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TECHNICAL NOTE NO. 1.

THE DEVELOPMENT AND DESIGN OF A POLARIMETER
FOR LUNAR STUDIES

by

DAVID CLARKE

Department of Astronomy,
University of Manchester.

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This paper consists of three parts describing the development of a polarimeter and includes the observational results that were obtained during the winter of 1961-1962.

The first part describes some of the difficulties of measuring the usually small degree of polarization of astronomical objects. The equations necessary for the evaluation of the degree of polarization by certain methods have been formulated and accuracies of these methods have been compared.

In order to gain experience in photometric and polarization experiments, a single-beamed device was designed and tested. Part II contains a description of this instrument. The second half of Part II contains a description of the double-beam polarimeter that is currently in use.

Part III lists the astronomical results that have been obtained at the telescope. Observations were made on several stars, Uranus and the Moon, including measurements of the degree of polarization at different wavelengths.
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THE DEVELOPMENT AND DESIGN
OF A POLARIMETER

INTRODUCTION

Our information and knowledge about astronomical bodies is obtained by analysing the light which is either emitted or reflected by them. This analysis of the light includes measuring the direction of propagation, the intensity, the spectral distribution, the state of polarization, perhaps as a function of wavelength, all as a function of time. Polarimetric analysis of the light can give us valuable information about the properties of the materials that scatter the light in our direction. For example, the sunlight which illuminates the Moon or a planet can be considered to be unpolarized (natural light) while the light we receive from these bodies, being just reflected sunlight, is usually polarized. By measuring the degree of polarization (defined later) against the phase angle, which is the angle formed between the direction of illumination, the scattering object and the direction of observation, we can, with relation to other optical properties, build up our knowledge about the surface properties of the Moon, or in the case of the planets, about the particles in their atmospheres which scatter the light.

Most of the scattering processes give rise to linear polarization and the polarimeters discussed below were designed to detect and measure linear polarization.

Considerations of problems of design and a certain amount of experience with D.C. techniques have influenced the designs of the two polarimeters. The analysis and formulation of equations later in this paper is from the D.C. standpoint and fulfils the means for converting the collected information into the required results.
Some Theoretical Considerations

Definitions

The essential requisite for any analyser which is to be used for detecting and measuring linear polarization is that the vibrations of the light must be resolved into two orthogonal components. By measuring the intensities of these components at different settings of the analyser relative to the incoming light beam, the degree of polarization can be determined. When the resolution into two components by an analyser is not perfect, a correction factor has to be applied to the results.

Perhaps the simplest form of analyser is 'polaroid'. This material acts as an analyser by absorbing one component (or at least the larger part of it) and allowing the other component to be transmitted. The degree of polarization can be determined by measuring the intensity of the transmitted light for at least three different rotational settings of the polaroid. The amount of light passing through the analyser is given by:

\[ I(a) = I_M \cos^2 (\alpha + \phi) + I_m \sin^2 (\alpha + \phi). \]  

(1)

where \( I_M \) and \( I_m \) are the maximum and minimum intensity components respectively of the partially polarized source, \( \phi \) is the angle that the maximum component makes with the main transmission axis of the polaroid and \( \alpha \) is the rotational setting of the analyser relative to some arbitrary zero.

Equation (1) can be rewritten in the form

\[ I(\alpha) = \frac{1}{2}(I_M + I_m) \{ \rho \cos 2 (\alpha + \phi) + 1 \}. \]  

(2)

where \( \rho \) represents the degree of polarization defined as

\[ \rho = \frac{I_M - I_m}{I_M + I_m}. \]  

(3)

By rearranging equation (2) we find that

\[ \rho \cos 2 (\alpha + \phi) = \frac{2 I(\alpha)}{I_M + I_m} - 1. \]  

(4)
By defining \( P(\alpha) = \rho \cos 2(\phi + \alpha) \) we can rewrite this last equation as

\[
P(\alpha) = \frac{2I(\alpha)}{I_m + I_m} - 1. \tag{5}
\]

There are several different methods of extracting values of the degree of polarization \( \rho \) from the measurements made of \( P(\alpha) \), according to the different values that are chosen for the angle \( \alpha \). Perhaps the most familiar is essentially Pickering's method where measurements of \( P(\alpha) \) are taken at intervals of \( \frac{\pi}{4} \). If one expands the cosine term of equation (4) it can be shown that

\[
\rho^2 = P^2(\alpha) + P^2(\alpha + \frac{\pi}{4}) \tag{6}
\]

and hence,

\[
\rho = \sqrt{P^2(\alpha) + P^2(\alpha + \frac{\pi}{4})}. \tag{7}
\]

For a complete determination, it is usual to obtain eight values of \( P(\alpha) \) at intervals of \( \frac{\pi}{4} \) between \( \alpha = 0 \) and \( 2\pi \) in order to form a mean value of \( \rho \). Other methods include the fitting of one of a series of master-curves to the values of \( P(\alpha) \) or obtaining a least squares solution by means of a computer.

Some other analysers have been designed and used which allow the intensities of both components to be measured. Analysers of this type, such as a Wollaston prism, have been termed double-beamed devices; while analysers such as polaroid or a Nicol prism have been termed single-beamed devices.

**Problems of Measuring the Polarization of Astronomical Objects**

1) **Choice of Telescope**

It is obvious that serious difficulties will arise if the telescope that is chosen is a Newtonian reflector. There seems to have been only one serious attempt to use such an instrument: namely, the 60" at the Boyden Station, Bloemfontein. The calibration curves for such an instrument were complicated [1] and obviously led to an increase in the error of a determination over the equivalent Cassegrain reflector.

The Cassegrain reflector gives rise to problems when the wavelength dependence of polarization is studied [2]. Aluminium surfaced mirrors generally exhibit polarizing patches and the amount of polarization introduced in this way is highly colour dependent. In some cases this can be so severe that if the whole light is analysed, different corrections have to be applied for stars of different colours.
If linear polarization is being investigated and the mirror or objective of the telescope introduces circular or elliptical polarization, the effect is equivalent to a depolarizing action, and results will be obtained which will be systematically too small. In the case of a reflector, this effect can be introduced if the mirror is not perfectly clean, or if the reflectivity of the aluminium is nonuniform over the surface. Glass under strain exhibits birefringence, introducing a phase change between resolved components, thus transforming linear into elliptical polarization. For example, if a telescope objective is under strain and giving rise to a relative retardation of $\lambda/16$, the observed linear polarization will be only 93% of its true value. To complicate things further, the amount of strain could depend on the orientation of the telescope.

Instrumental polarization effects are likely to occur in both types of telescope. If the wavelength dependence of polarization is to be investigated, corrections for the reflector may be colour-dependent; but there is the advantage over the refractor that measurements can be extended into the ultraviolet.

It is difficult to make comparisons between results obtained by refractors and results obtained by reflectors. It appears that the refractor does give slightly better results. However, this difference in accuracy need not necessarily be a consequence of the type of telescope used, but probably depends on the way the light has been analysed and measured.

(ii) Problems of Photometry

Any polarization measurements require accurate photometry. Most astronomical objects have only a small amount of polarization and, hence, the changes of intensity levels at different rotational settings of the analyser are small. These differences have to be measured against the background scintillation noise or changes in atmospheric transparency. Any polarimeter must have at least one rotatable part and rotation of the image of the aperture stop over a cathode that is non uniform in sensitivity or changes in the gain of a multiplier as its orientation changes, can give rise to systematic errors in the determinations.

There are very many ways of designing an instrument to measure the degree of polarization. Several different designs have been used by various astronomers. Because of the large number of variations it is very difficult to make comparisons.
Quite generally, polarimeters can be divided into two main groups; single-beamed and double-beamed devices. These two groups can be subdivided in many different ways. For example, it is possible to subdivide each main group into two groups under the headings of D.C. or A.C. techniques. These again could be subdivided into devices which produce some sort of signal (e.g. an intensity measurement) or devices which use an optical compensator which can be adjusted until a null signal is obtained. A.C. devices can be subdivided according to the way A.C. signal is produced. For example, it could be produced by a mechanical chopper, a revolving analyser or a revolving $\lambda/2$ plate followed by a fixed analyser. The A.C. signal might be measured or reduced to zero by adjusting an optical compensator.

It must be pointed out that the author has a basic dislike for any instrument that uses a polarizing plate as a calibration, or to produce a null method or if the calibration is performed using a depolarizer when its transmission coefficient has to be known. The introduction of parameters of this nature is quite likely to produce systematic errors. The 'ratio method', which is described later, is the only technique that does not suffer from these drawbacks.

**Double-Beamed Methods**

Hiltner [3] demonstrated in 1951 that if a double-beamed analyser, such as a Wollaston prism, were to be used, the rapid fluctuations in the output signals (scintillation) were coherent in both beams. A double-beamed analyser immediately lends itself to the idea of "seeing compensation". By fitting a conventional photometer to each beam, the ratio of the two outputs can be formed and displayed, or the outputs can be recorded simultaneously and the ratio formed subsequently, so removing scintillation noise or the effects of changes in atmospheric transparency.

In order to determine the degree of polarization it is not necessary to display all the information (i.e. the intensities in both beams). For example, there are methods which rely on a knowledge of the difference and sum of the two signals (see later) or on a knowledge of the difference signal only [4]. It is quite convenient, for instance, to form the difference electronically and to display this signal. Previous experimenters in this field have indicated that the difference signal formed in this way is "seeing-compensated". This is not strictly so, as seeing compensation can only be accomplished by forming the ratio between the two signals. There is, however, a factor of the order of 100 improvement by measuring the difference directly than by obtaining the difference from two readings which in the case of a single-beamed
analyser must have been taken at different times. The scintillation noise expressed as a percentage is the same on the displayed difference signal as it is on the output signal of either of the two beams.

Suppose that the difference is measured between two components 90° apart (i) directly and then (ii) by measuring the two signals at two settings of the analyser separated by 90°. If K is the fractional error in any measured quantity (it is an indication of the extent of the scintillation) the different determinations give:

1) \[ D = S_1 - S_2 \pm K(S_1 - S_2), \]  \hspace{1cm} (8)

2) \[ D = S_1 - S_2 \pm K\sqrt{S_1^2 + S_2^2}. \]  \hspace{1cm} (9)

For typical values of \( S_1 = 1.01 \) and \( S_2 = 1.00 \), the error of the first method is of the order 0.01 of that of the second. This is what has been implied by "the seeing compensation of the difference method".

Comparison between "ratio" and "difference" methods

At the present time there are two distinct methods of using a double-beamed device, and these will be discussed below. The first was used by Behr [4] and involves measuring difference signals with and without a known added amount of polarization, after making the difference zero at some particular angular setting of the analyser when the depolarizer is in the beam. He has performed experiments on unpolarized sources to show that only a sine wave is produced because of the non-uniform sensitivities of the cathodes when the instrument is rotated. When making an actual measurement, the phase of the sine wave is fixed at some angular setting by making the difference signal zero using a depolarizer. The criticisms of this type of instrument are that it is not seeing compensated; the difference curve need not necessarily be in the form of a sine wave when observing an unpolarized source (as was the experience of the author) and the instrument is calibrated using a known amount of polarization which is likely to introduce systematic errors in the results. The calibration can, however, be performed using only a depolarizer if both sum and difference signals are measured (see later).

Formation of values of the ratio of the two intensities can be performed in many ways. For example, Gehrels [5] integrates the signals from each beam, records the levels on a pen recorder and evaluates the
ratio from the two levels. This is done at each angular setting both
with and without depolarizer. The advantages of this method are that it
is seeing-compensated, and does not introduce possible systematic
errors such as in the value of the transmission factor of a filter etc.
However, in this particular design there is the disadvantage that the
ratio can only be obtained with an accuracy of about 1 per cent.

For a given change in the polarization of a source, the percentage
change in the difference signal is much greater (of the order of 100
times) than the percentage change in the ratio. Because of this, the
ratio method, the accuracy of which is limited by photon shot noise or
by the mode of display, only gives the same order of accuracy as the
difference method, the accuracy of which is limited either by photon
shot noise or scintillation.

The mathematical representation of the two methods

Consider the various methods of determining the degree of polar-
ization, taking into account the angular dependence of the overall gain
of the system caused by image-wander over a non uniformly sensitive
photocathode, changes in gain of the multiplier caused by the changes
in orientation with respect to the Earth's magnetic field, or changes in
the sensitivity of the multiplier at the different orientations of the
plane of polarization with respect to the photocathode. (In the case of
all known double-beamed polarimeters, the latter effect does not apply
as the multipliers rotate together with the analyser).

The output signals for each beam can be expressed in the form
\[ S_1(\phi) = T(t) G_1(\phi) \left\{ I_m \cos^2 (\phi) + I_n \sin^2 (\phi) \right\} , \]  (10)
\[ S_2(\phi) = T(t) G_2(\phi) \left\{ I_m \sin^2 (\phi) + I_n \cos^2 (\phi) \right\} . \]  (11)

where \( G_1(\phi) \) and \( G_2(\phi) \) represent the overall gain factors of each beam,
\( T(t) \) the atmospheric scintillation and transparency and the other
symbols are the same as defined previously.

These two equations can be rewritten in the form
\[ S_1(\phi) = \frac{1}{2} T(t) G_1(\phi) \left( I_m + I_n \right) (1 + P(\phi)) \]  (12)

and
\[ S_2(\phi) = \frac{1}{2} T(t) G_2(\phi) \left( I_m + I_n \right) (1 - P(\phi)) \]  (13)

where all the symbols have been defined previously.
The unwanted terms can be removed by affecting each beam in some way, such as by introducing a depolarizing filter before the analyser or by adding a known quantity of polarization. If we consider the first of these measures, equations (12) and (13) reduce to

\[ S_{1D} = \frac{4}{5} T_D(t) G_1(a) (I_M + I_s) \tau \]  

and

\[ S_{2D} = \frac{4}{5} T_D(t) G_2(a) (I_M + I_s) \tau \]  

where \( S_{1D}(a) \) and \( S_{2D}(a) \) are the signals, \( T_D(t) \) represents scintillation etc. when the depolarizer is in the main beam and \( \tau \) is the transmission factor of the depolarizing filter to unpolarized light.

**Single-Beamed Polarimeter**

For the case of a single beamed polarimeter, equations (12) and (14) can be used to determine values of \( P(a) \), giving

\[ P(a) = \frac{S_1(a) T_D(t)}{S_{1D}(a) T(t)} - 1. \]  

If it is assumed that the mean values of \( T(t) \) and \( T_D(t) \) are the same while the measurements of \( S_1(a) \) and \( S_{1D}(a) \) are being taken, this last equation reduces to

\[ P(a) = \frac{S_1(a) \tau}{S_{1D}(a)} - 1. \]  

**Double-Beamed Polarimeter**

By forming the ratio of the outputs either at the time of observation and displaying this as a signal or by forming the ratio subsequently after measuring the output signals simultaneously, the pairs of equations (12), (13) and (14), (15) give

\[ R(a) = \frac{G_1(a) (1 + P(a))}{G_2(a) (1 - P(a))} \]  

and

\[ R_D(a) = \frac{G_1(a)}{G_2(a)} \]
where \( R(a) \) and \( R_D(a) \) represent the ratios without and with the depolarizer, respectively. By combining equations (18) and (19)

\[
P(a) = \frac{R(a) - R_D(a)}{R(a) + R_D(a)}
\]

(20)

Suppose that the difference between the two outputs is formed and displayed as a signal. If we combine (12) with (13) and (14) with (15), we find that

\[
D(a) = \frac{T(t) \left( I_M + I_a \right)}{2} \left\{ G_1(a) - G_2(a) + P(a) \left[ G_1(a) + G_2(a) \right] \right\}
\]

(21)

and

\[
D_D(a) = \frac{T_D(t) \left( I_M + I_a \right)}{2} \left\{ G_1(a) - G_2(a) \right\}
\]

(22)

These two equations alone do not contain sufficient information to determine values of \( P(a) \). It is also not sufficient to repeat the readings after a known amount of polarization has been added; the forms of \( G_1(a) \) and \( G_2(a) \) must be known. Behr [4] has shown that, in his design of polarimeter, the forms of \( G_1(a) \) and \( G_2(a) \) are such that a sine wave is superimposed on the \( P(a) \) wave and that this can be accounted for and removed.

The addition of the two outputs can also be formed electronically and displayed. Combining (12) with (13) and (14) with (15) we find that

\[
A(a) = \frac{T(t) \left( I_M + I_a \right)}{2} \left\{ G_1(a) + G_2(a) + P(a) \left[ G_1(a) - G_2(a) \right] \right\}
\]

(23)

and

\[
A_D(a) = \frac{T_D(t) \left( I_M + I_a \right)}{2} \left\{ G_1(a) + G_2(a) \right\}
\]

(24)

Equations (23) and (24) are not useful in themselves, but can be combined with equations (21) and (22) to determine values of \( P(a) \). For example, using equations (21), (22) and (24) we find that

\[
P(a) = \frac{T_D(a) - D_D(a)}{A_D(a)}
\]

(25)

again on the assumption that the mean values of \( T(t) \) and \( T_D(t) \) are the same during the periods in which the observations are taken.
Consider now the estimates of the relative accuracies of the three methods of determining values of \( P(\alpha) \) which have been formulated in equations (17), (20), and (25). For the case of a single-beamed polarimeter, (equation 17), the final accuracy is determined by atmospheric seeing or by photon shot noise. For double-beamed polarimeters, the accuracy of the ratio method (equation 20) is limited by photon shot noise or mode of display, while in the case of the sum and difference method (equation 25), the limitation is set either by atmospheric seeing or by photon shot noise.

In order to obtain an estimate of the expected error for each case, it is sufficient to consider perfect polarimeters where \( G_1(\alpha) = G_2(\alpha) \) = constant; the necessary equations then reduce to

**Single beam:**

\[
P(\alpha) = \frac{S_1(\alpha)\tau}{S_{1D}(\alpha)} - 1.
\]

**Double-beamed-ratio:**

\[
P(\alpha) = \frac{R(\alpha) - 1}{R(\alpha) + 1};
\]

**Double-beamed sum and difference**

\[
P(\alpha) = \frac{\tau D(\alpha)}{A_D(\alpha)}
\]

By performing the necessary algebra on equation (26) we find the fractional error in \( P(\alpha) \) to be given by:

\[
\frac{\Delta P}{P} = \frac{P + 1}{P} \left( \frac{\Delta^2 S_1}{S_1^2} + \frac{\Delta^2 S_{1D}}{S_{1D}^2} \right)^{1/2},
\]

where the prefix \( \Delta \) indicates the error in the quantity associated with that symbol. When the output signal is limited by atmospheric seeing it can be assumed that \( \langle \Delta S/S_1 \rangle = \langle \Delta S_{1D}/S_{1D} \rangle = K \).
the magnitude of which depends on the scintillation noise at the time.

Equation (29) then reduces to

$$\frac{\Delta P}{P} = \frac{(P + 1)\sqrt{2}}{P} K.$$  \hspace{1cm} (30)

When the accuracy is limited by photon shot noise,

$$\Delta S / S_1 = S_1^{-1}, \Delta S_{1D} / S_{1D} = S_{1D}^{-1} \text{ and, hence,}$$

$$\frac{\Delta P}{P} = \frac{P + 1}{P} \left( \frac{1}{S_1} + \frac{1}{S_{1D}} \right)^{1/2}.$$  \hspace{1cm} (31)

which can be rewritten in the form

$$\frac{\Delta P}{P} = \frac{P + 1}{P} \left( \frac{\tau + P + 1}{\tau S_1} \right)^{1/2}.$$  \hspace{1cm} (32)

where $S_1$ can be considered as the number of primary photo-electrons emitted by the cathode of the multiplier.

For the ratio method, we have

$$\frac{\Delta P}{P} = \frac{(1 - P)^2}{2P} \Delta R.$$  \hspace{1cm} (33)

This can be rewritten in terms which are more convenient when the limitation is set by photon shot noise, giving

$$\frac{\Delta P}{P} = \frac{1}{P} \sqrt{\frac{1 - P^2}{I}}$$  \hspace{1cm} (34)

where $I = I_u + I_d$ is expressed in numbers of primary photo-electrons.

For the sum and difference method,

$$\frac{\Delta P}{P} = \sqrt{2} K.$$  \hspace{1cm} (35)

again on the assumption that $K = \Delta D / D = \Delta A / A_D$, giving an indication of the magnitude of the seeing noise. When photon shot noise is dominant
Comparison of Error Curves for Single and Double-Beamed Systems

Fig. 1.
Comparison of Error Curves for the Double-Beamed Methods (Equations (33), (34), (35) and (36))

Equation (33) \( \Delta R = 0.01 \)

Equation (34) \( I = 10^4 \)

Equation (34) \( I = 10^4 \)

Equation (35) \( K = 0.05 \)

Equation (36) \( I = 10^2 \)

\[ \text{Comparison of Error Curves for the Double-Beamed Methods (Equations (33), (34), (35) and (36))} \]
the fractional error can be written more conveniently in the form

\[ \frac{\Delta P}{P} = \sqrt{\frac{2}{\pi}}. \]  \hspace{1cm} (36)

In order to make comparisons between the various effects which influence the fractional errors, values of \( \frac{\Delta P}{P} \) were evaluated for equations (30), (32), (33), (34), (35), and (36) and appropriate curves have been drawn. To compare the single-beamed method with double-beamed methods curves of \( \log(\frac{\Delta P}{P}) \) have been plotted against \( P \) for the three cases, (30), (33), and (35) on Fig. 1, for typical values of \( \text{K}=5\% \) and \( \Delta R = 1 \) per cent. As was to be expected, the single beamed device is inferior to the double-beamed device by a factor of about 100, and for the majority of astronomical objects would be wholly inadequate.

In order to consider the relative merits of the two double-beamed methods in more detail, including the errors introduced by photon shot noise, Fig. 2 has been drawn illustrating typical values of \( \frac{\Delta P}{P} \) against \( P \) that have been obtained from equations (33), (34), (35), and (36). It can be seen that the ratio method will give approximately the same accuracy as the sum and difference method for typical values of \( \Delta R = 0.01 \) and \( \text{K}=0.05 \). For the ratio method to be an improvement, the error \( \Delta R \) would have to be reduced to much less than \( \Delta R = 0.01 \) and, hence, some mode of display other than a pen recorder would have to be used.

The curves indicate that there is a definite transition depending on the type of object under analysis and the seeing conditions at the time when the ratio method (seeing compensated) will give results superior to the sum and difference method.
PART II

The Design of the Polarimeter

The First Polarimeter

It was decided that it would be useful to design and construct a single-beamed polarimeter so that several design problems could be investigated and valuable experience gained in polarimetric studies, both in the laboratory and at the telescope.

This polarimeter was designed to be used on the 15″ f/6 photographic refractor at the Wilfred Hall Observatory, Longridge, Nr Preston, Lancashire; being about 40 miles from the University at Manchester. A simplified drawing of the components is shown on Fig. 3. The collimator had a focal length of 2″, thus giving a parallel beam of ½″ diameter through the analyser. The field lens had a focal length of 3″ and imaged the telescope objective on the photocathode of the multiplier. The analyser was obtained from Barr and Stroud, Ltd., Anniesland, Glasgow W.3. and was their first-quality polaroid, mounted on strain-free glass. White light transmission of this filter was 29.7 per cent. The depolarizer was the usual Lyot type and could be pushed in or out of the parallel beam, immediately before the analyser. It consisted of two quartz plates the first 1/16″ thick, the second 1/8″ thick, cut parallel to their optical axes and cemented together with Canada balsam, so that the axes of the plates were separated by 45°. This filter was also made by Barr and Stroud Ltd., and was dispatched on loan until its efficiency had been tested.

The iris diaphragm had a minimum diameter of 0.035″ which corresponded to 80 secs of the sky. Immediately behind the diaphragm was a tube that could be inserted into the main beam at right angles to the optical axis, so that the object under analysis could be viewed and centered. The magnification of this viewing system was eight times. Contrast between the iris diaphragm and the sky background was improved by illuminating the diaphragm from the inside by feeble light, the intensity of which was controlled by a 30 ohm potentiometer.

It is well known that the output of a photomultiplier to polarized light can depend on the angle between the maximum component and the photocathode, especially if the multiplier is of the side-window type (1P21). Effects of this nature were eliminated by making the photomultiplier rotate with the analyser.
The Single-Beamed Polarimeter

Fig. 3.
Two types of photomultiplier were used in this polarimeter: an E.M.I. 6094 and an E.M.I. 9558 (tri-alkali). Both these tubes are end-on-types, the photocathode being normal to the incident light. The dynode chain of the multiplier was fed by a neon stabilised high tension power pack. The output from the cell was fed directly to an Avo D.C. amplifier (type 1388B). The current range of this instrument is from $10^{-6}$ amps, to $3 \times 10^{-13}$ amps. for full scale meter deflection and there is a switch which provides seven input ranges. An output of 100 m.V. at full scale deflection is provided to operate a pen recorder. This output was fed to a Speedomax G pen recorder which requires 10 m.V. for full-scale deflection. The linearity of the combined electronic system was checked by putting a "potted" 10v source across the input and feedback connections of the amplifier. Readings were taken on the pen recorder for various voltages of this source.

Laboratory Experiments

These experiments included an investigation of the effects which give rise to deviations from a steady output level when observing unpolarized light, as the polarimeter is set at different angular positions. The sensitivity of the cell to polarized light was also investigated. Each experiment will be discussed below.

1 Rotational effects of the photomultiplier

It was found essential that any test source should be uniformly bright; otherwise there would be deviations on rotation because of the non-uniform sensitivity of the photocathode. This was achieved by fixing a diffusing screen in the plane of the iris diaphragm. When fixed on an optical bench, the polarimeter was sensitive to vibration; and it was found essential to attach the light source on to the polarimeter itself.

Without the multiplier in the system a diffusing screen was placed in the plane of the photocathode. The patch of light on this screen was viewed by means of a microscope and it was seen to move relatively to the screen, as the analyser was rotated. When the multiplier was attached to the system, the signal exhibited small deviations for the different angular settings of the analyser. The output signal against angular setting was not a pure sine wave as might have been expected from the analysis of Behr [4]. In order to investigate this further, and to differentiate between the efforts of image-wander and actual changes in the gain of the cell, the inside of the photomultiplier housing shutter was painted with luminous paint. In this way the light source was made to rotate with the multiplier. Deviations were still apparent.
suggesting that the gain of the cell changed with orientation. The possibility of a differential light leak was excluded as the polarimeter was wrapped in layers of black cloth and the experiment was carried out in a photographic darkroom. An experiment was also tried in which it was hoped to see the deviations in dark current for different angular settings; but the noise was too great for the results to be significant. When the second polarimeter was made (see later), one way of measuring the output signal was to integrate the pulse count over a period of time. The results from this apparatus showed that dark count did depend on the orientation of the multiplier, the value changing by as much as 20%, depending on the position.

The effect of the gain of a photomultiplier depending on its orientation relative to magnetic fields has already been noted [7]; and, in particular, the effect produced by the Earth’s or stray magnetic fields is not insignificant. Not only did mu-metal screening remove the deviations when the polarimeter was rotated, but increased the gain of the cell over its previous maximum value by a small amount (~3 per cent). In the case of the tri-alkali cells, the deviations were reduced by increasing the voltage between the cathode and first dynode. (There is a large distance (3 cms) between the cathode and the first dynode, and the manufacturers suggest that the voltage should be twice as much as between each of the other dynodes to prevent drifting of the electrons).

The experiments were carried out with the axis of rotation in directions, North-South, East-West and along the Earth’s magnetic field vector. In each case the deviations occurred but it was always possible to remove them by using the mu-metal cans. The paths of the electrons through the venetian blind system of the multiplier are very complicated. No orientation was found with respect to the Earth’s field which removed the deviations without mu-metal screening (See Fig. (4)). Behr [4] who used E.M.I. 6094 multipliers, found that effects of the Earth’s field were too small to have been of significance.

ii Effects of polarized light on the photocathode

By releasing the grub screws that clamped the multiplier housing to the tube containing the analyser, it was possible to rotate the analyser with respect to the photocathode. There were no significant changes in the apparent gain of the cell for different positions of the plane of polarization with respect to the photocathode. Side window types of cell, such as the 1P21, are handicapped by this effect.
Variation of output signal with orientation

Variation of output signal with orientation, mu-metal can added

Variation of dark count with orientation

Orientation Effects of Multipliers

Fig. 4.
iii Efficiency of the depolarizer

A strongly polarized source was imaged on to the iris diaphragm of the polarimeter and the signal was measured at angular positions separated by 30°, with and without the depolarizer in the beam. It was found that the depolarizing efficiency was of the order of 98 per cent; i.e. if a source is linearly polarized to the extent of 98 per cent, the filter would reduce the degree of polarization to 1 per cent. The value of $\tau$ was determined by analysing an unpolarized source.

Experiments on the telescope

The polarimeter was used at Preston from December 1960 until February 1961. The photograph in Fig. 5. shows the polarimeter attached to the 15" telescope. Objects that were investigated included the Moon, Mars, Venus, $\alpha$ Tauri (Aldebaran) and several fainter stars. In the case of the Moon, Mars and Venus the telescope was stopped down to obtain a manageable signal. Typical light levels were (E.H.T. 1100 V - dark current 10$^{-12}$ amps):

- Moon (3" aperture): 5 x 10$^{-8}$ amps.
- Mars (3" aperture): 6 x 10$^{-8}$ amps.
- Aldebaran (15" aperture): 8 x 10$^{-8}$ amps.

Guiding was effected by use of the twin 15" visual telescope. The image in the diaphragm was viewed through the side microscope and the telescope adjusted until the image was in the centre of the field. This was usually performed in stages, reducing the size of the iris each time. Adjustments were then made to the grill in the guiding telescope until the same object was at the centre of the crosswires.

The first analysing procedure was to measure the output signal on the pen recorder at angular settings 30° apart, between 0° and 360° and then repeat the process after the depolarizer had been interposed in the beam. This proved unsatisfactory because of the changes in the general atmospheric transparency while the experiment was in progress. At any particular angular setting it was found essential to obtain readings with and without the depolarizer with the minimum delay between the two.

Measurements were taken on $\alpha$ Tauri (Aldebaran) on several nights in order to test the reliability of the apparatus. This star was
Fig. 5. Single Beamed Polarimeter Attached to Telescope
expected to have zero polarization, certainly less than 0.1 per cent. The values of \( P(\alpha) \) obtained on this star agree very well with what had been predicted by equation (30). Several fainter stars, which have had their polarization measured by other astronomers were analysed in a similar way, but no reliable data could be obtained because of the effect scintillation was having on the results. It was obvious that little or no useful work could be performed on stellar sources in this way.

In the case of the disc of Mars, the situation was improved as scintillation was less serious. From a series of recordings taking about one hour (1961 Jan. 7. OOH-30M) the degree of polarization for the whole disc was found to be \( 1.2 \pm 0.2 \) per cent at the phase angle being \( 10^\circ \ 8' \). This result compares favourably with the previous results of B. Lyot \[6\] at the same phase angle.

No reliable measurements were made on the Moon with this instrument because of guiding difficulties. The R.A. slow motion was then controlled by two cords which seriously limited the ease and accuracy of the guiding. Before the double-beamed polarimeter was used, this guiding mechanism was replaced by a guiding motor which is controlled by push-buttons.

The Second Polarimeter

Several double-beamed polarimeters have been used by other astronomers \[3,4,5\]. All these devices were optically identical, using a Wollaston prism to separate the two components. This type of analyser has, however, disadvantages. The main disadvantage is that the angular separation between the two components is usually small and further separation has to be made by means of a silvered wedge or prism. The angular separation is also wavelength-dependent. If the Fresnel reflection coefficients are computed for each beam at each interface, the light losses can be considerable and the transmission coefficients for each beam are usually unequal. For example, a calcite prism of \( 25^\circ \) gives deviations of \( 3^\circ \ 30' \) and \( 3^\circ \ 40' \) for the two beams, with transmission coefficients of \( 80\% \) and \( 87\% \) at the wavelengths of the sodium D lines.

It was decided to investigate other double-beamed analysing systems; and after consulting R. and J. Beck Ltd., London, a new type of analysing prism was envisaged which would resolve the light into two components, giving a large separation, with the minimum of light loss in the beams. It is essentially based on the prism used by Foster \[8\]. The prism was made of glass with a slice of calcite separating the two pieces. (See fig. 6.) Separation into the two components depends on one of the components passing straight through the prism, while the other is totally internally reflected at the first glass-calcite interface.
The Analysing Prism

Fig. 6.
The calcite was cut so that component A was presented with the refractive index for the O-ray (~1.66), while component B was presented with the refractive index for the E-ray (~1.48). By having a glass with a refractive index which matched that of the O-ray, component A was made to pass straight through the prism while component B suffered total internal reflection and could be brought through the side of the prism. It was immediately obvious that in this design of prism, it would be possible to have the entrance face and both exit faces normal to the incoming and outgoing rays so that there would be only the minimum light losses. In this case, if a parallel unpolarized beam were to pass through the prism the resolved components would have the same intensity.

Consider the angle $\gamma$ which is required to obtain total internal reflection of the E-ray using typical values for the refractive index of calcite. For total internal reflection

$$\mu_0 \sin \gamma \geq \mu_e,$$

giving

$$\sin \gamma \geq \frac{1.486}{1.658}$$

or

$$\gamma \geq 63^\circ 42'.$$

Total reflection will occur if $\omega < 28^\circ 18'$. A convenient value of $\omega = 22^\circ 30'$ was chosen, so that the angular separation between the two beams was $45^\circ$.

For such a device to be perfectly efficient it is very important that the refractive index of the glass should match the refractive index of the O-ray of calcite throughout the visible range. The glass which was the nearest to fulfilling the required characteristics was a Chance type, DBC 657508. At the wavelength of the sodium D-lines this glass has a refractive index of 1.657 while the refractive index for the O-ray of calcite is 1.658. The magnitude of the refractive index difference is approximately the same over the visible region of the spectrum, although it does increase slightly in the ultra-violet. The prism was designed to have a 2 sq.cm entrance aperture. It was made by Hilger and Watts, Ltd. London.

As mentioned at the beginning of this paper corrections have to be applied to any measurements if the analyser is not perfectly efficient. Laboratory and telescope tests have shown that this particular prism is very efficient and that no corrections need be applied.
The Double-Beamed Polarimeter

Fig. 7.
The optical layout of the double-beamed polarimeter is shown in fig. 7. The lenses were high quality achromats made by Dallmeyer Ltd., London; the collimator having a focal length of 3" and the field lenses having focal lengths of 2".

The analysing prism was held in a rectangular metal tube by means of nylon screws. This tube was, in turn, suspended from a plate which fitted into a collar brazed to the outer tube. By adjusting the nylon screws the prism could be moved along the length of the tube and set at any orientation with respect to the rotation axis.

Spring retainers were fitted on the filter slots so that the depolarizing filter and colour filters would remain in position when the inside tube was rotated. The outer tube, holding the ball races, had a wide slot milled into it. The slot was covered by a light-tight lid so that the filters could be easily interchanged. This was an improvement over the first polarimeter where the filter slot was directly exposed, making it difficult to eliminate light leaks.

The ball races had an internal bore of 1 7/8", being the largest in the Aircraft series from the Hoffman catalogue. After the desired angular setting had been found, the polarimeter could be clamped by means of a lock screw. The plate containing this screw was calibrated with 7½° intervals.

The minimum size of the iris diaphragm was 0".025, which corresponds to 55 secs. of arc. In terms of lunar dimensions, this corresponds to a length of 100 kms (65 miles) on the lunar surface. The diaphragm was illuminated in a similar way to that used in the first polarimeter and the same procedure was used to centre any celestial object.

Alignment of the prism was carried out in the following manner. After the collimator had been set correctly relative to the iris diaphragm, a parallel beam of light was directed through the prism and the prism's position was altered until the beam passed centrally, the image formed by the field lens in beam 1 being on the axis of the tube. The prism was then moved along the main tube until the image formed by the second field lens was on the axis of the tube which contained the lens. When the polarimeter was rotated the images formed by the field lenses wandered slightly and this effect could not be removed by further adjustment of the prism.

Two different types of photomultiplier were used in conjunction with this apparatus. These were the E.M.I. 6256 in the preliminary
experiments and later the E.M.I. 9558B (tri-alkalis). Several tri-
alkali cells were used until a well matched pair was found. The cells
that were put into permanent use (serial numbers 5224 and 5273) both
had photosensitivities of 125μA/L and had similar magnitudes of dark
current.

Six colour filters have been used with the apparatus. These were
from the Kodak Wratten catalogue, numbers 94, 74, 99, 73, 72B and 89B.
All these filters have only one main transmission band. The effective
wavelength for each filter was evaluated graphically using the formula

\[ \bar{\lambda} = \frac{\int S(\lambda) \cdot \lambda \cdot d\lambda}{\int S(\lambda) \cdot d\lambda} \]

where \( S(\lambda) \) is the response of the tube to the part of the light trans-
mitted by the filter. The effective wavelengths and half-widths of
each filter are given below:

<table>
<thead>
<tr>
<th>Filter</th>
<th>( \bar{\lambda} )</th>
<th>( \Delta \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>94</td>
<td>4590</td>
<td>320</td>
</tr>
<tr>
<td>74</td>
<td>5370</td>
<td>320</td>
</tr>
<tr>
<td>99</td>
<td>5490</td>
<td>320</td>
</tr>
<tr>
<td>73</td>
<td>5760</td>
<td>300</td>
</tr>
<tr>
<td>72B</td>
<td>6080</td>
<td>300</td>
</tr>
<tr>
<td>89B</td>
<td>7390</td>
<td>650</td>
</tr>
</tbody>
</table>

The efficiency of the depolarizer was checked with each filter in turn
and it was found that there was little or no difference over the
efficiency when the depolarizer was used with white light.

Between the E.H.T. pack and the cathode of each multiplier, a
2 meg-ohm potentiometer was inserted, so that the gain of each cell
could be controlled independently. The base for the multiplier on
beam 1 was wired in the normal way while the base for the second
multiplier was wired without the last three resistors in the dynode
chain. Shielded leads were taken from the pins corresponding to the
last two dynodes, to a small box containing the necessary resistors.
Inputs were fitted to this box so that the multiplier could be used in
its normal condition or coupled with the other multiplier; the difference
current between the two cells could then be measured directly. This
difference circuit has been used previously by Thiessen [9] and the
diagram for this and the optional circuit is shown on fig.8.
Circuit for measurement of difference of two signals

Direct difference connections

Normal operation of cell 2.

Dynode Connections

Fig. 8.
Two different electronic systems were used in order to compare techniques of the sum and difference method and the ratio method. For the sum and difference method the output currents were measured by a D.C. amplifier and displayed on the Speedomax G pen recorder; while for the ratio method two pulse counting photometers were made which displayed the total number of individual photoelectrons counted in a pre-determined time. By forming the ratio of the digitised outputs, the accuracy of these latter measurements depends on the total number of pulses counted. The two systems are considered in greater detail below.

Sum and Difference Method

For this technique the dynode chain of the second multiplier was connected as described earlier, so that the difference signal could be measured directly by the D.C. amplifier.

The equation necessary to evaluate values of \( P(\alpha) \) by this method was derived in Part I of this paper and is written in the form

\[
P(\alpha) = \frac{T_D(\alpha) - D_D(\alpha)}{A_D(\alpha)}.
\]  

(35)

The method to obtain the information for this equation involved measuring the difference signals, with and without the depolarizer, and the isolated signal of one of the beams when the depolarizer was in the main beam. No actual addition signal was measured. With the polarimeter set at \( \alpha = 0 \) and with the depolarizer in the main beam, the magnitudes of the signals were adjusted by altering the E.H.T. of the cells until the difference signal was zero. In this state,

\[
S_{1D} = S_{2D} = \frac{1}{2}A_D
\]

Hence, \( A_D \) was determined by measuring the signal from one of the components. This was done, for example, by disconnecting the last dynode of cell 2. It had been hoped that this operation might have been performed more smoothly by having switches in the appropriate places in the dynode chains of the cells. Leakage across the switches that were used upset the measurements and it was found more reliable to disconnect a lead completely by unplugging.

Readings for \( S_{1D} \), \( D_D \) and \( D \) were taken for other values of \( \alpha \), usually separated by 45°. Because of the rotation of the images of the aperture stop over non-uniform photocathodes, \( D_D \) was not equal to zero at the
other angular settings and, in this case,

\[ 2 S_{1D} / A_D. \]

The error in assuming that \( 2S_{1D} - A_D \) is usually small, but can be eliminated if necessary since \( D_D \) has been measured. The value of \( A_D \) can be obtained from

\[ A_D = S_{1D} + S_{2D} \]

and therefore,

\[ A_D = 2S_{1D} - D_D. \]  \( (37) \)

It was found that the output signal of a cell settled down fairly quickly (~5 to 10 sec) if adjustments to the gain by means of changing the E.H.T. were small. In some cases the results were obtained by adjusting \( D_D \) to be zero for each angular setting, but this proved to be tedious.

Since the difference signals were usually small in comparison with the individual signals from the two components, it was sometimes found convenient to expand the difference signal by a factor of ten by using the switch on the D.C. amplifier. This was particularly useful when adjusting the value of \( D_D \) to zero.

Using a laboratory source, the stability of the difference signal was checked over periods of time which were of the order of 10 times greater than the actual time required for taking the measurements on the telescope. It was found that small drifts did occur, giving rise to changes of 1 to 2 per cent in the level of the difference signal over a period of about fifteen minutes. Each observation of a difference signal at the telescope took less than one minute; and as scintillation noise of this signal was of the order of 5 to 10 per cent, no changes in level due to drift were detectable. Any change in level that was detected was caused by changes in the atmospheric transparency.

**Ratio Method**

In order that this method might compete with the sum and difference method, it was realised that the accuracy of any determined ratio should be high, at least better than 1 per cent and this suggested that pulse-
counting should be a good technique, where the accuracy of the final display depends only on the number of pulses that have been counted.

Two similar pulse-counting photometers were designed and made so that the intensities of the components could be measured simultaneously. Each photometer consisted of a wide-band pulse amplifier, a pulse height discriminator and a decade scaler. Both photometers were controlled by the same time so that they integrated the pulse count over the same period of time.

General requirements of the wide-band amplifiers were that the gains should be of the order of $10^4$, and that the negative pulses supplied by the multiplier should be amplified and the polarity of the pulses inverted so that positive pulses of the order of 25 volts can be supplied to the discriminator and decade scalers. Each amplifier consisted of two "rings of three", each ring being mounted separately to avoid electrical oscillations. When the amplifiers were tested, the final output exhibited noise of the order of 1 volt superimposed on a 50 cycle ripple of about 2 volts which was certainly adequate as the discriminator is never set below 10 volts. The overall gains of the amplifiers were checked using a pulse generator and oscilloscope, and were found to be approximately $7 \times 10^3$. Although these amplifiers suffered less from microphonics than any other previously made by the author, precautions were taken by mounting their chassis on sponge. The chassis were also isolated from vibrating parts such as the relays belonging to the timing mechanism. In order to keep the gains of the amplifier constant, the H.T. pack for the amplifiers was stabilized by using neons; a 10 per cent change in mains voltage produced only a one per cent change in the H.T. voltage.

The decade scalers, including the discriminators, were built in the Physical Laboratories at Manchester, and based on a circuit of Mullards. They used E.I. T. type decade counting tubes and were designed to count positive pulses. Each complete scaler had five such tubes in series so that the maximum count that could be registered was 99,999. These units are standard items in the Physical Laboratories, and the only necessary modification was to provide outputs immediately after each decade tube, so that the scaler could be used to divide the pulse counting rate by $10$, $10^2$, $10^3$ etc. By choosing the output where the pulse rate had been reduced to a value less than 10 per sec, a mechanical register could then be driven without counting losses. By connecting the last output of the scaler to the mechanical register it was possible to increase the maximum integrated count number from $10^3$ to $10^5$. 
The original timer that was used for integrating the pulse count consisted of an ex-government clock and contactors. Every half second the clock makes a contact and this in turn causes a solenoid to operate in the contactor. By the action of this solenoid a cam is driven round, taking one minute for a revolution. A 24 volt D.C. supply is needed to operate this assembly. Two of these contactors were put into series; one for controlling the count on-off mechanism and the other controlling the set-zero of the scalers. The contacts from the first cam were included in another circuit which contained a double make relay. This relay controlled the inputs of the two scalers. The contacts from the second cam were included in the set-zero mechanisms of both scalers.

By shaping the cams the following sequence was obtained. At time zero both contactors were open and the scalers were in operating conditions. After 45 secs. the first contactor closed, thus operating the secondary relay so that both scalers were switched off simultaneously. At 59 secs. the second contactor would close and reset both scalers. At 60 secs. both contactors re-opened and the whole sequence repeated. During the interval from 45 secs. to 59 secs. the integrated count was read and recorded. On most occasions observations were performed single-handed and it was found convenient to stop the clock after the count was completed (i.e. immediately after 45 secs.). Readings of the integrated count were then taken and adjustments made to the polarimeter (i.e. remove or insert the depolarizer or alter the angular setting) before the clock was restarted.

Although this system worked satisfactorily, it was decided that a variable timer would give greater flexibility. When the design for such a timer was considered, another device was envisaged which would have the advantage that the ratio of the intensities would be displayed directly. By using one counting circuit as the control, it was possible to switch off the inputs to both scalers after the first scaler had counted a number of pulses which is divisible by ten depending on the accuracy that was required. The second scaler then gave the reading of the ratio of the two intensities.

This optional timer-divide by ten unit was completely transistorised. Any sequence was initiated by a push-button control. The timer had a range from 5 secs. to 2 mins. and allowed both scalers to count over a pre-selected period after pressing the start control. After the integrating period, the scalers were switched off simultaneously and could be later set to zero by hand or by a pulse from the control unit; the delay of this pulse after the stop count could be set again from 5 secs. to 2 mins.
The direct-ratio method suffers from the disadvantage that if the atmospheric transparency is varying during the count integration, the time for the count will differ for each integration. This method is obviously inaccurate if the dark current is so large that its subtraction interferes with the significant figures of the total count.

In normal practice, the timing circuit was used and the integration was adjusted so that of the order of $10^6$ pulses were counted on each beam. In this way the accuracy of the photometry was approximately 0.1 per cent on each beam.

Because of the finite resolving time of the amplifier, thus limiting the number of pulses that can be counted in a given time, it was necessary to reduce the aperture of the telescope for objects brighter than 5th magnitude. The constancy of repeated measurements of a given ratio was a good test to check that the amplifiers were not being overloaded by two high or too many pulses.

Experiments were carried out integrating of the order $10^4$, $10^5$ and $10^6$ pulses on each beam and the standard deviations of the ratio, as obtained from repeated measurements, went down by approximately the expected amount.

Values for the degree of polarization of celestial sources were obtained by taking an integrated count, with and without the depolarizing filter, at angular settings $45^\circ$ apart, between $0^\circ$ and $360^\circ$ and evaluating the results as before.

General lay-out of apparatus

A tall Dexion trolley was constructed to carry most of the electronic apparatus. One side of the trolley housed the E.H.T. pack, the pots. for gain adjustments, the D.C. amplifier and the Speedomax pen recorder. The other side held the stabilised power pack and the amplifiers for the pulse counting photometers. The amplifiers were put as close as possible to the multipliers so that the leads could be kept as short as possible. The remainder of the pulse counting apparatus, including the decade scalers, timer, transistorised unit and power supply were mounted on a permanent table. The photograph of the polarimeter itself is shown on fig.9.

Discussion of the two methods

Both electronic systems have been applied to various celestial objects and it has been found, in general, that the sum and difference method, displaying the results on a pen recorder, is more suitable for
Fig. 9. The Double-Beamed Polarimeter attached to the Telescope
bright sources exhibiting a relatively high degree of polarization (i.e. > 1 per cent). This method is always used when measuring the polarization of the planets or small regions on the Moon. It has been shown that this method is ineffective if the sky transparency is not reasonably constant.

When the pulse-counting apparatus was tested on the telescope, repeated experiments were made on the constancy of a given ratio. On several occasions thin cloud reduced the intensity of an object. A factor of five in reduction of intensity had little or no effect on the measured ratio. Greater intensity changes than this caused a deterioration in the accuracy of the measurement. Because of the brightness of the Moon and planets the use of this apparatus has been confined to stellar sources.

During the winter of 1961/1962 measurements were made on the Moon in white light and in narrower spectral ranges. These results together with others of various objects have been collected and are discussed together in Part III of this paper.
PART III

Preliminary Results

The reliability of the telescope objective

The single-beamed polarimeter described earlier had indicated that the telescope objective was not introducing parasitic polarization to any serious extent. These experiments had shown that any systematic polarization introduced in this way was certainly less than one per cent. By using the double-beamed device it was hoped that smaller differences than this might be detected over the lunar surface. Further experiments on the reliability of the objective were carried out by making measurements on stars that had been analysed previously by other observers.

A group of known stars was studied in Cassiopeia; and measurements were taken at various orientations of the telescope. No systematic polarization was detected at any orientation; and it can be said that the amount of polarization introduced by the objective was less than 0.1 per cent.

The angle of the arbitrary zero on the scale of the polarimeter relative to right ascension, which in turn determines the angle of the analyser relative to right ascension, was estimated to be 87° ± 3°. This could be determined more accurately by observing standard sources.

Sky background

On nearly all occasions of observations other than of the Moon, the intensity of the background sky in the neighbourhood of the object was measured. This background level was found to be unimportant for the stars that were analysed except when the background was due to scattered moonlight. For example, η Aur. (4.7m) was measured on Oct. 29 1961 when the Moon was only 15° away. The background light from the area of the sky filling the iris diaphragm had an intensity of about one-tenth that of the star. This background was highly polarized. It is possible by analysing a small region of the sky near to the star to compensate for the background and obtain corrected results for the star. It is advisable to avoid making stellar and planetary measurements when the Moon is above the horizon or if the sky is not completely dark.
Effects of guiding

If we compare the stellar and lunar results, no improvement in the accuracy is noted which would be due to the higher light level of the Moon. In the case of the Moon, the accuracy of the results is controlled by the accuracy of guiding (because of the sharp changes in albedo over the lunar surface) rather than by any other cause.

The effect of drift in the guiding on a stellar image was not noticed until the edge of the iris diaphragm began to cut the light off.

Results on comparison stars

Tests on the reliability of the polarimeter were made by observing some stars which had been measured previously by other astronomers. The results of four of these stars are tabulated below to give an idea of the accuracy of the observations.

<table>
<thead>
<tr>
<th>Star</th>
<th>Date</th>
<th>Time H M</th>
<th>$\rho$</th>
<th>Previous Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$ Cas.</td>
<td>1962 Jan 31</td>
<td>23 25</td>
<td>1.3 ± 0.2</td>
<td>1.28 (B)</td>
</tr>
<tr>
<td></td>
<td>1962 Feb 4</td>
<td>23 30</td>
<td>1.3 ± 0.1</td>
<td>1.5 (Hi)</td>
</tr>
<tr>
<td></td>
<td>1962 Feb 14</td>
<td>03 35</td>
<td>1.8 ± 0.3</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ Cas.</td>
<td>1962 Feb 14</td>
<td>05 00</td>
<td>0.7 ± 0.2</td>
<td>0.81 (B)</td>
</tr>
<tr>
<td></td>
<td>1962 Feb 14</td>
<td>05 00</td>
<td></td>
<td>0.8 (Ha)</td>
</tr>
<tr>
<td>$\rho$ Cas.</td>
<td>1962 Jan 27</td>
<td>22 45</td>
<td>1.2 ± 0.4</td>
<td>1.24 (B) 1.4 (Ha)</td>
</tr>
<tr>
<td></td>
<td>1962 Jan 27</td>
<td>20 40</td>
<td></td>
<td>1.4 (Hi)</td>
</tr>
<tr>
<td>$\alpha$ Tau.</td>
<td>1962 Jan 19</td>
<td>19 30</td>
<td>&lt;0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1962 Jan 19</td>
<td>22 30</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1962 Jan 27</td>
<td>20 40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The last column of the above table lists results that have been obtained by previous observers (B = Behr, Ha = Hall and Hi = Hiltner).

Determination of phase angles

When tabulating lunar and planetary results it is usual to refer to any measurements to the phase angle at any particular time. This angle can be determined using "The Astronomical Ephemeris".
Evaluation of Phase Angles

Fig. 10.
For the case of the planets, the phase angle can be evaluated easily. In view of fig.10, the cosine formula gives
\[ R^2 = r^2 + \Delta^2 - 2r\Delta \cos \theta, \]
whence
\[ \cos \theta = \frac{r^2 + \Delta^2 - R^2}{2r\Delta}. \tag{38} \]
The values of \( r, \Delta, \) and \( R \) are all given in the ephemeris tables; and, hence, the phase angle \( \theta \) can be determined.

In the case of the Moon, these formulae are not convenient, as values for \( r \) are not tabulated. From fig.10. it follows again that
\[ \theta = \Pi - E - S; \tag{39} \]
and the sine formula gives
\[ \sin S = (\sqrt{r}) \sin E. \tag{40} \]
Since, for the Moon, \( S \) is always a small quantity, equation (39) becomes
\[ \theta = \Pi - E - (6' 40'') \sin E. \tag{41} \]
To determine the angle \( E \), it is possible to use the right ascensions \((\alpha)\) and declinations \((\delta)\) of the Sun and the Moon and these are tabulated. Using the cosine formula of spherical trigonometry (see fig.10), we see
\[ \cos E = \sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha - \alpha') \tag{42} \]
the values of \( \alpha \) and \( \delta \) should still be corrected for the parallax.

The angle \( \theta \) can be found by using equations (42) to solve for \( E \) and then substituting this in equation (41).

 Corrections to the phase angle can also be applied for the different regions of the lunar surface but in normal circumstances
these corrections are insignificant. The rate of change of polarization with phase angle is small so that there would be no significant difference in the results if two different areas were to be compared at identical times or identical phase angles.

Lunar Observations

Some regions that have been studied are tabulated below. The co-ordinates of the areas that have been measured were obtained from the USAF Lunar Atlas [10]. The smallest opening of the iris diaphragm was always used (corresponding to ~100 kms on the lunar surface).

<table>
<thead>
<tr>
<th>Region No.</th>
<th>Description of Area</th>
<th>$\xi$</th>
<th>$\eta$</th>
<th>$\lambda$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Plato (walls not included in area)</td>
<td>-102</td>
<td>+783</td>
<td>+51° 30'</td>
<td>-9° 26'</td>
</tr>
<tr>
<td>3</td>
<td>Pico</td>
<td>-108</td>
<td>+715</td>
<td>+45° 38'</td>
<td>-8° 58'</td>
</tr>
<tr>
<td>4</td>
<td>Aritarchus</td>
<td>-674</td>
<td>+401</td>
<td>+23° 39'</td>
<td>-47° 23'</td>
</tr>
<tr>
<td>6</td>
<td>W. Walls of Copernicus</td>
<td>-349</td>
<td>+169</td>
<td>+9° 44'</td>
<td>-20° 44'</td>
</tr>
<tr>
<td>7</td>
<td>Small crater in Ptolemaeus</td>
<td>+013</td>
<td>-148</td>
<td>-8° 31'</td>
<td>+0° 46'</td>
</tr>
<tr>
<td>8</td>
<td>Central Peak Alponsus</td>
<td>-048</td>
<td>-232</td>
<td>-13° 25'</td>
<td>-2° 42'</td>
</tr>
<tr>
<td>9</td>
<td>White Spot</td>
<td>-175</td>
<td>-251</td>
<td>-14° 32'</td>
<td>-10° 25'</td>
</tr>
<tr>
<td>10</td>
<td>Floor of Lassell</td>
<td>-131</td>
<td>-265</td>
<td>-15° 23'</td>
<td>-7° 49'</td>
</tr>
<tr>
<td>11</td>
<td>Alpetragius</td>
<td>-075</td>
<td>-267</td>
<td>-16° 01'</td>
<td>-4° 29'</td>
</tr>
<tr>
<td>12</td>
<td>Central Peak Arzachel</td>
<td>-038</td>
<td>-312</td>
<td>-18° 11'</td>
<td>-2° 17'</td>
</tr>
</tbody>
</table>

White light results

The results obtained from the areas listed above together with
details of time of observation, and phase angle are tabulated below:

<table>
<thead>
<tr>
<th>Region</th>
<th>Date</th>
<th>Time</th>
<th>Phase Angle</th>
<th>(\rho) (per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1962 Jan 28</td>
<td>04 00</td>
<td>76° 32' 63° 40'</td>
<td>7.8 ± 0.2 6.2 ± 0.2</td>
</tr>
<tr>
<td>2</td>
<td>1962 Feb 13</td>
<td>20 40</td>
<td>76° 32' 63° 40'</td>
<td>7.8 ± 0.2 6.2 ± 0.2</td>
</tr>
<tr>
<td>3</td>
<td>1962 Jan 28</td>
<td>04 30</td>
<td>76° 35' 61° 19'</td>
<td>10.5 ± 0.5 6.7 ± 0.2</td>
</tr>
<tr>
<td>3</td>
<td>1962 Feb 14</td>
<td>01 20</td>
<td>76° 35' 61° 19'</td>
<td>10.5 ± 0.5 6.7 ± 0.2</td>
</tr>
<tr>
<td>4</td>
<td>1962 Jan 28</td>
<td>04 55</td>
<td>76° 32' 61° 19'</td>
<td>11.0 ± 0.5 6.7 ± 0.2</td>
</tr>
<tr>
<td>4</td>
<td>1962 Feb 14</td>
<td>01 25</td>
<td>76° 32' 61° 19'</td>
<td>11.0 ± 0.5 6.7 ± 0.2</td>
</tr>
<tr>
<td>5</td>
<td>1962 Feb 14</td>
<td>00 25</td>
<td>76° 35' 61° 19'</td>
<td>5.1 ± 0.2</td>
</tr>
<tr>
<td>5</td>
<td>1962 Feb 14</td>
<td>00 35</td>
<td>76° 35' 61° 19'</td>
<td>5.1 ± 0.2</td>
</tr>
<tr>
<td>6</td>
<td>1962 Feb 14</td>
<td>01 15</td>
<td>61° 28' 61° 40'</td>
<td>4.8 ± 0.2</td>
</tr>
<tr>
<td>6</td>
<td>1962 Feb 14</td>
<td>01 05</td>
<td>61° 28' 61° 40'</td>
<td>4.8 ± 0.2</td>
</tr>
<tr>
<td>7</td>
<td>1962 Feb 14</td>
<td>00 55</td>
<td>61° 35' 61° 40'</td>
<td>4.5 ± 0.2</td>
</tr>
<tr>
<td>7</td>
<td>1962 Feb 14</td>
<td>00 45</td>
<td>61° 35' 61° 40'</td>
<td>4.5 ± 0.2</td>
</tr>
</tbody>
</table>

On February 14, 1962, several regions of the lunar surface were analysed; the regions being as far apart as Alphonsus, Plato and the brightly illuminated walls of Copernicus. Since Copernicus was observed at lunar sunrise, the angle of illumination on the walls must have differed quite radically from the other areas, and even the angles of illumination for these areas varies considerably. The phase angle changes, however, only slightly from point to point; and since the polarization differences from region to region are small depending apparently on albedo, it can be said that polarization depends mainly on the phase angle rather than the angle of illumination. This confirms results of Dollfus on the cliff of the “Straight Wall” and on the rim of “Schroeter’s Valley”.

The photograph on fig.11 was taken by members of the Manchester University Lunar Group at Pic-du-Midi and shows the Alphonsus region at identical illumination conditions as on Feb.14, 1962.
Fig. 11. Areas analysed 1962 Feb. 14. (Photograph taken 1961 May 25. 21H 27M).
The central peak of Alphonsus showed no fundamental difference in the degree of polarization over the neighbouring regions and the values for the peaks in Alphonsus, Arzachel and Alpetragius are approximately the same. The value for the mountain Pico at approximately the same time was much higher but this might be a result of the inclusion of some maria in the field of view surrounding Pico.

Darker regions such as the floors of the craters Ptolemaeus and Lassell do show higher polarization than the mountain peaks. The white spot to the west of Lassell exhibited even higher polarization and this is exceptional since its albedo is higher than its immediate surroundings. But, again, this might have been caused by the inclusion of darker maria in the same field of view.
Observations of Wavelength Dependence of Polarization

Three areas that were measured in white light, were re-measured at different wavelengths, isolated by the fairly wide-band filters. The angle $\Phi$ in the angle of the plane of polarization relative to the telescope's arbitrary zero.

<table>
<thead>
<tr>
<th>Region No.</th>
<th>Date</th>
<th>Wavelength</th>
<th>$\rho$</th>
<th>$\Phi$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(Filter No.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1962</td>
<td>Jan 28</td>
<td>4590(94)</td>
<td>9.0 ± 0.2</td>
<td>Observations occasionally interrupted by cloud.</td>
</tr>
<tr>
<td>(Aristarchus)</td>
<td>05 15</td>
<td>7390(89B)</td>
<td>7.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>05 25</td>
<td>6080(72B)</td>
<td>7.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>05 55</td>
<td>5760(73)</td>
<td>8.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1962</td>
<td>Feb 13</td>
<td>7390(89B)</td>
<td>4.8 ± 0.1</td>
<td>84.5° ± 1°</td>
</tr>
<tr>
<td>(Plato)</td>
<td>21 25</td>
<td>6080(72B)</td>
<td>5.0</td>
<td>83</td>
<td>Mean phase</td>
</tr>
<tr>
<td></td>
<td>21 50</td>
<td>5760(73)</td>
<td>4.8</td>
<td>81.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>22 05</td>
<td>5490(99)</td>
<td>4.2</td>
<td>81.5</td>
<td>Angle =</td>
</tr>
<tr>
<td></td>
<td>22 25</td>
<td>5370(74)</td>
<td>5.1</td>
<td>81.7</td>
<td>63° 40'</td>
</tr>
<tr>
<td></td>
<td>22 35</td>
<td>4590(94)</td>
<td>4.7</td>
<td>82.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1962</td>
<td>Feb 13</td>
<td>4590(94)</td>
<td>5.2 ± 0.1</td>
<td>80.9° ± 1°</td>
</tr>
<tr>
<td>(Alphonsus)</td>
<td>22 45</td>
<td>5370(74)</td>
<td>4.4</td>
<td>81.5</td>
<td>Mean phase</td>
</tr>
<tr>
<td></td>
<td>23 05</td>
<td>5490(99)</td>
<td>4.7</td>
<td>82.5</td>
<td>Angle =</td>
</tr>
<tr>
<td></td>
<td>23 20</td>
<td>5760(73)</td>
<td>4.4</td>
<td>79.7</td>
<td>62° 16'</td>
</tr>
<tr>
<td></td>
<td>23 45</td>
<td>6080(72B)</td>
<td>3.6</td>
<td>79.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23 55</td>
<td>7390(89B)</td>
<td>3.8</td>
<td>82.1</td>
<td></td>
</tr>
</tbody>
</table>

The graphs displaying these results are plotted on Fig. 12.

It can be seen that $\Phi$ does not change appreciably with wavelength. From the results at the same phase angle on Feb. 13, 1962, the fractional change in polarization over the wavelength region is greater for Alphonsus than it is for the floor of Plato. This is in agreement with Teyfel [11] who found that, in general, the fractional change on mountainous regions is greater than on the maria. Teyfel's results were obtained by a photographic spectrograph and the accuracy of a determination was inferior to photoelectric methods. Further measurements of
Wavelength Dependence of Lunar Polarization

Fig. 12.
the wavelength-dependence of polarization in several regions of the lunar surface are being currently obtained using the double-beamed polarimeter.

**Planetary observations**

As mentioned earlier, the single-beamed polarimeter was used to determine the degree of polarization of Mars.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>( \rho ) (per cent)</th>
<th>Phase Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961 Jan 7</td>
<td>00 30</td>
<td>1.2 ± 0.2</td>
<td>10° 8'</td>
</tr>
</tbody>
</table>

Using the double-beamed polarimeter, the planet Uranus was analysed on two nights, and a small amount of polarization was detected on both occasions.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>( \rho ) (per cent)</th>
<th>( \phi ) (per cent)</th>
<th>Phase Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962 Jan 28</td>
<td>01 40</td>
<td>0.93 ± 0.25</td>
<td>53° ± 3°</td>
<td>1° 46'</td>
</tr>
<tr>
<td>1962 Feb 4</td>
<td>02 40</td>
<td>0.59 ± 0.30</td>
<td>48° ± 3°</td>
<td>0° 46'</td>
</tr>
</tbody>
</table>

**Future polarimetric studies**

An observing programme of the Moon, Jupiter and its satellites is already in progress. It is hoped that measurements will also be made on Mars and Venus, when these objects become more favourable during the winter of 1962-1963. Whenever possible the wavelength dependence of the polarization of all these objects will be studied.
Acknowledgements

Firstly, I wish to thank Professor J. Ring for his help and valuable discussion, and Professor Z. Kopal for his guidance, Dr. J. F. Grainger also proved most helpful in discussing some of the problems.

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References