at 1000 nautical miles is 6 nautical miles, then

$$\sigma_{\theta_{\text{max}}} = \frac{6}{1000 \sqrt{2}} = 0.00424 \text{ radians} \quad (81)$$

$$\sigma_{\theta_{\text{max}}} = 0.243^\circ \quad (82)$$

This angular error is much less than the error of foreseeable loop-type aircraft direction finders.

The relation between instrumental error and bearing error in the interferometer technique can be developed from Figure 35.

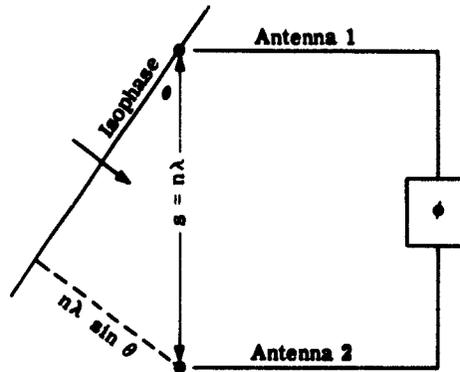


FIGURE 35. INTERFEROMETER

θ = the angle between the wavefront and the plane of the antennas, and the bearing angle

s = antenna spacing

ϕ = phase measurement between the signals from the two antennas

$$\phi = 2\pi ft \tag{83}$$

where t is the time delay in arrival at Antenna 2. Then

$$\phi = 2\pi f \frac{n\lambda \sin \theta}{c} \tag{84}$$

$$\lambda = \frac{c}{f}$$

$$\phi = 2\pi n \sin \theta \tag{85}$$

$$\frac{d\phi}{d\theta} = 2\pi n \cos \theta \tag{86}$$

The measurement of θ is most accurate when

$$\cos \theta = 0$$

then

$$\sigma_{\phi} = 2\pi n \sigma_{\theta} \tag{87}$$

If we assume a transmitter on the ground and two spaced antennas on the aircraft, the maximum spacing is about 140 feet (maximum dimension of a Boeing 707). If a frequency is 40 kc

$$\lambda = 24,590 \text{ feet} \tag{88}$$

$$n = \frac{d}{\lambda} = 57 \times 10^{-4} \tag{89}$$

Using Equation 87 and $\sigma_\theta = 0.243^\circ$, we find

$$\sigma_\phi = 87 \times 10^{-4} \quad (90)$$

This is an impossible phase measurement.

If we assume that the spaced antennas are the transmitting antennas on the ground, and that the phase measurement is made in the air (CONSOLAN operates similarly), then we can find the separation of antenna required to provide a 6-nautical mile d_{rms} position error at a range of 1000 nautical miles, if the phase measurement is accurate to 1° —a typical instrumental accuracy.

$$\sigma_\phi = 2\pi n \sigma_\theta \quad (\text{Equation 87})$$

$$\sigma_\phi = 1^\circ$$

$$n = \frac{\sigma_\phi}{2\pi \sigma_\theta} = \frac{1}{2\pi \cdot 0.243} = 0.655 \quad (91)$$

$$d = n\lambda = 3.04 \text{ nautical miles} \quad (92)$$

where d is the separation of antennas required. Because of the action of $\cos \theta$ in Equation 86 and the position error caused by skewed LOP's, this distance would have to be increased by a factor of at least 3 or 4 to provide coverage over any sizable area [23].

Such a measurement is dependent on a correlation coefficient of nearly one between the propagation times from the two antennas. Dr. Pierce has evidence that as the separation between antennas increases to about 3 to 5λ , the correlation coefficient drops to 0.5.¹⁸ The measurement of the bearing is less accurate than indicated in the above analysis because of the lack of the correlation; as the correlation falls off with increased antenna spacing, the degradation increases. The accuracy will not rise proportionally to the separation until after the correlation is zero (at about 40λ). If we then increase the separation to achieve the accuracies of a hyperbolic system, we have, indeed, a hyperbolic system. The interferometer system is simply a short baseline hyperbolic system.

The high accuracy of OMEGA results from the great increase in antenna spacing beyond the point where the path correlation falls to zero.

4.6.2. RHO-RHO AND HYPERBOLIC-ELLIPTIC. Both rho-rho and hyperbolic-elliptic systems have been proposed in the past and will undoubtedly be proposed in the future. There

¹⁸Personal communication with Dr. J. A. Pierce, Cruft Laboratory, Harvard University.

are no systems of either type now in use at VLF. During the course of this contract we learned that Pickard and Burns, Inc., worked on a related system, but information concerning this system is still classified.

The two systems are discussed together because they have much in common. Both require the measurement of absolute distance to at least one station; and both require the use of only two stations to provide a fix.

Several possible advantages of a rho-rho system have prompted its consideration. One of these is the saving in cost through the use of only two stations. Transforming system coordinates into geographic coordinates would be somewhat simpler for rho-rho systems than for hyperbolic systems, because the equations of rho-rho LOP's are much simpler than the equations of hyperbolic LOP's. The accuracies specified in this study, however, would require corrections for the ellipticity of the earth, and this removes some of the advantage. Computers of sufficient accuracy and range for either rho-rho or hyperbolic conversion are not now available. Since range from each station would be directly measured, rate information would be readily available, and diurnal shift corrections could be made with less computation.

Hyperbolic-elliptic systems are similar. Here one time difference and one sum are obtained. The LOP's thus established are a set of hyperbolas with the stations as foci, and a set of ellipses with the same foci. The LOP's are then orthogonal trajectories, and all crossing angles are right angles. Even though the time-difference loci on a spherical earth are not actually hyperbolas, the ellipses are also distorted and the LOP's still form an orthogonal family. The relative accuracy of the two systems (rho-rho, hyperbolic-elliptic) depends largely on the method of obtaining the time-difference information. If the time difference for a hyperbolic-elliptic configuration is derived from two independent range measurements, one on each station, then the d_{rms} error is the same as that of a rho-rho configuration. The two systems are, of course, equivalent under these conditions.

The only advantage of a truly hyperbolic-elliptic system (accuracy advantage) would be improved accuracy if the direct time-difference measurement were more accurate than the difference of two range measurements. This would seldom be true, however, because the local oscillator errors would cancel just as they do in OMEGA (see Appendix E). The measurement technique might be nearly identical to that used by OMEGA. There appears to be little relative merit between the two systems except that the rho-rho coordinate system is more convenient.

The large disadvantage of rho-rho systems in the past has been the inability to measure accurately the absolute ranges, which have to be measured by a phase comparison with a local stable oscillator. Hyperbolic systems have an accuracy advantage from geometry over rho-rho systems. Equations 156 and 173 of Appendix B give the d_{rms} formula for the pure-hyperbolic

and rho-rho configurations. From the plots of Appendix D, it can be seen that the constant K in defining the d_{rms} error does not change much from the minimum value of 0.8 ($\sin \theta = \sin \phi_1 = \sin \phi_2 = 1$ in Equation 156) over a very large area. Over a similar area, however, $\sin 2\phi$ (Equation 173) can widely vary and rapidly approach zero.

The time error of a clock (oscillator) ϵ_T after a time T is usually expressed as

$$\epsilon_T = t_0 + RT + \frac{1}{2}AT^2 \quad (93)$$

where t_0 is the initial "setting" error, R is the clock rate (initial frequency error or offset), and A is the acceleration or drift rate. The rate R and acceleration A are usually given by

$$R = \frac{\Delta f}{f} \quad (94)$$

$$A = \frac{\Delta f/f}{\text{Day}} \text{ or } \frac{\Delta f/f}{\text{Month}} \quad (95)$$

In most navigation systems the setting error t_0 is unimportant and can be corrected by a phase shifter if necessary. But the initial offset R is important; it must either be corrected by shifting frequency or be measured and accounted for. The acceleration, or drift rate, has normally been considered a random error, although the drift is usually constant over long periods of time. The random variable arises not from the drift rate itself, but from our lack of knowledge of it.

Past attempts to use local oscillators for measuring range have been based on quartz oscillators with a drift rate of about 1 part in 10^9 per day. If we assume that the oscillator was locked to the proper frequency at the beginning of a flight, the time error after a ten-hour flight would be

$$\epsilon_T = 7.2 \text{ } \mu\text{sec} \quad (96)$$

Usually, the clock rate cannot be set any better than 1 part in 10^9 (and often error would then be worse) at the beginning of a flight, and the total accumulated error would then be

$$\epsilon_T = 7.2 \text{ } \mu\text{sec} + 36 \text{ } \mu\text{sec} = 43.2 \text{ } \mu\text{sec} \quad (97)$$

The same type of error accrues because of errors in the station clock. Quartz oscillators have been improved until they now might have a stability of 2 to 5 parts in 10^{10} .¹⁹ The original setting problem is the same, however, except that frequency standards are available with low enough phase noise to permit 2×10^{-11} synchronization within a few minutes. However, the

¹⁹Personal communication with Mr. Peter Sulzer of Sulzer Laboratories, Washington, D. C., and Mr. Heaton, NBS, Boulder Laboratories, Boulder, Colo.

standard would have to be available at the departure point. It is conceivable that atomic standards located at each air terminal could be used for synchronization if each terminal standard were locked to the system VLF signal on a 24-hour-averaging basis.

In addition to the maintenance of a local "secondary frequency standard" at each terminus are operational problems. The oscillators must operate continuously; so some standby provision would be required. How well the oscillators can be maintained at the desired stability under the environment changes that are likely to occur is questionable.

The most objectionable characteristic of quartz oscillators, then, is their poor accuracy, or large value of R (Equation 93). On the other hand atomic frequency standards are now commercially available and provide a time measurement fundamentally related to a basic physical phenomenon. The AUTOMICHRON is probably the most familiar atomic standard and has been widely used and tested; but it is large and does not lend itself to airborne operation. Several small rubidium standards have been manufactured with advertised accuracies (R) of 1×10^{-10} or less, and stabilities of 5×10^{-11} or less. Such units have a volume of only one to two cubic feet. Their reliability is not yet as high as that of quartz oscillators, and, so far as we know, their airborne performance has not yet been fully evaluated.

4.7. SUMMARY AND CONCLUSIONS

The object of this study has been the evaluation of VLF radio navigation systems for use as a long range navaid (navigation aid) for civil aviation. We have evaluated the systems for an accuracy requirement of less than a 6-nautical-mile rms radial error at ranges up to 3000 nautical miles. We have assumed that operational capability is desirable in all types of aircraft up to and including Mach 3 jets and have emphasized existing systems — therefore, hyperbolic systems, such as OMEGA.

The accuracy analysis has shown that the maximum allowable radial rms error of 6 nautical miles can be provided by a three-station configuration with station baselines of from 3000 to 5000 nautical miles in length, provided that the standard deviation of the time-difference measurement is less than $37 \mu\text{sec}$.

The evaluation of propagation data taken in the OMEGA program indicates that the time-difference measurements may be expected to have a standard deviation of about $16 \mu\text{sec}$. However, the data from propagation phenomena used to estimate the component of this figure were gathered at fixed locations during times of a relatively quiet ionosphere.

Time-difference measurement errors caused by ionospheric disturbances have been treated as systematic errors, and, although the standard deviation given above includes uncertainty in

the phase velocity at VLF, it might also be considered a systematic error; systematic errors are likely to be constant over large areas for extended periods of time, have a definite upper bound, and are not normally distributed.

An excellent example of an error not normally distributed is that from SID's, which cause a maximum error in time difference of about 30 to 40 μ sec, or about 5 miles, but whose exact distribution is not yet known. The proper evaluation of errors not normally distributed and having a definite upper bound will depend upon the manner in which the accuracy evaluation is used. Systematic errors which are constant over large intervals of time and space should really be considered displacements of the means of normal bivariate distributions. Answers we might receive about the probabilities of such distributions might be quite different from those obtained by considering the errors of the means as random errors with a standard deviation, for example, of 1/3 of the maximum error.

Many errors that are not random (i.e., SID's, uncertain phase velocity, diurnal variation, magnetic storms, etc.) have a common effect on all users in a given area. If, then, the error analysis is used to determine minimum separation standards, these errors will not reduce the separation but merely shift the entire complex of aircraft. Although this may not create a hazard enroute, it will also not solve difficulties near terminals, where accurate reference to ground coordinates will finally be required.

Errors resulting from anomalous propagation due to a disturbed ionosphere may exceed the 6-mile accuracy requirement. Although such errors may not occur frequently, they may sometimes occur for several hours at a time or over an entire flight. They cannot be fully evaluated until more propagation data become available. Particular phenomena that require more investigation are SID's, magnetic storms, arctic and auroral propagation, meteor showers, and east-vs-west propagation time. The present OMEGA system has position ambiguities every 8 miles, and, although a system of ambiguity resolution has been proposed for the OMEGA airborne receiver, the dispersive properties of VLF propagation must be more thoroughly investigated before the reliability of resolution can be evaluated.

The readout of present experimental OMEGA airborne receivers is a strip chart recorder; transformation of hyperbolic coordinates to geographic coordinates is done on charts similar to LORAN-A charts. If the readout required in future airline operation is a real time indication of geographic coordinates, a computer will be required for coordinate conversion. Such a sufficiently accurate computer is not yet available. The present OMEGA airborne receiver requires external rate information on the radial velocity of the receiver relative to each station. The rate, or velocity information, must be accurate to ± 250 knots. Although this is not a stringent

requirement and can be furnished by standard airspeed indicators and heading references (including wind drift), a computer would again be required to compute the radial components of velocity and correct for diurnal variations.

The estimated cost of production models of the present OMEGA receiver is from \$8000 to \$10,000. A receiver with lone identification, or ambiguity resolution, would cost nearly twice this amount. The cost of a computer cannot be estimated since none have been developed, but it might be considerably more than that of the receiver. The costs of transmitter stations have been estimated at from ten to thirty million dollars.

Rho-rho systems have been considered briefly. Although the analysis of such systems is somewhat inconclusive because we lack information on airborne oscillator characteristics, it appears that the accuracy of such systems would be marginal. One attractive scheme is the use of existing VLF transmitters. Several such stations are now operated for communication and time standard purposes, and some of them are being frequency stabilized to a degree that meets station synchronization requirements. The use of existing stations would greatly reduce the cost of the system. The OMEGA program, however, has used the Navy NBA station in Balboa, Canal Zone, in such a dual role and, as a result, has encountered many operational problems.

Although the accuracy of a VLF system will often be very good and usually better than required to meet the requirements specified here, there are times during anomalous propagation conditions when the errors may exceed these requirements. Operational experience might reduce the errors, but the civil aviation community would hardly risk it. While the military might find value in a system that is usually good and only inaccurate a small percentage of the time, civil aviation would more likely accept mediocre accuracy if there were assurance that the errors would never exceed some outside limit. Propagation research is now lacking in accuracy and reliability. That is, we can show that the accuracy is usually very good, but we cannot prove errors will not be occasionally excessive. Unfortunately, the propagation investigation necessary for such proof may require a great deal of time.

Several problems make the introduction of VLF systems on an operational basis unlikely between 1965 and 1975. Propagation research necessary to insure acceptable operation under all states of the ionosphere, in all areas, may require several years. Solving the ambiguity resolution problem has not been conclusively demonstrated; here, again, considerable experimentation is required. The problem of providing an economical airborne computer for coordinate conversion is not solved and may require a long time. Finally, the requirement for stations on foreign soil will surely raise diplomatic problems.

5
INERTIAL NAVIGATION TECHNIQUES

This section presents the results of the study and analysis of inertial navigation techniques to determine their suitability for civil aircraft navigation over the ocean. Section 5.1 defines inertial navigation techniques and discusses the missions for which they are being considered. It also discusses inertial systems generally—their accuracy, reliability, cost, and time of availability.

Section 5.2 treats the accuracy requirement in detail, examines the error sources, and analyzes how well currently available inertial systems meet these requirements. It includes an analysis of possible uses of inertial systems with external fix correction.

Section 5.3 considers operating and maintenance characteristics, physical characteristics, and the compatibility of inertial platforms with commercial aircraft equipment; Sections 5.4 and 5.5, reliability and cost; and Section 5.6, the future availability of inertial systems.

5.1. INTRODUCTION

Carefully qualifying the term "inertial navigation technique" is necessary to define and limit the subject of this section. NAVIGATION is, of course, "the process of directing the movements of a craft from one point to another" [49]. INERTIAL NAVIGATION is navigation by a dead reckoning process performed by an electromechanical system containing instruments which mechanize Newton's laws of motion and gravitation; these are coupled with computing machinery which solves the equations of motion in an appropriate reference framework. Dead reckoning can be defined as navigation "in which position is determined from a record of distance from a known position" [50]. In general, precision inertial instruments (gyros and accelerometers) provide a spatial reference framework and measure vehicle accelerations. Sensed accelerations are integrated with respect to time to provide velocity; integration of velocity provides distance and, hence, position. Thus, usual dead reckoning navigation information—position, velocity, track, course and distance to destination—can be made available as a system output. The TECHNIQUES are "the methods or the details of procedures" in equipment systems to perform a navigation function [51].

The suitability of an inertial system for any application is determined principally by the requirements of the mission and its operating environment. The mission under consideration here is that of civil aircraft operating on overseas or high-altitude transcontinental routes. The environment comprises the routes and operation of American international commercial passenger and cargo air carriers; it includes all latitudes. The time period is the near future, with the introduction of equipment into service occurring between 1965 and 1975. Hence, the

inertial system must provide an air navigation capability for aircraft types ranging from four-engine, propeller-driven planes, such as the DC-6, DC-7, and Constellation, to subsonic turbojets and supersonic transports. Flight distances are between 1500 and 5000 nautical miles. Flight times are 12 hours or less; these may be either the time required for flight from departure point to destination or the time between navigation position fixes derived from sources other than the inertial navigator.

This report does not consider terminal area navigation; requirements are confined to enroute operation. Although air navigation is necessary in three dimensions, we have assumed that navigation in the vertical dimension is accomplished by other means and that a source of altitude information is aboard the airplane and provides an altitude input to the inertial system. This report does not consider inertial guidance, which implies a system performing additional functions, such as vehicle attitude control and steering. It is limited to inertial techniques for air navigation only. Occasionally it notes—but does not pursue—the capability of an inertial navigation system to provide such outputs as autopilot command signals. Many aircraft instruments employ inertial techniques in that they mechanize Newton's Second Law; attitude reference instruments, such as the directional gyro and the gyro horizon, are examples. Such devices are not considered here. Various heading-reference directional gyro systems are properly classed as component equipment of a navigation system; these have been treated in Section 3 of this report.

Suitability implies meeting certain requirements, primarily accuracy, reliability, and simplicity during flight. Other requirements include restrictions on system physical characteristics, such as size, weight, and power consumption; compatibility with other navigation and aircraft equipment; and necessity for indications of degraded operation or failure. Additional requirements relate to ground operations. Routine maintenance, checkout, and preflight operations of a navigation system must permit their orderly incorporation into existing ground operation routines. Demands for maintenance and repair facilities, spare parts, ground support equipment, and technical personnel should meet reasonable limitations.

Equipment suitable for commercial aviation must meet various economic limitations. There must be reasonable costs for systems, spare parts, maintenance facilities, ground equipment, aircraft installation, and personnel, as well as for initial and operating costs. Economic evaluation of inertial systems can be accomplished by comparing inertial system costs with those of other navigation systems. This section will offer only cost information and estimates of inertial systems, leaving comparisons to the reader.

Equipment must soon be available to resolve the many problems preceding regular operational usage. Unfortunately, recent classified military developments will not be declassified and

available for civilian use soon. Current research and development programs, mostly classified, will result in many new improved inertial devices and systems; however, their cost may be prohibitively high for civilian purposes, or their reliability and performance may not be sufficiently established. Since reliable equipment will be needed soon, this section pays particular attention to currently operational systems and available equipment, emphasizing inertial systems with an appreciable usage history in manned aircraft.

This section offers neither an introduction to, nor a detailed dissertation on, the theory and principles of operating inertial navigation systems. Current literature treating inertial systems and components is sufficiently abundant.⁵⁸ Therefore, attention may be directed immediately to the performance of inertial systems during flight.

Inertial navigation systems can be roughly classified by general application as space systems, missile systems, and terrestrial cruise systems. Since space navigation presents problems quite apart from commercial aviation, it can be dismissed from future consideration. The navigation systems for ballistic missile guidance are designed for operation during flights differing markedly from aircraft flights. Missiles require instruments with different characteristics and different equipment configurations. For example, accelerometers and gyros used in missile systems must perform satisfactorily for relatively short periods of time (seconds or minutes) under conditions of high vehicle thrust acceleration (e.g., 10 g's). An aircraft auto-navigator, on the other hand, will operate under very low thrust acceleration (fractions of a g) for extended periods of time (hours). Application and dynamic environment differences also result in different design requirements for computers, integrators, platform configuration, and stabilization. During this study many types of systems were investigated and found to have serious disadvantages when evaluated for commercial aviation requirements. This section omits discussion of such systems (for example, stop-down systems) and treats only those most suitable.

5.2. ACCURACY OF INERTIAL SYSTEMS

5.2.1. FLIGHT ENVIRONMENT AND ACCURACY CONSIDERATIONS. Enroute aircraft cruise navigation on international routes must be accomplished over distances ranging from 800 to 5000 nautical miles with flight times from one to 20 hours. Over-ocean flights are predominantly planned for minimum time routes (pressure pattern flights) with Flight Information Region clearances based on the following aircraft separation: 2000 feet vertically, 30 minutes of time along course, and two degrees of latitude in the cross course dimension (assuming an

⁵⁸See, for example, References 52 through 59. The Introduction to Inertial Guidance by Pitman, et al. [56], offers a current selected bibliography on inertial systems and components.

easterly and westerly flow of traffic). Though the protected block of airspace is approximately 2×10^4 cubic nautical miles in volume for turbojet operations, it is still necessary for flight deck navigators to maintain a position fix renewal rate of 20 to 35 minutes using celestial, LORAN, D/F and CONSOLAN lines of position in order to insure safe, expeditious flight and track maintenance.

Considering current navigation practices and accuracy, and future aircraft speed and traffic densities in the North Atlantic, the Federal Aviation Agency has specified an accuracy criterion for this inertial system study. The overall navigation process should provide "airborne knowledge of position with maximum radial error spread having d_{rms} values within the range from 2 nautical miles to 6 nautical miles."

5.2.2. DESCRIPTION OF A TYPICAL INERTIAL SYSTEM. Inertial systems vary widely in method of mechanization, choice of coordinate systems, treatment of gravity vector determination, and computation techniques. The accuracy of a specific system can be considered if its configuration and the nature of the error sources are specified. Since there are basic similarities, including error sources and error propagation, among systems applicable to cruise navigation in two dimensions, approximate accuracy can be discussed. A typical hypothetical system can be postulated as consisting of a gimballed gyro-stabilized platform mounting two accelerometers whose sensitive axes are orthogonal and lie in a plane maintained normal to the earth's local gravity vector. A pressure altimeter provides instrumentation in the vertical (z) dimension. Three single-degree-of-freedom gyros (or two 2-degree-of-freedom gyros) and associated platform-axis servos provide angular platform stabilization about the x, y, z Cartesian axes of the platform. The gyros are torqued to maintain the platform plane orientation (defined by the x and y axes) perpendicular to the local vertical. Computing machinery performs integration of the accelerometer outputs to provide displacement (distance traveled) information along the direction of the x and y axes. Additional computer functions provide gyro torquing signals and various gravitational, centripetal, and Coriolis "corrections." Schuler tuning is usually employed in such a local-level tracking system. Air navigation must be accomplished in a reference frame related to the earth; thus it is conducted in a rotating frame. The accelerometers sense both thrust acceleration and gravitational acceleration. Hence, to provide navigation information relative to the earth's surface, sensed acceleration must be corrected for gravity and vehicle motion in inertial space related to the desired motion in the rotating earth reference frame. Correction for gravitational, centripetal, and Coriolis accelerations must be mechanized or computationally treated.

5.2.3. ERROR SOURCES. Position errors arise from many sources: accelerometer errors, errors in the spatial reference in gyros, computer errors due either to basic computer

inaccuracy or to expedient approximations used in the computational procedure, and integrator errors in the computer or accelerometers—to mention a few.

The most significant errors in current cruise systems can be classified by source:

- (a) Incorrect initial position and velocity settings
- (b) Initial platform level and heading misalignment
- (c) Gyro drifts
- (d) Accelerometer bias and scale-factor errors

This list is neither complete nor all-inclusive, but it does represent the major sources of error common to all systems. Navigation accuracy is thus directly related to the initial conditions at the commencement of flight and to the performance of the inertial instruments (gyros and accelerometers) during flight.

5.2.4. TIME PROPAGATION OF ERRORS. The time propagation of these errors and the resulting position-indication errors are of interest. Though errors in indicated displacement along both x and y axes occur, we can consider a single axis (e.g., the x displacement). Indicated y displacement errors are similar, differing only in sign in some instances. Table VIII shows typical time propagation of these errors for one axis. The expressions are reasonable approximations for several hours of constant-velocity flight if one neglects cross coupling and other second order effects. This discussion emphasizes the fact that inertial system error is a function of time rather than distance. Depending on system configuration, errors arising from imperfections in components and imperfect initial alignment are characterized by various bounded oscillatory components with 84.4-minute Schuler periods and additional divergent terms.

Observations of the source and propagation of errors in a single channel provide some insight into the significant problems associated with inertial navigation systems. However, accuracy conclusions should be drawn not from single axis figures, but from resulting radial position errors. It is convenient to relate the stated accuracy criteria given in terms of d_{rms} error to corresponding x, y Cartesian coordinate values. In addition, a common statistic used in inertial literature is the "circular probable error," CEP or CPE. Noting the relationship of the CEP statistic to the d_{rms} statistic is useful.

If X_T, Y_T denote the Cartesian coordinates of the true position of the aircraft, and X_I, Y_I denote the coordinates indicated by the inertial system, then the radial error d is given by

$$d = \sqrt{(X_I - X_T)^2 + (Y_I - Y_T)^2} \quad (98)$$

TABLE VIII. TYPICAL SINGLE AXIS (x AXIS)*
ERROR PROPAGATION

Source	Resulting Position Error
Initial Platform-Tilt Angle $\phi_y(0)$	$-\phi_y(0)R[1 - \cos \omega t]$
Initial Platform-Heading-Angle Error $\phi_z(0)$	$\phi_z(0)v_y t$
Initial Position Error x_0	$x_0 \cos \omega t$
Initial Velocity Error v_{x0}	$v_{x0} \frac{1}{\omega} \sin \omega t$
Heading-Gyro-Constant Drift ϵ_z	$\epsilon_z v_y [t^2 - \frac{2}{\omega} (1 - \cos \omega t)]$
Level-Gyro-Constant Drift ϵ_y	$-\epsilon_y R [t - \frac{1}{\omega} \sin \omega t]$
Accelerometer Bias B_x	$B_x \frac{1}{\omega^2} [1 - \cos \omega t]$
Accelerometer-Scale-Factor Error K_x	$K_x v_x (t - \frac{1}{\omega} \sin \omega t)$

$R \approx$ earth radius

$v =$ velocity

$t =$ time

x, y, z subscripts denote x, y, z axes.

$\omega = \sqrt{g/R}$

*After [55 and 56].

The d_{rms} error (root mean square radial error) is by definition

$$d_{\text{rms}} = \sqrt[3]{E(d^2)} \quad (99)$$

where $E(d^2)$ denotes the expectation of the random variable d^2 .

If x and y denote the random errors in the two axes of the inertial system, then

$$d^2 = x^2 + y^2 \quad (100)$$

It is reasonable to assume that the errors in both the x and y channels of the inertial system are normally distributed with means $E(x)$ and $E(y)$ equal to zero. It is also reasonable to

assume that the standard deviations (σ_x, σ_y) of the distributions are equal. Having thus assumed that inertial position indication errors are described by a bivariate normal distribution where

$$E(x) = E(y) = 0 \quad (101)$$

and

$$\sigma_x = \sigma_y = \sigma \quad (102)$$

and again noting the d_{rms} error requirements (2 to 6 nautical miles), we can summarize the accuracy requirements as shown in Table IX. The relationships among the three statistics for a circular normal distribution are

$$d_{rms} = \sqrt{2}\sigma \quad (103)$$

$$CEP = \text{Median of } d = 1.1774\sigma \quad (104)$$

TABLE IX. ACCURACY CRITERIA

Statistic	Desired Error Range (n mi)	
	Minimum	Maximum
d_{rms}	2	6
$\sigma = \sigma_x = \sigma_y$	1.41	4.24
CEP	1.66	4.99

5.2.5. COMPUTATION OF INERTIAL POSITION ERROR. It is useful now to consider the total position error for a given time and path of flight of a typical current velocity-damped inertial navigation system. We have chosen a flight of one hour in an easterly direction at 200 knots. We will determine final position errors in the east coordinate (x) and the north coordinate (y) and then combine them as before to present a resultant radial position error. A thorough and exact analysis requires a detailed knowledge of the significant sources of error and the effect of their individual contributions to the total position error. Such knowledge is particularly important for the design of systems in which choice of components and necessary compensation must be made. However, a rough analysis is sufficiently accurate to validly indicate inertial system capabilities.

Three types of error are considered:

- (a) Errors in construction
- (b) Errors in mechanization and computation
- (c) Errors of false information [54]

In constructing the physical inertial system, the chief deviation from the ideal system arises in the mechanical alignment of the components. For an x,y,z coordinate system with horizontal axes x and y , and vertical axis z parallel to the gravitational field of the earth, a typical system would ideally provide true alignment of the following axes along the corresponding true-position axes:

- (a) Along the x axis
 - (1) Sensitive axis of the x -direction accelerometer
 - (2) Input axis of the roll gyro

- (b) Along the y axis
 - (1) Sensitive axis of the y -direction accelerometer
 - (2) Input axis of the pitch gyro

- (c) Along the z axis
 - Input axis of the yaw gyro.

However, in the actual physical system, deviations from these true alignments produce a principal source of error.

Errors in mechanization are attributed to the accelerometers, the gyroscopes, the position computer, and the gyro-torque computer. In the accelerometer the significant errors are the bias error (i.e., at zero input the output is not zero) and nonlinearity (when the scale factor is not constant with all magnitudes of input). This scale-factor error would be most significant during periods of high acceleration.

Errors in the output of the position computer depend on the order in which computations are performed, on amplifier drifts, nonlinearities, approximations in computation, etc. These errors will be grouped into an effective single component error. The gyro-torque computer possesses errors of the same nature as the position computer. The signals produced for the gyro torquer include errors both from inaccuracies inherent in the computer and from errors present in the information put into the computer.

False information errors can be classified in two types. The first consists of inaccurate information from the accelerometers and altimeter; even if the computer itself is "perfect," its output will be in error. The second stems from the lack of precise information about the size and shape of the earth and from the local anomalies in the gravitational field. Usually the earth is regarded as an oblate spheroid, only a close approximation to the actual shape. In addition, the computer can be mechanized to give only close approximations in performed calculations.

An exact error analysis would be lengthy and complex. The sets of differential equations involved are nonlinear; hence, analysis is complicated and cumbersome. Even when coupling terms are neglected and the equations are linearized, they are still complex enough to require computer solution. A less elegant error determination provides an order-of-magnitude result that sufficiently approximates the results of actual test conditions.

Neglecting coupling terms and expressing the contributions to total channel error in terms of the rms values of the individual error sources, we find it sufficient to compute the total-channel rms error as follows:

$$\sigma_x = (\sigma_{x1}^2 + \sigma_{x2}^2 + \dots + \sigma_{xn}^2)^{1/2} \quad (105)$$

$$\sigma_y = (\sigma_{y1}^2 + \sigma_{y2}^2 + \dots + \sigma_{yn}^2)^{1/2} \quad (106)$$

In these equations a channel has n sources of error and the terms $\sigma_{\alpha i}$ are the rms-error values of the individual error sources after specified flight times; α denotes channel (x or y), and i designates source. Independence of error sources and essentially zero means are implied.

Now the effect of an individual source of error on the total position error is apparent from its contributions to the rms values determined from Equations 105 and 106. Determining the rms value of each error source is thus of prime importance in considering the contribution of each source to the total position error. Since the errors generally vary with flight time, it is necessary to obtain an expression for each error source in terms of time so that the rms value of each can be computed for a specified time in a given flight. For this reason error equations are necessary, but since only an approximation of position is necessary, the equations can be in simple, uncoupled form, with all second-order effects neglected.

The equations are developed and treated elsewhere in the literature [60]. Table X summarizes these results, presenting error source data and resulting north and east position rms errors after one hour of flight. Resulting rms-position errors are $\sigma_x = 1.04$ nautical miles, $\sigma_y = 1.05$ nautical miles. The rms-error magnitudes indicated in Table X are typical of current velocity-damped systems. They were based on data for the LN-2C system. The d_{rms} error after one hour of flight is 1.485 nautical miles, and the $E(d)$ is 1.316 nautical miles. The method of analysis and results given here follow those presented by Litton in Reference 60. Results closely approximate empirical test results. Note that these results are for a doppler-inertial, not a pure-inertial, system. To date, systems intended for navigating over periods of time greater than one or two hours have been hybrid systems, such as doppler-inertial or astro-inertial, rather than pure inertial. Litton's results indicate that up to six hours of flight the error continues to propagate essentially linearly with time.

TABLE X. TYPICAL ERROR-SOURCE SUMMARY OF A VELOCITY-DAMPED INERTIAL SYSTEM FOR A 60-MINUTE FLIGHT EAST AT 200 KNOTS

Error Source	Function to Approximate Error	rms Error Magnitude	Position Error (n mi)	
			North	East
Accelerometer				
Bias	Step	$5 \times 10^{-5} g$	0.214	0.214
Bias Drift	Ramp	$5 \times 10^{-5} g/hr$	0.210	0.210
Nonlinearity and Scale Factor	Impulse	10^{-4}	0.000	0.004
Level Axis Gyros				
Random Drift	Ramp	$0.01^\circ/hr$	0.742	0.742
Mass Unbalance	Step	$0.08^\circ/hr-g$	0.018	0.018
Torquer Non-linearity	Ramp	$0.006^\circ/hr$	0.443	0.443
Initial Misalignment	Step	$0.05 \times 10^{-3} rad$	0.436	0.436
Azimuth Gyro				
Initial Misalignment		20 sec	0.020	0.000
Gyro Drift and Torquer Nonlinearity		$0.01^\circ/hr$	0.030	0.000
Computer				
Ellipticity	Step	$1.63 \times 10^{-5} g$	0.069	0.000
Centripetal	Step	$1.75 n mi/hr^2$	0.110	0.000
Coriolis	Step	Negligible	0.000	0.000
Integrator Bias	Step	$5 \times 10^{-5} g$	0.216	0.216
Integrator Gain	Impulse	10^{-3}	0.000	0.044
Latitude Computer		1%	0.117	0.000
rms Error			1.05	1.04

5.2.6. CURRENT SYSTEMS. In general, drawing upon the unclassified information available, we find that the accuracy of current systems ranges from 1 to 3 nautical miles/hour (CEP). Doppler inertial systems range from 1 to 2 nautical miles/hour (CEP), and pure inertial systems from 2 to 3 nautical miles/hour (CEP).

Litton's LN-3 system, which uses the Litton P-200 platform, is a proven example of a pure inertial system [61, 62]. Approximately 500 P-200 platforms have been built. LN-3 tests have been extensive and thorough. Analysis estimates of the CEP and experimental results compare closely. The system CEP can be realistically stated as 2 nautical miles/hour. But caution must be exercised in extrapolating these CEP error-propagation rates beyond two hours. Cross-coupling errors become more significant with time; also aircraft velocity, maneuvers, and flight path variations have a greater consequence. Error-propagation terms proportional to time squared become increasingly significant after two hours. Hence, linear projection of the CEP

rates to flight times over six hours provides only an approximation. The major source of errors in all cases is the gyro assembly. It is interesting to consider inertial capabilities in the light of the accuracy criteria for a New York-to-London North Atlantic turbojet flight. If the total flight time is slightly over six hours, inertial navigation time to land-based radio navigation aids at the destination is about six hours. A 1-nautical-mile/hour (CEP) system with optimistic linear extrapolation of this error to six hours results in a terminal d_{rms} of 7.2 nautical miles, which is slightly beyond the stated upper limit criterion of $d_{\text{rms}} = 6$ nautical miles. Although this is an optimistic approximation, it indicates that high-quality inertial systems are entering the realm of competition in accuracy. A Mach 2 transport on the New York-to-London routes will fly for approximately two hours and forty-five minutes before reaching land-based navigation aids in the British Isles. A 1-nautical-mile/hour (CEP) system would result in a d_{rms} error of 3.3 nautical miles. A 2-nautical-mile/hour (CEP) system, realizable today over three-hour periods, would achieve a d_{rms} error of only 6.6 nautical miles.

An inertial navigation system provided with periodic external position-fix information offers immediate promise of acceptable navigational accuracy. Complex hybrid systems such as Doppler-Inertial and Stellar-Inertial are currently employed for military applications. The cost and complexity of these systems will in all likelihood preclude their early introduction into commercial service. However, a simple hybrid form of an inertial system and an external position fix system, such as LORAN-A or LORAN-C, is immediately feasible. Since Doppler-LORAN-A combinations are currently in service in commercial aircraft [63, 64], a discussion of similar employment of inertial systems is in order.

For discussion purposes a very simple hybrid configuration can be postulated. The inertial system is the primary navigation system and provides continuous present position information in two horizontal coordinates (X_I, Y_I). The inertial system is specified only with respect to statistical-error propagation. We assume that the system is well aligned and trimmed prior to commencement of flight and that initial position and velocity are accurately set. No readjustments of the inertial system are made during flight. The along-track and cross-track inertial errors are each assumed to be normally distributed with zero means and equal standard deviations ($\sigma_{X_I}, \sigma_{Y_I}$). A nominal "1.8 nautical-mile/hour (CEP)" system will be considered and assumed to have an error propagation characteristic described by the following quadratic function of time:

$$\sigma_{X_I} = \sigma_{Y_I} = 1.1038t + 0.0678t^2 \quad (107)$$

Figure 36 graphically presents this error characteristic. Although the bivariate circular normal nature of the error distribution permits ready conversion between CEP, d_{rms} , and standard deviations, the figure shows curves indicating each parameter as a function of time for the reader's convenience.

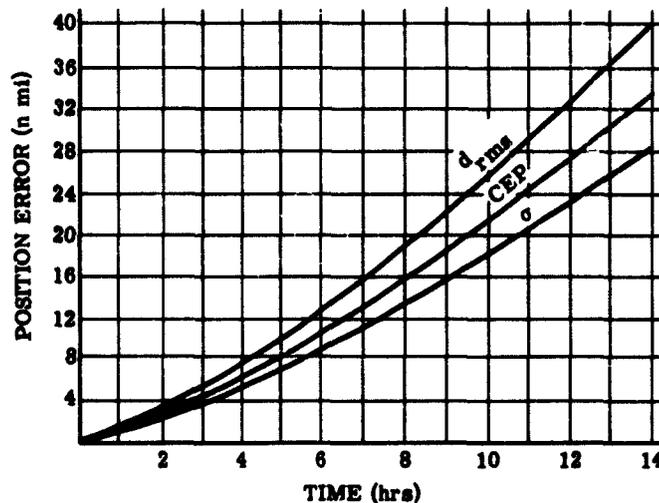


FIGURE 36. HYPOTHETICAL INERTIAL-SYSTEM POSITION-ERROR PROPAGATION

We assume that the external position-fix system has the following characteristics. System area coverage is such that it is possible to obtain a fix at any desired time during the flight. The d_{rms} error throughout the service area is 3 nautical miles. For simplification we suppose that a position fix can be obtained in Cartesian coordinates (X_E, Y_E) where the errors in each coordinate are normally distributed with zero means and equal standard deviations ($\sigma_{X_E} = \sigma_{Y_E} = 2.12$ nautical miles). We assume that fix-renewal periods as short as 30 minutes are possible.

Figure 37 is a functional block diagram of the components and operations of one channel (X) of the hybrid system. The Y channel is identical. The inertial system's present position indication X_I is continuously available. The external system provides periodic position information X_E . The corrected position X_C is continuously displayed for navigation purposes. X_C is the sum of the continuous inertial output X_I and a periodically inserted constant correction C_X . The value of C_X is zero at the time of take-off. When an external fix is obtained, the most probable position is determined by weighted combination of the inertial and external system

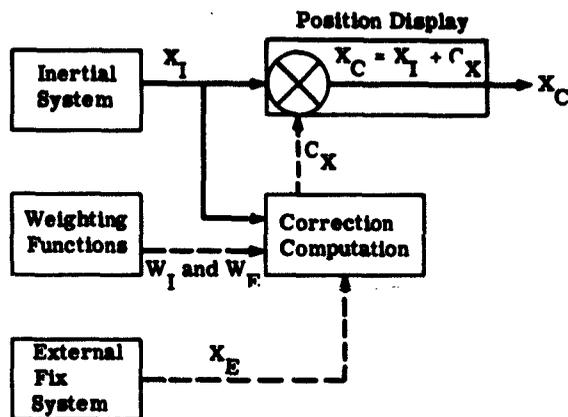


FIGURE 37. HYBRID SYSTEM BLOCK DIAGRAM

indications. The difference between the inertial indication and the weighted coordinates are applied as a correction to the display.

A priori knowledge of the inertial and external systems permits the determination of a weighting function for use in estimating the most probable position. Since the error distributions for both the inertial and external systems have been postulated as being circular-normal with zero means, the most probable position derived from simultaneous inertial and external system position indications will be on a line between the two positions. The most probable X and Y coordinates can be estimated independently. Hence, we need consider only one channel, the X channel. Given an X_I inertial reading and a simultaneous X_E external system reading, the most probable X coordinate can be estimated with an appropriate maximum likelihood estimator \hat{X} . It can be shown that such an estimator is

$$\hat{X} = X_W = \frac{\sigma_{X_E}^2}{\sigma_{X_I}^2 + \sigma_{X_E}^2} X_I + \frac{\sigma_{X_I}^2}{\sigma_{X_I}^2 + \sigma_{X_E}^2} X_E \quad (108)$$

where the maximum likelihood estimator \hat{X} is a weighted coordinate X_W and

$$X_W = W_I X_I + W_E X_E \quad (109)$$

where

$$W_I = \frac{\sigma_{X_E}^2}{\sigma_{X_I}^2 + \sigma_{X_E}^2} \quad (110)$$

and

$$W_E = \frac{\sigma_{X_I}^2}{\sigma_{X_I}^2 + \sigma_{X_E}^2} \quad (111)$$

and

$$W_I + W_E = 1 \quad (112)$$

It follows that

$$\sigma_{X_W} = \sqrt{W_I^2 \sigma_{X_I}^2 + W_E^2 \sigma_{X_E}^2} \quad (113)$$

Though σ_{X_E} is time invariant, σ_{X_I} is not. Hence, W_I and W_E are functions of time. Figure 38 shows W_I and W_E for the two systems as functions of time, from Equations 107, 110, and 111.

The operations indicated in Figure 37 are now evident. Upon obtaining an external system fix, we can obtain the weighted coordinate X_W by application of Equation 109 and weights selected from a table or graph such as shown in Figure 38. A correction C_X is then computed using

$$C_X = X_W - X_I \quad (114)$$

C_X is manually inserted in the position display. The display provides a continuous readout of X_C .

Consider now the overall system position accuracy. The position error of the navigation system must not exceed 6 nautical miles d_{rms} during flight. Figure 39 graphically presents the navigation accuracy achieved when external system fixes are taken at appropriate times and utilized in the manner described above. In order not to exceed a 6-nautical-mile d_{rms} error with a minimum number of external fixes, it is necessary to obtain fixes and redetermine and insert correction C_X at the times shown in Table XI. The sawtooth curve in Figure 39 shows the hybrid system error. The minimum realizable d_{rms} error with this postulated hybrid system is indicated by the dotted curve; to achieve this error, one must reduce all fix renewal intervals to zero time (i.e., continuous external system position information must be available, and the hybrid system must operate as a continuous process).

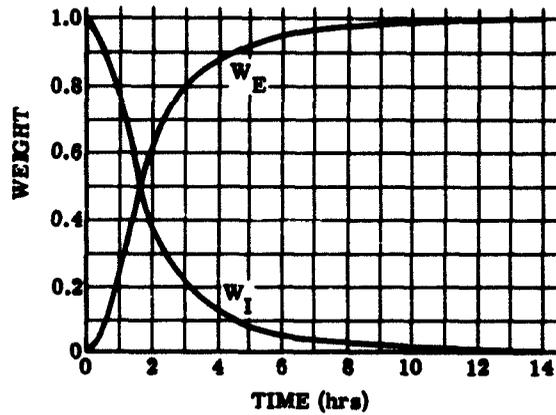


FIGURE 38. WEIGHTING FUNCTIONS

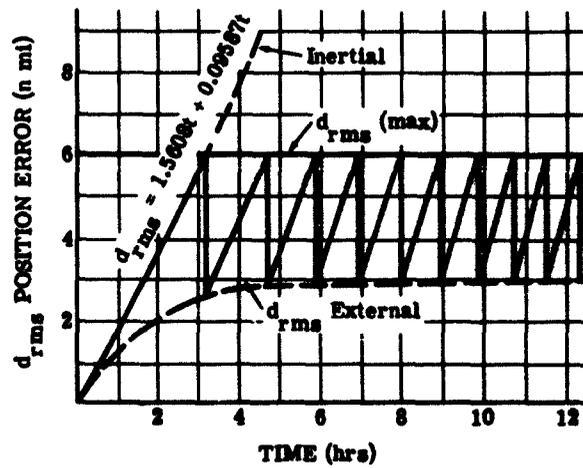


FIGURE 39. HYBRID-SYSTEM ERROR VS. TIME

TABLE XI. SYSTEM ERRORS VS. TIME

Fix Nos.	Fix Interval (hr)	Elapsed Time (hr)	Inertial Error (d_{rms})	Hybrid* Error (d_{rms})
0		0	0	
	3.21			
1		3.21	6.00	2.680
	1.43			
2		4.64	9.31	2.852
	1.22			
3		5.87	12.45	2.913
	1.10			
4		6.97	15.537	2.943
	1.02			
5		7.99	18.590	2.958
	0.96			
6		8.94	21.627	2.968
	0.90			
7		9.85	24.667	2.974
	0.86			
8		10.70	27.69	2.977
	0.82			
9		11.52	30.705	2.981
	0.78			
10		12.30	33.72	2.985

*Immediately following the application of correction C_X

The foregoing discussion admittedly suffers from oversimplification. We have chosen the simple hybrid configuration because of similarity to current employment of LORAN-A and doppler and ease of implementation in an aircraft for early test purposes. In addition, the configuration chosen and the accompanying simplifying assumptions permit a quantitative discussion of navigation accuracy without consideration of the detailed characteristics of the inertial and external systems. However, it does crudely indicate that in the near future some current inertial systems, while not satisfying stated accuracy requirements when employed independently, can be used in a simple hybrid configuration to obtain the required accuracy. The inertial system described by Equation 107 remains within error limitations for only 3.2 hours. After an elapsed time of 6 hours the pure inertial error would be approximately 13 nautical miles d_{rms} . With three external-system fixes of 3-nautical-mile- d_{rms} error, the combination system error at 6 hours is 2.97 nautical miles d_{rms} . A 1.8-nautical-mile/hour (CEP) inertial system is thus utilized with an external-reference hybrid partner to achieve 0.6 nautical mile/hour (CEP) results.

Table XII indicates flight times for various route lengths in propeller, jet, and Mach 2 aircraft. Associated with each route length is a weight showing the approximate relative frequency of over-ocean commercial flights on routes of given lengths. Flight times are approximate, being simply the distance-speed quotient, and thus do not reflect departure, terminal area, traffic pattern flight time, or other factors. The table of flight times is useful when used with Figure 39 to determine the number of external fixes required during a flight over a particular route in various aircraft. The table of flight times is roughly partitioned according to the number of fixes required. Turbojet transatlantic crossings require about 6 to 9 hours of flight. Of this period, about 3 hours are spent "over ocean" between 10° W and 50° W. During the remaining flight time the aircraft is usually close to domestic navigation aids. Thus we see that the inertial system must be used for about 4 1/2 to 7 1/2 hours, after which domestic aids on the destination continent become available. If the hypothetical hybrid system were employed and a navigation error of less than 6 nautical miles d_{rms} were maintained, crossing the North Atlantic would require two to four fixes.

More sophisticated hybrid configurations are possible. The hypothetical system discussed here is by no means an optimum system. It neither uses past-position information nor extrapolates inertial performances to the future. Hybrids are potentially more accurate and reliable. For example, should one of the component systems fail, the other system would still provide a capability to navigate to destination or to an alternate airport at reduced accuracy. In a hybrid configuration improved operation of the inertial system by inflight correction and adjustment of platform electronics is obvious. In general, hybrid systems offer immediate advantages: increased navigation system accuracy, reliability, and more efficient and safe flight operations.

TABLE XII. FLIGHT TIMES

NOTE: Fix Requirements Are Shown by Grouped Cells.

DISTANCE (n mi)	% OF TOTAL FLIGHTS	PROPELLER 245 knots	TURBOPROP 320 knots	TURBOJET 485 knots	MACH 2 1148 knots		
800	10	3.26 hr	2.5 hr	1.64 hr	0.69 hr		
1200	4	4.87	3.75	2.46	1.05		
1700	10	6.94	5.31	3.51			
2200	32	8.98	6.88	4.54			
3000	40	12.25	9.38	6.19	2.61		
3400	4	13.87	10.63	7.01	2.96		
		4 or more fixes		2 to 3 fixes		1 fix	No fixes

5.3. OTHER CHARACTERISTICS OF INERTIAL SYSTEMS

The preceding section has indicated that current inertial systems are accurate enough for transatlantic flights of supersonic aircraft without external fix correction, but that they require external fix correction for subsonic aircraft. In any case, inertial systems are competitive with other self-contained navigation systems even for slower aircraft. It is apparent that choice of an inertial navigation system must also rest upon factors other than accuracy—for example, operational convenience, reliability, cost, and maintenance requirements. In the following discussion we will consider a number of these factors.

5.3.1. PHYSICAL CHARACTERISTICS. Current inertial systems, intended for use in manned aircraft, have similar weight, size, and power requirement characteristics. These are about 90 pounds, 2 cubic feet, and 500 watts. The systems consist usually of four or five units. Platforms weigh about 30 to 50 pounds and occupy 0.8 to 1 cubic foot of space. Other units—computer, electronics, and controls—are also lightweight and small. Modular design is used, and the systems are well adapted for black box or module replacement flight-line repair. As previously stated, inertial systems pose no unique installation problems and require only reasonable placement for maintenance and checkout purposes. While small weight and size have always been required of airborne equipment, in modern commercial aircraft size is the predominant limitation. Two recently developed inertial systems are as small as 1 and 1.37 cubic feet. Hence, should inertial systems prove desirable because of navigation capabilities, physical characteristics will not prohibit their use.

5.3.2. OPERATIONAL SIMPLICITY. The simplicity of an inertial system is one of its more attractive characteristics. Navigation information can be displayed in the cockpit in a variety of forms: continuous bearing and range information to selected positions, distance to go and distance off intended track, or present position information. Display instruments that commonly serve the inertial system and other navigation devices, such as VOR, TACAN, and ADF, are available. Resetting the inertial indication of present position can be done by a separately determined position fix. Adjunct equipment can automatically compute and display wind vectors during inertial operation. A capability to insert wind vector information can be included; this feature would be advantageous if it were desired to use the computer and magnetic heading data from the aircraft compass system for emergency dead reckoning during "platform failure." The flight-operating controls of an inertial system are few and simple, demanding little more than setting the selection switch for mode of operation and setting the coordinate position of the flight log terminal. The system can be coupled to the autopilot, further reducing demands on the crew. Likewise, a few simple controls can be provided to meet the specifications dictated

by any particular navigation procedure. Thus, no crew-training problem will result from the introduction of inertial systems and no operational complexity will be encountered in the cockpit.

5.3.3. COMPATIBILITY. Because it is self-contained, passive, and nonradiating, a pure inertial system will not interfere with other radio or electronic equipment on board. Nor will it require structural changes to the airframe to accommodate an antenna. Installation must be planned carefully to facilitate preflight checks and ground maintenance. It is important to consider air cooling to insure uninterrupted operation for the prime power source. In general, the inertial system presents no special installation problems and has no operating characteristics that would conflict with the operation of other equipment.

5.3.4. GROUND OPERATIONS. Although accuracy, reliability, and cost will be the main determiners of the suitability of inertial systems for commercial aircraft, we must also consider navigation, maintenance, repair, and routine ground operations. Therefore, routine preflight operations, periodic maintenance, and base repair and maintenance have been investigated.

5.3.4.1. Routine Preflight Operations. Information on several available inertial systems has shown that preflight procedures and ground support equipment are similar. Two preflight operations are especially important. First, the operating condition of the system must be determined through some checkout procedure. Second, the system must be accurately aligned and leveled with exact initial position settings to achieve accurate navigation during flight.

Table VIII indicated the significant error sources. Four of these sources—initial platform tilt and heading, and initial position and velocity—obviously arise from ground operations. In commercial operation two of these error sources pose little or no problem: initial position at the airport gate can be accurately known, and the initial velocity is zero. Leveling the platform and aligning it in azimuth must be precise. Gyros and accelerometers must be properly biased; this is usually accomplished automatically during the level-and-alignment sequence and/or during base maintenance.

5.3.4.2. Flight Line Maintenance. Flight line maintenance includes routine upkeep, periodic and special inspections, replacement of components and accessories, and the daily preparation of aircraft for flight, including preflight and postflight inspections and corrections for minor discrepancies or troubles. This maintenance level uses portable test equipment capable of localizing system malfunction to one of the main units of the system, which may then be replaced on the spot. The tests are usually performed with the inertial system installed in the aircraft and can be executed by an operator in about eight minutes. It must be noted that this portable

test equipment is incapable of testing the inertial system for marginal conditions of operation. However, the combination of "go or no-go" self-test circuitry within the inertial system and the portable line test analyzer does provide a good capability for system checkout and isolates faults to replaceable units or modules.

5.3.4.3. Periodic Maintenance. Periodic maintenance requires more elaborate equipment to test all units of the inertial system. Malfunctions can be localized to a defective module or removable subassembly, and the necessary replacement can be made from spares. Also, at this level the inertial platform is periodically calibrated. This calibration is an electrical adjustment of the gyro-null errors which takes a skilled operator about 15 minutes. Experience to date has shown that each operational platform employing ball-spin-bearing gyros should be calibrated monthly to maintain specified system accuracy unless the system provides for automatic gyro biasing as part of its alignment procedure. However, the development of the gas-spin-bearing gyro, which features an extremely long operating life, lower drift rate, and improved stability, may eliminate the need for such frequent calibration or gyro biasing. A semiannual calibration may be sufficient.

The checkout and functional testing of platforms during periodic maintenance requires special facilities and skilled personnel. The platform components should not be repaired at this maintenance level since disassembly, repair, and reassembly of the mechanical and electro-mechanical components should be undertaken only by technicians skilled in instrument repair and testing, and in an ultraclean environment. Replacement of some platform components is occasionally possible, but general platform repair is not advisable. Units other than the platform are most easily repaired by module replacement with actual repair work confined to base operations. However, functional testing of the platform can be done only with special and expensive test equipment. It requires, in addition to a functional test console, a tilt table which will permit rotation of the platform through 360° in pitch or roll.

Associated test equipment must include the electronics necessary to control the platform, as well as a torque bias panel and sufficient isolation amplifiers and dummy loads to simulate the remainder of the equipment; it also must provide for the necessary instrumentation of the platform outputs. It should be noted that functional tests of the platform will require accurate reference information at the test site. In particular an azimuth reference to true north should be provided to an accuracy of 0.05° . In addition, a level reference to at least the same accuracy is desirable; this can be provided on the test fixture by the use of bubble levels and leveling screws.

5.3.4.4. Base Maintenance. Base maintenance, repair, and testing of inertial systems and components are expensive and complex. Highly skilled engineers and technicians are required to perform exacting and time-consuming operations. The facility must be specially constructed to meet cleanliness and vibration isolation requirements. Although repairing and testing system units, modules, subassemblies, etc., and testing the platform and complete system present difficult technical problems, servicing and calibrating the gyros and accelerometers are most difficult. Striving toward high accuracy and reliability has resulted in the development of miniature precision components, which have become increasingly sensitive to contamination and have made contamination control more difficult. Gyros and accelerometers must be repaired in white rooms to prevent contamination by dirt.

A white room requires many special kinds of equipment to closely control air temperature, relative humidity, pressure, and—most difficult—particle content, which must be measured manually (which is time consuming) or photometrically (which is costly). It often takes shrewd detection work to find the source of contamination.

The design of white rooms is expected to improve in the near future as the requirements for operating them become stricter with the trend towards subminiaturization of precision components. Portable or prefabricated white rooms have been considered for temporary requirements in field repair depots, for example. These are economical and can be used for a specific task, dismantled, moved to a new location, reassembled, cleaned, and put back into operation for much less than the cost of constructing and operating a permanent white room.

Base maintenance support will be a major problem of the commercial carrier adopting inertial systems. The carrier could acquire facilities and a staff to support systems at this maintenance level; or several airlines could support common facilities. Another alternative, one that should not be lightly dismissed, would be to contract all base maintenance work to the manufacturer.

5.3.4.5. Alignment. Aligning an inertial platform consists of leveling the platform and establishing an azimuth reference (usually true north). This procedure is controlled by a control unit usually mounted on the dash panel; it consists of one or more control switches, warning lights, and a display panel that shows the position and attitude of the aircraft.

Stationary ground alignment procedures are normally followed; however, some inertial systems also provide for in-flight alignment. In the normal mode, alignment procedures are preceded by a warm-up period ranging from 9 to 30 minutes in order to bring the system com-

ponents up to their optimum operating temperatures. Some systems permit fast alignment—that is, alignment initiated from a cold start—which will be discussed later.

Obviously, leveling and aligning will dictate certain procedures and impose certain requirements on handling the aircraft on the airport at the gate. However, these restrictions should not prove insurmountable. During leveling and aligning it is quite likely that the aircraft will not be perfectly motionless. Loading operations and wind buffeting will cause aircraft motion, but this should not hinder inertial start up. The gyrocompass alignment methods appear especially adaptable to commercial operations since they eliminate the need for external equipment such as that required for optical alignment. Fueling and other operations requiring power shut-down will have to be coordinated with inertial power continuity requirements.

5.3.4.5.1. Normal Alignment. Most inertial guidance systems feature both automatic and semiautomatic modes for alignment. Regardless of which mode is selected, the alignment is usually executed in two separate steps: (1) coarse level and azimuth alignment, followed by (2) fine level and azimuth alignment. In coarse level and azimuth alignment, the platform is leveled from within $\pm 5^\circ$ to within ± 20 seconds of level and aligned from within $\pm 1^\circ$ to within ± 15 minutes of true north. The time required for this mode of alignment ranges from 10 seconds to 1.5 minutes.

In fine level and azimuth alignment the platform is leveled from within ± 20 seconds to within ± 10 seconds of level and aligned from within ± 4.2 minutes to within ± 5 seconds of true north. The time required ranges from 2 to 25 minutes. The spreads in alignment accuracy and time can be attributed to the particular systems and to the methods used to align them (e.g., gyro and accelerometer biasing is required in the alignment of some systems, while gyro biasing, for example, is not required as often for those systems which employ air-bearing gyros and is, therefore, not part of their normal alignment).

The following is a partial list of methods that may be used to coarse-level an inertial platform.

- (a) Servo the platform to a level position, utilizing the pitch and roll synchro information.
- (b) Mount vertical pendulums on the platform and use their erection signals to torque the platform to a level position.
- (c) Use one or two theodolites in conjunction with computer control to level the platform.

A corresponding list for coarse-azimuth alignment is:

- (a) Servo the azimuth axis of the platform to the output of the aircraft magnetic compass, corrected for magnetic variation.
- (b) Store the known heading in the platform computer for a known position of the aircraft.
- (c) Aim the aircraft along a line of known azimuth and then manually set the azimuth indicator to this value, which in conjunction with the computer and azimuth servo loop will slew the platform to this azimuth.
- (d) Use a theodolite in conjunction with platform-mounted alignment mirrors to determine the angle at which the azimuth indicator should be set.
- (e) Employ rapid gyrocompassing techniques (i.e., start the azimuth alignment process before the platform has been brought to optimum operating temperature or before it has been in the fine-level mode).

Some methods for accomplishing fine-level and azimuth alignment of an inertial platform are:

- (a) Use the signals from the level-axis accelerometers to torque the platform (e.g., the accelerometer signals are null when the platform is normal to the local gravity vector).
- (b) Use two theodolites (equipped with Gauss eyepieces) and platform-mounted alignment mirrors in conjunction with computer control to level the platform optically.
- (c) Slave the platform-azimuth servo to an external highly accurate north-seeking gyro.
- (d) Store the aircraft heading in the systems computer after it has been determined to the desired accuracy for a known position of the aircraft.

The optical alignment method utilizing two theodolites probably provides the most accurate alignment and may even require the least time, provided that the time spent setting up the optical equipment is not counted. However, its disadvantages are that it is not fully automatic and requires additional optical equipment and personnel. Furthermore, the increased accuracy obtained may not be worthwhile when the normal alignment mode is used; but it is well worthwhile for the fast alignment mode. This point is illustrated in Figures 40 and 41, which are adopted from information in the General Electric Banir II Inertial Reference System, Manual [65].

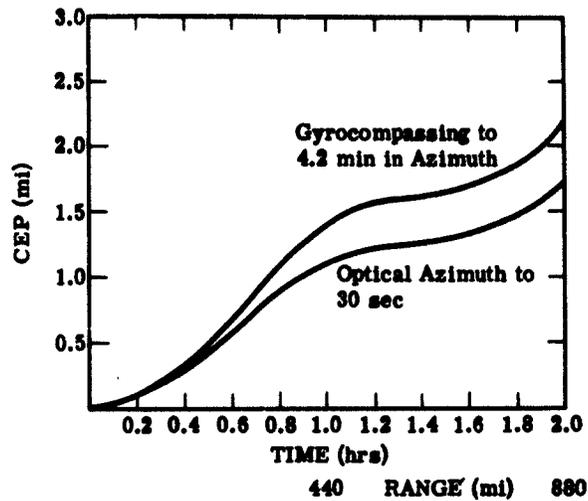


FIGURE 40. NAVIGATION ERROR NORMAL ALIGNMENT AND ERECTION

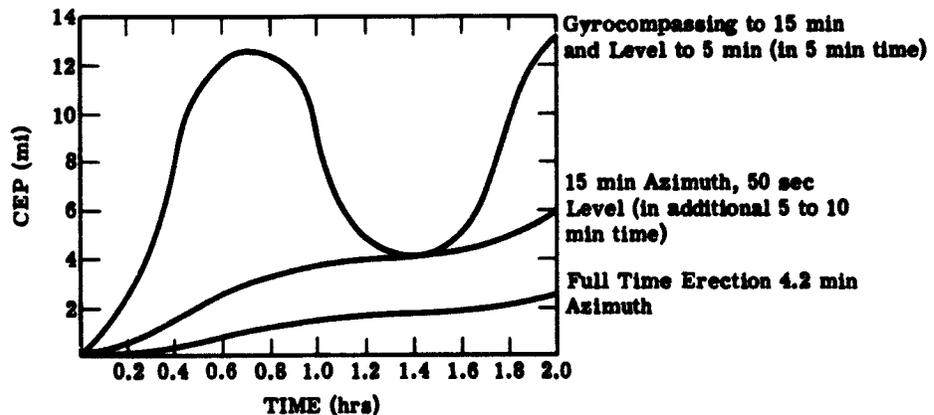


FIGURE 41. NAVIGATION ERROR FAST "SCRAMBLE" ERECTION AND ALIGNMENT

5.3.4.5.2. Fast Alignment. Several methods for fast alignment of inertial platforms have been developed for military aircraft that have to take off under scramble conditions. The method employed by Litton Industries for its LN-3 series of inertial platforms is that of stored heading. Here the aircraft that is on alert status is aligned in a given position, and then the power to the system is turned off. As long as the aircraft is not moved from this position the

platform will immediately align itself to the remembered heading after 1.5 minutes required for the gyro to come up to speed. The LN-3 system will maintain near-operating temperature for several hours after power has been turned off, so that the system can be aligned in 1.5 minutes even though the aircraft has been on the ground without power for up to three hours.

General Electric's Banir II Inertial Reference System, mentioned earlier, employs three single-degree-of-freedom gyros that are trimmed during normal alignment procedure; hence, this system requires between 37 to 45 minutes for normal platform alignment. For comparison, its fast alignment mode, which omits the gyrobiasing procedure, requires 6 minutes of time, during which the platform is leveled to ± 5 minutes and aligned to ± 15 minutes in azimuth.

Astro-Space Laboratories has proposed a pure inertial guidance system called the NAV-100 [86]. It uses air-bearing pendulums to establish the local vertical and attains azimuth alignment by slaving the NAV-100 system to an accurate north-seeking gyro unit. This could provide for fast alignment if one would not count the time spent by the north-seeking gyro in establishing the azimuth reference.

5.3.4.5.3. In-Flight Alignment. Because of the additional errors and noise, one should not expect the accuracy of an inertial guidance system aligned in flight to be as good as that obtained by normal alignment on the ground. Therefore, in-flight alignment is used primarily in emergencies as a safety factor for the navigation system.

An external source of horizontal velocity information is required for the alignment of and inertial platform in flight; the accuracy of the erection and alignment depends on the accuracy and noise level of the external velocity source, and on the time available. Possible sources of velocity inputs are air speed data, doppler radar velocity information, and velocity from another navigation system.

The method generally used for in-flight alignment is to erect the inertial platform as a first-order system, using the accelerometers to torque the gyros directly, and then slave the platform to the magnetic compass to obtain azimuth alignment. This method of erection and alignment is satisfactory for attitude and heading information, but is unsatisfactory for supplying velocity and position information.

More accurate erection and alignment can be obtained by utilizing an external source of velocity information, which permits aligning the platform in a gyrocompassing mode in flight and erecting it in a damped second- or third-order mode. Once aligned and erected, the system can be switched back to its normal mode of operation. A typical alignment accuracy with this method is around 10 minutes.

5.3.5. SAFETY FEATURES. Certain failure-indication features are included in the design of most inertial systems. Built-in test circuitry and flight-line test equipment are used before flight to determine if the system is operating properly. In flight, self-check circuitry is used to monitor power supply voltages and frequency, and gyro- and accelerometer-error signals. If these signals are within preset bounds, normal system operation is likely. Deviations beyond limits indicate a malfunction and activate suitable visual and/or audio signals in the cockpit. Thus an inertial system can indicate some failures and emergency modes of navigation can then be used. But, the absence of a failure warning does not insure that safe navigation is being accomplished; other failures can occur without being indicated. Marginal-heading gyro operation, for example, can result in a serious departure from course with no malfunction indicated. While the inertial system will indicate some catastrophic failures, it will not ordinarily indicate degraded operation and thus is not a failsafe navigation system. Additional independent navigation or position information will have to be available on the flight deck to permit the crew to monitor the inertial system for safe operation.

In the event of failure an emergency navigation capability is a necessity. A dual inertial system might be considered. However, this requires a flag indication to identify the defective unit when one unit fails. If a unit failure is evidenced only by a difference in position readings, and either indicated position could conceivably be correct, then the crew must decide which unit is correct (a chance decision) or refer to a third and perhaps different system to resolve the question. In the case of the chance decision in choosing the correctly operating unit in the dual system, the a priori probability of incorrect navigation is the same for the dual system as for a single system. Hence, in this instance dual equipment offers no safety advantage.

Hybrid systems based on dissimilar phenomena offer obvious advantages. In the case of LORAN-A and doppler the crew abides by a rule of thumb that dictates additional LORAN-A fixes and doppler system checks when a LORAN position differs from the Doppler by more than approximately 15 nautical miles. An inertial system and ground reference system hybrid offers similar capabilities.

Consider the simple hypothetical hybrid described and discussed in Section 5.9. Major malfunctions are detected by observing system operation indicators. More elusive is marginal system operation or an operator's blunder. However, the nature of the hybrid configuration permits certain checks that can significantly increase the detection of inaccurate hybrid system operation or blunders. The simple hybrid system serves as a convenient example.

Consider the simultaneous position indications of the inertial system and the external system. In Figure 42 P_I is the position indicated by the inertial system (coordinates X_I, Y_I),

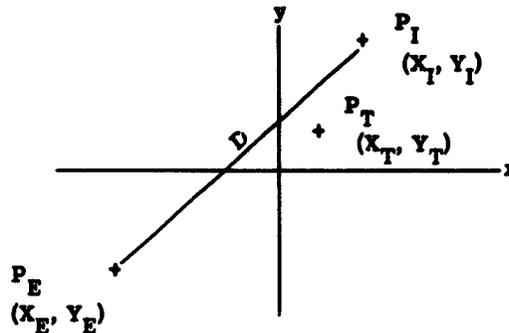


FIGURE 42. HYBRID SYSTEM POSITION INDICATED GEOMETRY

and P_E is the simultaneous indication of the external system (coordinates X_E, Y_E). Some P_T is the true position of the aircraft (coordinates X_T, Y_T). The distributions of position indications of both the inertial system and external system are assumed to be circular normal. X_I is $N(X_T, \sigma_{X_I})$, Y_I is $N(Y_T, \sigma_{Y_I})$, and $\sigma_{X_I} = \sigma_{Y_I} = \sigma_I$. X_E is $N(X_T, \sigma_{X_E})$, Y_E is $N(Y_T, \sigma_{Y_E})$, and $\sigma_{X_E} = \sigma_{Y_E} = \sigma_E$.

Consider now the distance D between the indicated inertial system position and the indicated external system position. In the operating system this quantity could be computed automatically or could easily be determined by the crew using graphical or mathematical methods.

$$D = [x_d^2 + y_d^2]^{1/2} \tag{115}$$

where

$$x_d = (X_I - X_T) - (X_E - X_T) = X_I - X_E \tag{116}$$

and

$$y_d = (Y_I - Y_T) - (Y_E - Y_T) = Y_I - Y_E \tag{117}$$

We note that

$$\begin{aligned} \frac{X_I - X_T}{\sigma_{X_I}} \text{ is } N(0, 1) & \quad \frac{Y_I - Y_T}{\sigma_{Y_I}} \text{ is } N(0, 1) \\ \frac{X_E - X_T}{\sigma_{X_E}} \text{ is } N(0, 1) & \quad \frac{Y_E - Y_T}{\sigma_{Y_E}} \text{ is } N(0, 1) \end{aligned}$$

and that the variances and expectations of x_d and y_d are given by

$$V(x_d) = \sigma_{x_I}^2 + \sigma_{x_E}^2 \quad (118)$$

$$V(y_d) = \sigma_{y_I}^2 + \sigma_{y_E}^2 \quad (119)$$

$$E(x_d) = E(y_d) = 0 \quad (120)$$

Let

$$\tilde{D}^2 = \frac{x_d^2}{\sigma_{x_I}^2 + \sigma_{x_E}^2} + \frac{y_d^2}{\sigma_{y_I}^2 + \sigma_{y_E}^2} = \chi_1^2 + \chi_1^2 = \chi_2^2 \quad (121)$$

\tilde{D}^2 has a chi square distribution with two degrees of freedom. It is obvious that

$$\tilde{D}^2 = \frac{x_d^2 + y_d^2}{\sigma_I^2 + \sigma_E^2} = \frac{x_d^2 + y_d^2}{\sigma^2} \quad (122)$$

where

$$\sigma^2 = \sigma_I^2 + \sigma_E^2 \quad (123)$$

and that

$$\sigma^2 \tilde{D}^2 = D^2 \quad (124)$$

or

$$D = \sigma \tilde{D} \quad (125)$$

The foregoing indicates that knowledge of the statistical nature of the errors in the inertial and external systems (bivariate circular-normal distributions with known means and standard deviations) results in the availability of statistical information concerning the distribution of D . Thus the probability α of D exceeding certain values $\sigma\sqrt{\epsilon}$ can be determined.

$$P_r\{\tilde{D}^2 > \epsilon\} = P_r\left\{\frac{D^2}{\sigma^2} > \epsilon\right\} = P_r\{D > \sigma\sqrt{\epsilon}\} = \alpha \quad (126)$$

From χ_2^2 tables we obtain the following values for α and ϵ .

α	ϵ	$\sqrt{\epsilon}\sigma$
0.001	13.815	3.717 σ
0.01	9.210	3.035 σ
0.05	5.991	2.448 σ
0.10	4.605	2.146 σ
0.368	2.0	1.414 σ
0.5	1.41	1.187 σ

Selecting α (for example $\alpha = 0.05$) and using Equation 107 or Figure 36 and Equation 123, we can construct the graph shown in Figure 43. The probability of obtaining a distance D for a corresponding time t so that the point t,D falls in the shaded area is less than or equal to 0.05, if both systems are operating properly. Figure 43 thus can serve as a graphical rule of thumb to reach a decision as to whether the systems are functioning correctly or not. Other values for α could be used. Additional curves in Figure 43 indicate the expected value of D and the median of D as a function of time.

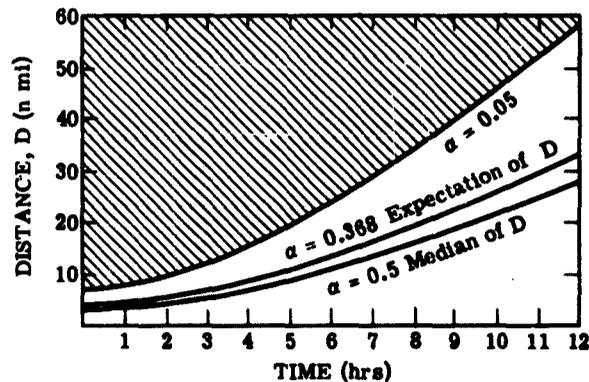


FIGURE 43. PERFORMANCE COMPARISON CHART

Consider inertial and external position indications obtained after 3.21 hours of flight. At this time the expected value of D is about 6 nautical miles, and the median of D is about 5 miles. The probability that D will be less than 5 miles is 1/2 the probability that D will be greater than 5 miles. There is one chance in twenty that D will be greater than about 12.5 miles. If D were found to be 16 miles, for example, the navigation system operator would take corrective action:

obtain additional fixes, check computations and the operation of both systems, and perhaps advise air traffic control of unsafe navigation. The penalty for concluding system failure when operation is normal is additional crew work. At $\alpha = 0.05$ this can be expected to occur once in twenty fixes. Of course the gain in safe operation due to detecting actual malfunctions depends on other considerations, such as traffic situations, and alternative navigation and fixing techniques available.

5.4. RELIABILITY

In reliability inertial techniques offer no distinct advantage or disadvantage over other self-contained navigation systems. The principal sources of failure are the electronic components and computers. Manufacturers' claims that pure inertial navigation system MTBF's (Mean Time Between Failures) of 250 hours seem reasonable and representative of the current state of the art. As early as 1960 platforms averaged over 550 hours between failures (e.g., 13 failures in 7300 hours). Since then, platform reliability has been, and continues to be, improved. The reliability of other components—amplifiers, power supplies, and control equipment—is similar and shares a common state of the art with other electronic equipment presently aboard commercial transports. Inertial system computers likewise compare with computers integral to other self-contained navigation systems and aids, such as doppler, radar, and star trackers.

The probability of "successful flights" on North Atlantic crossings for inertial systems is an interesting question. Here "successful" means without dependence on an auxiliary or backup navigation system. The available information on the failure behavior of inertial systems under certain operating conditions is in the form of MTBF, which is computed as total hours of use divided by the number of failures. As previously stated, a conservative value for this statistic is 250 hours. These operational data may comprise a mixture of failure data (e.g., time to first failure, time from first to second failure, etc.)

It seems reasonable to analyze the problem in the following manner. Consider n equipments which commence operation at time $t = 0$. At time $t = T$, the $MTBF = nT/f$, where f = number of failures. There may be a little ambiguity about when the MTBF should be computed; but apparently it is desirable to compute the MTBF at the time of a failure, not between failures. It appears reasonable to consider that the probability of failure in a small time interval Δt may follow the Poisson Law; that is, $P(x = \text{number of failures in } \Delta t) = e^{-\lambda} \lambda^x / x!$ where $\lambda = \mu \Delta t$ (μ = mean-failure rate per unit time). It can then be shown that the length of times between failures follows the exponential distribution. Let t_i be the length of time between occurrence of the $i-1$ and i failures. Thus, we may write for $f(t)$ the exponential distribution $f(t) = \theta^{-1} e^{-t/\theta}$. Accept-

ing this form, we find the expected value of t to be $E(t) = \theta$. Also, $V(t) = E(t-\theta)^2 = \theta^2$; so $\sigma(t) = \theta$. The distribution is easily integrated; the probability of failure before time t^* is

$$P(t^*) = 1 - e^{-t^*/\theta} \quad (127)$$

Now θ is the parameter, described above as the MTBF. In other situations θ may be called the Mean Time Before Failure, so careful labeling for a specific problem is required. Tables of the exponential function may be used to evaluate desired probabilities. The distribution also may be related to the chi square distribution with 2 degrees of freedom. Hence, after suitable transformation, the tables of the χ_2^2 distribution also may be used to evaluate desired probabilities. It should be recognized that a reasonably good empirical distribution for t would be preferable.

Assuming that a North Atlantic crossing requires 6 hours of inertial operation and using the crude datum given as $\theta = \text{MTBF} = 250$ hours, we may determine the probability of a single successful flight as

$$P_s = e^{-t^*/\theta} = e^{-6/250} = 0.979$$

For an airline flying 20 crossings per day (e.g., 10 round trips) the probability of a successful day's operation (no failures) is $(0.979)^{20} \approx 0.655$. It is doubtful that this figure is acceptable.

With a dual system installation in a single aircraft where each unit has an MTBF of 250 hours, the probability that neither unit fails during a 6-hour flight is $(0.979)^2 = 0.958$ (we assume that the two units are independent). The probability of both units failing is $(1 - 0.979)^2 = 0.00044$. The remaining case is that in which only one of the units fails. The probability of this occurring is $(1 - 0.958 - 0.00044) = 0.0417$. This was discussed in Section 5.3.5. If there is no way to determine which unit is faulty, then a chance decision must select a unit for continued navigational use (the probability of selecting the correct unit is $1/2$). Thus the probability of successful flight is the probability that neither unit fails plus $1/2$ the probability that exactly one unit fails: $0.958 + 0.0417/2 = 0.979$; this is the same probability of success as that for a single unit system.

For reliability, then, inertial systems compare favorably with doppler navigation systems. Duplicate equipment seems to be mandatory for commercial operation, and some external reference system will certainly be desirable.

5.5. ECONOMIC CONSIDERATIONS

Initial and operating costs connected with inertial systems are quite high. An inertial system might permit a reduction in flight crew by assigning the present functions of the specialist navigator to another crew member (e.g., the copilot), who would rely mainly on the inertial system; crew wages would thus be reduced. But it is evident that other operating costs would be increased, and it is possible that no real financial advantage would be realized.

At present, estimates of the cost of an inertial system, made by knowledgeable individuals in government laboratories and industry, range from \$40,000 to \$225,000. Conceivably, a rule of thumb for estimating inertial costs is to consider the initial cost (including installation in aircraft and procurement of support equipment and spares) as approximately three times that of currently flying doppler units. Dual doppler installations cost TWA approximately \$150,000 per aircraft (cost of the dual doppler equipment alone is approximately \$25,000). At present it is possible to procure a complete inertial system (platform, electronics, computer, and control and display units) for approximately \$110,000. Since price depends on the quantity of units ordered, some cost reduction should be possible. The typical "\$100,000 system" would be a 1.5- to 2-nautical-mile/hour (CEP) system weighing approximately 100 pounds and measuring about 2 cubic feet.

Developing and producing systems solely for commercial applications is almost prohibitive. In presenting the cost estimation above, we have assumed that the system is the product of a military research and development program, is in quantity production for military procurement, and that programs for testing, evaluating, and improving the system have been completed. The commercial procurement would thus involve only the production of additional copies, perhaps with slight modifications for commercial use.

Spare parts would be proportionately expensive. Gyros cost approximately \$8000 and accelerometers range from \$3000 to \$8000. At the level of base maintenance repair, equipment required to test and calibrate these parts would cost more than \$100,000. Test consoles and associated checkout equipment for work at both periodic maintenance and base maintenance levels are exceedingly expensive. Flight line checkout equipment can be as costly as \$50,000 per unit, depending upon the system in use and the quantity required.

Because of the highly skilled professional and technical personnel and the time-consuming nature of the work, high initial costs for maintenance and repair would be accompanied by high operating costs. Operating a white room is naturally more costly than operating a normal assembly or test line. Thus it is pertinent to point out some typical cost figures and to consider whether or not a white room can pay for itself. Exact costs for specific types of white rooms are not available, but ranges may be pointed out.

Construction of the bare room, with air treatment and utility lines supplied, will vary between \$30 and \$150 per square foot. Labor costs will average about \$5/hour/100 square feet²¹ before overhead [67]. This includes time spent in changing clothes and in the air shower at the start of a shift and time during lunch and coffee breaks; these are major cost factors. In addition, cost for special laundering of uniforms, extra blower capacity to keep the room pressurized through the high-efficiency filter, extra maintenance time for housekeeping, etc., must be considered.

Operating a white room of 10,000 square feet can involve an original investment of from \$300,000 to \$1,500,000 with operating costs of about \$2,000,000 per year. Obviously, operating maintenance cost will be high whether work is performed by the airline or under contract by the manufacturer.

5.8. TIMELY AVAILABILITY AND FUTURE PROJECTIONS

Table XIII lists some of the existing and proposed systems reviewed. Any of these or similar systems could be available for introduction in 1965 or 1966. The greatest problem would be that of providing appropriate maintenance and support. Installation and operation during flight pose no insurmountable problems. The Litton LN-3 systems have been in quantity production for several years and have seen extensive service in military aircraft; they could be placed in service immediately for operational testing by the airlines.

Table XIII indicates that the present state of the art is mainly a function of the accelerometers and gyros available. Currently available commercial accelerometers and gyros have the following characteristics.

Accelerometers

Threshold:	as low as 10^{-7} g
Linearity:	2% to 0.005% of applied acceleration, or some as good as ± 0.0001 g for range of ± 1 g
Ranges:	0 to ± 1 g, 0 to ± 10 g, and some in excess of 100 g
Weight:	0.25 to 4.5 lb
Price:	\$2500 to \$8900

Gyros (single degree of freedom)

Random Drift:	3°/hr to 0.003°/hr
Weight:	1 to 2 lb
Price:	\$8500

²¹This relatively odd unit to describe costs is evidently based upon an average room occupancy, but the reference does not specify what the average was.

TABLE XIII. INERTIAL NAVIGATION SYSTEMS

Manufacturer	System Name	Platform Name No. of Gimbals	Gyro Name No. Used Random Walk Sensitivity	Gyro Description	Accelerometer Name No. Used Sensitivity	Accelerometer Description	Position Accuracy	Altitude Accuracy Feet Miles Reading	Computer	Total Size Total Weight	No. of Units Delivered	Used In	Comments
LITTON	LM-3C	P-300	G-01-3-01	Two degree-of-freedom floated ball gimbals	A-300	Flushed pendulum, Torque-balance type	1 to 2 m/hr	3 m/s	Analog	2 ft ³	12	PSV	DOPPLER-INERTIAL. Used since 1960
		Four	Two 0.01 0/hr		Three 10 ⁻⁵ g			3 m/s		150 lb			
	LM-3	P-300	G-200	Two degree-of-freedom floated ball gimbals	A-300	Flushed pendulum, Torque-balance type	1 to 2 m/hr	3 m/s	Analog and Digital	1.87 ft ³	187	F-104G CF-104	Variations of the LM-3 are the LM-3-2A, LM-3-2B, LM-3D, LM-3E, LM-4, LM-4A
LITTON	LM-4	P-300	Two degree-of-freedom floated ball gimbals	Two degree-of-freedom inertial gas spin gyro	A-300	Miscellaneous, flushed pendulum, torque-balance type	1 to 2 m/hr (expected)	3 m/s	Digital	Approx. 1 ft ³	0	Orbital and sub-orbital space craft	Essentially a scaled-down version of the LM-3, with the improved gas spin gyro and use of solid state circuitry
		Four	Two 0.001 0/hr		Three 3 x 10 ⁻⁵ g			3 m/s		40 lb			
GENERAL ELECTRIC	RAMS E	(-)	KR-40	Single degree-of-freedom integrating gyro	LR-2	Flushed dc torque-balance type	CEP of 1.4 m/hr or 2.2 m/hr	(expected)	Analog and Digital	1.27 ft ³	0	Designed for use in manned aircraft	Combination of a geometric and an analytic system
		Four	Three 0.01 0/hr		Three 10 ⁻⁵ g			3 m/s		86 lb			
ASTRO-SPACE LABORATORIES	NAV-100	(-)	ANNG-6	Two degree-of-freedom air-suspended spherical gyros with optical pickoffs (nonintegrating)	(-)	Air bearing pendulum and addressal time drive	0.5 m/hr after 5 hrs (expected)		Digital	2.2 ft ³	0	Proposed for use in aircraft and space-craft	A geometric inertial navigator
		Four	Super-gyro (Less than 0.001 0/hr)		Two Used to establish vertical (nonintegrating)					84 lb			

Also in use are gyros with two degrees of freedom that have random drift rates of 0.01° /hour; some that are still in the proposal stage have expected random drift rates of 0.001° /hour. The latter gyros, sometimes referred to as super-gyros, usually have a more sensitive optical type of pickoff.

Considering the time frame of interest, we foresee no major technical breakthroughs that will radically improve the state of the art. Accuracy and reliability will be improved; costs will be reduced. But these changes will be evolutionary rather than revolutionary, the result of continuing product improvement and more precise manufacturing processes.

6
USE OF SATELLITES FOR LONG-DISTANCE NAVIGATION

6.1. INTRODUCTION

This section describes and analyzes three possible methods of applying earth satellites to the problem of long-distance navigation. It considers cost and performance, and presents conclusions.

Certain features are common to all methods of using satellites for navigation purposes. Information on the position of the satellite relative to fixed points on the earth as a function of time is communicated to the navigating vehicle in some manner (possibly from the satellite itself). The navigation system then determines the aircraft's position with respect to the satellite at a known time. By combining this with the known position of the satellite, we can compute the aircraft's position in earth coordinates. Any method of using navigation satellites thus requires accurate tracking of the satellite by a number of ground stations, so that the error in its known position can be kept less than the allowable error of aircraft navigation. Furthermore, in most navigation methods accurate values of aircraft velocity and altitude must be available to the navigator for use in the computation of position. Optical methods of determining the position of the satellite by the aircraft would be useful only in good weather. For an all-weather navigation system, some type of radar or radio link between the satellite and the aircraft is required.

There are two general methods of using a satellite for aircraft navigation purposes. One method obtains information relating aircraft position to satellite position. This may be information on aircraft position with respect to two or more satellites at a single instant, or a single satellite at two or more instants. From this information the position of the aircraft can be fixed with respect to the known satellite positions at the instants of measurement. The second method uses the satellite as a celestial body. The navigation system of the aircraft makes angular measurements to determine the satellite's direction relative to vertical or horizontal earth references. This is similar to conventional celestial navigation.

The first method may be varied in a number of ways. In one variation distances from aircraft to satellite may be measured by means of a radar system using either direct-ranging or a transponder in the satellite. In another, distances from the satellites to the aircraft may be measured by making time difference measurements, with accurate electronic clocks in both the aircraft and the satellites. This variation will be discussed in Section 6.4.

Still another variation uses measurements of the doppler shift of a radio signal transmitted from the satellite. This particular variation is the basis for the TRANSIT navigation satellite

system, which will be described in more detail in Section 6.2. The TRANSIT system is currently being developed and tested by the Applied Physics Laboratory of Johns Hopkins University under sponsorship of the U. S. Navy Bureau of Naval Weapons. The system is now undergoing tests using several orbiting TRANSIT satellites.

Navigation methods in which the satellite is used as a celestial body also have a number of possible variations. For example, measurement of the altitude angle of the satellite with respect to a stable vertical reference could be obtained at two instants of time. Alternatively, the azimuth of the satellite could be measured with respect to an azimuth heading reference. In other variations of the method, rate of change of altitude, azimuth, or range would be measured. Systems which might use the celestial navigation principle of operation are described in Section 6.3.

Methods of relating aircraft position to satellite position by a combination of range and angle measurements are also possible. Since no specific method of using such a combination has been proposed for implementation, this approach is not discussed in this report. However, the characteristics of such a method can probably be evaluated from information contained in the discussion of other methods.

The conclusions and recommendations reached in this report regarding satellite methods of navigation have necessarily been based on incomplete information. Except TRANSIT, the systems investigated herein have not been carried beyond the proposal stage. Facts are therefore lacking on many details upon which an evaluation of the system must be based. Where it has been necessary to define the system for evaluation purposes, assumptions have been made.

It should also be noted that conclusions reached in this report regarding the use of satellite methods of navigation have been based solely on conditions representing commercial aircraft flight. These conclusions cannot necessarily be extended to other applications.

6.2. TRANSIT NAVIGATION SATELLITE SYSTEM

6.2.1. **SYSTEM OPERATION.** In the TRANSIT system of navigation the navigating vehicle determines its position on the surface of the earth by locating itself with respect to the known position of a satellite. This technique uses the doppler shift in the frequency of the signal received from a stable radio transmitter in each satellite. A number of satellites containing radio transmitters with very stable frequencies are placed into approximately circular orbits at an altitude of about 600 nautical miles. The position of the orbiting satellites is obtained by means of four ground-based tracking stations, which determine the orbit of each satellite

by analyzing the doppler shift in the received radio signal. The data used by the tracking stations consist of the number of doppler cycles in each of many constant intervals of time—for example, one second. The data are digitized at the tracking stations and transmitted by teletype to a computing center. The satellite orbit is then computed from these data by a method described in Reference 68. The resulting set of parameters describing the computed orbit is sent to an injection station, which stores the computed information until the satellite is within range of its transmitter, and then transmits the orbit parameters to the satellite. In the satellite these parameters are stored in an electronic memory device and are broadcast at two-minute intervals.

An aircraft receiving the satellite orbit parameter data and measuring the doppler shift of the satellite transmitter can determine its own position with respect to known points on the surface of the earth. The frequency of the satellite transmitter is compared with the frequency of a precise local oscillator in the aircraft, and measurements of the doppler shift are used to compute the location of the aircraft with respect to the satellite. This information in combination with the information on the satellite's position provides the navigator with his position on the earth.

In the original TRANSIT system about 50 points of the doppler curve were recorded as the satellite passed, then compared with a theoretical doppler curve based on an assumed position of the navigator. From this comparison an improved estimate of aircraft position was made, and a new doppler curve, based on this modified position, was plotted. This process was continued until the best fix was obtained between the actual doppler curve and the theoretical curve. The navigator then used the last corrected position. Although this process is being used in Project DAMP with good results [69], the equipment required to plot the doppler curve and to make successive approximations is very large and bulky and thus has been abandoned in favor of the present TRANSIT system.

In the present system the latitude and longitude of the aircraft are determined by obtaining the integral of doppler shift over several precisely measured two-minute intervals. (The satellite transmits a digital code word every two minutes; these signals can be used as a very accurate timing reference.) In two minutes the satellite travels about 450 miles (Figure 44). In one pass of the satellite it is possible to obtain as many as five or six of these intervals, although not all passes provide this many usable intervals. However, only three intervals are required for a position determination.

The user's equipment integrates the beat frequency f_b with time to obtain a total count N_{b1} of the number of cycles occurring during a two-minute interval (t_1 to t_2):

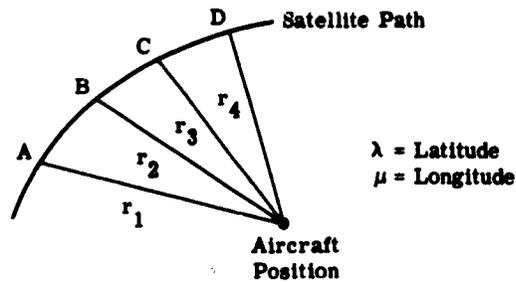


FIGURE 44. GEOMETRY OF TRANSIT POSITION—
DETERMINATION METHOD

$$N_{b1} = \int_{t_1}^{t_2} f_b dt$$

The beat frequency is produced by the time rate of change of the range r and the unknown difference δ between the transmitted and local frequencies

$$f_b = - (f/c) \dot{r} + \delta$$

Thus

$$\begin{aligned} N_{b1} &= - (f/c) \int_{t_1}^{t_2} \dot{r} dt + (t_2 - t_1)\delta \\ &= - (f/c) (r_2 - r_1) + (t_2 - t_1)\delta \end{aligned}$$

and

$$\begin{aligned} r_1 - r_2 &= \frac{c}{f} (N_{b1} + (t_1 - t_2)\delta) \\ &= \frac{c}{f} (N_{b1} + K) \end{aligned}$$

Similarly

$$r_2 - r_3 = \frac{c}{f} (N_{b2} + K)$$

and

$$r_3 - r_4 = \frac{c}{f} (N_{b3} + K)$$

The quantities r_1 , r_2 , r_3 , and r_4 can be expressed in terms of the known positions of the satellite at A, B, C, and D and the latitude, longitude, and altitude of the aircraft position. The

equations have as unknowns K , λ and μ . In solving these equations, we initially assume a latitude λ_0 and a longitude μ_0 of the aircraft. The equations are solved for K , $\Delta\lambda$, and $\Delta\mu$, where $\Delta\lambda$ and $\Delta\mu$ are the differences between the actual and assumed coordinates. An equation of the form

$$(r_1 - r_2) = \text{constant}$$

defines a hyperbolic surface with foci at A and B. The navigator's position is the intersection of such hyperbolic surfaces.

Present plans for the TRANSIT system are to continue with test flights and then to proceed to the operational satellite program beginning in 1962. Four operational satellite shots are to be made each year until four satellites are in orbit (since some of the shots may not be successful, it may require longer than one year to orbit four satellites). The satellites will be placed in polar orbits at altitudes of 600 nautical miles, their orbital planes separated by 45° . The orbital period will be 108 minutes.

6.2.1.1. Possibilities of System Simplification. In Section 6.2.2.3 the performance of combined satellite-inertial and satellite-doppler systems is assessed. For applications in which the one d_{rms} value of the total error may be allowed to reach six nautical miles, we find that a position fix error of as much as three nautical miles does not materially reduce the required fix renewal interval. For such position fix accuracies we may consider the use of simplified aircraft navigation equipment with the TRANSIT system to provide reduced performance at a saving in size, weight, and cost. Here we discuss several methods of simplifying the aircraft navigating equipment and operational procedures to indicate their possibilities rather than to completely evaluate their applicability. In stating estimates of savings in equipment, we usually give the fractional saving relative to the higher accuracy system.

One possibility would be to eliminate the requirement that the navigating aircraft receive time synchronizing information and orbital data from the satellite. Accurate time-synchronizing information could be provided by using a clock installed in the aircraft. Such a clock must be good to 0.1 or 0.2 second over the entire flight time. In addition, a measure of the 2-minute interval for collection of doppler information must be accurate to perhaps 1 msec. These requirements are well within the capabilities of digital counting equipment fed from the local oscillator used for doppler measurements.

To eliminate the transmission of orbital data from the satellite to the aircraft is also possible. Since the total time of aircraft flights will be 15 hours or less, the satellite ephemerical data may be provided in the form of printed copy carried aboard the aircraft. The

ephemeridal data required for each satellite for a period of 15 hours amounts to about 6000 decimal digits. Distributing orbital and time-synchronizing data in this manner could largely eliminate the memory equipment from the receiver which could thus be reduced in size and weight by one third. Furthermore, the data could be supplied in inertial rectangular coordinates, which is the form required in the subsequent computation. Supplying the data directly in the form needed would eliminate about half the required computation, with a consequent saving in equipment cost and computation time.

Some disadvantages to this simplified method would partially counteract its advantages. Preflight data could not be updated after the aircraft took off; hence the time over which the satellite orbit must be predicted would be greater than if the data were received via the satellite. However, this does not appear to be serious since the extrapolation error for a period of as much as two days has a standard deviation of only about 1.0 nautical mile. But the method would require additional computation and distribution of information on the ground, as well as additional data-handling by the aircraft crew. This would add to the work load of the crew and increase the possibility of mistakes.

A further simplification of position computation would be to precompute information in a form directly suitable for the final computation steps. By use of this method, the navigating equations might be reduced to the general form

$$\lambda = \lambda_0 + C_1 N_{b1} + C_2 N_{b2}$$

where C_1 and C_2 are functions of λ_0 and μ_0 . This could eliminate or greatly simplify the special-purpose computer required aboard the aircraft. The values of C_1 and C_2 would be pre-computed, a single computation being adequate for all airplanes flying within the area covered by the computation. Since the computation would be complex and required within a limited time, it would be performed by digital computer on the ground.

For example, an entire grid of 50-mile spacings across the Atlantic could be precomputed and made available to all navigators crossing the Atlantic. In order to determine a fix, an aircraft's position would be estimated to the nearest 50 miles, and the computation performed with the precomputed values of C_1 and C_2 corresponding to this estimated position.

Another possibility for simplifying aircraft equipment would be to use one frequency rather than two for measuring doppler shift. As discussed in Section 6.2.2.1, the use of two frequencies is required to account accurately for the effects of ionospheric refraction on the doppler measurement. The lower frequency gives a very large error, but the upper frequency (about 400 Mc) has given an error of 0.5 nautical mile. At night 0.1 nautical mile is a typical re-

fraction error. Unfortunately, these figures have not been confirmed during periods of magnetic storms or interference. Reference 70 indicates that the error introduced by the use of a single frequency would be about one nautical mile. It was estimated that using one rather than two frequencies would save \$2000 in the aircraft equipment.

The accuracy of the oscillator aboard the aircraft could be relaxed in hope of saving weight, size, and cost; current accuracies are a few parts in 10^{10} . However, the cost of a very accurate clock is not appreciably greater than the cost of a clock accurate to only 1 part in 10^9 or 10^8 . Therefore, we do not recommend the relaxation of frequency control. On the other hand, keeping the frequency of the local oscillators in the satellites and aircraft fixed with sufficient accuracy (about 1 part in 10^8 over the entire flight time) would simplify the computation by eliminating the constant K as an unknown in the equations. An unchecked fix of the navigating aircraft could then be obtained by using only two rather than three doppler measurement intervals of two minutes each. A third observation could be made as a check on the first two observations. It is estimated that this would cut the size and weight of the computing equipment by almost two.

Providing satisfactory computation at low cost is a problem that is still under development at the time of this writing and may be serious. One solution would be possible if the aircraft had a digital computer which could be shared by the TRANSIT system. Such an arrangement might make more rapid and accurate computation economically feasible. This would require that the TRANSIT system be integrated into the complete navigation system of the aircraft at an early stage in the design.

Choosing aircraft navigating equipment to achieve specific objectives in designing for accuracy would require a thorough analysis and design procedure beyond the scope of the present study. The intent of this section has been limited to indicating that appreciable savings are possible for less accurate systems. We refer to this again in Section 6.2.3.

6.2.2. SYSTEM PERFORMANCE

6.2.2.1. System Accuracy. The potential accuracy of the TRANSIT navigation system has been investigated both analytically and experimentally during the development of the system. This subsection describes the sources of error and gives information on their probable magnitude.

The question of the ultimate accuracy attainable with the TRANSIT system is rather controversial at the present time. Currently available technical information does not permit a complete analysis and evaluation of this aspect of the problem. Furthermore, the special conditions associated with aircraft navigation have not been given as much attention as those

of marine navigation. It has been possible, however, to make accuracy estimates which are sufficiently reliable to indicate the probable role of the TRANSIT system in aircraft navigation.

This section will first summarize the accuracy claimed for the TRANSIT system. Some of the special conditions and some of the questions of aircraft navigation will be discussed next. From this discussion suitable figures for performance analysis will then be derived.

The manner in which components of error are quoted in the references covering this subject is not always clearly defined, possibly because of the inadequacy of available data or because of the complexity of the relationships. Where information is not supplied, it has been assumed that the number quoted represents a $1-\sigma$ value for each coordinate (i.e., along-track and cross-track).

The inaccuracy in determining aircraft position consists of two major parts: the error in establishing the position of the satellite with respect to an earth-based coordinate system and the error in locating the aircraft with respect to the satellite.

An error in the assumed coordinates of the satellite will result in a computed aircraft position having an error of almost equal magnitude [70]. If there is no appreciable delay between the time at which tracking data are collected and the time for which the satellite orbit is being estimated, the errors in estimating instantaneous positions will have $1-\sigma$ values of 0.25 nautical mile. For navigation purposes, however, it is necessary to use position data which have been predicted in advance for periods ranging up to 24 hours by computations on the tracking data. From an analysis of test data obtained on the TRANSIT 4A satellite, launched in June 1961, Reference 70 states that the error resulting from satellite coordinate prediction for twelve hours will not exceed 0.5 nautical mile. As indicated previously, this is interpreted here as meaning that the $1-\sigma$ values of along-track and cross-track errors will each be 0.5 nautical mile.

In addition to the physical limitations on the accuracy with which satellite position can be determined, the distribution of information on satellite position will be restricted so that non-military users will be able to depend on it for an accuracy not better than 0.5 nautical mile. This would be done by using a special code for transmitting the less significant digits of the position data from the satellite. The rms combination of this round-off error (about 0.15 nautical mile) with the prediction error would not differ appreciably from the prediction error itself; therefore, for purposes of analysis, it is appropriate to use a $1-\sigma$ value of 0.5 nautical mile each for along-track and cross-track errors.

The errors in locating the position of the aircraft with respect to the satellite path will be discussed next. A possible source of error in all satellite radio tracking schemes is iono-

spheric refraction. Since the doppler shift of the satellite signal is basically the time rate of change of its electromagnetic path length, the doppler shift is altered from what it would be in the absence of the ionosphere. If a single frequency is transmitted, the effect of ionospheric refraction is to introduce an error into the measurement. Reference 70 summarizes test data and characterizes this error as "typically a mile."

To obtain improved results, the TRANSIT system uses measurements of the doppler shift at two harmonically related frequencies. A computation based on the measurements at the two frequencies then permits a large reduction in the error caused by refraction. The assumption here is that refraction in the ionosphere varies inversely with the square of the frequency.

Another component of navigational error is produced by instabilities in the frequency of the satellite transmitters and/or the navigator's local oscillator. A constant frequency error of one part in 10^8 would, if uncorrected, produce an error of one nautical mile in position. A method of computing an unknown but constant error has been described in Section 6.2.1. In addition to a frequency shift of constant amount, the satellite transmitter or the navigator's local oscillator may drift in frequency during the pass of the satellite. In this case the resulting navigation error is not easily eliminated. For an aircraft at a distance of 400 nautical miles from the subsatellite point, a constant drift rate of 1 part in 10^8 /hour would result in a total error of 0.20 nautical mile [71]. Actual test data, however, have shown that the frequency stability ranges from 1 part in 10^9 /hour for the first satellite, to 1 part in 10^{10} /hour for the third. These figures indicate that frequency stability in the satellite need not contribute any significant error in the navigation process, if a constant frequency shift is assumed and corrected for in the computation.

Simplified methods of computing aircraft position which integrate the total number of cycles of doppler shift over several time intervals are currently being developed. These methods will probably result in some increase in error as compared with the original method of using up to 50 data points, because some of the data available in the received signal are not used and because certain simplifications are made in computing. According to Reference 68, a computation of this type, using refraction-corrected data, should be possible with an accuracy of 1 to 2 nautical miles.

Another source of error is the uncertainty of the navigator's altitude. As a rough approximation, it may be assumed that an altitude uncertainty causes a position error of equal magnitude. The altitude which must be used in position determination is the distance from the center of the earth. The altitude uncertainty thus consists of two components, the deviation of mean sea level from a geocentric sphere and the altitude of the aircraft above mean sea level.

The deviation of mean sea level from a geocentric sphere is primarily a function of latitude. The latitude would be known, even before the position fix, within 10 miles. This would be good enough to prevent an appreciable error (i.e., more than 0.1 nautical mile) in this component of altitude.

It is also reasonable to assume that the standard deviation of the aircraft altimeter would be less than 600 feet (0.1 nautical mile). Hence, the total error in altitude may be assumed to be small in comparison with other sources of error.

In aircraft navigation the fixes would not be taken from a stationary position; instead, the observer would move a substantial distance while determining a fix. The component of aircraft motion due to the rotation of the earth can be determined accurately. However, an error will also be introduced into the position computation as a result of an uncertainty in the aircraft velocity relative to the earth. It should be noted that this error is not peculiar to the TRANSIT system, but occurs for any method of navigation which uses a running fix. The effect of an error in the navigator's velocity is particularly important in the present study because of the high speed of the aircraft and the corresponding magnitude of the velocity uncertainty.

This error can be estimated in the following manner [71]. An error in the assumed velocity of the aircraft will result in an error in the assumed distance traversed by the aircraft during the pass time of the satellite. For example, during a 15-minute pass interval, a 2-knot velocity error accumulates to 0.5 nautical mile. Typical navigational errors in estimated position appear to be about 1/2 to 2/3 of this accrued error. The error in estimated position resulting from the 2-knot velocity error would thus be about 0.25 to 0.33 nautical mile. The direction in which the assumed position is in error is generally different from the direction of the vector representing the velocity error.

An aircraft using TRANSIT should thus know its altitude and velocity vector as accurately as possible during the fix. To do this, it should fly a known path (preferably straight and level flight) while making a fix, and should use an altimeter which is accurate to 600 feet or better.

Errors can be introduced by multipath reception of the signal; that is, reflection of the signal from the satellite by the water when the satellite is near the horizon. Errors of this type can be eliminated by using antennas that have nulls at the horizon or by discarding data from the beginning and the end of a pass.

Tests have been made during the experimental program to determine the magnitude of error due to instrumentation errors in the navigation equipment itself. Reference 70 states that the errors due to this source do not exceed about 0.03 nautical mile in each coordinate.

In measuring the distance between tracking stations widely scattered around the world, there have been noticeable discrepancies between the data obtained from tracking the TRANSIT satellites and the data already available from previous geodetic measurements. The worst discrepancy, for the Australia tracking station, is over 0.5 nautical mile. The TRANSIT system is currently being used to establish these relative positions so that the navigator's position with respect to geographical points in various parts of the world can be obtained accurately. All differences between the position data provided by TRANSIT and those provided by other navigation means should, of course, be accounted for; but the quoted magnitude of the difference is not serious, even if uncorrected.

Reference 70 concludes that, if a single frequency is used, the mean navigation error will be about one nautical mile; with two frequencies, it can be reduced to 0.5 nautical mile. If elaborate gear is used within the navigating vehicle, it is claimed that an accuracy as high as 0.1 nautical mile can be obtained. However, this high accuracy would not be available to the nonmilitary user.

As indicated previously, general agreement does not yet exist concerning the potential accuracy of the TRANSIT system. Questions have been raised as to how well it is possible to predict the orbit of low-altitude satellites for periods of hours or days, in view of the effects of atmospheric drag and spatial variations in the earth's gravity field. The inherent accuracy of the navigation method is also questionable, particularly regarding the effects of atmospheric refraction and the size of error coefficients for aircraft at relatively large distances from the subsatellite track. These questions probably have more to do with the ability to achieve future accuracies of 0.1 nautical mile than with the ability to achieve the more modest accuracy requirements needed for commercial aircraft navigation. Nevertheless, it seems advisable to temper the accuracies claimed for the TRANSIT system until they have been demonstrated in actual use for aircraft navigation.

For purposes of analysis we will assume that it is possible to maintain a knowledge of the satellite orbit extrapolated up to 12 hours in advance with a resulting error in position determination having a $1-\sigma$ value in each coordinate of 0.5 nautical mile. We will also assume that the various errors in satellite-to-aircraft position (excluding the effect of aircraft velocity uncertainty) have a $1-\sigma$ value of 1.0 nautical mile in each coordinate. This figure is intended to represent an rms combination of errors due to atmospheric refraction (assuming the use of two frequencies), frequency noise, altitude uncertainty, instrumentation error, and approximations in position computation. The error in satellite-to-aircraft position would be greater than this because of errors introduced by aircraft motion. This component of error is assumed to be $1/6$ of the velocity error in knots, the higher value of the error range discussed previously.

The standard deviation of the total navigational error would be arrived at by taking the root-mean-square sum of the individual components of error.

It may be anticipated that additional research and development will result in some improvement of the currently attainable accuracies; but this will be offset by limitations in the accuracy of an operational system due to difficulties of operation and maintenance.

The components of error summarized above can be stated in the following terms. The rms combination of the satellite position error (0.5 nautical mile) and relative aircraft position error (1.0 nautical mile) amounts to a $1-\sigma$ value of 1.12 nautical miles in each coordinate. The corresponding d_{rms} value is 1.58 nautical miles. To this must be added a component whose magnitude in nautical miles is $1/6$ of the velocity uncertainty in knots. The total error computed from these figures will be substantially larger than that estimated in Reference 70.

6.2.2.2. Fix Renewal Interval. An analysis was made to determine the distribution of fix renewal intervals with the TRANSIT system for aircraft flights at various velocities both west and east. Cases were studied for flight occurring along the equator and at 45° latitude.

One part of this study was concerned with single coverage areas; that is, geographical areas lying in the coverage swath of only a single satellite. An aircraft would normally be able to obtain a position fix at 110-minute intervals as long as it remained in the coverage area of a single satellite. The cross-track component of the position fix would be missed, however, when the subsatellite track passed within about 100 miles of the aircraft; the interval between complete fix renewals would then be extended to two orbital passes requiring 220 minutes. The normal interval of 110 minutes would also be modified when the aircraft passed from the coverage zone of one satellite to that of a neighboring satellite. In this case the fix renewal interval might range anywhere from 0 to 220 minutes, depending on the phasing of the two satellite orbits. In order to determine the distribution of fix renewal intervals, the analysis had to account properly for the probabilities of each type of fix renewal. During the study this analysis was carried out for aircraft flying at various velocities east and west.

The distribution of fix renewal intervals is primarily a function of the absolute velocity of an aircraft with respect to the satellite orbits. This velocity is composed of two components, the earth's rotation and the relative motion of the aircraft with respect to the earth. In analyzing fix renewal intervals, the two extreme conditions are

- (a) A Mach 3 aircraft with an absolute velocity of about 2500 knots flying from west to east at the equator.
- (b) An aircraft flying east to west at a velocity corresponding to the earth's rotation and appearing to stand still with respect to the satellite orbital plane.

The results of the analysis are shown in Table XIV. For each velocity at least 5% of the fix renewal intervals would be as long as 220 minutes. The percentage of intervals exceeding 110 minutes would range from 6.7% to 51%. The average interval would lie between 110 and 115 minutes.

These results are somewhat oversimplified in that no allowance has been made for the period of about 20 minutes required for the observation of the satellite pass and computation of position. This factor would have the effect of causing a slight decrease in the percentage of intervals which are less than 110 minutes and a corresponding increase in the percentage of intervals between 110 and 220 minutes. The average fix renewal interval would be increased by 0 to 20 minutes for the various cases shown in Table XIV, the higher value occurring for the higher absolute velocities.

An investigation was also made for flights at 45° latitude, where the proposed TRANSIT operational system of four satellites would provide approximately double coverage; that is, at any point of 45° latitude it would be possible to observe the passage of two of the four satellites. Figures 45 and 46 show the distribution of fix renewal intervals not only for the double coverage which would actually exist (Figure 45), but single coverage, too (Figure 46). One of the cases represents a condition in which the absolute velocity of the aircraft with respect to the orbital plane is only 270 knots; hence the aircraft remains within the coverage of the same satellite for a number of satellite passes. Fix renewal intervals equal to the orbital period of 110 minutes would therefore predominate; this is indicated by the sharp peak in the curve of Figure 45. A considerable percentage of the intervals would have a value of 220 minutes (two orbital periods) when a fix is missed because the satellite passes directly over the aircraft. The remainder of the curve, which has a triangular shape, accounts for those cases which occur when the aircraft passes from the coverage of one satellite to that of an adjacent satellite.

The effect of adding the second coverage layer to that of the single coverage is shown in Figure 46. The frequency of occurrence is relatively constant over the interval from 0 to 110 minutes. A total of 5.9% of the cases have a 110-minute fix renewal interval, and almost no cases extend beyond 110 minutes. For this situation the average fix renewal interval is 56.4 minutes.

Flight at an absolute velocity of 990 knots presents a substantially different picture for single coverage. The aircraft now has a very substantial velocity with respect to the orbital plane, so that it passes through the coverage of a single satellite in less than the orbital period. Consequently, all fix renewal intervals consist of cases in which the aircraft passes from one coverage to another. For this condition, the shape of the curve is predominantly triangular.

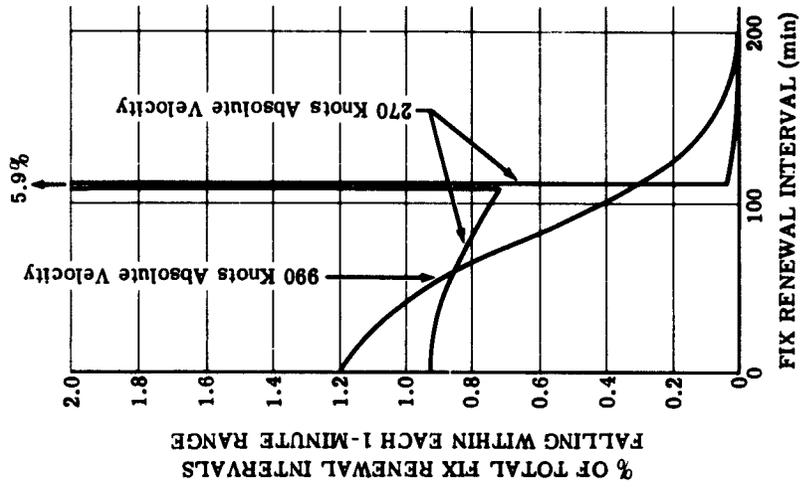


FIGURE 46. DISTRIBUTION OF FIX-RENEWAL INTERVALS WITH TRANSIT SYSTEM FOR DOUBLE COVERAGE AREAS

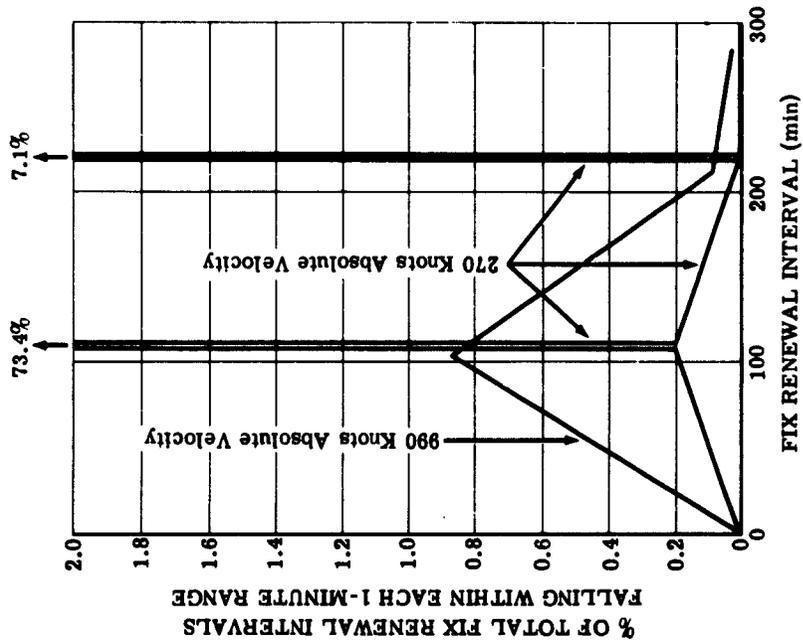


FIGURE 45. DISTRIBUTION OF FIX-RENEWAL INTERVALS WITH TRANSIT SYSTEM FOR SINGLE COVERAGE AREAS

For double coverage most fix renewal intervals fall below 110 minutes, the average wait being 52.5 minutes.

Distribution of fix renewal intervals for other aircraft velocities will not differ greatly from those shown. Therefore, we can state that for aircraft flights occurring in areas of double coverage the average fix renewal interval is 55 minutes, and that only a relatively small proportion of the fix renewal intervals exceeds 110 minutes.

If flight at 270 knots absolute velocity is extended to quadruple coverage, we find that the average interval is reduced to 26 minutes, and that only about 6% of the intervals are longer than 66 minutes.

If the effect of the 20-minute period required for the fix renewal is taken into account for double and quadruple coverage, the distribution of fix renewal intervals is shifted again toward increased times.

6.2.2.3. Performance Characteristics of Combined Systems. Because of its substantial fix renewal interval, the TRANSIT navigation method must be used in combination with a dead-reckoning system. This section discusses the performance characteristics of a satellite-inertial system and a satellite-doppler system.

Figure 47 shows the maximum allowable intervals at which position fixes can be renewed for a combination of a TRANSIT satellite system and an inertial system. The fix renewal interval required to maintain errors which reach values, at the end of the interval, ranging from 2 to 6 nautical miles d_{rms} is shown for various combinations of satellite error and inertial navigation error. Inertial navigation errors having d_{rms} values ranging from 1.414 nautical miles/hour to 12 nautical miles/hour are shown in combination with satellite d_{rms} errors which have fixed components of 1.58 and 3 nautical miles in addition to variable components caused by aircraft velocity uncertainty.

As indicated in Section 6.2.2.2, a geographical area which has simultaneous coverage by only one satellite may be expected to have an average interval of 110 minutes with 5% of the intervals being at least 220 minutes. A 1.414-nautical-mile/hour inertial system in combination with a 1.58-nautical-mile satellite system could maintain the maximum error within 6 nautical miles over a 220-minute period. For the average interval of 110 minutes, the maximum d_{rms} error (before fixing) could be kept within 3 nautical miles.

At the higher latitudes, where simultaneous coverage by two satellites is maintained, a 3-nautical-mile/hour inertial system combined with a 1.58-nautical-mile satellite system could maintain the maximum d_{rms} error within 6 nautical miles for the occasional 110-minute interval and within 3.5 nautical miles for the average interval of about 55 minutes. Operation with

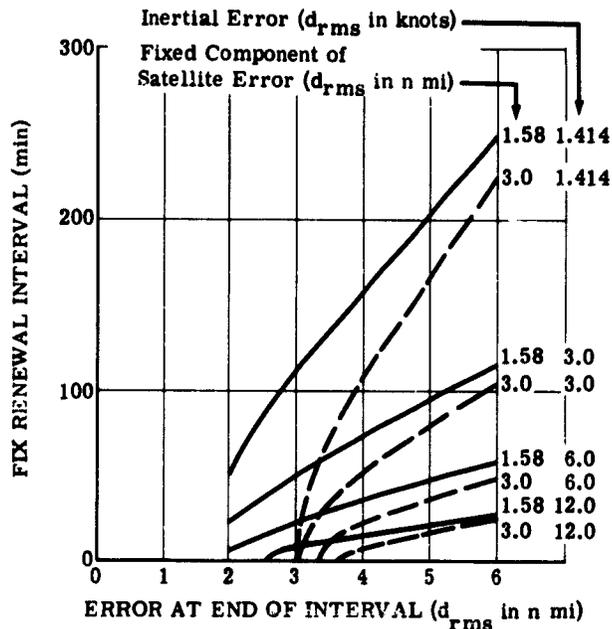


FIGURE 47. ALLOWABLE FIX-RENEWAL INTERVAL OF COMBINED TRANSIT AND INERTIAL SYSTEMS

still lower performance inertial systems would require increased satellite coverage beyond that presently planned for the TRANSIT operational system.

In Figure 48 the maximum allowable fix renewal interval to maintain errors of 2 to 6 nautical miles is shown for a navigation system which combines a 1.58-nautical-mile TRANSIT satellite with both a 1.5% and 1.0% doppler navigation system. A 1.5% doppler system represents the performance of present day systems. The information for a 1% doppler system shows the effect on system performance of a possible improvement in doppler equipment.

The doppler system is assumed to have an error whose d_{rms} value remains a constant percentage (either 1.5% or 1.0%) of the total flight distance. Although the d_{rms} error is known to vary with total flight distance, the use of a constant value simplifies the presentation of results without significantly affecting the conclusions reached.

Consider first the performance of a satellite-doppler system operating in areas of double coverage (i.e., at the equator with eight satellites in orbit or at high latitudes with four satellites in orbit). A satellite position-fixing system combined with a 1.5% doppler system could contain the d_{rms} error at the end of the fix renewal interval within 6 nautical miles for a pro-

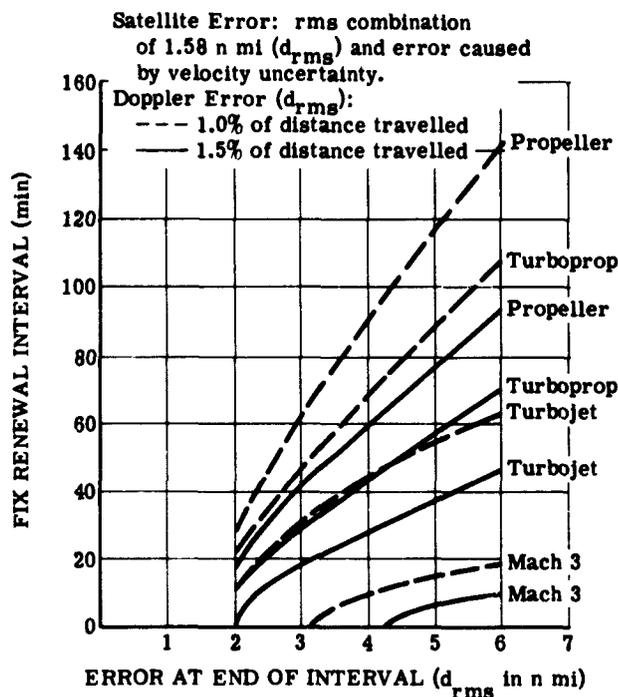


FIGURE 48. ALLOWABLE FIX-RENEWAL INTERVAL OF COMBINED TRANSIT AND DOPPLER SYSTEMS

propeller or turboprop aircraft. If a 1% doppler system were available, it also could accomplish this error limitation for a turbojet aircraft.

In areas of quadruple coverage (i.e., at high latitudes with eight satellites in orbit) the error at the end of the interval could be kept within 6 nautical miles for turbojet aircraft even with a 1.5% doppler system. But Mach 3 aircraft using either a 1.5% or a 1% doppler system, the error at the end of the fix renewal interval would greatly exceed 6 nautical miles.

We conclude that the TRANSIT satellite system using four operational satellites can be used in certain combinations with an inertial navigation system of relatively high performance to provide moderate overall accuracy. The required inertial-system performance could be substantially reduced if four additional satellites were available. The TRANSIT system using a total of four satellites with a 1.5% doppler system would be suitable to provide a system of low accuracy—but only for subsonic aircraft. The fix renewal interval required with a Mach 3 aircraft would be so short as to be unsuitable with the TRANSIT system.

6.2.2.4. Reliability. Limitations on the reliability of a navigation system are of two general types: failure of one of its parts (equipment breakdown or human error); or inherent limitations in the navigation technique, such as those resulting from radio interference or unfavorable satellite positions. Although the sources of the limitation may be basically different, we are primarily concerned with the net effect—that is, the reduction in the percentage of time during which the system provides usable results. In this section we will discuss the effects of equipment failure, human error, loss of phase-lock through radio interference, and unfavorable satellite position.

Specific information is lacking on the probability of failure in TRANSIT equipment, since the program is now under development. The circuitry is for the most part conventional; the only item which might be regarded as critical is the crystal oscillator. The equipment used with the simplified method of computation is somewhat complex, but not to such an extent as to appreciably degrade its reliability. Reference 70 states that experience with similar equipment indicates that the equipment "down time" can be kept to 20% or less without undue difficulty.

The likelihood of human errors will depend on the extent to which the crew members must participate in the operating procedure. The TRANSIT navigation method, as described in the references, would require at least some attention on the part of the crew, mostly in monitoring the operation of the equipment and detecting unfavorable conditions or incorrect operation. It is unlikely that a data-processing method which requires appreciable amounts of manual computation would be acceptable, particularly in view of the present tendency toward minimizing the navigator's work load. Certain minimum operations might be acceptable, however, such as manually inserting numerical data into the computer. If human participation is limited and if suitable training in equipment operation is provided, we anticipate no substantial reduction of system reliability due to human error.

An important possibility of operational failure is loss of phase-lock in the user's receiving equipment. According to Reference 70 the reliability of the prototype gear is currently about 75%; that is, a successful fix is obtained in about three out of four cases attempted. To obtain a fix with the present method of computing position, a navigator must lock in continuously for a least three 2-minute intervals. If contact with the satellite is lost even temporarily during an interval, the data from that interval are void. The 2-minute integration scheme is thus different from the original TRANSIT fix determination method in which 50 data points were used. Because of the great redundancy of information with the original method, the temporary loss of contact was not serious. Reference 70 states that the prototype gear loses about 20% of the passes because of phase-lock loss. This loss can probably be blamed on the isotropic

antennas of the satellites used initially. In the TRAAC satellite launched on November 15, 1961, a method of satellite attitude stabilization is being tested in the hope that this will permit the use of a directional antenna on the satellite to reduce the number of passes lost because of the loss of phase-lock.

If the ground track of the satellite is less than about 100 miles from the aircraft, the fix will not be satisfactory. Since this tends to occur at random intervals, we may consider it a reduction in system reliability. Therefore, it has been allowed for in computing fix renewal intervals in Section 6.2.2.2. The probability of missing a successful fix by being too near the satellite subtrack can be roughly estimated in the following manner. If the gross coverage of a satellite is assumed to be a swath 3000 nautical miles wide, and the net coverage, two strips, each 1400 nautical miles wide, then the probability that an aircraft within the gross coverage area will fail to obtain a successful fix on a particular satellite pass is 6.7%. However, it is still possible, even in this case, to obtain a single line of position.

In summary we may say that system failures due to equipment breakdown or human error are not expected to be excessive. The other types of failure are for the most part intermittent in nature, affecting only a single pass. They are therefore best interpreted in terms of their effect on the fix renewal interval, which is discussed elsewhere.

6.2.3. SIZE, WEIGHT, AND COST OF EQUIPMENT. The size and weight of the user's equipment will greatly affect its practicability for aircraft navigation purposes. The bread-board models of the receiver and the computer now being developed each occupy about 15 cubic feet and weigh about 100 pounds. Power requirements are approximately 1 kw. It should be noted that little has yet been done to miniaturize the navigating equipment. It is likely that when the system becomes operational, the size and weight would be considerably reduced. With miniaturization techniques that would be available by 1970, the size of the equipment capable of operating as described in Section 6.2.1 might be reduced to 3-5 cubic feet and the weight to 100-125 pounds. The lower limits would probably be typical of a system incorporating the simplifications discussed in Section 6.2.1.1.

The following cost estimates for the equipment now being developed are based on an operational system (i.e., research and development expenses are not indicated). Estimates were obtained from the Applied Physics Laboratory for a system which uses the computation method described in Section 6.2.1. Reference 70 gives a functional block diagram and general description of this system. It is understood that no provision has yet been made for computing the correction required to account for aircraft velocity. If equipment were to be produced in lots of 100, a receiver is estimated to cost about \$8000 and a special-purpose computer about

\$6000. To account for aircraft motion the computation of aircraft position would become more complex, perhaps by a factor of about 2. This would increase the cost of the computer somewhat, but would probably not double it. The solution time for a position fix would also be increased.

Since the system is under development at the time of this writing and has not yet been successfully demonstrated, it seems advisable to allow some increase in cost to provide for unforeseen contingencies. Therefore, we conclude that the production version of an aircraft installation of TRANSIT equipment capable of providing position fixes of not better than a 1.58-nautical-mile accuracy (d_{rms}) would cost between \$20,000 and \$30,000. This figure does not allow for spare equipment.

This cost range could be reduced if any of the simplifications described in Section 6.2.1.1 could be incorporated. For example, if the receiver were to be modified to receive only one frequency, the cost would be about \$2000 less; if the memory unit were omitted, an additional \$2000 could probably be cut. The use of a simplified method of computing based on the use of precomputed data would substantially simplify the airborne computer but would require added cost for using ground-based digital computers. Accurate figures on total potential savings are not available, but it seems possible that the previously mentioned cost could be cut to range from \$15,000 to \$25,000.

The costs of constructing, launching, and maintaining the satellites are roughly as follows. The Scout launching rocket will be used to place the operational satellite in orbit. It is estimated that the cost of making one shot amounts to \$1,000,000. The estimated life of a satellite in orbit is four years, so that the initial cost prorated over the satellite lifetime is \$250,000 per year. If all satellites are successfully placed in orbit, the cost of each satellite amounts to approximately \$500,000 per year; half of this is for the initial cost and half for the cost of ground tracking. Unsuccessful launchings would of course increase the annual cost chargeable to each successful satellite. Since this system is to be maintained by the U. S. Navy, we assume that the costs discussed here would not be borne by the commercial aircraft operators. However, the responsibility for any extra costs, such as for special distribution of orbital data, remains to be established.

6.2.4. PROBLEMS OF INTRODUCTION AND USE. The present plans of the U. S. Navy are to launch and operate a number of TRANSIT satellites which will be available for navigation purposes to both military and nonmilitary users. (The only limitation on the use of the satellites by nonmilitary users will be a restriction on the accuracy of the information made available to them concerning the location of the satellite.) As long as this policy remains in force,

the expense of launching and tracking the satellites would be borne by the Navy. On the other hand, FAA and its counterparts in other countries (and probably the operators of commercial aircraft) might have to provide for any extra satellites or the processing and distribution of data especially required by their use of the system.

It should be recognized that the operation of the satellite system would remain under the control of the Navy, so that policies and procedures regarding use would be set by that agency. Although no information is available on major changes anticipated in these policies, there is always a possibility that the requirements of the Navy might dictate certain modifications which would unfavorably affect the use of the system for aircraft navigation purposes or be incompatible with the policies and procedures of the operators of long-distance commercial aircraft.

Reliability of the satellite equipment must be very high in order to justify its use; there can be no provision for maintenance after launch. If a satellite is to have a lifetime of four years before atmospheric drag causes it to re-enter the atmosphere, it is reasonable to require that 90% of the satellites should last this long without an equipment failure. This implies that during these four years the mean time between failure should be ten times this long, or 40 years. This may be a difficult requirement, in view of the complexity of the equipment used in the satellite. Poor reliability, coupled with the high cost of building and launching a satellite, would make a system uneconomical.

Besides reliability of the satellite equipment, reliable performance must be exhibited by the user's equipment. This has been discussed in Section 6.2.2.4.

One of the operating problems of TRANSIT results from the fact that the error in position data increases significantly over periods as short as 12 hours. Consequently, operation of the system requires extensive ground-based facilities for tracking satellites, computing and predicting orbits, and transmitting ephemeridal data to the satellite. The operation of these facilities will represent a substantial addition to the system-operating costs.

6.3. ANGLE-MEASURING SYSTEMS

6.3.1. SYSTEM OPERATION. This section considers navigation systems which use angular measurements of high altitude satellites. This type of system is intended to provide world-wide, all-weather coverage. Since it is a passive system, it does not limit the volume of traffic which can be handled. In these respects, it is similar to the other systems described in this report.

The operation of an angle-measuring system is similar to conventional celestial navigation by optical means. The computation of position is based on the measurement of angular position

of one or more satellites with respect to vertical or horizontal references. Thus, similar problems are encountered in obtaining an accurate determination of the vertical. The essential difference is that using satellites permits operation at radio frequencies capable of providing all-weather operation.

As a specific example of angle-measurement systems, the navigation concept outlined in Reference 72 is described. This is based on the use of equipment similar to the Radio Sextant AN/SRN-4, developed for the U. S. Navy by the Collins Radio Company and designed to read out altitude and azimuth data. It has two major components. One is the tracking unit, consisting of the antenna and its scanning mechanism, the receiver, the tracking servos, and the angle readout system. The other is the Schuler-tuned stabilization system and north reference.

An aircraft navigation system would operate in the following manner. A number of satellites would be placed in polar orbits around the earth at altitudes lying somewhere between 4000 and 6000 nautical miles from the surface of the earth. In Reference 72 it is proposed that a total of four satellites be used. This question is considered further in Section 6.3.2.2.

A transmitter would be installed in each satellite to provide a CW signal of adequate power for tracking. Power for the transmitter would be supplied by a combination of solar cells and batteries, or possibly some form of nuclear power. In order to provide ephemeridal data, each satellite would be tracked by fixed ground-based stations using both optical and electronic methods. The resulting data would be used to compute the satellite orbits for future periods of a month or more.

The system to be installed in an aircraft for navigation purposes could be made smaller and lighter than the AN/SRN-4 by reducing the antenna size for shipboard use. It is estimated that an antenna having an 18-inch parabolic dish would be suitable not only for satellite tracking but for sun tracking as well.

The required size of the antenna is dictated by two considerations, sensitivity and pointing accuracy. With an 18-inch stabilized antenna operating at a wavelength of 8.7 mm to track a satellite at an altitude of 6000 nautical miles, the transmitted power required is 0.2 watt to a receiver with a noise figure of 8. Such transmitters are currently available. A solid-state transmitter capable of a 0.6-watt continuous output should be available for satellite use in the near future.

If a stabilized antenna were to be used, either the antenna or the complete satellite would have to be stabilized. As an alternative, an unstabilized antenna could be used if its radiation pattern were nondirectional. This would, however, increase the satellite power requirements by 17 db.

The user would track the satellite by means of its CW signal. For aircraft operation, the system would use the sun as a backup source for the satellites. This would add little to the complication of the system and would provide substantially continuous position-fixing capability, at least during daytime. The use of the moon is not recommended, however, since the required receiver sensitivity would be too high.

Optimum frequency for radio transmission from the satellite is in the neighborhood of 16 kMc (1.9 cm). Below this frequency sun-tracking accuracy decreases because the radio-metric center of the sun wanders appreciably. Above this frequency all-weather operation is impaired. The highest recommended operating frequency is 35 kMc (8.7 mm). At this frequency transmission through the atmosphere is satisfactory except in cases of heavy rain.

A system using an 18-inch dish could be practical for aircraft installation. This is somewhat smaller than some conventional weather radar dishes. However, placing the antenna within the aircraft may be a problem, especially for high-speed aircraft. Since it must be able to look over an almost complete hemisphere, it would have to be placed at the top of the aircraft; for example, in the position occupied by an astrodome. For some of the higher-speed aircraft, a retractable system having a hemispherical radome with a 22-inch diameter might be used. For Mach 3 aircraft, however, it appears that the system must be flush-mounted. This may cause appreciable refraction problems, but these might be alleviated if measurements at low altitudes (for example, below 10^0 to 15^0) are not used.

A digital computer is required to operate the inertial navigation and antenna stabilization servo systems. With only small additional complications (probably the addition of more memory capacity), this same digital computer would be used for the the navigation computation.

A time standard would be provided for determining the location of the satellite at the time a fix on it is obtained. This could be periodically reset by means of standard time broadcasts. For aircraft navigation purposes, however, the total time of flight would be short enough that a clock with an accuracy of at least 1 part in 10^6 would not need resetting.

The airborne equipment used for radio celestial navigation necessarily contains an inertial navigation system to provide vertical-reference and aircraft-velocity-vector information for use in the radio celestial position-fixing process. This same equipment can perform functions beyond that of position fixing. The inertial navigation system furnishes a dead-reckoning mode of navigation which can supplement the position-fixing capability of the celestial navigation mode. It also contains a vertical reference system for indicating or controlling aircraft attitude. In addition, certain methods of operating the celestial navigation system can be used to compute not only position-fixing data, but high-accuracy heading-reference data as well. The fact that

these multiple functions are inherently available from the system should be considered in comparing cost and performance of different methods of satellite navigation and of alternate types of navigation as well.

A considerable number of methods could be used to compute position fixes with angle measurement techniques, but only a limited number of these methods are suitable for aircraft navigation.

In some methods of navigation, it is necessary to know the azimuth of the celestial body being observed. For the present application such methods would be undesirable because of the difficulty of obtaining this information from an independent source. In fact, a distinct advantage of the system is that it might be used to supply azimuth information for other purposes.

Certain methods of navigation require the simultaneous observation of two celestial bodies in order to obtain a fix. During daytime the two bodies might be the sun and a navigation satellite; at night two satellites would be necessary. This would increase the total number of satellites required to give substantially continuous coverage. For a smaller number of satellites such methods could still be used when two bodies are observable.

However, position-fixing by methods needing only a single satellite would also have to be available. Some methods which can be used with a single satellite require a running fix with a considerable period of time elapsing between the individual observations. This method would introduce substantial errors into the fix from the uncertainty in aircraft velocity, as in the TRANSIT system. Furthermore, a substantial period of time required for the position fix would be incompatible with the short fix-renewal intervals required to maintain specified aircraft navigation accuracy.

It appears, therefore, that navigation methods using angle measurement techniques should

- a. Use a relatively large number of satellites, so that two satellites are in view almost continuously, or
- b. Use a smaller number of satellites, and depend primarily on position-fixing methods which can be completed in a short time with only a single satellite.

These methods are considered further in Section 6.3.2.

The design and operation of the ground and satellite portions of an angle-measuring system are relatively simple and straightforward, as compared to other types of navigation satellites discussed in this report. The satellite requires only a CW radio transmitter, an omnidirectional antenna, and the necessary power supply. The total weight of a satellite providing this equipment has been estimated in Reference 73 as not exceeding 50 pounds.

The relatively small size of the receiving antenna dish required for practicable installation in an aircraft may be limited by the receiver sensitivity. To provide sufficient signal strength without exceeding available power capacities in the satellite might require a stabilized directional antenna, which would in turn require some additional complication of the satellite design for attitude control.

Even with this added complication the satellite equipment could be expected to be relatively simple and sufficiently reliable. For the most part it would employ state-of-the-art techniques and would have a low rate of obsolescence.

The ground-tracking-and-computing equipment would also be comparatively simple. Since the satellites operate at relatively high altitudes, they could be observed and tracked from a small number of stations and they would be subjected to a minimum of perturbing forces due to atmospheric drag, variations of the earth's gravitational field, and effects of the gravitational fields of the sun and moon. Consequently, it is claimed that orbits could be predicted for a month or more, thus avoiding the necessity for rapid computation and distribution of predicted orbital data. The distribution of information could be performed without using the satellite itself for this purpose; therefore, ground-transmitting and satellite-receiving equipment are not required. This expected orbital stability has not yet been experimentally verified.

6.3.2. SYSTEM PERFORMANCE

6.3.2.1. System Accuracy. Detailed accuracy figures on existing radio sextants are classified and therefore not given in this report. It is possible, however, to indicate the general nature of the inaccuracies and to estimate aircraft navigation system performance.

System errors arise from both equipment limitations and the propagation characteristics of electromagnetic radiation. One of the major limitations of the equipment has been the inaccuracy of the vertical reference system. Any error in the vertical reference system contributes directly to errors in reading altitude angles. In order to minimize such errors, the vertical reference system should be built as an integral part of the antenna system, to eliminate relative angular motion due to limited mechanical stiffness of the intervening structure.

Other equipment limitations, both electrical and mechanical, may exist. Mechanical deflection or irregularities, electrical drift, bias or noise, and servo performance require very careful system design and adjustment to maintain accuracy.

The propagation characteristics of electromagnetic radiation can affect accuracy in a number of ways. The precision of angular measurement is, of course, affected by antenna dish size, operating frequency band, scan method, available signal, and amplifier sensitivity. In

addition, accuracy may be limited by nonhomogeneity of radome dimensions or material, atmospheric refraction, and, in the case of sun tracking, shifts in the apparent center of the sun due to dynamic solar activity.

The radio sextant system built by Collins Radio Company for the U. S. Navy is claimed to have a potential system accuracy of less than a nautical mile in each coordinate. A major limitation in obtaining this high accuracy has been the error in the vertical reference system [74]. The vertical reference data were supplied from an external source located at a considerable distance from the antenna; therefore, flexure of the ship's structure introduced errors.

It is claimed that a radio celestial-navigation system installed in an aircraft could measure altitude angles with respect to the vertical with a standard deviation of 1 minute of arc. However, this accuracy has not yet been demonstrated in a working system suitable for aircraft installation and therefore remains to be verified. The major sources of difficulty in achieving such accuracies appear to lie in the reduced antenna size, nonhomogeneity of the radome, and errors in the vertical reference system.

For purposes of discussion, a standard deviation of 1 minute of arc will be assumed, but the above reservations should be kept in mind.

If position-fixing could be accomplished by the use of a single satellite, it would be possible to use a method consisting essentially of determining altitude and altitude rate. The satellite would be tracked for a brief period—for example, 10 minutes. Two readings of altitude angle would be obtained at precisely determined instants of time during the tracking period. These two values could be interpreted either as two distinct measurements of altitude, or as a single altitude and an average rate of altitude change.

A single altitude reading might be accomplished with an error having a standard deviation of 1 minute of arc. If the error in the second reading has the same standard deviation and is substantially independent of the error in the first reading, the system accuracy may be analyzed as if a running-fix method using two separate altitude readings were used. If an attempt is made to derive an altitude rate, difficulties would apparently be encountered in establishing the instant at which the mean value of the rate occurs. Consequently, it is best to think of the method as a means of obtaining two altitude readings at short time intervals. By thus limiting the duration of the measurement interval, the method has the advantage of minimizing those errors which are the result of uncertainty in aircraft motion and drift of the vertical reference. At the same time, even a period as short as 10 minutes permits a sufficient change in altitude of a rapidly moving satellite to achieve reasonable accuracy. The errors in obtaining a position fix by this method may now be examined.

In ordinary celestial navigation the celestial bodies are at a great distance relative to the diameter of the earth. When satellites are used, however, their altitude is roughly comparable to the radius of the earth (Figure 49). The altitude angle h at which the satellite is observed

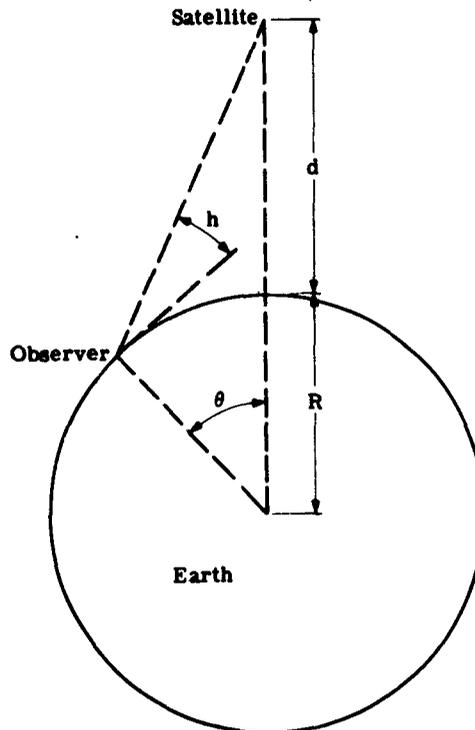


FIGURE 49. GEOMETRY OF SATELLITE OBSERVATION

at a given instant is related to the angle θ which represents the distance between the observer and the subsatellite point in the equation

$$h = \frac{\pi}{2} - \theta - \arctan \left[\frac{\sin \theta}{1 + \frac{d}{R} - \cos \theta} \right]$$

An angular error in reading h produces a smaller error in θ . Figure 50 shows the ratio of errors $\Delta\theta/\Delta h$ as a function of θ for a satellite at an altitude of 4000 nautical miles. The parallax effect is therefore favorable to obtaining increased accuracy.

Figure 51 shows a spherical triangle representing the geometry of a position fix obtained from two altitude readings of a satellite by an observer. From the two separate readings of

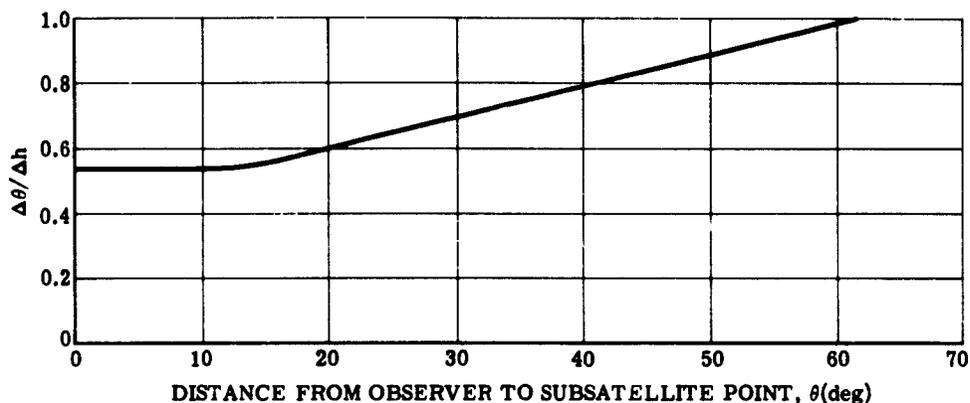


FIGURE 50. EFFECT OF SATELLITE PARALLAX IN REDUCING ERROR IN POSITION FIX

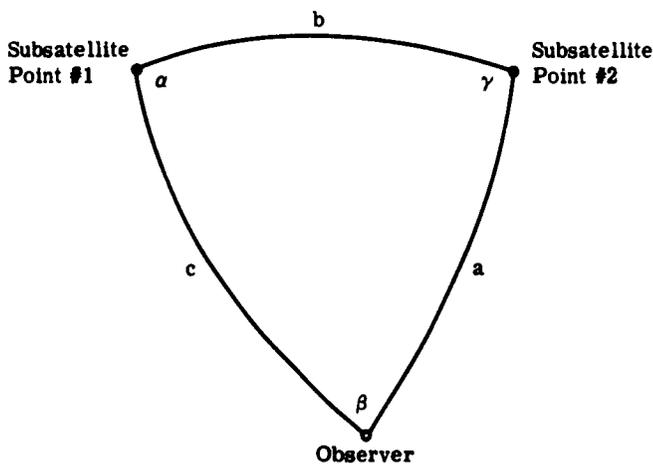


FIGURE 51. GEOMETRY OF POSITION FIX

h when the satellite is at points 1 and 2, respectively, the spherical angles c and a can be computed. The location of points 1 and 2, and the spherical angle b are also known from the ephemeridal data available on the satellite and the exact instants of observation. Hence, the observer's position can be determined. In general, the error in the position fix tends to be inversely proportional to $\sin \beta$. Ideally, β should be 90° for a measurement, but generally this can only be approximated. A satisfactory value for β will usually be difficult to obtain if the observer is a great distance from the subsatellite track. It will also be difficult when the observer is very close to the subsatellite track, unless one of the measurements is made when the satellite is nearest the zenith. (As an alternative to this latter case, greater accuracy could be obtained

by measuring an altitude and a rate of change of azimuth.) If the angle β can be kept to values between 30° and 150° during a position fix, the value of $\sin \beta$ will not fall below 0.500. This fact, in combination with the effect of satellite parallax, will then maintain the position fix error in each coordinate resulting from errors in altitude readings at one or two nautical miles/minute of arc error in h .

An additional component of error which can be introduced into the position fix results from the fact that a running fix has been made. An uncertainty in the velocity of the aircraft during the interval of time between the two altitude readings will cause an error in the position fix, as noted with the TRANSIT system.

For an angle-measuring system this error may be estimated in the following manner. The position fix would be obtained by establishing two Sumner circles, each representing a line of position at two instants of reading (LOP No. 1 and LOP No. 2 of Figure 52). In order to fix

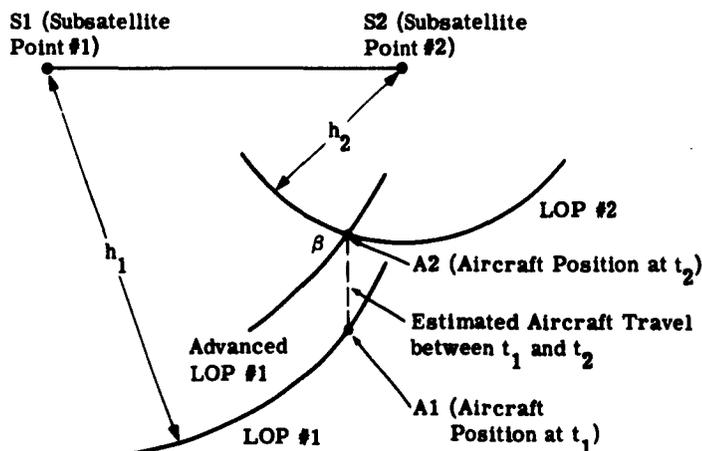


FIGURE 52. POSITION FIX FOR MOVING AIRCRAFT

the aircraft's position at the time of the second observation, LOP No. 1 would be moved along by the assumed distance traveled by the aircraft during the interval, to advanced LOP No. 1. The point at which this translated line of position crossed LOP No. 2 would represent the computed position of the aircraft at t_2 . An error in either speed or heading in the estimated distance traveled during the interval could result in an error in the computed position. A component of the velocity error which was normal to the line $S1 - A1$ would shift the line of position parallel to itself at $A2$ and hence would introduce no error. On the other hand, a component of velocity error Δd along $S1 - A1$ would shift the position fix by an amount equal to $\Delta d / \sin \beta$.

The direction of the velocity error will generally have a random relation to the S1 - A1 line. As an approximation, we may take a fraction of the total error Δd corresponding to $\sin 45^\circ$ to estimate the error due to velocity uncertainty. If the velocity uncertainty accumulates at a rate of Δv knots, its total value in a 10-minute interval will be $0.167 \Delta v$. Thus, the error component due to velocity uncertainty is

$$e = \frac{0.707 \times 0.167 \times \Delta v}{\sin \beta} = \frac{0.118 \Delta v}{\sin \beta}$$

For values of β lying between 30° to 90° , this would result in an error ranging from $0.236 \Delta v$ to $0.118 \Delta v$. Thus, a 3-knot inertial navigation system would introduce errors in the position fix amounting to 0.35 to 0.70 nautical mile. These errors are in terms of d_{rms} values and should be added to errors in altitude reading on an rms basis.

From the preceding analysis we conclude that a navigation system based on angular measurements of satellites having a standard deviation of 1 minute of arc would have a d_{rms} error of about 2 nautical miles.

6.3.2.2. Satellite Coverage. Consideration will now be given to the continuity of coverage which can be obtained from various numbers of satellites operating in 4-hour polar orbits at altitudes of 3440 nautical miles. The somewhat optimistic assumption that successful observations are possible if the satellite is anywhere above the observer's horizon simplifies the analysis. Thus, a satellite at 3440 nautical miles covers a circular area constituting 25% of the earth's surface. With its apex at the center of the earth a cone intersecting this circle would have a half-angle of 60° . We also assume that the phases of the various satellites in their orbits are randomly distributed. Otherwise the satellite would have to carry additional equipment and fuel to provide for control of its flight path in order to maintain the desired phasing among the various satellites.

If an observer near the earth's surface could lie directly on a subsatellite track, he would observe the satellite for 1/3 of its total orbital period, the gap in coverage thus being 66.7% of the total time. If two satellites passed over his position in the same manner, the percentage of time during which neither one would be in view would range from 33% to 67% (depending on the relative phase relationship of the two satellites), but on the average would be 44.4%. For three and four satellites the gap in coverage would average 29.6% and 19.8%, respectively. If path control of the satellites were maintained, the phase of the individual orbits could be controlled so that only three satellites passing over a given point would reduce the coverage gap to 0%.

The distribution of time intervals during which coverage gaps exist is also of interest. In an area covered by a single satellite these intervals would all be equal to 67% of the orbital

period, or 160 minutes. In an area covered by two satellites the intervals would range from 0 to 160 minutes, with a constant probability throughout this range. Frequency of gaps in coverage and their duration are shown for various numbers of satellites in Table XV and Figure 53.

TABLE XV. COVERAGE GAPS WITH 3440-NAUTICAL-MILE-ALTITUDE SATELLITES

No. of Satellites	Coverage Gap (% of total time)	Total No. of Gaps/Day	Ave. Length of Gap (min)	Total No. of Gaps/Day Longer Than 1 Hr
1	66.7	6.0	160	6.0
2	44.4	8.0	80	5.0
3	29.6	8.0	53	3.1
4	19.8	7.1	40	1.7
8	3.9	2.0	28	0.2

Note: This graph applies to angle-measuring satellites operating at 3440 n mi altitude

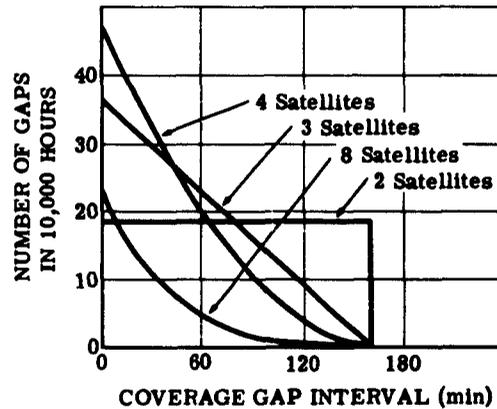


FIGURE 53. DISTRIBUTION OF COVERAGE-GAP INTERVALS FOR VARIOUS NUMBERS OF SATELLITES PASSING OVER AIRCRAFT

If four satellites were placed in polar orbits with random phasing, the geographical area near the North Pole would have a coverage pattern similar to that shown for four satellites. The geographical area at latitudes of less than 30° would have a coverage pattern similar to that shown for two satellites.

The coverage pattern just discussed applies to a position fix method which would require only a single satellite. For this method the sun would not be useful. Its slow rate of angular movement would require an excessive time for obtaining a position fix.

As an example of the use of Table XIII and Figure 47, consider an aircraft flying in an area passed over by four of the total number of satellites in orbit. Coverage gaps ranging in length from 0 to 160 minutes would occur. According to Table XV, 7.1 gaps occur per 24-hour day, or 2960 per 10,000 hours. Figure 47 indicates that 19 of these 2960 gaps would have a total duration between 60 and 61 minutes.

Let us next consider the case in which a position fix is obtained by observing two celestial bodies simultaneously. We assume that the sun would be available 50% of the time and that the satellite would be randomly phased. The results are shown in Table XVI; even at a location passed over by four satellites, substantial periods would exist during which double coverage would not be available.

TABLE XVI. GAP IN DOUBLE COVERAGE

No. of Satellites To Be Used in Conjunction with Sun*	% of Time Two Bodies Are Not Available
1	83.3
2	66.7
3	52.0
4	39.7
5	30.2
6	21.9
7	16.0
8	11.7

*The sun is assumed to be available 50% of the time.

From the data presented above, it is possible to draw conclusions concerning the number of satellites required to give satisfactory system performance. If four satellites and the sun were used, the number of gaps occurring for two-body navigation methods would require a 3-nautical-mile/hour d_{rms} inertial system to keep the total navigation error within 6 nautical miles d_{rms} . By increasing the number of satellites to eight, the double coverage is available all but 11.7% of the time. A 6-nautical-mile/hour inertial system would probably be adequate.

If a fix method requiring only a single satellite were used, four satellites would give results roughly equivalent to eight satellites and the sun for two-body methods. For four satellites a gap longer than 100 minutes would occur at high latitudes once every 24 to 36 hours. At lower latitudes it would occur once every 8 hours. With eight satellites gaps of 100 minutes or longer would occur less than once a month at high latitudes and once every 36 hours at low latitudes. The system performance can be determined by relating these results to Figure 54. We conclude

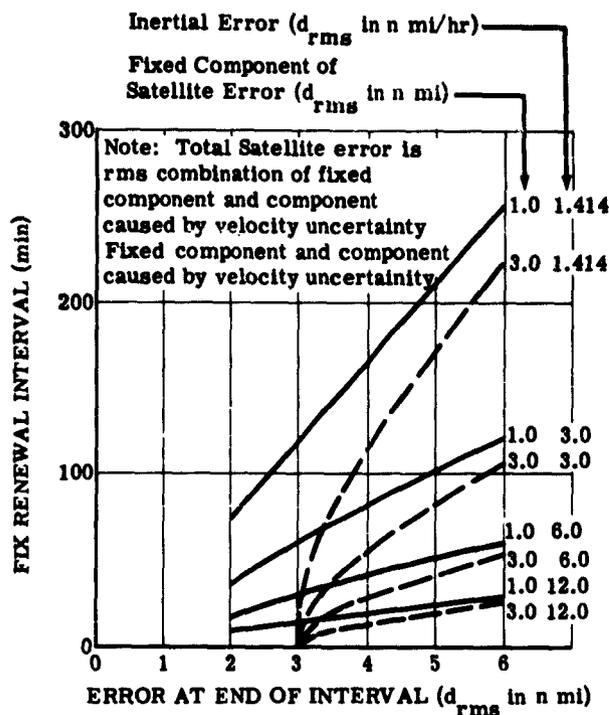


FIGURE 54. ALLOWABLE FIX-RENEWAL INTERVAL OF ANGLE-MEASUREMENT SATELLITE SYSTEM COMBINED WITH INERTIAL SYSTEM

that for a given number of satellites, a navigation system using angle-measuring techniques would give substantially better coverage, than the TRANSIT system. The angle-measuring system is workable with four satellites, but should use at least eight satellites in order to give coverage which could reasonably be referred to as "continuous." With this continuous coverage a relatively low performance inertial system would be adequate.

6.3.2.3. Reliability. In considering the reliability of the system, we should recall that it is intended to be used not only for the primary purpose of radio celestial position-fixing navigation, but also as a supplementary inertial navigation system, as a north reference, and as a vertical reference for attitude control. Although the system is capable of performing these multiple functions when it is operating normally, certain types of malfunctions would interrupt all of them. In particular, a failure of the inertial navigation portion of the system would result in a complete interruption of service. Consequently, in assessing the cost and reliability of the system, it would not be justifiable to consider one type of navigation as a backup source for the other, unless duplicate equipment were provided.

The user's navigation equipment is a relatively complex electromechanical system as compared with that required for other satellite navigation systems; the accuracy of the entire system depends on maintaining accurate adjustment. Its reliability as an accurate navigation device would therefore demand consistently high-grade maintenance and operating procedures.

6.3.3. SIZE, WEIGHT, AND COST. The complete cost of a radio-celestial system in an aircraft has been roughly estimated as \$20,000 to \$30,000. This would include the electronics (consisting of antenna receiver and pointing mechanism), but not the vertical reference system.

The vertical reference system would not only be used in connection with the observation of satellite position, but would also supply an inertial navigation capability. Cost data on inertial navigation systems indicate that the cost of the vertical reference system would probably be considerably more than the cost estimated above for the remainder of the system.

The figures quoted are only for the equipment itself; if the aircraft structure had to be modified to accommodate the receiving antenna, the installed cost, size, and weight of the system might be substantially increased.

The weight of all equipment, including the vertical reference system, is claimed to be 300 to 375 pounds; about 75 pounds is required for the vertical reference system.

6.3.4. PROBLEMS OF INTRODUCTION AND USE. In the development and application of the angle-measuring method of navigation, three distinct phases can be identified. First, feasibility of the system would be demonstrated analytically and experimentally. Then, if this demonstration were successful, prototype equipment would be developed. Finally, a complete system of ground-based equipment and satellites would be installed with coincident construction and installation of the user's equipment.

Realistic estimates can be made only for the first phase of such a program. An analytical study would be performed to establish a strong presumption of feasibility and to evaluate the system objectively. In the following experimental program several satellites would be constructed and launched, with the intention that one or two of them would be successfully placed in orbit. Experiments could then determine the accuracy with which the orbit of each satellite could be tracked and predicted. At the same time the user's equipment would be developed and tested to determine the potential accuracy of the system, as well as its other characteristics.

Three or four satellites would be launched at a complete cost (including the launch vehicle) of \$1,000,000 each. Studies of tracking and orbit prediction, made in sufficient detail to demonstrate feasibility, would cost perhaps \$500,000. Development and test of user's equipment would require an additional \$1,000,000 to \$2,000,000. Thus, a feasibility demonstration program would appear to cost from \$5,000,000 to \$7,000,000.

Although a full-scale experimental project designed to demonstrate feasibility is necessarily a multi-million dollar program, the initial step, consisting of an analytical evaluation, can be performed at relatively little expense. This part of the program could be completed and reviewed before commitments were made for carrying out the remainder of the program. An experimental program could also be conducted, at very limited expense, if intermediate-altitude satellites orbited for other programs could be used for preliminary studies of angle-measurement system performance.

6.4. RANGE-MEASURING SYSTEMS

6.4.1. SYSTEM OPERATION. Aircraft navigation systems have been proposed which use radar ranging from satellites to obtain position fixes. Like other navigation satellite systems, this technique is intended to provide all-weather navigation capability and largely continuous coverage at any point in the world.

One representative system has been proposed by the Fairchild Stratos Corp. [75]. Position of the aircraft in the proposed system would be determined by using information on range obtained by means of pulse signals transmitted between the aircraft and two or more satellites. Several variations of the method would be possible:

- (a) Ranging by using a pulse from the aircraft to trigger a responder in the satellite.
- (b) Ranging by comparing an accurate clock installed in the aircraft with one installed in the satellite.
- (c) Evaluating the differences in arrival time of accurately timed pulses from the various satellites.

The third of these methods is the one recommended by Fairchild for civilian application. It would work in the following manner. A total of twelve satellites would be placed in 24-hour orbits in a configuration permitting complete coverage of the earth. Each satellite would carry a clock having an accuracy of 1 part in 10^{10} . The satellite would also carry a receiver so that this clock could be reset from the ground every 500 seconds. The maximum drift of the satellite clock would therefore not exceed $0.05 \mu\text{sec}$, corresponding to a range error of 0.009 nautical mile.

Each satellite would use a small stabilized antenna to send out a 1000-Mc signal in a train of pulses $2 \mu\text{sec}$ long emitted once a second. An average radiation power of 10 watts and a 20° beamwidth would be used. To permit accurate ranging, the satellite clock would control the time of occurrence of each pulse. Each pulse train would consist of 500 pulses, somewhat irregularly spaced within the pulse train, so that an electronic gating circuit within the aircraft would lock onto the pulse train in only one position of the gate.

The pulse trains of the various satellites would be emitted in a rigidly maintained sequence in order to permit the aircraft navigation system to identify the source of a given pulse train by its time of arrival. If 12 satellites were used, each of the 12 would emit a pulse train in sequence, one pulse train being emitted by a satellite every 1/12 second.

The navigating aircraft would receive the pulse signals by means of an unstabilized antenna and a receiver. The aircraft would also carry a clock with an accuracy of 1 part in 10^8 for use in range determination. The aircraft position would be determined by measuring the difference in the arrival times of the pulses from two or more satellites. As indicated previously, the satellite from which a given pulse was emitted could be determined by its time of arrival. The time interval between pulses from different satellites could also be determined. For both of these purposes a clock of relatively low accuracy would be adequate.

If accurate information were available on the time interval between the arrival of pulses from two satellites, on the position of the satellites, and on the altitude of the aircraft, the position of the aircraft could be established as lying along a curved line on a geocentric sphere at the aircraft's altitude. Observation of this time interval for two different pairs of satellites would thus provide a position fix for the aircraft. A minimum of three satellites is required to provide such a fix.

In order to compute aircraft position, the data on position and pulse timing of each satellite and the flight program of the aircraft would be inserted, before the flight, into the memory of a computer carried by the aircraft. For the selected flight program a set of desired range differences for given satellite pairs could be defined as a function of the aircraft position along the flight path. In the computer the two measured range differences would be compared to the desired differences, giving two range-difference errors. A coordinate resolver would use these errors to determine the deviation of aircraft position from its desired path in earth coordinates. This deviation from the desired path could then be corrected either manually or automatically. If the aircraft position could be maintained close to the original program, the aircraft computer could be considerably simplified. It is claimed that the memory requirements would then not exceed a few thousand bits per hour.

Strictly speaking, the position fixing method just described is not a range-measuring method, but a range-difference-measuring method. The measurement technique, based on the accurate determination of time intervals, is common to both methods.

The position of a 24-hour satellite in an equatorial orbit would remain fixed with respect to earth coordinates for any observer on the earth's surface. If the 24-hour satellite were in an inclined orbit, its position would appear to follow a figure eight with a period of 24 hours.

Its ability to maintain fixed or repetitive positions with respect to the earth will be affected by several factors. If the orbit deviates from exact circularity, does not have exactly a 24-hour period, or does not have an inclination of exactly 0° , its position will not remain fixed and must therefore be represented by more complex ephemerical data. Another factor affecting the satellite orbit is the perturbations caused by the tidal effects of the sun, the moon, and other planets. Some or all of these effects could be avoided if station-keeping were adopted to maintain the satellite in a fixed orbit. Although a detailed study might indicate that the use of station-keeping methods would be justified, the present analysis will be limited to the consideration of systems using randomly spaced satellites in order to be consistent with the analysis of other satellite methods in this report.

The complexity of the satellite portion of the range-measuring system would be about the same as that of the TRANSIT system and greater than that of the angle-measuring system. The satellites would have the functions of antenna stabilization, reception of information from the earth, accurate time measurement, pulse generation and transmission, and power supply. They would have to be placed in relatively high orbits with very good accuracy.

6.4.2. SYSTEM PERFORMANCE

6.4.2.1. System Accuracy. The sources of error in this system include the errors in knowledge of satellite position and in the process of determining range differences.

The satellites would be placed in 24-hour orbits. It should be possible to predict these orbits for long periods of time, since atmospheric drag is absent and the satellites would be outside the range of appreciable variations in the earth's gravitational field. Gravitational effects of the sun and moon would cause slight perturbations of the ideal circular orbit which should theoretically be corrected in the navigation process; but they could be ignored, if they were not required to obtain the accuracy needed for navigation purposes.

Although no detailed data have been obtained on the accuracy of 24-hour satellite-position determination and prediction, there is no reason to doubt that this can be kept high enough to be used for navigation purposes.

Errors in range-difference determination would be caused primarily by drift of the satellite and aircraft clocks, limitations of the range gating circuits, and atmospheric effects. It is claimed that this error would not exceed 0.05 nautical mile. It should be kept in mind that the error referred to is the discrepancy in range-difference. The error in determining aircraft position will generally be larger than this. For certain unfavorable relative positions of the satellites or aircraft the range-difference discrepancy could be multiplied many times.

For example, consider two satellites separated by only 1000 nautical miles. A range-difference error of 0.05 nautical mile would move the line of position about 1 nautical mile. This represents an extreme case; in general, position-fixing errors of as much as 0.5 nautical mile due to unfavorable geometry would be unlikely if a total of 12 satellites were used.

The effect of errors in aircraft altitude could be eliminated by making an additional range-difference measurement, but it is unlikely that this additional complexity would be justified for the accuracies required for commercial navigation. Another possible source of error would result from attempts to simplify the equations used in position-fixing.

Without attempting to perform detailed error analyses, we think that this method would be capable of navigation accuracies at least equal to and possibly better than the other methods considered in this report. A 1-nautical-mile d_{rms} may be a good estimate.

6.4.2.2. Fix-Renewal Intervals. The range-measuring method described in this report requires that three satellites be in view simultaneously. A method suggested for providing this coverage would use 12 satellites. Three satellites would be placed in each of four orbital planes parallel to the four surfaces of a tetrahedron. If the satellite positions were not controlled, an average of four satellites would be visible from any point on the earth's surface, but variations in this coverage would occur. Fewer than three satellites would be observable 15% to 20% of the time. During these gaps a dead-reckoning method of navigation would be required.

If the 12 satellites were maintained in synchronized rather than random orbits, it would be possible to keep three continuously within view of any point on the earth's surface. Position fixes could then be provided continuously, eliminating the need for a complementary dead-reckoning system. Relative positions of at least three satellites favorable to high-accuracy position fixes could also be maintained by this means. Because of these substantial improvements in system performance, there is a strong incentive for adopting synchronization of satellite orbits; a detailed study is required to determine the feasibility and economy. But, as indicated previously, our analysis is limited to satellites using random rather than synchronized orbits.

6.4.3. SIZE, WEIGHT, AND COST. At this time it is difficult to provide firm information concerning size, weight, and cost of range-measuring systems, since these systems have not yet passed beyond the preliminary concept stage. The data of Table XVII were supplied by Fairchild Stratos Corp. as very rough preliminary estimates for production versions of the airborne equipment.

TABLE XVII. ESTIMATES OF RANGE-MEASURING SYSTEM

	Cu Ft	Weight (lb)	Watts	Cost
Receiver	0.5	20	25	\$3000
Computer	1.0	30	50	3000
Display	<u>0.05</u>	<u>2</u>	<u>0.5</u>	<u>250</u>
	1.55	52	75.5	\$6250

The estimates exclude the altimeter and aircraft controls of the airplane and are limited to those items directly used for aircraft navigation. The size, weight, and cost figures represent the characteristics of a system using advanced methods of miniaturization; therefore, the figures refer to equipment which would probably be available about the year 1970.

Because of the preliminary nature of these estimates, it is not clear that they allow adequately for unforeseen problems which are almost certain to appear in the detailed design and operation of the system. In addition to the primary task of solving a set of hyperbolic equations, provision must be made for such features as identification of satellites; insertion, storage, and retrieval of ephemerical data and aircraft altitude; abnormal flight paths and conditions; and special corrections. Equipment which provided for these and other unforeseen requirements would very likely exceed the figures quoted. Instead, a system costing from \$10,000 to \$12,000 might be called for. Corresponding increases in equipment size and weight (of 2 to 3 cu ft and 75 to 100 pounds) should also be realistically expected.

Equipment for a range-measuring system is in many respects similar to that used in TRANSIT. It contains many of the same components (e.g., receiver, clock, memory, and computer), and computing position for two range differences from three satellites is probably similar in complexity, if not in the detailed statement of equations, to that of TRANSIT. On the other hand, the equipment of the range-measuring system is somewhat simpler than that of the TRANSIT system. To make corrections for ionospheric refraction, the TRANSIT system requires the use of two receivers. It also requires additional computation of correction data for ionospheric refraction and for aircraft motion.

From these considerations, one would expect that the size, weight, and cost of the equipment for the range-measuring system would be appreciably less than that for the TRANSIT system. This is borne out by the difference in costs quoted for production versions of the two systems, although the amount of the difference between the two quoted costs seems excessive. Comparing quoted size and weight is inapplicable because of the different assumptions on which the estimates were based.

6.4.4. PROBLEMS OF INTRODUCTION AND USE. Certain features of the range-measuring system require techniques which are near or somewhat beyond the limit of the present state of the art, particularly in connection with the accurate clocks and 24-hour satellites.

The method depends on the use of very accurate time references which are just now becoming available commercially. For example, the National Company, Inc., of Malden, Mass., makes the automichron, based on the use of a cesium beam. This has a long-time accuracy of 1 part in 10^{11} . It can be placed in a cylinder 4 inches in diameter by 2 feet long, which is small enough to put into a satellite. But the performance of such devices under the environmental conditions associated with launch and space flight remains to be demonstrated.

Another problem is the cost and difficulty of getting a satellite into a 24-hour orbit. The satellite for the range-measuring system is estimated to weigh about 200 pounds. The relatively great altitude of the orbit (20,000 nautical miles) requires a large booster. There are also difficulties in establishing a satellite in an equatorial orbit from a launching point which is far from the equator.

If synchronized satellite orbits were adopted, methods of station-keeping would have to be incorporated in the satellite so that it would maintain a fixed position with respect to the rotating earth. Such methods would require applying thrust to the satellite in accordance with tracking data on its varying position. If such thrust were provided by chemical propulsion units, the life of the satellite would be limited by the amount of fuel which could be carried. On the other hand, if methods of ionic or electric propulsion were adopted, the total amount of mass required for reaction purposes would be small enough to permit a long life.

It should be noted that problems of launching and controlling the orbits of 24-hour satellites are actively being worked on in connection with other space programs for meteorological and communications satellites. Although some time may be required before 24-hour satellites can be used successfully, it is anticipated that such problems will eventually be solved.

Additional investigation of range-measuring methods would preferably begin with a feasibility demonstration. The simplest and most economical method of demonstrating feasibility experimentally would be to make use of 24-hour satellites launched in connection with meteorological or communications programs. If it were not necessary to test the feasibility of satellite-borne high-accuracy clocks, the only modification necessary for using satellites from other programs would be to specially modulate their radio signals. Although the complete system would require the simultaneous observation of three satellites, it would be possible to conduct experiments with only one satellite available. For this situation, range measurements rather than range-difference measurements would be made, and position would be determined

in terms of a line of position rather than a single point. From this measurement an error in a single coordinate of the position fix could be established at a given point on the earth's surface. From a number of measurements at different distances from the sub-satellite point, sufficient information could be obtained to establish the accuracy of the system. A program of this type, making use of satellites launched for other purposes, is estimated to cost between \$250,000 and \$350,000.

If feasibility could be demonstrated in this manner, a more elaborate program could follow to provide a full-scale system test. Estimates given in Section 6.3.4 of \$5,000,000 to \$7,000,000 for an experimental program designed to demonstrate the feasibility of an angle-measuring system can reasonably be assumed to apply to the range-measuring system, too, although the distribution of costs in the development of the satellite's and user's equipment might be considerably different.

6.5. CONCLUSIONS AND RECOMMENDATIONS

6.5.1. SYSTEM CHARACTERISTICS

6.5.1.1. TRANSIT System. A navigation system based on the observation of the TRANSIT satellite whose characteristics might make it suitable for aircraft installation is currently under development. It appears that such a system would be capable of providing aircraft position fixes with an error whose d_{rms} value is the rms combination of 1.58 nautical miles and 0.24 times the velocity uncertainty in knots. Based primarily on information obtained from the Applied Physics Laboratory of Johns Hopkins University, the estimate of the cost of equipment installed in the aircraft ranges between \$20,000 and \$30,000. If miniaturized components are used, the size and weight of production versions of the equipment are expected to range from 3 to 5 cu ft, and 100 to 125 pounds.

The operational system presently planned will place four TRANSIT satellites in orbit. At the equator, this number would provide fix renewal intervals which would average about 120 minutes and range from 20 to 220 minutes. At 45° latitude the intervals would average about 60 minutes and range from 20 to 110 minutes.

Performance characteristics of the TRANSIT system in combination with an inertial system and a doppler system are shown in Figures 47 and 48, respectively. For flights at 45° latitude the TRANSIT system, in combination with an inertial system having an accuracy of 3 nautical miles/hour d_{rms} , could keep the navigation accuracy to an average d_{rms} value of 3.5 nautical miles with occasional excursions to 6 nautical miles. For the fix-renewal intervals characteristic of the TRANSIT system a position fixing accuracy better than the 1.58

nautical miles available from the TRANSIT system would provide no significant improvement in performance.

If eight satellites were used instead of four, the average fix-renewal intervals would be approximately cut in half. The TRANSIT system in combination with a 3-nautical-mile/hour inertial system would then keep the d_{rms} navigation accuracy to an average value of 3 nautical miles, with occasional excursions to 4.5 nautical miles.

Because of the low operating altitude of the satellites, an extensive ground-based network of tracking, computing, and data distribution equipment is required. Operating costs would be largely covered if the Navy were to continue maintaining the system. Otherwise, the operating cost would add appreciably to the expense of this navigation method, even if distributed over the many aircraft which might use it. To maintain eight satellites in orbit would cost from \$4000 to \$8000 annually per aircraft.

Thus, the TRANSIT system, combined with a relatively high-performance inertial navigation system, is capable of providing aircraft navigation of moderate accuracy, but is compatible with a doppler system of 1% or 1.5% accuracy only for subsonic aircraft. The primary disadvantages of the system are its relatively long and uncertain fix renewal interval, and the extensive ground-based network required for it. Major design problems which remain to be solved at the time of this writing are the loss of signal continuity during a satellite pass, the problems of orbit prediction, and the need for a simple but accurate method of position computation.

6.5.1.2. Angle-measuring Systems. Navigation systems based on the use of angle measurement of satellites have been proposed. The equipment would be similar in nature to the radio sextant AN/SRN-4 developed by the Collins Radio Co. for the U. S. Navy, but would have to be reduced in size and weight. A navigation system of this type might be capable of providing position-fix measurements to an accuracy of 2 nautical miles d_{rms} , but this would require great care in the design, operation, and maintenance of the system, particularly with respect to the vertical reference and radome equipment. Although the accuracy of the angle-measurement system is not significantly different from that of the TRANSIT system, it would provide much better coverage with a given number of satellites. If eight satellites were provided and a one-body method of position-fixing were adopted, the coverage would be largely continuous. If four satellites were provided and the position-fixing capability were combined with the dead-reckoning capability of a 3-nautical-mile/hour d_{rms} system, the accuracy would remain in the neighborhood of 3 nautical miles/hour d_{rms} except for one or two occasions for 24 hours of flight time when coverage gaps in excess of one hour would be experienced.

The satellite and ground-based equipment required for an angle-measuring system is relatively simple, but the aircraft equipment is complex and bulky. The complete cost has been roughly estimated at \$20,000 to \$30,000 (exclusive of the vertical reference system). All equipment, including the vertical reference system, is estimated to weigh 300 to 375 pounds, and to occupy 15 to 20 cu ft. Additional costs would be incurred if substantial modification of the aircraft were required to accommodate the equipment installation.

In addition to the position-fixing capability, an angle-measuring system of the type described here would also provide an inertial navigation capability and would permit very accurate heading and altitude measurements. These added capabilities should be taken into account in comparing the system with other types of satellite navigation systems. On the other hand, certain malfunctions of the system would have the effect of simultaneously eliminating several capabilities from the overall navigation system, unless portions of the aircraft equipment were carried in duplicate.

Although some of the equipment concepts of the angle-measuring system are already under development, its feasibility remains to be established. Questions remain to be resolved concerning the potential accuracy of a system whose size, weight, and cost must be tailored to the limitations of aircraft installation.

6.5.1.3. Range-Measuring Systems. Navigation systems based on the use of range-measurement methods have been proposed but are not yet in development. One method which has been suggested would fix the position of the aircraft by observing two sets of instantaneous range differences between pairs of satellites, these range differences being measured as time intervals between the arrival of a pulse train from each of the satellites.

This system should be capable of providing position fixes with a d_{rms} error of about one nautical mile. Instantaneous rather than running fixes would be available. It would require a total of 12 satellites (assuming random phasing) operating in 24-hour orbits to keep the coverage gap down to 15 to 20% of the total time. The aircraft equipment should be somewhat simpler than that used with the TRANSIT system; it is estimated to cost \$10,000 to \$12,000, to weigh 75 to 100 pounds, and to occupy 2 to 3 cu ft.

This type of system should be at least as accurate as TRANSIT and would be much more rapid and continuous in fixing position. On the other hand, it requires a total of 12 satellites operating at the high altitudes required for 24-hour orbits. Since its development has not been initiated, its technical feasibility has not yet been demonstrated, but no exceptionally difficult problems are anticipated. A development program would take several years to reach the present status of the TRANSIT system.

6.5.2. APPLICATION TO AIRCRAFT NAVIGATION. The current stage of development of the satellite concept of navigation makes it unlikely that such methods could seriously be considered for installation in new aircraft which will be flying during the early years of the 1965 to 1975 time frame with which this study is concerned. For example, the present schedule for the development and construction of the supersonic transport calls for its introduction to begin around 1970. In order to be accepted for use as a part of its navigation system, equipment must be completely developed by about 1964 or 1965. This is clearly out of the question for angle- or range-measuring systems. The development of the TRANSIT system is much farther along. It is conceivable but unlikely that it could be ready by such a date; its accuracy remains to be completely evaluated, and there are still a number of system design problems to be resolved.

If we are considering the latter portion of the 1965 to 1975 period, the major emphasis in new aircraft construction is likely to be placed on supersonic types. Since new, rather than existing, aircraft offer the best possibility for application of radically new navigation methods, it is instructive to consider the satellite methods of navigation in terms of their application to supersonic aircraft.

One conclusion reached by most operators of surface vessels is that any newly developed navigation system must be accepted by merchant marines of all nations and that the user's equipment must be established as standard requirements within the international "Rule of the Road" agreement [76]. This involves general agreement on policy matters, operating procedures, frequency band assignments, etc. We may assume that a similar policy would apply to the use of satellites for air navigation purposes.

Reference 77 notes that the introduction of the supersonic transport would be the result of a joint government-industry team effort on one type of vehicle for each country involved. The navigation equipment of the transport would therefore be selected in accordance with the recommendations of the governmental regulatory agency concerned with the aircraft's development. This would be favorable to the introduction of a satellite system, since a coordinated effort could be made to fit the satellite and ground-based system to a specific design for the aircraft equipment. But it would still be necessary to obtain agreement among countries on a specific system.

Reference 75 notes that a new system must be able to show very substantial economic and technical advantages (when compared to other existing and proposed long-distance navigation aids) before it will be favorably received and seriously considered for adoption. The general requirements for a navigation system suitable for supersonic transports are discussed in Reference 77. The system, consisting of both equipment and operator, must, of

course, be capable of providing navigation information accurate enough to meet the specifications laid down for both safety and economy. The equipment must not only have the inherent capability for such accuracy, but must operate reliably: by using reliable equipment and techniques, providing redundant information through several types of navigation equipment which complement each other, and permitting the operator to judge the usefulness of the available information. At least some of the equipment should be very simple and trouble-free, even though it has limited accuracy. This simple equipment may be supplemented with more sophisticated equipment with limited reliability but much higher accuracy. The information presented directly by the more accurate equipment should be optionally available in terms of heading and air speed, as well as in combined form. This would permit the navigator to monitor the operation of the equipment and check the validity against output data of the simpler equipment, usually provided as heading and air speed. The navigation system must also be capable of considerable automation, in order to minimize the work load placed on the crew members and lessen possible mistakes (blunders) and delays.

6.5.2.1. Position-Fixing Performance. Satellite navigation systems can provide position fixes whose d_{rms} errors are generally in the range of 1 to 3 nautical miles. This performance is sufficient for the navigation of all types of over-ocean aircraft.

The rapidity and continuity of position-fix information vary considerably among the three types of system discussed. The TRANSIT system with eight satellites would provide marginal fix-continuity performance, but would nevertheless be suitable for use with a relatively high-performance inertial system. The angle-measuring and range-measuring methods provide much better continuity, which could be considered quite satisfactory for supersonic aircraft navigation.

In operating supersonic aircraft, great importance is likely to be attached to maintaining preassigned flight schedules and routes. This should be facilitated by the short flight time and the high altitude which keeps the aircraft above most of the bad weather. The regularity of the schedule could simplify the computation procedure and equipment by permitting pre-computation or minimizing the amount of data storage. Provision would still be required for unusual flight conditions, but somewhat reduced convenience, accuracy, or speed of position fix might be acceptable if these conditions did not occur frequently.

6.5.2.2. Reliability. Reference 77 states that the more complex and accurate types of aircraft navigation equipment in use today, though potentially capable of much higher precision than a human navigator, have only about 60% reliability in the most precise modes of operation.

For an additional 20% of the total time information can be provided with about the same accuracy as by the human navigator, and for the remaining time it is much less accurate.

The reliability performance of the aircraft-equipment portion of the TRANSIT system would appear to be about the same as that of the existing equipment described in Section 6.2.2.4. In the absence of more detailed information, range-measuring systems are estimated to have about the same reliability as the TRANSIT system. Angle-measurement systems are rather complex and would, if anything, be somewhat less reliable than the TRANSIT system. Also, the vulnerability of their position-fixing, inertial-navigation, and heading-reference capabilities to a single malfunction has been noted previously.

6.5.2.3. Automaticity. Each of the navigation methods investigated in this report incorporates automatic equipment which computes position. However, it is unlikely that they could operate without some attention on the part of the crew. This might consist of calibrating and adjusting the equipment, inserting data, checking the validity of the solution, and handling special conditions. Good judgment would thus be required in using the equipment. The methods do have the advantage that they will be able to provide output data in a form which can be directly compared with position fixes provided by other methods; hence, their performance can be directly monitored and major mistakes avoided.

6.5.2.4. Size and Weight. The estimated size and weight of the aircraft's equipment may be compared with the total size and weight of the present navigation system of the B-58, whose design may be considered representative of a supersonic commercial aircraft. The present B-58 navigation system weighs approximately 600 to 700 pounds and occupies approximately 80 cu ft [77]. If the present analog system is converted to a digital system, it should be possible to reduce its size and weight still further. For a commercial application, the total size and weight might require a total allowance of 200 to 300 pounds and a size of 50 cu ft.

The range-measuring system with an estimated weight of 75 to 100 pounds and size of 2 to 3 cu ft for the miniaturized version of the equipment would constitute a relatively large part of the weight allowance mentioned above. An angle-measuring system estimated at 300 to 375 pounds (including the vertical reference system) is very heavy; this is a serious disadvantage.

6.5.2.5. Costs. In evaluating satellite navigation methods the total costs to be considered include the research and development and the installation and operation of the ground-based, satellite, and airborne equipment.

Estimated costs of the aircraft's equipment generally appear to be moderate for the TRANSIT and range-measuring systems. They are much higher for the complete angle-measuring system, but this may be justified by the additional functions performed.

The significance of the costs of constructing and launching navigation satellites, tracking them, and computing and distributing ephemeridal data can best be judged in terms of the cost distributed over the total number of aircraft which would make use of satellite methods of navigation. The fleet of major U. S. carriers currently assigned to long-distance over-ocean routes is estimated to consist of 116 turbojet aircraft and 194 turboprop and propeller-driven aircraft. If the fleets of other countries are also considered, these figures would be increased by 50% to 100%. A greater proportion of the foreign fleet consists of turboprop and propeller-driven aircraft. It has been estimated that by 1975 the total number of flights across the North Atlantic will increase by a factor of 3 or 4. Most over-ocean flights will use supersonic aircraft. For rough estimates we may assume that a successfully operating navigational satellite system might be used by 500 to 1000 over-ocean aircraft. Thus, in a system using eight satellites, with the initial and operating costs for each satellite amounting to \$500,000 per year, the cost of the system per aircraft (exclusive of the cost of the user's equipment) would range from \$4000 to \$8000 per year. These figures could be substantially reduced if the use of this navigation system were shared with aircraft and ship operators for both military and civilian applications.

6.5.2.6. Other Characteristics. All the methods of satellite navigation described here are capable of all-weather operation, since they operate at radio frequencies which are able to penetrate clouds and rain. But for supersonic aircraft this advantage may be of limited importance. Mach 3 aircraft are expected to cruise at altitudes normally between 60,000 and 80,000 feet. According to Reference 78, cloud cover at these altitudes is relatively infrequent. For example, above 35,000 feet on the North Atlantic route and 50,000 feet at the lower latitudes there is 95% probability of having less than 1/10 sky cover. As far as weather conditions are concerned, manual or automatic methods of optical celestial navigation are clearly capable of being used almost without interruption. Satellite methods therefore have no special advantage over such methods, with respect to all-weather operation.

The capability of satellite navigation systems to cover the world can be a very distinct advantage over other methods of position-fixing with ground-based installations covering only a limited area of the earth's surface. Once in operation, the satellite system would cover the entire surface of the earth. Other methods would require many installations, some of them in distant and inaccessible locations.

The satellite methods discussed here are passive. Since the user's equipment would not interrogate the satellite, it could handle any amount of traffic volume without saturation.

6.5.3. SUMMARY. A decision to adopt satellite methods must be made on the basis of an overall view of the entire navigation problem; satellite methods must be compared with alternative methods of position-fixing to arrive at the most suitable means of over-ocean navigation.

Aircraft navigation by satellites appears to be economically and technically feasible. Satellites would be capable of handling any volume of traffic and would provide world-wide all-weather coverage. Used with random, rather than synchronized, orbits, they would provide a position-fixing capability which is not continuous but must be combined with dead-reckoning methods. In this manner it would be possible to navigate the aircraft with a d_{rms} error from 2 to 6 nautical miles. For certain methods the user's equipment would not be excessively large or expensive. Overall system reliability may be open to some question, pending an actual system demonstration. All systems would require many installations of ground-based and satellite equipment.

The TRANSIT system is now under development, and one using four satellites is scheduled to be operational in 1963. More detailed information concerning its cost and performance will become available as the program proceeds. For each of the other methods, a program of research and development would be needed to bring the system to its full capabilities. Detailed analytical studies of particular systems can be made at limited expense. An experimental demonstration using satellites launched for other purposes would cost from \$250,000 to \$350,000. A complete experimental demonstration of feasibility would cost from \$5,000,000 to \$7,000,000.

The characteristics of the three navigation methods discussed in this report are presented in Table XVIII.

We may also state the conclusions of this report in terms of the specifications contained in the contract statement of work. Satellite methods of navigation should be available for introduction and use only during the latter part of the 1965-1975 time period; TRANSIT would be available earliest. With respect to fix-renewal interval, the TRANSIT system, even if expanded to eight satellites, would not meet the requirement for a fix renewal every 500 nautical miles for aircraft at speeds higher than Mach 1. The other methods considered would in general meet this requirement except for occasional coverage gaps. All systems would be capable of providing position fixes with d_{rms} values of 3 nautical miles or less.

TABLE XVIII. COMPARISON OF NAVIGATION SATELLITE SYSTEMS

NOTE: Coverage data are based on random, rather than synchronized, orbits. Spare equipment is not included in cost estimates.

Measure Method	Doppler	Angle	Range-Difference
Status	Under development	Proposal (user's equipment partly developed)	Proposal
Number of Satellites	8	8	12
Satellite Altitude (n mi)	600	4000 to 6000	20,000
Traffic-handling Capability	Unlimited	Unlimited	Unlimited
Weather Capability	All-weather	All-weather	All-weather
Geographical Coverage	World-wide; best at high latitudes	World-wide; best at high latitudes	World-wide; equal coverage at all latitudes
Performance at 45° Latitude			
Accuracy of Fix (d_{rms} in n mi)	1.5 to 2.5	2	1
Continuous Coverage (%)	0	96	80 to 85
Gaps per 24 Hours	25 to 30	2	No data
Average Length of Gap (min)	45 to 50	35 to 40	No data
Accuracy with 3 n mi/hr Inertial System (d_{rms})	3.0	2.5	2.0
Satellite Characteristics	Moderately complex	Simple	Moderately complex
Ground-based System	Extensive	Moderate	Moderate
User's Equipment (Airborne)			
Size (cu ft)	3 to 5	15 to 20	2 to 3
Weight (lb)	100 to 125	300 to 375	75 to 100
Cost (\$)	20,000 to 30,000	20,000 to 30,000 (exclusive of vertical reference system)	10,000 to 12,000
Complexity	Moderate	High	Moderate

In addition to its uses for aircraft navigation, satellite navigation systems would possibly have two other applications. One would be to provide geodetic positioning, which would allow very accurate determination of the relative location of points on the earth's surface. Another would be the synchronization of clocks throughout the world. It is also possible that satellites could combine navigation with other functions, such as communication; this would reduce construction and launching costs.

The results and conclusions of the study of navigational satellites presented in this report are necessarily based on incomplete information; hence, it is desirable to continuously review new information. Since the development and evaluation of navigational satellites are being actively pursued, more detailed and conclusive data will be forthcoming in the near future. But unless major advances are reported, it is unlikely that the results and conclusions reached in this report will be greatly changed.

If future research and development of satellite navigation methods is to be undertaken, we recommend that these be initially confined to small-scale efforts primarily directed toward investigating the critical factors likely to affect the success of proposed systems. This type of investigation can be performed at a cost relatively small compared to that of a full-scale test program.

Appendix A

RADIATION OF VERY-LOW-FREQUENCY ENERGY FROM AIRCRAFT ANTENNAS

The problem of radiating significant energy in the VLF spectrum can be demonstrated by considering a simple idealized antenna. This will be a vertical antenna top loaded by a single horizontal wire with the following dimensions:

$$f = 40 \text{ kc}$$

$$\text{height of vertical} = 3 \text{ meters}$$

$$\text{length of horizontal} = 30.5 \text{ meters}$$

$$\text{diameter of horizontal} = 0.052 \text{ inches}$$

We will assume that the top loading is ideal, so that the top current is equal to the base current; and that the antenna is above a perfectly conducting ground plane.

With these assumptions the effective height (h_e) of the antenna is equal to its physical height and the radiation resistance R_{ar} can be computed [81]

$$R_{ar} = 160 \pi^2 \left(\frac{h_e}{\lambda} \right)^2 \quad (128)$$

From the above data

$$R_{ar} = 25.3 \times 10^{-5} \text{ ohm} \quad (129)$$

The reactance of a very small antenna is capacitive, and in this case, can be assumed due to the capacitance between the horizontal wire and ground. The capacitance of such a wire is given by Reference 82

$$C = \frac{7.354 l}{\log_{10} (4h/d) - S} \quad (130)$$

If

$$h = 9.85 \text{ feet}$$

$$d = 0.052 \text{ feet}$$

$$l = 100 \text{ feet}$$

$$S = 0.082$$

then

$$C = 190 \text{ pf} \quad (131)$$

Then, at 40 kc

$$X_c = 21,000 \text{ ohm} \quad (132)$$

The aircraft antenna can be represented then as a resistance of 25.3×10^{-5} ohms in series with a capacitive reactance of 21,000 ohms. The maximum power that can be radiated is limited by both the efficiency and breakdown voltage across the antenna terminals. The efficiency is difficult to predict since it depends upon the particular aircraft installation, but the voltage limitation itself will provide an upper limit on power.

If the breakdown voltage is 20,000 volts maximum peak [83], the maximum current is

$$I_{\max} = \frac{V_{\max \text{ peak}}}{\sqrt{2} Z} \quad (133)$$

$$I_{\max} = \frac{20,000}{\sqrt{2} \times 21,000} = 0.674 \text{ amp} \quad (134)$$

The maximum radiated power is then

$$P_{\max} = I_{\max}^2 R_{ar} \quad (135)$$

$$P_{\max} = 115 \times 10^{-6} \text{ watts} \quad (136)$$

Although the antenna has been idealized, the idealizations are optimistic and illustrate the problems of very high reactance and extremely low radiation resistances typical of aircraft VLF antennas.

Appendix 3
DERIVATION OF THE d_{rms} ERROR OF HYPERBOLIC-HYPERBOLIC,
HYPERBOLIC-ELLIPTIC, AND RHO-RHO NAVIGATION SYSTEMS

B.1. HYPERBOLIC-HYPERBOLIC SYSTEMS

A position fix is obtained from the intersection of two lines of position. In the vicinity of the fix the lines of position may be assumed to be straight lines intersecting at an angle θ . If each line of position is in error by a small distance ϵ , then the indicated position will be in error by a distance d as shown in Figure 55. R is the true or actual position, and I is the indicated position. The displacement or error in the individual lines of position are ϵ_1 and ϵ_2 , and the sides of the parallelogram are then

$$p = \frac{\epsilon_1}{\sin \theta} \tag{137}$$

$$q = \frac{\epsilon_2}{\sin \theta} \tag{138}$$

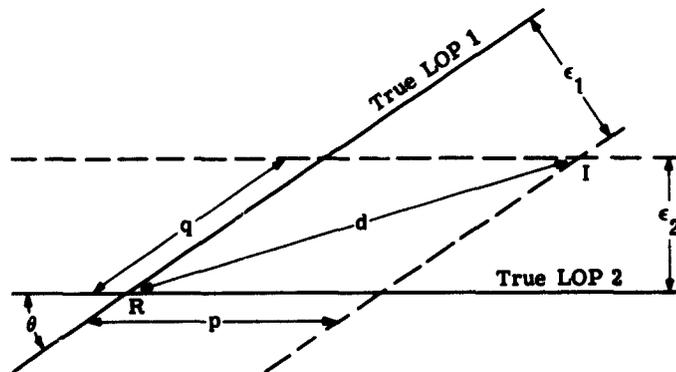


FIGURE 55. HYPERBOLIC-SYSTEM ERROR PARALLELOGRAM

From the law of cosines

$$d^2 = p^2 + q^2 + 2pq \cos \theta \tag{139}$$

$$d^2 = \frac{1}{\sin^2 \theta} (\epsilon_1^2 + \epsilon_2^2 + 2\epsilon_1\epsilon_2 \cos \theta) \tag{140}$$

The error of each line of position ϵ can be related to an equivalent error in the distance difference or time-difference measurement. If we assume a receiver at location R and a transmitter at M and S_1 , as shown in Figure 56, the line of position at R defined by a time-difference

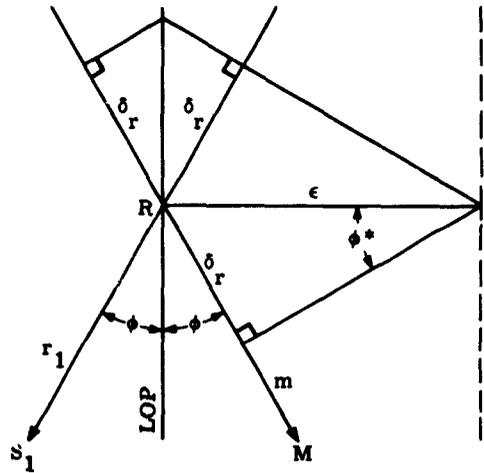


FIGURE 56. LOP ERROR DIAGRAM

measurement on the signals from stations M and S₁ will be that line through R which is the locus of points with an equal propagation-time difference from M and S₁. On a plane or at short ranges on the earth such a locus is a hyperbola, but on a spherical earth it is not a simple curve.

If the distance from R to S₁ is r₁, and the distance from R to M is m, then the distance difference at R is

$$\Delta R = r_1 - m \tag{141}$$

If we change the distance from R to each station by an amount δr , which is very small compared to r₁ and m, then the distance difference at I is

$$\Delta I = (r_1 + \delta r) - (m - \delta r) = r_1 - m + 2\delta r \tag{142}$$

The time differences corresponding to ΔR and ΔI are

$$\Delta T_R = \Delta R / V_p \tag{143}$$

and

$$\Delta T_I = \Delta I / V_p \tag{144}$$

where V_p is the phase velocity which can be assumed equal to c, the universal constant for the velocity of light.

From Figure 56, by plane geometry, since $\phi^* = \phi$

$$\epsilon = \frac{\delta r}{\sin \phi} \tag{145}$$

and if we let δt be the change in time difference corresponding to the displacement ϵ , then

$$\delta t = \Delta T_I - \Delta T_R = \frac{2\delta r}{c} \quad (146)$$

or

$$\delta r = \frac{c\delta t}{2} \quad (147)$$

and

$$\epsilon = \frac{c\delta t}{2 \sin \phi} \quad (148)$$

Equation 140 can now be written as

$$d^2 = \frac{c^2}{4 \sin^2 \theta} \left[\frac{(\delta t_1)^2}{\sin^2 \phi_1} + \frac{(\delta t_2)^2}{\sin^2 \phi_2} + \frac{2 \delta t_1 \delta t_2 \cos \theta}{\sin \phi_1 \sin \phi_2} \right] \quad (149)$$

At each receiver location R we assume that many pairs of time-difference readings (ΔT_{S_1} and ΔT_{S_2}) can be made to establish the receiver position. Each determination will be accompanied by some radial-position error d_1 obtainable from Equation 149. The d_{rms} radial error of the system at each location will be defined as the square root of the following quantity:

$$d_{rms}^2 = \frac{1}{n} \sum_{i=1}^n d_i^2 \quad (150)$$

This measure of accuracy is the same d_{rms} error defined and used by Crichtlow [25] and Jessel and Trow [23].

Combining Equations 149 and 150 we find

$$d_{rms}^2 = \frac{c^2}{4 \sin^2 \theta} \left[\frac{\Sigma(\delta t_1)^2}{n \sin^2 \phi_1} + \frac{\Sigma(\delta t_2)^2}{n \sin^2 \phi_2} + \frac{2 \Sigma(\delta t_1 \delta t_2) \cos \theta}{n \sin \phi_1 \sin \phi_2} \right] \quad (151)$$

If the population of time-difference measurements ΔT_I is a normally distributed random variable with mean ΔT_R , then the variance of ΔT_I is given by

$$V(\Delta T_I) = \frac{1}{n} \sum_{i=1}^n (\Delta T_{I_i} - \Delta T_R)^2 \quad (152)$$

but

$$\Delta T_{I_1} - \Delta T_R = \delta t_1 \quad (153)$$

or

$$\sigma^2(\Delta T_I) = \frac{1}{n} \sum_{i=1}^n (\delta t_i)^2 \quad (154)$$

If we now refer to each time-difference measurement as ΔT_1 and ΔT_2 for measurements involving slave stations S_1 and S_2 , then

$$\frac{\Sigma(\delta t_1 \delta t_2)}{n \sigma_{\Delta T_1} \sigma_{\Delta T_2}} = \rho \quad (155)$$

where ρ is the correlation coefficient for the time-difference measurements on the two slave stations S_1 and S_2 .

Finally

$$d_{\text{rms}} = \frac{c}{2 \sin \theta} \left[\frac{\sigma_{\Delta T_1}^2}{\sin^2 \phi_1} + \frac{\sigma_{\Delta T_2}^2}{\sin^2 \phi_2} + \frac{2\rho \sigma_{\Delta T_1} \sigma_{\Delta T_2} \cos \theta}{\sin \phi_1 \sin \phi_2} \right]^{1/2} \quad (156)$$

B.2. RHO-RHO SYSTEMS

The derivation for the d_{rms} error of a rho-rho system follows closely that of the last section. In Figure 57, range measurements are made on stations S_1 and S_2 . If the receiver is at R, the ranges to the two stations are r_1 and r_2 . The lines of position are actually circles but may be approximated as straight lines near the receiver R.

Following the method of Section B.1, we find

$$p = \frac{\epsilon_1}{\sin \theta} \quad (157)$$

$$q = \frac{\epsilon_2}{\sin \theta} \quad (158)$$

If the range measurements are made by measuring the time of propagation of a signal from each station to the receiver R,

$$TR_1 = \frac{r_1}{c} \quad (159)$$

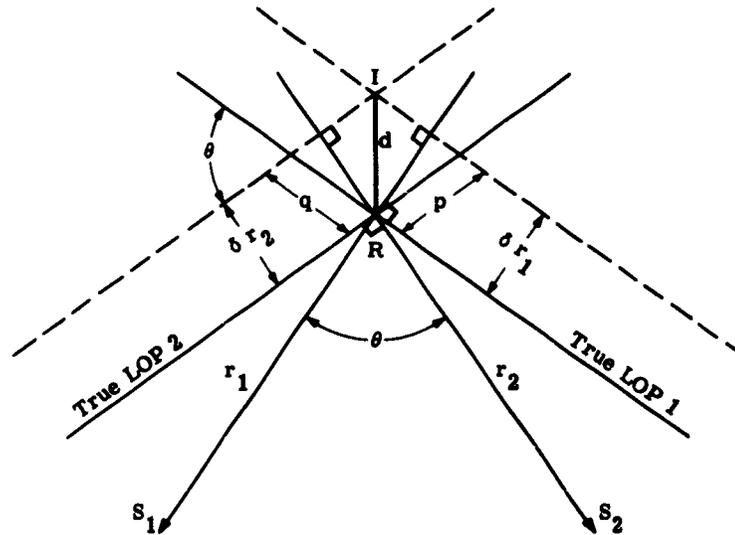


FIGURE 57. RHO-RHO SYSTEM ERROR DIAGRAM

$$TR_2 = \frac{r_2}{c} \tag{160}$$

If the range or time measurement is in error, we will have

$$TI_1 = \frac{r_1 + \delta r_1}{c} \tag{161}$$

$$TI_2 = \frac{r_2 + \delta r_2}{c} \tag{162}$$

or

$$TI_1 - TR_1 = \delta t_1 = \frac{\delta r_1}{c} \tag{163}$$

$$TI_2 - TR_2 = \delta t_2 = \frac{\delta r_2}{c} \tag{164}$$

If both errors are positive (an increase in range) the erroneous position given by TI_1 and TI_2 will be at I with a position error of d. The distance d in this case represents the short diagonal of the error parallelogram and is given by

$$d^2 = p^2 + q^2 - 2pq \cos \theta \tag{165}$$

However, in this case the lateral displacement of the LOP's, ϵ , is simply δr or

$$\begin{aligned}\epsilon_1 &= \delta r_1 \\ \epsilon_2 &= \delta r_2\end{aligned}\tag{166}$$

and

$$p = \frac{c \delta t_1}{\sin \theta}\tag{167}$$

$$q = \frac{c \delta t_2}{\sin \theta}\tag{168}$$

Using the same definition of d_{rms} as in Section B.1 and, as before, letting

$$T_1 = TI_1\tag{169}$$

$$T_2 = TI_2\tag{170}$$

we find

$$d_{\text{rms}} = \frac{c}{\sin \theta} \left(\sigma_{T_1}^2 + \sigma_{T_2}^2 - 2\rho \sigma_{T_1} \sigma_{T_2} \cos \theta \right)^{1/2}\tag{171}$$

where ρ is the correlation coefficient between T_1 and T_2 . The angle θ is the angle subtended by the two stations at the receiver, and, if we follow the notation of the last section and let this angle be 2ϕ , then

$$\theta = 2\phi\tag{172}$$

and

$$d_{\text{rms}} = \frac{c}{\sin 2\phi} \left(\sigma_{T_1}^2 + \sigma_{T_2}^2 - 2\rho \sigma_{T_1} \sigma_{T_2} \cos 2\phi \right)^{1/2}\tag{173}$$

B.3. HYPERBOLIC-ELLIPTIC SYSTEMS

At times in the past, consideration has been given to the possibility of using a hyperbolic-elliptic system. Such a system would measure the time-of-arrival difference of signals from two stations as well as the sum of the times of propagation from each station. This system, like rho-rho, would require only two stations, but might benefit from the fact that the hyperbolic and elliptical lines of position always cross at right angles.

The d_{rms} error will be derived for three implementations of such a system. Figure 58 shows the three cases.

(a) The difference between r_1 and r_2 is measured directly as a time difference in a hyperbolic system, and the sum is derived from the difference measurement and one range measurement.

(b) The difference between r_1 and r_2 is measured directly, but the sum is the sum of two range measurements ($r_1 + r_2$).

(c) Both the difference and the sum are derived from a range measurement of r_1 and r_2 separately.

We will show that the third implementation gives a d_{rms} error identical to that of a rho-rho system.

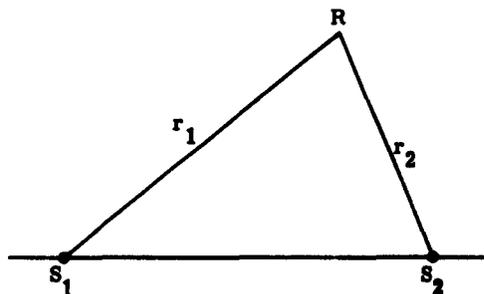


FIGURE 58. HYPERBOLIC-ELLIPTIC SYSTEM GEOMETRY

B.3.1. DIRECT DIFFERENCE MEASUREMENT AND ONE-RANGE MEASUREMENT. The method used here will again follow closely the method of Section B.1. Since hyperbolas and ellipses generated from the same two stations or foci are orthogonal trajectories, the error parallelogram will be as shown in Figure 59 where the angle of cut between lines of position (θ) is always 90° .

The position error can be expressed simply as

$$d^2 = p^2 + q^2 \tag{174}$$

or

$$d^2 = \epsilon_h^2 + \epsilon_e^2 \tag{175}$$

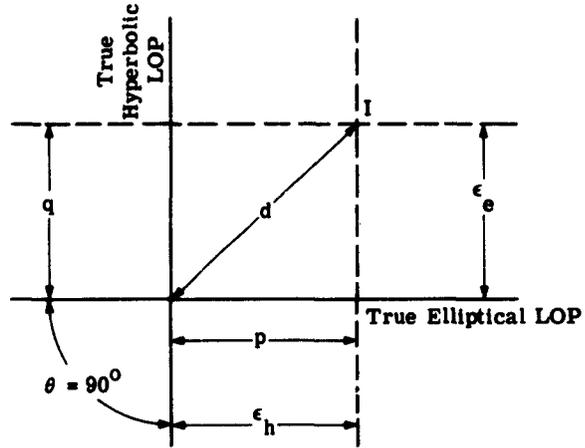


FIGURE 59. HYPERBOLIC-ELLIPTIC SYSTEM ERROR PARALLELOGRAM

where ϵ_h and ϵ_e are the displacement error in the hyperbolic and elliptic lines of position. Again, from Equation 144,

$$\epsilon_h = \frac{\delta r_h}{\sin \phi} \tag{176}$$

From Figure 60

$$\epsilon_e = \frac{\delta r_e}{\cos \phi} \tag{177}$$

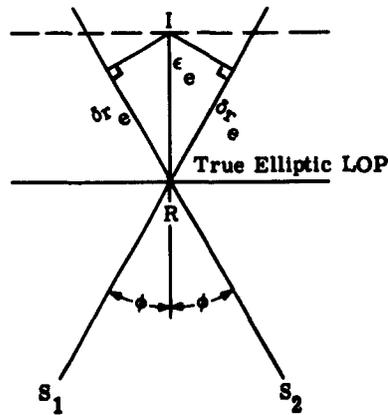


FIGURE 60. HYPERBOLIC-ELLIPTIC SYSTEM ERROR DIAGRAM

and from a development similar to Equations 140 through 147,

$$\epsilon_h = \frac{c \delta t_h}{2 \sin \phi} \quad (178)$$

$$\epsilon_e = \frac{c \delta t_e}{2 \cos \phi} \quad (179)$$

and

$$d^2 = \frac{c^2}{4} \left[\frac{\delta t_h^2}{\sin^2 \phi} + \frac{\delta t_e^2}{\cos^2 \phi} \right] \quad (180)$$

The sum of the distances to the stations can be derived from the time difference measurement and one-range measurement as follows:

$$r_1 + r_2 = 2r_1 - (r_1 - r_2) \quad (181)$$

$$\Sigma TR = \frac{r_1 + r_2}{c} \quad (182)$$

$$\Sigma TR = \frac{2r_1}{c} - \Delta TR \quad (183)$$

If an error is made in measuring either the time difference or the range time ($2r_1/c$), then the time measurement of the sum will change by

$$\delta t_e = 2\delta t_{r_1} + \delta t_h \quad (184)$$

$$\delta t_e^2 = 4(\delta t_{r_1})^2 - 4\delta t_{r_1} \delta t_h + (\delta t_h)^2 \quad (185)$$

Now

$$d^2 = \frac{c^2}{4} \left[\frac{(\delta t_h)^2}{\sin^2 \phi} + \frac{4(\delta t_{r_1})^2}{\cos^2 \phi} - \frac{4\delta t_{r_1} \delta t_h}{\cos^2 \phi} + \frac{(\delta t_h)^2}{\cos^2 \phi} \right] \quad (186)$$

From Equation 149 and with the method of Equations 152 through 154

$$d_{\text{rms}} = \frac{c}{2} \left[\frac{\sigma_{\Delta T}^2}{\sin^2 \phi} + \frac{4\sigma_{T_1}^2}{\cos^2 \phi} - \frac{4\rho\sigma_{T_1}\sigma_{\Delta T}}{\cos^2 \phi} + \frac{\sigma_{\Delta T}^2}{\cos^2 \phi} \right]^{1/2} \quad (187)$$

$$d_{\text{rms}} = c \left[\frac{\sigma_{\Delta T}^2}{\sin^2 2\phi} + \frac{\sigma_{T_1}^2}{\cos^2 \phi} - \frac{\rho \sigma_{T_1} \sigma_{\Delta T}}{\cos^2 \phi} \right]^{1/2} \quad (188)$$

where ρ is the correlation coefficient between the time-difference measurement ΔT and the range measurement T_1 .

B.3.2. DIRECT-DIFFERENCE MEASUREMENT AND TWO-RANGE MEASUREMENTS. If both r_1 and r_2 are measured separately to obtain $r_1 + r_2$, then

$$(r_1 + r_2) = r_1 + r_2 \quad (189)$$

and

$$\delta t_e = \delta t_{r_1} + \delta t_{r_2} \quad (190)$$

$$\delta t_e^2 = (\delta t_{r_1})^2 + 2\delta t_{r_1} \delta t_{r_2} + (\delta t_{r_2})^2 \quad (191)$$

From Equation 180

$$d^2 = \frac{c^2}{4} \left[\frac{(\delta t_h)^2}{\sin^2 \phi} + \frac{(\delta t_{r_1})^2}{\cos^2 \phi} + \frac{2(\delta t_{r_1} \delta t_{r_2})}{\cos^2 \phi} + \frac{(\delta t_{r_2})^2}{\cos^2 \phi} \right] \quad (192)$$

and

$$d_{\text{rms}} = \frac{c}{2} \left[\frac{\sigma_{\Delta T}^2}{\sin^2 \phi} + \frac{\sigma_{T_1}^2}{\cos^2 \phi} + \frac{\sigma_{T_2}^2}{\cos^2 \phi} + \frac{2\rho \sigma_{T_1} \sigma_{T_2}}{\cos^2 \phi} \right]^{1/2} \quad (193)$$

where ρ is the correlation coefficient between T_1 and T_2 .

B.3.3. ONLY TWO-RANGE MEASUREMENTS ARE USED TO DERIVE BOTH THE SUM AND DIFFERENCE. In a manner similar to that above,

$$\delta t_h = \delta t_{r_1} - \delta t_{r_2} \quad (194)$$

$$(\delta t_h)^2 = (\delta t_{r_1})^2 - 2\delta t_{r_1} \delta t_{r_2} + (\delta t_{r_2})^2 \quad (195)$$

Combining Equation 180 with 188 and 192, we obtain the d_{rms} error as before

$$d_{\text{rms}} = \frac{c}{2} \left[\frac{\sigma_{T_1}^2}{\sin^2 \phi} - \frac{2\rho\sigma_{T_1}\sigma_{T_2}}{\sin^2 \phi} + \frac{\sigma_{T_2}^2}{\sin^2 \phi} + \frac{\sigma_{T_1}^2}{\cos^2 \phi} + \frac{2\rho\sigma_{T_1}\sigma_{T_2}}{\cos^2 \phi} + \frac{\sigma_{T_2}^2}{\cos^2 \phi} \right]^{1/2} \quad (196)$$

$$d_{\text{rms}} = \frac{c}{\sin 2\phi} \left[\sigma_{T_1}^2 + \sigma_{T_2}^2 - 2\rho\sigma_{T_1}\sigma_{T_2} \cos 2\phi \right]^{1/2} \quad (197)$$

It should be noted that this equation is identical to Equation 173, derived for a rho-rho system.

Appendix C
CORRELATION COEFFICIENT BETWEEN TIME DIFFERENCES FOR A THREE-STATION FIX

When three stations, one master and two slaves, are used to obtain a fix by measuring two time differences ($\Delta T_1, \Delta T_2$), there is some correlation between the readings because the master-receiver path is common to both.

Let the transmission time t of each signal be a normally distributed random variable with mean μ_1 and variance σ^2 —that is, t is $N(\mu_1, \sigma)$. Let each time difference ΔT be represented here for simplicity as T . Each T will be normally distributed. Let

$$T_1 = t_1 - t_M \quad (198)$$

$$T_2 = t_2 - t_M \quad (199)$$

where the subscripts 1 and 2 refer to slave stations 1 and 2, and M to the master station. The correlation coefficient $\rho_{1,2}$ is then the correlation between T_1 and T_2 and is given by

$$\rho_{T_1, T_2} = \frac{E[(T_1 - \mu_{T_1})(T_2 - \mu_{T_2})]}{\sigma_{T_1} \sigma_{T_2}} \quad (200)$$

Since the variance σ^2 of the sum or difference of two random variables is the sum of their variances,

$$\sigma_{T_1}^2 = \sigma_{t_1}^2 + \sigma_{t_M}^2 \quad (201)$$

$$\sigma_{T_2}^2 = \sigma_{t_2}^2 + \sigma_{t_M}^2 \quad (202)$$

then

$$\sigma_{T_1} \sigma_{T_2} = \sqrt{(\sigma_{t_1}^2 + \sigma_{t_M}^2)(\sigma_{t_2}^2 + \sigma_{t_M}^2)} \quad (203)$$

Now if we consider the numerator

$$\begin{aligned} E[(T_1 - \mu_{T_1})(T_2 - \mu_{T_2})] &= E(T_1 T_2) - \mu_{T_2} E(T_1) - \mu_{T_1} E(T_2) + \mu_{T_1} \mu_{T_2} \quad (204) \\ &= E(T_1 T_2) - \mu_{T_1} \mu_{T_2} = E(t_1 - t_M)(t_2 - t_M) - (\mu_1 - \mu_M)(\mu_2 - \mu_M) \\ &= E(t_1 t_2) - E(t_2 t_M) - E(t_1 t_M) + E(t_M^2) - (\mu_1 - \mu_M)(\mu_2 - \mu_M) \end{aligned}$$

(Equation 204 continued)

$$\begin{aligned}
 &= \mu_1\mu_2 - \mu_2\mu_M - \mu_1\mu_M + \sigma_M^2 + \mu_M^2 - \mu_1\mu_2 + \mu_2\mu_M + \mu_1\mu_M - \mu_M^2 \\
 &= \sigma_M^2
 \end{aligned}$$

Now we can write

$$\rho_{T_1, T_2} = \frac{\sigma_M^2}{\sqrt{(\sigma_{t_1}^2 + \sigma_{t_M}^2)(\sigma_{t_2}^2 + \sigma_{t_M}^2)}} \tag{205}$$

$$\rho_{T_1, T_2} = \frac{1}{\sqrt{\left[\left(\frac{\sigma_{t_1}}{\sigma_M}\right)^2 + 1\right]\left[\left(\frac{\sigma_{t_2}}{\sigma_M}\right)^2 + 1\right]}} \tag{206}$$

In a three-station chain σ_{t_1} and σ_{t_2} must include the paths from the master to the slave as well as from slave to receiver. If all five paths are independent and of similar length, we might assume

$$\sigma_{t_1} = \sigma_{t_2} = \sqrt{2} \sigma_M \tag{207}$$

and

$$\rho_{T_1, T_2} = 0.3 \tag{208}$$

Near Slave 1 we might assume

$$\begin{aligned}
 \sigma_{t_1} &= \sigma_M \\
 \sigma_{t_2} &= \sqrt{2} \sigma_M
 \end{aligned} \tag{209}$$

or

$$\rho_{T_1, T_2} = 0.41 \tag{210}$$

Near the master station

$$\rho_{T_1, T_2} \rightarrow 0 \tag{211}$$

Appendix D
SYSTEM ACCURACY CONTOURS FOR A THREE-STATION HYPERBOLIC SYSTEM
ON A SPHERICAL EARTH

D.1. d_{rms} ERROR ON A SPHERICAL EARTH

The d_{rms} error of a three-station hyperbolic system as developed in Appendix B is,

$$d_{\text{rms}} = \frac{c}{2 \sin \theta} \left[\frac{\sigma_{\Delta T_1}^2}{\sin^2 \phi_1} + \frac{\sigma_{\Delta T_2}^2}{\sin^2 \phi_2} + \frac{2\rho \sigma_{\Delta T_1} \sigma_{\Delta T_2} \cos \theta}{\sin \phi_1 \sin \phi_2} \right]^{1/2} \quad (212)$$

where

c = velocity of light

θ = angle of "cut" between the two lines of position

$2\phi_1$ and $2\phi_2$ = the angle subtended at the receiver by the master and Slave Station 1, and master and Slave Station 2

$\sigma_{\Delta T_1}$ and $\sigma_{\Delta T_2}$ = the standard deviations of the time difference measurements corresponding to Slave 1 and Slave 2, and

ρ = correlation coefficient between ΔT_1 and ΔT_2

For a three-station triad the master station is used for both time-difference measurements and

$$\theta = \phi_1 + \phi_2$$

The d_{rms} error at a given location is a function of the geometrical relation between the receiver and the three transmitting stations. When a system is designed and installed to provide navigation within a certain "service area," it is usually desirable to locate the stations so that the maximum d_{rms} error anywhere in the service area is less than some specified amount. A convenient way of determining the location for the stations is to prepare graphs showing lines of constant d_{rms} error for several station configurations. Such graphs can then be used as overlays on maps to find the configuration giving the optimum results for the chosen service area.

This method has been used frequently in the past, and many plots of constant d_{rms} error have been drawn for the LORAN-A, Decca, and LORAN-C systems. In all cases, however, it has been assumed that the system was used on a flat earth, and calculations were based on a system of plane hyperbolas. Jessell and Trow give a method for drawing such curves [23].

Although the computation of error contours based on the assumption that the earth is flat does not result in serious error for systems with baseline lengths of less than 1000 miles, the errors become serious when the method is applied to VLF systems with baseline lengths of several thousand miles. When a hyperbolic system is used on a spherical earth and the lengths of the baselines are large in comparison to the radius of the earth, the lines of position are not

hyperbolas, nor are they the intersections of hyperboloids of revolution with a sphere. Under such conditions the lines of position must be described as the loci of points equidistant from two fixed points where the distances are great-circle distances. The departure of such loci from hyperbolic lines can best be visualized by considering the extreme example, in which one station is located at the North Pole and one at the South Pole. The loci of all points with a constant distance difference from each station are then circles corresponding to the parallels of latitude.

The solution of Equation 212 at any point on a spherical earth requires the computation of ϕ_1 , ϕ_2 , and hence θ , by the use of spherical trigonometry. The problem has been solved by the use of an analog computer, and the results are given in Figures 61 through 95.

D.2. EXPLANATION OF ACCURACY CONTOURS

Plots have been made for systems with the following baseline lengths (in nautical miles): 1000, 2000, 3000, 4000, and 5000. For each baseline length plots were made with the following baseline angles (angle subtended by the two slave stations at the master station): 90° , 105° , 120° , 135° , 150° , 165° , and 180° . In all the computations we have assumed that ρ , the correlation coefficient between ΔT_1 and ΔT_2 , equals 0.3. Figure 77 shows computations based on both $\rho = 0.3$ and $\rho = 0$ to show the lack of sensitivity of d_{rms} error to ρ .

We have also assumed that $\sigma_{\Delta T_1} = \sigma_{\Delta T_2}$. Actually the standard deviation of each time difference might be expected to be a function of the range to each station. The equality assumption was used here because it greatly simplified the computation problem.

Further experimental data are not yet available in sufficient quantity to establish a functional relationship between the standard deviation of propagation time and range. The data that have been taken indicate that the correlation between the standard deviation of a time difference and the distance to the location of a receiver is not important.

If we assume that

$$\sigma_{\Delta T_1} = \sigma_{\Delta T_2} = \sigma_{\Delta T}$$

we can rewrite Equation 212 in the following form:

$$K = \frac{d_{\text{rms}}}{c \sigma_{\Delta T}} = \frac{1}{2 \sin \theta} \left(\frac{1}{\sin^2 \phi_1} + \frac{1}{\sin^2 \phi_2} + \frac{2\rho \cos \theta}{\sin \phi_1 \sin \phi_2} \right)^{1/2} \quad (213)$$

The values assigned to the error contours in Figures 61 through 95 are values of K . For any curve shown, the value of the d_{rms} error represented by that curve can be found by

$$d_{\text{rms}} = K c \sigma_{\Delta T} \quad (214)$$

where c is the velocity of light and $\sigma_{\Delta T}$ is the standard deviation of the time-difference measurement. If $\sigma_{\Delta T}$ is given in μsec , and the d_{rms} error in nautical miles, then

$$c = 0.1618 \text{ nautical miles}/\mu\text{sec} \quad (215)$$

Figure 77 contains K contours plotted for both $\rho = 0.3$ and $\rho = 0$. Inside the normal service area (receiver to master distance less than about 3500 miles) the curves are not greatly sensitive to ρ ; so the use of

$$\rho = 0.3 \quad (216)$$

over the entire service area seems justified (see Appendix C). The actual variation in ρ would only be over a range of about

$$\rho \approx 0.2 \text{ to } 0.4 \quad (217)$$

The figures show "azimuthal equidistant projections." The distances as measured radially from the master station are correct and constant in scale. Azimuths measured at the master station are also correctly represented. But all distances measured other than radially from the master station are incorrect, as are all angles measured about any point other than the origin or master station. This distortion of areas presents a problem of interpretation and stems from the age-old problem of representation of large areas of a globe on a flat plane. Cartographers have not solved the problem, which we are now faced with. For example, a circle drawn with a radius of 10,800 nautical miles with the master station at its center actually represents a single point on the earth, the point antipodal to the master station.

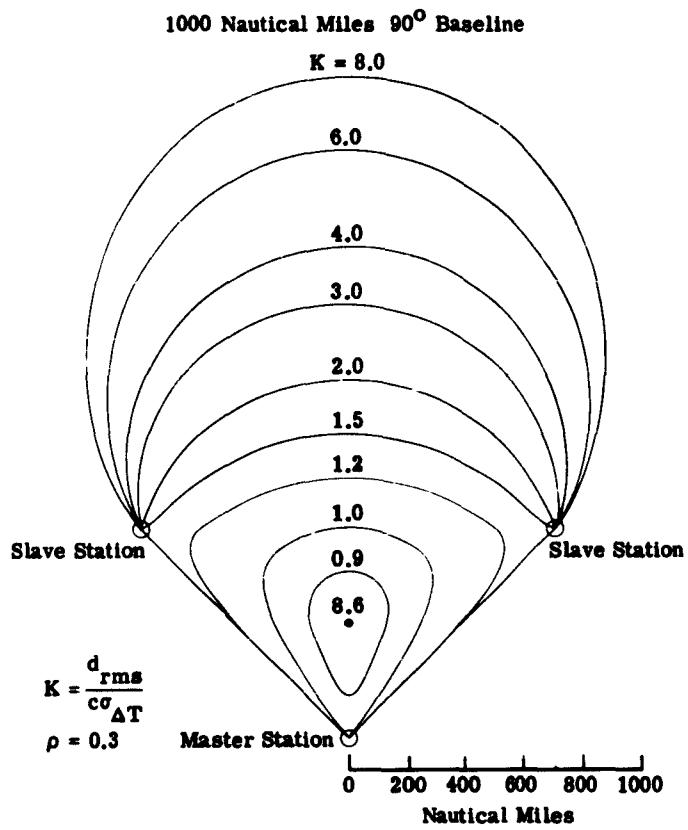


FIGURE 61. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

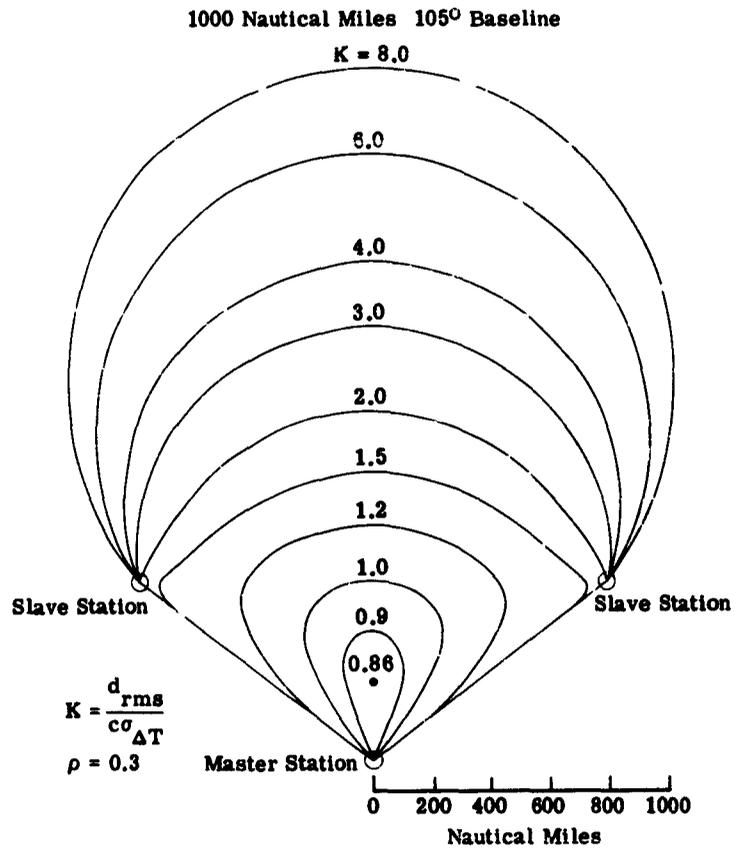


FIGURE 62. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

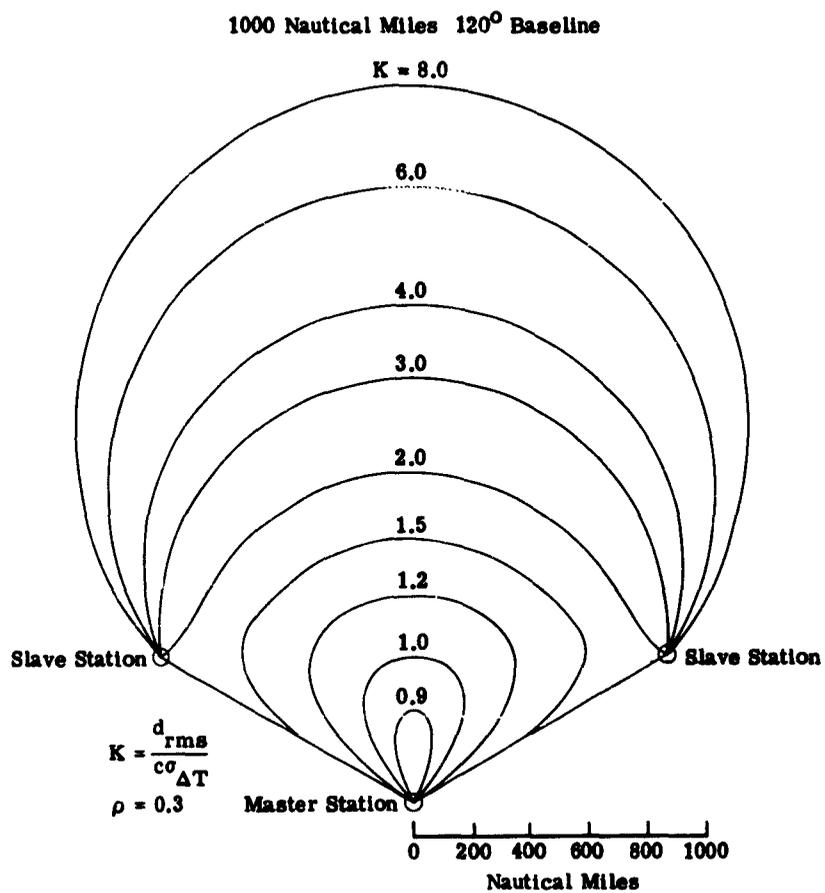


FIGURE 63. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

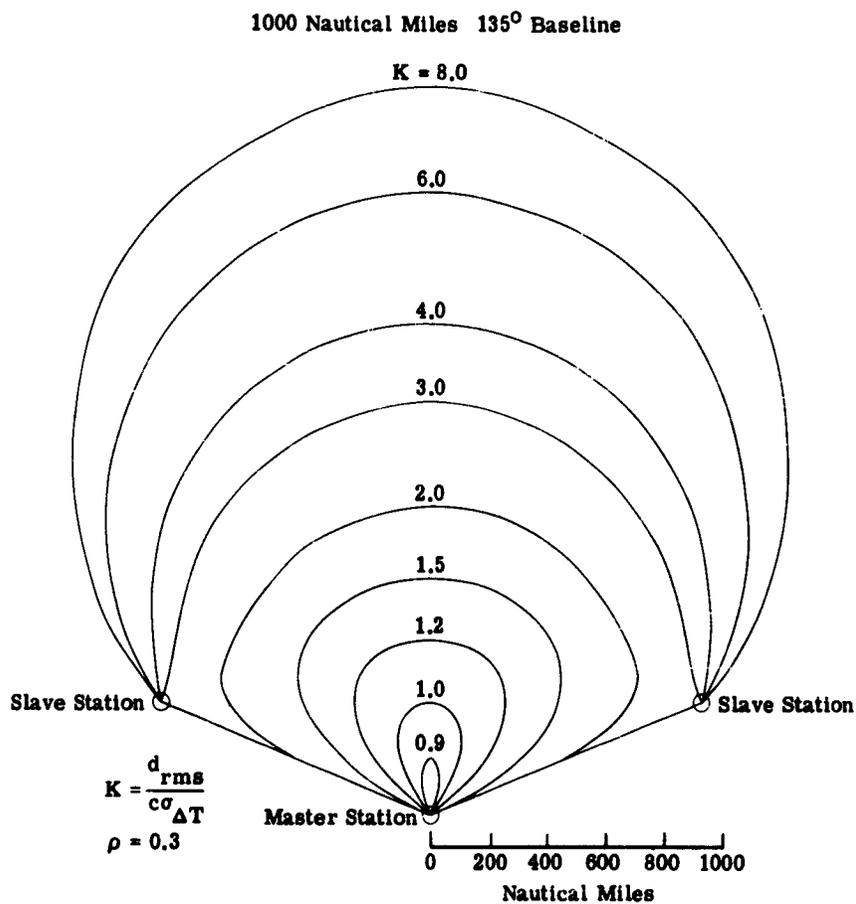


FIGURE 64. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

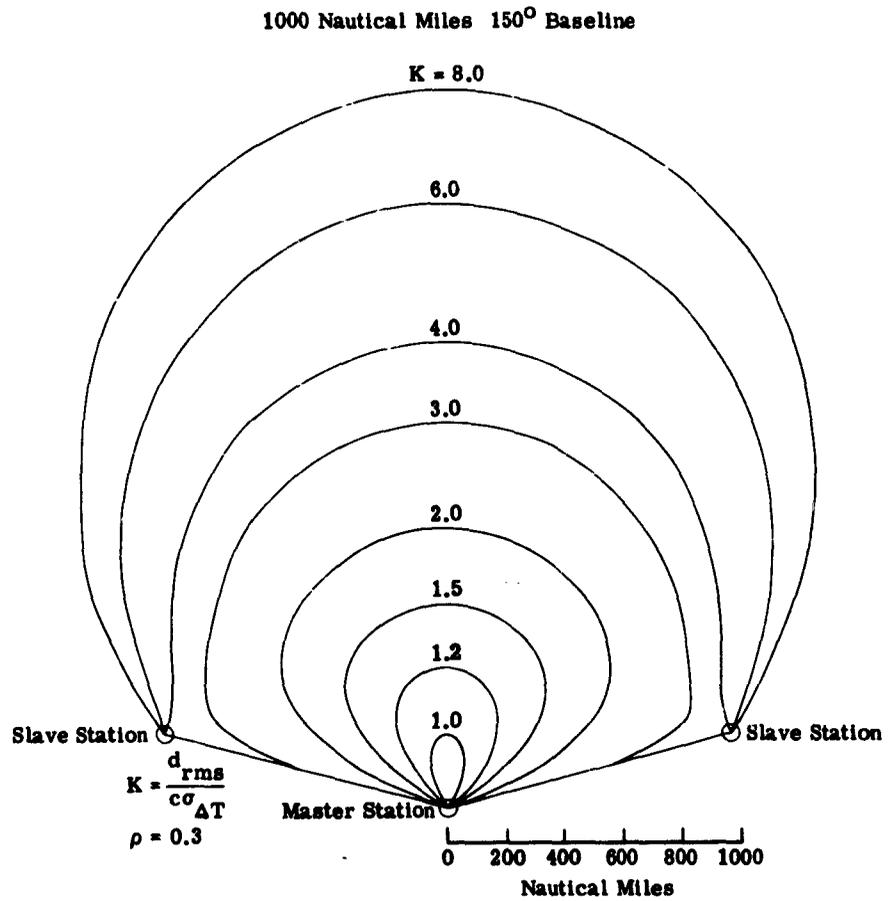


FIGURE 65. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

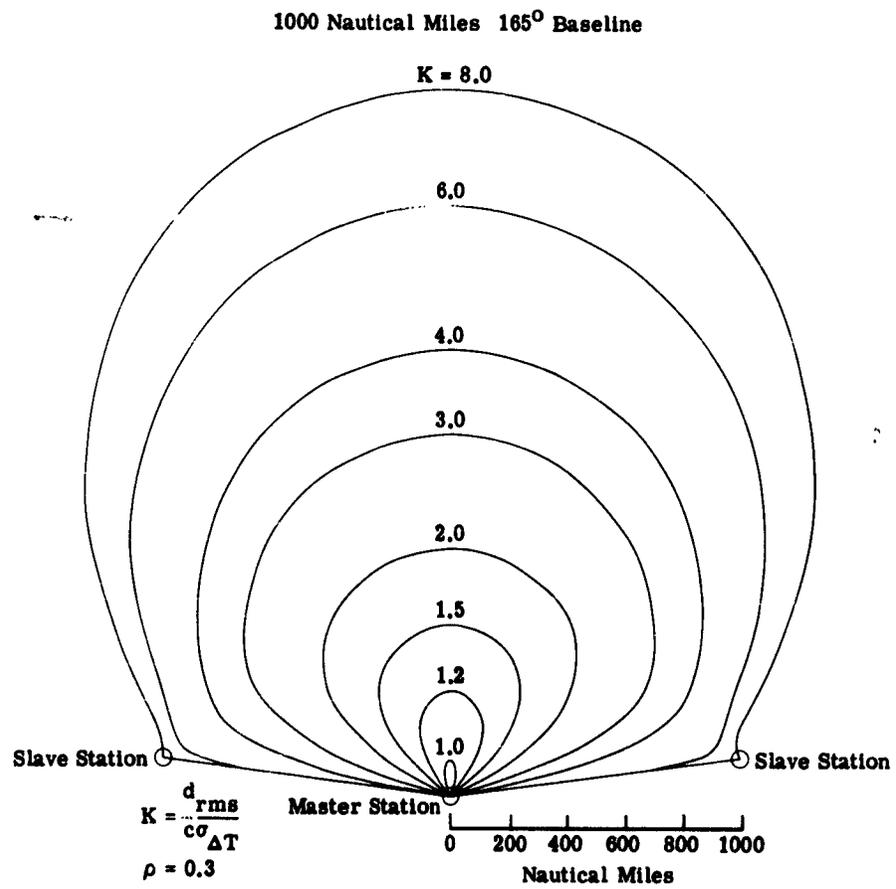


FIGURE 66. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

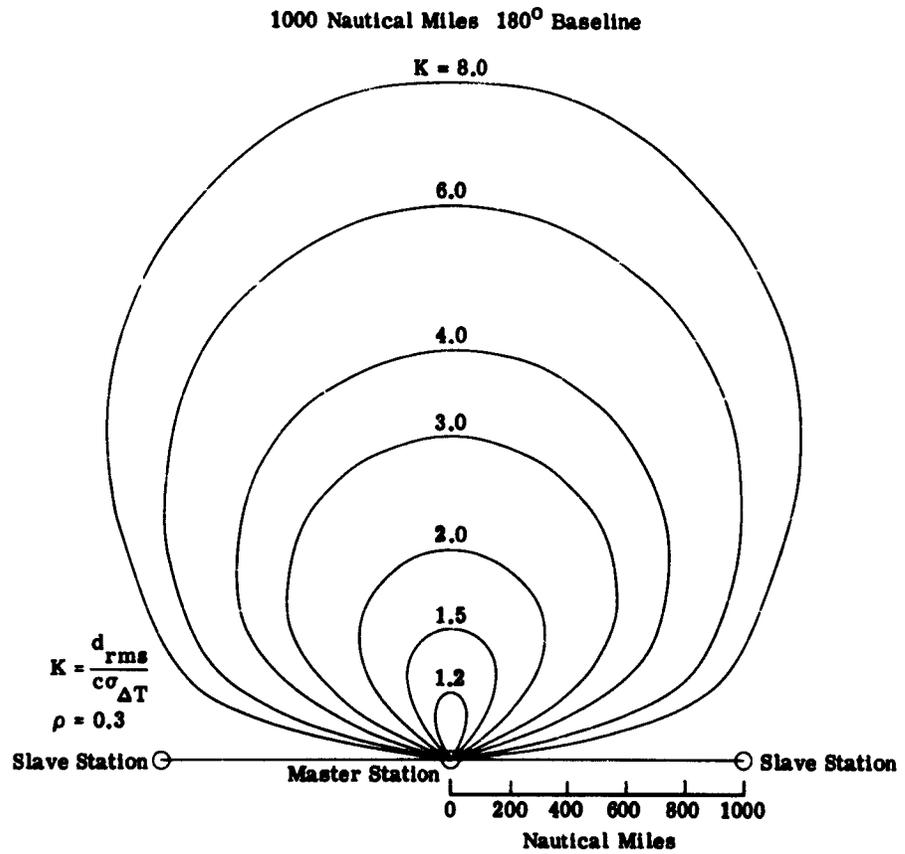


FIGURE 67. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

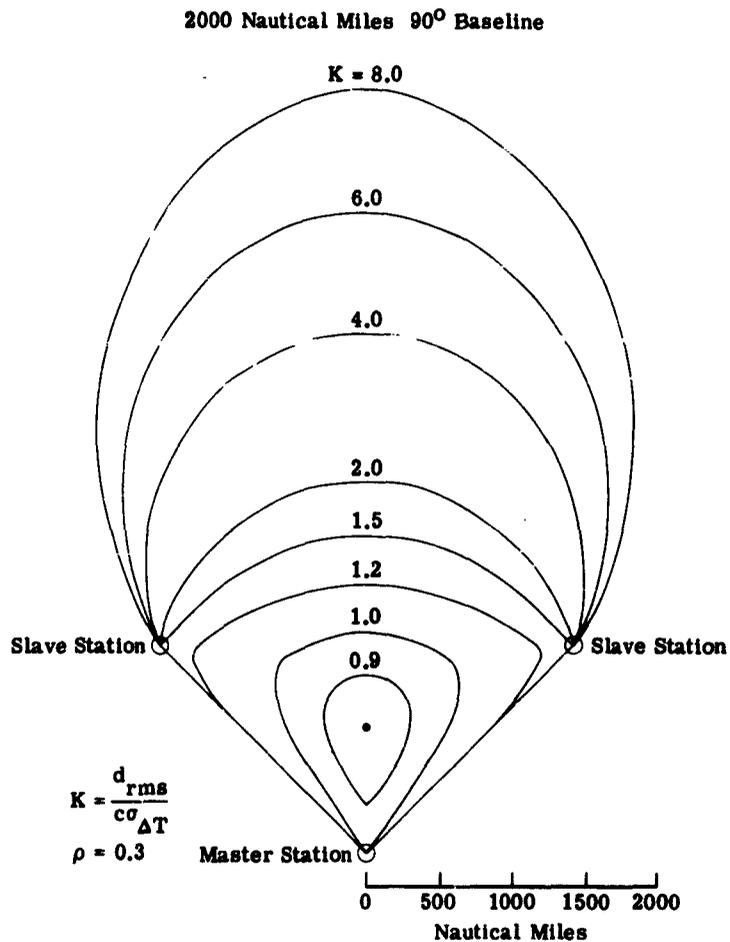


FIGURE 68. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

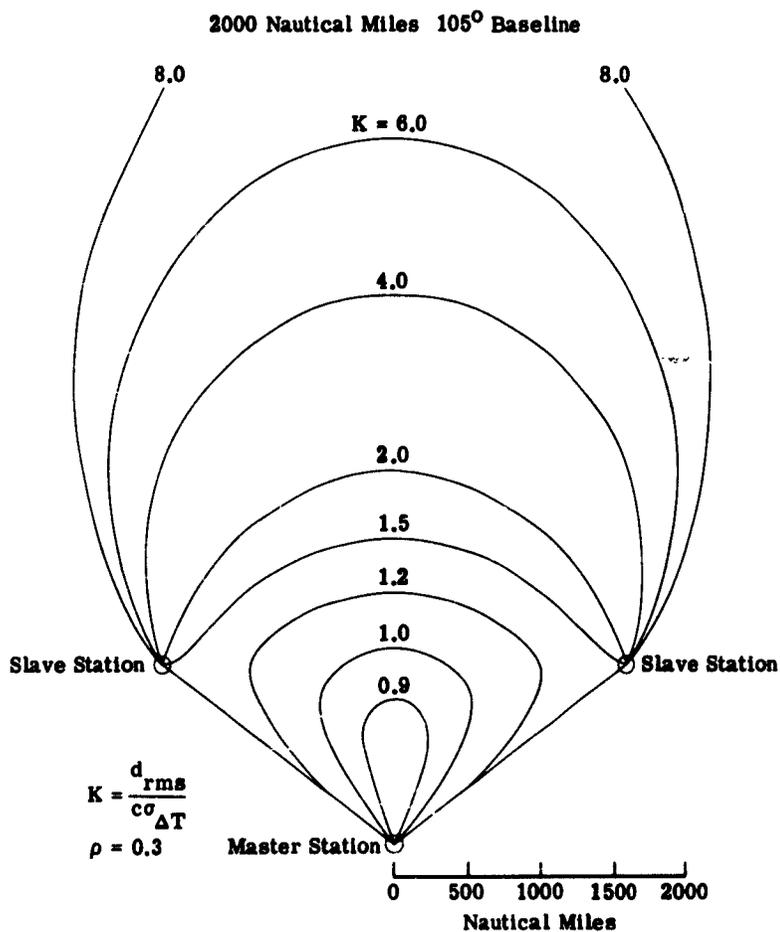


FIGURE 69. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

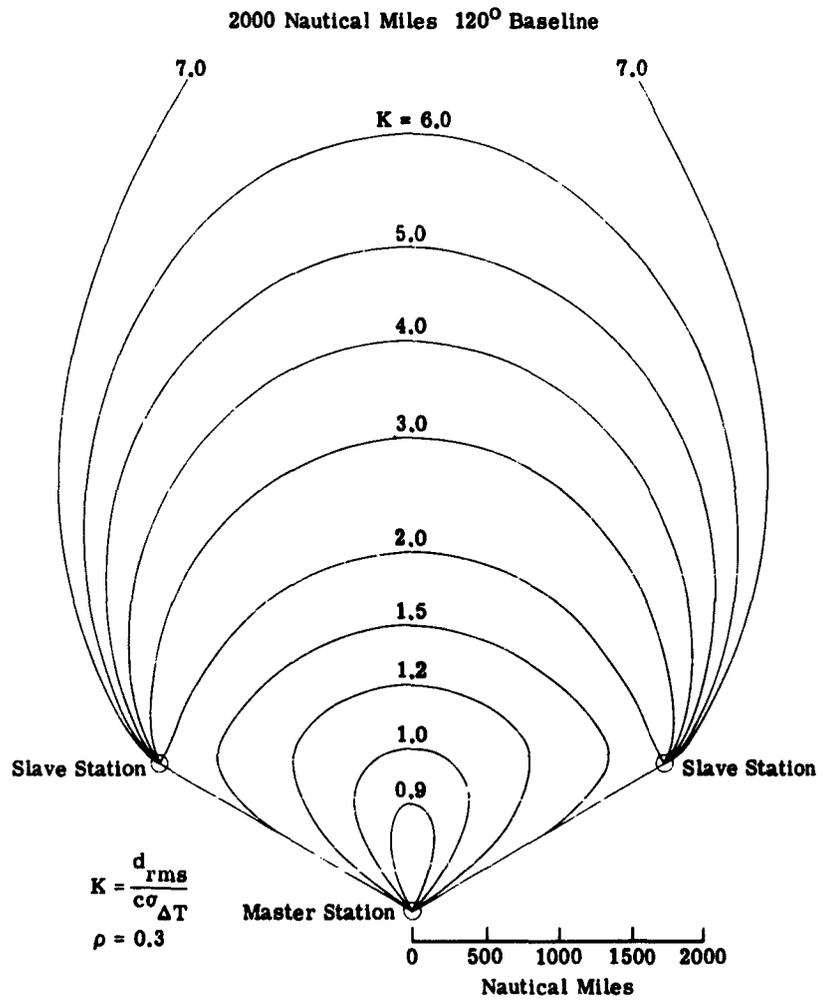


FIGURE 70. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

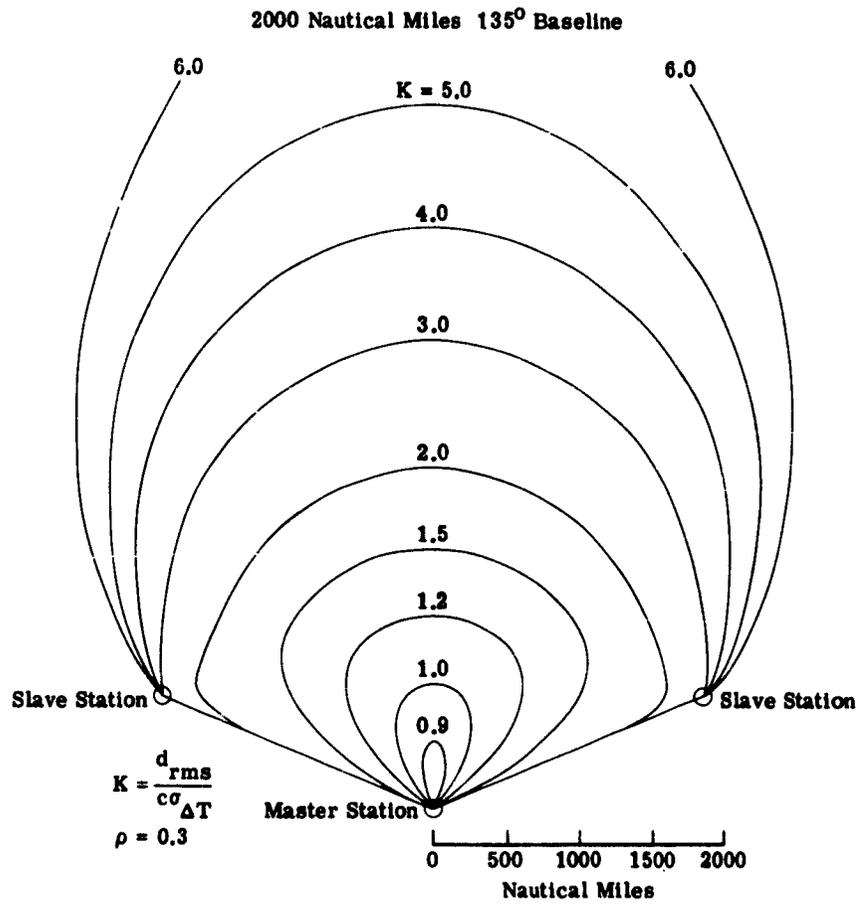


FIGURE 71. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

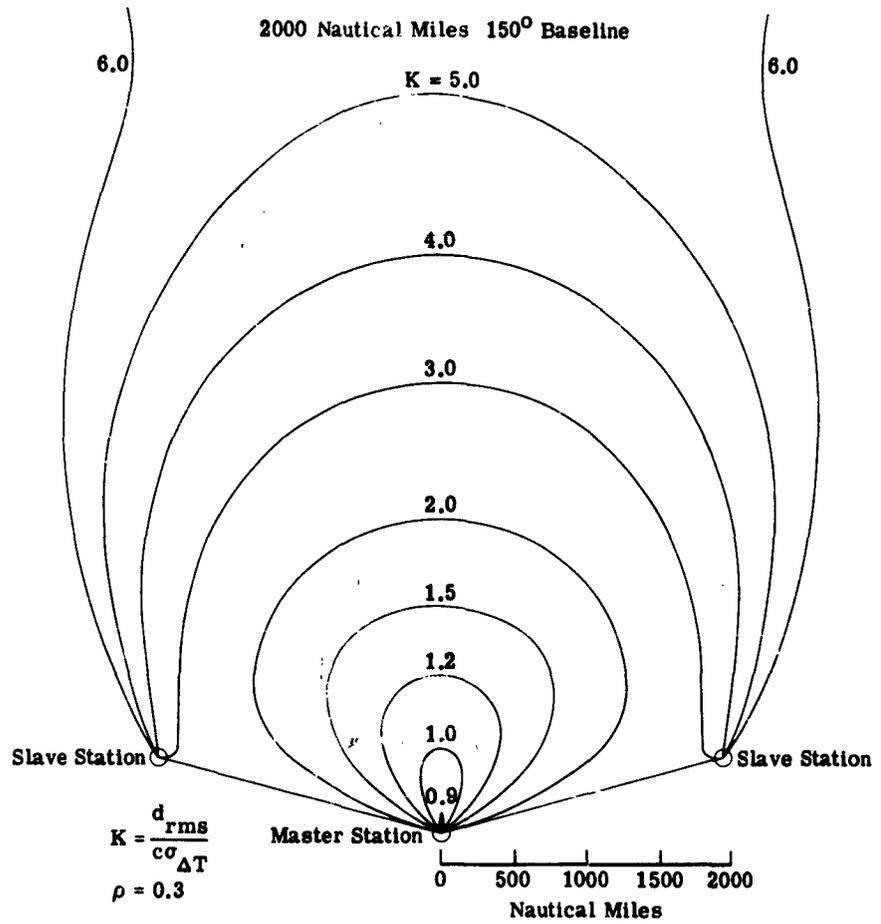


FIGURE 72. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

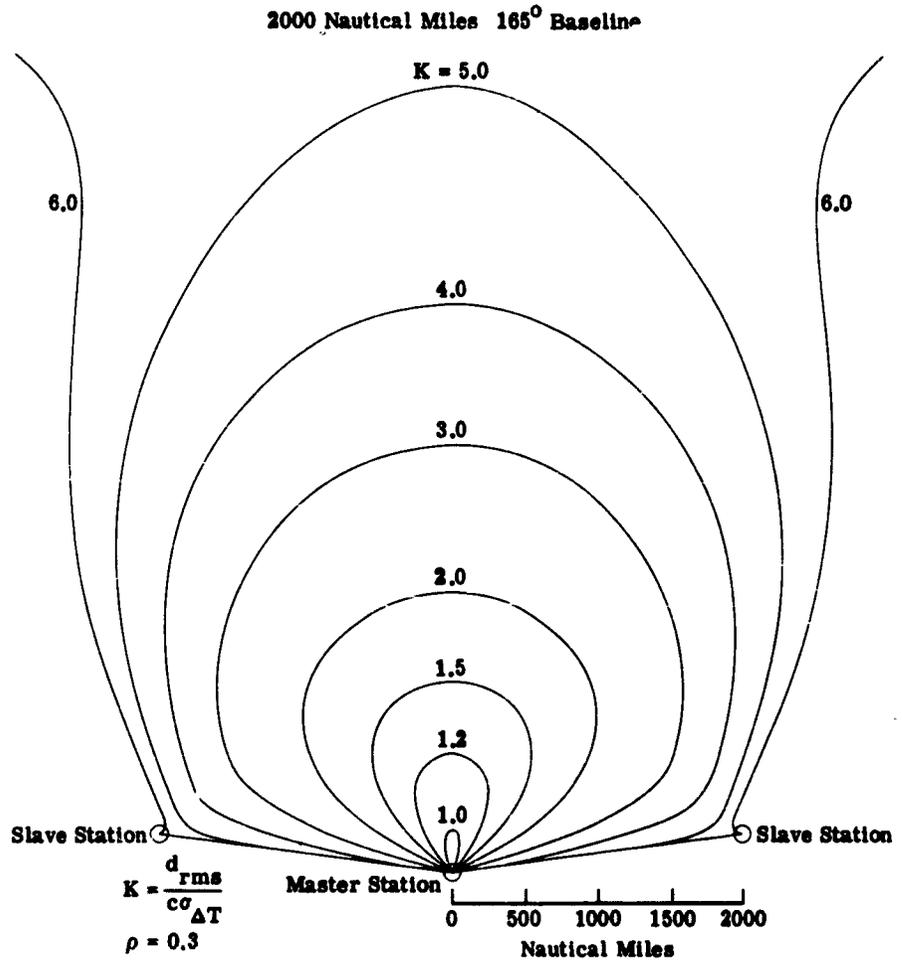


FIGURE 73. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

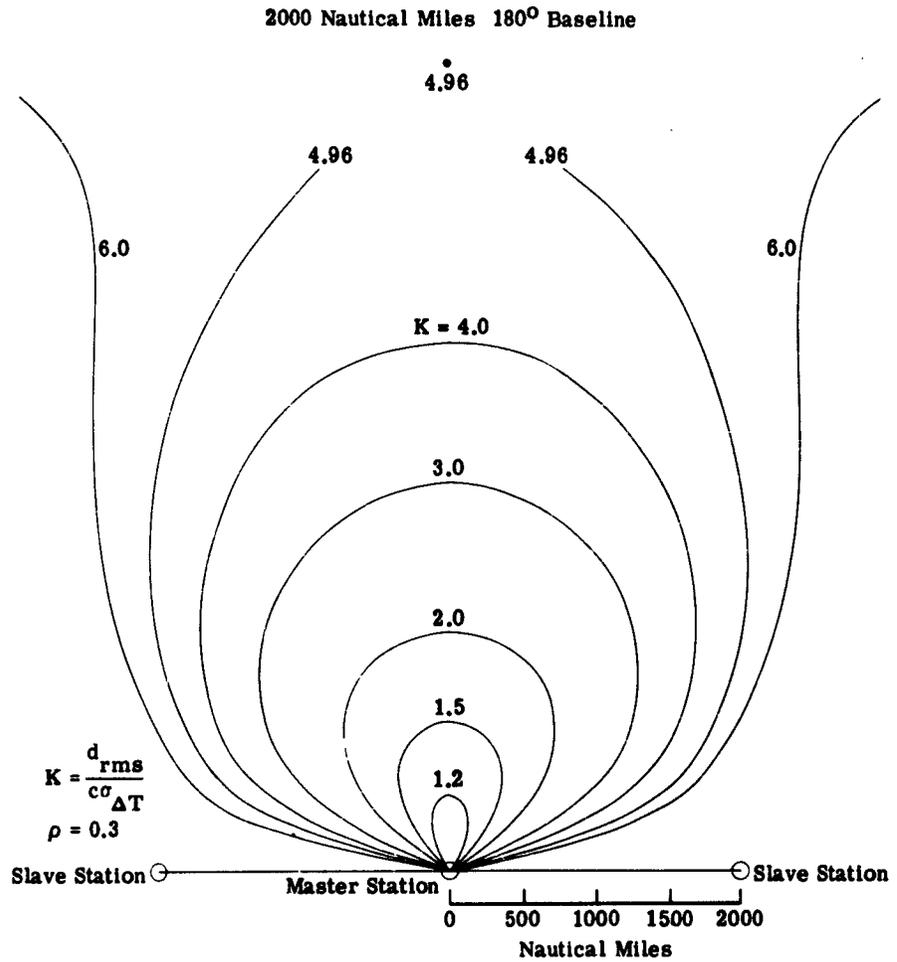


FIGURE 74. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

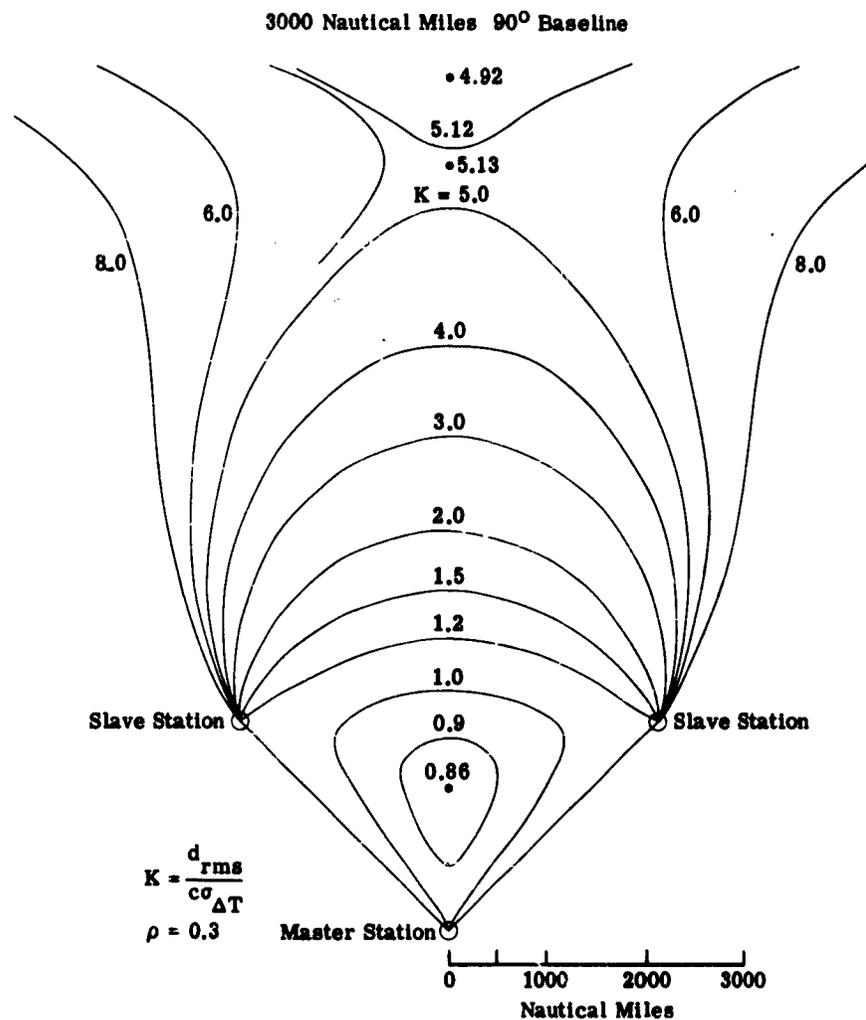


FIGURE 75. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

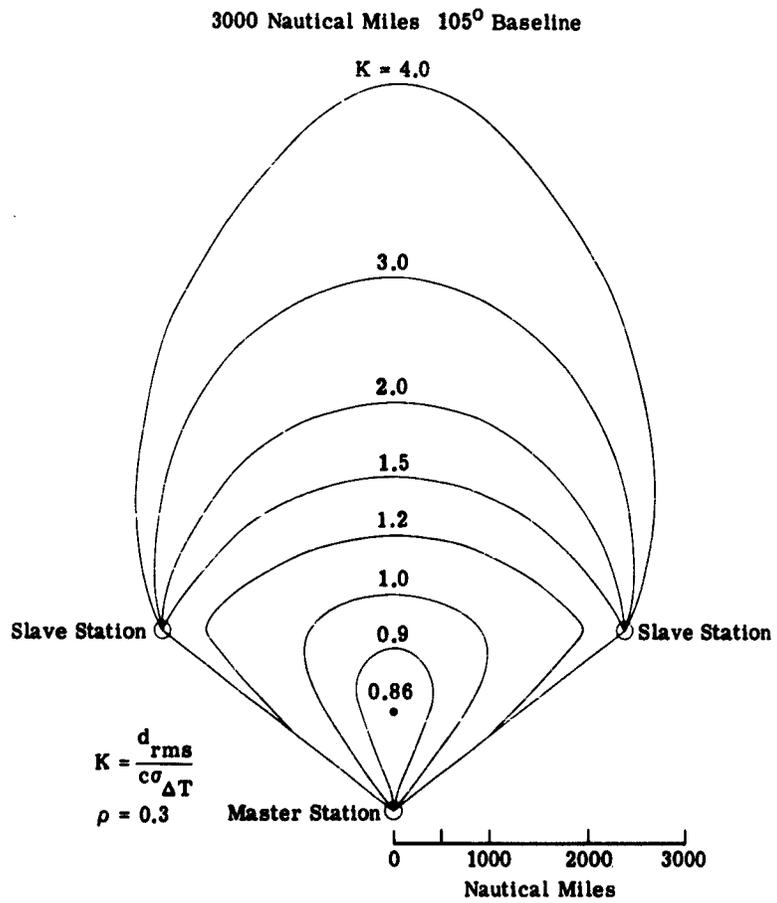


FIGURE 76. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

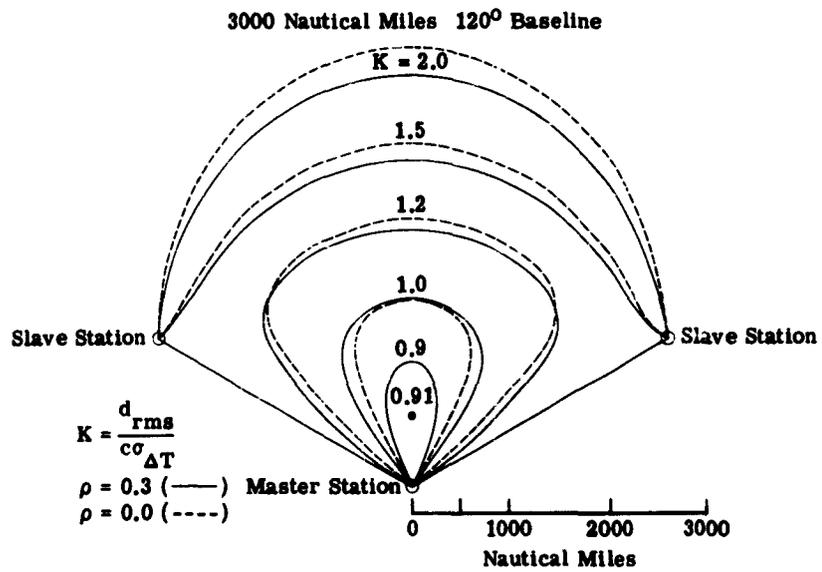


FIGURE 77. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

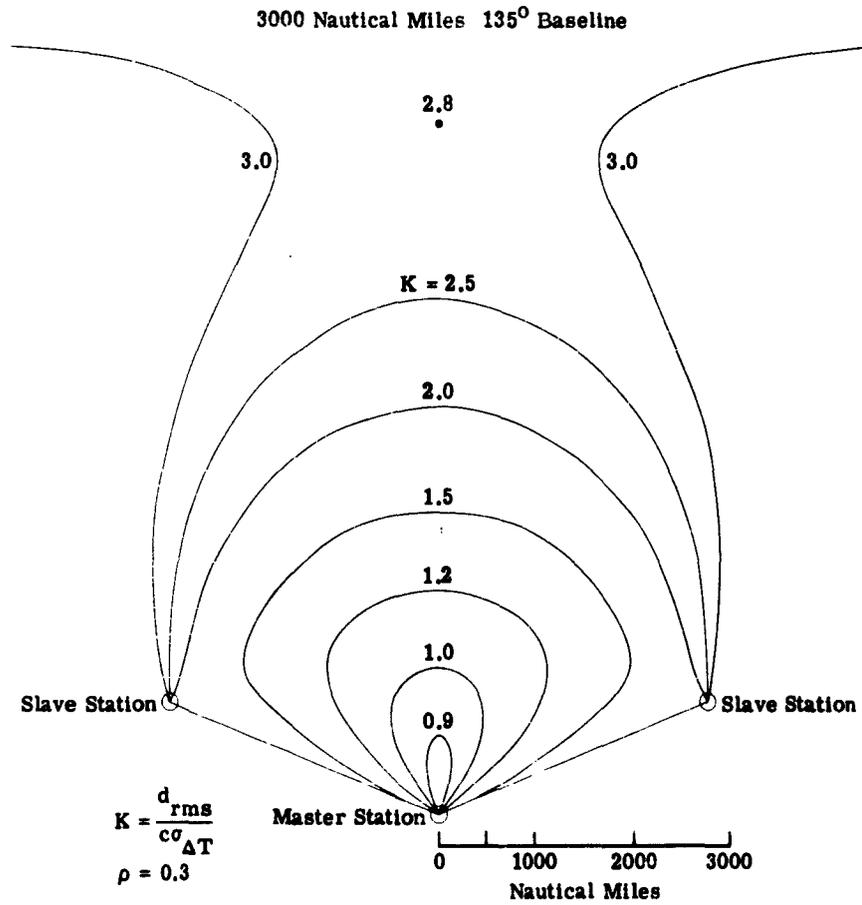


FIGURE 78. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

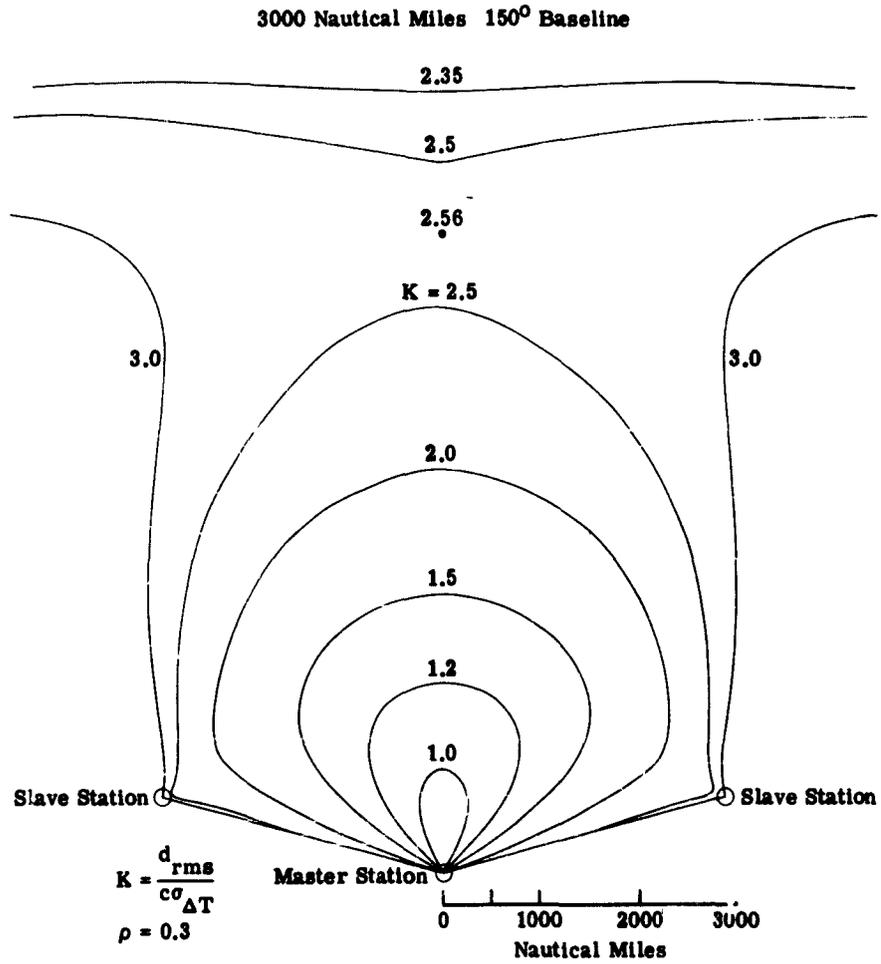


FIGURE 79. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

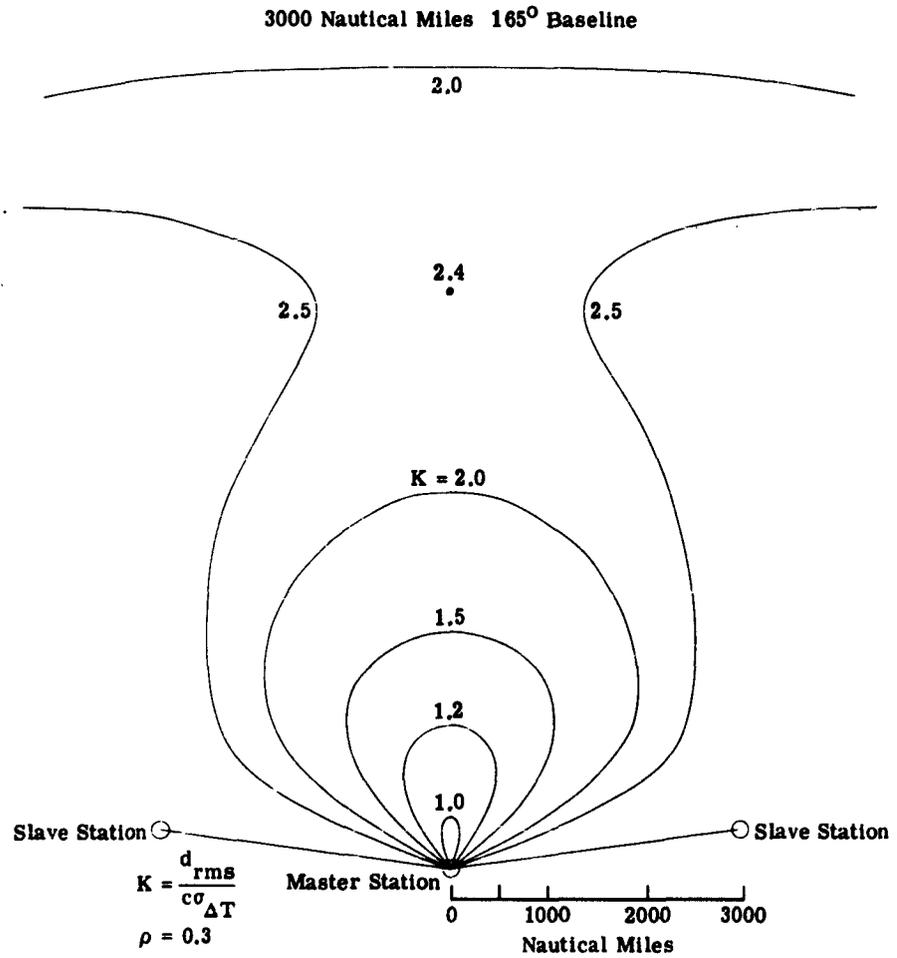


FIGURE 80. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

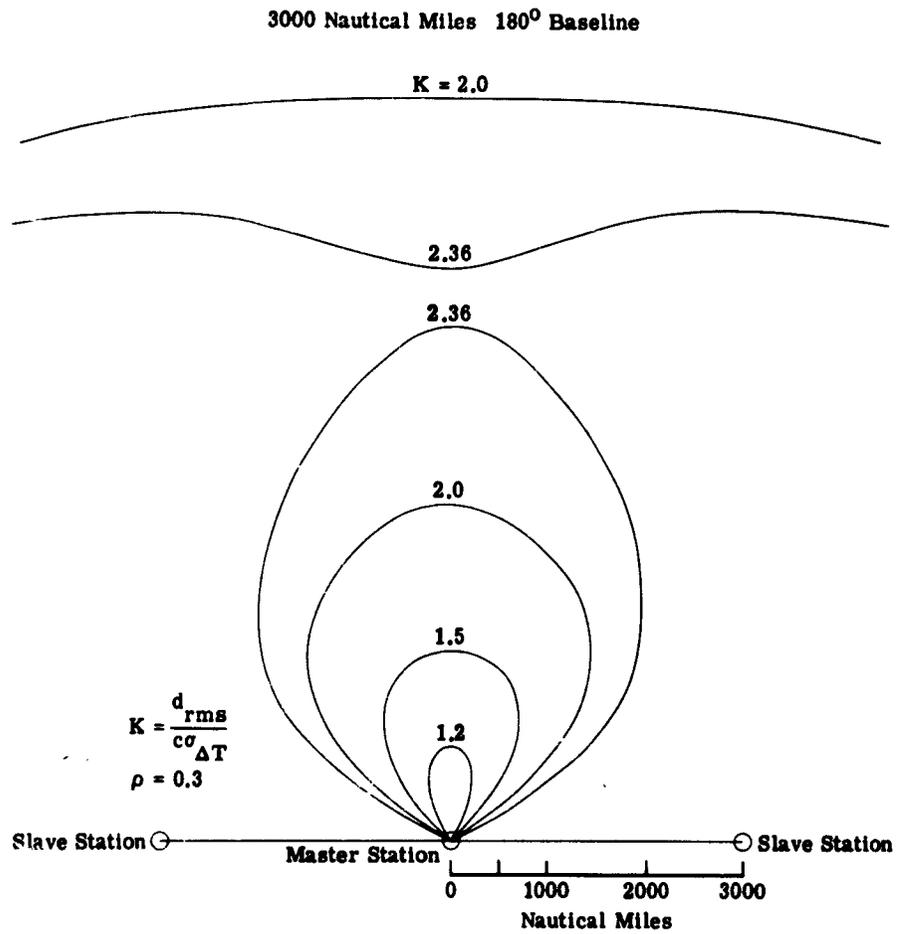


FIGURE 81. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

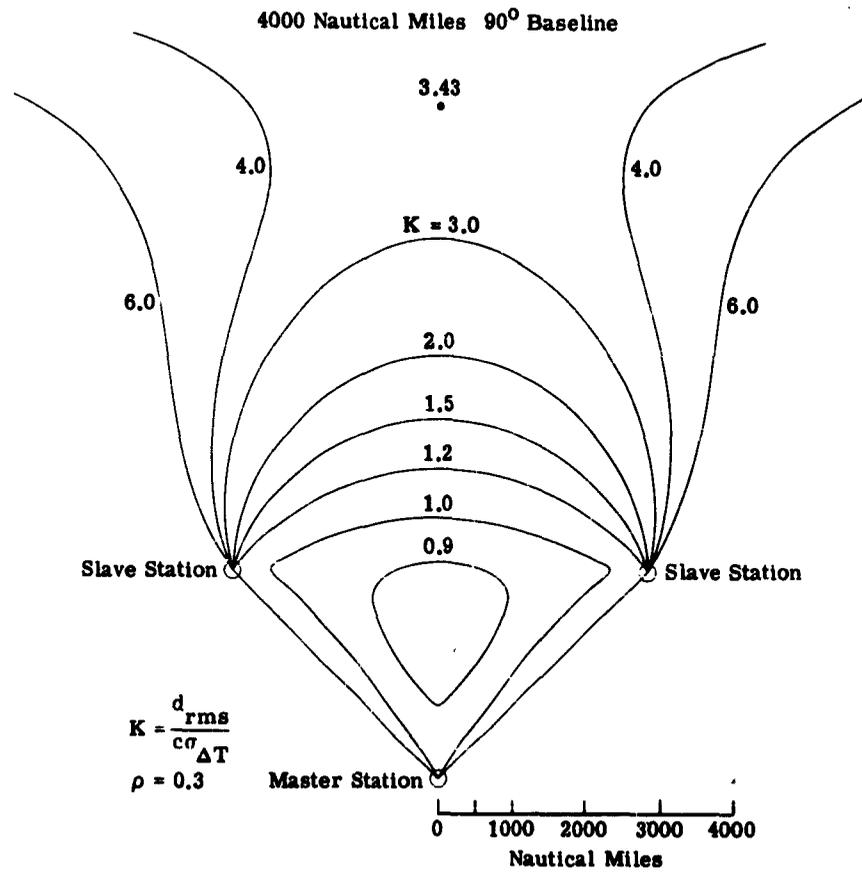


FIGURE 82. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

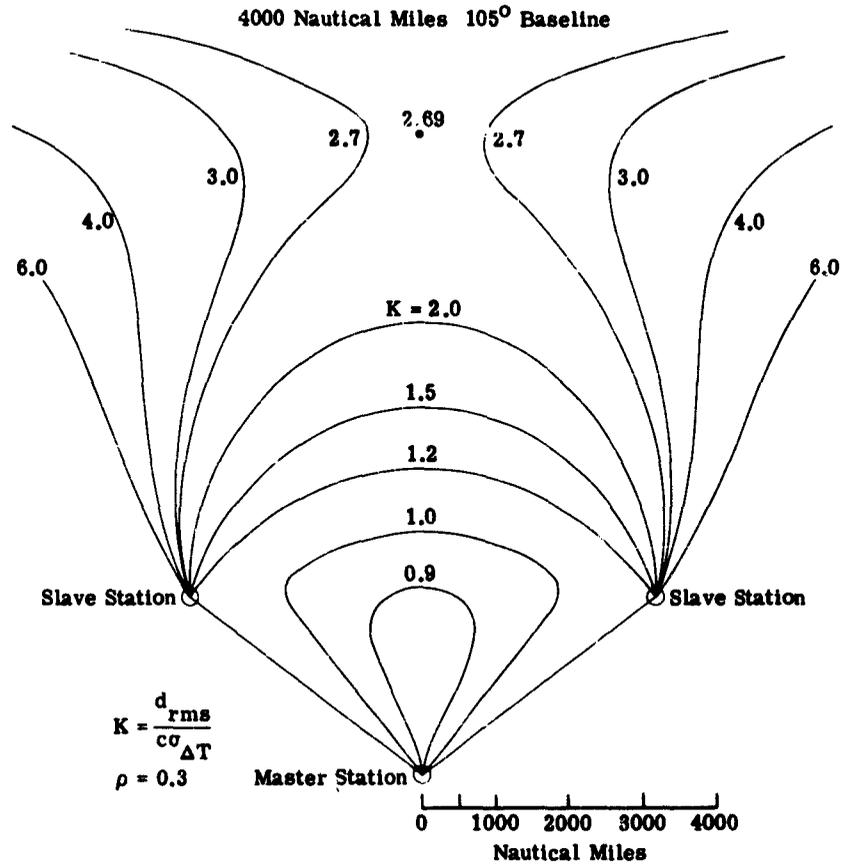


FIGURE 83. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

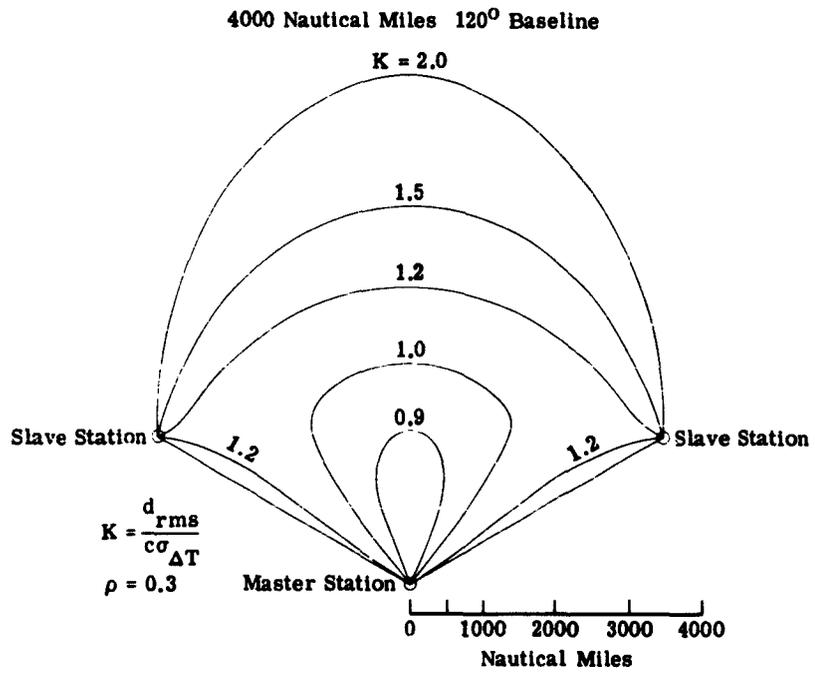


FIGURE 84. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

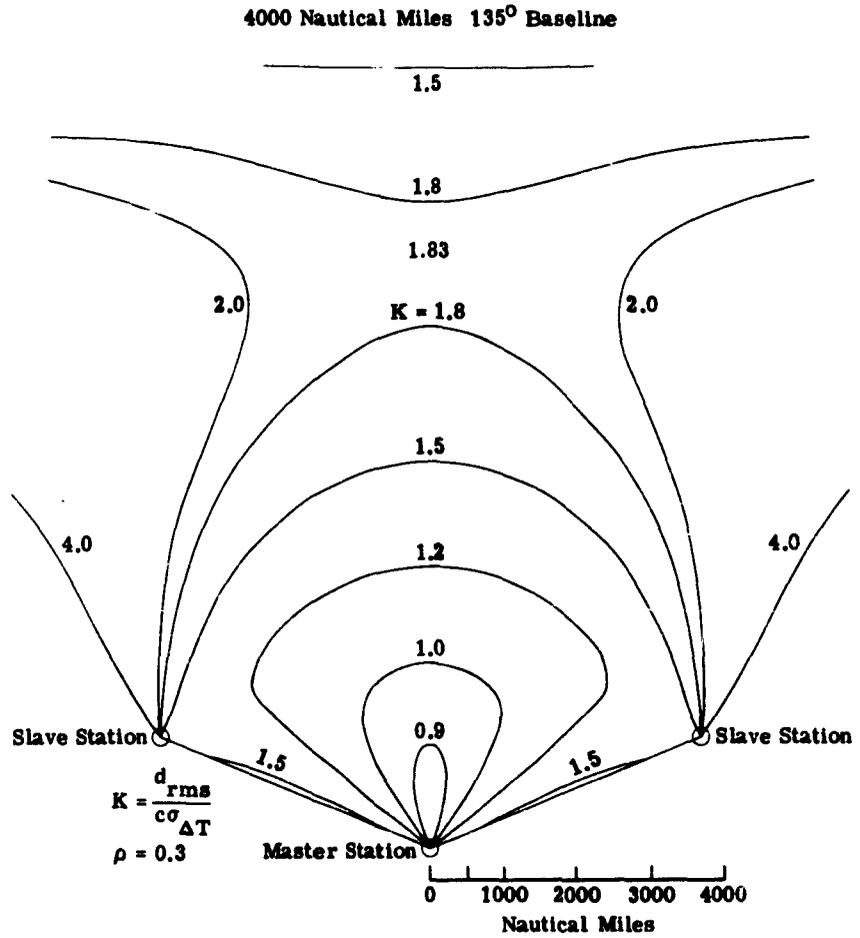


FIGURE 85. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

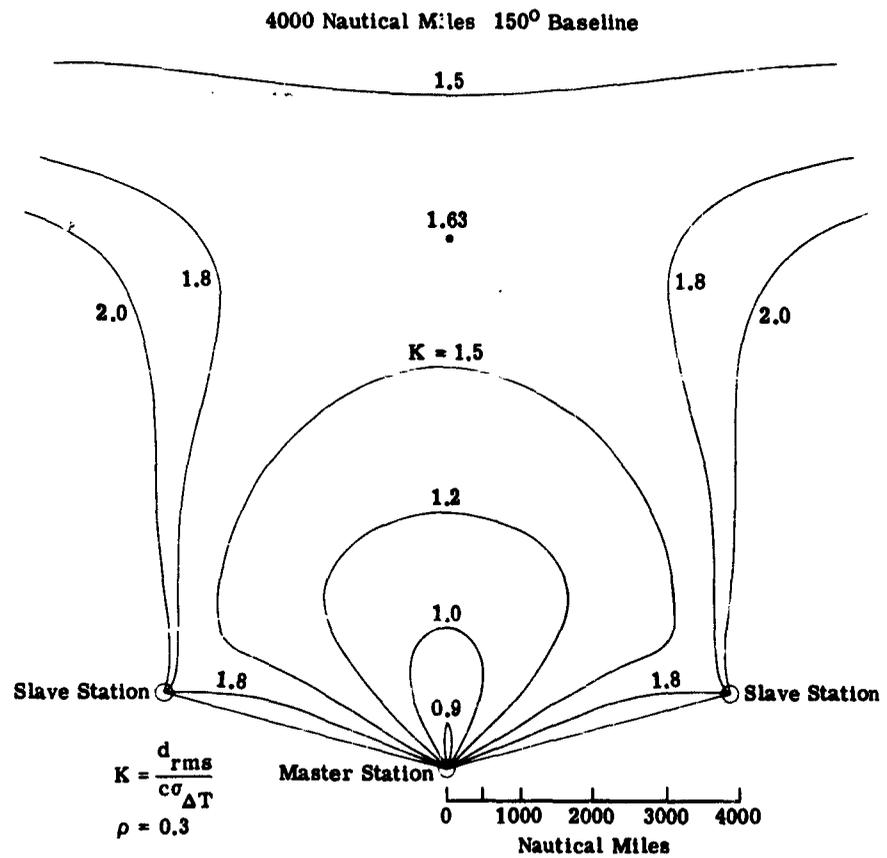


FIGURE 86. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

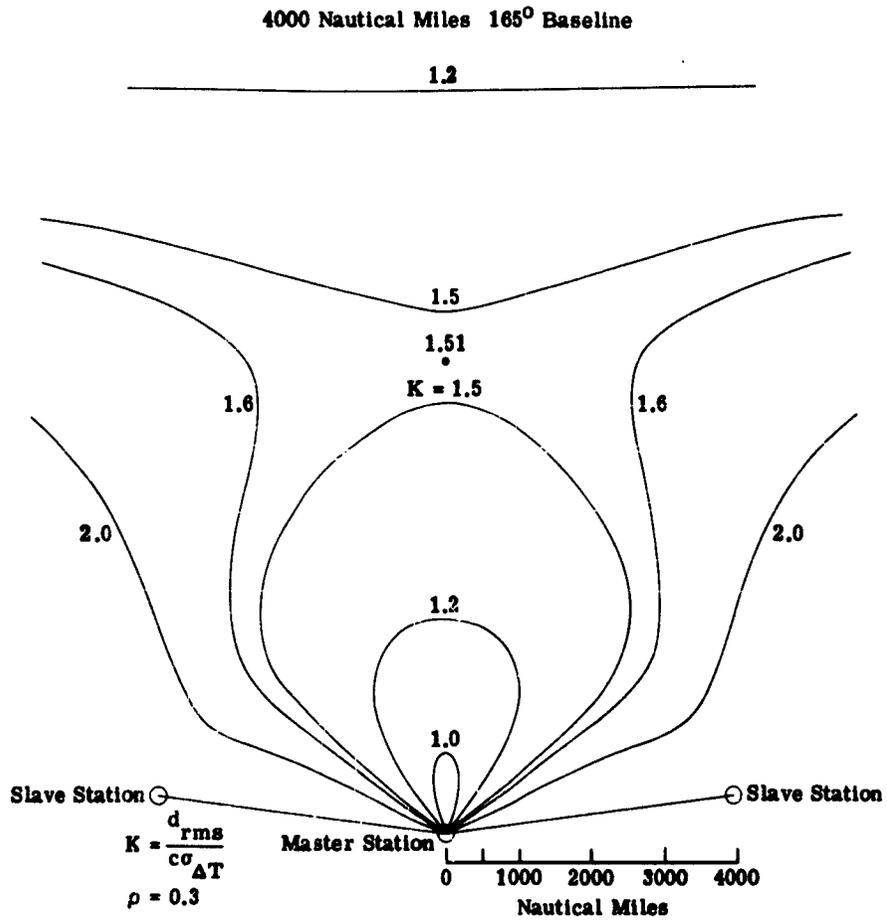


FIGURE 87. d_{rms} - ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

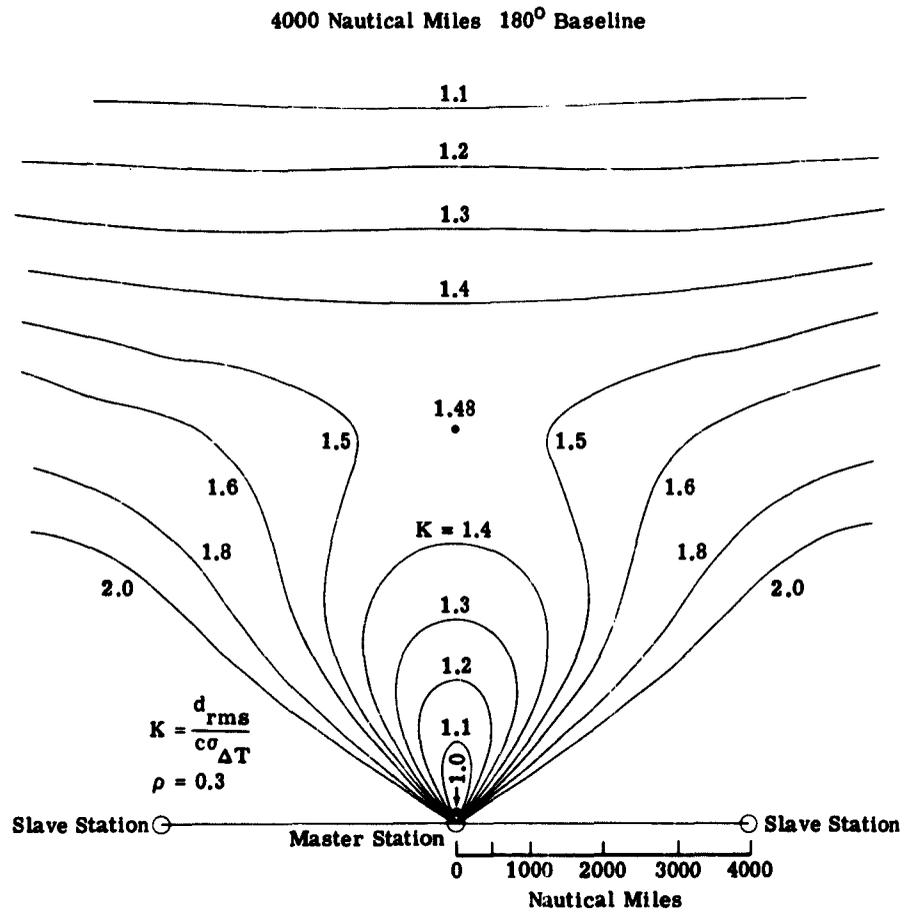


FIGURE 88. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

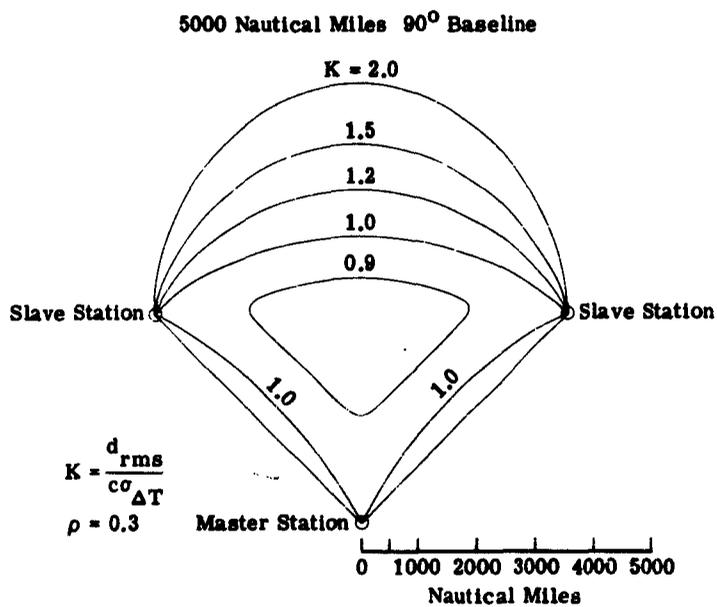


FIGURE 89. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

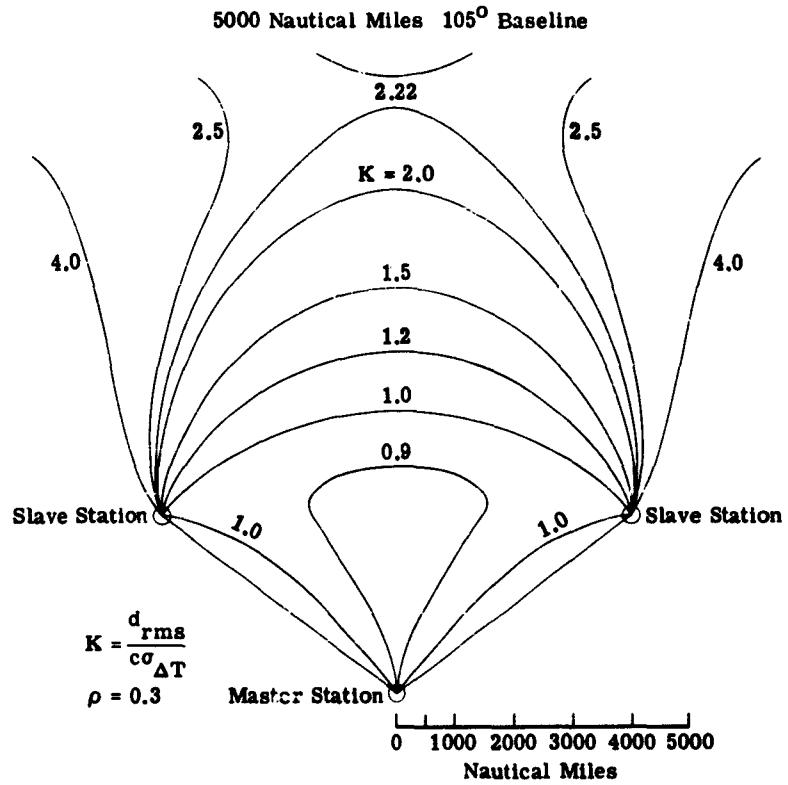


FIGURE 90. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

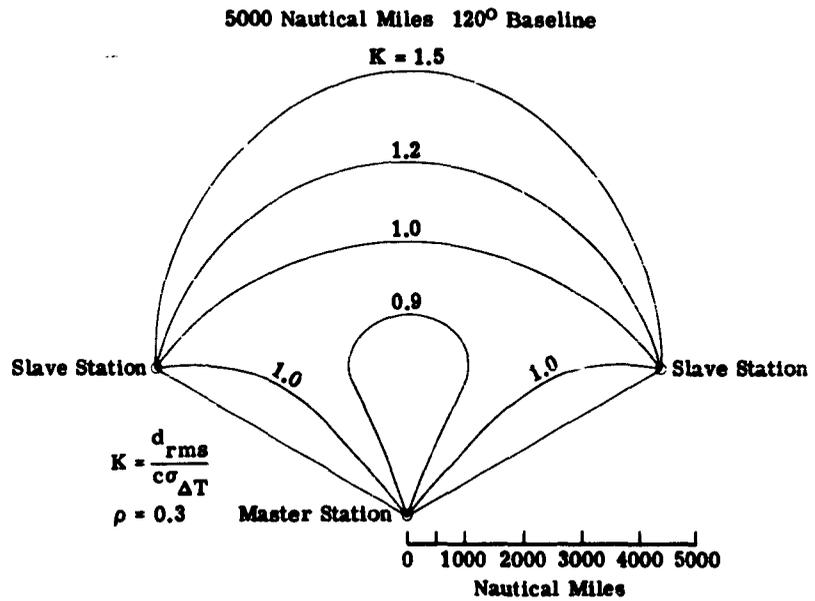


FIGURE 91. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

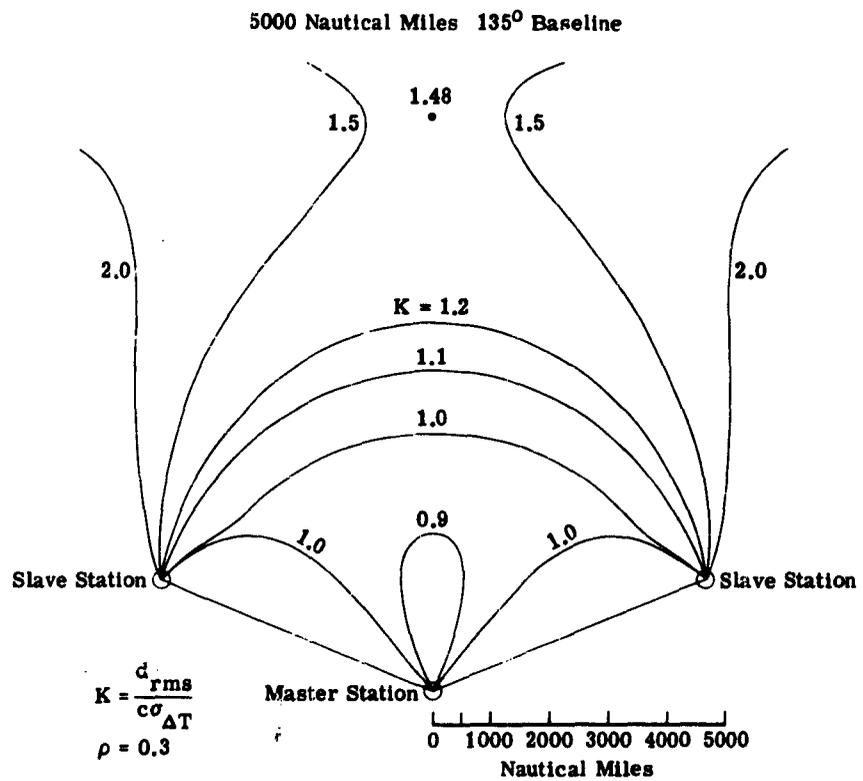


FIGURE 92. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

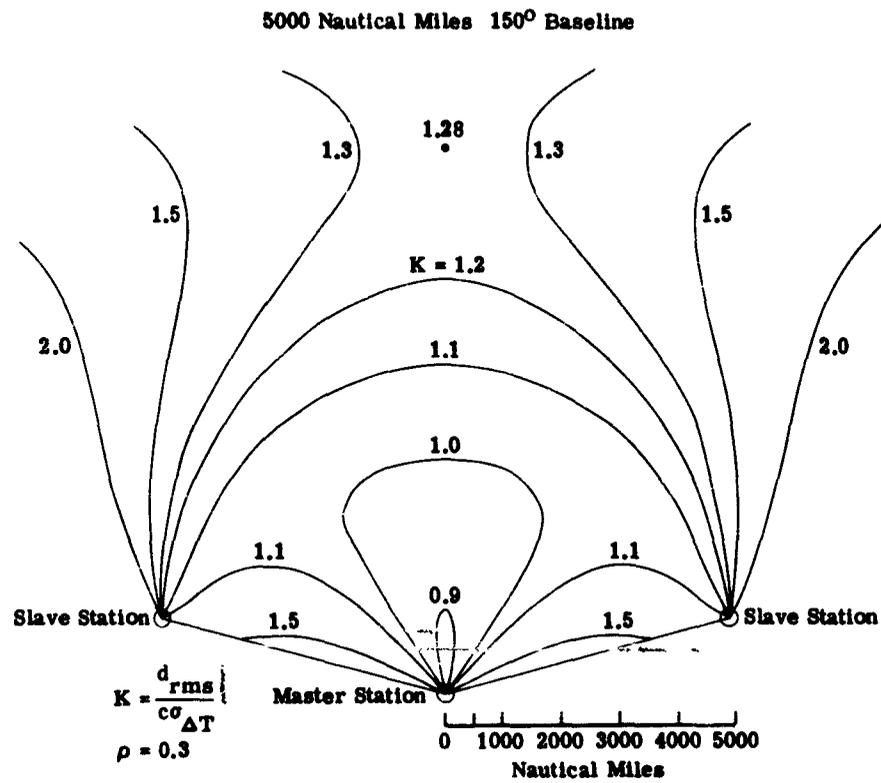


FIGURE 93. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

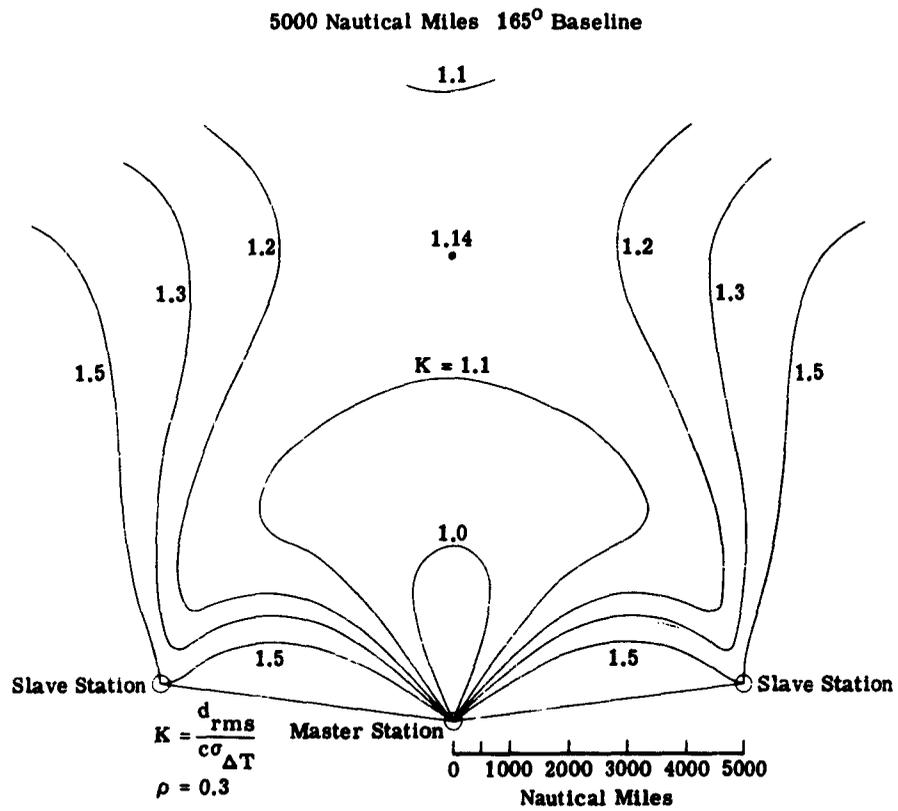


FIGURE 94. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

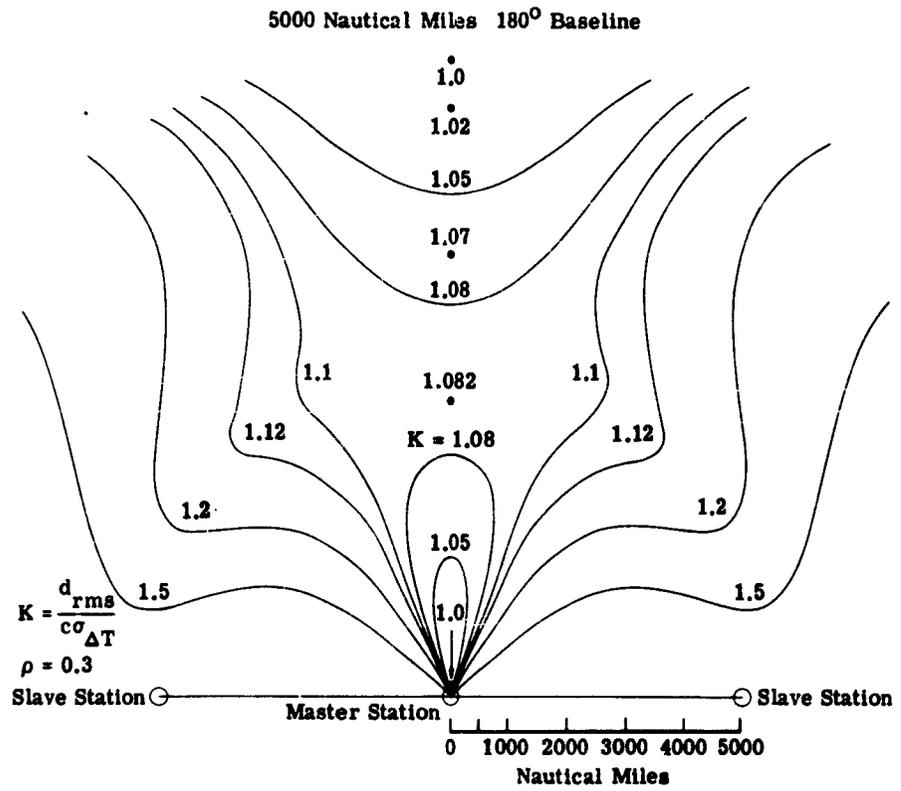


FIGURE 95. d_{rms} -ERROR ISOGRAM FOR A HYPERBOLIC NAVIGATION SYSTEM ON A SPHERICAL EARTH

D.3. COMPUTATION OF ACCURACY CONTOURS WITH AN ANALOG COMPUTER

The analog computer used in the solution of this problem operated in the following manner. An appropriate d_{rms} value was set into the computer; then as one parameter was changed, the computer chose the other parameter to make the d_{rms} a constant.

The plotting was direct and fast with a high degree of accuracy maintained by eliminating any forcing function having or approaching a zero slope.

With a master station M and slave stations S_1 and S_2 specified, we know triangle S_2MS_1 of Figure 96. By adjusting r and C_1 or C_2 where

$$C_1 + C_2 = C = \angle S_2MS_1 \tag{218}$$

the d_{rms} error of position determination of point P can be computed for any position of point P coupled with any chosen configuration of triangle S_2MS_1 . By placing triangle S_2MS_1 of Figure 96 on the surface of a sphere, such as the earth, we can determine the d_{rms} error for any point on the surface of the sphere.

Figure 97 shows this triangle on the surface of a sphere where now the sides of the triangles are the great circle route between two vertices of the triangle, and the three vertices of the triangle no longer add up to 180° . Triangle MPS_1 is redrawn in Figure 98 for analysis by spherical trigonometry. Specified in the triangle are base line b_1 , angle C_1 , and range r, leaving angle $2\phi_1$ to be determined for the solution of d_{rms} .

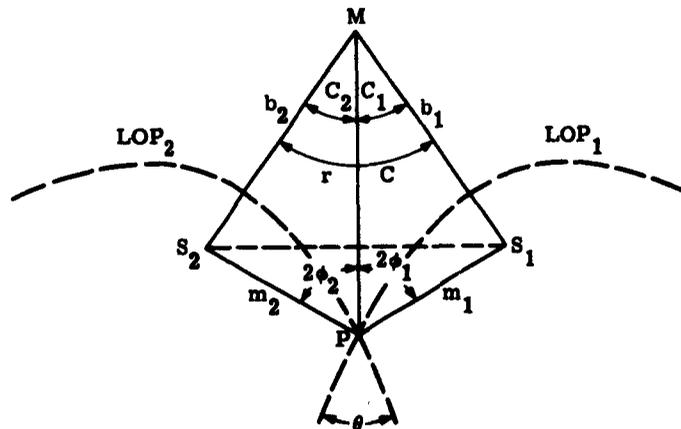


FIGURE 96. INTERSECTION OF LOP'S 1 AND 2 WHICH DETERMINE POSITION OF POINT P

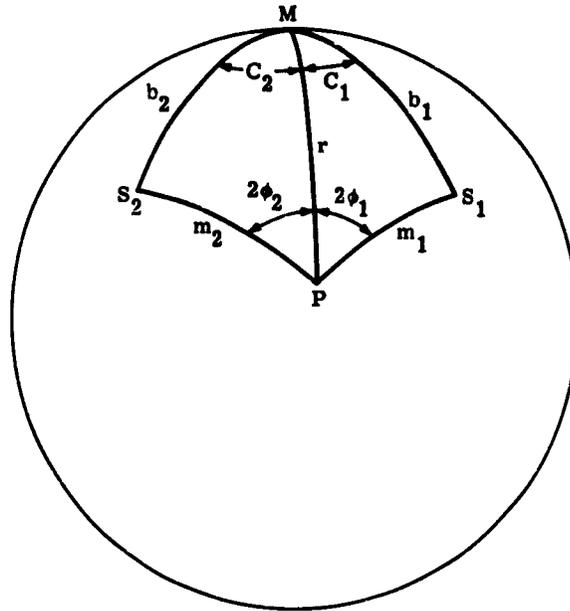


FIGURE 97. TRIANGLE S_2MS_1 LOCATED ON A SPHERICAL EARTH

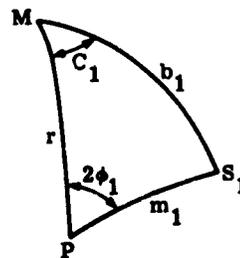


FIGURE 98. TRIANGLE MPS_1 OF FIGURE 91

Spherical trigonometry equations for solving this triangle are

$$\frac{\sin C_1}{\sin m_1} = \frac{\sin S_1}{\sin r} = \frac{\sin 2\phi_1}{\sin b_1} \quad (219)$$

$$\cos m_1 = \cos r \cos b_1 + \sin r \sin b_1 \cos C_1 \quad (220)$$

$$\cos r = \cos m_1 \cos b_1 + \sin m_1 \sin b_1 \cos S_1 \quad (221)$$

$$\cos b_1 = \cos m_1 \cos r + \sin m_1 \sin r \cos 2\phi_1 \quad (222)$$

$$\cos 2\phi_1 = -\cos C_1 \cos S_1 + \sin C_1 \sin S_1 \cos b_1 \quad (223)$$

$$\cos C_1 = -\cos 2\phi_1 \cos S_1 + \sin 2\phi_1 \sin S_1 \cos m_1 \quad (224)$$

$$\cos S_1 = -\cos 2\phi_1 \cos C_1 + \sin 2\phi_1 \sin C_1 \cos r \quad (225)$$

From Identity 219 extract the equation

$$\sin m_1 \sin 2\phi_1 = \sin b_1 \sin C_1 \quad (226)$$

and Equation 222 rearranged gives

$$\sin m_1 \sin r \cos 2\phi_1 = \cos b_1 - \cos m_1 \cos r \quad (227)$$

Substituting Equation 220 for unknown $\cos m_1$ in the right hand part of Equation 227 gives

$$\sin m_1 \sin r \cos 2\phi_1 = \cos b_1 - \cos^2 r \cos b_1 - \cos r \sin r \sin b_1 \cos C_1$$

which can be reduced to

$$\sin m_1 \sin r \cos 2\phi_1 = \cos b_1 (1 - \cos^2 r) - \cos r \sin r \sin b_1 \cos C_1$$

or

$$\sin m_1 \sin r \cos 2\phi_1 = \cos b_1 \sin^2 r - \cos r \sin r \sin b_1 \cos C_1 \quad (228)$$

Dividing by $\sin r$ in Equation 228 produces

$$\sin m_1 \cos 2\phi_1 = \cos b_1 \sin r - \cos r \sin b_1 \cos C_1 \quad (229)$$

which when combined with

$$\sin m_1 \sin 2\phi_1 = \sin b_1 \sin C_1 \quad (230)$$

produces two equations having the same form as the plane trigonometry pair of

$$\rho \sin a = Y \quad (231)$$

and

$$\rho \cos a = X \quad (232)$$

where $\rho_1 = \sin m_1$

$$a_1 = 2\phi_1$$

$$X_1 = \cos b_1 \sin r - \cos r \sin b_1 \cos C_1$$

$$Y_1 = \sin b_1 \sin C_1$$

Equations 228 and 229 readily lend themselves to plane trigonometry from which we obtain the standard analog computer rectangular to polar transformation

$$X \sin a - Y \cos a = 0 \quad (233)$$

or

$$X_1 \sin 2\phi_1 - Y_1 \cos 2\phi_1 = 0 \quad (234)$$

Equation 234 makes an ideal forcing function in that when $\sin 2\phi_1$ has a zero slope, $\cos 2\phi_1$ has a maximum slope; and when $\cos 2\phi_1$ has a zero slope, $\sin 2\phi_1$ has a maximum slope. Hence, the accuracy of the determination of $2\phi_1$ will be good for the entire 360° of a circle with a degeneration in accuracy occurring only when X_1 and Y_1 approach zero at the same time.

D.4. ANALOG COMPUTER CIRCUIT

The analog computer circuitry for the solution and plotting of constant d_{rms} error contours is shown in the block diagram of Figure 99. The operating principle is as follows.

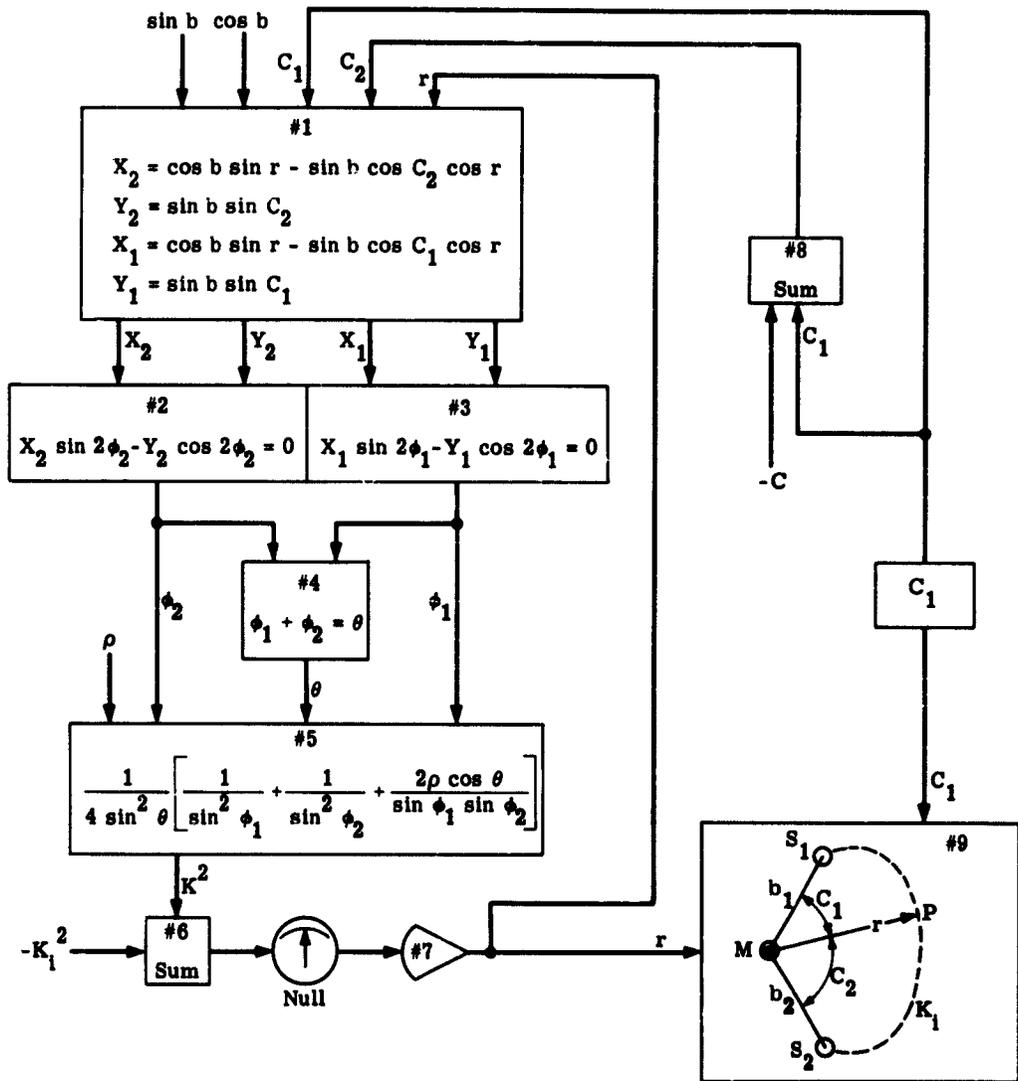


FIGURE 99. COMPUTER BLOCK DIAGRAM FOR SOLVING AND PLOTTING CONSTANT $d_{rms}(K_1)$ ERROR CONTOURS

- (a) Block #1 combines the necessary sines and cosines to produce X_1 , X_2 , Y_1 , and Y_2 .
(NOTE: Subscripts are not shown on b and r , because in this problem $b_1 = b_2$ while side r is common to both triangles.)
- (b) Block #2 and Block #3 are simulated polar resolvers which force the proper $2\phi_1$ and $2\phi_2$ to satisfy Equation 234.

$$X_1 \sin 2\phi_1 - Y_1 \cos 2\phi_1 = 0 \quad (235)$$

- (c) The values of ϕ_1 and ϕ_2 are summed in Block #4 to produce θ .
- (d) Block #5 operates on ϕ_1 , ϕ_2 , and θ plus an added ρ to produce the value K^2 by solving (see Equation 213)

$$\frac{d_{\text{rms}}^2}{c^2 \sigma^2} = K^2 = \frac{1}{4 \sin^2 \theta} \left[\frac{1}{\sin^2 \phi_1} + \frac{1}{\sin^2 \phi_2} + \frac{2\rho \cos \theta}{\sin \phi_1 \sin \phi_2} \right] \quad (236)$$

- (e) A value $-K_1^2$ is chosen as a desired value to be plotted for K^2 . This value is compared in Block #6 with the value of K^2 that was produced in Block #5, and the difference is shown on a null meter.
- (f) Block #7 is a high-gain amplifier (HG) which insists on its input being equal to zero. By feeding the difference $K^2 - K_1^2$ from summing Block #6 into this high-gain amplifier and using its output as the value of r , this output will be adjusted so that $K^2 - K_1^2 = 0$.
- (g) From the triangle S_1MS_2 of Figure 91 formed by the location of the master station M and slave stations S_1 and S_2 , the angle $C = \angle S_1MS_2$ is chosen for the configuration in question. The negative of this quantity is fed to summing Block #8, which compares this value with C_1 to produce C_2 at its output. $C = C_1 + C_2$
- (h) Block #9 is a plotting table where the constant K_1 or $\frac{d_{\text{rms}}}{c\sigma}$ is plotted relative to the previous manually drawn master station, the two slave stations, and the connecting base lines.
- (i) The values for C_1 , C_2 , and r along with chosen $\sin b$ and $\cos b$ are fed back to Block #1 to complete the loop.

The actual operation of the computer can be done in a number of ways. C_1 can be a gradually increasing value from an integrator, with the proper r being produced by high-gain amplifier #7 of Figure 90. But because of discontinuities found in the curves, the procedure was to choose a C_1 value, vary r until a null was found on the meter, and then plot that point. In this way it was possible to pass a discontinuity and find the curve again. Also, it was possible to

search suspected areas without needing a constant null, thereby avoiding the possibility of missing some of the curve.

The actual analog computer circuitry for generating the values of the block diagram of Figure 99 is as follows.

(1) The generation of $Y_1 = \sin b \sin C_1$ is shown in Figure 100 where the proper voltage representing the angle C_1 is fed to diode function generator (DFG) which has been adjusted to produce at its output the function $\sin C_1$. The base line between the master station and slave station S_1 does not change in value; hence $\sin b$ is computed and manually set on a multiplying potentiometer, so that the output of this potentiometer is $\sin C_1 \sin b = Y_1$.

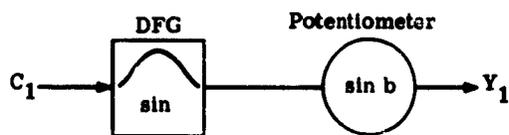


FIGURE 100. COMPUTER CIRCUIT FOR GENERATING Y_1

(2) The generation of $X_1 = \cos b \sin r - \sin b \cos C_1 \cos r$ is shown in Figure 101 where three diode function generators, two potentiometers, a servo multiplier and a summing amplifier are used to complete the generation of X_1 . The upper path generates $-\cos C_1 \sin b$, which is one input to the multiplier, while the center path produces $\cos r$, which is the other input. The output of the multiplier is then $-\cos C_1 \sin b \cos r$. The output of the lower path $\sin r \cos b$ is

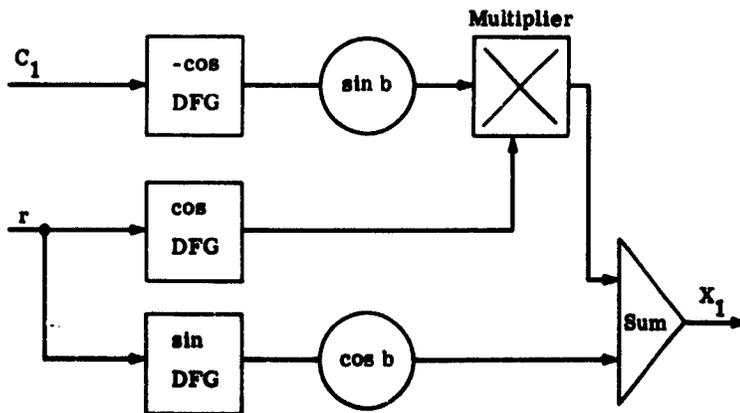


FIGURE 101. COMPUTER CIRCUIT FOR GENERATING X_1

summed with the output of the multiplier by a summing amplifier to produce $X_1 = \sin r \cos b - \cos C_1 \sin b \cos r$. Similar circuitry generates X_2 and Y_2 .

(3) The circuitry used to generate $2\phi_1$ from the values X_1 and Y_1 is shown in Figure 102 and comprises two diode-function generators, two servo multipliers, and a high-gain amplifier.

The equation

$$X_1 \sin 2\phi_1 - Y_1 \cos 2\phi_1 = 0$$

is used here to force the high-gain amplifier to produce $2\phi_1$ at its output. If we assume that this output is already $2\phi_1$, then DFG #1 will generate $\sin 2\phi_1$ for one input to multiplier #1. With X_1 as the other input, multiplier #1 produces $(X_1 \sin 2\phi_1)$ for the upper input to the high-gain amplifier (HG). Similarly, the lower path generates $-Y_1 \cos 2\phi_1$ for the lower input to the amplifier. Since the total input to the high-gain amplifier $X_1 \sin 2\phi_1 - Y_1 \cos 2\phi_1$ is zero, the output of the high-gain amplifier will continually adjust its output $2\phi_1$ so that its input remains equal to zero, thus the high-gain amplifier has been forced to produce $(2\phi_1)$. The quantity $2\phi_2$ is produced by a similar circuit.

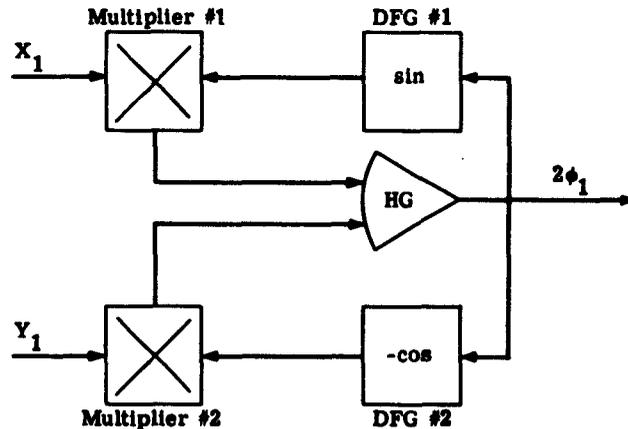


FIGURE 102. COMPUTER CIRCUIT FOR GENERATING $2\phi_1$

(4) The generation of

$$\frac{d_{rms}^2}{c^2 \sigma^2} = \frac{1}{4 \sin^2 \theta} \left[\frac{1}{\sin^2 \phi_1} + \frac{1}{\sin^2 \phi_2} + \frac{2\rho \cos \theta}{\sin \phi_1 \sin \phi_2} \right] \quad (237)$$

is accomplished by multiplying the $2\phi_1$ and $2\phi_2$ by $1/2$ to produce the desired ϕ_1 and ϕ_2 . θ is obtained by summing ϕ_1 and ϕ_2 to satisfy the equation $(\phi_1 + \phi_2 = \theta)$. By the use of diode-

function generators $\sin \theta$, $\cos \theta$, $\sin \phi_1$, and $\sin \phi_2$ are obtained; thus we have all the terms necessary to solve Equation 236 for the determination of d_{rms} error. Because of the many terms in the equation, Figure 103 is the complete circuit diagram for the solution of Equation 237. $\sin \phi_1$, $\sin \phi_2$, and $\sin \theta$ are fed directly to servo multipliers SM1, SM2, SM3, respec-

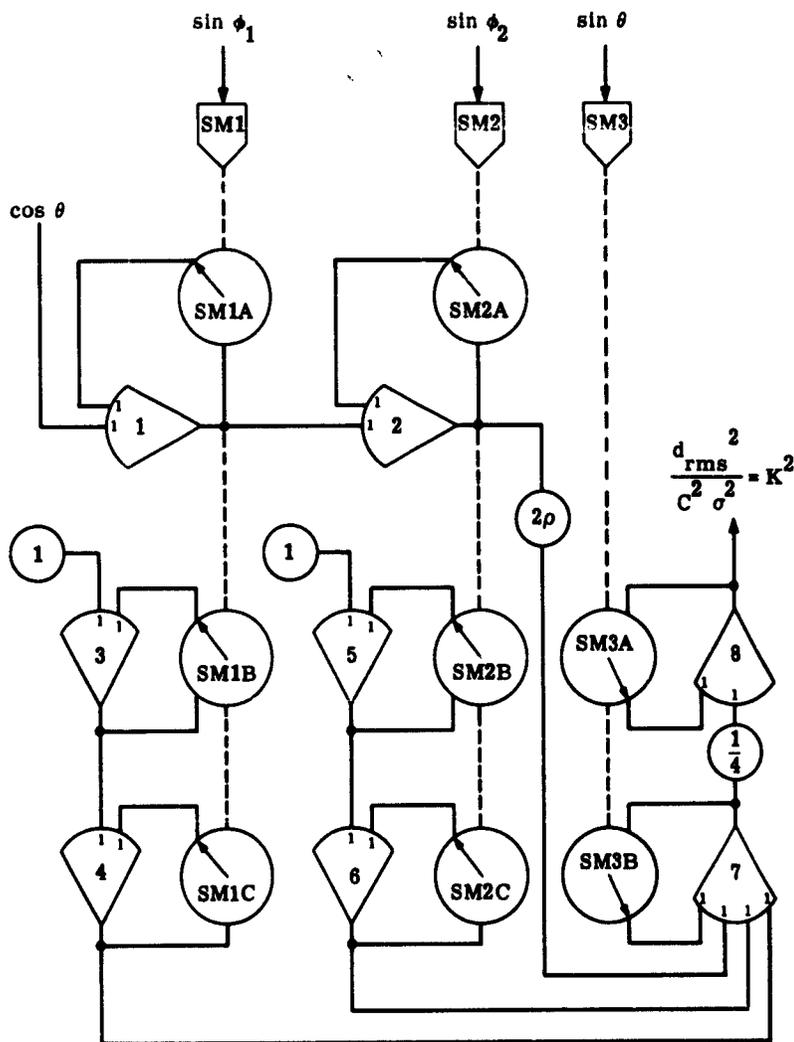


FIGURE 103. COMPUTER CIRCUIT FOR SOLVING

$$\frac{d_{rms}^2}{C^2 \sigma^2} = \frac{1}{4 \sin^2 \theta} \left[\frac{1}{\sin^2 \phi_1} + \frac{1}{\sin^2 \phi_2} + \frac{2\rho \cos \theta}{\sin \phi_1 \sin \phi_2} \right]$$

tively, which in turn position banks of multiplying potentiometers to multiply the potentiometer input by its respective sine function setting.

In Figure 104, high-gain amplifier #1 is shown in detail along with servo-set potentiometer SM1A. The action is as follows. The output of amplifier #1 ($-E_o$) is multiplied by the

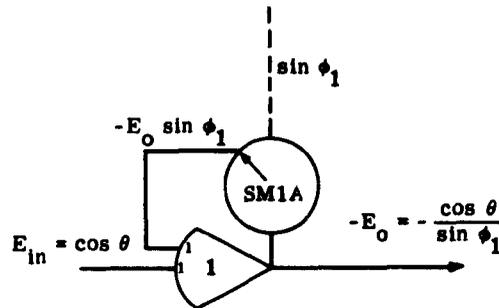


FIGURE 104. ANALOG HIGH-GAIN AMPLIFIER WITH A SERVO-SET POTENTIOMETER IN THE FEEDBACK PATH FOR DIVISION

$\sin \phi_1$ setting of potentiometer SM1A to produce $-E_o \sin \phi_1$ as the feedback value around amplifier #1. Remembering that the inputs of a high-gain amplifier must add to zero and that the sign is inverted by an amplifier, we find

$$-E_o \sin \phi_1 + \cos \theta = 0$$

or

$$E_o \sin \phi_1 = \cos \theta$$

Dividing both sides by $\sin \phi_1$ produces

$$E_o = \frac{\cos \theta}{\sin \phi_1}$$

Thus, in Figure 103 $\cos \theta$ is divided by $\sin \phi_1$ by amplifier 1 and servo-set potentiometer SM1A. By a similar arrangement of amplifier 2 and servo-set potentiometer SM2A this output of amplifier 1 is further divided by $\sin \phi_2$ at the output of amplifier 2, and this in turn is multiplied by 2ρ by the manually set potentiometer marked 2ρ such that the output of this circuit is now $\frac{2\rho \cos \theta}{\sin \phi_1 \sin \phi_2}$ (the last term in the bracket of Equation 237). The circuit of amplifiers 3 and 4 divides by $\sin \phi_1$ twice to produce $\frac{1}{\sin^2 \phi_1}$ while amplifiers 5 and 6 produce $\frac{1}{\sin^2 \phi_1}$. The

sum of these three terms comprises the entire bracketed quantity of Equation 237 and is summed at the input of amplifier 7. Amplifier 7, potentiometer 1/4, and amplifier 8 multiply the bracketed quantity by $\frac{1}{4 \sin^2 \theta}$ which completes the solution for $\frac{d_{rms}^2}{c^2 \sigma^2}$ at the output of amplifier 8.

From here $\frac{d_{rms}^2}{c^2 \sigma^2}$ or K^2 is compared with manually set K_1^2 to generate range vector r for plotting in the previously described circuit of Figure 99.

D.5. CONCLUSIONS

The practice of avoiding any forcing function having or approaching a zero slope was maintained even to finding a null on the null meter manually. When a point was reached in the plotting of constant d_{rms} or K where $\left| \frac{dK}{dr} \right|$ was less than $\left| \frac{dK}{dC_1} \right|$, then the practice of setting C_1 and nulling with r was changed to setting in r and nulling with C_1 . It should be mentioned that this practice broke down in a few plots having a maximum and/or a minimum d_{rms} because for these isolated areas the slopes $\left| \frac{dK}{dr} \right|$ and $\left| \frac{dK}{dC_1} \right|$ approached zero at the same time. With the exception of the maximum and minimum d_{rms} areas the nulling technique was fast and definite.

Appendix E
DESCRIPTION OF THE OMEGA AND DELTA VERY-LOW-FREQUENCY
HYPERBOLIC RADIO NAVIGATION SYSTEMS

E.1. OMEGA

The OMEGA system is a time-shared cw hyperbolic phase comparison system operating in the 10- to 14-kc navigation band. The particular implementation described here was that envisioned in 1958 and presented in Reference 37. Since then certain minor changes have been made which will be mentioned later. The system is designed to provide world-wide coverage with about six to eight transmitters on baselines of about 5000 nautical miles. Each transmitter radiates the same time-shared frequency. The receiver compares the phase of the received signal from each station to the phase of a local oscillator. Each phase comparison is stored by a servo until it can be compared with that from another station and thus provide a measurement of phase between the two received signals. Each phase comparison determines an LOP. The lines of position thus determined are spaced about 7.9 nautical miles apart on the baseline for a 10.2-kc primary frequency (f_1). The 7.9-mile ambiguities would be resolved by the transmission of a second frequency (f_2) from each station. The receiver could then make a "coarse" measurement on the difference frequency. The ambiguities of the "coarse" readings would be spaced along the baseline by

$$d = \frac{c}{2(f_1 - f_2)} \tag{238}$$

If $c = 162,000$ nautical miles/second and $f_1 - f_2 = 1000$ cps, the ambiguities would be spaced by approximately 81 nautical miles, a distance that could be resolved by dead reckoning.

A station sequence providing eight-station coverage of the earth and two-frequency lane resolution is shown in Figure 105. The pattern of pulse deviations provides synchronization

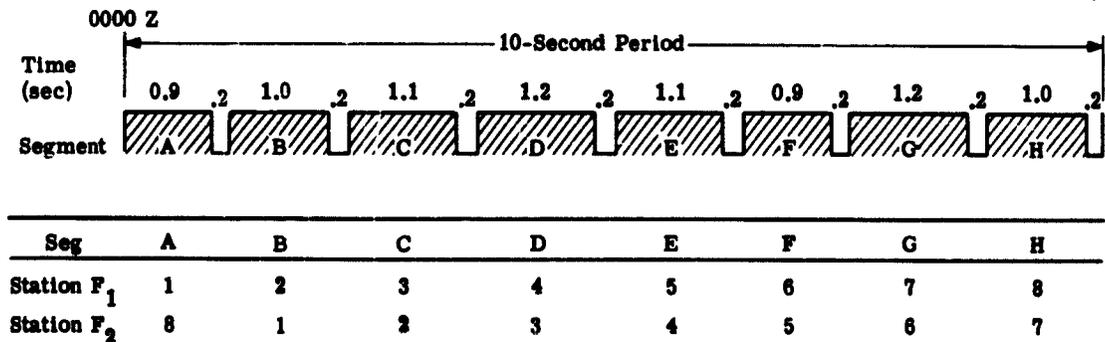


FIGURE 105. OMEGA TRANSMISSION SEQUENCE

information for the receivers. Since each station transmits only one frequency at a time, the transmitters and antennas can be sequentially tuned and thus serve both frequencies. A block diagram of the master station is shown in Figure 106. Each slave station is similar to the master except that f_1 and f_2 at the slaves are phase-locked to the master as shown in Figure 107.

A block diagram of the navigation receiver is shown in Figure 108. As stated before, the phase of each signal is compared with the local oscillator before the phase difference between the two signals is taken. If the phase of the f_m master signal is θ_m , the phase of the slave signal is θ_s , and the phase of the local oscillator is ψ , the outputs from the two phase shifters in the f_1 channel are

$$\phi_m = \theta_m - \psi \tag{239}$$

$$\phi_s = \theta_s - \psi \tag{240}$$

After the outputs of the phase shifters are subtracted in the differential, however, the measured phase difference between signals will be

$$\Delta = \phi_m - \phi_s = \theta_m - \theta_s \tag{241}$$

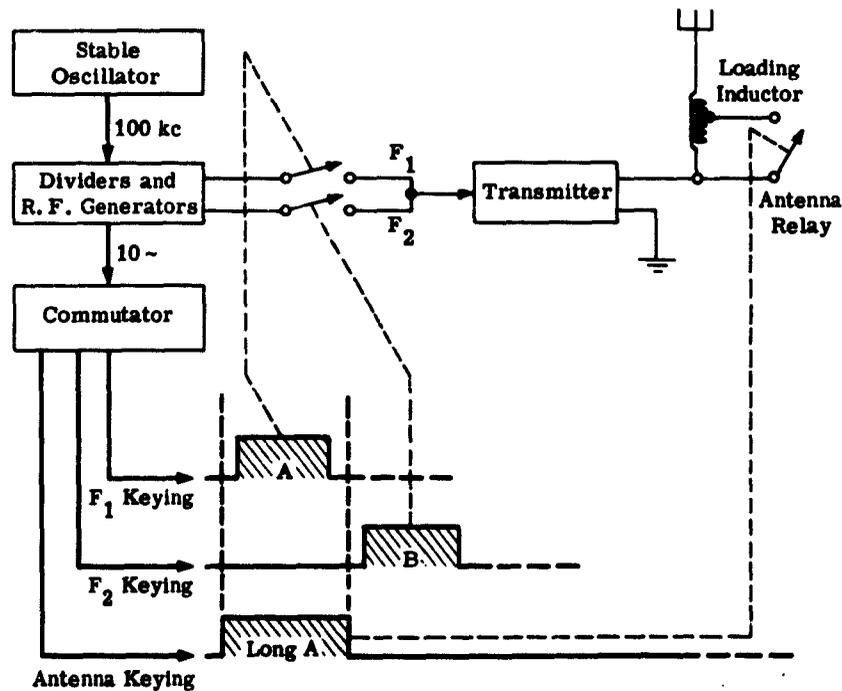


FIGURE 106. OMEGA MASTER STATION BLOCK DIAGRAM

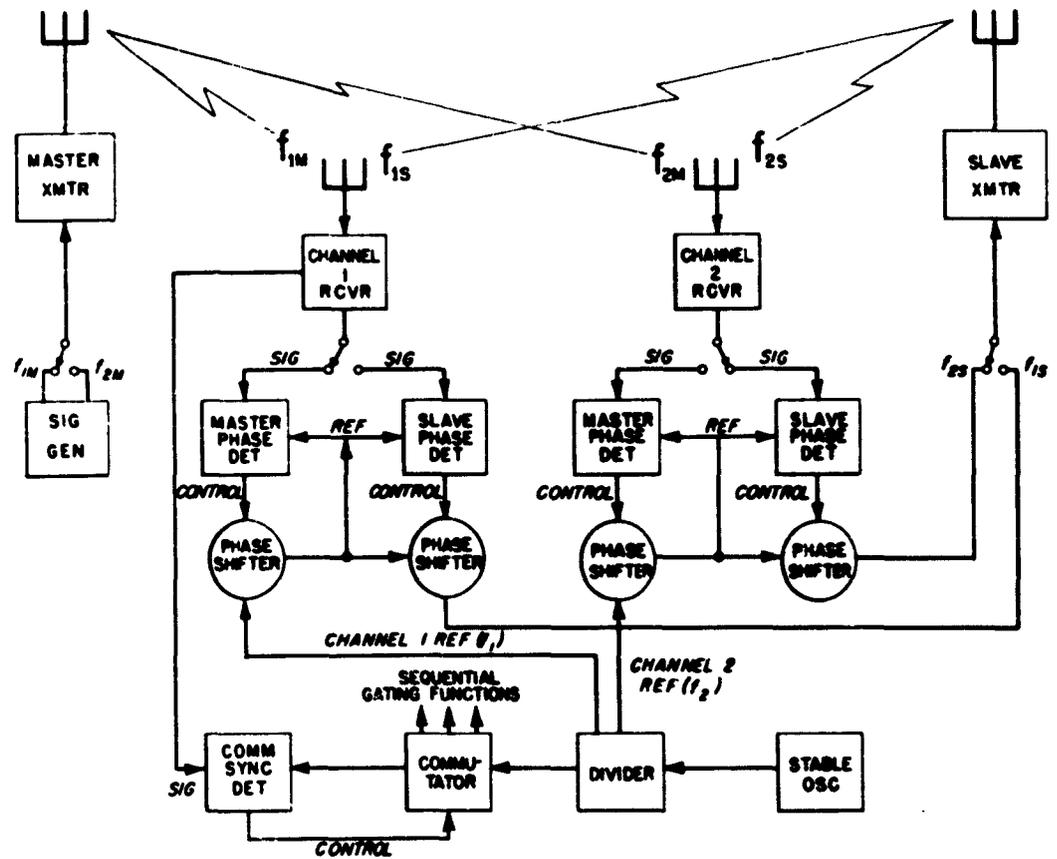


FIGURE 107. OMEGA SLAVE-STATION SYNCHRONIZER BLOCK DIAGRAM

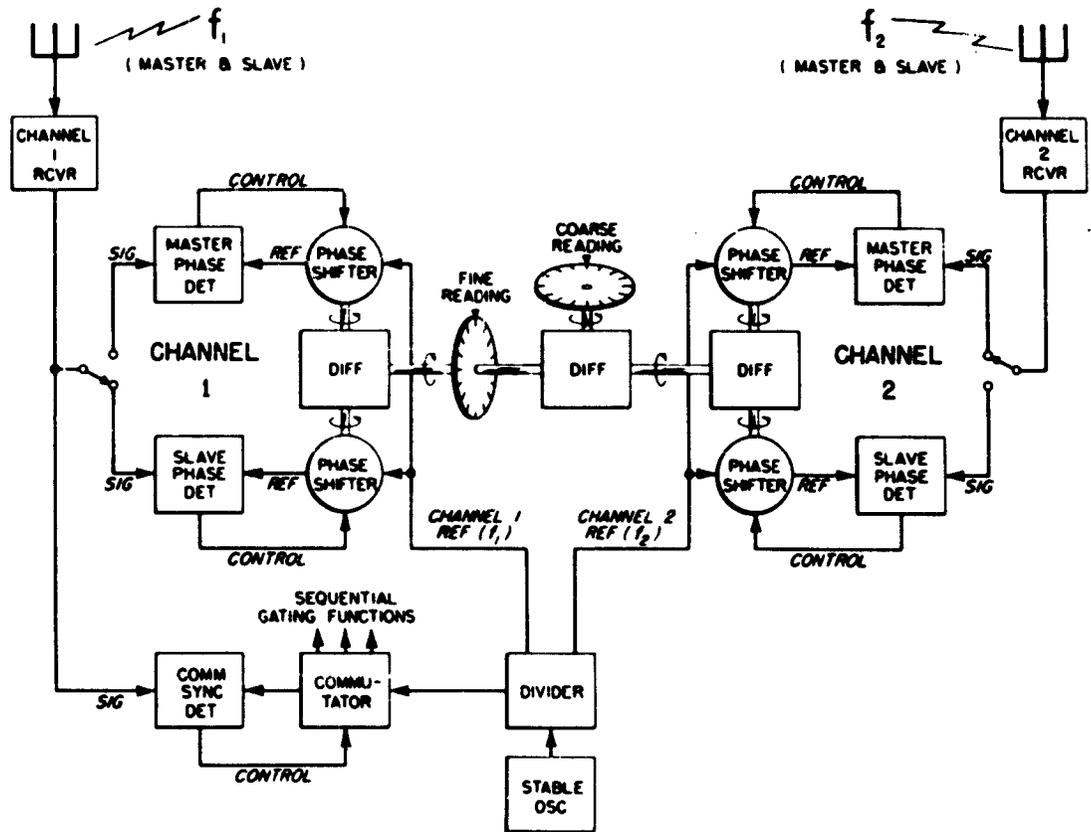


FIGURE 108. OMEGA NAVIGATION RECEIVER BLOCK DIAGRAM

The local oscillator phase has now cancelled out. If the local oscillator is off frequency, however, ψ will change between station sampling periods. Since the sampling periods are up to 10 seconds apart, local oscillator drift of 1 part in 10^7 will cause an error in $\Delta\theta$ of

$$\epsilon_{\Delta\theta} = 10 \times 10^{-7} \times 10^6 = 1 \mu\text{sec} \quad (242)$$

The phase-shifter servo systems are rate-aided so that the phase shifters will track the drift of ψ , and the resulting error is actually near zero as long as the servos can track the local oscillator drift. Since the rate of change of the local oscillator phase is small compared to the signal phase rate, this is easily done. The oscillator drift appears similar to a change in vehicle velocity, and the oscillator stability specifications are relaxed in accordance with receiver bandwidth requirements and servo characteristics. Either the f_1 or f_2 phase difference can be used as the fine reading; the choice is arbitrary. The baseline lanewidths will be different, of course, and are related in the following manner

$$\frac{f_1 \text{ lanewidth}}{f_2 \text{ lanewidth}} = \frac{f_2}{f_1} = 1 + \frac{f_2 - f_1}{f_1} \quad (243)$$

If $\frac{f_2 - f_1}{f_1}$ is small, the lanewidths are nearly equal. It should be noted that each LOP is actually sampled at a 10-second rate. While this means that the phase servos are data sampling systems, they will nevertheless provide a continuous indication if rate information is available either as part of the servo or externally.

E.2. THE AIRBORNE OMEGA RECEIVER

The Naval Research Laboratory has developed and built the only existing model of an airborne OMEGA receiver [40-45]. The model has recently been used on test program flights to South America. The airborne model deviates somewhat from the earlier description given above. The eight-station, 10-second transmission sequence has been reduced to a four-station, 5-second sequence. At present three stations are operating on 10.2 kc and are located at Haiku, Hawaii (slave); Summit, Canal Zone (Balboa) (master); Forestport, New York (slave). The Forestport station transmits twice during each sequence, thus giving the four-station sequence; the synchronizer of the airborne receiver has been designed for this sequence although it is not restricted to 10.2 kc. Expansion of the receiver synchronizer to include a larger sequence would not add greatly to the complexity of the unit. A more serious limitation of the synchronizer is the lack of search mode. Initially the receiver is synchronized with the stations manually and maintains the synch by oscillator stability, which will maintain synch for periods of nearly one

week. An automatic synchronizer which allegedly will establish and maintain synch has been built, but it is not used because the complexity of the receiving equipment is nearly doubled and a signal-to-noise ratio of more than one is required for reliable operation.²²

Lane identification has also been omitted from the airborne model presumably for three reasons:

- (a) NRL thinks that lane information can be obtained by continuous tracking from a known initial position.
- (b) The necessary circuitry would double or triple the size and complexity of the receiver.
- (c) The reliability of lane resolution using multiple frequencies has not been demonstrated.

The maximum aircraft velocity that can be accommodated by the receiver is 1200 knots. However, to operate at this velocity, the receiver must be supplied with rate information manually. This is done by setting dials to the indicated airspeed and heading of the aircraft in relation to the hyperbolic position lines. The acceptable error of rate information is ± 240 knots radial from any station. If the receiver is given no rate information, the maximum aircraft velocity is 240 knots.

The Navy expects to build models with automatic rate inputs coupled to standard flight instruments. The output of the receiver is the hyperbolic lane number of each of two position lines presented on counter type readouts and on a strip chart recorder. The least count of each output is 0.01 lane.

The total size of the NRL receiver is about 3 cu ft, and it weighs about 60 pounds. Power requirements are 100 watts of 400 cps primary power. It appears to us that the NRL receiver could be repackaged in about half its present volume. The present model was constructed in a laboratory, and evidently no attempt was made to fully use cabinet space.

E.3. DELRAC

The DELRAC system, a proposal of the Decca Navigator Company, Ltd., is based on the DECCA and DECTRA systems and uses techniques from both [79]. The system philosophy for DELRAC is almost identical to OMEGA, but the systems differ largely in instrumentation technique. In the DELRAC system each LOP would be obtained by a set of measurements on one station pair (master and slave). The second LOP would use a separate pair. Decca Navigator

²²Personal communication with A. F. Thornhill and R. J. Lampkin of NRL.

Company, Ltd., envisions 12 such pairs providing world-wide coverage with an accuracy of 10-nautical-mile maximum error at the 95% reliability level. The DELRAC system would also operate in the 10- to 14-kc navigation band with lanewidths of about 6.7 nautical miles. Lanes would be resolved by the use of multiple frequencies as in the originally proposed OMEGA system. The lanes would always be resolved in three steps, requiring at least two steps to enlarge the region of ambiguity to a size resolvable by dead reckoning. If the lanewidths are obtained by mixing each of several frequencies with the fundamental frequency $f = 10$ to 14 kc, and the ratio of successive lanewidths is n , then

$$F_1 - F = \frac{F}{n} \quad (244)$$

$$F_2 - F = \frac{F}{n^2} \quad (245)$$

$$F_3 - F = \frac{F}{n^3} \quad (246)$$

or

$$F_1 = F \left(1 + \frac{1}{n} \right) \quad (247)$$

$$F_2 = F \left(1 + \frac{1}{n^2} \right) \quad (248)$$

If three steps are used to reduce further the level of ambiguity

$$F_3 = F \left(1 + \frac{1}{n^3} \right) \quad (249)$$

If $F = 12$ kc, the frequencies used would be $F_1 = 16$ kc, $F_2 = 13 \frac{1}{3}$ kc, and $F_3 = 12 \frac{12}{27}$ kc, with successive lanewidths of 6.7, 20.1, 60.3, and 180.9 nautical miles.

The transmissions would be shared, as they are in OMEGA, with a sequence as shown in Figure 109. The individual phase measurements would be made with discriminators with the readout on rotating dials (deconimeters), as in the DECCA system. The lane resolution circuits would be unique, however. Rather than presenting the phase difference for each frequency difference in the readout, they would display only the "fine" readings and one "coarse" reading for the largest lane. The frequency differences corresponding to the intermediate-size lanes would be used to stabilize the circuits for the frequency difference corresponding to the largest power of n used. Thus only two deconimeters would be used to define an LOP regardless of the number of steps taken to resolve ambiguities. A block diagram illustrating this method is shown in Figure 110.

SCHEDULE

Time →

Periods →	1	2	3	4	5	6	1
{ Master A	F	F ₁	F ₂				
{ Slave B		F	F ₁	F ₂			
{ Master C			F	F ₁	F ₂		
{ Slave D				F	F ₁	F ₂	
{ Master E	F ₂				F	F ₁	F ₂
{ Slave F	F ₁	F ₂				F	F ₁

→ and so on

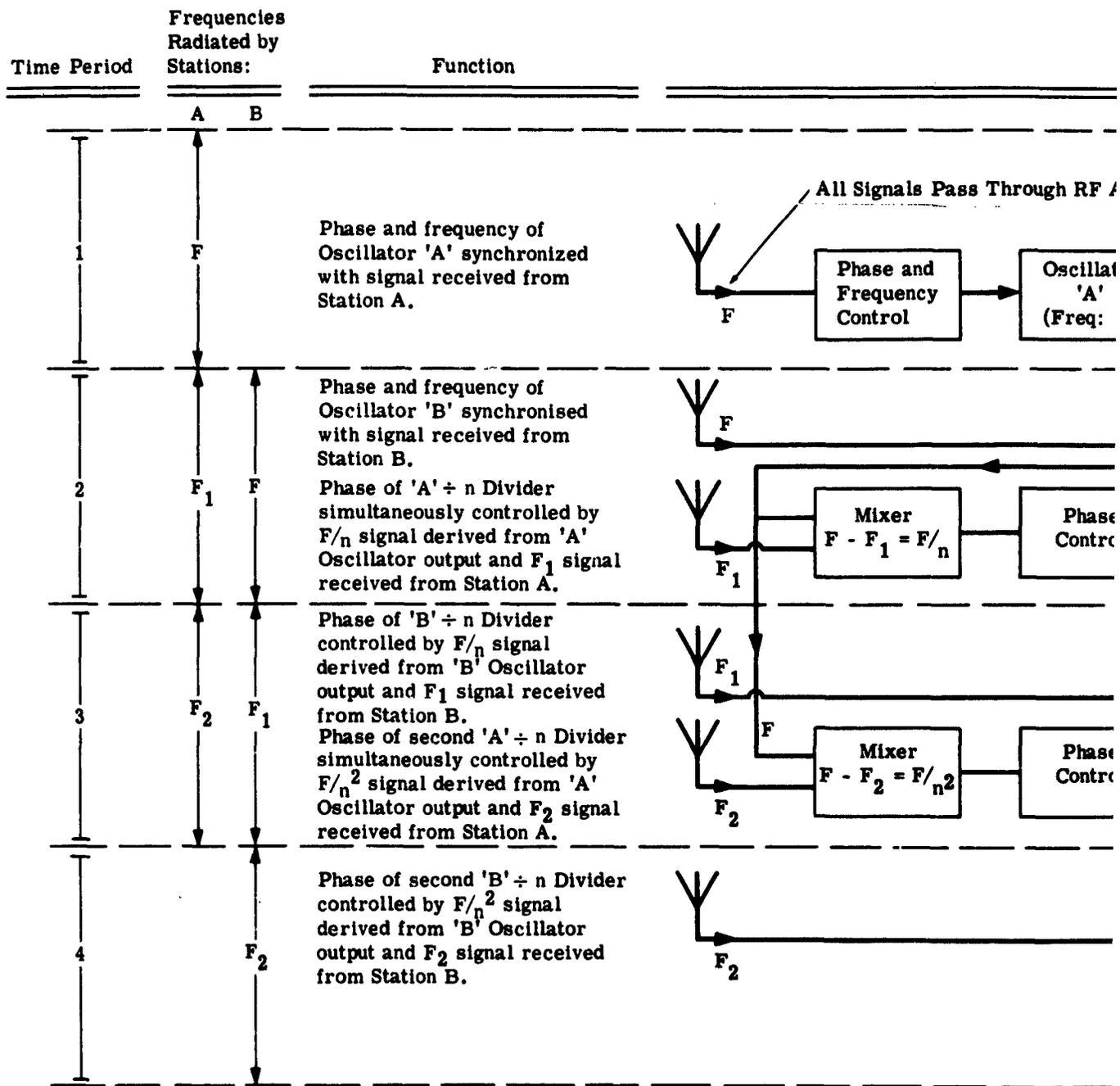
→

FIGURE 109. DELRAC TRANSMISSION SEQUENCE

The Decca Navigator Company, Ltd., envisions such a receiver with a 5-cps bandwidth operating at ranges up to 3000 nautical miles, with 5 kw radiated power.

At this time no reliable information on the physical characteristics of the DELRAC equipment is available. The following estimates were made in 1958 [80]:

- Weight of airborne equipment—88 pounds
- Cost of airborne equipment, approx.—\$8000
- Cost of one transmitter pair, approx.—\$1,000,000



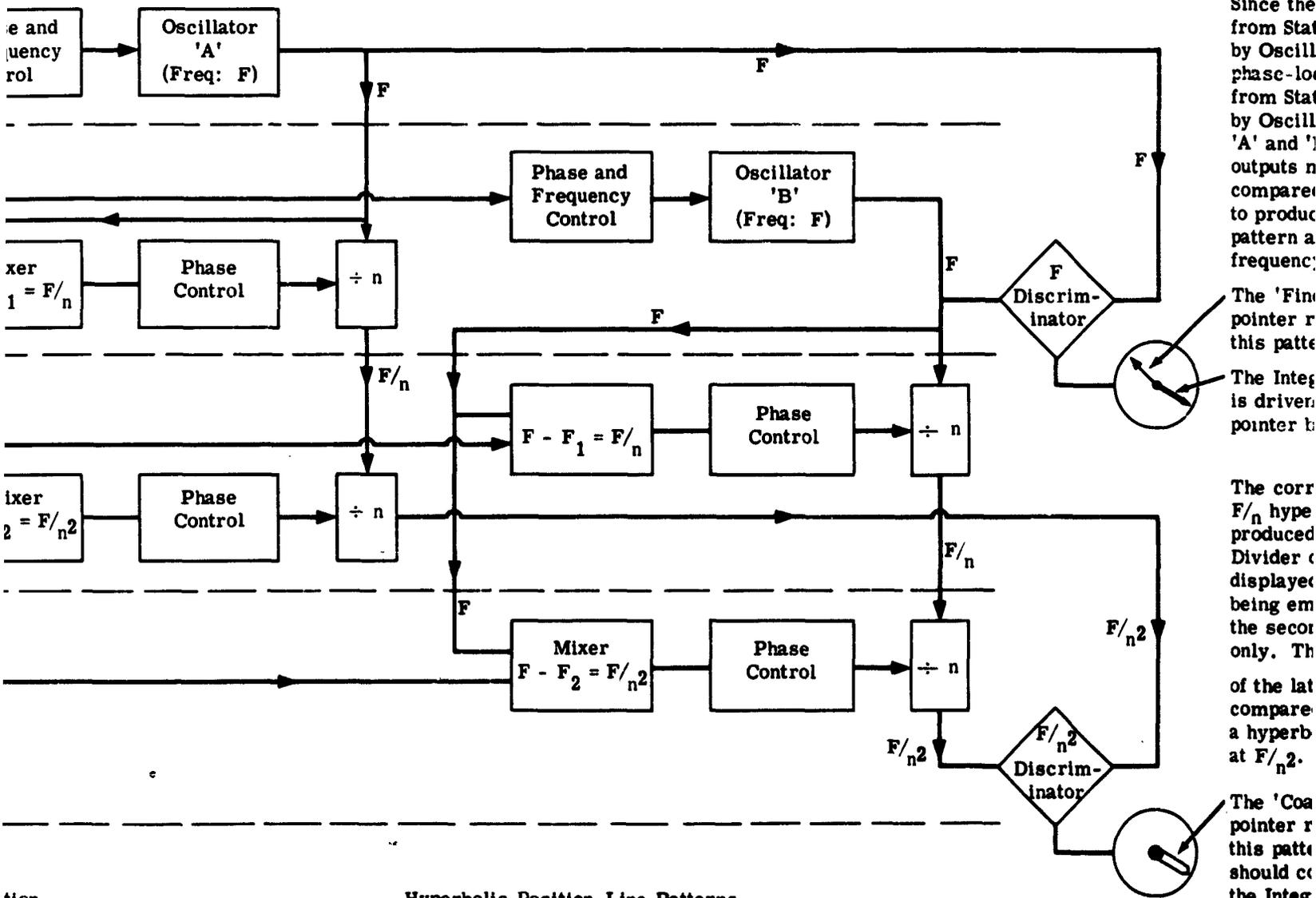
Typical Carrier Frequency Allocation
(Assuming $n = 3$)

$$\begin{aligned}
 F &= 12 \text{ kcs} \\
 F_1 &= F(1 + 1/n) = 16 \text{ kcs} \\
 F_2 &= F(1 + 1/n^2) = 13\text{-}1/3\text{rd kcs}
 \end{aligned}$$

FIGURE 110. SIMPLIFIED B

Receiver Elements Operative

signals Pass Through RF Amplifiers at this Point



Since the
from Stat
by Oscill
phase-loc
from Stat
by Oscill
'A' and 'B'
outputs n
compare
to produc
pattern a
frequency

The 'Fine'
pointer r
this patte
The Integ
is driven
pointer h

The corr
F/n hype
produced
Divider
displaye
being em
the secor
only. Th
of the lat
compare
a hyperb
at F/n².

The 'Coa
pointer r
this patte
should co
the Integ

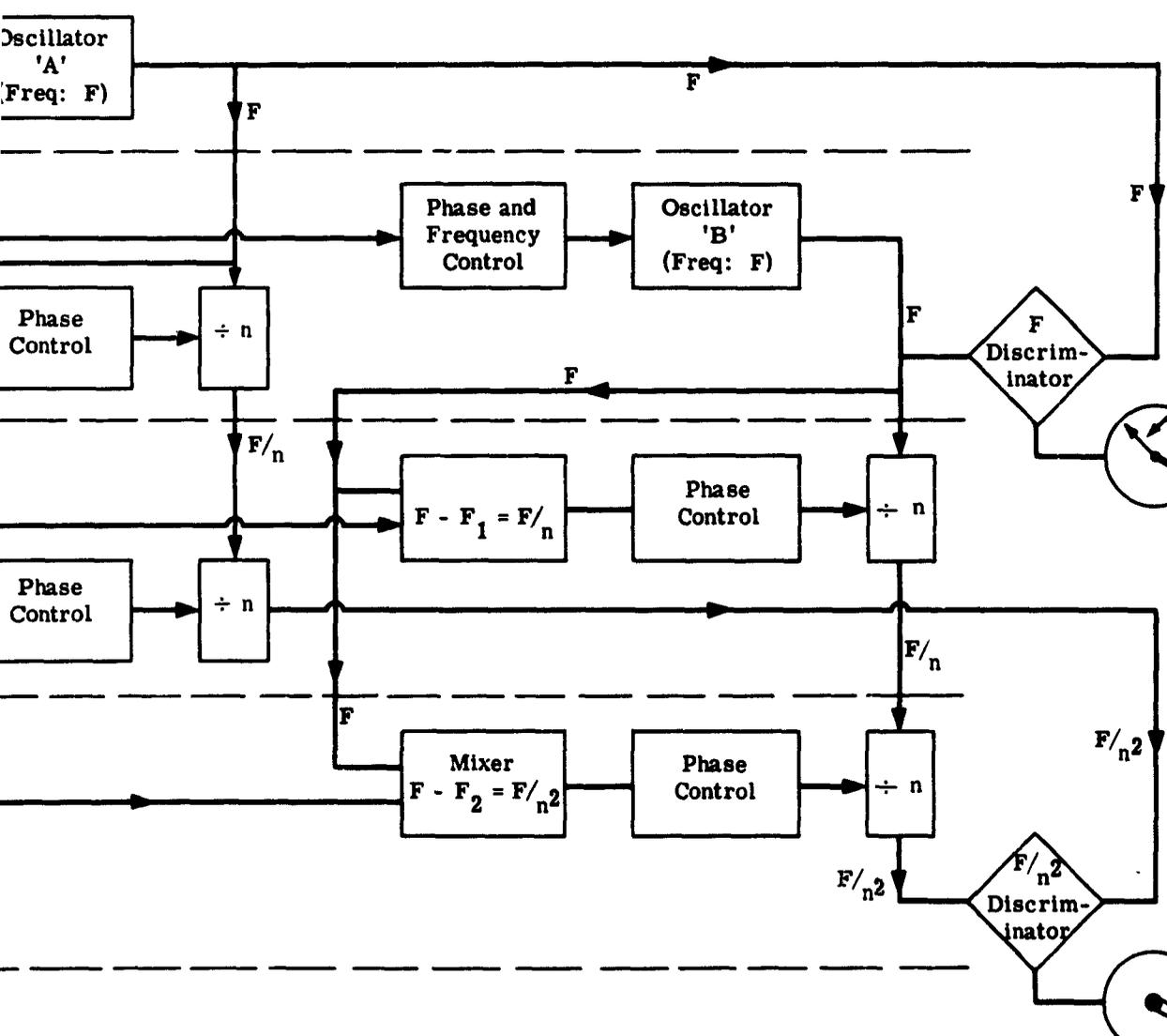
Hyperbolic Position-Line Patterns
(With F = 12 kcs)

Pattern	Comparison Frequency	Lane Width on Baseline
'Fine'	12 kcs	12.5 km (13,670 yds)
'Coarse'	1-1/3rd kcs	112.5 km (69.90 st. miles)

FIGURE 110. SIMPLIFIED BLOCK DIAGRAM OF DELRAC RECEIVER



High RF Amplifiers at this Point



Since the F transmission from Station B (stored by Oscillator 'B') is phase-locked to that from Station A (stored by Oscillator 'A'), the 'A' and 'B' Oscillator outputs may be phase-compared after period 2 to produce a hyperbolic pattern at comparison frequency F.

The 'Fine' Decometer pointer responds to this pattern.

The Integrating pointer is driven from the 'Fine' pointer by $n^2:1$ gearing.

The corresponding F/n hyperbolic pattern produced by the $\div n$ Divider outputs is not displayed, these outputs being employed to feed the second $\div n$ Dividers only. The F/n^2 outputs of the latter are phase-compared to produce a hyperbolic pattern at F/n^2 .

The 'Coarse' Decometer pointer responds to this pattern and should coincide with the Integrating pointer.

Hyperbolic Position-Line Patterns (With F = 12 kcs)

Pattern	Comparison Frequency	Lane Width on Baseline
'Fine'	12 kcs	12.5 km (13,670 yds)
'Coarse'	1-1/3rd kcs	112.5 km (69.90 st. miles)



REFERENCES

1. Precise Compass Swinging and Compensation Techniques, Final Report, Contract No. AF 33(606)-2382, Sperry Gyroscope, Great Neck, N. Y., 1955.
2. "Air Navigation," Department of the Air Force Manual 51-40, 1960, Vol. II.
3. N. Bowditch, American Practical Navigator, H. O. Publ. No. 9, U. S. Navy, Hydrographic Office, Washington, D. C., 1958.
4. O. E. Augustin, Magnetic Components and Compensation Study, WADD Technical Note 61-61, Aeronautical Systems Div., Air Force Systems Command, U. S. A. F., Wright-Patterson AFB, Ohio.
5. C. G. Kohler, Employment and Suitability Test of the MC-1 Magnetic Compass Calibrator Set, Rept. No. APGC-TR-58-21, Air Research and Development Command, U. S. A. F., Eglin AFB, Fla., Feb. 1958.
6. E. Luber, and M. Klemes, Study and Development of a Non-Floated Inertial Quality Gyroscope, ASD-TDR-62-80, Sperry Gyroscope, Great Neck, N. Y., Feb. 1962.
7. A. L. Rawlings, The Theory of the Gyroscopic Compass, MacMillan, New York, 1944.
8. J. L. R. Cudaback, Earth Rate Directional Reference for Aircraft, Thesis, U. S. A. F. Inst. Tech., Air University, March 1956.
9. Flight Evaluation Program Earth Rate Directional Reference, Final Report, ID-551/APN-105, Lab. Electron., Boston, Mass., April 1961.
10. S. A. Deel, Magnetic Declination in the United States, 1945, U. S. Coast and Geodetic Survey, 1946.
11. H. H. Howe, Magnetic Poles and the Compass, U. S. Coast and Geodetic Survey, 1949.
12. H. H. Howe, and L. Hurwitz, Magnetic Surveys, U. S. Coast and Geodetic Survey, 1956.
13. J. A. Jacobs, Research Concerning Geomagnetic Fluctuations, Final Report, Contract No. AF 19(604)-4068, Geophys. Lab., Univ. British Columbia, Jan. 1961.
14. D. G. Knapp, Practical Uses of the Earth's Magnetism, Coast and Geodetic Pamphlet, U. S. Gov. Printing Office, Washington, D. C.
15. A. K. Ludy, and H. H. Howe, Magnetism of the Earth, Serial 663-1945, U. S. Dep. Comm.
16. G. E. Roberts, "Aerial Magnetic Survey," J. Inst. Navigation, Vol. III, No. 2, p. 121.
17. G. B. Walker, and P. I. O'Dea, "Geomagnetic Secular-Change Impulses," Trans. Am. Geophys. Union, Vol. 33, No. 6, p. 797.
18. A. M. Weber and E. B. Roberts, "The 1950 World Isogonic Chart," J. Geophys. Res., Vol. 56, No. 1, p. 81.
19. T. C. Lyon, Practical Air Navigation, Special Publ. No. 197, U. S. Coast and Geodetic Survey, 1939.

20. N. Marchand, "The Evaluation of Error Standard Deviation in the Accuracy Evaluation of the Doppler Navigator," IRE Trans. Aerospace Navigational Electron., Sept. 1961, Vol. ANE-8, No. 3.
21. N. Braverman and N. Marchand, "Propagation-of-Error Equations for Airborne Doppler Navigation," to be published in IRE Trans. Aerospace Navigational Electron.
22. W. Q. Crichlow, J. W. Herbstreit, E. M. Johnson, K. A. Norton, and C. E. Smith, The Range Reliability and Accuracy of a Low-Frequency LORAN System, Rept. No. ORS-P23, Operational Research Staff, Office of the Chief Signal Officer, The Pentagon, Washington, D. C.
23. A. H. Jessell and G. H. Trow, "The Presentation of the Firing Accuracy of Navigation Systems," J. Inst. Navigation, Oct. 1948, Vol. I, No. 4, pp. 313-341.
24. A. Norton, "Low and Medium Frequency Radio Propagation," in Electromagnetic Wave Propagation, Academic Press, New York, 1960, pp. 375-444.
25. W. Q. Crichlow, The Comparative Accuracy of Various Existing and Proposed Radio Navigation Systems, Rept. No. CRPL-41, Nat. Bur. Std., Dec. 30, 1946.
26. J. A. Pierce, "Intercontinental Frequency Comparison by Low-Frequency Radio Transmission," Proc. IRE, June 1957, Vol. 45, No. 6, pp. 794-803.
27. C. J. Casselman, D. P. Heritage, and M. L. Tibbals, "VLF Measurements for the Radux-Omega Navigation System," Proc. IRE, May 1959, Vol. 47, No. 5, pp. 829-839.
28. A. D. Watt, R. W. Plush, "Power Requirements and Choice of an Optimum Frequency for a Worldwide Standard-Frequency Broadcasting Station," J. Res. Nat. Bur. Std., July-Aug. 1959, Vol. 63D, No. 1, pp. 35-44.
29. W. D. Westfall, "Prediction of VLF Diurnal Phase Changes and Solar Flare Effects," J. Geophys. Res., Sept. 1961, Vol. 66, No. 9.
30. J. R. Wait, "Diurnal Change of Ionospheric Heights Deduced from Phase Velocity Measurements at VLF," Proc. IRE, 1959, Vol. 47, p. 589.
31. J. R. Wait, "Comments on Paper by W. D. Westfall," J. Geophys. Res., Feb. 1962, Vol. 62, No. 2, p. 916.
32. K. P. Spies, and J. R. Wait, Mode Calculations for VLF Propagation in the Earth-Ionosphere Waveguide, Nat. Bur. Std. Tech. Note No. 114, July 17, 1961.
33. J. R. Wait, "A Comparison Between Theoretical and Experimental Data on Phase Velocity of VLF Radio Waves," Proc. IRE, June 1961, Vol. 49, No. 6, p. 1089.
34. W. T. Blackband, "Effects of the Ionosphere on VLF Navigational Aids," J. Res. Nat. Bur. Std., Nov.-Dec. 1961, Vol. 65D, No. 6, p. 575.
35. W. L. Taylor, "VLF Attenuation for East-West and West-East Daytime Propagation Using Atmospheric," J. Geophys. Res., July 1960, Vol. 65, No. 7, pp. 1933-1938.
36. W. Q. Crichlow, C. J. Doubique, A. D. Spaulding, and W. M. Beerg, "Determination of the Amplitude-Probability Distribution of Atmospheric Radio Noise from Statistical Moments," J. Res. Nat. Bur. Std., Jan.-Feb. 1960, Vol. 64D, No. 1, pp. 49-56.

37. C. J. Casselman and M. L. Tibbals, The Radux-Omega Long Range Navigation System Second National Convention on Military Electronics, June 16-18 1958.
38. W. Q. Crichlow, and C. E. Smith, Worldwide Radio Noise Levels Expected in the Frequency Band 10 Kilocycles to 100 Megacycles, Nat. Bur. Std. Circular 557, Aug. 1955.
39. A. D. Watt, W. W. Plush, W. W. Brown, and A. H. Morgan, "Worldwide VLF Standard Frequency and Time Signal Broadcasting," J. Res. Nat. Bur. Std., Nov.-Dec. 1961, Vol. 65D, No. 6, pp. 617-628.
40. A. F. Thornhill, "Interim Report Number 1 on Airborne Omega Receiver," NRL Memo. Rept. No. 959, 28 July 1959.
41. A. F. Thornhill, "Interim Report Number 2 on Airborne Omega Receiver," NRL Memo. Rept. No. 1029, 29 Feb. 1960.
42. R. J. Lampkin and A. F. Thornhill, "Interim Report Number 3 on Airborne Omega Receiver," NRL Memo. Rept. No. 1118, 16 Nov. 1960.
43. R. J. Lampkin, T. E. Leonard, and A. F. Thornhill, "Interim Report Number 4 on Airborne Omega Receiver," NRL Memo. Rept. No. 1182, 15 June 1961.
44. Evans L. Klein, "Interim Report Number 5 on Airborne Omega Receiver," NRL Memo. Rept. No. 1268, 5 Feb. 1962.
45. A. F. Thornhill and M. F. Williams, "Long Range Flight Tests of the Omega Aircraft Receiver," Rept. No. 6, NRL Memo. Rept. No. 1325, 28 May 1962.
46. F. H. Reder, M. R. Winkler, and C. Bickart, "Results of a Long-Range Clock Synchronization Experiment," Proc. IRE, June 1961, Vol. 49, No. 6, pp. 1028-1032.
47. S. M. Serebreny, E. J. Wiegmon, and R. G. Hadfield, The Feasibility of the Use of Minimum Time Track Data in Airways Design for Long Range and Oceanic Routes, Stanford Res. Inst., Contract FAA/BRD-420, 15 Oct. 1961.
48. R. R. Stone, W. Markowitz, and R. G. Hall, "Time and Frequency Synchronization of Navy VLF Transmissions," IRE Trans. Instr., Sept. 1960, Vol. 1-9, No. 2, p. 155.
49. N. Bowditch, American Practical Navigator, H. O. Publ. No. 9, U. S. Navy Hydrographic Office, Washington, D. C., 1958.
50. T. C. Lyon, Practical Air Navigation, 7th ed., Weems System of Navigation, Inc., Annapolis, 1958.
51. Webster's New Collegiate Dictionary, G. & C. Merriam Co., Springfield, Mass., 1953.
52. C. Draper, W. Wrigley, and J. Hovorka, "Inertial Guidance," Pergamon Press, New York, 1960.
53. C. L. McClure, "Theory of Inertial Guidance," Prentice-Hall, Inc., Englewood Cliffs, 1960.
54. C. F. Savant, Jr., R. C. Howard, C. B. Solloway, and C. A. Savant, "Principles of Inertial Navigation," McGraw-Hill, New York, 1961.
55. Gyroscopes: Theory and Design, ed. by Paul H. Savet, McGraw-Hill, New York, 1961.

56. G. R. Pitman, Jr., "Inertial Guidance," John Wiley and Sons, New York, 1962.
57. C. F. O'Donnell, "How Inertial Navigation Works," Electronic Equipment, Sept., 1957, Vol. 5, pp. 42-49.
58. C. F. O'Donnell, "Inertial Navigation," Autonetics, Oct., 1958, Vol. 266, No. 4.
59. Bernard Lichtenstein, Gyros, 4th ed., Kearfott Division, General Precision, Inc., June 1961.
60. Litton LN-2C Inertial-Doppler ASW Navigation System, Publ. No. 1196-A, Litton Industries, Electronic Equipments Division, Beverly Hills, Calif., March 20, 1960.
61. Litton LN-3 Inertial Navigation System, Publ. No. 1394, Litton Systems, Inc., Guidance and Control Systems Division, Woodland Hills, Calif., Jan. 23, 1961.
62. Litton LN-3 Status Summary, Publ. No. 2071, Litton Systems, Inc., Guidance and Control Systems Division, Woodland Hills, Calif., Feb. 1, 1962.
63. R. N. White, "Airborne Doppler Radar Navigation of Jet Transport Aircraft," IRE Trans. Aerospace Navigational Electron., March 1962, Vol. ANE-9, No. 1.
64. TWA, Dual Doppler Radar, Pilot-Operated Navigation System, presented at the Doppler Navigation Symposium, Kansas City, Mo., Jan. 8, 1962.
65. The Inertial Navigator, Publ. No. IMEJ 4695, General Electric Light Military Electronics Department, Johnson City, N. Y.
66. USAF NAV-100 Briefing, Astro Space Labs, Inc., Huntsville, Ala.
67. Alvin Lieberman, "Clean Rooms — White to Whiter," Research and Development, Armour Research Foundation, Jan. 1962, Vol. 13, No. 1.
68. A. M. Stone and G. C. Weiffenbach, Radio Doppler Method of Using Satellites for Geodesy, Navigation, and Geophysics Within the Planetary System, Applied Physics Lab. Rept. RG-385, Jan. 1961.
69. Down-Range Anti-Ballistic Measurement Program, Quarterly Technical Progress Rept. No. 5, AD 3 22 788, Radio Corporation of America, Missile and Surface Radar Division, Moorestown, N. J.
70. R. B. Kershner and R. R. Newton, "The Transit System," J. Inst. Navigation, Vol. 15, No. 2, April 1962.
71. W. H. Guier and G. C. Weiffenbach, "A Satellite Doppler Navigation System," Proc. I.R.E., April 1960, p. 507.
72. A. B. Moody, "Navigation Using Signals from High-Altitude Satellites," Proc. I.R.E., April 1960, p. 500.
73. Requirements for a High-Altitude Satellite All-Weather Celestial Navigation System to be known as the PATHFINDER NAVIGATION SYSTEM, 3 April 1959, U. S. Navy Hydrographic Office and U. S. Naval Observatory, Washington, D. C., 3 April 1959, (CONFIDENTIAL).
74. G. R. Marner, "Automatic Radio Celestial Navigation," Navigation, July/October 1959, Vol. 12, Nos. 3 and 4.
75. Aircraft Navigation Using Pulse Signals from High Satellites, Rept. No. OR-47C, Fairchild Stratos Corp., Sept. 1961.
76. C. C. Wan, "Application of a Satellite System to Marine and Air Navigation," IRE Trans. Space Electron Telemetry, 1960.

77. W. L. Polhemus, "The Navigation and Control of Supersonic Aircraft," J. Inst. Navigation, April 1962, Vol. 15, No. 2, pp. 179-219.
78. I. Solomon, "Estimates of Altitudes with Specified Probabilities of Being Above All Clouds," USAF Air Weather Service Tech. Rept. 159, Oct. 1961.
79. Delrac, A Long Range VLF Area Coverage Navigational Aid, Memo Plans 154, The Decca Navigator Company, Ltd., May 1954.
80. E. H. Leo Brandt, Herausgeber, Radio Navigation Systems for Aviation and Maritime Use, Nachrichtentechnische Gesellschaft and Navigationsverfahren und Technik of the Ausschusz für Funkortung, Jan. 1959.
81. C. A. Martin and P. S. Carter, "Low-Frequency Antennas," in H. Jasik (ed.), Antenna Engineering Handbook, McGraw-Hill, New York, 1943, p. 114.
82. F. E. Terman, Radio Engineers' Handbook, McGraw-Hill, New York, 1943, p. 114.
83. A. L. Whitson and R. B. Battelle, Suitability of the 200 to 400 kc Frequency Band for Air Traffic Control Signalling System, Final Report, Contract AF 33(616)-5346, Stanford Research Institute, Menlo Park, Calif., Sept. 1957.

BIBLIOGRAPHY

- Cade, C. M., "The Use of Radio Sextants in Automatic Navigation Systems," J. Inst. Navigation, Vol. XIII, p. 70.
- Frazer, D. O., and J. A. Lee, "The Rate-Integrating Gyro as an Aircraft Heading Reference," J. Inst. Navigation, April 1960, Vol. XIII, No. 2, p. 219.
- Fried, Walter R., "Performance Analysis and Future Outlook of Doppler Navigation Systems," Navigation, Vol. 6, p. 284.
- Green, J. F., "Some Consideration of Free Gyro Failures," J. Inst. Navigation, Vol. XIII, p. 78.
- Green, J. F., and A. P. Glenn, "Heading Definition in Commercial Aircraft," J. Inst. Navigation, April, 1960, Vol. XIII, No. 2, p. 196.
- Harter, H. L., "Circular Error Probabilities," J. Am. Statistical Assoc., Dec. 1960, Vol. 55, pp. 723-731.
- Jones, Sir H. S., "The Earth's Magnetic Field," J. Inst. Navigation, Vol. III, No. 1, p. 1.
- Kershner, R. B., "Transit Program Results," Astronautics, May 1960, p. 30.
- Marner, G. R., "Plotting the Future Course of Marine Celestial Navigation," Navigation, Winter, 1960-61, Vol. 7, No. 4.
- Nunn, J. L., "The Air Accuracy of Compasses," J. Inst. Navigation, Vol. III, No. 4, p. 404.
- Pedersen, E. S., "Self Contained Polar Navigation," J. Inst. Navigation, Vol. XIII, p. 78.
- Pneumo-Dynamic Corporation, Standby, Two-Inch Directional Indicator and Compass System, Rept. No. PDC-0626-C-I-1 c. I.
- Richardson, K. I. T., The Gyroscope Applied, Hutchinson's Scientific and Technical Publications, New York, 1954.

Spencer, N. S., and G. F. Kucera, Handbook of Magnetic Compass Adjustment and Compensation, H. O. Publ. No. 226, U. S. Navy Hydrographic Office, Washington, D. C., April 1944.

Williams, Wentworth, Jr., "A Doppler Navigation System Using Earth Satellites," IRE Trans. Space Electron. Telemetry, Fifth National Symposium, 1960.

White, R. N., "Airborne Doppler Radar Navigation of Jet Transport Aircraft," IRE Trans. Aerospace Navigational Electron., Vol. ANE-9, No. 1, March 1962.

Willis, D. C., Operational Experience with Doppler, Paper, Institute of Navigation Meeting, June 1962.

DISTRIBUTION LIST

<u>Copy No.</u>	<u>Addressee</u>
1-200	Chief
(1 repro)	General Service Branch
	RD-94
	Aviation Research and Development Service
	Washington 25, D. C.

Div. 19

Inst. of Science and Technology U. of Mich., Ann Arbor
**STUDY AND ANALYSIS OF SELECTED LONG-DISTANCE
NAVIGATION TECHNIQUES**, Volume II: Summary,
Final Report, by James O'Day, Irvin Sattinger, Robert
Scott, and Joseph Sullivan. Mar. 63. 370 p. incl. illus.,
tables, 80 refs.
(Report No. 4761-10-F(III))
(Contract ARDS-436)

The state of the art and the development potential of heading
references, VLF radio systems, inertial techniques, and
satellite systems have been considered for their applicability
to long-range ocean-crossing nonmilitary aircraft from 1965
to 1975. The navigation systems discussed here are by no
means the only competitors for position and course deter-
mination over transoceanic and high-altitude transcontinent-
al regions; our data and information should permit further
comparison with other systems. (over)

UNCLASSIFIED
I. O'Day, James;
Sattinger, Irvin;
Scott, Robert; and
Sullivan, Joseph
II. Federal Aviation
Agency
III. Contract ARDS-
436

UNCLASSIFIED

Div. 19

Inst. of Science and Technology U. of Mich., Ann Arbor
**STUDY AND ANALYSIS OF SELECTED LONG-DISTANCE
NAVIGATION TECHNIQUES**, Volume II: Summary,
Final Report, by James O'Day, Irvin Sattinger, Robert
Scott, and Joseph Sullivan. Mar. 63. 370 p. incl. illus.,
tables, 80 refs.
(Report No. 4761-10-F(III))
(Contract ARDS-436)

The state of the art and the development potential of heading
references, VLF radio systems, inertial techniques, and
satellite systems have been considered for their applicability
to long-range ocean-crossing nonmilitary aircraft from 1965
to 1975. The navigation systems discussed here are by no
means the only competitors for position and course deter-
mination over transoceanic and high-altitude transcontinent-
al regions; our data and information should permit further
comparison with other systems. (over)

UNCLASSIFIED
I. O'Day, James;
Sattinger, Irvin;
Scott, Robert; and
Sullivan, Joseph
II. Federal Aviation
Agency
III. Contract ARDS-
436

UNCLASSIFIED

Div. 19

Inst. of Science and Technology U. of Mich., Ann Arbor
**STUDY AND ANALYSIS OF SELECTED LONG-DISTANCE
NAVIGATION TECHNIQUES**, Volume II: Summary,
Final Report, by James O'Day, Irvin Sattinger, Robert
Scott, and Joseph Sullivan. Mar. 63. 370 p. incl. illus.,
tables, 80 refs.
(Report No. 4761-10-F(III))
(Contract ARDS-436)

The state of the art and the development potential of heading
references, VLF radio systems, inertial techniques, and
satellite systems have been considered for their applicability
to long-range ocean-crossing nonmilitary aircraft from 1965
to 1975. The navigation systems discussed here are by no
means the only competitors for position and course deter-
mination over transoceanic and high-altitude transcontinent-
al regions; our data and information should permit further
comparison with other systems. (over)

UNCLASSIFIED
I. O'Day, James;
Sattinger, Irvin;
Scott, Robert; and
Sullivan, Joseph
II. Federal Aviation
Agency
III. Contract ARDS-
436

UNCLASSIFIED

Div. 19

Inst. of Science and Technology U. of Mich., Ann Arbor
**STUDY AND ANALYSIS OF SELECTED LONG-DISTANCE
NAVIGATION TECHNIQUES**, Volume II: Summary,
Final Report, by James O'Day, Irvin Sattinger, Robert
Scott, and Joseph Sullivan. Mar. 63. 370 p. incl. illus.,
tables, 80 refs.
(Report No. 4761-10-F(III))
(Contract ARDS-436)

The state of the art and the development potential of heading
references, VLF radio systems, inertial techniques, and
satellite systems have been considered for their applicability
to long-range ocean-crossing nonmilitary aircraft from 1965
to 1975. The navigation systems discussed here are by no
means the only competitors for position and course deter-
mination over transoceanic and high-altitude transcontinent-
al regions; our data and information should permit further
comparison with other systems. (over)

UNCLASSIFIED
I. O'Day, James;
Sattinger, Irvin;
Scott, Robert; and
Sullivan, Joseph
II. Federal Aviation
Agency
III. Contract ARDS-
436

UNCLASSIFIED

UNCLASSIFIED

DESCRIPTORS
Automatic navigators
Bearing finding
Celestial navigation
Doppler navigation
Hyperbolic navigation
Inertial navigation
Radio navigation
Space navigation

We conclude that magnetically slaved and good free gyroscopes will continue to be the principal heading references for commercial aircraft; we recommend improvement programs in magnetic compasses and the use of nonfloated friction-averaging gyros. We do not expect that VLF systems will have been sufficiently operated to be acceptable for commercial aviation before 1975. However, their inherent accuracy and ability to cover large areas make them attractive if certain propagation and instrumentation errors can be solved. In their present form inertial systems are competitive with doppler navigation systems, at least in accuracy. The choice of an inertial system for commercial flight depends on cost, reliability, and convenience. Product improvement and the recently lowered cost of inertial platforms may make these systems attractive in the near future, particularly for higher speed aircraft. The present configuration of TRANSIT, the only satellite reference system scheduled for implementation, exhibits time gaps which would be serious for aircraft use. The single-fix accuracy appears adequate if additional satellites are orbited to provide more frequent fixes.

UNCLASSIFIED

UNCLASSIFIED
DESCRIPTORS
Automatic navigators
Bearing finding
Celestial navigation
Doppler navigation
Hyperbolic navigation
Inertial navigation
Radio navigation
Space navigation

We conclude that magnetically slaved and good free gyroscopes will continue to be the principal heading references for commercial aircraft; we recommend improvement programs in magnetic compasses and the use of nonfloated friction-averaging gyros. We do not expect that VLF systems will have been sufficiently operated to be acceptable for commercial aviation before 1975. However, their inherent accuracy and ability to cover large areas make them attractive if certain propagation and instrumentation errors can be solved. In their present form inertial systems are competitive with doppler navigation systems, at least in accuracy. The choice of an inertial system for commercial flight depends on cost, reliability, and convenience. Product improvement and the recently lowered cost of inertial platforms may make these systems attractive in the near future, particularly for higher speed aircraft. The present configuration of TRANSIT, the only satellite reference system scheduled for implementation, exhibits time gaps which would be serious for aircraft use. The single-fix accuracy appears adequate if additional satellites are orbited to provide more frequent fixes.

UNCLASSIFIED

DESCRIPTORS
Automatic navigators
Bearing finding
Celestial navigation
Doppler navigation
Hyperbolic navigation
Inertial navigation
Radio navigation
Space navigation

We conclude that magnetically slaved and good free gyroscopes will continue to be the principal heading references for commercial aircraft; we recommend improvement programs in magnetic compasses and the use of nonfloated friction-averaging gyros. We do not expect that VLF systems will have been sufficiently operated to be acceptable for commercial aviation before 1975. However, their inherent accuracy and ability to cover large areas make them attractive if certain propagation and instrumentation errors can be solved. In their present form inertial systems are competitive with doppler navigation systems, at least in accuracy. The choice of an inertial system for commercial flight depends on cost, reliability, and convenience. Product improvement and the recently lowered cost of inertial platforms may make these systems attractive in the near future, particularly for higher speed aircraft. The present configuration of TRANSIT, the only satellite reference system scheduled for implementation, exhibits time gaps which would be serious for aircraft use. The single-fix accuracy appears adequate if additional satellites are orbited to provide more frequent fixes.

UNCLASSIFIED

UNCLASSIFIED
DESCRIPTORS
Automatic navigators
Bearing finding
Celestial navigation
Doppler navigation
Hyperbolic navigation
Inertial navigation
Radio navigation
Space navigation

We conclude that magnetically slaved and good free gyroscopes will continue to be the principal heading references for commercial aircraft; we recommend improvement programs in magnetic compasses and the use of nonfloated friction-averaging gyros. We do not expect that VLF systems will have been sufficiently operated to be acceptable for commercial aviation before 1975. However, their inherent accuracy and ability to cover large areas make them attractive if certain propagation and instrumentation errors can be solved. In their present form inertial systems are competitive with doppler navigation systems, at least in accuracy. The choice of an inertial system for commercial flight depends on cost, reliability, and convenience. Product improvement and the recently lowered cost of inertial platforms may make these systems attractive in the near future, particularly for higher speed aircraft. The present configuration of TRANSIT, the only satellite reference system scheduled for implementation, exhibits time gaps which would be serious for aircraft use. The single-fix accuracy appears adequate if additional satellites are orbited to provide more frequent fixes.

UNCLASSIFIED