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AFCRL-63-18

Final Report
Contract No. AF19(604)-7316
January 1963

• *applied • mathematics* • •

Study of the Properties of Matter

Using Differential Geometric Methods

Electronics Material Sciences Laboratory
Electronics Research Directorate
Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
Bedford, Massachusetts

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PARKE MATHEMATICAL LABORATORIES, Inc.
Bedford Road ● Carlisle, Massachusetts

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ABSTRACT

Summary of work performed under the contract AF19(604)-7316.

The Research in Retrospect

The broad objective of the research was the study and correlation of the uses of geometry in theoretical physics and mathematical engineering. A preliminary literature survey, early in the contract brought to light an unexpectedly large number of papers and books reflecting the central role of the "geometric attitude" in modern physics and applied mathematics. As recently as July 1962 in SIAM Review, F. Joachim Weyl, in his retiring President's address: "The Perplexing Case of Geometry", emphasizes the importance of geometric pictures as the triggering ideas behind the scenery of contemporary mathematical research. In the field of theoretical physics there has just appeared "Geometrodynamics" by J. A. Wheeler, Academic Press, 1962. This is the first of a series of monographs, "Topics of Modern Physics" sponsored by the Italian Physical Society. In short, the far reaching importance and the naturalness of the geometric frame of mind has received more and more emphasis from many and unexpected sources during the period of the contract. We submit that geometrization and geometrical languages offer the best possibility of a common denominator for the collective use of ideas and results that should flow freely in research that crosses disciplinary barriers.

In 1960 a cursory survey of the materials brought to light in the preliminary literature search showed three recognizable uses of geometry which may be characterized as follows:

Geometry I .=. The use of space and space-time to describe physical configurations and events, e.g., Euclidean geometry and Minkowski space-time.

Geometry II .=. The equating of physics and geometry, e.g., Ricci-flat Riemannian geometry and classical gravitation, Rainich geometry - a purely geometric description of classical gravitation and electromagnetic radiation.

Geometry III   .—. Geometrization of the Analytical Formulas
of physics or engineering, e.g., the geo-
metrization of mechanics of Synge, the
stability of dynamical systems as a pro-
blem in the geometry of paths, the topology
of networks, diakoptics of Kron, etc.

The primary interest of this contract was Geometry III, outstandingly
illustrated in Synge: "Tensorial Methods in Dynamics", and in the two
volume RAAG MEMOIRS: "Unifying study of basic problems in engineering and
physical sciences by means of geometry", by a group of Japanese scientists
and engineers.

The first accomplishment was to explore the importance of Lagrangian
methods and Finsler space in the geometrization of a wide class of classi-
cal mechanical systems. This was prophetic because it emphasized the
importance of John L. Synge's recent article "Classical Dynamics" in
Volume III/1 of the Encyclopedia of Physics, S. Flugge, ed., Springer,
Berlin, 1960, especially of Section E, "General Dynamical Theory". In this
article, Synge has worked out in detail what were only hints in his earlier
published work. Our contribution was to first observe that Synge's ideas
provided a natural geometrization in terms of action of the dynamical
system that underlies a given quantum mechanical problem. The second
step was to observe that the Feynman-Dirac realization of the quantum mech-
anical Green's function as the integral of the functional exp(i x [action]/h)
provides a natural method of quantizing the underlying geometrical
manifold. The result is a new Poincare epistemological sum that adds up
to a geometrization of quantum mechanics. (cf. G. Rosen: "Poincare's
Epistemological Sum", Il Nuovo Cimento $\underline{16}$ 966 (1960)).

During 1961 and 1962 the research effort turned from a broad survey
and analysis of the natural use of geometrical methods in theoretical physics
to their specific application in several problems of special interest in the
electronic materials sciences e.g., zone melting purification, coherency in
optics and quantum theory, electron straggling and radiation damage to crystals

The application of SFP-geometrization of quantum mechanics to the theory of
Brillouin Zones remains for the future due to intervention of such more
immediate problems.

Geometrical methods enter coherency problems through such notion as
Stokes' parameters, the Poincare sphere, orthogonal polarizations. General-
ized harmonic analysis was brought to a culmination by Norbert Wiener in his
classic paper (Acta Math. 55 118-258 (1930)). It is the natural wedding of
the ideas of Fourier analysis and probability theory. In it he already
introduces coherency matrices, shown by Parke (M. I. T. Jour. Math. and Phys.
28 131-139 (1949)) to have elements that are linear combinations of Stokes'
parameters. A review of the present status of this topic has been given in
SR 11. It should be said parenthetically that generalized harmonic analysis
is the natural tool to handling the linear-quadratic duality between theoret-
ical quantities and observables e.g., the intensity of a light vector and
what a photoelectric cell measures in terms of energy. Not only time
correlation but space correlation becomes important for relativistic
phenomena. Here one is lead to correlation theory in space time manifolds
and Lorentz-invariance of the relevant formulations. This is an open
problem.

An accomplishment that resulted from a literature search was an
unexpected and extensive "Annotated Bibliography on Geometrical Methods in
Modern Physics". This has been widely distributed and will have the same
strong influence on the work of others it has had on ours.

The broad accomplishment spread over the several years of the contract
has been the application of the geometric viewpoint to many diverse
theoretical problems arising in the several electronic material sciences
groups. These problems range from electron straggling and radiation damage
theory to coherency problems in classical and quantum radiation theory,
circuit minimization and topology of networks. Some of this effort is
reported elsewhere by the groups involved.

These years of work, in close contact with many of the day to day
theoretical problems of the Electronic Materials Sciences Laboratory
leave us, as applied mathematicians, in no doubt of the usefulness, importanc

and communicability of the geometric attitude and its ability to reduce
many diverse problems and ideas to an intelluctually transportable
common denominator.

PUBLICATIONS

SR 1:   F. Erdogan : Evaluation of Feynman Path Integrals, September 1960.

After a technical description of the Feynman path integral formu-
lation of quantum mechanics the report treats several practical
methods of evaluating path integrals: (i) parameterization by
means of orthonormal functions, with examples, (ii) approxima-
tion by n-parameter family of curves. The determination of nor-
malizing factors is treated, with illustrative examples. Energy
eigenvalues and a quasi-classical approximation are treated in
terms of path integrals. A first crude step is taken toward a
geometrization of quantum mechanics in terms of path integrals
in action space.

SR 2:   N. G. Parke III : Differential Geometry and Lagrangian Methods in
Quantum Mechanics, October 1960.

The importance and place of geometry in quantum mechanics is
discussed in terms of the Poincare epistomological sums and
relevant work in the geometrization of classical physics, es-
pecially mechanics. The report goes on to single out Finsler
space as the natural space of configuration and time on which to
introduce Feynman quantization. A survey of the Lagrangians
associated by a Legendre transformation with the Hamiltonians
underlying quantum mechanical problems of practical and histori-
cal interest leads to questions as to the natural geometrical
representation of spin in the sense of Wheeler's quantum geo-
metrodynamics. The report closes with an exposition of the
elements of the differential geometry of Finsler space and an
illustration, the harmonic oscillator as a Finsler Space-Time
Manifold.

SR 3:   N. G. Parke III : Geometry in Theoretical Physics and Mathematical
Engineering, I, October 1961.

An analysis is made of the three uses of geometry in Physics, i.e.:
(1) as the framework of events and configurations, (2) equating
physics and geometry and (3) the geometrization of mathematical
formulas. This analysis is illustrated with a survey of the
work of Horvath, Kron, Kondo, etc., in the fields of physics and
engineering.

SR 4,5,6:   A. Kahan : An Annotated Bibliography on Geometrical Methods in
Modern Physics, 3 Vols., October 1961.

This annotated bibliography on the application of geometrical
methods in modern physics and engineering consists of 835
references, and is organized in three volumes.
Volume I covers: 1. General formulation. 2. Quantum mechanics
of particles.
Volume II covers: 3. Classical and quantum field dynamics.
4. Mathematical methods. 5. Relativity. 6. Elementary
particles. 7. Miscellaneous.
Volume III covers: 8. Solid State physics. 9. Electronic
and band structure. 10. Magnetism. 11. Superconductivity.

SR 7:   J. Hutchinson : Synge's Geometrization of Mechanics: a Precis,
October 1961.

Historically and practically, Synge's work on geometrization
crowns an effort that goes back to Lagrange (1736-1813) in
his general treatment of dynamical systems and to Riemann
(1826-1866) for the idea of multidimensional geometry. This
working precis of Synge's work is essential background for
application of geometrical thinking to modern physics.

SR 8:   F. Rowsome : Wave Geometry: Ideas of the Hiroshima School and
of H. T. Flint, September 1961.

This report brings together a sequence of study memoranda on
Wave Geometry; i.e., Ideas of the Hiroshima School and of
H. T. Flint. The results are summarized at the end with a
critique of these theories.

SR 9:   P. Scop : The Lorentz Transformation in Accelerating Coordinate
Systems - With Examples, October 1961.

This brief report discusses and brings together points which
clarify the applicability of the Lorentz transformation in
accelerating coordinate systems. It begins with a kinematical
technique for relating accelerations in non-inertial frames
to those in inertial frames. It outlines briefly the differ-
ences between special and general relativity, illustrating that
these differences are nothing to do with accelerating frames
of reference.

SR 10: **N. G. Parke III : Electron Straggling Theory and Calculation, August 1962.**

Some conceptual difficulties in the theory of straggling are discussed and Landau's 1934 result is compared with that obtained by the method of random energy walk over successive impacts. Landau's differential equation is rederived and basic assumptions stated.

SR 11: **F. Rowsome : Mathematical Descriptions of Polarization and Related Phenomena. Part I. Classical Description, August 1962.**

This paper constitutes a brief review of the mathematical descriptions of polarization which have grown out of experimental optics and classical electromagnetic radiation theory, as well as a review of the classical optical theories relating to polarization phenomena. Emphasis is placed on the interrelation of the formulations.

SR 12: **F. Rowsome : Mathematical Descriptions of Polarization and Related Phenomena. Part II. Quantum Mechanical Description, December 1962.**

This paper is a review of current theories of the quantum mechanical property of polarization of beams of electrons and photons, and of the algebraic techniques describing polarization phenomena. The subject is approached in the perspective of measurement theory. Particular emphasis is placed on the mathematical foundations, the interrelations and the conceptual structure of the theories.