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TECHNICAL REPORT

WVT-RR-6302

α-CLASSIFICATION OF MICROFIBERS - PART II

M. A. SADOWSKY
M. A. HUSSAIN

FEBRUARY 1963

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RESEARCH & ENGINEERING DIVISION

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ABSTRACT

The δ-ring has been put through the acceptance test with $\alpha = 1$, and a linear elastically extensible, limp flexible fiber has been formed in the result.

Cross Reference Data

Basic Research	99
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CONCLUSIONS

The fiber $\alpha = 1$ passed the acceptance test and is available for use in analytic mechanics as an elastically extensible, limp flexible fiber.

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LIST OF SYMBOLS

x, y	=	cartesian coordinates
p	=	tension force within the fiber
Q	=	shearing force within the fiber
M	=	bending moment within the fiber
σ	=	normal stress on the fiber
τ	=	tangential stress on the fiber
ϕ	=	direction angle of the fiber (with respect to the x-axis). (Also see next entry.)
Φ	=	biharmonic stress functions. (Also see preceding entry.)
R	=	radius of curvature
I	=	moment of inertia of the δ-fiber
ϵ	=	relative elongation
E	=	modulus of elasticity in tension and compression
G	=	modulus of elasticity in shear
α	=	fiber characteristic (stiffness exponent)
ν	=	Poisson's ratio of the filler body
'	=	primed quantities refer to the δ-ring material
a	=	outer radius of the δ-ring
b	=	inner radius of the δ-ring
δ	=	(a-b) thickness of the δ-ring
n	=	as defined by equation (2)
r, θ	=	polar coordinates

σ_r, σ_θ, $\tau_{r\theta}$	=	polar stress components
u_r, u_θ	=	polar components of displacement
B, D	=	superposition constants for the outer body
A', B', C', D'	=	superposition constants for the δ-ring

OBJECTIVE

In Part I of the sequence of reports under the present title we have established the concept of an elastic δ-fiber and of a linear microfiber as the limit of a δ-fiber for $\delta \rightarrow o$. A simple classification of the linear microfibers was arrived at by means of mentally submitting the fibers to uniaxial tensile tests and to pure bending tests (bending by couples only.) As a result we obtained four different types of linear microfibers characterized by values of α

$$\alpha = 1, 2, 3 \text{ and } 4 \tag{1}$$

α is a stiffness characteristic of the microfiber appearing in equations (29) and (32), Part I:

$$n = \frac{E'}{E} = \frac{G'}{G} = \left(\frac{L}{\delta}\right)^{\alpha} = \left(\frac{a^2+b^2}{a^2-b^2}\right)^{\alpha} \tag{2}$$

α relates the elasticity moduli E' and G' of the δ-fiber variable during the execution of the limiting process $\delta \rightarrow o$ to the constant moduli E, G of the matrix filler. The values in equation (1) correspond to a plane strain conception, and the properties of the four microfibers are as follows (Part I, equations 22):

α = 1 elastically extensible, limp flexible

α = 2 inextensible, limp flexible

α = 3 inextensible, elastically flexible (3)

α = 4 inextensible and inflexible (rigid)

We note, again, the absence of an elastically extensible, elastically flexible linear microfiber.

The compliance of a linear fiber with the behavior determined by elastic extensibility or inextensibility, and by limp or elastic flexibility or inflexibility has been established in Part I on the basis of a uniaxial tensile test and a pure bending test (bending by moments only). These tests alone are insufficient to claim the presence of the attributed properties in general. A test involving a curved δ-fiber embedded in a filler matrix in a state of pure shear at infinity has been designed in Part I to investigate the fibers under the general conditions of curvature and nonuniformity of the stress field. It is now our intention to test each one of the four fibers of equation (1) by the curved δ-fiber test and to report the findings. The present Part II of the report sequence deals with testing the elastically extensible, limp flexible linear microfiber characterized by

$$\alpha = 1 \tag{4}$$

RECAPITULATION OF PART I OF THE REPORT SEQUENCE

Part II of the report is a development of the material established in Part I. For a complete orientation Part I should be read first. For convenience the salient features of Part I are repeated in the following recapitulation. The equations cited in the recapitulation refer to the Part I numbering.

We consider an infinite elastic matrix body filling the exterior of the circle $r = a$ (Fig. 2). Bonded to it along the circle $r = a$ is the δ-body which is a ring of outer radius a and inner radius b. Obviously,

$$\delta = a - b \qquad \text{(26 Part I)}$$

The elasticity constants of the matrix are denoted by E, G and ν; those of the δ-ring by E', G' and ν'. In the stiffening process of the δ-body, Eq. (15 Part I), we assume that ν' remains constant (it would be limited to the range $0 \leq \nu' \leq 1/2$ anyway). The analytical part of the test is simplified if we put $\nu = \nu'$. We then have

$$E = 2(1+\nu)G \quad , \quad E' = 2(1+\nu)G' \qquad \text{(27 Part I)}$$

and

$$\frac{E'}{E} = \frac{G'}{G} \qquad \text{(28 Part I)}$$

The stiffening equation (15 Part I) is set up best in dimensionless form

$$\frac{G'}{G} = \left(\frac{L}{\delta}\right)^{\alpha} \qquad \text{(29 Part I)}$$

in which L has the dimension of a length, same as δ, by Eq.(26 Part I) Numerous exploratory computations undertaken for $\alpha = 1, 2, 3$ have shown that the optimum values of L from the point of view of algebraic simplicity seems to be

$$L = \frac{a^2 + b^2}{a + b} \qquad \text{(30 Part I)}$$

This value won the competition as against a, b, $\frac{a+b}{2}$, $\frac{a^2}{a+b}$, $\frac{ab}{a+b}$, $\frac{b^2}{a+b}$. The competition was necessary because of the heavy algebraic involvements capable of discouraging the computational office. Accepting Eq. (30 Part I), we have from Eq. (29 Part I)

$$\frac{G'}{G} = \left(\frac{a^2+b^2}{a^2-b^2}\right)^{\alpha} \qquad \text{(31 Part I)}$$

In intermediate algebraic work this ratio was denoted by n:

$$n = \frac{G'}{G} = \left(\frac{a^2+b^2}{a^2-b^2}\right)^{\alpha} \qquad \text{(32 Part I)}$$

In testing, the matrix substance is subjected to the state of pure shearing stress at infinity:

$$\sigma_x = 1 \;, \quad \sigma_y = -1 \;, \quad \tau_{xy} = 0 \qquad \text{(33 Part I)}$$

which reads, transformed to polar coordinates r, θ

$$\sigma_r = \cos 2\theta \qquad \sigma_\theta = -\cos 2\theta \;, \quad \tau_{r\theta} = -\sin 2\theta \qquad \text{(34 Part I)}$$

while the inner surface of the δ-ring (at $r = b$) remains free from boundary tractions σ_r and $\tau_{r\theta}$. We have a problem of linear elasticity for the composite body consisting of the matrix substance with the δ-ring bonded to it along $r = a$. Priming quantities referring to the ring, and using polar coordinates r, θ as arguments, we have the following boundary and regularity conditions for our problem applicable for all values of θ:

Regularity conditions at infinity (for $r \rightarrow \infty$):

$$\begin{aligned} \sigma_r(r,\theta) &\rightarrow \cos 2\theta \\ \sigma_\theta(r,\theta) &\rightarrow -\cos 2\theta \\ \tau_{r\theta}(r,\theta) &\rightarrow -\sin 2\theta \end{aligned} \qquad \text{(35 Part I)}$$

Continuity conditions at r = a:

$$
\begin{aligned}
u_r(a,\theta) &= u_r'(a,\theta) \\
v_r(a,\theta) &= v_r'(a,\theta) \\
\sigma_r(a,\theta) &= \sigma_r'(a,\theta) \\
\tau_{r\theta}(a,\theta) &= \tau_{r\theta}'(a,\theta)
\end{aligned}
\qquad \text{(36 Part I)}
$$

Free boundary conditions at r = b:

$$
\begin{aligned}
\sigma_r'(b,\theta) &= 0 \\
\tau_{r\theta}'(b,\theta) &= 0
\end{aligned}
\qquad \text{(37 Part I)}
$$

For the solution of the problem we introduce two symbols $[S]$ and $[S']$ as follows: $[S]$ is the solution applicable to the matrix filler (range of r: $a \leq r < \infty$) and $[S']$ is the solution applicable to the δ-ring (range of r: $b \leq r \leq a$). To obtain $[S]$ and $[S']$ we need biharmonic stress functions of frequency 2 in θ and even in θ . There are four such functions, namely (as given with their systematic catalog numbers):

$$
\begin{aligned}
\Phi_{16} &= r^2 \cos 2\theta \\
\Phi_{20} &= \frac{1}{r^2} \cos 2\theta \\
\Phi_{22} &= r^4 \cos 2\theta \\
\Phi_{24} &= \cos 2\theta
\end{aligned}
\qquad \text{(38 Part I)}
$$

The displacement and stress fields generated are tabulated in Tables 2-5 at the end of Part I. ϕ_{18} generates a uniform stress field which is a -2 times multiple of the right hand members in Eq. (35 Part I). Denoting the biharmonic stress functions producing the solutions $[S]$ and $[S']$ by ϕ and ϕ', we have

$$\phi = -\frac{1}{2}\phi_{18} + B\phi_{20} + D\phi_{24} \tag{39 Part I}$$

$$\phi' = A'\phi_{18} + B'\phi_{20} + C'\phi_{22} + D'\phi_{24} \tag{40 Part I}$$

ϕ_{22} is not admitted into the ϕ superposition because of its divergence at infinity. The coefficient $-\frac{1}{2}$ in Eq. (39 Part I) is determined by the regularity conditions at infinity, Eq. (35 Part I) which are now satisfied. The boundary conditions yet to satisfy form a set of six equations (36 Part I) and (37 Part I) in six unknown superposition coefficients B, D, A', B', C', D'.

After the equations are solved algebraically in terms of the parameters a, b, G, for a numerical value of α of the set α = 1, 2, 3, 4, the following quantities pertaining to the δ-body (part of δ-ring Fig. 3) are computed:

$$P(a,b) = \int_b^a \sigma_\theta' \, dr \tag{41 Part I}$$

$$Q(a,b) = \int_b^a \tau_{r\theta}' \, dr \tag{42 Part I}$$

$$M(a,b) = \int_b^a \sigma_\theta' \left(r - \frac{a+b}{2}\right) dr \tag{43 Part I}$$

$$\sigma(a,b) = \sigma_r'(a,\theta) = \sigma_r(a,\theta) \tag{44 Part I}$$

$$\tau(a,b) = \tau_{r\theta}'(a,\theta) = \tau_{r\theta}(a,\theta) \tag{45 Part I}$$

Now - and not an instant earlier - comes the limiting process $\delta \to 0$ which leads to the formation of a linear microfiber. To fix ideas, we may say that a will be kept at a constant value while b will be made variable approaching a as a limit from below. The limits of the quantities defined in Eqs. (41-45 Part I) are the specific linear fiber quantities P, Q, M σ, T:

$$P = \lim_{b \to a} P(a,b) \tag{46 Part I}$$

$$Q = \lim_{b \to a} Q(a,b) \tag{47 Part I}$$

$$M = \lim_{b \to a} M(a,b) \tag{48 Part I}$$

$$\sigma = \lim_{b \to a} \sigma_r(a,b) \tag{49 Part I}$$

$$\tau = \lim_{b \to a} \tau_{r\theta}(a,b) \tag{50 Part I}$$

We can also compute, after the limiting is done, the relative elongation ϵ and the increment of the curvature $\Delta\, 1/R$ of the linear fiber by computing them for the edge of the matrix filler at r = a. The formulas follow:

$$\epsilon = \epsilon_\theta(a,\theta) = \frac{1}{a}\left(u_r + \frac{\partial u_\theta}{\partial \theta}\right) \tag{51 Part I}$$

$$\Delta \frac{1}{R} = \frac{1}{a^2}\left(u_r + \frac{\partial^2 u_r}{\partial \theta^2}\right) \tag{52 Part I}$$

With all those limiting values available for $\alpha = 1, 2, 3$ and 4 the characterizations Eq. (22 Part I) can be checked as to their general validity. Eqs. (9, 10, and 12 Part I) can be used for independent checking. The work planned along the lines indicated is algebraically taxing and will be communicated in forthcoming reports.

The Algebraic Matrix of the Linear System of Equations

The boundary conditions Eqs. (36 and 37 Part I) form a set of six equations in six unknowns B, D, A', B', C', D'. The coefficients of the equations are supplied by Tables 2-5 at the end of this report. The elastic moduli G and G' of the filler material and the δ-ring appear in a way which makes it possible to introduce the ratio n of them at once, Eq. (32 Part I). The contributions from the term $-\frac{1}{2}\Phi_{18}$, Eq (39 Part I) have been transposed as right hand members of the algebraic system. The algebraic matrix of the linear system of equations is given in Table (6 Part I). It is not practical to insist on obtaining a general solution for n left in letter form as defined by Eq. (32 Part I) Individual solutions are to be carried out for $\alpha = 1, 2, 3$ and 4. As evidence of complications arising if n is left in letter form we give the result for the coefficient D computed with n left in:

$$D = \frac{a^2\left[-n^2(a^2-b^2)^4 - 4(1-\nu)na^6(a^2-b^2) + 2n(a^2-b^2)^4 + 4(1-\nu)nb^2(a^6-b^6) - 4(1-\nu)b^2(a^6-b^6) + 4(1-\nu)a^6(a^2-b^2) + 16(1-\nu)^2a^6b^2 - (a^2-b^2)^4\right]}{(3-4\nu)n^2(a^2-b^2)^4 + 4(1-\nu)(3-4\nu)n(a^2-b^2)a^6 - 2(1-2\nu)n(a^2-b^2)^4 + 4(1-\nu)nb^2(a^6-b^6) - 4(1-\nu)b^2(a^6-b^6) + 4(1-\nu)a^6(a^2-b^2) + 16(1-\nu)^2a^6b^2 - (a^2-b^2)^4} \tag{53 Part I}$$

The subsequent substitutions of $\left(\frac{a^2+b^2}{a^2-b^2}\right)^{\alpha}$ for n in D and in the remaining coefficients would not be practical.

THE COEFFICIENTS OF SUPERPOSITION

We have six unknown coefficients B, D, A', B', C', D' in the superpositions for Φ and Φ', equations (39) and (40), Part I. The six boundary conditions were stated in equations (36) and (37), Part I. With the use of stress and displacement tables Nos. 2, 3, 4, 5, Part I, the boundary conditions were formulated as a linear non-homogeneous 6 x 6 system, Table 6, Part I. An attempt to solve formally for the six unknowns at this stage was dropped after the value of D was obtained. Equation (53), Part I shows that value to be algebraically overcomplicated and not conducive to a transparent analysis of forthcoming displacements and stresses. For this reason, it was decided to continue from there on individually for $\alpha = 1$, $\alpha = 2$, $\alpha = 3$ and $\alpha = 4$. In the present report we are concerned with the value $\alpha = 1$. The solutions for all six coefficients have been obtained for $\alpha = 1$ in terms of a, b, and ν as follows:

$$B = \frac{a^4b^2\left[2(1-\nu)a^6+(3-6\nu+4\nu^2)a^4b^2-2(2-\nu)a^2b^4-(1-2\nu)b^6\right]}{2\Delta} \tag{5}$$

$$D = \frac{a^2b^2\left[2(1-2\nu)(1-\nu)a^6+(1-2\nu)a^4b^2+2(2-\nu)a^2b^4+(1-2\nu)b^6\right]}{\Delta} \tag{6}$$

$$A' = -\frac{(1-\nu)a^2(a^2+b^2)\left[2(1-\nu)a^6+a^4b^2+a^2b^4+4b^6\right]}{(a^2-b^2)\Delta} \tag{7}$$

$$B' = -\frac{(1-\nu)a^4 b^4 (a^2+b^2)\left[2(1-\nu)a^4 + a^2 b^2 + b^4\right]}{(a^2-b^2)\Delta} \tag{8}$$

$$C' = \frac{2(1-\nu)a^2 b^4 (a^2+b^2)}{(a^2-b^2)\Delta} \tag{9}$$

$$D' = \frac{2(1-\nu)a^2 b^2 (a^2+b^2)\left[2(1-\nu)a^6 + a^4 b^2 + a^2 b^4 + b^6\right]}{(a^2-b^2)\Delta} \tag{10}$$

In the above equations, Δ is an abbreviation for

$$\Delta = \left[4(1-\nu)^2 a^8 + 8(1-\nu)^2 a^6 b^2 - a^4 b^4 + 2a^2 b^6 + (3-4\nu)b^8\right] \tag{11}$$

THE DISPLACEMENTS AT THE BOND $r = a$

The displacements u_r', u_θ' of the δ-ring and the displacements u_r, u_θ of the matrix join continuously at the bond $r = a$. This was expressed by the boundary conditions equations (36), Part I. The common values $u_r' = u_r$, $u_\theta' = u_\theta$ are given by

$$\frac{2G u_r}{\cos 2\theta} = \frac{2(1-\nu)a\left[2(1-\nu)a^8 + (9-16\nu+8\nu^2)a^6 b^2 + 3(1-2\nu)a^4 b^4 + (7-4\nu)a^2 b^6 + (3-4\nu)b^8\right]}{\Delta} \tag{12}$$

$$\frac{2G\,u_\theta}{\sin 2\theta} = \frac{2(1+\nu)a\left[-2(1-\nu)a^8-(5-12\nu+8\nu^2)a^6b^2+(1+2\nu)a^4b^4-(7-4\nu)a^2b^6-(3-4\nu)b^8\right]}{\Delta} \tag{13}$$

STRESSES IN THE δ -RING

The stresses in the δ-ring are given by

$$\frac{\sigma_r'}{\cos 2\theta} = \frac{2(1-\nu)(a^2+b^2)a^2(r^2-b^2)}{(a^2-b^2)\Delta\, r^4}\left\{r^2\left[2(1-\nu)a^6+a^4b^2+a^2b^4+4b^6\right]-3a^2b^2\left[2(1-\nu)a^4+a^2b^2+b^4\right]\right\} \tag{14}$$

$$\frac{\sigma_\theta'}{\cos 2\theta} = -\frac{2(1-\nu)(a^2+b^2)a^2}{(a^2-b^2)\Delta\, r^4}\left\{r^4\left[2(1-\nu)a^6+a^4b^2+a^2b^4+4b^6\right]+3a^2b^4\left[2(1-\nu)a^4+a^2b^2+b^4\right]-12r^6b^4\right\} \tag{15}$$

$$\frac{\tau_{r\theta}'}{\sin 2\theta} = \frac{2(1+\nu)(a^2+b^2)a^2(r^2-b^2)}{(a^2-b^2)\Delta\, r^4}\left\{6r^4b^4-r^2\left[2(1-\nu)a^6+a^4b^2+a^2b^4-2b^6\right]-3b^2\left[2(1-\nu)a^6+a^4b^2+a^2b^4\right]\right\} \tag{16}$$

We also include some additional information for the stress σ_θ' for $r = a$ and for $r = b$. For $r = a$,

$$\frac{\sigma_\theta'}{\cos 2\theta} = -\frac{2(1-\nu)(a^2+b^2)\left[2(1-\nu)a^8+a^6b^2-(5+6\nu)a^4b^4+7a^2b^6+3b^8\right]}{(a^2-b^2)\Delta} \tag{17}$$

and
$$\lim_{b \to a} \frac{\sigma_\theta'}{\cos 2\theta} \cdot \frac{(a-b)}{a} = - \frac{4(1-\nu)}{(4-3\nu)} \tag{18}$$

For $r = b$,

$$\frac{\sigma_\theta'}{\cos 2\theta} = - \frac{8(1-\nu)a^2(a^2+b^2)\left[2(1-\nu)a^6 + a^4b^2 + a^2b^4 - 2b^6\right]}{(a^2-b^2)\Delta} \tag{19}$$

and
$$\lim_{b \to a} \frac{\sigma_\theta'}{\cos 2\theta} \cdot \frac{(a-b)}{a} = - \frac{4(1-\nu)}{(4-3\nu)} \tag{20}$$

The equality of the limits in equations (18) and (20) shows that for a small value of $\delta = a - b$ the stresses σ_θ' at $r = a$ and $r = b$ are nearly equal. The graphs of the stresses σ_r', σ_θ', $\tau_{r\theta}'$ for $b \leq r \leq a$ are shown in Figures 4, 5, 6.

STRESS RESULTANTS IN THE δ-RING

With reference to notations adopted in equations (41)-(43), Part I, we have the following stress resultants:

$$P\,(a, b) = \int_b^a \sigma_\theta' \, dr =$$

$$= \frac{-2(1-\nu)(a^2+b^2)a\left[2(1-\nu)a^6 + (3-2\nu)a^4b^2 - 2a^2b^4 + b^6\right]}{\Delta} \cdot \cos 2\theta \tag{21}$$

$$Q(a, b) = \int_b^a \tau_{r\theta}' \, dr =$$

$$= -\frac{2(1-\nu)(a^4-b^4)a\left[2(1-\nu)a^4+a^2b^2-b^4\right]}{\Delta} \cdot \sin 2\theta \tag{22}$$

$$M(a, b) = \int_b^a \sigma_\theta' \cdot \left(r - \frac{a+b}{2}\right) dr =$$

$$= -\frac{(1-\nu)(a^2+b^2)a^2\left[2(1-\nu)a^6+(7-6\nu)a^4b^2-2a^2b^4+b^6\right]}{\Delta} \cdot \cos 2\theta - \frac{a+b}{2} P(a,b) \tag{23}$$

With reference to equations (44) and (45), Part I, we note

$\sigma(a,b) = \sigma_r'$ from equations (14) for $r = a$

$$\sigma(a,b) = \frac{2(1-\nu)(a^2+b^2)\left[2(1-\nu)a^6-(5-6\nu)a^4b^2-2a^2b^4+b^6\right]}{\Delta} \cdot \cos 2\theta \tag{24}$$

$\tau(a, b) = \tau_{r\theta}'$ from equation (16) for $r = a$

$$\tau(a,b) = \frac{2(1-\nu)(a^2+b^2)\left[-2(1-\nu)a^6-(7-6\nu)a^4b^2+2a^2b^4-b^6\right]}{\Delta} \cdot \sin 2\theta \qquad (25)$$

CHARACTERISTIC RESULTS FOR THE LINEAR FIBER

From equations (46)-(50), Part I, we obtain for the limiting case $\delta \to 0$ representing the linear fiber:

$$P = \lim_{b \to a} P(a, b) = -\frac{4a(1-\nu)}{(4-3\nu)} \cos 2\theta \qquad (26)$$

$$Q = \lim_{b \to a} Q(a, b) = 0 \qquad (27)$$

$$M = \lim_{b \to a} M(a, b) = 0 \qquad (28)$$

$$\sigma = \lim_{b \to a} \sigma(a, b) = -\frac{4(1-\nu)}{(4-3\nu)} \cos 2\theta \qquad (29)$$

$$\tau = \lim_{b \to a} \tau(a, b) = -\frac{8(1-\nu)}{(4-3\nu)} \sin 2\theta \qquad (30)$$

The displacements are obtained from equations (12) and (13) by putting $b = a$:

$$2Gu_r = \frac{4a(1-\nu)(3-\nu)}{(4-3\nu)} \cos 2\theta \tag{31}$$

$$2Gu_\theta = -\frac{4a(1-\nu)(2-\nu)}{(4-3\nu)} \sin 2\theta \tag{32}$$

The relative elongation ϵ_θ of the fiber is a function of θ alone. It is computed in polar coordinates as follows:

$$2G\epsilon_\theta = \frac{1}{a}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{a} \tag{33}$$

$$= -\frac{4(1-\nu)^2}{(4-3\nu)} \cos 2\theta \tag{34}$$

If the matrix were unstressed (no stresses whatsoever), the linear fiber would be in shape of a circular arc of curvature $1/a$. In presence of a stressfield in the matrix, the fiber is deformed and bent to a curved arc with a radius of curvature $\rho = \rho(\theta)$. The increment of the curvature ΔK is computed by the formula

$$\Delta K = \frac{1}{\rho} - \frac{1}{a} = -\frac{1}{a^2}\left(\frac{\partial^2 u_r}{\partial \theta^2} + u_r\right) \tag{35}$$

Using equation (31), we find

$$\Delta K = \frac{6(1-\nu)(3-\nu)}{(4-3\nu)aG} \cdot \cos 2\theta \tag{36}$$

INTERPRETATION OF RESULTS

We see from equation (34) that the fiber is extensible. From equations (26) and (34) we find

$$P = \frac{2Ga}{1-\nu}\epsilon_\theta \tag{37}$$

which shows that the extensibility is elastic. From equation (36) we see that the fiber is flexible. Equation (28) tells us that the flexibility is limp. Summing up, we do have an elastically extensible, limp flexible fiber, as expected and predicted by equation (22), Part 1 for $\alpha = 1$.

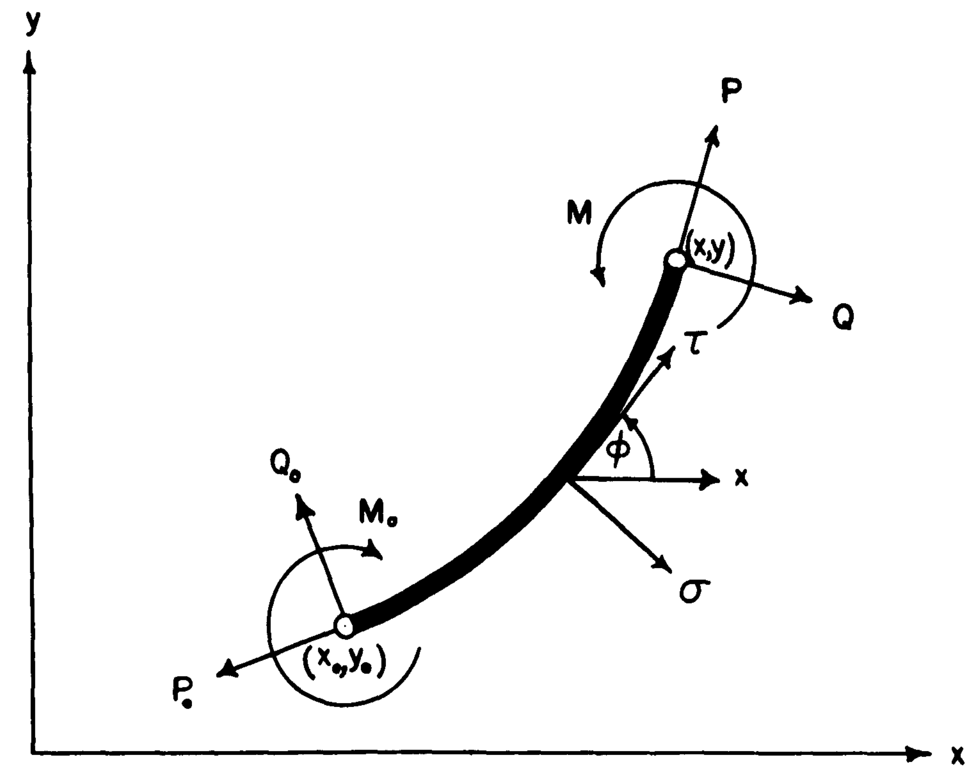

Figure 1. Free Body Diagram for a Fiber

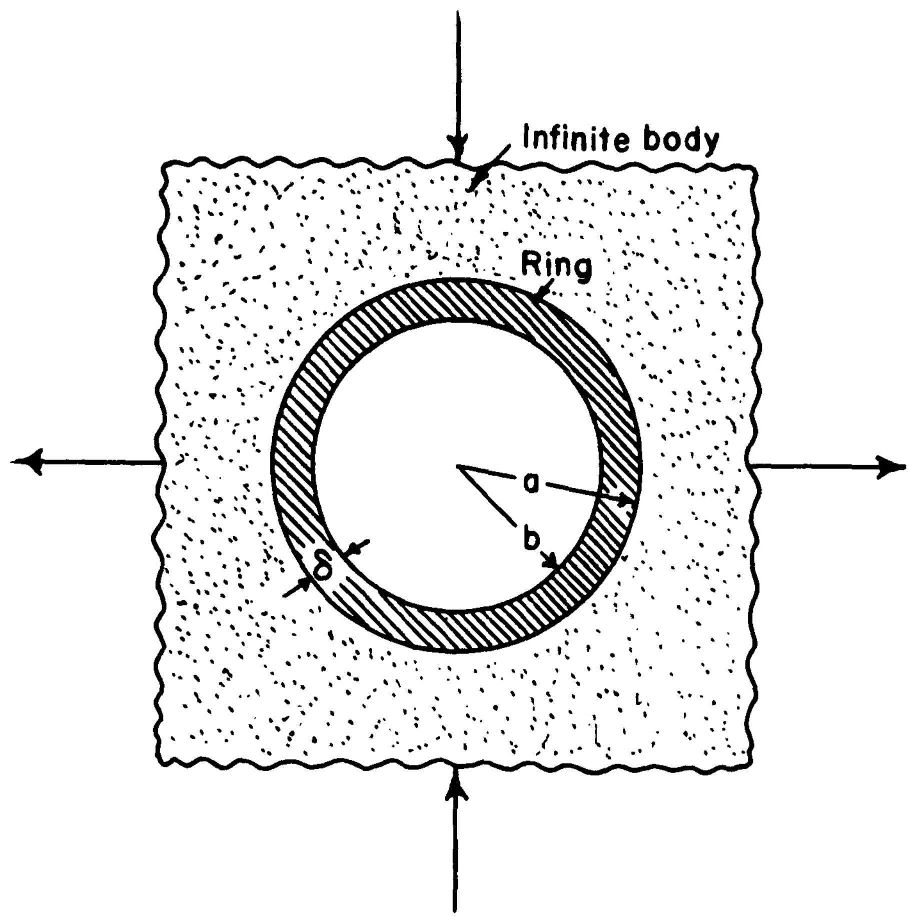

Figure 2. Composition of δ-Ring and the Filler Body

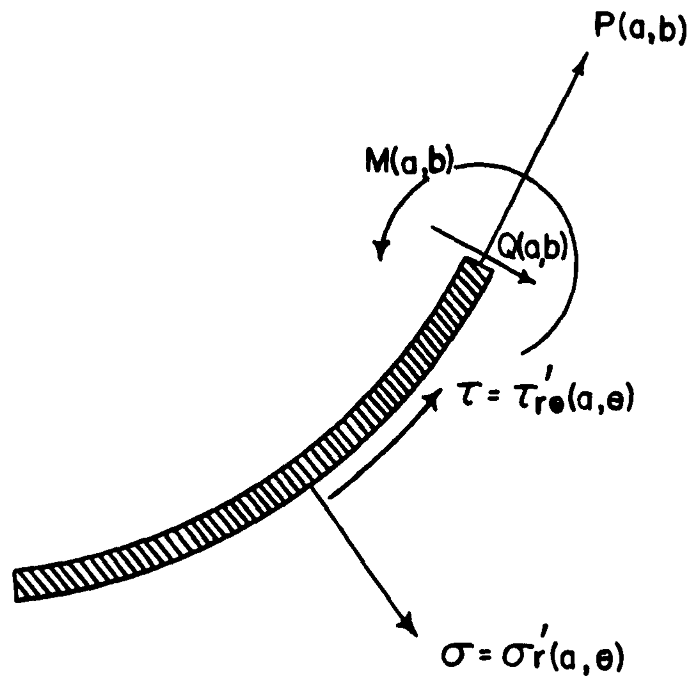

Figure 3. Interpretation of σ and τ in Terms of σ_r' and $\tau_{r\theta}'$

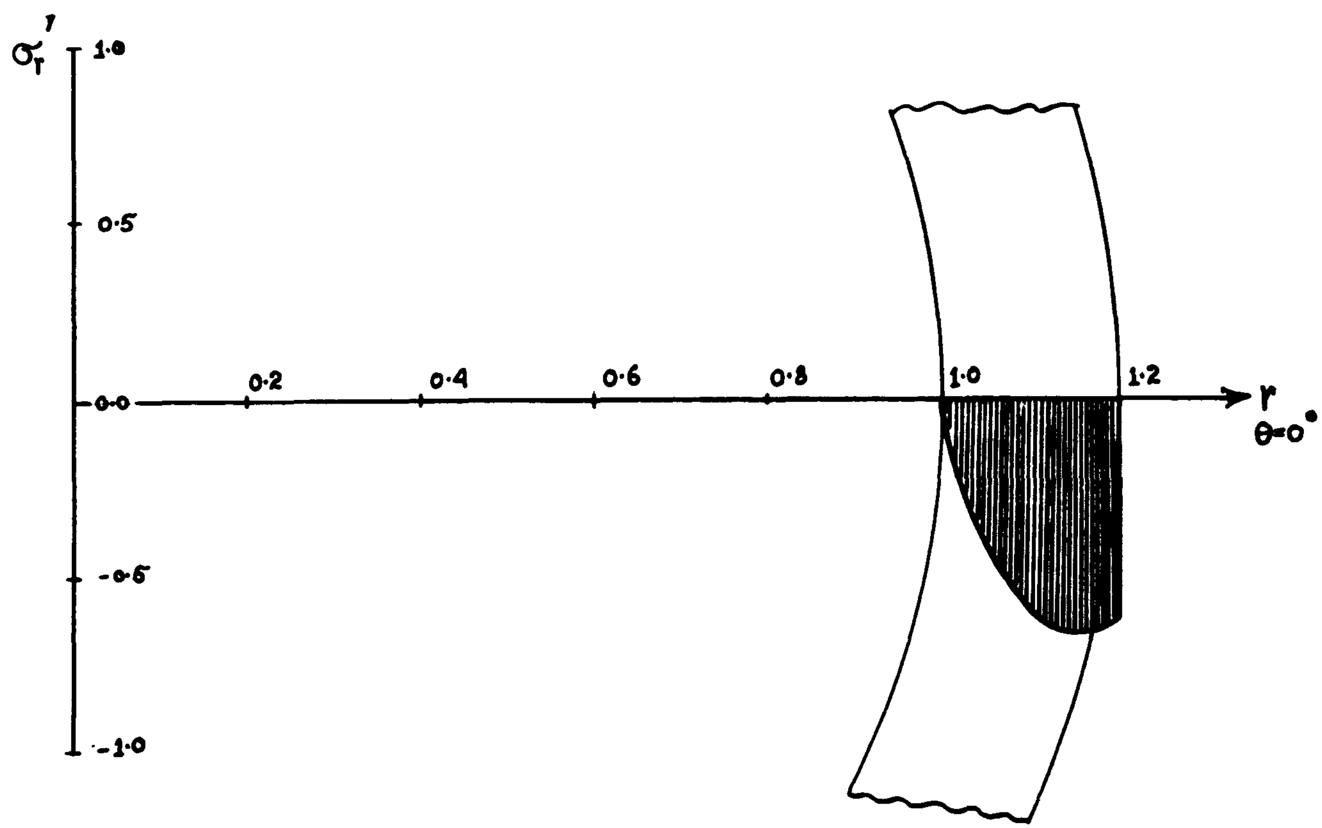

Figure 4. Graph of σ_r' for the δ-Ring: b = 1, a = 1.2 and ν = .3

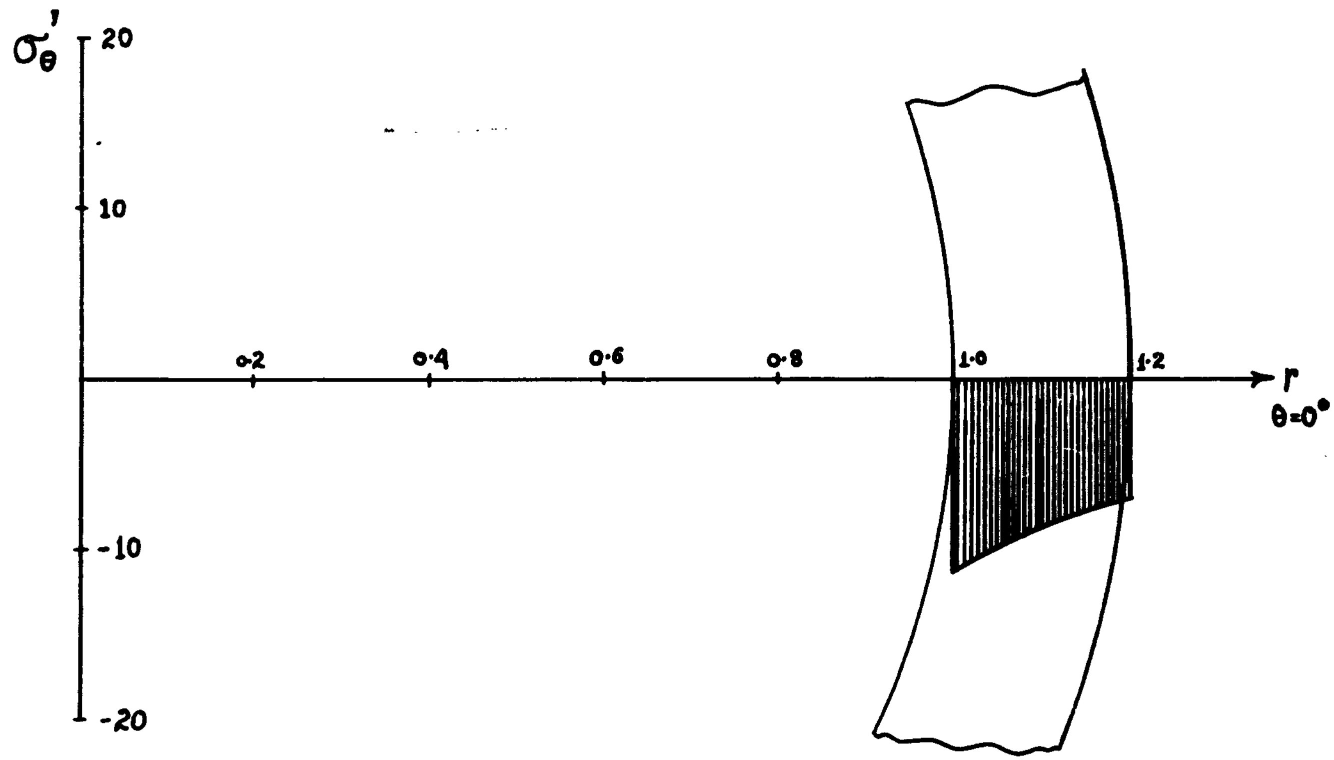

Figure 5. Graph of σ_θ' for the δ-Ring: b = 1, a = 1.2 and ν = .3

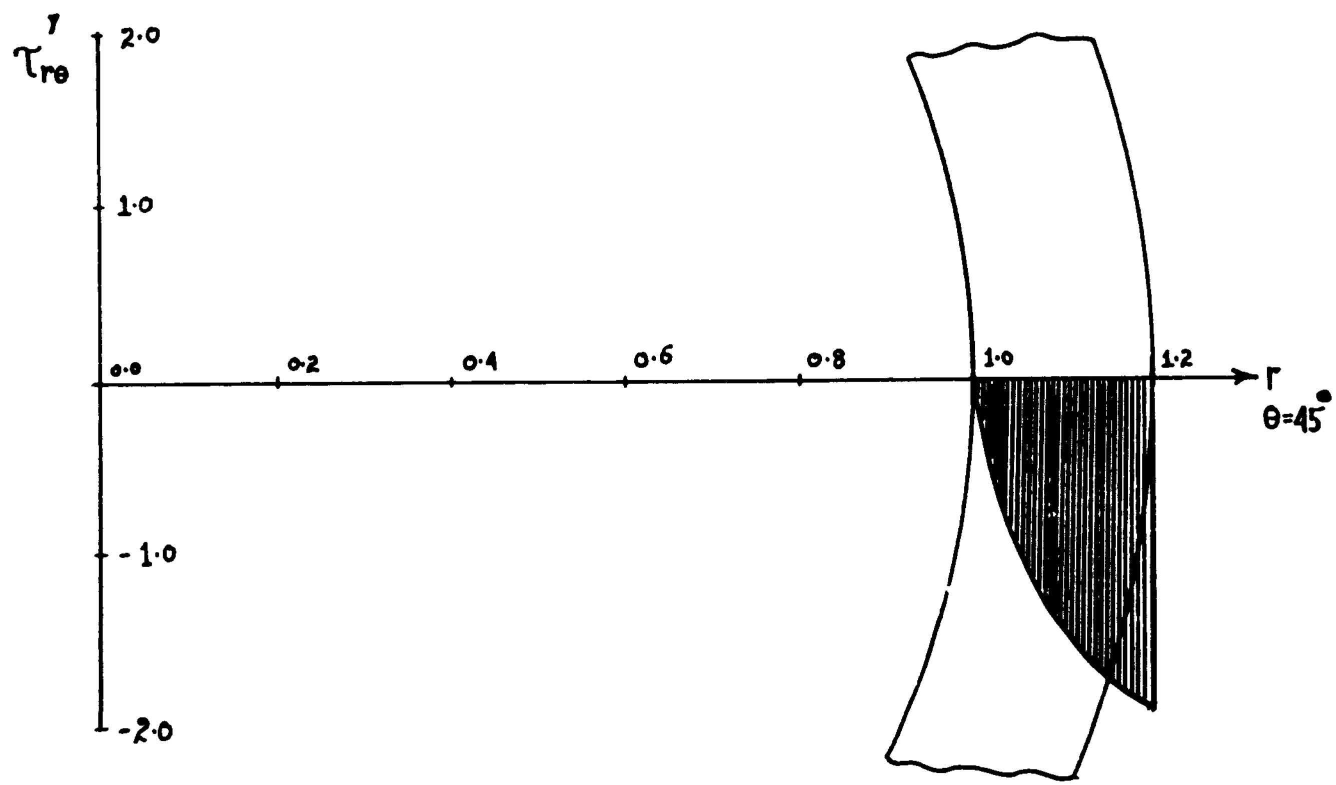

Figure 6. Graph of $\tau'_{r\theta}$ for the δ-Ring: b = 1, a = 1.2 and ν = .3

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