NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.
Reflection and transmission of a plane-parallel layer in the scope of non-linear optics

The subject of investigation is a plate of thickness 1 and of small luminance characterized by the absorption coefficient $k_0$ and the reflection coefficient on the face $r$. A luminous flux $S_0$ is incident perpendicularly. Owing to multiple reflection there exist internally two kinds of flux at any point $x$: $S_1$, moving parallel to the incident flux, and $S_2$, moving in the opposite direction. These are described by the differential equations

$$\frac{dS_1}{dx} = -kS_1,$$
$$\frac{dS_2}{dx} = kS_2$$

(1) with the boundary conditions $S_1(x=0) = S_0(1-r) + rS_2(x=0)$, $S_2(x=1) = rS_1(x=1)$ (2). The absorption coefficient can be expressed by $k = k_0 / (1 + \alpha S_1 + S_2)$, where $\alpha$ is the parameter of non-linearity.

The system (1) is solved by

$$\frac{d}{dx} \left( \frac{S_1}{S_2} \right) = \frac{C_1}{S_0A}$$

(4)

and the relation $S_1S_2 = C_1/\alpha^2$ can be derived additionally from (1), stating that the product of two oppositely directed fluxes is constant at any depth. Hence the reflection coefficient $R$ is obtained by

$$R = \frac{(1-r)S_1}{S_2} + r$$

(8)

and the transmission factor $T$ by

$$T = \frac{1-r}{S_0} \sqrt{\frac{C_1}{C_2}}$$

(9).

On the basis of these formulas the light field was studied inside and outside the medium. For the region where $k_0$ is positive $R$ and $T$ are calculated by

$$R = r + \frac{(1-r)S_1\exp(-2k_0)}{1-r^2\exp(-2k_0)}$$
$$T = \frac{(1-r)r\exp(-k_0)}{1-r^2\exp(-2k_0)}$$

(10)
Reflection and transmission...

for the condition $\alpha S_0 \ll 1$, and by

$$R = \frac{2r}{1+r} \frac{2r}{\alpha S_0 (1+r)^2} k_f, \quad (11)$$

$$T = \frac{1-r}{1+r} \frac{1}{\alpha S_0}$$

for the condition $\alpha S_0 \gg 1$. For the region of negative values of $k$, $R$ and $T$

$$R = \frac{2\alpha S_0 r}{\alpha S_0 [2k_f r - \alpha S_0 (1 - r^2)]} - \frac{2r (1 - r)^2 (\alpha S_0)^2 - r (k_f)^2}{2\alpha S_0 [2k_f r - \alpha S_0 (1 - r^2)]}$$

$$T = \frac{1 - r^4}{\alpha S_0 [2k_f r - \alpha S_0 (1 - r^2)]}$$

holds for high luminances. At high values of $r$ the energy density distribution in the plate is virtually constant. At small values, this distribution has a minimum in the interior of the plate which vanishes if $r \to 1$. There are 4 figures.

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