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TRANSLATION

ON THE KINETICS OF DISRUPTING
SUPERCONDUCTIVITY BY A VARIABLE FIELD

By

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On the Kinetics of Disrupting Superconductivity by a Variable Field

by

I. M. Lifshits and F. I. Tsikovich

Report by [1] contains equations, determining the kinetics of disruption of superconductivity of a cylindrical sample by a longitudinal magnetic field, and the case of a constant field is discussed in detail. In experiment [2] were obtained basic results for a variable field with consideration of the relaxation effect. This report has the purpose of giving a more thorough explanation of these results with consideration of sample curvature and thermal effects.

Par.1. Basic Equations

In the plane case, when the thickness zeta of the normal phase layer is small in comparison with the radius R of the sample, the process is described by following equations (see [1], [2]):

$$\frac{\partial H}{\partial t} = \frac{c^2}{4\pi\sigma}\frac{\partial^2 H}{\partial z^2},\quad 0 \leqslant z \leqslant \zeta(t); \qquad (1)$$

$$\zeta(t_0) = 0; \qquad (2)$$

$$H|_{z=0} = H_0(t); \qquad (3)$$

$$H|_{z=\zeta(t)} = H_k(T)\left[1 + \frac{\zeta'(t)}{V_0}\right]^2; \qquad (4)$$

$$\left.\frac{\partial H}{\partial z}\right|_{z=\zeta(t)} = -\frac{4\pi\sigma}{c^2} H|_{z=\zeta(t)}\zeta'(t). \qquad (6)$$

Here a - speed of light; σ - normal conduction; V_0 - relaxation coefficient of speed dimension; z-distance from surface of sample; $H_0(t)$ - outer field; $H_k(T)$ - critical field; T - temperature on boundary of phases, which is considered so far as a definite function of time; t_0 - moment, when the outer field, as it rises, it attains for the first time values $H_k(T_0)$, where T_0 = T(to) at $t < t_0$ the sample is in superconductive state).

The outer field
$$H_0(t) = H_c + H_a \sin \omega t \qquad (6)$$

is periodically acquiring values, higher and lower than the critical

$$(H_a > /H_k(T_0) - H_c /)$$

Thanks to the additional boundary condition (5) taking into consideration the effect of superconductivity zone in disregarding the depth δ_0 of field's penetration into the superconductor (at temperatures not very close to critical, $\delta_0 \sim$ 10-5 cm), the combination of equations (1)-(5) synonymously determines the magnetic field in a normal layer, because the kinetics of phase transition does not depend upon the electrodynamics of superconductive state. The use of the Ohm law in equations (1), (5) requires that zeta should be greater than the length L of the free migration of electrons[1] (at temperatures under consideration $L \sim 10^{-3}$ cm). Consequently, the plane approximation has a zone of applicability at $R \gg L$ (in the known to us experiments $R \sim 10^{-1}$ - 10^{-2} cm).

The mechanism of superconductive transition for further action is non-essential; it is only sufficient to assume that superconductivity succeeds in fully recovering within a part of the period, when $H_0 < H_k$. This assumption needs experimental confirmation, or it is principally possible , that at $H_0 < H_k$ on the surface is formed a thin superconductive film, isolating the magnetic flux in normal layer and preventing in this way further restoration of superconductivity.

Equations (1), (3), (4) and (5) can be written in form of one integro-differential equation. By determining the electric field E from equations of electromagnetic induction and substituting the current $j = \sigma E$ in the Maxwell equation for rot $\vec{H}$, we have

$$\frac{\partial H}{\partial z} = -\frac{4\pi\sigma}{c^2}\frac{d}{dt}\int_z^{\zeta} H\,dz. \qquad (6a)$$

Footnote to eq.(4)...1...at zeta $<$ V_0, which in case of low supercriticalness is always fulfilled (see (24), (22)).

1. Case where zeta $<$ L was discussed in [3]

Integration of this equation by z by the use of boundary conditions (3), (4) gives

$$H_k(T)\left(1+\frac{\zeta'}{V_0}\right)-H_0=-\frac{4\pi\sigma}{c^2}\int_0^{\zeta}\frac{d}{dt}\left\{\int_0^{z} H(z',t)\,dz'\right\}dz. \qquad (7)$$

From here it is easy and physically descriptively to derive an equation of motion of the phase boundary in case of small supercriticalness, if we would write approximately $H \cong H_k(T_0)^2$:

$$u=\frac{H_0-H_k(T_0)}{H_k(T_0)}\quad\left(u_c=\frac{H_c-H_k(T_0)}{H_k(T_0)},\ s=u_{max}\right),$$
$$v=\frac{H_k(T)-H_k(T_0)}{H_k(T_0)}=\frac{(T-T_0)\,dH_k/dT}{H_k(T_0)},\quad \tau=\frac{t}{t_1},\quad x=\frac{z}{r_1}, \qquad (8)$$
$$\xi=\frac{\zeta}{r_1},\quad \lambda=\frac{l}{r_1}\quad\left(\frac{r_1^2}{t_1}=\frac{c^2}{4\pi\sigma},\quad l=\frac{c^2}{4\pi\sigma V_0}\right)$$

and disregarding member $\lambda\xi'v$ of second smallness magnitude (delda xi' $=\frac{\text{zeta}'}{V_0} \ll 1$, $v < u \ll 1$), we obtain

$$\xi\xi'+\lambda\xi'=u-v,\quad \xi(\tau_0)=0\quad\left(\xi'=\frac{d\xi}{d\tau}\right),$$
$$\zeta\zeta'+l\zeta'=\frac{c^2}{4\pi\sigma}(u-v),\quad \zeta(t_0)=0\quad\left(\zeta'=\frac{d\zeta}{dt}\right). \qquad (9)$$

Here u describes the change in outer field, causing a phase transition, the remaining members - internal phenomena in the superconductor, determining the rate of the boundary: xi xi'-electromagnetic "braking " (Foucault currents), lambda xi'-relaxation, v-thermal effects.

In inertialess ($V_0 \to \infty$, $l = 0$, $\lambda = 0$) and isothermal ($T \equiv T_0$, $v \equiv 0$) cases (9) coincides with equations, obtained in [1] and [2] by decomposing into steps of small parameter. If for the obtainment of these equations the field distribution in normal layer had to be approximated by a linear function

$$H=H_k(T_0)\left[1+u-(u-v-\lambda\xi')\frac{x}{\xi}\right]. \qquad (9a)$$

then in this report it was found sufficient a more rough approximation $H \cong$ const. This is explained by the fact, that in (7) the field H stands under the sign of the integral, so that its values are "used " over the entire thickness of the normal layer $0 \leq z \leq$ zeta, while in (3) - (5), appearing as the basic ones in [1] and [2] the boundary values H only "participate".

~~2. This method~~ was applied in [4] to the exceptional case of a constant field ,isothermal condition and inertialess motion.

Par.2. Isothermal Case

This case takes place at sufficient heat transfer either into the surrounding medium (e.g. when flushing the sample He-II) or in the deep areas of the sample, having considerable specific heat[1]. Since it is experimentally realizable, it permits full mathematical investigation, and that is why it is of most interest.

In the role of basic unit it is proper to select a period of the external field, $t_1 = 2\pi/\omega$; then $r_1 = \sqrt{\pi}\delta$, where

$$\delta = \frac{c}{\sqrt{2\pi\sigma\omega}} \tag{10}$$

depth of penetration of the variable field of frequency ω into normal phase ("skin depth"). The equation of motion of the boundary

$$\xi\xi' + \lambda\xi' = u, \quad \xi(\tau_0) = 0 \tag{11}$$

is integrated in general form

$$\xi = \sqrt{2\int_{\tau_0}^{\tau} u\,d\tau + \lambda^2} - \lambda, \quad \text{или} \quad \zeta = \sqrt{\frac{c^2}{\sigma\omega}\int_{\tau_0}^{\tau} u\,d\tau + l^2} - l. \tag{12}$$

The normal layer reaches maximum thickness $s = r_1 xi_{max}$ at the moment τ_m, determinable as the first one different from τ_0 root of equation $u(\tau) = 0$ ($H_0(t) = H_k$), afterwhich starts the restoration of superconductivity. If it does take place by reverse motion the boundaries in conditions of "magnetic" supercooling the normal phase without formation of new nuclei[2], remain in force the equations (11) and (12). Then, if the constant component of the outer field is greater than the critical value ($u_0 > 0$) the result of each cycle is irreversible disruption of superconductivity in the layer of definite thickness, so that during the period of the final number of cycles there is total disruption in superconductivity. In an opposite case $H_0 \leq H_k$ ($b = -u_0 \geq 0$) toward the initial moment of the second cycle superconductivity succeeds in fully restoring itself, and the process is periodic. When $H_a > H_0 + H_k$ (especially, when there is no

1. Isothermicity criterion see par.4.
2. For sufficiently high frequencies this hypothesis is highly probable, because the origination of a nucleus requires finite time.

constant component) in each ceriod appears a cycle of disruption-restoration of super conductivity by the field of opposite direction. In the discussed case $\xi \ll 1$ both cycles do not cover each other.

From equation (12) are obtained two specific cases: purely electromagnetic at zeta $\gg 1$ and purely relaxation at zeta $\ll 1$. It can hardly be expected that l should be too much greater than the length of the free run of electrons (because V_0 by the order of magnitude is not smaller than $c^2/4\pi\sigma L' \sim 10^2$ cm/sec[1]); it is most probable that $l \ll L$ ($l \sim 10^{-5} \div 10^{-6}$ cm). Consequently, the purely relaxation case takes place, apparently when zeta $< L$, i.e. not in the zone of applicability of equation (12); consequently, the actual condition of realizing a purely relaxation case differing from zeta $\ll 1$, obtained from (12).

In purely electromagnetic case

$$\xi = \sqrt{2\int_{\tau_0}^{\tau} u\, d\tau} \tag{13}$$

the frequency dependence of depth and rate of disruption of superconductivity

$$\zeta \sim \frac{1}{\sqrt{\omega}}, \quad V \sim \sqrt{\omega} \quad (13a)$$

is obtained from deliberations of dimensionality and is not connected with the small ness of supercriticalness (see [2]). We will underline, that these ratios take place only upon disregard of sample curvature, by a change in its temperature and relaxation, thanks to which the nondimensional formulation of the problem does not include R/r_1 and λ, depending upon ω. The obtained frequency dependence corresponds to the time dependence for the case of constant field (see [1]), if it is assumed that $t \sim 1/\omega$.

The maximum depth of superconductivity disruption

$$s = c\sqrt{\frac{\lambda_{\xi s}}{\sigma\omega}} = \sqrt{2\pi\lambda_{\xi s}\delta}; \tag{14}$$

1. Here, as well as for all other numerical evaluations and calculations, the conductivity value was taken for tin, $\sigma = 5 \cdot 10^{20}$ sec^{-1}. For a majority of pure metals σ is the same.

Here, $\varphi = \tau_m - \tau_0$ - fraction of the period, which constitutes the time of disruption in superconductivity, $\chi \approx 0.65$ (see (18))- mean value of the function $u/_t$ in this interval, s - smaller or of the magnitude δ, whereby its ratio is not disrupted even at greater supercriticalnesses, when $\sqrt{\xi}$ changes into $\sqrt{\ln \xi}$ (see[2]), which grows extremely slowly, remaining of the order of 1 at all real values ξ. The proportionality of the depth s of skin depth δ indicates similarity in the disruption of superconductivity and penetration of the field into normal metal. The fact is, both phenomena are described by one and the very same equation (1) and the boundary condition (3) on the surface; only the initial condition (2) and conditions (4) and (5) on the phase boundary differ from the corresponding conditions $H/_{t=0}$ 0 and $H/_{z=\infty}$ 0 for the case of normal metal. However, if the field in normal metal decreases, as it gets away farther from the surface, quite rapidly, as is the case with the skin-effect, it is possible to introduce by the order of magnitude thickness values zeta of the layer, in which the field is localized, and then the phenomenon will be described by equations (1) - (5), where it is only necessary to write $H_k = 0$. The latter explains the dependence of s upon the supercriticalness ξ.

The rate of the phase boundary

$$V = \frac{c^2}{4\pi\sigma}\frac{u}{\zeta}. \qquad (15)$$

The inverse dependence of V upon zeta explains the basic experimental result - divergence of orders of speed magnitudes at total transformation of the specimen in to normal state (see[5]) and during disruption of superconductivity by a high frequency field in a thin surface layer (see [6]). Physically this can be interpreted, as a much easier "reduction" by the electromagnetic field of a thin normal layer. The average rate

$$V_{cp} = \frac{s}{t_m - t_0} = \frac{c}{2\pi}\sqrt{\frac{\chi\xi\omega}{\varphi\sigma}} = \sqrt{\frac{\chi\xi}{2\pi\varphi}}\,\omega\delta. \qquad (16)$$

To give an idea about the order of values s and V_{aver} at various frequencies we present the following table:

ω SEC^{-1}

ω сек$^{-1}$	10^{-1}	1	10	10^{2}	10^{3}	10^{4}	10^{5}
$s/\sqrt{2\pi\chi\varphi\varepsilon}=\delta=\frac{0{,}54}{\sqrt{\omega}}$ см	1,7	$5{,}4\cdot10^{-1}$	$1{,}7\cdot10^{-1}$	$5{,}4\cdot10^{-2}$	$1{,}7\cdot10^{-2}$	$5{,}4\cdot10^{-3}$	$1{,}7\cdot10^{-3}$
$V_{cp}\Big/\sqrt{\frac{\chi\varepsilon}{2\pi\varphi}}=\omega\delta=0{,}54\sqrt{\omega}\,\frac{см}{сек}$	$1{,}7\cdot10^{-1}$	$5{,}4\cdot10^{-1}$	1,7	5,4	$1{,}7\cdot10$	$5{,}4\cdot10$	$1{,}7\cdot10^{2}$

The range of frequencies in which formulas (14) and (16) are applicable, is limited by requirements $L \ll s \ll R$, i.e. Omega $(R) \ll \omega \ll$ Omega (L)[1], where

$$\Omega(s)=\frac{c^2\chi\varphi\varepsilon}{\sigma s^2}\sim\frac{\varphi\varepsilon}{s^2\,(см)}\ сек^{-1} \tag{17}$$

the frequency at which maximum depth of superconductivity disruption equals s. At $R \sim 10^{-1}$- 10^{-2} cm. $L \sim 10^{-3}$ cm, it gives

$$10\div10^{3}\ll\omega\,(сек^{-1})/\varphi\varepsilon\ll10^{5}. \quad (17a)$$

The dependence upon relative values of constant and variable components and the critical value of the field (i.e.upon b and ε) is contained in χ, φ and ε. In this case φ and χ are the function of $\theta = \frac{\varepsilon}{b}$ only:

$$\varphi=\frac{1}{\pi}\arccos\frac{1}{1+\theta}\simeq\begin{cases}\sqrt{2\theta}/\pi & (\theta\ll1),\\ 1/2 & (\theta\ll1);\end{cases}$$

$$\chi=\frac{\sqrt{1+\frac{2}{\theta}}}{\arccos\frac{1}{1+\theta}}-\frac{1}{\theta}\simeq\begin{cases}\frac{2}{3}=0{,}667 & (\theta\ll1),\\ \frac{2}{\pi}=0{,}637 & (\theta\gg1).\end{cases} \tag{18}$$

In fig.1 are given graphs of functions $s/\sqrt{\varepsilon\delta} = \sqrt{2\pi\chi\varphi}$ and $V_{aver}/\sqrt{\varepsilon\omega\delta} = \sqrt{\chi/2\pi\varphi}$. The dotted line shows asymptotic curves.The corresponding approximated formulas:

See page 7a for equasions 19 and 20

1. In addition to isothermal conditions, which also represent a frequency limitation (see (53), (55),(56)). The conditions of inertialessness $s \gg 1$ i.e. $\omega \ll$ Omega (1), in conformity with above made statements, apperently, is covered by requirement $s \gg L$.

$$s \simeq \begin{cases} \sqrt{2\varepsilon}\,\delta & (b \ll \varepsilon), \\ \sqrt{\frac{4}{3}\sqrt{2}\,\frac{\varepsilon^{3/2}}{b^{1/2}}}\,\delta & (b \gg \varepsilon); \end{cases} \qquad V_{\text{cp}} \simeq \begin{cases} \frac{\sqrt{2\varepsilon}}{\pi}\,\omega\delta & (b \ll \varepsilon), \\ \frac{1}{\sqrt{3\sqrt{2}}}\,\varepsilon^{1/2} b^{1/2}\,\omega\delta & (b \gg \varepsilon); \end{cases} \tag{19}$$

$$\Omega(s) \simeq \begin{cases} \frac{c^2\varepsilon}{\pi e\, s^2} & (b \ll \varepsilon), \\ \frac{2\sqrt{2}\,c^2\varepsilon^{3/2}}{3\pi e b^{1/2} s^2} & (b \gg \varepsilon). \end{cases} \tag{20}$$

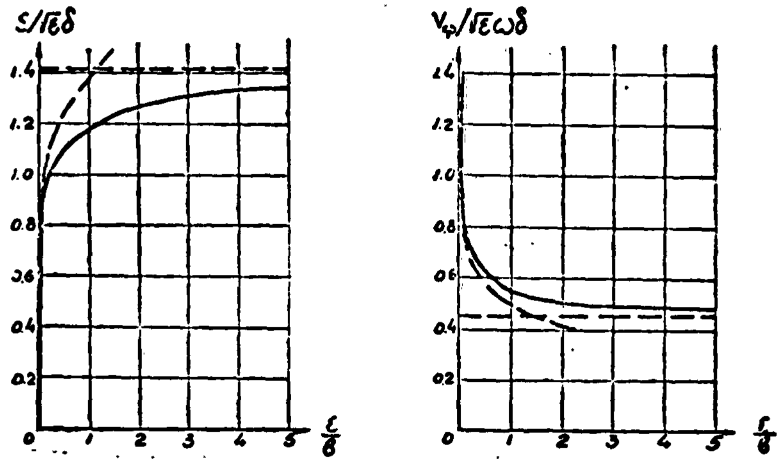

Fig.1.

In purely relaxation case

$$\xi = \frac{1}{\lambda}\int_{\tau_0}^{\tau} u\,d\tau, \quad \zeta = V_0\int_{t_0}^{t} u\,dt. \tag{21}$$

The physical sense of this approximation is explained, if we compute the rate of the boundary

$$V = uV_0 = V_0\frac{H_0 - H_k}{H_k}. \tag{22}$$

A comparison with formula $V = V_0 \frac{H/_{z=\text{zeta}} - H_k}{H_k}$ of ordinary case shows, that equation (21) disregards the difference between the values of the field on the phase boundary and on the surface of the sample.

Maximum depth and average rate of superconductivity disruption are determined by the following formulas:

$$s = \frac{c^2\chi_{\varphi s}}{2\pi l\omega} = \frac{2\pi\chi_{\varphi s}V_0}{\omega} \approx \begin{cases} \frac{2sV_0}{\omega} \ (b < s), \\ \frac{4\sqrt{2}}{3}\frac{s^{1/2}V_0}{b^{1/2}\omega} \ (b \gg s); \end{cases} \quad V_{cp} = \chi_s V_0. \tag{23}$$

Actually, the dependence upon frequency, supercriticalness and parameter Θ was obtained different, than in purely electromagnetic case (compare (14), (16), (19), instead of the dependence upon conductivity a dependence upon V_0 appeared.

In general case (12) (zeta$\sim$1) the rate

$$V = \frac{1}{\sqrt{(1/V_{\text{эм}})^2 + (1/V_p)^2}} \quad (\text{at given } t) \; (\text{при данном } t) \tag{24}$$

(see (15), (22)). Maximum depth and average rate

$$s = l\left[\sqrt{\frac{\Omega(l)}{\omega}+1}-1\right], \; V_{\text{ср}} = \frac{l\omega}{2\pi\varphi}\left[\sqrt{\frac{\Omega(l)}{\omega}+1}-1\right] = 2\chi\varepsilon V_0 \frac{\omega}{\Omega(l)}\left[\sqrt{\frac{\Omega(l)}{\omega}+1}-1\right] \quad (\Omega(l) = 16\pi^2\chi\xi\varepsilon\sigma(V_0/c)^2) \tag{25}$$

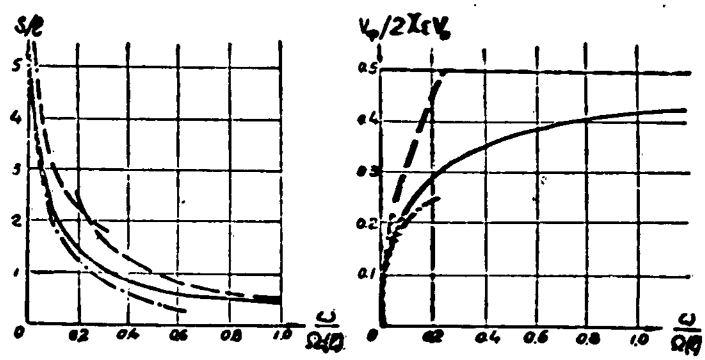

Fig.2.

are presented graphically in fig.2.The dotted lines represent the above discussed specific formulas (14), (16 and (23). The dash-dots show curves, approximately considering relaxation correction ($\simeq 1/s_{em} = \sqrt{\omega/\Omega(1)}$) to formulas of purely electromagnetic instance. The relaxation coefficient, presenting considerable interest for the microscopic theory of superconductivity, can be determined with the aid of measuring s or V_{aver} in the range of frequencies, permissible for the present theory, provided it will be possible to make them quite accurately (summary relative error, including also the errors of the experiment as well as the inaccuracy of our formulas, should be, in any event, much lower 1/L).

Par.3.Calculation of Curvature

In the case of a cylinder it is suitable to adopt as basic unit its radius, r_1 = R; then $t_1 = 4\pi\sigma R^2/c^2$, λ = 1/R. In dimensionless variables the peoblem acquires the form of:

$$\frac{\partial U}{\partial \tau} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial U}{\partial \rho}\right), \quad \eta(\tau) \leqslant \rho \leqslant 1; \qquad (26)$$

$$U|_{\rho=1} = u(\tau); \qquad (27)$$

$$U|_{\rho=\eta(\tau)} = v - \lambda\eta'(\tau); \qquad (28)$$

$$\left.\frac{\partial U}{\partial \rho}\right|_{\rho=\eta(\tau)} = -\eta'(\tau); \qquad (29)$$

$$\eta(\tau_0) = 1. \qquad (30)$$

Here

$$U(\rho, \tau) = \frac{H(r,t) - H_k(T_0)}{H_k(T_0)}, \quad \rho = \frac{r}{R}, \quad \eta = \frac{a}{R}; \qquad (31)$$

r-current coordinate (radius in cylindrical system), a=a(t) - radius of superconductive core; disregarding the very same items, as in plane case.

To derive an approximate equation of motion of the boundary in case of low supercriticalness we are employing the method [2]. Transforming in equations (26) to independent variables ρ, η we have

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial U}{\partial \rho}\right) + \left.\frac{\partial U}{\partial \eta}\frac{\partial U}{\partial \rho}\right|_{\rho=\eta} = 0. \quad (31a)$$

Eliminating the nonlinear member $\frac{\partial U}{\partial \eta} \cdot \left.\frac{\partial U}{\partial \rho}\right|_{\rho=\delta} \sim \varepsilon^2$ and by integrating twice with consideration of the boundary conditions (27), (28) we obtain

$$\frac{\partial U}{\partial \rho} = -\frac{u - v + \lambda\eta'}{\rho \ln \eta}, \quad U = u - (u - v + \lambda\eta')\frac{\ln \rho}{\ln \eta}. \qquad (32)$$

Now (30), (31) give

$$(\eta \ln \eta - \lambda)\eta' = u - v, \quad \eta(\tau_0) = 1. \qquad (33)$$

The very same result can be obtained by the method described in par.1 of this report.

Confining ourselves to the isomeric case, we integrate (33):

$$1 - \eta^2(1 - 2\ln \eta) + 4\lambda(1 - \eta) = 4\int_{\tau_0}^{\tau} u\, d\tau. \qquad (34)$$

As it actually shoul be , at $1 - \eta \ll 1$ (33) and (34) are converted into equations (9) and (12) of the plane approximation, as discussed above. If $1 - \eta \sim 1$, at $\lambda \ll 1$

the relaxation member can be disregarded. Consequently, if for a sample of ordinary dimensions ($R \gtrsim 1$) arises the need of considering the curvature , we have then a purely electromagnetic case

$$\eta = \Psi\left(4\int_{\tau_0}^{\tau} u\,d\tau\right) = \Psi(2\xi^2_{\max}) \qquad (35)$$

(see (13)), where ψ (w) is determined by the ratio

$$1 - \Psi^2(1 - 2\ln\Psi) = w. \qquad (36)$$

The graphs of functions Omega(w) is shown in [4] . It decreases monotonously from 1 at w = 0 to 0 at w = 1. dOmega/dw at w = 0 and w = 1 transforms into infinity, and at Omega = $\frac{1}{e}$ = 0.37, w=1 $-\frac{3}{e^2}$ = 0.59 (bending point) has a min, equalling e/4 = 0.68.

Maximum thickness of normal layer

$$S = R[1 - \Psi(2s^2/R^2)] = R\{1 - \Psi[2\Omega(R)/\omega]\} \qquad (37)$$

solid curve in fig.3) is obtained greater,than in the plane case (dotted curve). In this respect there is an increase in V_{aver}.The coefficient of increase rise monotonously from 1 at S = 0 to $\sqrt{2}$ at S = R. Total disruption in superconductivity within a period of one cycle takes place at frequencies ,smaller than

$$2\Omega(R) \sim \begin{cases} \frac{\epsilon}{R^2\,(cm)}\,ce\kappa^{-1}\,(b \ll \epsilon), \\ \frac{\epsilon^{3/2}}{b^{1/2}\,R^2\,(cm)}\,ce\kappa^{-1}\,(b \gg \epsilon). \end{cases} \qquad (37a)$$

(see (20)).

In a purely relaxation case ($R \lesssim 1, \lambda \gg 1$)

$$\eta = 1 - \frac{1}{\lambda}\int_{\tau_0}^{\tau} u\,d\tau, \qquad (38)$$

which coincides with equation (21) of plane approximation in conformity with the physical nature of this case. Total disruption in superconductivity occurs at frequencies, smaller than

$$\frac{1}{2}\Omega(\sqrt{\lambda}R) = \frac{2\pi\lambda\tau_s V_0}{R}. \qquad (39)$$

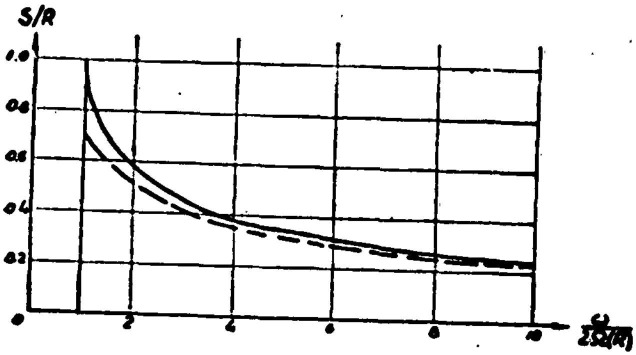

Fig.3.

Par.4.Thermal Effects

As result of enormous electro-and heat conductivity of a majority of pure metals at discussed temperatures ($x^2 = 4\pi\sigma D/C^2 \sim 10^{-4} - 10^5$, where D - heat conductivity coefficient), the change in temperature can be determined from the thermal balance equation. The fact is: by solving equations of heat conductivity, carried out by [1] for the special case of semispace, it becomes evident, that a change in temperature is practically localized in a layer with a thickness $\sim x$ zeta/$\sqrt{u}$ and is insignificantly low; consequently, at zeta $\gg \sqrt{uR/x}$ the temperature within the limits of the sample is practically constant, and at zeta $\lesssim \sqrt{uR/x}$, when the thermal balance equation gives an understated value v, this error is not substantial, because they are all equal to $v \ll u$.

In this way

$$\frac{dT}{dt} = \frac{-2\pi q a \frac{da}{dt} + \frac{c^2}{16\pi^2\sigma} \int_a^R \left(\frac{\partial H}{\partial r}\right)^2 2\pi r\, dr - 2\pi R K g (T - T_0)}{\pi a^2 C_s + \pi (R^2 - a^2) C_n}, \quad T(t_0) = T_0. \quad (40)$$

Here in the numerator is given the amount of heat, liberated per unit of time in a unit of sample length thanks to phase conversion, Foucault currents and heat exchange with outer medium (for the latter was adopted the Newton law $\left.\frac{\delta T}{\delta r}\right|_r = R - g(T'_{r=R} - T_0)$), in the denominator - specific heat per unit of length; q = (1/4π)

$TH_k dH_k/dT$ - heat of superconductivity disruption per unit of volume; K - heat conduction coefficient; C_s and C_n - specific heat per unit of volume of superconductive and normal phases respectively.

$$C_s - C_n = \frac{T}{4\pi}\left[\left(\frac{dH_k}{dT}\right)^2 + H_k \frac{d^2H_k}{dT^2}\right], \quad C_{s,n}|_{T=T_k} \sim 10^4 \frac{\text{erg}}{\text{cm}^3\ \text{degr.}} \quad (40a)$$

(T_k - critical temperature). The temperature change of all introduced values, with the exception of q, is disregarded. Considerations of the temperature dependence $q(T) \sim 1 + v$ does away with the limitation $v \ll 1$ (in this paragraph it was not assumed that $\bar{u} \ll 1$), which offers the possibility of obtaining proper formulas (44)-(46).

In the dimensionless variables (r_1 = R, as in previous paragraph)

$$\frac{dv}{d\tau} = -\frac{\alpha(1+v)\eta\frac{d\eta}{d\tau} + \beta\int_\eta^1 \rho\left(\frac{\partial U}{\partial\rho}\right)^2 d\rho + \gamma v}{\mu + (1-\mu)\eta^2}, \quad v(\tau_0) = 0. \quad (41)$$

Here

$$\mu = \frac{C_n}{C_s}, \quad \alpha = \frac{T(dH_k/dT)^2}{2\pi C_s}, \quad \beta = -\frac{H_k dH_k/dT}{2\pi C_s}, \quad \gamma = \frac{8\pi\sigma RKg}{c^2 C_s} = 2x^2 Rg \quad (42)$$

dimensionless positive constants. At $T_k - T \ll T$ $\mu \sim 1$, $\alpha \sim 1$ ($\mu|_T = 1 - \frac{\alpha}{2}$; for tin $\alpha/T = T_k = 3.7° = 0.67$, $\alpha/_{T=2°} \approx 0.2$). The ratio β/α decreases from infinity at T = 0 to 0 at $T = T_k$; $\beta/\alpha \sim 1$ at $T_k - T \sim T$; $\beta/\alpha \simeq (T_k - T)/T$ at $T_k - T \ll T$.

In case of low supercriticalness $\delta U/\delta\rho = -\eta\dot{\eta}/\rho$ (see (32), (33) and the Joule heat is given by member $\beta\eta^2\dot{\eta}^2 \ln\eta$ of much higher order of smallness, than the heat of phase transition:

$$-\frac{Q_{\text{Дж}}}{Q_\Phi} = \frac{\beta}{\alpha}\frac{\eta\eta'\ln\eta}{1+v} = \frac{\beta}{\alpha}\frac{u - v + \lambda\eta'}{1+v}. \quad (43)$$

Nevertheless, by virtue of its irreversibility, the Joule heat, together with other thermal effects of second magnitude, at an ideal thermoinsulation of the specimen would have qualitatively changed the process of the phenomenon over a duration of a greater number of cycles. Consequently, disregarding later on these effects, we will (even when discussing an instance of adiabatic condition) assume heat transfer as sufficient for the lead out into the outer medium a small irreversible part of the liberated (absorbed) heat. In case where $u \sim 1$ (43) gives $-Q_J/Q_T \sim \beta/\chi$, so that formulas (44)-(46), derived with disregard of Joule heat, are valid only at $\beta/\alpha \ll 1$ i.e. $T_k - T \ll T_k$.

During thermal insulation of the sample ($\gamma = 0$)

$$\frac{dv}{d\eta} = -\frac{(1+v)\eta}{\mu + (1-\mu)\eta^2}, \quad v|_{\eta=1} = 0; \tag{44}$$

$$1 + v = [\mu + (1-\mu)\eta^2]^{-\frac{\alpha}{2(1-\mu)}}.$$

To totally disrupt superconductivity it is necessary, that the outer field should exceed H_k ($1 + v/\eta = 0$). Consequently, at an adiabatic condition a second critical value of the magnetic field appears.

$$H'_k = H_k \mu^{-\frac{\alpha}{2(1-\mu)}} = H_k \left(\frac{C_s}{C_n}\right)^{\overline{1 + H_k \frac{d^2 H_k}{dT^2} / \left(\frac{dH_k}{dT}\right)^2}}, \tag{45}$$

so that for $H_k < H_0 < H'_k$ the equilibrium appears to be a "mixed" state, when the specimen consists of superconductive core with a radius

$$\eta_\infty = \sqrt{\left[\left(\frac{H_k}{H_0}\right)^{\frac{2(1-\mu)}{\alpha}} - \mu\right] \Big/ (1-\mu)} \tag{46}$$

and normal outer layer.

In plane case ($1-\eta \ll 1$) $v \ll \alpha$, $u \ll \alpha$ and formulae (44), (46) acquire the form of

$$v = \alpha\xi; \tag{47}$$

$$\xi_\infty = u/\alpha. \tag{48}$$

Equation (41) is simplified considerably:

$$v' = \alpha\xi' - \gamma v, \quad v(\tau_0) = 0 \tag{49}$$

and is integrated in common form

$$v = \alpha \int_{\tau_0}^{\tau} e^{-\gamma(\tau-\vartheta)} \xi'(\vartheta)\, d\vartheta = \frac{\alpha}{\gamma}\left\{\xi' - \int_{\tau_0}^{\tau} e^{-\gamma(\tau-\vartheta)} \xi''(\vartheta)\, d\vartheta\right\} =$$

$$= \alpha\left\{\xi - \gamma \int_{\tau_0}^{\tau} e^{-\gamma(\tau-\vartheta)} \xi(\vartheta)\, d\vartheta\right\}. \tag{50}$$

Substituting in (49) $v = u - \xi\xi' - \lambda\xi'$ (9), we obtain an equation of motion of the boundary with consideration of thermal effects:

$$(u - \xi\xi' - \lambda\xi')' + \gamma(u - \xi\xi' - \lambda\xi') - \alpha\xi' = 0, \quad \xi'(\tau_0) = \xi(\tau_0) = 0. \tag{50a}$$

It permits the first integral

$$u - \xi\xi' - \lambda\xi' + \gamma\left(\int_{\tau_0}^{\tau} u\, d\tau - \frac{1}{2}\xi^2 - \lambda\xi\right) - \alpha\xi = 0, \quad \xi(\tau_0) = 0. \tag{51}$$

Unfortunately, at finite gamma values integration of (51) in quadratures is impossible. We will therefore confine ourselves to specific cases.

When gamma $\to \infty$ (51) gives

$$\int_{\tau_0}^{\tau} u\,d\tau - \frac{1}{2}\xi^2 - \lambda\xi = 0, \quad (51a)$$

which represents already a known isothermal approximation (12). Considering in (51) the alpha xi member, we obtain the following approximation:

$$\xi = \sqrt{\xi_{\text{эм}}^2 + (\lambda + \alpha/\gamma)^2} - (\lambda + \alpha/\gamma). \quad (52)$$

This equation is valid when u - xi xi' - lambda xi' $\ll$ alpha xi, i.e. xi' $\ll$ gamma xi, which is fulfilled within a period including almost the entire time of disrupting superconductivity (with exception of the initial stage) at

$$\gamma \gg \frac{2\sigma R^2\omega}{c^2\varphi}, \text{ или } \omega \ll \frac{c^2\varphi\gamma}{2\sigma R^2} \sim \Omega\left(\sqrt{\frac{\varepsilon}{\gamma}}R\right). \quad (53)$$

It is evident from (52), that the thermal effects are not essential during

$$\gamma \gg \frac{\alpha}{\lambda} \quad (54)$$

or gamma $\gg$ alpha/xi_{em}. The latter is realized practically in time at

$$\gamma \gg \frac{\sigma R\sqrt{a\omega}}{c\sqrt{\chi\varphi\varepsilon}}, \text{ или } \omega \ll \Omega\left(\frac{a}{\gamma}R\right). \quad (55)$$

In this way (53) and (54) or (53) and (55) present sufficiently valid conditions for isothermal description of the process as a whole on account of heat transfer into the outer medium. This criterion is obtained from requirement v $\ll$ u (see (50)).

The change in temperature can be small also on account of greater specific heat value, by which the absorbed heat is being distributed. Even at thermal insulation of the sample v $\ll$ u, if xi $\ll$ xiφ (see (47), (48). The latter is fulfilled within almost the entire period os superconductivity disruption (with exception of the final stage) at

$$\omega \gg \begin{cases} \Omega\left(\frac{\varepsilon}{a}R\right) & \left(\frac{\varepsilon}{a} \gtrsim \lambda\right), \\ \Omega\left(\sqrt{\frac{\varepsilon}{a}\lambda}R\right)^{1}\left(\frac{\varepsilon}{a} \lesssim \lambda\right). \end{cases} \quad (56)$$

The equation of motion of the boundary permits integration in general form, when in (9)· the member xi xi' is less negligible, i.e. retardation with Foucault currents is not substantial

$$u - \lambda\xi' + \gamma\left(\int_{\tau_0}^{\tau} u\,d\tau - \lambda\xi\right) - \alpha\xi = 0, \quad \xi(\tau_0) = 0;$$
$$\xi = \frac{1}{\lambda}\int_{\tau_0}^{\tau} e^{-(\alpha/\lambda+\gamma)(\tau-\vartheta)}\left[u(\vartheta) + \gamma\int_{\tau_0}^{\vartheta} u\,d\tau\right]d\vartheta. \tag{57}$$

Such a cumbersome equation is obtained only in case where lambda xi' $\sim$ v (consequently, xi $\ll$ lambda), which finds itself beyond the zone of applicability of the present theory. But if in (9) the basic role is assumed by one of the remaining members , (57) is simplified considerably, transforming at v $\ll$ lambda xi' (purely relaxation case) into equation (21), and at v $\gg$ lambda xi' (purely thermal case) into

$$\xi = \frac{1}{\alpha}\left(u + \gamma\int_{\tau_0}^{\tau} u\,d\tau\right). \tag{58}$$

Under an adiabatic condition (58) gives xi = xi-, i.e. the process represents a sequence of equilibrium states. The thickness of the normal layer rises, and reaches maximum value

$$s = \frac{\xi}{\alpha}R = \frac{2\pi C_0 R\xi}{T(dH_N/dT)^2}, \tag{59}$$

it decreases and turns into zero together with supercriticalness. In this case no difficulty is involved in considering the curvature [see (46)].

The criterion of the quasistationary process has the form of u' $\ll \alpha^2$ and u' $\ll \alpha$ u/lambda. The first one of these conditions is equivalent

$$\omega \ll \frac{c^2\alpha^2}{4\pi\sigma R^2\xi\sqrt{1+2/\varphi}}; \tag{60}$$

the second one is fulfilled within almost the entire time of superconductivity disruption (with exception of the initial and final stages) at

$$\omega \ll \frac{c^2\alpha}{4\pi\sigma R^2\lambda\sqrt{1+2/\varphi}}. \tag{61}$$

Since $4\pi\chi\varphi\sqrt{1+2/\varphi} \sim 2$ [(see (18)], conditions (60) and (61), as it was to be expected, are directly opposite (56).

For a periodic field with rectangular form of pulse it is possible to examine also the case of frequencies, intermediate between (56) and (60), (61). Integration of

1. These frequencies lie beyond the zone of applicability of the given theory.

equation (51) at gamma = 0, u = const gives for the maximum depth s = Rxi $(2\pi\varphi/\omega)$

$$\frac{s}{(l/2)R} + \left(1 + \frac{\lambda s}{l}\right) \ln\left[1 - \frac{s}{(l/2)R}\right] = -\frac{c^2 a^2 \varphi}{2\sigma R^2 l \omega}. \qquad (62)$$

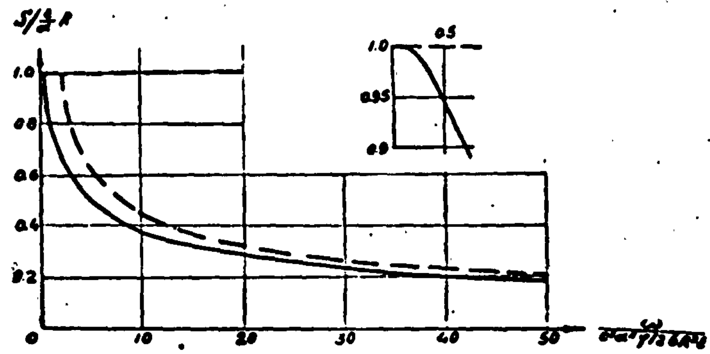

Fig.4

The frequency dependence s in inertialess case is presented graphically in fig.4. The dotted line indicates specific cases: isothermal (higher frequencies) and purely thermal (low frequencies).

Literature

1. I.M.Lifshits Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki, 20, 834, 1950

2. I.M.Lifshits, Doklady Akademii Nauk SSSR, 90, 363, 1953

3. I.M.Lifshits and M.I.Kaganov, Doklady Akademii Nauk SSSR, 90, 529, 1953

4. A.B.Pippard, Phil.Mag.41, 243, 1950

5. N.Ye.Alekseyevskiy, Doklady Akademii Nauk SSSR, 60, 37, 1948

6. A.A.Galkin; B.G.Lazarev; P.A.Bezuglyy, Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki, 20, 987, 1950

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