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Symmetrical One-Frequency Forced Oscillations in Nonlinear Automatic Control Systems

by

V. V. Pavlov

Greater attention is devoted to approximate investigation of symmetrical one-frequency forced oscillations in automatic Control Systems with substantially nonlinear link [1 - 3].

The basis for approximate determination of forced oscillations in nonlinear systems is constituted by the expression for equivalent linearization of nonlinear functions f(x,px)

$$f(x, px) = -\frac{q'(A)}{\Omega} px + q(A) x \qquad (1)$$

where

$$q(A) = \frac{1}{\pi A} \int_0^{2\pi} f(A \cos \psi, -\Omega A \sin \psi) \cos \psi d\psi \qquad (1A)$$

$$q'(A) = \frac{1}{\pi A} \int_0^{2\pi} f(A \cos \psi, -\Omega A \sin \psi) \sin \psi d\psi \qquad (1b)$$

obtained by N.M.Krylov and N.N.Bogolyubov [4] for the case, when the nonlinear function in the equation, describing an automatic control system, is proportional to the small parameter ε. Using expression (1), they obtain [3] a formula with the aid of which one-frequency forced oscillations are investigated

$$A \frac{Q(j\Omega) + q - jq'}{S(j\Omega)} = B e^{-j\varphi} \qquad (2)$$

where A- amplitude of forced oscillations, B- amplitude of the perturbation effect, Q(p) and S(p) - operational polynomials of any arbitrary degree, include in the following equation, describing the investigated automatic control system

$$Q(p) x + f(x, px) = S(p) U(t) \qquad (3)$$

where U(t) = B sin omega t.

When deriving expression (2) it is assumed, that in the system are realized the properties of a filter.

However, as shown by practice, not in all instances when studying one-frequency forced oscillations does this expression give a satisfactory result. Here, as a rule, considerable discrepancies between the approximated result according to (2) and the accurate one are observed in the region of low frequencies.

To study one-frequency forced oscillations in automatic control systems with substantially nonlinear link, the characteristic of which can be written in form of

$$F(x, px) = h_1 x + h_2 px + f(x, px) \qquad (4)$$

where h_1x+h_2px - linear part of the characteristic $F(x,px)$, it is possible to utilize the results, obtained in [5].

According to [5], an approximate investigation of essentially nonlinear systems can be made by an ordinary method [3], but by replacing the essentially nonlinear function f(x,px) with a more definite expression for equivalent linearization, having the form of

$$f(x, px) = -\left\{\frac{q'(a)}{\omega} px + \left[-q(a) + \frac{\lambda}{\omega} q'(a) + \left(\frac{q(a)}{2\omega}\right)^2 + \left(\frac{q'(a)}{2\omega}\right)^2\right] x\right\} \qquad (5)$$

where

$$q(a) = \frac{1}{\pi a}\int_0^{2\pi} f(a\cos\psi, -\lambda a\cos\psi - \omega a\sin\psi)\cos\psi\, d\psi \qquad (5a)$$

$$q'(a) = \frac{1}{\pi a}\int_0^{2\pi} f(a\cos\psi, -\lambda a\cos\psi - \omega a\sin\psi)\sin\psi\, d\psi \qquad (5b)$$

lambda and omega - attenuation ratio and the frequency of solving the "originating" equation, which is found from the basic differential equation at $f(x,px) \equiv 0$.

The use of term (5) for studying the transient processes in automatic control systems with essentially nonlinear link gives in individual cases a more accurate result [5], than when using the expression for equivalent linearization according to [4]. This closer definition is brought in by the correcting member

$$\Delta q(a) = \left(\frac{q(a)}{2\omega}\right)^2 + \left(\frac{q'(a)}{2\omega}\right)^2 \qquad (6)$$

We shall discuss ways of utilizing the more defined expression (5) for equivalent linearization of essentially nonlinear function f(x, px) for the purpose of studying one-frequency forced oscillations, excited in nonlinear automatic control systems. We will assume here that the investigated system is also described by equation (3), assuming, that Q(p) includes the linearized part of F(x, px).

Then, if the linear part of the automatic control system possesses the property of a filter [3], then in first approximation it is possible to seek a solution for the established forced oscillations in form of

$$x = A\sin(\Omega t + \varphi) \tag{7}$$

If we were to consider (7), then equation (3) can be presented [3] in form of

$$\left[Q(p) - S(p)\frac{B}{A}\left(\cos\varphi - \frac{\sin\varphi}{\Omega}p\right)\right]x = -f(x, px) \tag{8}$$

For equivalent linearization of a nonlinear function f(x, px) in accordance with term (5) it is necessary at first to determine the value of the attenuation ratio lambda and the frequency omega of the solution of the "originating" equation. The "originating" equation in this case is obtained from (8) at f(x, px) ≡ 0.

$$\left[Q(p) - S(p)\frac{B}{A}\left(\cos\varphi - \frac{\sin\varphi}{\Omega}p\right)\right]x = 0 \tag{9}$$

It is apparent, that a solution of equation (9) will be (7), because we are interested only in set solutions. Hence it is evident, that the values, characterizing the "originating" solution, will be

$$\lambda = 0, \qquad \omega = \Omega \tag{10}$$

then

$$f_1(x, px) = -f(x, px) = -\left\{-\frac{q'(A)}{\Omega}px + \left[q(A) + \left(\frac{q(A)}{2\Omega}\right)^2 + \left(\frac{q'(A)}{2\Omega}\right)^2\right]x\right\} \tag{11}$$

where

$$\begin{aligned} q(A) &= \frac{1}{\pi A}\int_0^{2\pi} f(A\cos\psi, -A\Omega\sin\psi)\cos\psi\,d\psi \\ q'(A) &= \frac{1}{\pi A}\int_0^{2\pi} f(A\cos\psi, -A\Omega\sin\psi)\sin\psi\,d\psi \end{aligned} \tag{11a}$$

Comparing the obtained more defined expression (11) (more accurate formula (11) for equivalent linearization of nonlinear function with (1), we point out, that the difference between them lies in the additional member

$$\Delta q = \left(\frac{q}{2\Omega}\right)^2 + \left(\frac{q'}{2\Omega}\right)^2. \qquad (12)$$

the magnitude of which depends substantially upon the frequency of forced oscillations omega, values q(A) and q'(A) and their ratio.

Next, by substituting (11) in (8) we arrive at a more accurate expression for studying one-frequency forced oscillations

$$A\frac{Q(j\Omega) + (q + \Delta q) - jq'}{S(j\Omega)} = Be^{-j\varphi}. \qquad (13)$$

whereby its distinction from ordinary terms used in this case lies in the correction member Δq and, as is evident from (12), it can be substantially different in the zone of low frequencies from expression (2).

As an illustration for the above stated we shall discuss certain examples. We will consider here, that in the examples under question the properties of a filter are being realized.

Example 1. Assuming a certain automatic control system is described by equation (3), whereby

$$\begin{aligned} Q(p) &= p^2 + p + 1.96 \\ S(p) &= 1 \end{aligned} \qquad (14)$$

$$f(x) = \begin{cases} 5 & \text{at } x > 1 \\ 0 & -1 < x < 1 \\ -5 & x > -1 \end{cases}$$

It is necessary its amplitude frequency characteristic at different amplitude values of the perturbation effect U(t)= B sin omegat.

To do this, by using (13) and considering (14) we will obtain an expression, with the aid of which it is easy to plot the frequency characteristic

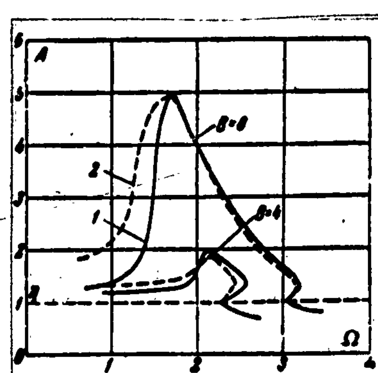


Fig.1.Approximate amplitude-frequency characteristic; 1-with consideration of correction, 2-without consideration of correction.

$$\frac{A}{B} = \frac{1}{\sqrt{(1.96 + q + \Delta q - \Omega^2)^2 + \Omega^2}} \qquad (15)$$

where

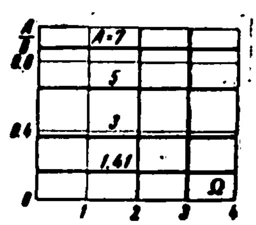
$$q = \frac{4S}{\pi A^2}\sqrt{A^2 - \eta^2}, \quad S = 3, \quad \eta = 1 \qquad (15a)$$

In fig.1 was obtained the amplitude-frequency characteristic by combining on one graph families of curves, determinable by left and right parts of expression (15), i.e.

$$f_B(A_i, B_j, \Omega) = \frac{A}{B} \qquad (16)$$

$$M(A_i, \Omega) = \frac{1}{\sqrt{(1.96 + q + \Delta q - \Omega^2)^2 + \Omega^2}} \qquad (17)$$

which have the form, shown in fig.2 and 3 respectively. By the intersection points are determined the necessary dependence data A = f(omega) at B_j = const (fig.1.curve 1). For the purpose of comparison on the very same graph was plotted the amplitude frequency characteristic without consideration of correction (curve 2).



As is evident from fig.1.a substantial correction takes place in the zone of low frequencies. However, as already mentioned before, the magnitude of the correction depends not only upon the frequency , but also upon the value q and q', i.e.upon the concrete form of linearization coefficients, which must be taken under consideration in every concrete case.

Example 2. In role of second example we shall discuss a nonlinear automatic system, investigated in [6].

The system is described by the following equation:

$$(Tp + 1)x + kf(x) = kU(t) \qquad (18)$$

whereby T=k=1, f(x)-relay characteristic without insensitivity zone with k_p=1.

It is necessary to determine, the dependence of the threshold value of the amplitude B of outer sinusoidal perturbation U(t) = B sin omega upon the frequency omega.

Taking into consideration, that the linearization coefficient of the nonlinear member f(x) is determined by the ratio $q(A) = 4k_p/\pi A$, we will present (13) in form of

$$\frac{A}{W(j\Omega)} + C + \frac{C^2}{4A\Omega^2} = Be^{-j\varphi} \qquad (19)$$

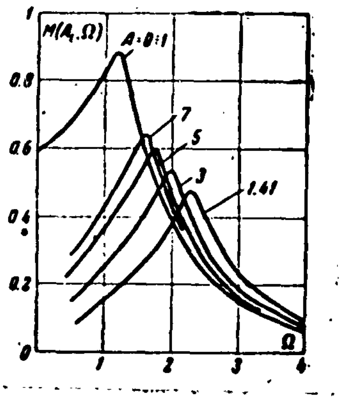


Fig.3.

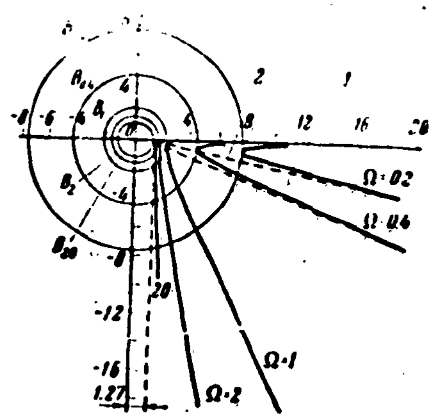


Fig.4

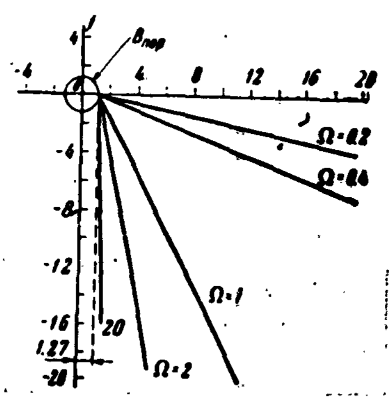


Fig.5

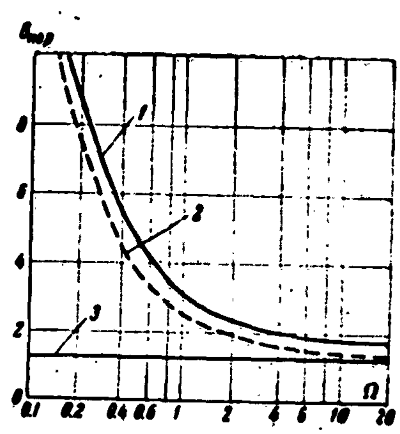


Fig.6

Fig.4. Determining V_{por} in an automatic control system: 1-more accurate value Z(A, jomega), 2-value Z (A, jomega)without correction.

Fig.5. Determining V_{por} in automatic control system by Z (A, jomega)without correction

Fig.6. Dependence of amplitude V_{por} upon frequency of forced oscillations omega: 1-accurate value, 2-approximate, more accurate, 3-approximated.

where

$$C = \frac{4k_p}{\pi} = 1.27, \qquad W(j\Omega) = \frac{k}{Q(j\Omega)} = \frac{1}{jT\Omega + 1} \qquad (19a)$$

It is apparent, that $C^2/4A\omega^2$ appears in this way to be the correcting member Δ, which is due to the presence of Δq in (11).

After, having formulated on a complex surface the expression

$$Z(A, \Omega_i) = \frac{A}{W(j\Omega_i)} + C + \frac{C^2}{4A\Omega_i^2} \quad (20)$$

at various values ω_i = const, it is not difficult to determine the threshold value V_{por} ($V_{threshold}$) (fig.4). In fig.5 is given an analogous formulation without consideration of correction and it was determined one solely possible thereat value $V_{thr} = 1.27$, which does not depend upon the frequency omega.

The dependences $V_{thr} = f_p(\omega)$ obtained in this way are given in fig.6. On the very same fig is given the accurate dependence of amplitude threshold value V_{thr} upon the outer effect frequency omega, borrowed from report [6].

It is evident from fig.6, that the approximate dependence V_{thr}, was plotted by the ordinary approximation method [3], but with the utilization of a more accurate term for equivalent linearization (11), is in excellent conformity (i.e. with a sufficient degree of practical accuracy) with the accurate dependence of the amplitude threshold value upon frequency. This shows, that the employment of term (11) in an ordinary approximated method allows to more closely define the result of investigations in these individual, single cases, in which the ordinary form of linearization (1) gives a less accurate result.

In this way it is possible to make a conclusion, that the use in an ordinary approximated method of calculating the more accurate expression (11) for equivalent linearization of an essentially nonlinear member offers a perfectly satisfactory result when investigating one-frequency forced oscillations in nonlinear automatic control systems.

Submitted May 5, 1961

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Principle of Constructing Self-Adjusting Systems

by

V. A. Gordeyev

Discussed the circuit of self-adjusting, which consists of basic circuit, device, determining the quality characteristic, and operational element. In the role of basic circuit is accepted an oscillatory link of second magnitude.

Generally speaking the syetem of controlling an object can have the form of, as shown in fig.1. The coefficients of the W_o function change at will. At this change there may originate such a combination of object parameters, that at unchanged coefficients of the regulator W_c the system becomes unstable or does not yield the given quality. To satisfy these these requirements is possible by introducing an additional self-adjustment circuit, which by a selected algorithm would change the parameters W_c (fig.2). The output signal, e.g. controllable coordinate, is fed to the analyzer, which represents a metering-computing device, determining any given quality characteristic. If the value of the calculated quality index differs from its given value, then an error value appears, which goes to the control device. It changes the value of the control parameter a_i until the current value of the quality index, computed in the analyzer, will be equal to its fixed value. The double arrow in fig.2 indicates that not the coordinate, but the parameter, is changing.

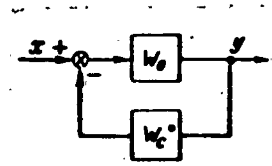


Fig.1.Control system

The analyzed circuit presents a parametric feedback by the quality characteristic.

We shall discuss a number of possible quality indices and principles of constructing self-adjustment circuits for systems with rapid change in the parameter of the basic circuit. Rapid is considered such a change in parameters, when within the period of the transient process of the basic circuit the parameters do change to such an extent, that it is impossible during the investigation to use the method of "frozen coefficients".

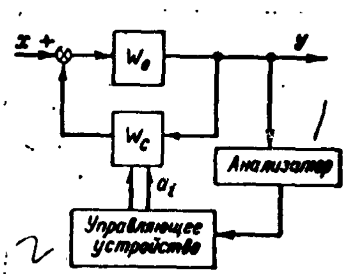


Fig.2.Self-adjustment circuit
1-analyzer; 2-control device

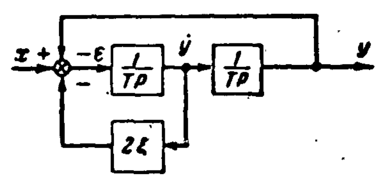


Fig.3.Structural arrangement of oscillatory link.

We shall analyze certain quality characteristics using an oscillatory link as an example. The basic form can then be replaced by a structural form (fig.3),having a transmission function

$$\Phi(p) = \frac{y}{x} = \frac{1}{T^2p^2 + 2\xi Tp + 1}$$

Assuming that there are convergent (fig.4),or divergent (fig.5) oscillations of of coordinate y, caused either by the change in parameters, or by reactions. The analyzer is constructed so, that at its output during the period of oscillations should originate a value, equal to the difference of positive and negative half-waves.

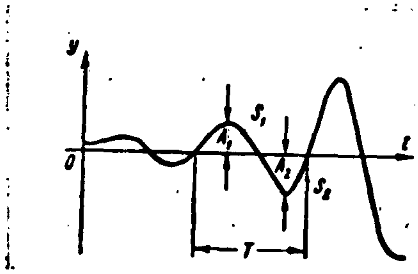


Fig.4.Convergent oscillations of coordinate y

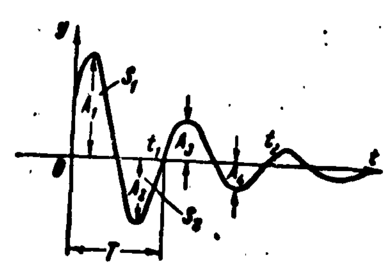


Fig.5. Divergent oscillations of coordinate y.

$$\varkappa = A_1 - A_2, \quad \varkappa > 0 \text{ при } A_1 > A_2, \quad \varkappa < 0 \text{ при } A_1 < A_2 \qquad (2)$$

where A_1- first half wave of i-period, A_2 - second half wave of i-period.

The obtained signal χ is accepted to be the controlling one for the self-adjustment circuit. The scheme is set up so that the attenuation xi changes in dependence upon the value χ. These changes will take place in such direction as to make χ equal to zero; in this case the system will be on the boundary of stability.

When working out the initial condition with respect to speed $\dot{y}_0$ the transient process by the coordinate for (1) is determined by formula [1].

$$y(t) = \frac{T\dot{y}_0}{\sqrt{1-\xi^2}} \exp\left(-\frac{\xi}{T}t\right) \sin\frac{t}{T}\sqrt{1-\xi^2} \qquad (3)$$

Through the duration of the first period $\chi = 0$ there is no correcting signal. This signal on the basic of (2) is emitted through the period T and is equal to

$$\varkappa = \frac{T\dot{y}_0 \exp\left(-\frac{\pi\xi}{2\sqrt{1-\xi^2}}\right)}{\sqrt{1-\xi^2}} \left[1 - \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)\right] \qquad (4)$$

After each following period is emitted a value, determinable by equation (4), but $\dot{y}_0$ in this case will correspond to the value of speed at the beginning of each period. It is evident from formula (4) that the greater the amplitude of oscillations, the greater will be the value of the correcting action χ at the very same xi and T.

Dependence graphs $\chi = f(xi, T)$ at $\dot{y}_0 = 1$ are given in fig.6, whence it is evident, that the value χ is greater at negative xi, than at positive. Consequently from the zone of instability the system will come out more energetically, than from the zone of stability.

If in the role of controlling device we would take an integrating link with amplification factor k, then at above mentioned conditions the equation of oscillations y for the n-period will acquire the form of

$$T^2\ddot{y} + \left[2\xi_0 + \sum_{i=0}^{n} k\varkappa_i (t - t_i)\right] T\dot{y} + y = 0, \quad \text{при } \dot{y}(t_n) = \dot{y}_n, (y(t_n) = y_n, \qquad (5)$$

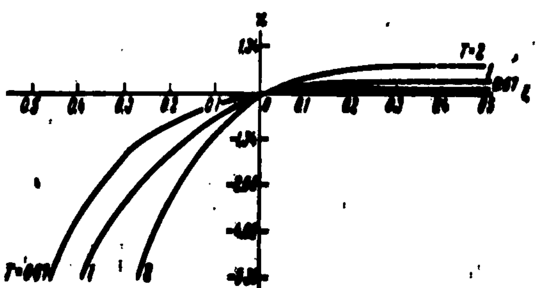
where xi_0 - initial attenuation value, and t changes from $t - t_i = 0$ at $t < t_i$ to $t = T$.

In stable state oscillations y are described by equation of conservative link.

Modeling was done on an IPT-5 type model. Structural arrangement of analyzer is shown in fig.7.On the first memory devices [ZU=MD] are measured the amplitude values.

Within a period the commutating device [KU=CD] transmits these values to second ZU, where they are retained for the duration of the period, and the first ZU at that moment become discharged, getting ready for the following cycle. The commutating device consists of differentiating link, detector and a series of relays.

The circuits of memory devices with commutation contacts are given in fig.8. In the role of ZU were used integrators working in condition of memorizing the initial conditions [2].



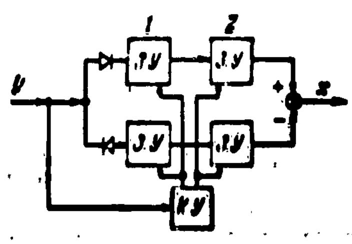


Fig.6.Dependence graphs χ =f(xi,T)   Fig.7.Structural arrangement of analyzer

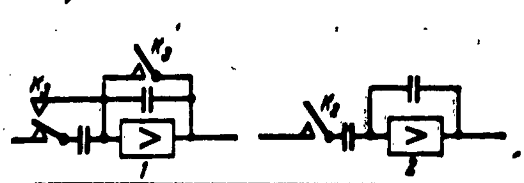


Fig.8.Arrangements of memory devices.

A change in xi was made by changing one of the co-multiples, fed to the multiplication link. Initial conditions were fixed in both circuits $\dot{y}(0)$ and 2xi(0). We took xi(0) $>$ 0 and xi(0) $<$ 0. The value $\dot{y}$ (0) was selected so that the system should not reach saturation. During the modeling was changed the coefficient k in the self-adjustment circuit. Fig.9 show oscillograms of processes at various k. As is evident, an increase in k raises the oscillatory nature, a reduction - restricts the working process.

Graphs clearly show the performance of commutating and memory devices and the obtainment of value χ .

In some instances the quality index of the form (2) is inapplicable because of the presence of oscillations y in stable state.

It is then possible to utilize the quality criterion in form of

$$\varkappa = A_1 - rA_2, \quad \text{при } \varkappa = 0 \quad r = \frac{A_1}{A_2} \qquad (6)$$

In this way, when taking up the value r, we are taking up the desired amplitude ratio of the oscillatory process. If $r > 1$, then the given conditions will be the damping oscillations. The dependence $\chi = f(xi, T=1)$ for oscillatory link at various r is given in fig.10.

The self-adjustment circuit, utilizing criteria (6) operates well in the presence of oscillations. If we strayed into a zone where $xi > xi_3$, then in the absence of

action there will be no oscillations and the self-adjustment system will not bring out the basic circuit onto a given value xi_3. Ordinarily there is always action at really functioning system, and the origination of a similar situation is not dangerous.

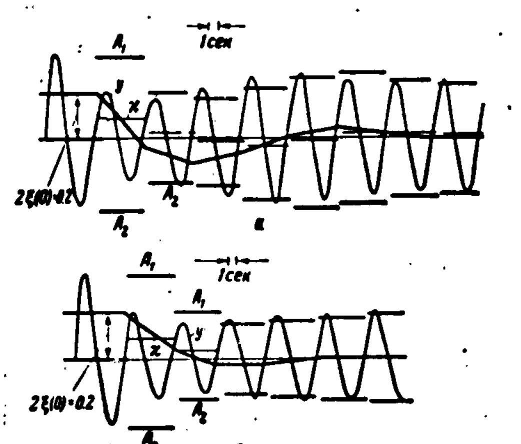


Fig.9

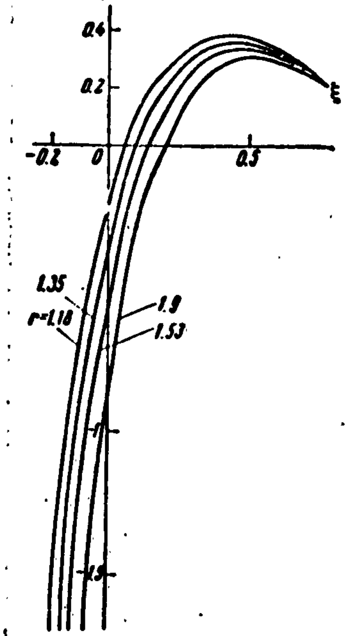


Fig.10

Fig.9.Oscillograms of transient processes at $\chi = A_1 - A_2$: a) k = 0.33, a) k = 0.142
Fig.10. Dependence graphs $\chi = f(xi, r)$.

However, it is necessary, it is possible to provide a device which would forcibly reduce the value xi in the absence of oscillations, or it is possible to feed periodically artificial, for example perturbation pulses.

The arrangement of the analyzer, giving value (6), differs from such for (2) only by the introduction into one of its arms a constant coefficient, equalling r.

Adjustment oscillograms xi(0)=-0.1 with a system of reducing xi in the absence of oscillations at r = 1.35 (xi_3 = 0.1) is shown in fig.11. In view of the considerable amplification factor in the self-adjustment circuit takes place a rapid neglect in the zone of positive xi. And upon discontinuation of oscillations the auxiliary system causes a reduction in xi (fig.11a). The reduction, when the value xi reaches a value xi = - 0.24, the internal fluctuations of the model disrupt the system to such an extent, that the reduction in xi is interrupted and the analyzer begins working, assuring attenuation of the system (fig.11,b). In fig.11 a,b, xi and y are designated, the remaining curves were recorded on a loop to check the operational accuracy of sections of the system. In fig.11c is presented the operation of self-adjustment during linear change in xi_v at a rate of 0.05 units/period. The perturbation along xi was fed following the control device. The broken xi_u characterizes the voltage at the output of the control device. Under xi_{osh} is understood a value, fed to the control link. It is evident from the oscillogram, that inspite of the change in xi_v, self-adjustment does not give ẏ to oscillations and breaks up.

The amplification factor in the self-adjustment circuit should be selected when applying the discussed principle to a real system, because this coefficient is limited not only by the stability conditions of the self-adjustment circuit, as by the permissible value of skipping into the zone of positive xi. In addition, this value depends upon the action and permissible oscillation amplitudes of the control parameter.

It is possible to improve the proposed quality index in the aspect of reducing time during its calculation and increasing the interference resistance.

The first one can be obtained, if the value χ is calculated not by the period, but by the half-period. It is necessary to compare each two adjoining amplitudes continuously, and not by the period in pairs. This calls for a change in analyzer, and to increase the rapid action of the self-adjustment circuit.

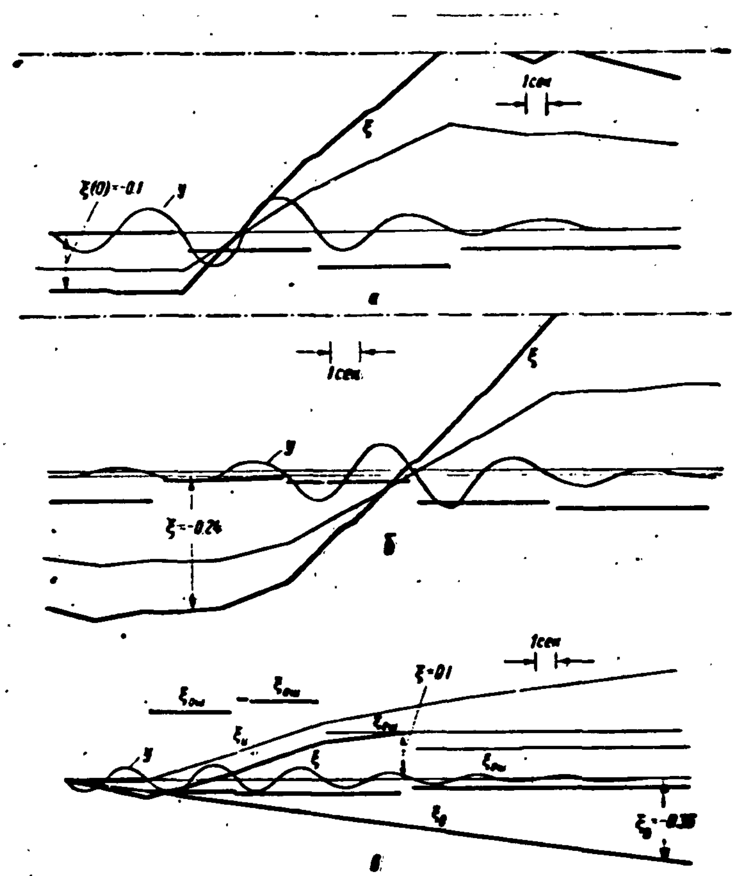


Fig. 11, Oscillograms of transient processes at $\chi = A_1 - rA_2$

To raise the interference resistance of the analyzer it is possible instead of amplitudes (2) to take the area (integrals) of S_1 and S_2 half waves. The interferences imposed on the oscillations, are smoothened out.

The value χ is then equal

$$\varkappa = S_1 - rS_2 = C_0\left[1 + \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)\right]\left[1 - r\exp\left(\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)\right] \quad (7)$$

As a deficiency should be mentioned the drift in the integrators, calculating the areas. It is therefore necessary to apply special measures for the elimination of same. To calculate χ, according to expression (7), only the first ZU in the analyzer are substituted by integrators.

Quality indices (7) and (6) assure identical damping, because at χ = 0 we have

$$r = \frac{A_1}{A_2} = \frac{S_1}{S_2} = \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right). \qquad (8)$$

The degree of attenuation for the oscillatory link at T = 1 equals [1]

$$j = \frac{y_{1m} - y_{2m}}{y_{1m}} = 1 - \exp\left(-\frac{2\pi\xi}{\sqrt{1-\xi^2}}\right)$$

where y_{1m}, y_{2m} - amplitudes of two neighboring half waves of one polarity. Then, by comparing equations (8) and (9), we obtain

$$r = \frac{1}{\sqrt{1-j}} \qquad (10)$$

In this way, r characterizes the degree of attenuation of the transient process.

Oscillograms at a quality index of the form of (7) are analogous to graphs of transient processes in previous instances.

In all cases under discussion in the role of regulating parameter was taken the value xi. If we change the amplification factor of the second integrating link or the amplification factor of feedback, encompassing both integrating links (fig.3), then we will obtain, respectively, transmission functions

$$W(p) = \frac{1}{\frac{T^2}{m^2}p^2 + \frac{2\xi T}{m}p + 1}, \qquad W(p) = \frac{\frac{1}{m}}{\frac{T^2}{m^2}p^2 + \frac{2\xi T}{m}p + 1} \qquad (11)$$

and the roots

$$\lambda_{1,2} = -\frac{\xi m}{T} \pm j\frac{m}{T}\sqrt{1-\xi^2} \qquad (12)$$

where m = var.

It is evident herefrom, that it is necessary to construct the system so, that the value m should never be equal to zero, otherwise there will appear two zero roots, and the application of the discussed criteria is impossible.

In conclusion it should be underlined, that the above mentioned quality indices can be utilized for self-adjustment only at an oscillatory stability boundary.

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About a Type of Self-Adjusting Control System

by

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1. Arrangement of the problem. Equations of perturbation movement of an automatic control system in most general case can be presented in vector form as

$$\frac{dX}{dt} = F_1(t, X, Y, U)$$
$$\frac{dY}{dt} = F_2(t, X, Y, W, P) \qquad (1.1)$$
$$t = 0, \quad X = X_0, \quad Y = Y_0.$$

where X - vector of object's control coordinates; Y- vector of regulator control coordinates; U - vector of inner and outer noncontrollable perturbations affecting the object of control W - vector of inner and outer uncontrolled perturbations, affecting the regulator; X_0, Y_0- initial values of vectors X and Y, F_1, F_2 - nonlinear functions.

The coordinates of vectors X,Y,W and U in general case appear to be random functions.

In the system (1.1) the first equation binds the controllable coordinates of the object (vector X) with other coordinates, in the second equation are bound the coordinates of the regulator (vector Y) with coordinates of the object and controlling parameters (vector P).

The nature of change and the values of perturbations U and W, which reflect the internal (deviation of parameter values from calculated), as well as external perturbations, can not be accurately forseen when designing and producing the system. That is why a self-adjusting control system is necessary.

We will assume, that the self-adjustment system should meet given requirements with respect to control accuracy .

$$|x_i| \leqslant \varepsilon_{x_i}$$
$$|\dot{x}_i| \leqslant \varepsilon_{\dot{x}_i} \quad (i = 1, 2, \ldots, r) \quad \text{при } t \in [t_0, T] \qquad (1.2)$$

where x_i, $\dot{x}_i$ ($i = 1, 2, \ldots, r$) - coordinates of vectors X and $\dot{X}$, ε_{x_i}, $\varepsilon_{\dot{x}_i}$ - given constant numbers, $[t_0, T]$ - operating interval of the object.

To fulfill the conditions (1,2) for the basic system (1.1) with any one of the available methods (analytical, by modeling, etc) even in the presence of full information about the U and W vectors is practically impossible because of the complexity of the system (1.1). Making a whole series of assumptions, we simplify the mathematical model of the system. The simplified system of equations is used as standard. The equation coefficients of this system are computed in the assumption, that the object in the interval $[t_0, T]$ is affected by known perturbations U_3 W_3. Judging by the results of measuring the sensitivity of elements we calculate the current (real) coefficients of the approximating system of equations. The coefficients obtained in such a way are compared with standard ones and their difference are used in the regulator to bring down the values of current coefficients to standard. A comparison of current and standard coefficients is made on partial intervals, into which the basic interval $[t_0, T]$ is broken down.

2. Formulation of a standard system. At the first step of simplifying the system of equations (1.1) the indidental factors are replaced by their mathematical expectations. We will obtain the following regular model of the real control system:

$$\frac{dX}{dt} = F_1(t, X, Y, \overline{U})$$
$$\frac{dY}{dt} = F_2(t, \dot{X}, Y, \overline{W}, P) \qquad (2.1)$$
$$t = t_0, \quad X = \overline{X}_0, \quad Y = \overline{Y}_0$$

In the equations (2.1) the top row designates that the mathematical expectation is taken from each vector coordinate. We assume, that in these equations the values of the vectors $\overline{U}$ and $\overline{W}$ in the interval $[t_0, T]$ are known.

Next we will write, that $\overline{U} = 0$ at $t \in [t_0, T]$.

When formulating a self-adjusting system of equations (2.1) will serve as initial base for the obtainment of standard. But the use of systems (2.1) directly in the role of standard is not-advisable by virtue of its sufficient complexity (greater order, presence of nonlinearity etc.). In addition, sufficiently complete information regarding control can be obtained on a much simpler model, which, in the final count, will considerably reduced the volume of computation operations upon the formulation of a standard and during the functioning of a selfadjusting system.

Let us assume that as result of linearization, the disregarding of second degree bonds and elimination of regulator variable (vector Y) the system (2.1) is reduced into form

$$\overline{R}_i\left(t, \frac{d}{dt}, P\right) x_i(t) = \overline{\psi}_i\left(t, \frac{d}{dt}, P\right) \quad (i = 1, 2, \ldots, r) \qquad (2.2)$$

where

$$R_i\left(t, \frac{d}{dt}, P\right) = a_{ni}(t, P) \frac{d^{n_i}}{dt^{n_i}} + \ldots + a_{0i}(t, P) \qquad (2.2.a)$$

ψ_i (t, d/dt, P) - combines useful and harmful perturbations with corresponding operators.

For the system (2.2) is possible to attain fulfillment of conditions (1.2) on account of properly selecting the vector of the control parameters P, the coordinates of which may appear to be variable in time. The coefficients $a_{0_i}, a_{1_i}, \ldots, a_{n_i}$ will also be certain functions of time.

To simplify further we will try, if possible, to reduce the order of equations of the system (2.2) by disregarding small parameters. We can guide ourselves by ideas of reducing the order of systems with constant coefficients (2.1), (2.2). Since there are no analytical methods of reducing the order for systems with variable coefficients, then such a problem can presently be solved only with the aid of modern computing devices by the method of multiple repetition of the solution.

If the entire interval $[t_0, T]$ is broken down into intervals, where the coefficients change slowly, then in these intervals, having taken constant coefficients (averaged), the reduction in the orders of equations of system (2.2) can also be carried out by already existing methods (2.1), (2.2).

Assuming that as result of disregarding small parameters the system of equations (2.2) acquires the form of

$$\bar{R}_i\left(t, \frac{d}{dt}, P\right) x_i(t) = \psi_i\left(t, \frac{d}{dt}, P\right) \quad (i = 1, 2, \ldots, r) \qquad (2.3)$$

where

$$\bar{n}_i < n_i$$

$$\bar{R}_i\left(t, \frac{d}{dt}, P\right) = a_{\bar{n}_i}(t, P)\frac{d^{\bar{n}_i}}{dt^{\bar{n}_i}} + \ldots + a_{\bar{0}_i}(t, P) \qquad (2.3a)$$

To simplify the investigations we assume, that each coordinate of the vector of control parameters is included in only one of the equations of system (2.3), i.e. system (2.3) is broken down into a series of independent channels by each one of the controllable variables. Intersecting connections between channels can be realized only through perturbations.

For the case of simplicity we shall further discuss a case of controlling only one variable, designating it by x, and the corresponding standard differential equation by

$$a_n^0(t, P_x)\frac{d^n x}{dt^n} + a_{n-1}^0(t, P_x)\frac{d^{n-1}x}{dt^{n-1}} + \ldots + a_0^0(t, P_x)x = \psi_0(t) \qquad (2.4)$$

Here P_x - vector of control parameters, affecting the variable x

3. Linking Mathematical Model of the Control System with Standard. The real process will be approximated by equation

$$a_n(t)\frac{d^n x}{dt^n} + a_{n-1}(t)\frac{d^{n-1}x}{dt^{n-1}} + \ldots + a_0(t)x = \psi_0(t) \qquad t \in [t_0, T] \qquad (3.1)$$

In equation (3.1) the right part is taken as standard. This eliminates the necessity of measuring the perturbation effects, which on some real objects appears to be quite difficult and a practically nonrealizable problem. Deviations of actual perturbations and values of operator coefficients of the right side of equation (3.1) from standard will be considered in a self-adjustment system through values of coefficients $a_n(t), \ldots, a_0(t)$. Furthermore, the mentioned assumption (about the effect on the system of standard actions) offers the possibility of selecting a quite easy algorithm to calculate the coefficients $a_n(t), \dots, a_0(t)$ of the approximating equation (3.1).

But it appears, that there is no need for calculating all the coefficients $a_n(t), \dots, a_0(t)$ of equation (3.1). The truth is, upon the parameters of the regulator ordinarily essentially depends only a part of these coefficients. Consequently, calculating all coefficients, we can anyway not distinguish all their values from standard. But for satisfactory operation of the system it is necessary somehow to consider the deviations of values of all coefficients from standard. We do this in the following manner.

Not reducing the generality of estimates, we will assume, that upon the controlling parameters depends only m of the first coefficients ($a_0(t)$, $a_1(t), \dots, a_{m-1}(t)$). We will transfer into the right side of equation (3.1) the members containing coefficients $a_m(t)$, $a_{m+1}(t), \dots, a_n(t)$

$$a_{m-1}(t)\frac{d^{m-1}x}{dt^{m-1}}+\dots+a_0(t)x=\psi_2(t)-\left[a_n(t)\frac{d^nx}{dt^n}+\dots+a_m(t)\frac{d^mx}{dt^m}\right] \quad (3.1a)$$

Since the information about coefficients $a_n(t), \dots, a_m(t)$ cannot be used by us in the regulator, then we will not directly determine these coefficients.

But when determining coefficients $a_0(t), \dots, a_{m-1}(t)$ we will assume that the values of the coefficients $a_n(t), \dots, a_m(t)$ are to standard

$$a_{m-1}(t)\frac{d^{m-1}x}{dt^{m-1}}+\dots a_0(t)x=\psi_2(t)-\left[a_n{}^0(t)\frac{d^nx}{dt^n}+\dots+a_m{}^0(t)\frac{d^mx}{dt^m}\right] \quad (3.2)$$

The deviations of values of coefficients $a_n(t), \dots, a_m(t)$ from standard will be considered through values of coefficients $a_0(t), \dots, a_{m-1}(t)$.

The above mentioned method of direct consideration of deviations of values of noncontrollable coefficients considerably reduces the volume of computing operations, since it eliminates the necessity of calculating n + 1-m coefficients.

4. Determining the coefficients of approximating equation. In the role of approximating equation we will take for generality equation (3.1). We will assume, that at a certain interval $[t_k, t_{k+1}]$ were obtained values $x(t)$, $\dot{x}(t), \dots, x^n(t)$ of real process in points $t_k = \tau_1, \tau_2, \dots, \tau_s = t_{k+1}$. Then, assuming, that the interval $[t_k, t_{k+1}]$

was selected in such a way, that the coefficients $a_n(t),\ldots,a_o(t)$ can be replaced with sufficient accuracy (within limits of this interval)by constant numbers, we will obtain a system s of linear heterogeneous algebraic equations to determine n + 1 of unknown coefficients

$$a_n^{(k)}\frac{d^n x(\tau_j)}{dt^n}+a_{n-1}^{(k)}\frac{d^{n-1}x(\tau_j)}{dt^{n-1}}+\ldots+a_0^{(k)}x(\tau_j)=\psi_0^{(k)}(\tau_j)\qquad (j=1,2,\ldots,s) \tag{4.1}$$

In the system (4.1) the index k designates conformity of coefficients $a_n,\ldots,a_o$ and perturbations ψ_j (t) of the interval $[t_k, t_{k+1}]$.

We wish to point out, that the selection of the magnitude of the interval $[t_k, t_{k+1}]$ represents a very complex problem. It is also necessary to consider here the rate of change in coefficients a_n $(t),\ldots,a_o(t)$, and the rate of change in perturbations, and the time necessary for measuring and analyzing data about the process etc.

We will generally assume that $s \geq n + 1$.

Since the measurements , carried out at the moment of time, situated closer to the right end of the interval $[t_k, t_{k+1}]$, bear information, which shows maximum effect on further process of controlling, then it appears to be advisable to introduce weight coefficients ρ (τ_j), determining the value of each measurement and of each one of the equations (4.1). The weight coefficients should satisfy the conditions:

$$1)\ \rho(\tau_j) > 0 \qquad (j=1,2,\ldots,s)$$

$$2)\ \sum_{1=j}^{s}\rho(\tau_j)=1 \tag{4.1a}$$

And so, from the assumption about linearity of rise in weight of measurements with time it is possible to select the weight coefficients in the following manner:

$$\rho(\tau_j)=\frac{2j}{s(s+1)}\qquad (j=1,2,\ldots,s) \tag{4.2}$$

The determination of coefficients $a_o(k)$, $a_1^{(k)},\ldots,a_n^{(k)}$ will be done by the method of least squares, minimizing the function

$$L=\sum_{j=1}^{s}\rho(\tau_j)L_j^2 \tag{4.3}$$

where

$$L_j=\sum_{i=0}^{n}x^{(i)}(\tau_j)\,a_i^{(k)}-\varphi_0^{(k)}(\tau_j) \tag{4.3a}$$

The necessary minimum condition of function L is the equality to zero of its partial derivatives of first magnitude.

Having calculated the partial derivatives $\partial L/\partial a_i^{(k)}$ $(i = 0,1,\ldots,n)$ and equating same to zero, we will obtain a system n+1 of linear algebraic equations to determine the unknown coefficients

$$a_0^{(k)} \sum_{j=1}^{s} [\rho(\tau_j) x(\tau_j) x^{(i)}(\tau_j)] + a_1^{(k)} \sum_{j=1}^{s} [\rho(\tau_j) x^{(i)}(\tau_j) \dot{x}(\tau_j)] + \ldots$$
$$\ldots + a_n^{(k)} \sum_{j=1}^{s} [\rho(\tau_j) x^{(i)}(\tau_j) x^{(n)}(\tau_j)] =$$
$$= \sum_{j=1}^{s} [\rho(\tau_j) x^{i}(\tau_j) \psi_0^{(k)}(\tau_j)] \qquad (i = 0, 1, \ldots, n) \qquad (4.4)$$

If vectors are introduced

$$\vec{x} = (x(\tau_1)\sqrt{\rho(\tau_1)}, x(\tau_2)\sqrt{\rho(\tau_2)}, \ldots, x(\tau_s)\sqrt{\rho(\tau_s)})$$
$$\dot{\vec{x}} = (\dot{x}(\tau_1)\sqrt{\rho(\tau_1)}, \dot{x}(\tau_2)\sqrt{\rho(\tau_2)}, \ldots \dot{x}(\tau_s)\sqrt{\rho(\tau_s)})$$
$$\vec{x}^{(n)} = (x^{(n)}(\tau_1)\sqrt{\rho(\tau_1)}, x^{(n)}(\tau_2)\sqrt{\rho(\tau_2)}, \ldots, x^{(n)}(\tau_s)\sqrt{\rho(\tau_s)}) \qquad (4.4a)$$
$$\vec{\psi}_0^{(k)} = (\psi_0^{k}(\tau_1)\sqrt{\rho(\tau_1)}, \ldots, \psi_0^{k}(\tau_s)\sqrt{\rho(\tau_s)})$$

Then system (4.4) can be rewritten into form of

$$a_0^{(k)} (\vec{x}^{(i)}, \vec{x}) + a_1^{(k)} (\vec{x}^{(i)} \dot{\vec{x}}) + \ldots + a_n^{(k)} (\vec{x}^{(i)}, \vec{x}^{(n)}) = (\vec{x}^{(i)}, \vec{\psi}_0^{(k)}) \qquad (4.5)$$
$$(i = 0, 1, \ldots, n)$$

where the expressions in round parentheses designate scalar derivatives of corresponding vectors.

In order that system (4.5) should have a single solution relative to the coefficients $a_0^{(k)},\ldots,a_n^{(k)}$, it is necessary and it is sufficient that the determinant of this system should be different from zero. Since the determinant of the system

$$\Gamma = \begin{vmatrix} (\vec{x}, \vec{x}) & (\vec{x}, \dot{\vec{x}}) & \ldots & (\vec{x}, \vec{x}^{(n)}) \\ (\dot{\vec{x}}, \vec{x}) & (\dot{\vec{x}}, \dot{\vec{x}}) & \ldots & (\dot{\vec{x}}, \vec{x}^{(n)}) \\ \ldots & \ldots & \ldots & \ldots \\ (\vec{x}^{(n)}, \vec{x}) & (\vec{x}^{(n)}, \dot{\vec{x}}) & \ldots & (\vec{x}^{(n)}, \vec{x}^{(n)}) \end{vmatrix} \qquad (4.5a)$$

is the gram determinant, then it will not be equal to zero then and only then when the system of vectors $\vec{x}, \dot{\vec{x}}, \ldots, \vec{x}^{(n)}$ will be linearly independent.

Since in practice are possible cases of linear dependence of these vectors, it is necessary to provide methods for synonymous determination of the sought for coefficients also for these cases.

In the role of one such method it is possible to introduce the following.

At each interval $[t_k, t_{k+1}]$ after obtaining s measurements we will calculate the Gram determinant in all its main minors. Assuming there is a minor of maximum power l, differing from zero ($l \leq s$). Then all coefficients $a_n^{(k)}$, $a_{n-1}^{(k)}, \dots, a_{n-l+1}^{k}$ are assumed to be equal to their standard values, and the remaining ones are determined by solving with any one known method a linear system with determinant, differing from zero.

If $l = n + 1$, then the very Gram determinant differs from zero and all sought for coefficients are determined synonymously.

But if $l = 0$, i.e. $(\vec{x}, \vec{x}) = 0$, then it can be assumed that all coefficients have standard values and no adjustment of same is necessary.

To reduce the volume of calculations it is possible to use the property of the Gram determinant, consisting in the fact, that if any one major minor is different from zero, then all main minors of lower power are also different from zero. Consequently it is possible to calculate the main minor in a sequence, beginning with the minor of maximum power (the very Gram determinant), and continuing the calculation to the first minor not equalling zero.

It should be pointed out, that with the aid of the described method it is possible to make a synonymous determination of coefficients $a_0^{(k)}, a_1^{(k)}, \dots, a_n^{(k)}$ and in case $s < n + 1$.

In some instances deviations of values of coefficients $a_0^{(k)}, \dots, a_n^{(k)}$ are best determined by minimizing function (4.3) with the aid of gradient methods. For zero approximations in this case it is possible to take standard values. And so, in the case of using just one step along the correction gradient to standard values they are calculated by values(formulas)

$$\Delta a_i^{(k)} = -\varepsilon^{(k)} \frac{\left(\frac{\partial L}{\partial a_i^{(k)}}\right)_0}{\sqrt{\sum_{i=0}^{n}\left(\frac{\partial L}{\partial a_i^{(k)}}\right)_0^2}} \qquad (4.6)$$

where

$$\frac{\partial L}{\partial a_i^{(k)}} = 2 \sum_{j=1}^{s} \rho(\tau_j) L_j x^{(i)}(\tau_j) \quad (4.6a)$$

$\varepsilon^{(k)} > 0$ are determined for each interval $[t_k, t_{k+1}]$.

Using analogoys formula when necessary it is also possible to calculate the following approximations.

5. The law governing changes in control coefficients. In general case each oneof the coefficients $a_0(t)$, $a_1(t), \ldots, a_n(t)$ may depend upon the control parameters $p_1, p_2, \ldots, p_g$ ($P_x = (p_1, p_2, \ldots, p_g)$) in an arbitrary manner. Because of this it is practically impossible by changing g of the regulator parameters to accurately adjust the values n+1 of the coefficients to standard. At an accurate adjustment of several coefficients the remaining ones can still to a large extent increase their deviations from standard. That is why a proposal is made to select the values of the control xoefficients at each interval $[t_k, t_{k+1}]$ using the method of least squares of the minimizing sum

$$L = \sum_{i=0}^{n} (\Delta a_i)^2 = \sum_{i=0}^{n} [a_i(p_1, \ldots, p_g) - a_i^0]^2. \quad (5.1)$$

Using the necessary condition of the presence of a minimum for functions (5.1) we will obtain the following system of algebraic equations to determine $p_1, \ldots, p_g$:

$$\sum_{i=0}^{n} [a_i(p_1, \ldots, p_g) - a_i^0] \frac{\partial a_i(p_1, \ldots, p_g)}{\partial p_j} = 0 \qquad (j = 1, 2, \ldots, g) \quad (5.2)$$

Actually the values of parameters $p_1, \ldots, p_g$ (amplification factors and time constants) are limited by certain restrictions

$$A_q < P_q < B_q \quad (q = 1, 2, \ldots, g) \quad (5.3)$$

Consequently the minimization of functions (3.1) must generally be carried out with consideration of conditions (5.3). In this case will be obtained values of parameters $p_1, \ldots, p_g$, the determination of which leads to the attainment of an approximation of the real system to the standard.

6. Integral form of equations to determine the coefficients of approximating equation. All above mentioned deliberations and operations were done under the assumption, that we have to our disposal a magnitude of control value at the interesting

us moment of time and the necessary number of its derivatives.

Even though there are certain assumptions on the obtainment of derivatives of any arbitrary order from the controlled value [5], but so far these assumptions are little realized. Consequently it is necessary to provide certain methods of realizing a self-adjusting system of control in the presence of a limited number of derivatives (ordinarily with permissible accuracy it is possible to obtain the first two derivatives), which can be practically measured.

The problem of determining the coefficients of approximating equation can be solved when measuring a limited number of derivatives if the integral form of an algebraic equation system (3.1) is used. This form of equations can be obtained if each member of equation (3.1) is integrated n-m times within corresponding limits

$$a_n^{(k)} \underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n-m} \frac{d^n x}{dt^n}\underbrace{dt\ldots dt}_{n-m} + a_{n-1}^{(k)} \underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n-m} \frac{d^{n-1}x}{dt^{n-1}}\underbrace{dt\ldots dt}_{n-m} + \ldots$$
$$\ldots + a_0^{(k)} \underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n-m} x\,dt\ldots dt = \underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\int_{t_k}^{t}}_{n-m} \psi_0^{(k)}(t)\underbrace{dt\ldots dt}_{n-m} \qquad (6.1)$$
$$(j = 1, 2, \ldots, s)$$

Here m - number of derivatives of controllable value, which can be measured with required accuracy.

Assuming that m = 2. Then after integration we will obtain

$$a_n^{(k)}\Big(x^{\mathrm{II}}(\tau_j) - x^{\mathrm{II}}(t_k) - x^{\mathrm{III}}(t_k)\Delta\tau_j - x^{\mathrm{IV}}(t_k)\frac{\Delta\tau_j^2}{2!} - \ldots$$
$$- x^{(n-1)}(t_k)\frac{\Delta\tau_j^{n-3}}{(n-3)!}\Big] + a_{n-1}^{(k)}\Big[x^{\mathrm{I}}(\tau_j) - x^{\mathrm{I}}(t_k) - x^{\mathrm{II}}(t_k)\Delta\tau_j -$$
$$- x^{\mathrm{III}}(t_k)\frac{\Delta\tau_j^2}{2!} - \ldots - x^{(n-2)}(t_k)\frac{\Delta\tau_j^{n-3}}{(n-3)!}\Big] + \cdots \qquad (6.1a)$$
$$\ldots + a_0^{(k)} \underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n-2} x(t)\underbrace{dt\ldots dt}_{n-2} = \underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n-2} \psi_0^{(k)}(t)\underbrace{dt\ldots dt}_{n-2}$$

where

$$(\Delta\tau_j = \tau_j \pm t_k,\ j = 1, 2, \ldots, s) \qquad (6.2)$$

But in the system (6.2) we have still not freed ourselves from the necessity of measuring high power derivatives, although they have to be measured not in points s but in one point t_k, i.e. at the beginning of interval $[t_k, t_{k+1}]$.

Since to our disposal are only values of the very variable and first and second derivatives, then we shall attach to the derivatives a power higher than the second standard value.

$$a_n^{(k)}\left[x^{II}(\tau_j) - x^{II}(t_k) - (x^{III}(t_k))_s\,\Delta\tau_j - \ldots - (x^{(n-1)}(t_k))_s\,\frac{\Delta\tau_j^{\,n-3}}{(n-3)!}\right] + \ldots$$
$$\ldots + a_0^{(k)}\underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n-2} x(t)\underbrace{dt\ldots dt}_{n-2} = \underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n-2}\psi_s^{(k)}(t)\underbrace{dt\ldots dt}_{n-2} \qquad (j=1,2,\ldots,S) \qquad (6.3)$$

From (6.3) by the explained method is possible to determine coefficients.

When assuming the smallness of the interval $[t_k, t_{k+1}]$, with members, containing the degrees $\Delta\tau_j = \tau_j - t_k$ higher than second power, can be disregarded. And so for example, at n-multiple integration and at mentioned assumption about the smallness of the interval $[t_k, t_{k+1}]$ the system of equations for determining the sought for coefficients acquires the form of

$$a_n^{(k)}\left[x(\tau_j) - x(t_k) - x^{I}(t_k)\,\Delta\tau_j - x^{II}(t_k)\,\frac{\Delta\tau_j^2}{2}\right] + a_{n-1}^{(k)}\left[\int_{t_k}^{\tau_j} x(t)\,dt - x(t_k)\,\Delta\tau_j - x^{I}(t_k)\,\frac{\Delta\tau_j^2}{2}\right] + \ldots + a_0^{(k)}\underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n} x(t)\underbrace{dt\ldots dt}_{n} =$$
$$= \underbrace{\int_{t_k}^{\tau_j}\int_{t_k}^{t}\cdots\int_{t_k}^{t}}_{n}\psi_s^{(k)}(t)\underbrace{dt\ldots dt}_{n}$$
$$(j = 1, 2, \ldots, s)$$

To reduce the order of the derivatives necessary for measuring derivatives it is also possible to use the following methods.

First of all, in equation (3.2) in the right side the values of derivatives $x^n(t), \ldots, x^m(t)$ can be taken as standard. The order of the higher derivative, which is necessary to measure, decreases from n to m - 1.

Secondly, standard values can be given to all members of the initial approximating equation, in addition to these members, the control value derivatives at which can be measured practically with required accuracy. It is assumed that there is a possibility of measuring the very controllable value in its two first derivatives. Equation (3.1) will then acquire the form of

$$a_2(t)\frac{d^2x}{dt^2}+a_1(t)\frac{dx}{dt}+a_0(t)x(t)=$$
$$=\psi_0(t)-\left[a_n^0(t)\left(\frac{d^nx}{dt^n}\right)_0+\ldots+a_3^0(t)\left(\frac{d^3x}{dt^3}\right)_0\right] \quad (6.5)$$

This approximation sometimes gives satisfactory results.

Conclusion: Self-adjusting systems created by the above explained principle, can be used for controlling a wide class of objects. The basic advantage of these systems is the consideration of noncontrolled perturbations, outer as well as inner (by maintaining the parameters of the object and regulator).

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