Some transverse space-charge effects in cyclic systems, especially, the nonlinear isoenergetic oscillations of particles in the radial direction of a beam, are studied. In a linear treatment of this problem (A. A. Kolomenskiy, A. N. Lebedev, CERN Symposium, 1959, p. 115) no account is taken of the nonlinear effects of the "negative mass". The amplitude of the radial oscillations $\ddot{x} + \omega^2 x = g_1 x^2 + g_2 x^3 + (e^2/m_c f^2)$ is assumed to be great enough to excite nonlinear effects. $\xi$ denotes the field of the fluctuations, $f = E/E'$, $E$ is the total particle energy. The amplitude $A$ and the phase $\varphi$ of these nonlinear oscillations depend on time to a relatively slight extent. The perturbation $f \ll f_0$ in the distribution function $F = F_0 (z) + f(t, z, \varphi)$ for the state of the beam satisfies a linear kinetic equation. The solutions of the dispersion relation resulting from this kinetic equation determine the asymptotic behavior of the perturbations. $F_0$ is the equilibrium distribution which is homogeneous with respect to the phase. The study of the perturbations uniformly distributed in the vertical sense is significant for every interval of $x$ and $y$. The two kernels of the integral equations for $f$ correspond with the two extreme cases where the dimensions of the perturbations along the beam axis are much greater or much smaller than the radial dimensions of the beam. The frequencies $\beta$ of the plasma oscillations for a given $F_0 (z)$ follow from this integral equation. With an accuracy of $(\Delta z/z_0)$, the formulas

$$\beta = \text{Im} \gamma = \text{Im} \left( \sqrt{1 + \frac{1}{\Delta z^2}} \right), \quad \text{Im} \gamma = \text{Im} \sqrt{1 + \frac{1}{\Delta z^2}},$$

are obtained for the first type of perturbation. For $\beta > 0$ the distribution is always unstable, for $\beta < 0$ the density performs small steady oscillations of the frequency, determined by Im$\gamma$. For this case, $\beta = 5g_1^2/12\omega + 3g_1^2/8\omega$. The duration of the development of the instability in an electron accumulator with $f = 60$, $R = 10^2$ cm, $N = 5 \cdot 10^{13}$, field index $n = 2/3$ has the value of $T \sim 6 \times 10^{-7}$ sec which is of the order of several revolutions. The phenomenon discussed here has to be taken into account in the accumulation of particles, in reactions in opposed beams, during an injection, etc. It can also be used for the self-control of beams.

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