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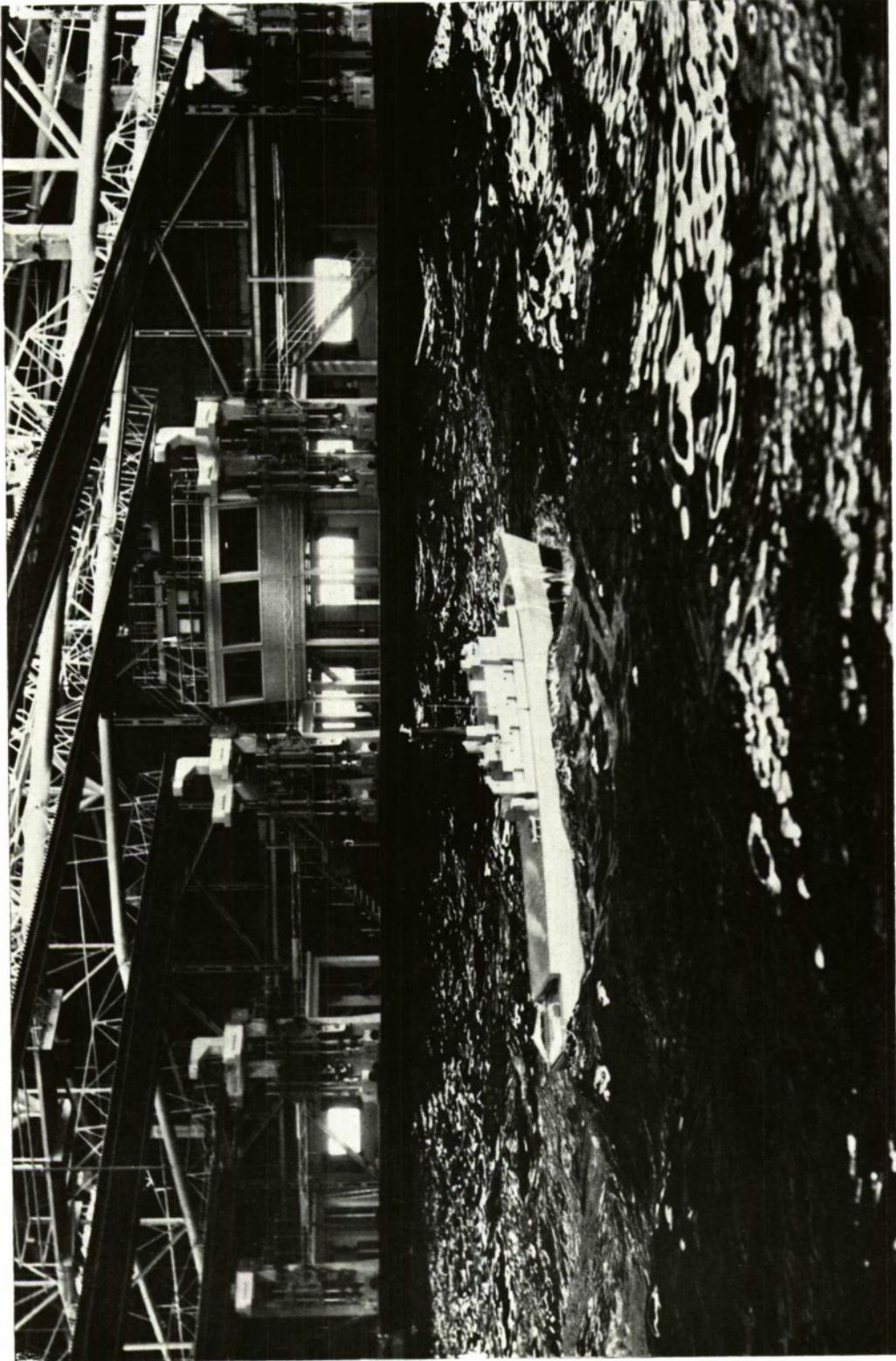


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#### THE MANOEUVRING TANK AT THE ADMIRALTY EXPERIMENT WORKS

The Manoeuvring Tank Waterway measures 400ft  $\times$  200ft with, at one end, a cantilever arm which rotates from an island, permitting circling experiments with models at any radius within the 95ft length of the arm. At the end remote from the rotating arm five wavemaker plungers span the 200ft width of the tank whilst five annular plungers adjoin these along one side to a distance of 200ft. Electronically controlled, hydraulically operated

pistons reciprocate each plunger separately at any chosen velocity to a maximum of 60 strokes per minute. These wamemakers can produce a maximum wave-height of 20in and can operate at varying degrees of disharmony to generate confused seas. The radio-controlled model in the foreground of the illustration is approximately 14ft. in length and is about 60ft from the end wamemakers.

# **SERVICES ELECTRONICS RESEARCH LABORATORY**

## ***In Its 21st Anniversary Year***

**M. S. Wills, B.Sc., M.A., A.Inst.P.**

### **Introduction**

This year Services Electronics Research Laboratory is celebrating its twenty-first anniversary, which seems an appropriate occasion to comment on its history and current work. The importance of electronics for defence in modern times can hardly be exaggerated. This was recognized long ago by the Admiralty, which explains why S.E.R.L. is a naval establishment although it has an inter-service responsibility. The laboratory was founded in 1945 to give permanent support to the already existing inter-service Co-ordination of Valve Development Organization (C.V.D.). It does this in three principal ways, by its internal research programme, by sponsoring C.V.D. contracts with industry and the universities, and by acting as the main source of people with the necessary specialized experience for staffing C.V.D. headquarters.

The laboratory was established by amalgamating several teams of scientists and engineers who had been working under the sponsorship of the Admiralty throughout the war. These teams were located in various branches of Admiralty Signal Establishment, as it then was, notably at Bristol University, and also at Bristol under Director of Naval Ordnance. Quite a number of the present staff of the laboratory are foundation members, having started in the wartime teams, and this has resulted in the existence of a valuable continuity of experience and tradition.

The special field of the laboratory has always been the so-called active electronic components. This term originally covered valves and gas discharge devices, but nowadays includes solid state devices also. A large variety of these now exists, the best known perhaps being the transistor. Much of the laboratory's effort is currently going into lasers, and the laboratory is the centre for government work on two of the main types of laser.

Two main principles have guided the development of the laboratory during the first twenty-one years of its existence. One was to maintain the continuity of the projects through the research, development and engineering phases and to collaborate closely with industrial firms in the stages leading up to production. Experimental production of a device is often undertaken in the course of its transfer to industry. This system of step-by-step R & D under one roof has obvious advantages and the laboratory is staffed and equipped appropriately to carry it through. The other was to build up a wide-ranging technological expertise so that work could proceed without the delays which tend to arise from relying entirely on other people to do special processes. The available techniques are continually being added to by the development of new methods. These are two important factors which over the years have facilitated rapid and efficient progress.

An outline account will now be given of some of the unclassified items of work going on recently in the laboratory. The main subjects are microwaves, neutron tubes, gas discharge lasers, ring lasers, semiconductor devices and semiconducting materials.

### Microwave electronics

A large proportion of the programme is concerned with high power travelling-wave amplifiers, the general objectives being to improve bandwidth, linearity and efficiency. In addition there is a growing activity on solid state oscillators and amplifiers.

#### Travelling-wave amplifiers

The Goonhilly Downs transmitter for the Telstar project used a high power tube which was specially developed for the purpose (Bryant 1962). This tube is illustrated in Fig. 1. At the Telstar transmission frequency of 6390 Mc/s the tube delivered a power

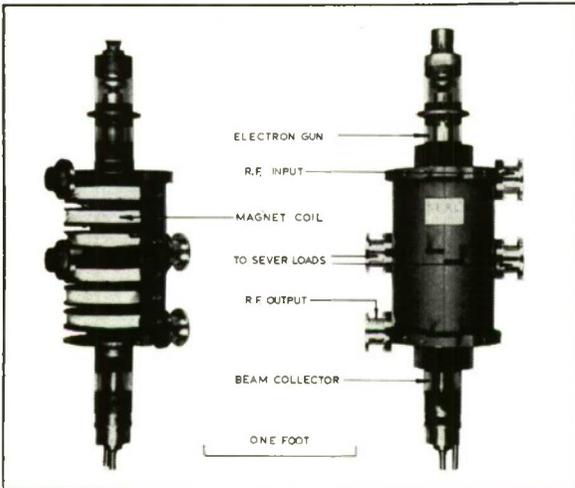


FIG. 1. Travelling-wave amplifier tube for 6275-6550 Mc/s.

output of 4 kw for an r.f. drive power of about 3 w. Present work is directed towards the design of a new tube to give approximately twice the output power.

Low power travelling-wave tubes use a simple helix for the slow-wave structure which gives very wide bandwidth and good coupling with the electron beam. However, for an output power of several kilowatts, thermal dissipation in the helix is a serious problem. Up to recently the only solution was to use instead of the helix a periodically-loaded waveguide structure and to accept a restricted bandwidth.

A new approach to the problem of heat dissipation in the helix has recently been tried, namely to support it in close contact with several rods of beryllium oxide, a newly developed ceramic which

combines high thermal conductivity with fairly low permittivity. The first experimental tube made in this way produced an output power of 1 kw at 10 Gc/s and had a bandwidth of 5 - 10 Gc/s (Wray and Dawsey 1965). This result encourages the hope that a solution has been found to the problem of designing high power travelling-wave tubes with large bandwidth.

#### Solid state amplifiers and oscillators

A junction diode biased in the backward direction has a nonlinear capacitance the value of which depends on the biasing voltage. This property can be made use of in several microwave circuits including parametric amplifiers. In the latter the varactor, as it is called, is excited with a large current at a high frequency  $\omega_p$ , so that the capacitance is varying at this frequency, the signal being at a lower frequency  $\omega_s$ . It turns out that if the circuit is resonant at the so-called idler frequency  $\omega_p - \omega_s$ , the varactor behaves towards the signal as though it had a negative resistance and therefore gives gain. There is considerable scope for ingenuity in the design of the microwave circuits for these amplifiers. One of the problems is to achieve low noise, and one way in which noise is reduced is by cooling the diode to liquid nitrogen temperature. The main effort is concentrated on improving the design of the diodes so that their parasitic resistance is made as low as possible. Both diffused p-n junction and Schottky barrier gallium arsenide diodes are being developed for this application. If this approach is successful it may be possible to relax the cooling requirements and hence simplify the equipment.

The recently discovered Gunn effect is a method of generating microwave oscillations simply by applying a suitable voltage to a crystal of a semiconductor such as gallium arsenide. If practical generators working on this principle can be developed they will be very suitable for use as local oscillators or as pumps for parametric amplifiers.

The ultimate goal of all this work is the designing of all-solid-state microwave receivers, which would have the obvious advantages of compactness and robustness.

### Neutron tubes

These are sealed-off tubes in which the nuclear reaction between deuterium and tritium is made use of to generate neutrons of energy 14 mev. Tubes of this sort are similar in size to x-ray tubes and may be operated by similar equipment and in similar environments, notably in industrial or clinical situations. Typically a tube contains an r.f. ion source from which ions are accelerated to a gas-loaded target. With the  $T(d,n)$  reaction only about 100 kv is needed for the accelerating voltage. The

gas pressure in the ion source and throughout the tube is maintained at 15 mtorr by a heated replenisher made of titanium.

In conventional accelerator-type neutron sources with differential pumping between the ion source and the high voltage regions, and with a tritiated target and deuterium in the ion source, the neutron output decreases rapidly owing to dilution of the target gas by deuterium. The useful life of a target may be less than one hour at outputs of the order of  $10^{10}$  neutrons per sec. In a sealed-off tube a 1 : 1 mixture of deuterium and tritium can be used in both ion source and target so that a continuous circulation of the mixture goes on with the proportions maintained constant in both regions. By this means a life of over 100 hours has been achieved at a level of  $10^{10}$  neutrons per sec. The tube referred to, shown diagrammatically in Fig. 2 (Bounden, Lomer and Wood 1965), is about 56 cm long and 6 cm in diameter. It has now reached the stage of commercial production and is finding application in reactivity measurements and activation analysis.

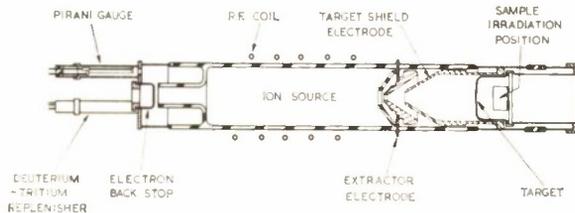


FIG. 2. Sealed-off neutron tube. Gas pressure:  $15\mu\text{m}$ ; ion source power: 500 w at 20 Mc/s; target voltage: 110 kv; total tube current: 1.5 mA; suppression voltage: 375 v; neutron output:  $10^{10}$  neutrons per sec; life: 100 hours.

The usefulness of a fast-neutron activation analysis system based on this tube has been demonstrated in two instances (Wood, Downton and Bakes 1966). The oxygen content of steel samples down to the 10 parts per million level has been determined within a time of one minute, using a fast pneumatic transfer system for the sample. It is hoped that this measurement will have a useful application in steel manufacture. Certain bulk constituents of minerals, namely oxygen, silicon, aluminium and iron, have been rapidly determined with an accuracy to 2%. This work was done in collaboration with the Warren Spring Laboratory of the Ministry of Technology.

The development of tubes with increased output is continuing. An output of  $10^{11}$  neutrons per sec. should be available soon and a further extension to  $10^{12}$  neutrons per sec. is foreseen. At this level the power supplied to the tube will be about 15 kw and the total neutron output power 1 w. With increased output levels more sensitive analysis will become possible and also the new field of neutron

radiotherapy will be opened up, as an alternative to x-ray therapy (Lomer and Greene 1963, Greene 1964). It is hoped that neutrons will have a substantial advantage in the treatment of oxygen-deficient tumours. In this work the laboratory is co-operating with Christie Hospital, Manchester, where a neutron tube system has been installed.

The experimental work on these tubes has required special facilities which have been built up at the laboratory, firstly to handle the tritium of which a tube contains 30 Ci, and secondly to permit running the tubes under observation behind shielding. In a standard installation, however, these tubes are lowered into a hole 12 ft. deep in the ground below the laboratory floor and this provides satisfactory shielding in a simple way.

A research which contributed a great deal towards the success of this programme was the work on rare earth targets which were pioneered in the laboratory (Redstone and Rowland 1964). The great advantage of these targets is that they have higher thermal stability than conventional targets made of titanium or zirconium (Large and Hill 1965a).

## Gas discharge lasers

### Lasers for holography

Holography is a fascinating subject which also shows promise of having unique applications. First proposed by Gabor nearly 20 years ago, it could not be developed until laser light sources were available. To make a hologram of a three-dimensional object, light from a laser is split at a semi-silvered mirror; the reflected component is directed towards the photographic plate forming the reference beam, and the transmitted component is used to illuminate the object. Light reflected from the object forms an interference pattern with the reference beam on the plate, which is then developed in the usual way. Such a hologram usually shows a gross pattern of spurious fringes due to diffraction of the coherent light by dust particles and to interference within the optical system. The true interference pattern is of microscopic scale and invisible to the naked eye.

To obtain a visual image from the hologram it is illuminated with laser light, when an image of the original object can be seen through it. Hologram images show parallax, exactly as if the original object were being examined; thus when viewed with both eyes the images display complete solidity. This is illustrated in Fig. 3, which shows two images given by the same hologram from different viewpoints.

It seems likely that for a long time to come laboratory conditions, such as extreme stability of the equipment, will be needed for three-dimensional work. Nevertheless there are possible applications where the conditions might be relaxed, for example



FIG. 3. Images given by the same hologram from different view-points.

pattern recognition in which the hologram of a specimen is compared with that of a standard. Slight dissimilarities show up immediately as interference patterns at the points where they exist. It has been suggested that this could be used for engineering inspection or for the recognition of fingerprints. Microscopy is another possible application which was the object of the original proposal by Gabor to circumvent the difficulties due to aberrations in electron microscopy. Magnification may be obtained with a hologram both by increasing the viewing distance and also by using a longer wavelength for viewing than was used to form the hologram in the first place.

The contribution of S.E.R.L. in this field is primarily the development of lasers suitable for use in holography. Emission in the blue or green is naturally preferable for photography. At present the argon ion laser is the best available source, but it has a very low efficiency, and there is considerable scope for work either to improve it or to find better sources.

*Molecular gas discharge lasers*

Molecular gases, those with more than one atom in the molecule, offer new possibilities for laser action because vibrational and rotational transitions are possible as well as the electronic transitions obtained with monatomic gases. Laser research on molecular gases was pioneered at S.E.R.L. by the investigation of nitrogen (Mathias and Parker 1963) and work on many other gases followed. In a joint project with N.P.L. and R.R.E., stimulated emission in the far infra-red out to 79  $\mu\text{m}$  was discovered, using water vapour (Crocker *et al.* 1964). About 150 lines are now known extending to 538  $\mu\text{m}$ . Fourteen new lines between 126 and 373  $\mu\text{m}$  were obtained with certain compounds containing hydrogen, carbon and nitrogen (Mathias, Crocker and Wills 1965). Several of the spectra are shown diagrammatically in Fig. 4.

The strongest stimulated emission line from water vapour was found to be at 27.9  $\mu\text{m}$  and it was realized that radiation of this wavelength would be very suitable for the investigation of discharge

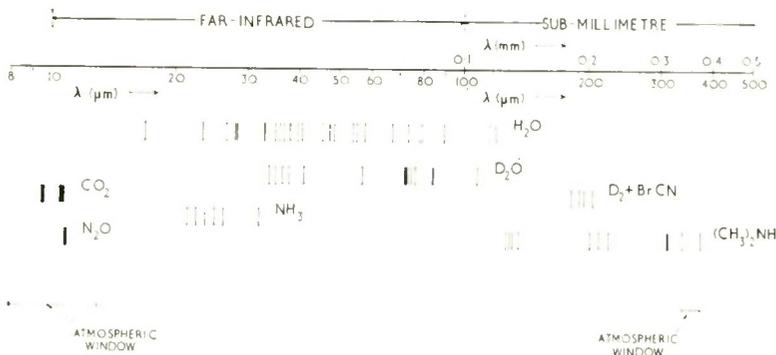


FIG. 4. Stimulated emission spectra observed from pulsed molecular gas lasers.

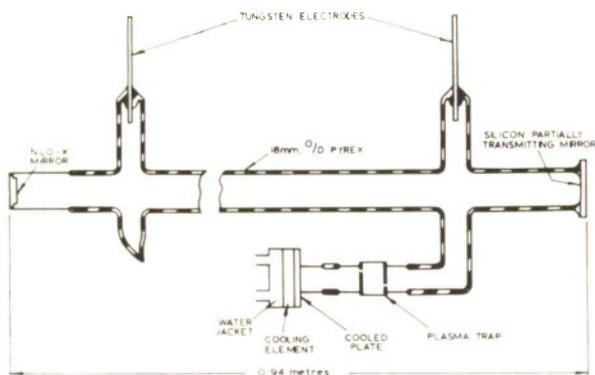


FIG. 5. Sealed-off water vapour laser.

plasmas, by the Faraday effect, of densities which are of interest from the point of view of thermonuclear power studies and magnetohydrodynamic generation. A fully engineered sealed-off water vapour laser has been developed with a life of over 300 hours (Large and Hill 1965b) which has been used for Faraday effect experiments at the Culham laboratory of U.K.A.E.A. An outline diagram of the laser tube is shown in Fig. 5. The long life is obtained by means of a water vapour replenisher which utilizes a Peltier cooling element. The correct pressure of vapour in the tube, about 1 torr, is obtained by adjusting the temperature of ice formed in the replenisher.

One of the latest developments which originated in France and the U.S.A. is a laser containing a mixture of carbon dioxide and nitrogen which operates at wavelengths in the region of  $10\ \mu\text{m}$ . The infra-red mean power output of the laser is in the order of 10 w with an efficiency of several per cent, which is remarkably high for a gas laser. The action of the laser is believed to take advantage of a resonance between the fundamental vibrational frequency of the nitrogen molecule and one of the fundamental modes of the  $\text{CO}_2$  molecule. Vibrationally excited nitrogen molecules formed in the discharge collide with unexcited carbon dioxide molecules and transfer their energy to produce carbon dioxide molecules in a single vibrational state. In this way inversions are produced in the low-lying vibrational levels of the carbon dioxide molecules. The laser emits a narrow beam of infra-red radiation which is strong enough to set fire to wood.

This development is of special interest partly because of the high efficiency of the device as a source of coherent radiation, and partly because the wavelength lies in a region of low atmospheric absorption. It seems likely that many applications will be found for the  $\text{N}_2 - \text{CO}_2$  laser where controllable intense local heating is required. In addition communications and radar-type applications are

being looked at. The development of molecular gas lasers has opened up a field of great interest to physicists and physical chemists, since it is now possible to generate strong radiation at wavelengths which are fairly closely spaced over the whole gap between the infra-red and microwave regions of the spectrum. Most of the transitions which are observed in stimulated emission are far too weak in spontaneous emission to have been observed before, so that a new field of molecular spectroscopy has been opened up. There is every reason to think that far-reaching applications will be found for these recent advances.

### Ring lasers

In 1925 Michelson and Gale in a classical experiment detected the rotation of the earth using a ring interferometer with a perimeter of one mile. Gas discharge lasers, with their extreme purity of optical oscillation, allow greatly increased sensitivity to be achieved, so that the ring can be made of manageable size; and, as suggested by Sperry, the device merits consideration as an alternative to the gyroscope for inertial navigation systems (King 1964).

In Fig. 6 is shown diagrammatically an early type of ring laser which was built at S.E.R.L. and demonstrated in action at the Physics Exhibition in 1964. The gas filling the discharge tubes is the usual mixture of helium and neon, the discharge is r.f. excited and the wavelength is  $6328\ \text{\AA}$ . Two independent oscillations propagate round the ring in opposite directions; if the ring has no absolute rotation the two oscillations are of equal frequency, determined primarily by the energy levels of neon and ultimately by the condition that the perimeter of the ring must equal a whole number of wavelengths, such that the wavelength lies within the Doppler-broadened line width. If the ring rotates

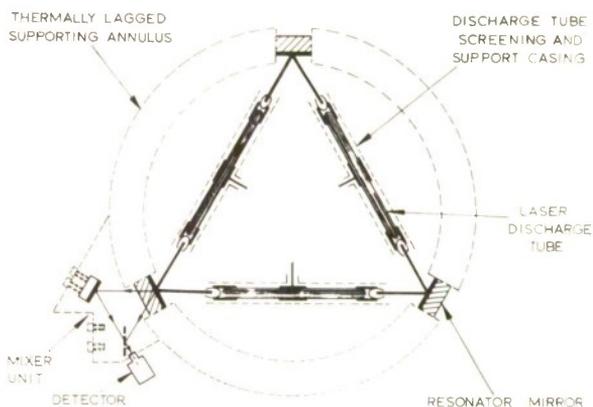


FIG. 6. An early type of ring laser.

about an axis normal to its plane, the two oscillations differ in frequency by an amount given to the first order by the expression  $\Delta f = 4A\omega/\lambda P$ , where  $A$  is the area of the ring,  $P$  its perimeter,  $\omega$  its angular velocity of rotation and  $\lambda$  the wavelength. This formula can be deduced by a simple classical argument, but an exact discussion of how ring lasers work would go too deep to be attempted here. The difference frequency is monitored and gives a measure of the rate of rotation; for instance in the laser illustrated the arms are of length 87 cm and a rotation rate of 1 rev  $h^{-1}$  gives a beat frequency of about 1.4 kc/s. If the number of cycles of the beat signal is counted a measure of angular rotation is obtained. One cycle of the beat frequency corresponds to a rotation of  $1.2 \times 10^{-6}$  rad. This result illustrates one possible application of ring lasers which is still being explored: the accurate measurement of angles in metrology.

The rotation of the earth should give, in the latitude of the U.K., a difference frequency of 46 c/s, but unfortunately, as usually happens with coupled oscillators, the two frequencies lock together below a certain beat frequency, in this case about 250 c/s. To measure rotations near or below the locking rate some form of bias must be introduced which displaces the beat frequency, for example a known superimposed rotation. Of course it is desirable to reduce the coupling as far as possible. It has been suggested that this coupling is due to light which originally belongs to one oscillation but is scattered so as to go round the ring in the opposite direction, each mirror or window contributing an increment of scattered light.

Ring lasers present many more problems than those touched on here, which is not surprising when it is remembered that differences of less than 250 c/s are being looked for between frequencies of  $5 \times 10^{14}$  c/s. Gyroscopes have many problems too and a great deal of development work has already been put into them. It is still too soon to decide whether a form of ring laser can make some useful contribution to navigation or metrology.

## Semiconductor devices

### Crystal lamps

Work is in progress on several different devices based on gallium arsenide. The parametric amplifier and the Gunn effect oscillator for microwaves have already been mentioned. In addition there is the injection laser which is itself a development from the 'crystal lamp.' The latter contains a p-n junction through which a current is passed in the forward direction. Electron-hole recombination results in the emission of photons whose energy depends on the height of the energy gap between the valence and conduction bands. In GaAs this is

1.38 eV and the emission is in the near infra-red at 0.9  $\mu\text{m}$ . It so happens that this wavelength can be detected very efficiently with a silicon p-i-n photodiode, a very fast acting detector. This combination of source and detector has been used for the purpose of reading punched tape. This equipment is more compact and robust than a tungsten filament lamp and photocell, and troubles due to filament displacement are eliminated. In addition, the discrimination ratio between light and dark signals is increased from something like 7 : 1 to 500 : 1. This is not because the paper is more opaque to infra-red radiation but because it works by scattering. With the much smaller solid state lamp a better optical and geometrical arrangement can be devised so that much less of the scattered light falls on the detector (Broom and Hilsum 1963). The convenient combination of gallium arsenide lamp and silicon photodiode will probably have other applications in systems which include light beams as linkages. Such links have a rapid response and provide electrical isolation. They are capable of giving added versatility to the design of electronic systems and such systems are coming to be called 'opto-electronic.'

To obtain visible light from a crystal lamp a wider energy gap is needed, and gallium phosphide is suitable (Allen *et al.* 1963). In this semiconductor the gap is 2.24 eV but direct band-to-band recombination is very improbable, and the colour of the emitted light depends on what impurity levels are present. By suitable doping both red and green lamps have been made. These are very small, 0.75 mm in diameter, rugged, reliable and fast acting, and have been used for high speed digital recording on photographic film. The efficiency of such lamps is only about 0.1%, and they would be much more attractive for use as indicator lamps or in small displays if the efficiency could be raised to, say, 1%. The aim of the research is to do this by improving the purity of the basic material as described later on.

### Injection lasers

If a gallium arsenide lamp is made with suitably disposed reflecting surfaces, as illustrated in Fig. 7, laser emission can be obtained. Typically, the laser crystal is a 0.4 mm cube, driven with a pulse of a few hundred amps peak, and gives a light output of 50 w peak, 1 w mean, with an efficiency of 30%. The lasers are usually operated at 77° K, but with sufficiently short pulses they have been operated at room temperature (Broom 1965). One of the aims of the programme is to attain higher efficiency at room temperature so as to simplify the operating conditions.

A gallium arsenide laser operated at room temperature has been made the basis of a range-finding

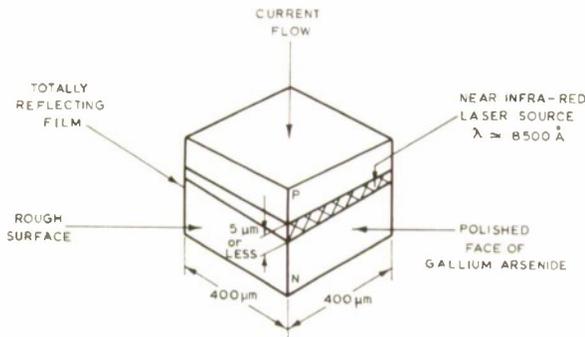


FIG. 7. Gallium arsenide injection laser.

system (Birbeck and Hambleton 1965). The combined transmitter and receiver is shown diagrammatically in Fig. 8. A range-finder working at optical frequencies has certain advantages over a microwave system. The much shorter wavelengths enable small beamwidths to be obtained with a compact transmitter, and this makes it easy to select and range on specific objects, without spurious echoes from the surroundings. Aircraft radio-altimeters are affected by spurious signals over certain types of terrain and it was thought that an optical range-finder might be able to resolve some of the anomalies. It must be emphasized that an optical system can only operate in good visibility and so could not be relied upon as an altimeter in itself, but it might be used to calibrate other types of altimeter.

The instrument was found to be capable of measuring heights up to 1000 feet to within about five feet over a wide variety of terrain. Echoes were obtained from the tops of trees as well as from the ground. This is an example of the way in which an optical altimeter can emphasize features which may not be resolved or even detected by a radio altimeter.

### Semiconducting materials

Closely linked with the work on devices described above is a programme for the preparation of new and improved materials. A new method (Effer 1965) is being applied to the preparation of gallium arsenide, namely vapour phase epitaxial deposition. This method employs the reaction between arsenic trichloride and gallium in a stream of hydrogen at 900°C, where arsenic and gallium chloride are generated. These constituents are volatile and are transported by the hydrogen stream to a cooler part of the system (700-800°C) where gallium arsenide is deposited on a less pure GaAs single crystal substrate. The resulting material has a purity two orders of magnitude higher than material made previously by growth

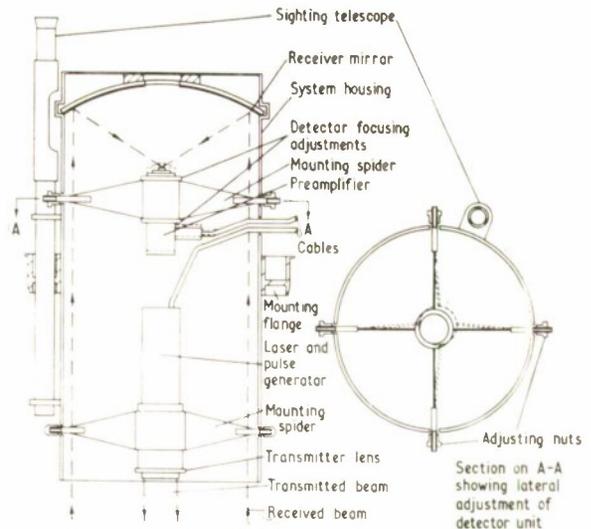


FIG. 8. Infra-red laser rangefinder.

from the melt at a higher temperature (1235°C). By the inclusion of suitable doping elements in the gallium, n-type epitaxial layers have been produced with controlled carrier concentrations from  $10^{14}$  to  $10^{19}$  carriers per  $\text{cm}^3$ . The epitaxial technique is in process of being applied to gallium phosphide in an attack on the problem of making more efficient crystal lamps for the visible spectrum.

With the aim of generating light of still shorter wavelength than can be obtained with gallium phosphide work has been started on zinc selenide, which has an energy gap of 2.66 eV. So far it has been shown that this substance can be made in single crystal form as a highly efficient photoluminescent material, and it is almost certain that it would have high efficiency as an injection luminescent material. Attempts to make low resistance p-type material and hence good p-n junctions have so far been unsuccessful, but it is possible that this can be achieved by a new technique called ion implantation which will now be described.

### Ion implantation in semiconductors

Transistors and many other devices, several of which have been mentioned already, depend for their action on p-n junctions formed in a semiconducting crystal. The doping of semiconductors with impurities is normally done by diffusion into the crystal at high temperatures, but a new method on which work has only recently started offers several advantages. Ions of the required impurity are accelerated in an electric field and driven into the surface of the semiconductor. The particles come to rest in the body of the material at a fairly definite distance from the surface. It is true that

they damage the crystal structure to some extent but subsequent annealing at temperatures around 500°C restores the structure sufficiently for devices to work satisfactorily.

It is possible by careful alignment to inject the high velocity ions down preferred directions in a crystal so that very little energy is lost by collisions. The resulting penetration is deep and the damage to the crystal very limited.

The advantages of the new method are: firstly, a high degree of control can be maintained both on the ionic penetration and also on the lateral extent and position of the ion beam. This may have important applications in micro-electronics—the building up of whole circuits of active and passive components on a single piece of material. Secondly, it is a low temperature process compared with diffusion, during which imperfections and unwanted impurities can be introduced into the crystal. Lastly, it is possible to introduce impurities which cannot be put in by diffusion or other methods.

This picture of the current work of the laboratory is necessarily incomplete but perhaps enough has been said to indicate that useful contributions are being made in a number of diverse fields.

## References

- J. W. Allen, M. E. Moncaster, and J. Starkiewicz, *Solid St. Electron.*, **6** (1963), 95.  
 F. E. Birbeck, and K. G. Hambleton, *J. Sci. Instrum.*, **42** (1965), 541.  
 J. E. Bounden, P. D. Lomer, and J. D. L. H. Wood, *Nucl. Instrum. Meth.*, **33** (1965), 283.  
 R. F. J. Broom, *J. Sci. Instrum.*, **42** (1965), 123.  
 R. F. Broom, and C. Hilsun, *Electronics*, (May 1963), 44.  
 M. O. Bryant, *Brit. Commun. Electron.*, **9** (1962), 590.  
 A. Crocker, H. A. Gebbie, M. F. Kimmitt, and L. E. S. Mathias, *Nature, Lond.*, **201** (1964), 250.  
 D. Effer, *J. Electrochem. Soc.*, **112** (1965), 1020.  
 D. Greene, *Nature, Lond.*, **202** (1964), 204.  
 P. G. R. King, *J. Inst. Navigation*, **17** (1964), 289.  
 L. N. Large, and H. Hill, *Nucl. Instrum. Meth.*, **34** (1965a), 100.  
 L. N. Large, and H. Hill, *Appl. Optics*, **4** (1965b), 625.  
 P. D. Lomer, and D. Greene, *Nature, Lond.*, **198** (1963), 200.  
 L. E. S. Mathias, and J. Parker, *Appl. Phys. Letters*, **3** (1963), 16.  
 L. E. S. Mathias, A. Crocker, and M. S. Wills, *Electronics Letters*, **1** (1965), 45.  
 R. Redstone, and M. C. Rowland, *Nature, Lond.*, **201** (1964), 1115.  
 J. D. L. H. Wood, D. W. Downton, and J. M. Bakes, 1966, *Proe. Int. Conf. on Modern Trends in Activation Analysis, Texas* (1965), McGraw-Hill, New York.  
 F. Wray, and J. R. Dawsey, *Electronics Letters*, **1** (1965), 226.



# COMPATIBILITY OF PAINTS with CATHODIC PROTECTION

A. F. Routley

Central Dockyard Laboratory

## SUMMARY

*The factors affecting a fully integrated anticorrosive system of cathodic protection and paint are discussed and the means by which paints can be assessed are described. The particular functions of a paint in a cathodically protected environment and the special forms of attack encountered are discussed in relation to the test procedures used and the application of the results to practical conditions.*

## Introduction

The advent of cathodic protection has given a new dimension to the technology of combating corrosion. Its application particularly to buried structures has contributed considerably to the economics of maintenance engineering and made it possible to extend the useful life of many components subjected to corrosive environments. However, its technique cannot be used without a detailed study of the particular problem to which it is to be applied.

Cathodic protection is a means of resisting corrosion which does not necessarily need any additional aid to function correctly, but in considering its use on ships other factors have to be taken into account. For example, bare steel requires about 20mA/sq. ft. for protection in still sea water which is equivalent to 200 amps for a medium size ship. In moving water or when the ship is steaming this can increase tenfold<sup>1</sup>. Obviously it is not a practical requirement for a ship to have to supply in excess of 2,000 amps for an impressed current system, or be equipped with zinc anodes to the extent of at least 2,000 sq. ft. surface area of zinc. The figures quoted are a maximum requirement, as the chalking that occurs would greatly reduce the current needed.

Painting is a convenient way of reducing the current demand required, achieving a more even

spread of current and at the same time providing protection of the hull when a cathodic protection system is inoperative, such as during refit periods in dock. As it is desirable to paint the ships outer-bottom because of antifouling requirements, the optimum protection must be given by a properly integrated system of cathodic protection and paint.

Not all underwater paints are suitable for use with a cathodic protection, because at the cathode, alkali is produced at a rate proportional to the amount of current used. Paints which are not alkali resistant are easily attacked and degenerate quickly giving rise to serious softening, blistering and detachment. Included in this category would be materials based on varnishes containing esters and acidic components such as linseed oil, alkyd resins, etc. Similarly alkali sensitive pigments would contribute to paint breakdown. Theoretically, alkali resistance is a purely relative term as no criteria for establishing it have been formulated. Current usage of paints has been based on the expected complete imperviousness to strong alkaline conditions and this has worked well in practice. However, it is not economic to continually use materials which have a high standard of chemical resistance, which is consequently expensive when a lower order of stability would have been sufficient. Other factors such as the type of cathodic protection system used and operating conditions of the ship can also influence the choice of paint system.

### Assessing Suitability of Paint Systems

Before deciding on the form of testing procedure to be adopted in the laboratory it is necessary to be quite clear on the service requirements of the cathodic protection system and paint coatings to be employed.

In the Royal Navy there are at present three different applications of cathodic protection to its ships.

1. All new construction shipping is cathodically protected during the fitting out period by suspending anodes over the ship's side, usually on booms to keep the anodes at about six feet away from the hull.
2. Most cathodically protected ships in service are fitted with an impressed current system, generally automatically compensated, using lead alloy or other suitable anodes.
3. A few ships are protected by zinc anodes fixed directly to the hull.

Each of these procedures require analysing before experimental work can be decided upon.

A common factor with these techniques is the level of protection given by the system. Normally the design requirements are that the ships' hull should be held at a potential of  $-0.8$  volts with reference to a silver/silver chloride reference electrode, but in each of the three cases the method of achieving this varies.

With new construction the paint systems will be new and the few anodes required comparatively remote from the hull. As the potential/distance relationship is as shown in Fig. 1 it is clear that the potential variation over the surface of the hull will be very small, so that the correct number of anodes of the right surface area equidistant around the hull will give a fairly constant potential in the range of  $-0.75$ v to  $-0.8$ v. Also as the paint coating is new it will not be fully water soaked, therefore the current demand will be low. Consequently any paint system will not be subjected anywhere to excessive alkaline conditions.

In the case of impressed current systems, the current per sq. ft. of anode surface area is capable of being much greater, and coupled with the cost per anode these features produce a design composed of a few anodes spaced around the hull. In practice, the anode is mounted on a shield the size of which is designed to protect the paint systems from potentials in excess of about  $-1.0$ v, but this still allows a large range of potential values to be formed on the hull.

Where ships are protected by zinc anodes coupled directly to the outer-bottom similar circumstances exist, but as the maximum output

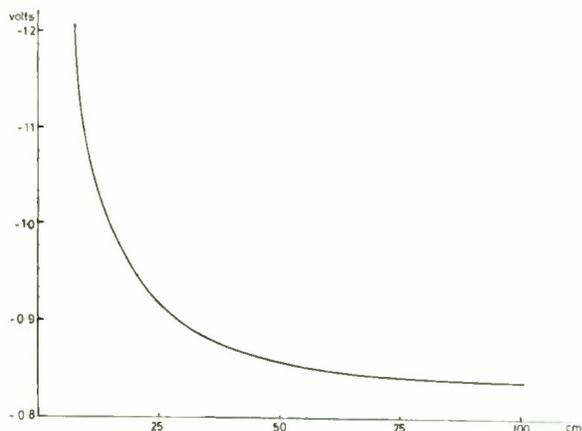


FIG. 1. Change in potential of cathode with increasing distance from anode (assuming output of 1 amp giving  $-0.75$ v. at infinity).

of individual anodes is low by comparison, and the maximum potential obtainable adjacent to a zinc anode is  $-1.05$ v, the severity of the conditions are less. In practice only small shields are needed around the anodes to give the necessary protection against the higher potentials in excess of  $-0.9$ v.

When deciding on laboratory techniques for testing it can be seen that primarily only two separate conditions of usage exist. These are subjected to a potential of up to say  $-0.85$  volts and potentials up to  $-1.0$  volt. That these conditions should be separated like this is important as paints found unsuitable for the higher potentials may well be satisfactory for the less arduous conditions and should not be dismissed out of hand as being incompatible with cathodic protection. (The third condition above can be taken as being a combination of the other two, particularly if it is acknowledged that shielding for zinc anodes can be given by a suitable paint).

### Testing Procedures

The method found most suitable for assessing paints under cathodic protection conditions consists basically of applying cathodic protection by means of a platinum wire anode and an external source of low voltage D.C. to paint systems applied to 6 in.  $\times$  4 in.  $\times$   $\frac{1}{8}$  in. gritblasted mild steel panels. The coated panels are placed around the periphery of a glass tank with the test surfaces facing inwards towards the centrally placed anode. The tank is filled with natural sea water which is continually replaced at a rate equal to a complete change of sea water every 24 hours. A schematic diagram of the apparatus is shown in Fig. 2. The level of protection is maintained by a potentiostat at the value required, *i.e.*  $-0.85$  volts or  $-1.0$  volts.

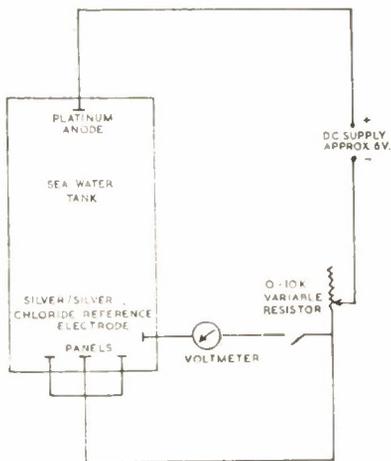


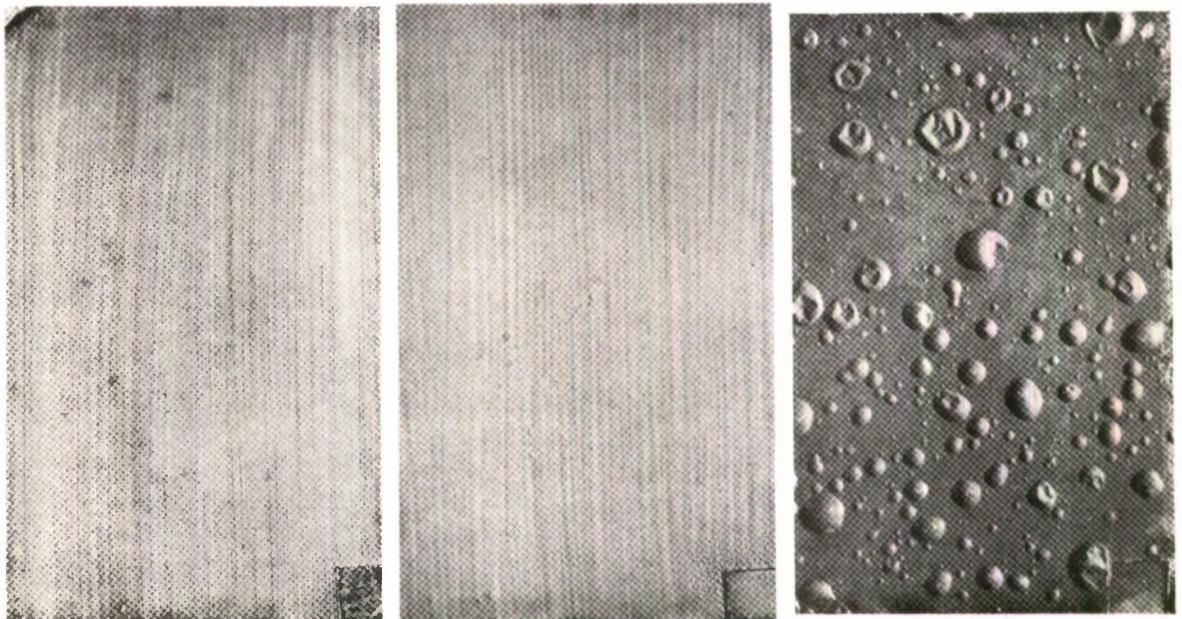
FIG. 2. Circuit diagram for cathodic protection apparatus.

To introduce a factor of acceleration to the method a 'holiday' or bare area of steel equivalent to 0.5% of the surface area is created on the painted panel adjacent to one corner. This has the effect of producing a part of the panel having a higher current density without altering the potential of the whole. The paint adjacent to the bare area is subjected to a greater rate of alkaline formation and so gives a measure of its tolerance to a slight

increase in the severity of the conditions. In practice where mechanical damage to the film occurs similar conditions will be created. Fig. 3 shows three examples of paint systems assessed by this method, which are rated as very good, satisfactory and unsuitable.

Using this method it has been found desirable to reduce the length of time required for full assessment by reducing the number of coats in the system to be tested. For example the normal system of a coal tar epoxy paint on a ship would employ three coats and this would be expected to last for two years. In the laboratory it is convenient to use two coats and test for six months, as any incompatibility is revealed fairly quickly.

An ancillary test which has given good indications of the likely performance of a paint film is one based on the immersion of the paint coating in an alkaline solution. Measurement made on the water extracts of blisters formed during cathodic protection experiments have shown that alkali strengths of pH 12 are commonly formed. This has been confirmed with experiments conducted on the permeability rates of various coatings under cathodic protection using the apparatus diagrammatically shown in Fig. 3. The pH value of the solution contained within the coated porous pot changes from seven to about 12.5 fairly rapidly and then remains almost constant over a long period



(a) No breakdown

(b) Fine blistering only around bare area

(c) Considerable blistering overall

FIG. 3. Examples of the effect of cathodic protection on paint systems.

TABLE I. Intercoat adhesion/cohesion values of paints in differing environments.

Paint system	Dry new coating	After immersion in seawater six months	After cathodic protection six months (-0.85v)
Coal Tar Epoxy A	1750 lbs. sq. in.	870 lbs. sq. in.	830 lbs. sq. in.
Coal Tar Epoxy B	2700 lbs. sq. in.	970 lbs. sq. in.	1000 lbs. sq. in.
DEF. 1441	2450 lbs. sq. in.	750 lbs. sq. in.	600 lbs. sq. in.
Oleoresinous primer	2350 lbs. sq. in.	730 lbs. sq. in.	100 lbs. sq. in.

of time. Assuming that the main cause of degradation of the paint system is due to alkali attack, it is reasonable to use a simple immersion test in alkali at pH 12 ( $N/100$ ) as a preliminary sorting method. Using this method an immersion period of eight weeks is employed, after which the coating can be examined for softening, blistering and other forms of attack. It is not fair to expect too much from this test, although generally agreement with cathodic protection tests is acceptable.

Other methods of test can be of great assistance in deciding on the suitability of a paint. The direct pull-off method for measuring adhesion<sup>2</sup> has been successfully used to determine the extent of breakdown within a paint system. Adhesion/cohesion measurements are made on coatings after subjection to cathodic protection and compared with results from the same system which has been immersed in sea water for the same period without cathodic protection. Examples of such comparison are given in Table I and from this it is clear that the oleoresinous primer is unsuited for cathodically protected ships.

From the results obtained using these methods it has been established that certain coatings previously considered unsuitable for cathodically protected ships have been quite satisfactory under test at -0.85 volts. Such paints include DEF. 1441 (A.C.C. 655) which is based on a linseed/tung oil phenolic varnish, aluminium/bituminised oil paints and epoxy ester metallic pigmented materials. It is interesting to note that zinc and aluminium pigmented paints are not usually attacked by the alkaline conditions produced. This is probably due to the electrical conducting nature of these materials which when applied direct to steel become part of the cathode so that alkali is produced at the surface of the paint and not at the steel/paint interface. This means that the alkali is immediately diluted with sea water and not forced to permeate through the paint film as is the normal case.

As it is the normal practice with new construction to shop prime all steel, a series of experiments have been conducted to assess the prefabrication primers' suitability for cathodically protected ships, on their own and when overcoated with the normal

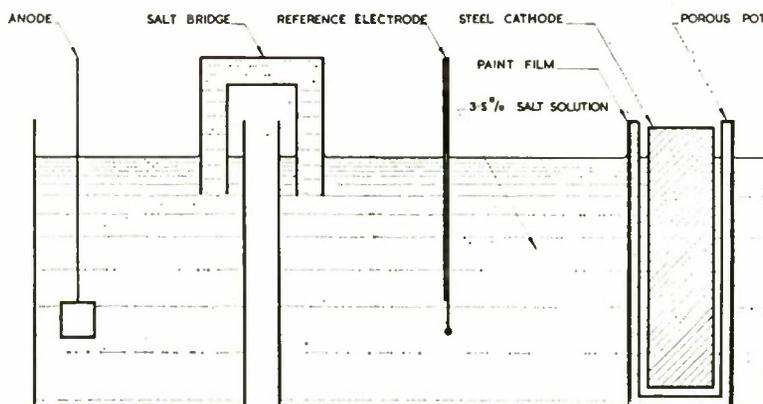


FIG. 4. Apparatus for experiments with paint films under conditions of cathodic protection.

outerbottom system. Generally these prefabrication primers are metallic pigmented two pack epoxy coatings although other materials such as etching primers are also used.

It has been noticed that if breakdown occurs with these systems it is usually due to blistering of the outerbottom paint from the prefabrication primer, particularly if the latter is a metallic pigmented one. This observation is further evidence to support the idea that the alkali is formed at the surface of the conducting prefabrication primer causing breakdown of the system applied to it. By contrast, the same paint systems tested by immersion in sea water without cathodic protection either show no blistering or blister at the prefabrication primer/steel interface.

The electrical conductivity of the coating when immersed in sea water obviously plays a part in the mechanism associated with life of the coating. Such measurements that have been made have not as yet established any relationship between breakdown and conductivity, although it seems that a rapid change in values is often connected with the degradation of the coating.

#### Application of Results

The main concern with the conduct of these tests is the general requirement of compatibility with cathodic protection systems, and as such is readily established. However, such decisions would only allow the very best of coatings to be accepted, perhaps only one or two, whereas there are many paint systems which although not resistant to conditions at  $-1.0$  volt are quite satisfactory at lower potentials, in the range of  $-0.8$  to  $-0.85$  volt. For impressed current systems whether automatically controlled or not, conditions are such that potentials in the region of  $-1.0$  volt will be obtained, particularly close to the anode, and only the few coatings found satisfactory by laboratory test methods can be used. When cathodic protection is being used for fitting out only or is being applied

by zinc anodes it is good economics to use coatings which are acceptable under these conditions of lower potentials.

In practice, ships fitted with impressed current systems are coated with coal tar epoxy paints and those with zinc anodes are painted with DEF. 1441 material as are those ships which will only be cathodically protected during building. Where zinc anodes are fitted, shields of coal tar epoxy paint are used to protect the DEF. 1441 paint from the higher potentials immediately adjacent to the anode.

#### Conclusions

Alkaline attack appears to be the main cause of breakdown of paint systems used in conjunction with cathodic protection, and tests based on this assumption give a reasonable prediction of performance. Other factors such as induced permeability caused by the passage of current and rate of change of conductivity under immersion conditions obviously play a part but as yet are undefined. It is thought that such empirical tests as those described form the best procedure for assessing paints, though more research is required into the fundamental forms of breakdown that occur in coatings during applied cathodic protection.

Research with improved designs of impressed current systems to provide a more even distribution of current is continuing and if successful would also help the paint problem, as most failures in practise with approved coatings have been associated with very high potentials in the region of anodes.

A future possibility in the paint field would be to use electrically conducting coatings which would then minimise the problem of providing coatings which are resistant to alkali.

#### References

- (1) L. Sawyer, A. Routley, D. Chapman and J. Crennell, *Journal of Applied Chemistry*, 15 (1965), 182.
- (2) *J.O.C.C.A.*, 46 (1963), 4.



# TRANSPORTATION TECHNIQUE IN LINEAR FRACTIONAL FUNCTIONALS PROGRAMMING

**Kanti Swarup**

*Faculty of Mathematics, University of Delhi*

**ABSTRACT**

*A transportation technique for linear fractional functionals programming problem is proposed, viz:*

Minimize

$$Z = \frac{\sum_{i=1}^m \sum_{j=1}^n C_{ij} x_{ij}}{\sum_{i=1}^m \sum_{j=1}^n d_{ij} x_{ij}}$$

subject to

$$\sum_{j=1}^n x_{ij} = a_i \quad i=1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j \quad j=1, 2, \dots, n$$

$$x_{ij} \geq 0$$

*Conditions for optimality are derived.*

**Introduction**

This paper is devoted to the special structure of linear fractional functionals programming problems. These special problems are important from a theoretical point of view. Theoretical importance of problems stems from the fact that the computational procedure that has been developed for their solution is an example of the simplifications that result, if advantage can be taken of structure within the problem matrix. The paper is the outcome of the main result of the linear fractional programming viz, absolute optimum occurs at a basic feasible solution.

The mathematical formulation of the problem considered here, is particularly simple and can be given as:—

Minimize

$$Z = \frac{\sum_{i=1}^m \sum_{j=1}^n C_{ij} x_{ij}}{\sum_{i=1}^m \sum_{j=1}^n d_{ij} x_{ij}}$$

subject to

$$\sum_{j=1}^n x_{ij} = a_i \quad i=1, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j \quad j=1, \dots, n$$

$$x_{ij} \geq 0$$

(A)

In view of the very special nature of the problem, it is possible to modify available methods<sup>(1, 5)</sup> for this particular problem.

The paper is divided into four sections. Section I consists of preliminaries. Simplex multipliers are determined in Section II. Optimality conditions are obtained in Section III. Section IV contains a numerical example in support of the theory.

**SECTION I**

**Preliminaries**

(i) 
$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

this is a consistency condition for the existence of the solution to the problem.

(ii) Out of  $m+n$  equations in (A), one equation is redundant, hence any basis will involve not  $m+n$  variables but only  $m+n-1$ <sup>(6)</sup>.

(iii) The special structure of (A) allows to represent the system by the array as shown in Fig. 1. Each of the boxes corresponds to a variable.

	$a_1 \downarrow$			
	$x_{11}$	$x_{12}$	$x_{1n}$	$a_1$
	$d_{11} \quad C_{11}$	$d_{12} \quad C_{12}$	$d_{1n} \quad C_{1n}$	
	$x_{21}$	$x_{22}$	$x_{2n}$	$a_2$
	$d_{21} \quad C_{21}$	$d_{22} \quad C_{22}$	$d_{2n} \quad C_{2n}$	
	$x_{m1}$	$x_{m2}$	$x_{mn}$	$a_m$
	$d_{m1} \quad C_{m1}$	$d_{m2} \quad C_{m2}$	$d_{mn} \quad C_{mn}$	
$b_j \rightarrow$	$b_1$	$b_2$	$b_n$	TOTAL
				$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

Fig. 1.

(iv) Initial feasible solution can be obtained with the help of well-known North-West corner rule or with other procedure which are generally used for obtaining initial feasible solution to transportation problem in linear programming, and does not depend on unit cost.

(v) All basis for the problem are triangular<sup>(6)</sup>.

(vi) Set of feasible solutions is regular.

**SECTION II**

We determine  $u_i^1, u_i^2$  ( $i=1, 2, \dots, m$ ) and  $v_j^1, v_j^2$  ( $j=1, \dots, n$ ) called simplex multipliers from the equations:

(1)  $C_{ij} + u_i^1 + v_j^1 = 0$   $i, j$  takes suffixes of basic variable.

(2)  $d_{ij} + u_i^2 + v_j^2 = 0$   $i, j$  takes suffixes of basic variable.

also let

(3)  $C_{ij}^1 = C_{ij} + u_i^1 + v_j^1$  ( $i=1, \dots, m, j=1, \dots, n$ )

(4)  $d_{ij}^1 = d_{ij} + u_i^2 + v_j^2$  ( $i=1, \dots, m, j=1, \dots, n$ )

Now one equation of (A) is redundant. As the redundant equation really does not exist, we can arbitrarily assign one of the  $u_i^1$  or  $v_j^1$  equal to zero in (1) and one of the  $u_i^2$  or  $v_j^2$  equal to zero in (2) and solve for the other multipliers. These multipliers can be determined uniquely because the system (1) and (2) each consists  $m+n-1$

independent equations. Having determined  $u_i$  and  $v_j$  we can easily compute  $C_{ij}^1$  and  $d_{ij}^1$  for the non-basic variables from (3) and (4) respectively.

**SECTION III**

In this section we proceed to obtain an improved basic feasible solution which requires the determin-

ation of entering and departing variables. We firstly express  $Z$  in terms of the non-basic variables only.

Now

$$(5) \quad Z = \frac{\sum_{i=1}^m \sum_{j=1}^n C_{ij} x_{ij}}{\sum_{i=1}^m \sum_{j=1}^n d_{ij} x_{ij}}$$

$$(6) \quad = \frac{\sum_{i=1}^m \sum_{j=1}^n C_{ij} x_{ij} + \sum_{i=1}^m u_i^1 \left( \sum_{j=1}^n x_{ij} - a_i \right)}{\sum_{i=1}^m \sum_{j=1}^n d_{ij} x_{ij} + \sum_{i=1}^m u_i^2 \left( \sum_{j=1}^n x_{ij} - a_i \right) + \sum_{j=1}^n v_j^1 \left( \sum_{i=1}^m x_{ij} - b_j \right) + \sum_{j=1}^n v_j^2 \left( \sum_{i=1}^m x_{ij} - b_j \right)}$$

Since the quantities in parentheses are zero therefore (5) is equal to (6), whatever values we give to  $u$ 's and  $v$ 's. Thus we have

$$(7) \quad Z =$$

$$\frac{\sum_{i=1}^m \sum_{j=1}^n \left( C_{ij} + u_i^1 + v_j^1 \right) x_{ij} - \sum_{i=1}^m u_i^1 a_i - \sum_{j=1}^n v_j^1 b_j}{\sum_{i=1}^m \sum_{j=1}^n \left( d_{ij} + u_i^2 + v_j^2 \right) x_{ij} - \sum_{i=1}^m u_i^2 a_i - \sum_{j=1}^n v_j^2 b_j}$$

As  $u_i^1, u_i^2, v_j^1$  and  $v_j^2$  are chosen such that

$$C_{ij} + u_i^1 + v_j^1 = 0$$

$$d_{ij} + u_i^2 + v_j^2 = 0$$

whenever  $x_{ij}$  belongs to the basic set which contain  $m+n-1$  basic variables.

Thus (7) reduces to

$$(8) \quad \frac{\sum_{i,j \in S} C_{ij}^1 x_{ij} - \sum_{i=1}^m u_i^1 a_i - \sum_{j=1}^n v_j^1 b_j}{\sum_{i,j \in S} d_{ij}^1 x_{ij} - \sum_{i=1}^m u_i^2 a_i - \sum_{j=1}^n v_j^2 b_j}$$

where  $\sum_{i,j \in S}$  means that summation extends over the set of non-basic variables.

The value of the objective function at this basic feasible solution is:—

$$(9) \quad Z = \frac{- \sum_{i=1}^m u_i^1 a_i - \sum_{j=1}^n v_j^1 b_j}{- \sum_{i=1}^m u_i^2 a_i - \sum_{j=1}^n v_j^2 b_j} = \frac{z^{(1)}}{z^{(2)}}$$

where

$$(10) \quad - \sum_{i=1}^m u_i^1 a_i - \sum_{j=1}^n v_j^1 b_j = z^{(1)}$$

$$(11) \quad - \sum_{i=1}^m u_i^2 a_i - \sum_{j=1}^n v_j^2 b_j = z^{(2)}$$

Therefore (8) is

$$(12) \quad Z = \frac{\sum_{i,j \in S} C_{ij}^1 x_{ij} + z^{(1)}}{\sum_{i,j \in S} d_{ij}^1 x_{ij} + z^{(2)}}$$

### Determination of Entering Variable

At the given basic feasible solution, all the non-basic variables are at zero-level. Let  $X_{st}$  be chosen to be variable to become basic *i.e.* to enter the basic set, at a value  $\theta$  and  $x_{vk}$  be the basic variable which is driven to zero *i.e.* departing variable. Then value of the objective function at the changed basic feasible solution will be

$$(13) \quad \bar{Z} = \frac{z^{(1)} + \theta C_{st}^1}{z^{(2)} + \theta d_{st}^1}$$

The objective function will improve if

$$(14) \quad \frac{z^{(1)}}{z^{(2)}} - \frac{z^{(1)} + \theta C_{st}^{(1)}}{z^{(2)} + \theta d_{st}^{(1)}} > 0$$

Let solution be non-degenerate and also  $d_{ij} > 0$  for all  $i, j$

(14) is equivalent to

$$(15) \quad \theta(z^{(1)}d_{st}^{(1)} - z^{(2)}C_{st}^{(1)}) > 0 \quad (\theta > 0)$$

or 
$$C_{st}^{(1)}z^{(2)} - d_{st}^{(1)}z^{(1)} < 0$$

where  $C_{st}^{(1)}$  and  $d_{st}^{(1)}$  refer to previous basic feasible solution.

Let

$$(16) \quad \Delta_{ij} = z^{(2)}C_{ij}^{(1)} - z^{(1)}d_{ij}^{(1)}$$

once  $u_i^1, v_j^1, u_i^2$  and  $v_j^2$  are computed it is easy matter to calculate  $\Delta_{ij}$  for non-basic variables.

Choose

$$(17) \quad \Delta_{st} = \min \Delta_{ij} \quad (\Delta_{ij} < 0)$$

i.e. we picked out most negative  $\Delta_{ij}$  to determine  $x_{st}$ . If all  $\Delta_{ij} \geq 0$ , then current basis is optimal.

The departing variable  $x_{vk}$  and the values of the basic variables for the new basis can exactly be determined in the same way as in case of transportation problem in linear programming.

### SECTION IV

#### Numerical Example

We explain our method by considering the following numerical example.

	$a_i \downarrow$								
	1	4	1	2	1	3	1	1	5
	1	1	2	3	1	2	1	3	7
	1	5	1	1	2	4	1	2	8
$b_j \rightarrow$	9	4	6	1	20				

$$\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j$$

In what follows, the numbers with asterisks denote the values of the basic variables, while numbers with daggers denote the values of  $\Delta_{ij}$ . Only negative values of  $\Delta_{ij}$  have been shown in the tables as these are the only values required for the decision of entering variables.

#### ITERATION I

		$a_i \downarrow$				$u_i^1 \downarrow$		$u_i^2 \downarrow$	
	5*	-56†	-54†	-114†	5	-5	-1		
	1	4	1	2	1	3	1	1	
	4*	3*			7	-2	-1		
	1	1	2	3	1	2	1	3	
		1*	6*	1*	8	0	0		
	1	5	1	1	2	4	1	2	
$b_j \rightarrow$	9	4	6	1	20				

$$v_j^1 \rightarrow \quad 1 \quad -1 \quad -4 \quad -2$$

$$v_j^2 \rightarrow \quad 0 \quad -1 \quad -2 \quad -1$$

$$z^{(1)} = 60, \quad z^{(2)} = 29, \quad Z = \frac{60}{29}$$

#### ITERATION II

		$a_i \downarrow$				$u_i^1 \downarrow$		$u_i^2 \downarrow$	
	4*	-58†	-60†	1*	5	-4	-1		
	1	4	1	2	1	3	1	1	
	5*	2*	-4†		7	-1	-1		
	1	1	2	3	1	2	1	3	
		2*	6*		8	1	0		
	1	5	1	1	2	4	1	2	
$b_j \rightarrow$	9	4	6	1	20				

$$v_j^1 \rightarrow \quad 0 \quad -2 \quad -5 \quad 3$$

$$v_j^2 \rightarrow \quad 0 \quad -1 \quad -2 \quad 0$$

$$z^{(1)} = 54, \quad z^{(2)} = 28, \quad Z = \frac{27}{14}$$

ITERATION III

			$a_i \downarrow$	$u_i^1 \downarrow$	$u_i^2 \downarrow$		
	2*		2*	1*	5	0	0
	1 4	1 2	1 3	1 1			
	7*				7	3	0
	1 1	2 3	1 2	1 3			
		4*	4*		8	-1	-1
	1 5	1 1	2 4	1 2			
$b_j \rightarrow$	9	4	6	1	20		
$v_j^1 \rightarrow$	-4	0	-3	-1			
$v_j^2 \rightarrow$	-1	0	-1	-1			

$$z^{(1)}=42, \quad z^{(2)}=24 \quad Z = \frac{7}{4}$$

Thus minimum value of the objective function is 7/4, at the optimum solution:

$$\begin{matrix} x_{11}=2 & x_{13}=2 & x_{14}=1 \\ x_{21}=7 & x_{32}=4 & x_{33}=4 \end{matrix}$$

**Acknowledgement**

I am extremely grateful to Prof. R. S. Varma, D.Sc., F.N.I., Head of the Department of Mathematics and Statistics, Faculty of Mathematics, University of Delhi, for continued help and inspiring guidance throughout the preparation of the paper.

**References**

- (1) B. Martos, "Hyperbolic Programming," *Pub. Math. Inst. Hung. Ac. Sc. 5, Series B*, 4 (1960), 383-406.
- (2) A. Charnes, and W. W. Cooper, "Programming with Linear Fractional Functionals," *Nav. Res. Log. Quart.* 9, Nos. 3-4 (1962), 181-186.
- (3) W. S. Dorn, "Linear Fractional Programming," *I.B.M. Corporation Research Report RC-830* (Nov. 1962).
- (4) W. Dinkelbach, "Die Maximierung eines Quotienten zweier linearer Funktionen unter linearen Nebenbedingungen," *Z. Wahrscheinlichkeitstheorie*, 1 (1962), 141-145.
- (5) Kanti Swarup, "Linear Fractional Functionals Programming," *O.R.S.A.*, 6, 13 (1965), 1029-1036.
- (6) W. W. Garvin, "Introduction to Linear Programming," McGraw-Hill Book Company, Inc. (1960).
- (7) G. Hadley, "Linear Programming," Addison-Wesley Publishing Company, Inc. (1963).
- (8) Kanti Swarup, "Some Aspects of Linear Fractional Functional Programming," *Australian J. Stats.*, 7, 3 (1955), 90-104.



Senior staff at the Admiralty Oil Laboratory on the occasion of a visit by Rear Admiral R. G. Draper, Director of Marine Engineering on 28th June.

Front row: Mr. E. P. Langston, Chief Scientist, A.O.L.; Rear Admiral R. G. Draper; Mr. E. J. Widdowson, retiring Chief Scientist.

Back row: Mr. P. W. Harrison, Dr. D. Wyllie.

# Problems Associated with the Use of Copper Alloy Castings in Marine Service \*

W. G. Beynon, B.Sc., F.I.M.

Ship Department, Ministry of Defence

## SUMMARY

*This Paper describes some of the problems associated with copper alloy castings encountered by the Ministry of Defence, Navy Department, and methods which are being taken to overcome them.*

*Gunmetal castings in LG4 (modified) have been found satisfactory for all general-purpose castings, and for high strength applications AB2 or a recently developed high strength cupro-nickel are satisfactory. The problems associated with shock loading are mentioned and the defects encountered in sand castings summarized. Future requirements are also briefly considered.*

## Introduction

This Paper, in the main, is the author's personal assessment of the problems associated with the use of copper alloy castings in marine service based on many years service with the Admiralty, now Ministry of Defence, Navy Department. This service has included periods in inspection laboratories, research laboratories, dockyard metallurgical laboratories, design departments, both in the constructive and engineering field and at Headquarters, Department of Materials Research. In these assignments good opportunities of assessing the problems associated with cast components have been afforded.

In the preparation of this Paper discussions have been held on the question of quality of copper alloy castings with metallurgists associated with commercial shipping. From these discussions it is clear that casting problems have a lesser effect on productivity than is the case with Naval ships.

The problems in question may be conveniently divided into those originating from two main aspects namely, metallurgical, *i.e.* associated with a particular alloy itself, and those derived from the casting process.

## Metallurgical

### Corrosion Resistance

A problem formerly of considerable importance was the dezincification and stress corrosion cracking of high tensile brass components. This phenomenon culminated in a decision to abolish the use of high tensile brass in Naval Marine Service in the early 1950's.

### Impingement Resistance

Failure of valve seats has been a continuing problem when using gunmetal for valves in sea-water systems. Valves have hitherto been made in 86/7/5/2 gunmetal but the use of LG4(mod)† is at the moment being introduced for general purposes. Recognition of the improved impingement resistance of aluminium bronzes led to the introduction of aluminium bronzes of AB1 and AB2 types. These alloys also have the advantage of increased strength.

### Dealuminification

Shortly after the introduction of AB1 castings evidence of the dealuminification tendencies of AB1 type alloys became available and the use of this alloy has been stopped. Although AB2 castings can readily suffer corrosion or dealuminification under crevice conditions, experience in H.M. ships has been satisfactory. This covers the construction of over 150 minesweepers which are fitted with cast AB2 shaft brackets, chain cable, stern tubes and

\* Paper presented at a Symposium on Copper Alloy Castings held at Birmingham, 25th-26th April 1966 by the Copper Development Association.

† This refers to B.S. 1400-LG4 with a restricted range for certain alloying elements as laid down by the Ministry of Defence, Navy Department.

rudders, none of which have shown any corrosion tendency in any form. The lesson here is to keep away from AB1 type castings when contact with seawater is involved.

### Strength

The ever-growing demand for increased strength in copper alloy castings has expanded the use of aluminium bronze where a 0.1% proof stress of 16 ton/in.<sup>2</sup> compared with 8 ton/in.<sup>2</sup> for gunmetals is of considerable advantage.

Latterly, the Navy Department has had under development a cast alloy based on 70/30 cupronickel. This has increased strength and toughness compared with AB2. Both factors are advantageous in Naval service where shock and high rates of loading have to be considered.

### Pressure-tightness

The inherent solidification characteristics of aluminium bronze and gunmetal suggest that the former should produce a better casting than the latter when compared on a basis of pressure-tightness. Experience shows that aluminium bronze can give considerable trouble from defects which arise from unsuitable pouring, running, gating and feeding arrangements and, once these are overcome by appropriate means, pressure-tightness can be obtained. However, techniques for readily achiev-

ing pressure-tightness and a consistently high standard of general soundness have yet to be evolved. In the case of gunmetal, the addition of lead and nickel to the traditional 88/10/2 alloy considerably improves pressure-tightness and present plans within the Navy Department are to standardize on the use of LG4 (mod) with a nickel addition for all gunmetal castings. About a year's experience with the alloy has resulted in favourable reports from all the Dockyards, both with regard to meeting tensile requirements and pressure-tightness. If the satisfactory performance of the alloy for general purposes, including those involving bearing requirements, is maintained, this will mean the use of only one gunmetal within the Navy Department. This would represent a commendable degree of standardization when one considers that there now exists a complete range of gunmetals within B.S.1400.

Tables 1, 2 and 3 give some of the results of tests relating to LG4 (mod) castings produced at the various Dockyards.

### Ductility

A measure of ductility in all engineering components is of importance, but the minimum acceptable in each case is open to discussion. However, for various reasons, a value of 11% on a 2 in.

TABLE 1a. Composition and Properties of Leaded Gunmetal (LG4 Mod) Castings\* (Dockyard No. 1).

<i>Chemical composition per cent</i>					<i>Mechanical Properties</i>				
<i>Cu</i>	<i>Sn</i>	<i>Zn</i>	<i>Pb</i>	<i>Ni</i>	<i>0.1% Proof Stress</i>		<i>Tensile Strength</i>		<i>Elongation</i>
					<i>ton/in<sup>2</sup></i>	<i>kg/mm<sup>2</sup></i>	<i>ton/in<sup>2</sup></i>	<i>kg/mm<sup>2</sup></i>	<i>per cent</i>
87.78	7.53	1.57	2.88	0.24	8.4	13.2	15.6	24.6	25.6
87.55	7.07	2.41	2.61	0.24	8.4	13.2	18.4	30.0	37.5
84.55	7.60	2.58	3.68	1.79	8.8	13.9	18.0	28.3	29.7
87.06	7.34	2.64	2.53	0.43	8.4	13.2	18.0	28.3	42.2
85.51	7.27	2.08	3.03	2.11	9.0	14.2	18.0	28.3	34.4
87.02	7.31	2.36	3.12	0.19	9.0	14.2	18.6	29.3	45.3
87.32	6.91	2.54	2.53	0.709	8.0	12.6	13.8	21.7	28.1
86.90	7.43	1.58	2.66	1.43	8.4	13.2	18.5	29.1	37.5
87.86	7.65	1.70	2.87	0.10	9.2	14.5	18.0	28.3	35.9
85.66	7.15	2.33	2.88	1.98	9.2	14.5	17.6	27.7	28.1
87.24	7.35	2.22	3.19	0.10	8.8	13.9	19.2	30.2	57.3

\* Specification requirements

Cu      Sn      Zn      Pb      Ni  
Rem. 7-7.5 1.5-3.0 2.5-3.5 2.0 max.

### Comment

Experiments carried out in 1962-3 indicated that LG4 castings gave a higher percentage of pressure-tight castings than LG3 and GIC alloys.

TABLE 1b. Composition and Properties of Leaded Gunmetal (LG4 Mod) Castings (Dockyard No. 1).

Chemical composition per cent					Mechanical Properties				
Cu	Sn	Zn	Pb	Ni	0.1% Proof Stress ton/in <sup>2</sup> kg/mm <sup>2</sup>		Tensile Strength ton/in <sup>2</sup> kg/mm <sup>2</sup>		Elongation per cent
87.51	7.80	2.28	1.84	0.17	8.4	13.2	16.4	25.8	29.1
86.13	7.21	1.67	2.94	2.05	10.4	16.4	16.6	26.1	21.9
86.80	7.81	1.97	3.29	0.15	9.0	14.2	19.2	30.2	45.3
84.88	7.80	2.05	3.23	2.09	10.5	16.5	18.8	29.6	34.4
88.72	7.54	2.55	1.21	0.10	9.2	14.5	19.8	31.2	62.5
85.97	7.57	2.90	3.04	1.12	10.0	15.7	19.8	31.2	43.8

*Comment*

Nickel only added for pressure-tightness.

TABLE 2. Composition and Properties of Leaded Gunmetal (LG4 Mod) Castings\* (Dockyard No. 2).

Chemical Composition per cent					Mechanical Properties		
Cu	Sn	Zn	Pb	Ni	Tensile Strength ton/in <sup>2</sup> kg/mm <sup>2</sup>		Elongation per cent
Rem.	7.00	2.53	3.38	1.89	18.65	29.3	27.5
Rem.	6.95	2.68	3.58	2.02	18.35	28.9	31.0
Rem.	6.95	2.94	3.69	1.79	18.00	28.3	31.0
Rem.	7.00	2.78	3.48	1.92	17.49	27.6	22.0
Rem.	8.00	1.37	3.25	2.03	17.60	27.7	24.5
Rem.	8.00	1.03	3.08	2.02	19.27	30.3	29.5
Rem.	7.87	1.06	3.17	2.02	17.70	27.9	27.9
Rem.	6.90	2.15	3.16	1.62	19.28	30.3	33.5
Rem.	6.75	1.53	2.91	2.16	17.46	27.6	29.0
Rem.	7.25	2.28	3.18	1.67	17.08	26.9	27.5
Rem.	7.00	2.39	3.42	2.29	16.49	25.9	18.5

\* Specification requirements

Cu Rem. 7-7.5    Sn 1.5-3.0    Zn 2.5-3.5    Pb 2 max.

Impurities as for LG4

Mechanical properties

0.1 per cent proof stress 8 ton/in<sup>2</sup>  
tensile strength 16 ton/in<sup>2</sup>  
elongation 18 per cent

TABLE 3. Composition and Properties of Leaded Gunmetal (LG4 Mod) Castings (Dockyard No. 3).

Chemical Composition per cent					Mechanical Properties				
Cu	Sn	Zn	Pb	Ni	0.1% Proof Stress ton/in <sup>2</sup> kg/mm <sup>2</sup>		Tensile Strength ton/in <sup>2</sup> kg/mm <sup>2</sup>		Elongation per cent
Rem.	6.8	2.5	3.08	2.0	{ 9.1 9.4	{ 14.3 14.8	17.4	27.4	18.0
							17.6	27.7	18.3

*Comment*

To date approximately 1,000 castings have been completed of which an estimated 10 per cent failed the water pressure test.

TABLE 4. Impact Properties of Nickel Aluminium Bronze.

<i>Composition per cent</i>					<i>Mechanical Properties</i>					
<i>Cu</i>	<i>Al</i>	<i>Fe</i>	<i>Ni</i>	<i>Mn</i>	<i>0.1% Proof Stress ton/in<sup>2</sup></i>	<i>kg/mm<sup>2</sup></i>	<i>Tensile Strength ton/in<sup>2</sup></i>	<i>kg/mm<sup>2</sup></i>	<i>Elongation per cent</i>	<i>Izod Values ft lbs*</i>
Rem.	9.70	5.05	4.90	1.23	19.0	29.9	47.8	75.3	25	24.3
Rem.	9.60	4.95	4.90	1.37	18.6	29.3	44.2	69.6	23	25.7
Rem.	9.60	4.95	4.90	1.06	18.0	28.3	45.2	71.2	25	26
Rem.	9.62	4.91	4.79	1.15	20.0	31.5	46.0	72.4	23	20
Rem.	9.69	4.80	5.48	1.44	20.2	31.8	45.6	71.8	21	20
Rem.	9.60	4.84	5.20	1.25	20.6	32.4	46.6	73.4	24	21
Rem.	9.57	4.83	5.00	1.33	20.6	32.4	44.8	70.5	22	22.7
Rem.	9.67	4.85	4.70	1.32	19.0	29.9	44.4	70.0	21	21.2
Rem.	9.79	4.85	5.10	1.02	19.6	30.9	45.0	70.9	21	21.5
Rem.	9.56	4.75	5.26	1.55	20.4	32.1	42.0	66.1	17	19.5
Rem.	9.72	4.75	5.16	0.87	19.0	29.9	44.0	69.3	16	20.5
Rem.	9.67	4.74	5.05	1.39	19.0	29.9	45.8	72.1	22	20.2
Rem.	9.76	4.70	5.05	1.45	19.0	29.9	44.6	70.2	22	22.3
Rem.	9.57	4.70	4.85	1.37	19.2	30.2	43.0	67.7	18	21.7
Rem.	9.76	4.75	4.93	1.18	19.0	29.9	44.8	70.5	19	18.8
Rem.	9.52	4.64	5.06	1.46	20.0	31.5	45.0	70.9	21	22.5
Rem.	9.47	4.60	4.90	1.50	20.0	31.5	43.6	68.7	19	21.3
Rem.	9.54	4.65	4.85	1.33	19.4	30.6	41.0	64.6	18	24.8
Rem.	9.58	4.55	5.12	1.70	21.0	33.1	43.6	68.7	17	18.3
Rem.	9.57	4.56	5.05	1.22	19.0	29.9	45.6	71.8	24	19.2
Rem.	9.53	3.46	5.04	1.30	18.0	28.3	45.0	70.9	24	16
Rem.	9.50	4.39	5.32	1.80	21.0	33.1	45.0	70.9	20	19
Rem.	9.58	4.34	4.67	1.48	20.0	31.5	44.4	70.0	22	21
Rem.	9.64	4.17	5.77	1.24	20.0	31.5	45.6	71.8	20	19.2
Rem.	9.50	4.01	5.30	1.38	19.8	31.2	45.4	71.5	18	20
Rem.	9.60	3.88	5.60	1.48	19.0	29.9	44.0	69.3	21	21
Rem.	9.63	3.85	5.50	1.40	18.8	29.6	45.6	71.8	26	24.3
Rem.	9.44	5.46	4.62	1.58	—	—	—	—	—	22
Rem.	9.68	5.04	4.54	0.08	—	—	—	—	—	20.3

\* 1 ft lb = 0.138 kg m

gauge length is considered the minimum allowable in Naval marine castings. Ductility in aluminium bronze castings is closely related to composition and, in particular, to aluminium content and as long as this value is kept below 10%, adequate ductility is achieved. Tables 4 and 5 from various sources indicate the relationship of mechanical properties to chemical composition.

It should be emphasized that the tensile properties relate to separately cast test bars and the elongation values are higher than would be obtained in actual castings.

#### Toughness

Notch toughness as measured by a notched impact test is of great importance in cast components. The minimum acceptable value for service application is open to discussion and work is in hand at the present time to gain further information on this particular point.

About two years ago the Navy Department asked for a minimum of 20 ft. lb. Izod in aluminium bronze castings. This caused quite a stir in foundry circles. However, surveys taken during the intervening period show that a figure of 15—17 ft. lb.

TABLE 5. Mechanical Properties obtained on Aluminium Bronze to DGS 8520.

Al per cent	0.1% Proof Stress		Tensile Strength		Elongation per cent	Izod Values ft. lb*
	ton/in <sup>2</sup>	kg/mm <sup>2</sup>	ton/in <sup>2</sup>	kg/mm <sup>2</sup>		
8.92	15.0	23.6	43.2	68.0	30.0	29.5, 30, 30.5
8.97	15.8	24.9	40.8	64.3	22.0	22, 23.5, 24
9.00	16.2	25.5	42.0	66.1	23.5	22.5, 22, 22.5
9.01	15.8	24.9	43.2	68.0	28.0	25, 25, 26
9.12	16.2	25.5	44.0	69.3	28.0	26, 27, 26
9.15	17.4	27.4	43.4	68.4	23.0	24, 24, 25
9.19	17.0	26.8	44.0	69.3	25.0	22, 22.5, 22.5
9.22	17.2	27.1	44.0	69.3	23.0	22.5, 22.5, 23
9.26	16.0	25.2	43.8	69.0	25.0	23.5, 23.5, 24.5
9.28	16.2	25.5	43.0	67.7	23.0	23, 23, 23.5
9.39	17.2	27.1	41.2	64.9	16.0	21, 21, 21.5
9.46	18.25	28.7	43.5	68.5	19.0	18, 18, 19
9.47	16.4	25.8	44.4	70.0	16.0	21, 17, 18
9.58	17.8	28.0	43.2	68.0	16.0	16.5, 17, 17

\* 1 ft. lb. = 0.138 kg m

is readily obtainable and it is likely that a figure of this order will be included into the Naval specification for aluminium bronze castings.

#### Shock Resistance

The functions of Naval vessels are such that they encounter shock loads of varying magnitude and these require that components have adequate shock resistance. In general terms good values in the mechanical properties already mentioned ensure reasonable shock resistance but in addition, it is the policy to check components under simulated shock conditions where both adequacy of design and material properties can be assessed. The new alloy based on 70/30 cupro-nickel has given encouraging results in this context.

#### Weldability

In view of the inherent tendency of the casting process to produce defects of one form or another, it is highly desirable that cast alloys should have adequate weldability. This is important from two aspects; first, the need to maintain adequate strength when a major repair is carried out and, secondly, the need to avoid introducing another defect such as heat affected zone cracking or impaired corrosion resistance which are of great importance even in a minor weld repair. The experience of the Navy Department arising from a vast number of weld repairs is satisfactory but at the present time the British Welding Research Association is examining all aspects of the welding

of aluminium bronze and cupro-nickel with the aim of producing a code of practice for welding these alloys.

Hot tensile properties are of great importance in welding behaviour, and for the production of crack-free joints the choice of weld metal for use with a particular parent material must be based on an adequate knowledge of the high temperature properties of the parent material and of the various weld materials under consideration. By such an approach it should be possible to deal satisfactorily with parent material which has poor high temperature tensile properties.

#### Problems Associated with the Sand Casting Process

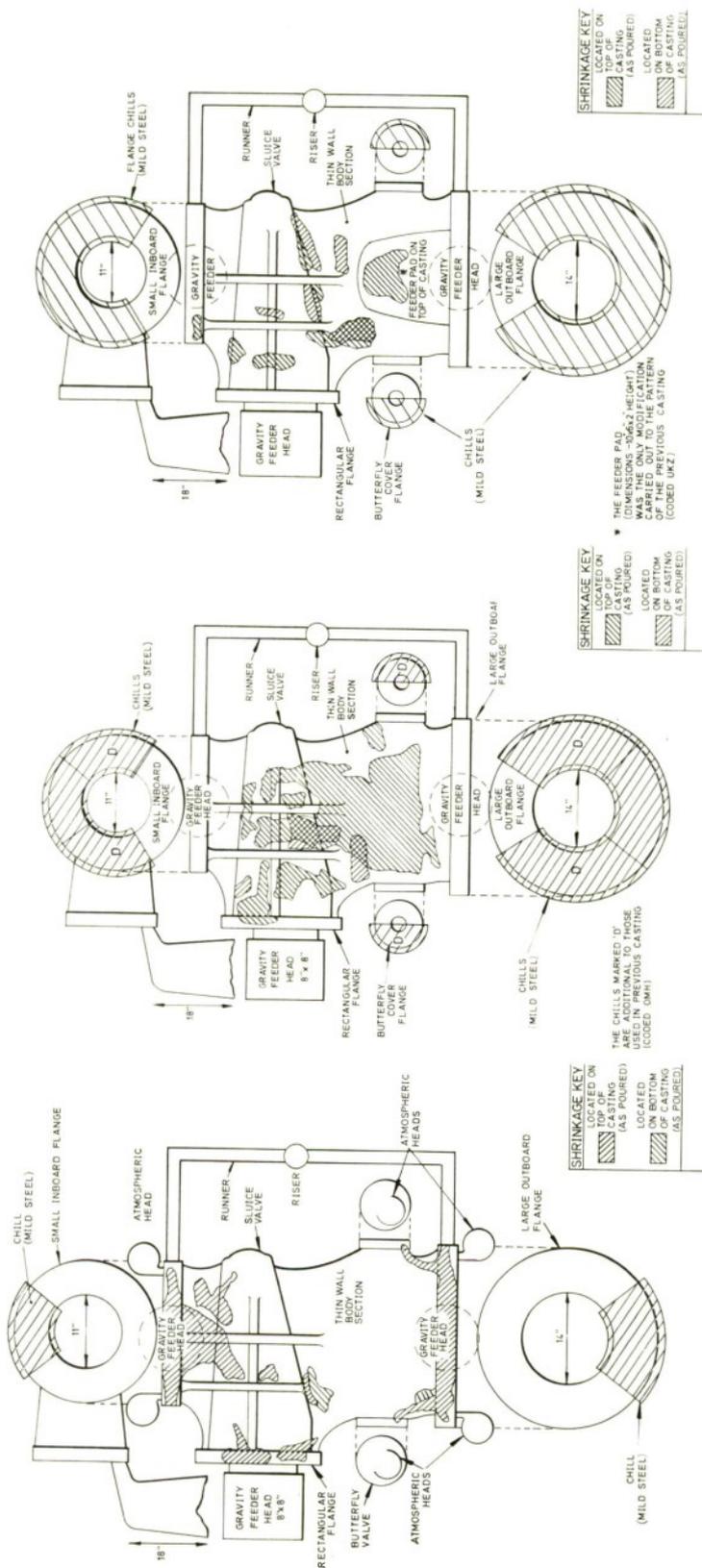
There are three major types of defects in sand castings, namely gas porosity, shrinkage and non-metallic inclusion. These are discussed separately.

#### Gas porosity

This arises from evolution of gas from the metal during cooling and solidification. The origin of the gas is the atmosphere, products of combustion of fuel used for heating, and mould and core materials. Control of defects due to gas is largely under the control of the foundry.

#### Shrinkage Cavities

These arise from the natural shrinkage in cooling through the liquid stage and from the shrinkage associated with the liquid to solid change. Shrinkage can largely be eliminated by examination



By the replacement of atmospheric heads with very heavy chilling of the flanges and finally by the addition of a feeding pad on top of the casting, the majority of the shrinkage porosity was eliminated.

FIG. 1 (above). Stages in the development of a satisfactory method of casting a large sluice valve body in the recently introduced high-strength cupro-nickel alloy. The diagrams show the successive modifications of the method and the defects located by radiography.

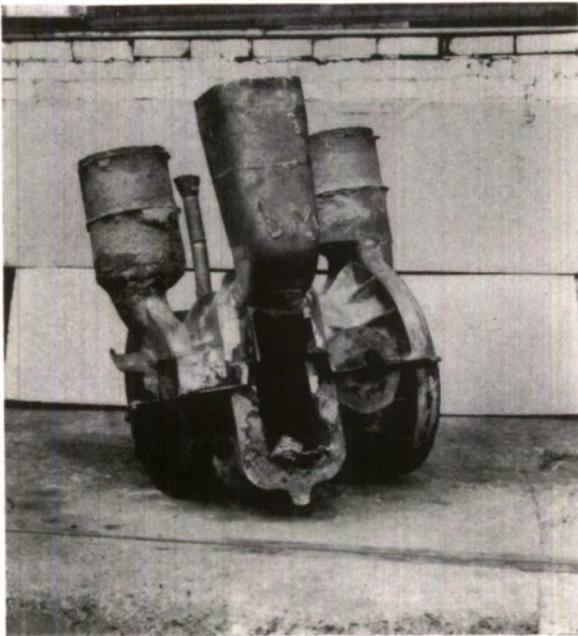


FIG. 2. The second cupro-nickel casting shown in the drawing B on the left.

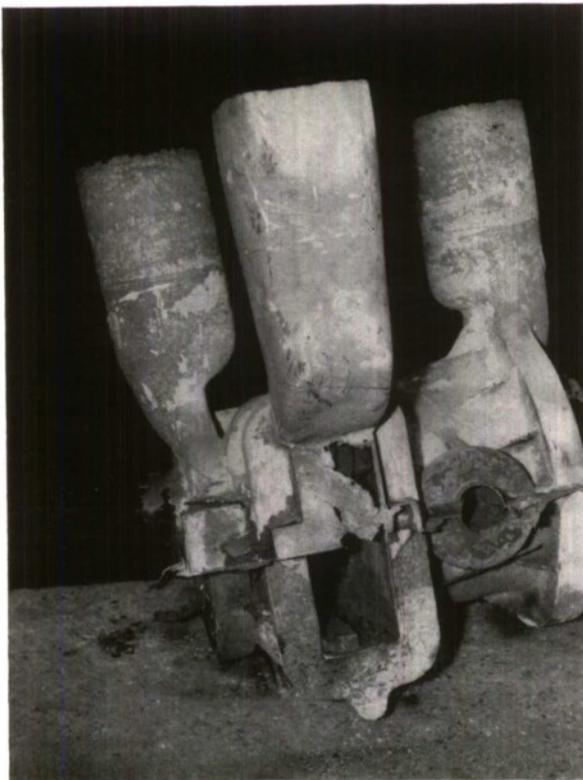


FIG. 3. The third cupro-nickel casting shown in drawing C.

of prototype castings by radiography, and cut up tests and the subsequent use of modified foundry techniques for overcoming these defects. Where they persist this is generally due to complexity of castings, size and unsuitable design. These are matters which should be tackled by consultation between the designer, the foundry and laboratory.

Elimination of inter-dendritic shrinkage in long freezing range alloys is difficult, but the retention of the as-cast columnar skin after machining goes some way to ensuring pressure tightness.

#### Non-Metallic Inclusions

These arise in the main from the mould materials and in the case of aluminium bronze from alumina derived from the aluminium in the alloys. If care is not taken to control turbulence during pouring, the oxide formed on the metal in passing from the ladle to the mould is drawn into the mould and becomes distributed throughout the casting. This entrapped oxide is a source of weakness and has a marked effect on ductility and tensile strength. Minimizing oxide entrapment is largely under control of the foundry through the use of correct pouring, gating and feeding arrangements.

Recent developments of the use of cast silicon aluminium bronze has shown that this material suffers less from entrapped alumina than complex aluminium bronze and this factor, among others, may lead to its greater application.

#### Acceptance Standards

In view of the inherent defects likely to occur in any sand casting, standards of acceptance based on radiographic quality are necessary. At the present time the standards are laid down in the appropriate documents which give the maximum size and frequency of defect acceptable.

Although this is the best that can be done at the present time, it is possible that castings suitable for their intended use are being rejected. In an endeavour to produce more realistic acceptance standards, work is in hand in which strength and radiographic appearance are being correlated. As may be expected a considerable number of castings will have to be assessed before current acceptance standards can be relaxed. It is also considered that composition and mechanical properties should be taken into account in assessing borderline cases.

#### The foundry of the future

Casting is a very old process and by a process of evolution the present quality standards have been attained which show a marked improvement on those, for example, of 15 years ago. This has been largely brought about by the use of radiography, whereby the quality of a casting is revealed and the position of defects indicated. By the co-operation of founders, radiographers and designers the

current high standards have been achieved. Nevertheless, rejection of castings continues and is a costly matter. It would appear that by the use of traditional foundry methods, *i.e.* pouring metal into a static mould, the hope of achieving castings always 'right first time' will never be realized and therefore one should consider other methods which are not new but at the present time only applied in a specialized way. There is the use of centrifugal casting which produces a very high yield of serviceable products when adopted for symmetrical casting. Centrifuging small non-symmetrical castings produces a high percentage of good quality castings and the application of centrifugal force over a wider range of component sizes merits attention. The use of pressure may achieve the desired ends more readily.

In another direction it might be possible to dispense with the traditional mould and produce castings by spraying metal onto an appropriate pattern which moves relative to the metal source. These techniques are already used for tungsten rocket nozzles. By appropriate control of metal temperature and atmosphere, the major defects encountered in present-day castings should be avoided.

#### **Acknowledgement**

This paper is published by permission of the Director General, Ships. Any views expressed are those of the author and do not necessarily represent official views.



## **Discussion on Fuel Cells at A.M.L.**

**Reported by J. McFadyen**

*Admiralty Materials Laboratory*

The present 'state of the art' of fuel cells was reviewed recently in this Journal (Watson, R. G. H., *J.R.N.S.S.*, 21, 1966, 110) where the latest developments in fuel cell materials, fuel batteries and power plants were discussed in detail.

With the establishment of a Fuel Cell Group at the Admiralty Materials Laboratory having responsibility for fuel cell research within the Ministry of Defence it was thought desirable that an opportunity should be given for interchange of thinking between those people in the United Kingdom with an interest in fuel cells so that the most rapid progress in their development might be achieved. An informal discussion on fuel cells was therefore arranged at the Admiralty Materials Laboratory on the 21st and 22nd April, 1966, so that members attending could give (or hear) short talks on various aspects of fuel cell research and development. The discussion, which was held from lunch time on the 21st April until after lunch on the following day, was attended by forty-seven visitors to the Laboratory representing Defence, Academic, Nationalised and Industrial Organisations. It was gratifying that Mr. E. J. Vaughan, C.B.E., Director of Materials Research (Naval) and Mr. F. T. Bacon, were able to attend the discussion.

The contributions offered by members of the discussion were conveniently grouped into four

sessions, two on each day, and details of the programmed contributions are given at the end of this note.

In opening the meeting, Dr. Ovenston, Superintendent, Admiralty Materials Laboratory, welcomed the visitors and reviewed in some detail the Navy Department Fuel Cell Research Programme, noting that contracts had been placed with industrial organisations in the United Kingdom for development of hydrogen and hydrazine fuel cells with air and/or oxygen as the oxidant. He also described the intra-mural programme in which, among others, problems of materials, catalyst poisoning, heat and mass transfer and life testing would be studied.

In the first session, the speakers dealt essentially with catalysts and catalyst supports for low temperature fuel cells. Mr. Connor surveyed the requirements of catalysts for fuel cell application and discussed the production and use of high surface area deposits of the platinum metals for this purpose. The techniques involved in the production of monoporous and biporous nickel sheet for use as fuel cell electrodes were described by Mr. Dunton. Mr. Walkiden stressed the need for localisation of the expensive electrocatalyst at or near the three phase interface which obtains in a cell with liquid electrolyte and presented results of experi-

ments where the catalyst was deposited at the boundary layer in a biporous electrode. Mr. Askew described work which had recently been initiated at the Admiralty Materials Laboratory on the examination of fuel cell electrodes by optical and electron microscopy and demonstrated how this could be used to measure surface areas of catalysts.

Dr. Hart opened the second session, which was devoted to applications of fuel cells, by detailing the economic reasons why fuel cells could not compete with conventional power plants for the bulk generation of electricity. In the remainder of this session, Service interest in fuel cells were discussed by Dr. Orr, Miss Macnair and Mr. Mitchell representing branches of the Army Department, Navy Department and Ministry of Aviation respectively. The silent operation of a fuel cell was shown by Dr. Orr to be of particular interest for small boat propulsion for military use and the freedom from vibration of fuel cell power plants was one of the points stressed by Miss Macnair reviewing Ship Department interests. The last contribution to the session, in which the four kilowatt power plant operating on methanol fuel and developed by "Shell" Research Limited for the Ministry of Aviation was compared with a modern diesel generator led to lively discussion of the basis on which comparison of fuel cells with conventional generators can be made.

At the end of the first day of the discussion, members were invited to visit the Chemical Engineering Division of the Laboratory to see and discuss with A.M.L. staff the work of the Fuel Cell Group. A demonstration of the fuel-cell-powered G.R.P. dinghy, the fuel cell for which was constructed at A.M.L. from electrodes manufactured by "Shell" Research Ltd., was viewed with much interest by the visitors.

The programmed contributions in the third session were given by representatives of industrial organisations in the United Kingdom engaged in the development of fuel cells. Mr. Williams discussed the performance of electrodes operating on the gaseous products obtained from steam reformed hydrocarbon fuels, one method of utilising cheaper, more common fuels in low temperature systems. The important aspects of electrode and cell life were discussed by Mr. Lomax and Mr. Gray and it was of interest to note that failure occurred both during the tests on individual cells and on a one kilowatt battery by degradation of the materials of construction in the strong alkaline environment and not by degradation of the electrode materials. Dr. Gregory described the recently developed commercially available "screen" electrodes where catalyst and hydrophobic agent mixtures are compressed on to a fine metal mesh, or screen. When

used in conjunction with an electrolyte held in a matrix, very high, reproducible performances are obtained.

The final session was devoted to fundamental research related to fuel cells. Dr. Parsons discussed the effect of the adsorption of intermediates in a complex reaction on the exchange current. Dr. Fleischmann described work on the growth of catalyst films and the activity of "edge" sites, while Professor Hills presented results which suggested that in aqueous electrolyte with a mercury electrode, electron transfer across the double layer may occur to give discharge of hydrogen at the cathode. A useful summary of methods of determining surface areas of electrodes was given by Dr. Packer to conclude this session.

The aims of the discussion were amply fulfilled. The topics covered proved of much interest to all members and this and the informal arrangements of the meeting contributed to active discussion periods after each session. As most of the visitors during their overnight stay were accommodated in the same hotel, members were able to continue the discussion into the evening and many ideas on promising lines for future research were exchanged in this even more informal atmosphere.

In accordance with the desire to maintain informality, no formal record of the proceedings were produced. It is hoped that a second conference along the same lines can be arranged in the autumn of 1967.

## Programme

*Thursday, 21st April, 1966*

Introductory Address by Dr. T. C. J. Ovenston, Superintendent, Admiralty Materials Laboratory.

*Session 1. Chairman—Dr. R. G. H. Watson, A.M.L.*

"The role of the platinum group metals in low temperature fuel cells." Mr. H. Connor, Johnson Matthey and Co. Ltd.

"The production and properties of porous nickel electrodes." Mr. T. A. Dunton, International Nickel Ltd.

"Platinum metal activation of fuel cell electrodes." Mr. G. W. Walkiden, International Nickel Ltd.

"Microscopy of fuel cell electrodes." Mr. B. A. Askew, A.M.L.

*Session 2. Chairman—Dr. E. C. Wadlow, D.M.R.(N).*

"An assessment of fuel cells for bulk power generation or storage." Dr. A. B. Hart, Central Electricity Research Laboratories.

“Silent propulsion applications.” Dr. B. H. Orr, Military Engineering Experimental Establishment.

“Possible Ship Department interests in fuel cells.” Miss E. J. Macnair, M.O.D.(N), Ship Department.

“The ‘Shell’ 4 kW fuel cell.” Mr. S. F. Mitchell, Signals Research and Development Establishment.

*Friday, 22nd April, 1966*

*Session 3. Chairman—Dr. E. C. Wadlow, D.M.R.(N).*

“Performance of fuel cell electrodes on impure hydrogen.” Mr. K. R. Williams, “Shell” Research Ltd.

“Life tests on electrodes and cells.” Mr. G. R. Lomax, Electric Power Storage Ltd.

“A 1 kW hydrogen battery.” Mr. J. Gray, Electric Power Storage Ltd.

“Screen Electrodes.” Dr. D. Gregory, Energy Conversion Ltd.

*Session 4. Chairman—Dr. R. G. H. Watson, A.M.L.*

“The relation between rates of electrode reactions and adsorption of intermediates.” Dr. R. Parsons, University of Bristol.

“Catalytic activity of metal films.” Dr. M. Fleischmann, University of Newcastle-upon-Tyne.

“Electron transfer in aqueous solutions.” Professor G. J. Hills, University of Southampton.

“Surface area measurement on electrodes.” Dr. R. K. Packer, Admiralty Underwater Weapons Establishment.



## RETIREMENTS

### P. L. TA'BOIS, Dip.Eng., M.I.E.E.

P. L. Ta'Bois formerly Principal Scientific Officer, retired on the 30th April at the age of 67 after 30 years of Admiralty service. To mark the occasion he was presented with a slide projector and carrying case by the Chief Scientist of A.S.W.E., Mr. D. S. Watson, on behalf of his many friends in the Establishment.

Peter Ta'Bois received his formal education at St. Paul's School and Kings College, London. His practical education continued throughout his long and varied professional career. For a while he worked as a junior accountant in New York. From those early days he recalls the advice he was given—“always get the petty cash right first”—still good advice to any engineer who might be tempted to indulge in flights of fancy. He joined Standard Telephones and Cables in 1925 and remained with them until 1936. During this time he travelled widely, installing and commissioning radio stations from the north of Scandinavia to Africa. He joined H.M. Signal School as a Technical Officer in 1936 and remained with the Establishment through all its changes or organisation and name until he retired.

For much of his Service career he continued to work in the world of communications. During the war years he was concerned with the Home and Overseas Communications Network but his particular contribution was the major part which he played in setting up the South Coast Radio Service. The experimental discovery that V.H.F. communication was not limited strictly to the horizon was exploited, by the use of directional aerials, to provide

a system of point-to-point radio telephone links, first along the South Coast of England, then to ships in the Channel and eventually, during the invasion of Continental Europe, into France and the Low Countries. He was promoted to Senior Technical Officer in 1944 and to Principal Scientific Officer in 1947. After the war he became Head of the Shore Installation Planning Division, dealing with Shore Wireless Stations and the communications installations of Naval Air Stations.

In 1954 he began a new phase in his career when he moved to the Electronic Warfare Division. For a time he continued to work on equipment for Shore Stations. Then, when he was already past the normal age of retirement, he became directly involved for the first time with seagoing equipment for the Navy. He was charged with supervising the development of a data processing sub-system which was to use the new solid state integrated circuit components. It was the first time that these had been used in Naval equipment or, indeed, anywhere else in Britain on a significant scale and there were many problems. He stayed on to see the testing of the prototype equipment successfully finished before leaving, even then rather reluctantly, to begin his well-earned retirement.

Peter laid up the familiar vintage Alvis a year or so ago when it became too temperamental for use as everyday transport. He hopes to restore it “now that he has more time.” However, since another major interest is the yacht, younger than the car but even more demanding, it would not be disloyal to doubt whether this particular task will ever be accomplished. When visited recently, Peter was fitting-out for the annual cruise to France and complained happily that he had never been busier in his life. We wish him and his wife many more seasons of such full employment and enjoyment.



## MR. G. C. J. THREADINGHAM OBITUARY



Mr. G. C. J. Threadingham retired from the Navy Department, in which he was widely known and respected, on 20th June, 1966, after 25 years' service. He had already been in business, as a salesman and buyer with the Gaumont British Picture Corporation, for twenty years before joining the Production Division of H.M. Signal School at Haslemere in 1941. He moved to Headquarters, then known as the Scientific Research and Experiment Department, in a section later to become part of the Department of Research Programmes and Planning, in 1945. Apart from an interlude as Supplies Officer at the Admiralty Photographic and Instrument Research Laboratory, also in London, "Thread" remained in the Contracts Section of D.R.P.P. until July, 1954, when he went to A.R.L. as Contracts Officer. Here he was responsible for the placing and progressing of research and other contracts involving very large sums of money, over a period of 13 years. His skilled and firm handling of situations involving perhaps rather awkward individuals and sometimes very unbusiness-like firms brought him real respect from people within and without A.R.L. and incidentally saved the taxpayer many thousands of pounds. Another activity which brought him into contact with a great many people was his work for UNICEF. By his sales of Christmas cards and calendars he raised about £2,000 over the years for that very worthy cause.

A large audience of his colleagues gathered for the presentation of a radio set to Mr. Threadingham by Superintendent, A.R.L., on behalf of the staff.



## OBITUARY JAMES D. MACPHERSON, Ph.D., B.Sc.

The death of Dr. James D. Macpherson occurred in Dartmouth, N.S. on June 14th, 1966 following an illness of a few months. At the time of his death he was the head of the Acoustics Section of the Defence Research Board's Naval Research Establishment. Although only thirty-three years of age, he had established himself as one of Canada's foremost specialists in underwater acoustics and had in his relatively short research career published eighteen papers or reports.

Dr. Macpherson was born in London, England and graduated from Imperial College with a B.Sc. Honours degree in 1953 and Ph.D. in 1956 for acoustics research in the effect of gas bubbles on sound propagation in water. He was awarded a National Research Council of Canada post-doctoral fellowship at Dalhousie University, Halifax, N.S. where he studied the thermal expansion of barium titanate using microwave resonance techniques. In 1958 he joined the Naval Research Establishment and during



the past eight years personally concentrated on studies of acoustic propagation and reverberation in shallow water. Latterly, he was also responsible for other work in various fields from propagation and reverberation in the deep ocean to noise generated by high speed flow.

Dr. Macpherson will be greatly missed by all at N.R.E. and by his many scientific associates both in Canada and in the U.K., U.S. and Europe. He was an inspiration to work alongside with his boundless energy and curiosity and his ability to define the crux of a scientific problem. He had a remarkable combination of practical and theoretical talent but he was probably happiest under the pressure of carrying out field experiments. He was a member of the Acoustical Society of America.

He led an active life at work and at play; he liked swimming, squash, sailing, photography and woodworking. Dr. Macpherson is survived by his wife and five young children, to whom we offer our deepest sympathy.



**ALFRED HINE, B.Sc., R.N.S.S.**

The many friends and colleagues of Alfred Hine at the Admiralty Compass Observatory as well as many more in the wider sphere of his activities were saddened to hear of his sudden death on 4th August as a result of a heart-attack at the relatively early age of 64, less than four years after his retirement. All at A.C.O. and others in the Service who knew him would wish to extend to Mrs. Hine their heartfelt sympathy in her sad loss, which they also share.

The major part of Alfred Hine's career, a full record of which was published in the *J.R.N.S.S.* of May 1963, was spent at A.C.O., which he joined as a Technical Assistant in 1935 and where he remained until he retired as a Principal Scientific Officer in 1963.

Although in his early days he was engaged in gyro compass development, he soon transferred to the magnetic department. There, for some twenty years, he headed the Magnetic Compass Group, concerned with all types of magnetic compasses for land, air and marine use. On the marine side perhaps his contribution was greatest, where he was the leading figure in the keen but healthy rivalry with what some were tempted to call the gyro compass "opposition." During this period, he and his colleagues of the "Hinery" brought the magnetic compass, which had played only an auxiliary role in major warships, into a position of parity with the gyro "master." This was achieved by a series of ingenious devices and mechanisms designed to minimise or eliminate what had hitherto been errors or shortcomings of conventional magnetic compasses. First among these came the use of a conducting compass fluid, and a bridge circuit made it possible to measure the deflection of the compass card as an electrical signal and, with suitable intermediate equipment, to obtain multiple transmissions of ship's head in any part of a ship,

including positions where a magnetic compass could not be used. Subsequently, he introduced a slave gyro to damp oscillations of the compass card from its transmissions and provide stability in azimuth. He also introduced automatic and simple methods of correcting the errors of deviation and variation. Today, the Admiralty Gyro Magnetic Compass plays a complementary role with the gyro compass in many major warships.

Alfred Hine became an international authority on magnetic compasses and magnetometers and helped to make A.C.O. a mecca of learning in this field. Much of this knowledge will become available to posterity, as his book on Magnetic Compasses and Magnetometers is to be published shortly and is likely to become a standard work of reference.

The achievements of Alfred Hine in his official life were balanced by so many notable off-duty activities that he seemed to personify the Kiplingesque definition of a man. Indeed, he gave others the impression that sixty seconds to the minute—temporal of course—were just not enough for him. Prominent among his outside activities was his Church work, but his many and diverse interests included radio, youth organisations, choral music, Toc H, sailing, beekeeping and other rural pursuits. Ranking higher in his affections was A.C.O. itself, where he occupied a flat for many years and of which he wrote a history after considerable painstaking research.

Almost all those at A.C.O. who knew him have their own particular memories of him, not the least of his keen sense of humour, even if a joke was against himself. The anecdotes in circulation, by him and about him, real and apocryphal, were so numerous that one sometimes found it difficult to differentiate between fact and fantasy.

Alf has gone, but his work lives on in many ways, not the least among mariners who, albeit unknowingly, see his memorials around them.



# LIMITED SPACE QUEUEING WITH ARRIVALS CORRELATED

**M. L. Chaudhry**

*Department of Mathematics, Kurukshetra University, India*

## SUMMARY

*This paper deals with a steady state behaviour of a discrete-time, single channel, first-come-first-served, limited space queueing problem. Probability generating functions for the queue lengths have been derived under two models each with the assumptions that the arrivals at two consecutive time-marks are (i) correlated and (ii) uncorrelated. Particular cases of importance have been listed. In the end, mean queue lengths together with their special cases have been determined.*

## Introduction

Usually in queueing problems the arrivals in two successive intervals are considered to be statistically independent. However, there may be situations where arrivals at two consecutive time-marks (defined later) are statistically dependent. Chaudhry (1965) has investigated a queueing process wherein

the departures have been correlated.

In this paper we study the steady state behaviour of a single channel queueing process in discrete time. The arrivals at two consecutive time marks are correlated whereas the departures are statistically independent. The queue discipline is first-come-first-served. The system (queue + service) can

accommodate only  $N$  units so that when a unit comes and finds  $N$  units already in the system, it goes away and is thus lost to the system. Heathcote and Moyal (1959) study the queueing process in which the arriving unit does not join the queue in case there are  $N$  units already present awaiting service.

It is not difficult to visualize situations wherein the idea of correlation can be applied. For example, let us consider two anti-aircraft gunners A and B whose job is to shoot down enemy aircraft. If we now identify an arrival (or no arrival) at a service channel with the shooting down of an aircraft by A, then

- (1) If an aircraft was shot down by A at a previous time-mark, then
  - (a) Probability (next aircraft will be shot down by him at the next time-mark) =  $p$ , say
  - (b) Probability (next aircraft will be shot down by B at the next time-mark) =  $q$ , say
- (2) If an aircraft was shot down by B at a previous time-mark, then
  - (a) Probability (next aircraft will be shot down by him at the next time-mark) =  $q$ , say
  - (b) Probability (next aircraft will be shot down by A at the next time-mark) =  $p$ , say

where it seems natural to assume  $q > p$  with  $p + q = 1$ .

In a general problem like this let the time be divided into denumerable set of time-marks  $t_0, t_1, t_2, \dots$  and assume that the various events involved in the process occur only at these time-marks. The chance of more than one arrival or of completion of more than one service at a time-mark is of negligible infinitesimal order. Consider two consecutive time-marks  $t_{r-1}, t_r$ . Then the probability of an arrival or no arrival at  $t_r$  is governed by the transition probability matrix (t.p.m.).

$$\begin{array}{cc}
 & \text{To} \\
 & \text{Arrival at } t_r \quad \text{No arrival at } t_r \\
 \text{From} & \begin{array}{cc}
 \text{Arrival at } t_{r-1} & \begin{pmatrix} p & q \\ q & p \end{pmatrix} \\
 \text{No arrival at } t_{r-1} & \\
 \end{array} \quad (1) \\
 & p + q = 1
 \end{array}$$

Hence there exists a correlation between arrivals at two consecutive time-marks. The coefficient of correlation can easily be shown to be equal to  $p - q = r$ , only when the probability of an arrival at  $t_0$  is  $\frac{1}{2}$  [Seth (1963)]. Instead of the doubly stochastic matrix (1), one could use the methods of this paper to deal with the more general t.p.ms. of the type

$$\begin{array}{cc}
 & \text{To} \\
 & \text{Arrival at } t_r \quad \text{No arrival at } t_r \\
 \text{From} & \begin{array}{cc}
 \text{Arrival at } t_{r-1} & \begin{pmatrix} p_1 & 1 - p_1 \\ 1 - p_2 & p_2 \end{pmatrix} \\
 \text{No arrival at } t_{r-1} & \\
 \end{array}
 \end{array}$$

but the results would be cumbersome.

Let the probability for a departure be  $\lambda$  and consequently for no departure  $\mu (= 1 - \lambda)$ . Also

- (1) If there is at least one unit in the system, just before  $t_r$ , then
  - Probability (a unit departs at  $t_r$ ) =  $\lambda$
  - Probability (no unit departs at  $t_r$ ) =  $\mu (= 1 - \lambda)$
- (2) If the maximum allowed length for the queue is  $N$ , just before  $t_r$ , the new arrival at  $t$  will not join if  $n = N$  but will join if  $n < N$ .

Two models 'A' and 'B' have been considered which differ from each other, in a sense, that if, just before a time-mark, the system is empty and if an arrival occurs at the time-mark, then

- (a) In Model A an arrival at the time-mark cannot depart at that time-mark.
- (b) In Model B an arrival at the time-mark can depart at that time-mark.

For each case the probability generating function for the queue length has been derived. The corresponding probability generating functions for the two models have also been worked out in the case when the arrivals at two consecutive time-marks are uncorrelated and the results have been compared with the correlated case for particular values of the operational parameters.

### Probability Generating Functions for the Queue Length in the Steady State

#### MODEL A

##### Arrivals Correlated

In this model we assume further that if there is no unit in the system, *i.e.*, the system is empty, just before  $t_r$ , then whether or not an arrival occurs at  $t_r$

- (a) Probability (the unit departs at  $t_r$ ) = 0
- (b) Probability (the unit does not depart at  $t_r$ ) = 1

Define

$P_n$  = Probability [queue is in state  $n^*$  at  $t_{r+1} - 0$ , an arrival occurs at  $t_r$ ]

$Q_n$  = Probability [queue is in state  $n$  at  $t_{r+1} - 0$ , no arrival occurs at  $t_r$ ]

Elementary probability reasoning gives the following mutually exclusive ways through which  $P_n$  is attained.

\*By state  $n$  we mean that there are  $n$  units waiting including the one in service, if there is any, just before a time-mark.

- (a) There are  $n$  units in the system, just before  $t_r$ , an arrival has occurred at  $t_{r-1}$  and at  $t_r$ , an arrival as well as a departure takes place.
- (b) There are  $(n-1)$  units in the system, just before  $t_r$ , an arrival has occurred at  $t_{r-1}$ , and at  $t_r$  an arrival but no departure takes place.
- (c) There are  $n$  units in the system, just before  $t_r$ , no arrival has occurred at  $t_{r-1}$  and at  $t_r$  an arrival as well as a departure takes place.
- (d) There are  $(n-1)$  units in the system, just before  $t_r$ , no arrival has occurred at  $t_{r-1}$  and at  $t_r$  an arrival but no departure takes place.

Thus

$$P_n = p(P_n\lambda + P_{n-1}\mu) + q(Q_n\lambda + Q_{n-1}\mu) \quad 1 < n \leq N \quad (2)$$

Also

$$P_1 = pP_1\lambda + q(Q_1\lambda + Q_0) \quad (3)$$

Observe

$$P_0 = 0 \quad (4)$$

Similarly

$$Q_N = P_N\mu + Q_N\mu \quad (5)$$

$$Q_n = q(P_{n+1}\lambda + P_n\mu) + p(Q_{n+1}\lambda + Q_n\mu) \quad 0 < n < N \quad (6)$$

$$Q_0 = qP_1\lambda + p(Q_1\lambda + Q_0) \quad (7)$$

Define the generating functions

$$P_A(s) = \sum_{n=0}^N s^n P_n \quad \text{and} \quad Q_A(s) = \sum_{n=0}^N s^n Q_n \quad (8)$$

Multiplying (2) by  $s^n$ , (3) by  $s$  and using (4) and (8) we get on re-arrangement

$$P_A(s)[1 - (\lambda + \mu s)p] + Q_A(s)[-q(\lambda + \mu s) + \mu s^{N+1}(pP_N + qQ_N) + \lambda qQ_0(1-s)] = 0 \quad (9)$$

Similarly from (5), (6), (7) and (8)

$$P_A(s)[q(\lambda + \mu s)] + Q_A(s)[p(\lambda + \mu s) - s + \mu s^{N+1}(pP_N + qQ_N) - p\lambda Q_0(1-s)] = 0 \quad (10)$$

Subtracting (10) from (9)

$$P_A(s)\mu - Q_A(s)\lambda + \lambda Q_0 = 0 \quad (11)$$

(10) and (11) on solution give

$$P_A(s) = \frac{N_{A1}}{D} \quad \text{and} \quad Q_A(s) = \frac{N_{A2}}{D}$$

where

$$N_{A1} = \lambda[p(\lambda + \mu s) - s]Q_0 + \lambda [\mu s^{N+1}(pP_N + qQ_N) - p\lambda Q_0(1-s)]$$

$$N_{A2} = \lambda q(\lambda + \mu s)Q_0 + \mu [\mu s^{N+1}(pP_N + qQ_N) - p\lambda Q_0(1-s)]$$

$$\text{and } D = \mu s - (\lambda + \mu s)(\lambda q + \mu p)$$

whence

$$R_A(s) = P_A(s) + Q_A(s) = \frac{\mu s^{N+1}(pP_N + qQ_N) + \lambda[(\lambda + \mu s)(p - q) - (p + qs)]Q_0}{\mu s - (\lambda + \mu s)(\lambda q + \mu p)} \quad (12)$$

The condition  $R_A(1) = 1$  gives

$$\mu(pP_N + qQ_N) + \lambda Q_0(-2q) - (\mu - \lambda)q = 0 \quad (13)$$

The denominator of (12) has a zero within the unit circle  $|s| = 1$ . Let this zero be denoted by  $\beta$  where

$$\beta = \frac{\lambda(\lambda q + \mu p)}{\mu(\lambda p + \mu q)} \quad \text{which } < 1 \text{ if } \mu > \frac{1}{2}.$$

Now  $R_A(s)$ , being a probability generating function, is analytic within its radius of convergence and thus it follows that

$$\mu\beta^{N+1}(pP_N + qQ_N) + \lambda Q_0 [(\lambda + \mu\beta)(p - q) - (p + q\beta)] = 0 \quad (14)$$

(13) and (14) on solution give

$$\mu(pP_N + qQ_N) = \frac{q(\mu - \lambda)[(\lambda + \mu\beta)(p - q) - (p + q\beta)]}{2q\beta^{N+1} + [(\lambda + \mu\beta)(p - q) - (p + q\beta)]}$$

and

$$\lambda Q_0 = \frac{-q(\mu - \lambda)\beta^{N+1}}{2q\beta^{N+1} + [(\lambda + \mu\beta)(p - q) - (p + q\beta)]}$$

Finally  $R_A(s) =$

$$\frac{q(\mu - \lambda)[\{(\lambda + \mu\beta)(p - q) - (p + q\beta)\}S^{N+1} - \beta^{N+1}\{(\lambda + \mu s)(p - q) - (p + qs)\}]}{[2q\beta^{N+1} + (\lambda + \mu\beta)(p - q) - (p + q\beta)] [\mu s - (\lambda + \mu s)(\lambda q + \mu p)]} \quad (15)$$

### Arrivals Uncorrelated

Let the arrivals be independent, that is, let  $p$  be the probability for an arrival, the probability for no arrival being  $q (= 1 - p)$  irrespective of the fact whether an arrival has or has not occurred at the previous time-mark. Assume  $P_n$  to be the probability that the queue is in state  $n$ , just before  $t_r$ . Relating the probabilities at  $t_r$ , with those at  $t_{r-1}$ , we see that

$$P_N = p(P_N\lambda + P_{N-1}\mu) + P_N\mu$$

$$P_n = p(P_n\lambda + P_{n-1}\mu) + q(P_n\mu + P_{n+1}\lambda) \quad 1 < n < N$$

$$P_0 = q(P_0 + P_1\lambda)$$

Define

$$G_A(s) = \sum_{n=0}^N s^n P_n$$

This set of equations give the solution

$$G_A(s) = \frac{\lambda(q + ps)P_0 - p\mu s^{N+1}P_N}{(\lambda q - p\mu s)}$$

The condition  $G_A(1) = 1$  gives

$$\lambda p_0 - p\mu P_N + (p - \lambda) = 0 \quad (16)$$

By a method similar to the one followed for the correlated cases we also have

$$\lambda(q + p\beta)P_0 - p\mu\beta^{N+1}P_N = 0 \quad (17)$$

where

$$\beta = \frac{\lambda q}{\mu p}, \text{ is the zero within the unit circle } |s| = 1$$

if  $q < \mu$

(16) and (17) give

$$P_0 = \frac{p\mu(p - \lambda)\beta^{N+1}}{p\lambda\mu(q + p\beta - \beta^{N+1})} \text{ and } P_N = \frac{\lambda(p - \lambda)(q + p\beta)}{p\lambda\mu(q + p\beta - \beta^{N+1})}$$

whence

$$G_A(s) = \frac{(p - \lambda)[(q + ps)\beta^{N+1} - (q + p\beta)s^{N+1}]}{(\lambda q - p\mu s)(q + p\beta - \beta^{N+1})} \quad (18)$$

### MODEL B

The assumptions in this model are the same as in Model A with the exception that when the system is empty, just before  $t_r$ , and if an arrival occurs at  $t_r$ , then

Probability (the arrival departs at  $t_r$ ) =  $\lambda$

Probability (the arrival does not depart at  $t_r$ ) =  $\mu (= 1 - \lambda)$

Using normal methods it can be shown that the following set of equations characterize the system

$$P_n = p(P_n\lambda + P_{n-1}\mu) + q(Q_n\lambda + Q_{n-1}\mu) \quad 1 \leq n \leq N \quad (19)$$

$$P_0 = pP_0\lambda + qQ_0\lambda \quad (20)$$

$$Q_N = P_N\mu + Q_N\mu \quad (21)$$

$$Q_n = q(P_{n+1}\lambda + P_n\mu) + p(Q_{n+1}\lambda + Q_n\mu) \quad 0 < n < N \quad (22)$$

$$Q_0 = qP_1\lambda + qP_0 + pQ_1\lambda + pQ_0 \quad (23)$$

Define the generating functions

$$P_B(s) = \sum_{n=0}^N s^n P_n \text{ and } Q_B(s) = \sum_{n=0}^N s^n Q_n \quad (24)$$

Equations (19), (20) and (24) give

$$P_B(s)[1 - p(\lambda + \mu s)] + Q_B(s)[-q(\lambda + \mu s) + \mu s^{N+1}(pP_N + qQ_N)] = 0 \quad (25)$$

Similarly (21), (23) and (24) give

$$P_B(s)[q(\lambda + s)] + Q_B(s)[p(\lambda + \mu s) - s] + \mu s^{N+1}(pP_N + qQ_N) - \mu(1 - s)(qP_0 + pQ_0) = 0 \quad (26)$$

Subtract (26) from (25)

$$\mu P_B(s) - \lambda Q_B(s) + \mu(qP_0 + pQ_0) = 0 \quad (27)$$

(26) and (27) on solution give

$$P_B(s) = \frac{N_{B1}}{D} \text{ and } Q_B(s) = \frac{N_{B2}}{D}$$

where  $N_{B1} =$

$$\mu(qP_0 + pQ_0)[p(\lambda + \mu s) - s] + \lambda[\mu s^{N+1}(pP_N + qQ_N) - \mu(1 - s)(qP_0 + pQ_0)]$$

$$N_{B2} = -\mu(qP_0 + pQ_0)(\lambda + \mu s)q + \mu[\mu s^{N+1}(pP_N + qQ_N) - \mu(1 - s)(qP_0 + pQ_0)]$$

and  $D = \mu s - (\lambda + \mu s)(\lambda q + \mu p)$

whence

$$R_B(s) = P_B(s) + Q_B(s) = \frac{\mu s^{N+1}(pP_N + qQ_N) + \mu(qP_0 + pQ_0)[(\lambda + \mu s)(p - q) - 1]}{\mu s - (\lambda + \mu s)(\lambda q + \mu p)} \quad (28)$$

The condition  $R_B(1) = 1$ , gives

$$\mu(pP_N + qQ_N) + \mu(qP_0 + pQ_0)(-2q) - (\mu - \lambda)q = 0 \quad (29)$$

The denominator of the right hand side of (28) is the same as in Model A. Therefore if  $\beta$  is the zero of the denominator and  $R_B(s)$  being a definite quantity the numerator must vanish or  $s = \beta$

Thus

$$\mu\beta^{N+1}(pP_N + qQ_N) + \mu(qP_0 + pQ_0)[\lambda + \mu\beta](p - q) - 1 = 0 \quad (30)$$

(29) and (30) on solution give

$$\mu(pP_N + qQ_N) = \frac{(\mu - \lambda)q[(\lambda + \mu\beta)(p - q) - 1]}{[(\lambda + \mu\beta)(p - q) - 1 + 2q\beta^{N+1}]}$$

$$\mu(qP_0 + pQ_0) = \frac{-q(\mu - \lambda)\beta^{N+1}}{[(\lambda + \mu\beta)(p - q) - 1 + 2q\beta^{N+1}]}$$

Finally we have  $R_B(s) =$

$$\frac{q(\mu - \lambda)s^{N+1}\{(\lambda + \mu\beta)(p - q) - 1\} - \beta^{N+1}\{(\lambda + \mu s)[2q\beta^{N+1} + (\lambda + \mu\beta)(p - q) - 1]\}[\mu s - (\lambda + \mu s)]}{[\mu s - (\lambda + \mu s)]} \frac{(p - q) - 1}{(\lambda q + \mu p)} \quad (31)$$

*Arrivals Uncorrelated*

By a similar reasoning as in Model A (uncorrelated arrivals) we have, in the present case, the following equations

$$P_N = p(P_N\lambda + P_{N-1}\mu) + P_N\mu$$

$$P_n = p(P_n\lambda + P_{n-1}\mu) + q(P_n\mu + P_{n+1}\lambda) \quad 0 < n < N$$

$$P_0 = pP_0\lambda + q(P_0 + P_1\lambda)$$

Define

$$G_B(s) = \sum_{n=0}^N s^n P_n$$

The above set of equations give the following solution

$$G_B(s) = \frac{\lambda q P_0 - p\mu s^{N+1} P_N}{(q\lambda - p\mu s)} \tag{32}$$

$G_B(1) = 1$  gives

$$\lambda q P_0 - p\mu P_N + (p - \lambda) = 0 \tag{33}$$

Also if  $\beta$  (which  $< 1$ , if  $q < \mu$ ) is the zero of the denominator of the expression in (32), then

$$\lambda q P_0 - p\mu \beta^{N+1} P_N = 0 \tag{34}$$

(33) and (34) on solution give

$$\lambda q P_0 = \frac{(p - \lambda)\beta^{N+1}}{(1 - \beta^{N+1})} \text{ and } p\mu P_N = \frac{(p - \lambda)}{(1 - \beta^{N+1})}$$

whence

$$G_B(s) = \frac{(p - \lambda)(\beta^{N+1} - s^{N+1})}{(1 - \beta^{N+1})(\lambda q - \mu ps)} \tag{35}$$

**Particular Cases for the Generating Functions**

*MODEL A*

*Arrivals Correlated*

(i) If the barrier at N is removed,

$$R_A(s) = \frac{\lambda - \mu}{2} \frac{(\lambda + \mu s)(p - q) - (p + qs)}{\mu s - (\lambda + \mu s)(\lambda q + \mu p)} \tag{36}$$

If, in addition,  $r=0$ ;  $p=q=\frac{1}{2}$  i.e., the arrivals are uncorrelated

$$R_A(s) = \frac{(\lambda - \mu)(1 + s)}{2(\lambda - \mu s)} \tag{37}$$

(ii) When  $r=1$ ;  $p=1$ ,  $q=0$  i.e., when there is a perfect correlation  $R_A(s)=0$ , in other words, the system degenerates.

(iii) When  $r=-1$ ;  $p=0$ ,  $q=1$ , i.e., when there is a perfect negative correlation

$$R_A(s) = \frac{(\mu - \lambda)[(\lambda + \mu\beta + \beta)s^{N+1} - \beta^{N+1}(\lambda + \mu s + s)]}{[2\beta^{N+1} - (\lambda + \mu\beta) - \beta][\lambda^2 - \mu^2 s]}$$

(iv) When  $r=0$ ;  $p=q=\frac{1}{2}$

$$R_A(s) = (\mu - \lambda) \frac{(1 + \beta)s^{N+1} - (1 + s)\beta^{N+1}}{[2\beta^{N+1} - (1 + \beta)][\lambda - \mu s]} \tag{38}$$

*Arrivals Uncorrelated*

(i) When the barrier at N is removed

$$G_A(s) = \frac{(\lambda - p)(q + ps)}{q\lambda - p\mu s}$$

If, however,  $p=q=\frac{1}{2}$

$$G_A(s) = \frac{(\lambda - \mu)(1 + s)}{2(\lambda - \mu s)}$$

This agrees with the result in equation (37) which was found by making the arrivals uncorrelated.

(ii) When  $p=q=\frac{1}{2}$

$$G_A(s) = (\mu - \lambda) \frac{[(1 + \beta)s^{N+1} - (1 + s)\beta^{N+1}]}{(\lambda - \mu s)[2\beta^{N+1} - (1 + \beta)]}$$

which is in conformity with the expression given in (38). Also note that the value of  $\beta = \frac{\mu}{\lambda}$ , the same as in the correlated case when  $p=q=\frac{1}{2}$ .

*MODEL B*

*Arrivals Correlated*

(i) When the barrier at N is removed

$$R_B(s) = \frac{\lambda - \mu}{2} \frac{(\lambda + \mu s)(p - q) - 1}{\mu s - (\lambda + \mu s)(\lambda q + \mu p)}$$

Also when  $p=q=\frac{1}{2}$

$$R_B(s) = \frac{\lambda - \mu}{\lambda - \mu s} \tag{39}$$

(ii) When  $r=1$ ;  $p=1$ ,  $q=0$

$R_B(s)=0$ , in other words, the system degenerates.

(iii) When  $r=-1$ ;  $p=0$ ,  $q=1$

$$R_B(s) = \frac{(\mu - \lambda)[(\lambda + \mu\beta + 1)s^{N+1} - (\lambda + \mu s + 1)\beta^{N+1}]}{(\lambda^2 - \mu^2 s)[2\beta^{N+1} - (\lambda + \mu\beta + 1)]}$$

(iv) When  $r=0$ ;  $p=q=\frac{1}{2}$

$$R_B(s) = \frac{(\mu - \lambda)(s^{N+1} - \beta^{N+1})}{(\lambda - \mu s)(\beta^{N+1} - 1)} \tag{40}$$

Arrivals Uncorrelated

(i) When the barrier at N is removed

$$G_B(s) = \frac{(\lambda - p)(q + ps)}{q\lambda - \mu ps}$$

If, however,  $p = q = \frac{1}{2}$

$$G_B(s) = \frac{\lambda - \mu}{\lambda - \mu s}$$

which is in agreement with the result found in equation (39)

(ii) When  $p = q = \frac{1}{2}$

$$G_B(s) = \frac{(\mu - \lambda)(s^{N+1} - \beta^{N+1})}{(\lambda - \mu s)(\beta^{N+1} - 1)} \tag{41}$$

The expression in equation (41) tallies with the one given in (40) which was found by making the arrivals uncorrelated. Also the value of  $\beta$  is the same as in the correlated case.

Mean Queue Length

MODEL A

Arrivals Correlated

The mean queue length is obtained by differentiating the expression in (15) with respect to s and then putting  $s=1$  therein. If  $L_A$  stands for the mean queue length

$$L_A = R'_A(l) = \frac{[(\lambda + \mu\beta)(p - q) - (p + q\beta)]q(\mu - \lambda)(N + 1)}{q(\mu - \lambda)[2q\beta^{N+1}$$

$$- \mu(\lambda p + \mu q)] - q\beta^{N+1}[(\mu - \lambda)\{\mu(p - q) - q\} + 2\mu(\lambda p + \mu q)] + (\lambda + \mu\beta)(p - q) - (p + q\beta)}$$

Arrivals Uncorrelated

Employing the procedure of the correlated case to the expression in (18) we get the mean queue length as

$$L_A = G'_A(l) = \frac{pq\beta^{N+1} + (q + p\beta)(q + p\beta)[(N + 1)(p - \lambda) - \mu p]}{[q + p\beta - \beta^{N+1}](p - \lambda)}$$

MODEL B

Arrivals Correlated

Differentiating the expression in (31) in the same manner as in Model A, we have the mean queue length,  $L_B$ , as

$$L_B = R'_B(l) = \frac{[\lambda + \mu\beta](p - q) - 1][q(\mu - \lambda)(N + 1) - \mu(\lambda p + \mu q)]}{q(\mu - \lambda)[2q\beta^{N+1} + (\lambda + \mu\beta) - \mu\beta^{N+1}[2q(\lambda p + \mu q) + (p - q)](p - q) - 1]}$$

Arrivals Uncorrelated

In the present case the mean queue length has been obtained from the expression in equation (35)

$$L_B = G'_B(l) = \frac{\mu p(\beta^{N+1} + 1) + (p - \lambda)(N + 1)}{(\beta^{N+1} - 1)(\lambda - p)}$$

Special Cases for the Mean Queue Lengths

MODEL A

Arrivals Correlated

(i) When  $r=1; p=1, q=0$

$$L_A = \infty$$

(ii) When  $r = -1; p=0, q=1$

$$L_A = \frac{(\lambda + \mu\beta + \beta)[(\mu - \lambda)(N + 1) - \mu^2] + \beta^{N+1}[(\lambda - \mu)(N + 1) - \mu^2]}{(\lambda + \mu\beta + \beta - 2\beta^{N+1})(\mu - \lambda)}$$

(iii) When  $r=0; p=q=\frac{1}{2}$

$$L_A = \frac{\beta^{N+1} + [(N + 1)(\mu - \lambda) - \mu](1 + \beta)}{(1 + \beta - 2\beta^{N+1})(\mu - \lambda)} \tag{42}$$

Arrivals Uncorrelated

When  $p = q = \frac{1}{2}$

$$L_A = \frac{\beta^{N+1} + [(N + 1)(\mu - \lambda) - \mu](1 + \beta)}{(1 + \beta - 2\beta^{N+1})(\mu - \lambda)}$$

This is in conformity with the expression given in (42) which was found by making the arrivals uncorrelated.

MODEL B

Arrivals Correlated

(i) When  $r=1; p=1, q=0$

$$L_B = \infty$$

(ii) When  $r = -1; p=0, q=1$

$$L_B = \frac{\mu\beta^{N+1}(\mu - \lambda) + (\lambda + \mu\beta + 1)[(N + 1)(\mu - \lambda) - \mu^2]}{(1 + \lambda + \mu\beta - 2\beta^{N+1})(\mu - \lambda)}$$

(iii) When  $r=0$ ;  $p=q=\frac{1}{2}$ 

$$L_B = \frac{\mu(\beta^{N+1}-1) + (\mu-\lambda)(N+1)}{(\beta^{N+1}-1)(\lambda-\mu)} \quad (43)$$

*Arrivals Uncorrelated*When  $p=q=\frac{1}{2}$ 

$$L_B = \frac{\mu(\beta^{N+1}-1) + (\mu-\lambda)(N+1)}{(\beta^{N+1}-1)(\lambda-\mu)}$$

which tallies with the expression in equation (43).

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**References**

- <sup>(1)</sup> Chaudhry, M. L. (1965). "Correlated Queueing," *C.O.R.S.*, **3**, 3, 142-151.
- <sup>(2)</sup> Heathcote, C. R. and Moyal, J. E. (1959). "The Random Walk in continuous time and its application to the theory of queues," *Biometrika*, **46**, 400-411.
- <sup>(3)</sup> Seth, Asha (1963). "The Correlated Unrestricted Random Walk," *J.R.S.S. Series B*, **25**, 2.



## Books available for Review

*Offers to review should be addressed to the Editor*

**The Scientist.**

H. Margenau and D. Bergamini.  
Time Life International. 1966. 35s. (No. 1448).

**A Simple Approach to Electronic Computers.** 2nd Edition.  
E. H. W. Hersee.  
Blackie & Son Ltd. 1966. 15s. p.b. 25s. bound. (No. 1449).

**Introduction to Transistor Electronics.**

R. L. Walker.  
Blackie & Son Ltd. 1966. 27s. 6d. p.b. 50s. bound. (No. 1450).

**Applied Electricity.** 4th Edition

A. W. Hirst.  
Blackie & Son Ltd. 1966. 35s. p.b. 55s. bound. (No. 1451).

**Linear Automatic Control Systems with Varying Parameters.**

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Blackie & Son Ltd. 1966. 75s. (No. 1452).

**Dynamical Theory of Groups and Fields.**

B. S. DeWitt.  
Blackie & Son Ltd. 1965. 22s. 6d. p.b. 40s. bound. (No. 1453).

**The Dynamics of Linear and Non-linear Systems.**

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Blackie & Son Ltd. 1965. 105s. (No. 1454).

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Ed. H. Barnes.  
George Allen & Unwin Ltd. 1966. 147s. (No. 1462).

**Principles of Physical Oceanography.**

G. Neumann and W. J. Pierson Jr.  
Prentice/Hall International Inc. 1966. 180s. (No. 1463).

**Marine Radio Manual.**

G. L. Danielson and F. C. Mayoh.  
George Newnes Ltd. 1966. 80s. (No. 1464).

# Book Reviews

**Fundamental Analogue Techniques.** By R. J. A. Paul.  
Pp. x + 216. London and Glasgow; Blackie and Sons  
Ltd. 1965. Price 35s.

This book is one of the "Electronic User Series," a series published for the benefit of the qualified scientist and engineer for whom electronics is a fringe activity. Professor Paul writes particularly for the scientist (or engineer) who perhaps uses the analogue computer, but may not fully appreciate the basic principles involved. The author is aware that there are omissions, but hopes he has treated a limited coverage in sufficient detail, rather than a superficial treatment of a wider field.

The usual brief introduction to the different types of computers is given in Chapter 1 before attention is directed to the differential analyser, with which the remainder of the book is concerned. The basic principle of computation by indirect analogy (*i.e.* the use of a computer as a graphical equation solver) is introduced in Chapter 2. Equations are transformed from physical variables to computer variables, and block diagrams are drawn indicating the mathematical processes required for solution. A list of requirements for the mechanisation of the blocks is then drawn up, and the way in which these requirements are met by the electronic machine is discussed. Details of the construction of the various electronic components are not included, but adequate references are given.

In the use of any analogue machine the selection of the scaling factors which are used to transform the mathematical equations is dictated by the limits of the computing elements. Programming and scaling is dealt with in Chapter 3, including the use of time as a parametric variable, to avoid division.

Historically, electrical systems were studied by investigating the behaviour of the analogous mechanical systems. However, in more recent years the opposite technique has developed rapidly due to the ease of constructing analogous electrical systems. In Chapter 4 of this book dynamic analogies are discussed and the fundamental concepts of a potential and a flow variable introduced. It is possible using this technique to construct an electrical network analogous to a mechanical system without first deriving the equations describing the mechanical system. The last section of this chapter describes the application of this technique to direct simulation on the analogue computer.

Integrators in the "hold" condition can be used for a limited period as memory devices, and comparators (or electronic switches) can be used to control the mode of the integrator (*i.e.* reset, hold or compute). Using these techniques it is possible on a purely analogue machine to undertake certain iterative computations. The solution of a number of problems is described in Chapter 5, for example, the evaluation of a definite integral, a split-boundary value problem, and an optimisation problem.

The engineer can in many cases draw a block diagram of the system he is studying, and he can determine the output/input relation for each block in transfer function form. It is often of advantage to simulate such a system on the computer in a similar manner, so that a block of computer components can be more closely identified with its physical counterpart. This method of simulation is of particular advantage if it is later proposed to replace blocks by the actual components of the physical system. To apply this technique it is necessary to be able to simulate rational transfer functions; there are several methods and the basic ideas are described in the final chapter of this book.

There are three appendices, on the Laplace transform, generalised two-port networks, and the symmetrical lattice network.

Professor Paul's stated aims in writing this book are quoted in the first paragraph of this review, and one could not do other than say that he fulfils these objectives. However, the fulfilment of these objectives, together with the limitation in the size, limits the number of readers who will find this book useful. It is a book for the hitherto casual user of the analogue computer, who, after reading the book, should certainly have a better understanding of the fundamental principles involved, and may be able to make more effective use of the computer in the future.

J. B. Spencer

**Concise Intermediate Physics.** By H. V. Pilling. Pp. xiii + 393. London; The English Universities Press Ltd. 1965. Price 21s.

The author states that this textbook is written for Advanced Level G.C.E. and University Entrance Candidates who are preparing for careers in medicine and branches of science other than physics. These students do not normally make a special study of mathematics and for this reason an attempt has been made to present the subject matter without using calculus. The reader is assumed to have a knowledge of mathematics only to the standard demanded by the Ordinary Level G.C.E. and with this limitation the author has tried to impart an appreciation of some fairly advanced physical principles.

The book is written in two parts. Part I (270 pages) contains the main text, while Part II (120 pages) consists of a wide variety of worked examples and exercises. Corresponding chapters in each part deal with the same topics so that it is a very easy matter to find the problems associated with any particular section of theory.

It is inevitable that with a relatively short text such as this, aiming to cover the whole 'A' Level physics course, many topics are discussed rather briefly and others are completely omitted. The author does mitigate himself to a certain extent by stating in the preface that the student should be concurrently pursuing an organised course of practical work and that descriptions of experiments which belong to such a course are not included. However, the practical course would have to be very comprehensive in order to fill all the gaps that appear in this text. Particularly brief sections include those dealing with Thermal Conductivity, Latent Heat, Spiral Springs and the Electrical Discharge in Gases at Low Pressures. There are also sections where the author demands rather a lot from the 'O' Level mathematics with which he assumes his readers to be acquainted. This applies particularly to the chapter dealing with "Electromagnetism and Dynamics." A good feature of the book is its inclusion of some of the recommendations made in the Science and Education Report on Physics in 1961. These are particularly evident in the chapter dealing with "Atomic and

Nuclear Physics" which should be very useful to all 'A' Level physics students. The book is also well illustrated throughout. However, after reading the first part, one is left with the feeling that the average student relying on it as his main source of knowledge would not do very well in an examination.

Part II of the text consists of a really useful collection of worked and unworked examples taken from past papers of various examination boards and universities. The questions are graded in each chapter beginning with almost trivial exercises and finishing with problems of some considerable difficulty. The worked examples are also graded and have their solutions presented clearly and logically.

To sum up, this book can really only be used for an 'A' Level physics course in conjunction with other texts which will provide the necessary extra details. Although the author states that the student should be concurrently pursuing a practical physics course, he gives no indication as to where the student should look in order to find these details. One assumes he should refer to one of the well known standard texts on physics, in which case he may as well buy that book instead of this one! It seems to me that the disadvantages of having a much restricted content far outweigh any advantage the student may hope to gain from simplified mathematics.

**R. G. F. Taylor**

**Classical Electromagnetic Radiation.** By J. B. Marion. Pp. xv + 479. London; Academic Press Inc. (London) Ltd. 1965. Price \$10.75.

This is a book on classical electrodynamics with special emphasis on radiation. The choice of contents and presentation are intended for the present-day student of physics.

Maxwell's field equations are developed at an early stage from the fundamental experimental laws, in more or less the historical order. There is a chapter on the Laplace and Poisson equations, and a useful and unusual chapter on multipole fields; the usual well-worn exposition on two dimensional electrostatics is omitted. The propagation of waves in both free space and material media is considered, and then the influence of boundaries including refraction, reflection and guided waves. A chapter on the Lienard-Wiechert potentials and the classical theory of radiation from moving particles, is followed by another on dipole and more complicated radiating systems. The latter seems to stray a little into antenna engineering, without being sufficiently comprehensive to be useful. We return to physics with a short chapter on classical electron theory, including dispersion theory, Thomson scattering, and other phenomena bordering upon the limits of classical theory. Three chapters follow on interference phenomena and scalar diffraction theory including, as we would expect, Rayleigh and associated macroscopic scattering phenomena. Again there is a little straying into the field of interference optics with insufficient detail to justify its inclusion. Finally there is a concentrated chapter on relativistic electrodynamics, which includes the Lagrangian formulation of the field equations.

A feature of this book is its interesting historical notes and references. There is a good supply of problems for the student throughout, many of which illustrate important phenomena not covered in the text. Gaussian units are used.

This book is intended for the student up to at least degree level, and we consider this aim is well fulfilled.

In general the presentation is clear and logical. The more lengthy or complicated problems included in advanced texts are either omitted or merely mentioned. However, basic mathematical methods are not shirked, and the student who has mastered this book will have a valuable groundwork for more specialised study. The book has an index, and can also be used as a reference book by the non-specialist.

The book is clearly and accurately printed on nice paper, which makes it pleasant to use. The price seems well justified, although we wonder whether the student would not have preferred a cheaper edition.

**R. B. R. Shersby-Harvie**

**The Theoretical Significance of Experimental Relativity.** By R. H. Dicke. Pp. xii + 153. London & Glasgow; Blackie & Son Ltd. 1965. Price 32s. 6d.

This book is the outcome of a series of lectures delivered by the author in July, 1963 at a Summer School in Les Houches. The main object is stated to have been the determination of those relativistic field theories of gravitation which were compatible with the principal observational results, viz.:

- i. Null experiments like those of Eötvös, and Michelson & Morley.
- ii. The three famous tests of General Relativity (Gravitational red shift, gravitational bending of light, and precession of Mercury's perihelion).
- iii. Cosmological observations.

The first 30 pages, which are substantially the Les Houches lectures, are devoted to a discussion of these observations, and are followed in the remaining 120 or so pages by 12 Appendices which are, in fact, reprints of papers published by the author either alone or in conjunction with others. The conclusions, on p. 30, classify possible theories under the headings of "cannot be excluded," "unlikely," or "can be excluded."

The language and notation throughout are those of the Tensor Calculus, and the distinguished authorship is a guarantee of a high level of scholarship. However, apart from the addition of up-to-date references to the relevant literature, the book remains simply a reprint of the lectures and published papers, any adequate idea of which would be difficult to convey in the space of a brief review; it will most likely be valuable to specialists actually working in this field, but one suspects that the general reader would find it rather hard going, and possibly disappointing.

**W. E. Dawson**

**Polarized Light.** By W. A. Shurcliff and S. S. Ballard. Number 7 in Momentum Series. Van Nostrand, 1965. \$1.50. 144 pp. Price 12s.

The first impression is of a handy paperback monograph, but this is belied by the easy style of writing, the absence of difficult mathematics and excursions from the limited field of the title.

The nature and behaviour of electromagnetic waves are particularly well described and a few pages are devoted to analytical methods and short cuts. The principles and practical aspects of the better known types of polarizer are quite well covered with the aid of illustrations. Two chapters totalling some 35 pages are devoted to the application of polarized light, the examples being chosen

to illustrate their diversity. The final chapter includes brief descriptions of the available methods of generating polarized light. There is a short bibliography, an index and a list of contents that includes section as well as chapter headings.

Editing and proof reading have been a little careless; one notices the occasional mis-spelling and that illustrations do not always face the relevant page of text. The chatty style is mostly effective, but it is a difficult technique to maintain and the authors lapse at times into the spoken aside. It is a mark of the success with which so much information is put across so painlessly that repetitions (e.g. of the reference to Chapter 2 in paragraph 2 on page 9) and long-windedness (e.g. page 103) are infrequent but particularly annoying. Most professional and amateur scientists would enjoy reading this book, and many would find that a subject they had regarded as curiously obscure finally made sense. Your reviewer has tried out samples on half-a-dozen people ranging from a physiologist working on human vision to a 15 year old budding scientist and they all said that they would be buying their own copy.

J. P. Grantham

**The Physics of Musical Sound.** By C. A. Taylor. Pp. xii + 196. London; The English Universities Press Ltd. 1965. Price: Book 35s., Record 7s. 6d. (not sold separately).

Musical acoustics is one of the most fascinating subjects, and if the teaching of sound in schools and universities (which is said to have languished in recent years) is to be re-vitalised, a course based on the relationship between physics and music would seem an ideal choice. Professor Taylor is certainly of this opinion and he quotes his own experience of being taught physics and music by the same schoolmaster, and later himself teaching the physics of sound to university students of whom many had been at schools in which the subject was neglected. However, although the overall level of this book is approximately that of a first year University course, it is intended that the book should appeal to anyone interested in the physics of musical sounds. Consequently the mathematics has been arranged in sections marked with an asterisk, and these can be omitted by the less-mathematically minded readers without any loss in continuity.

The theme connecting the chapters of the book is a hypothetical research project—the design of a new musical instrument. To this end the characteristics of musical sounds are investigated, and existing musical instruments analysed. The book begins with a study of the generation of simple tones, and the origin of differences in tonal quality. The analysis of more complex sounds is then discussed and the Fourier transformation method introduced; for the benefit of the non-mathematician analysis by optical transformation methods is described. Further chapters deal with the coupling of systems, the starting transients which are so important in the recognition of particular instruments, and other influences on the character of musical sounds (formants, etc.). At this stage in the collection of design data, the production of single notes has been investigated quite thoroughly. Most music, however, involves harmony and the simultaneous sounding of different notes. The combination of notes, and the concept and development of the musical scales are the next topics to be discussed. The survey of the basic physical phenomena is concluded with chapters on the influences on the perception of sound, and the physical characteristics of conventional musical instruments.

If the hypothetical research project were to be completed one would expect the last chapter to contain details of some new musical instrument. In fact it is admitted that the project is unfinished; the last chapter does, however, deal with attempts that have been made in recent years to produce new instruments (e.g. "Les Structures Sonores Lasry-Baschet").

By any standards this is an excellent book, beautifully produced, and it is worthy of the highest recommendation. Non-scientific readers may find some sections hard going (particularly those dealing with optical transforms), but this small reservation does not apply to RNSS members.

A 7-inch extended play record is sold with this book (the two are not in fact sold separately). This record was made by Professor Taylor using amateur recording equipment, and consists of illustrations of some of the physical principles involved in producing musical sounds. On first hearing the record one may think that a great deal more could have been demonstrated on a longer, professionally made record. On the other hand, the very fact that the recording is an amateur production stimulates the listener into doing experiments of his own.

J. B. Spencer

**Introduction to Chemical Thermodynamics.** By M. H. Everdell. Pp. xxviii + 436. London; The English Universities Press Ltd. 1965. Price 50s.

Although this book has been written for the undergraduate reading for an honours degree in chemistry, a large proportion of it will also be useful to students studying for a similar qualification in physics. It gives a very clear account of what tends to be a rather difficult subject and is a very easy book to read compared with many others that cover the same topic. The author has tried to show the importance of thermodynamics by explaining generally its foundations and applications before continuing with the more detailed theory.

The book is written in four parts. The first introduces the reader to the language of thermodynamics and some of its concepts, such as entropy. The nature of thermodynamics is also briefly described and the author distinguishes between classical and statistical thermodynamics. Classical thermodynamics is based on four laws which provide the rules governing the energy and entropy changes associated with any process. These laws are each stated, amplified and some simple applications discussed. The chapter finishes with a brief outline of statistical thermodynamics and how it differs from the classical case. At this point in the book the reader has been given a sufficient introduction to the subject to enable him to cope with the more detailed theory which follows.

Part II is entitled "Classical Thermodynamics" and gives a very thorough and detailed account of the laws of thermodynamics and their applications. It is shown how the Zeroth law leads to the establishment of temperature as a property of a system and the First law is then discussed with particular reference to its application to gases and the energetics of chemical reactions. The Second law is introduced and the Carnot cycle explained. This is then followed by the calculation of entropy changes in both reversible and irreversible processes. Finally it is shown how thermodynamics can explain the behaviour of solids and also predict possible chemical reactions.

The statistical approach to thermodynamics is covered in Part III. It begins with a discussion of thermodynamic probability and shows how this is related to the entropy of a system. The application of statistics to systems of

various types is considered and the basic analysis is given for the formulation of Bose-Einstein and Fermi-Dirac statistics. The Maxwell-Boltzmann distribution function is derived for an ideal gas and this leads to the idea of a partition function. The calculation of the thermodynamic properties of gases is given in terms of the rational partition function of the molecule and a method for the evaluation of this partition function is also described.

Part IV is concerned with the application of thermodynamics to special systems. Topics included are the phase relations and thermodynamics of solutions, a discussion of the properties of electrolytes and the thermodynamics of electro-chemical cells. The final chapter gives some methods for the determination of the fugacity coefficients of the components of a solution.

The whole book is very well written and although of high mathematical content there appear to be very few errors. Each chapter finishes with a set of questions and exercises and these are discussed, with answers, in an appendix. There is also a short section at the beginning of the book which deals with all the mathematical theorems that are used. The general plan of the book is good. The author builds up the subject from the beginning, assuming little previous knowledge on the part of the reader and has made a very useful and worthwhile contribution to the literature on this subject.

**R. G. F. Taylor**

**A New Approach to Chemistry.** By E. E. Cook. Pp. viii + 498. London, Longmans, Green & Co. Ltd. 1965. Price 13s. 6d.

The pattern of Introductory Texts is inevitably influenced by the gradually changing aspect of the relevant discipline, and it is of interest to compare the present volume with, for example, the school "Holmyard" of a generation ago. As the author states, an attempt has been made "to discard much of the out-of-date chemistry which has been accepted for too long, and to concentrate on the study of the subject from modern fundamental concepts." This must not, however, be construed too literally: the change is rather one of emphasis than pruning.

Present-day pupils, raised on "Dan Dare" can certainly accept atomic disintegration and electronics as every day terms, and the author is undoubtedly justified in introducing the concepts of the nuclear atom and electronic reaction at the outset. The first section of the book accordingly deals with fundamental principles such as symbols, formulae, atomic weights and equivalents, and then leads into a description of the modern ideas of the structure of matter and their bearing on the mechanism of chemical changes. At this point quantitative considerations are discussed in relation to equations and calculations therefrom, and this is followed by a classification of the various types of chemical change and a description of the factors which influence them. The ideas of catalysis and of chemical equilibrium are introduced without being developed formally.

The second and third parts deal with the metals and non-metals respectively. Some apology is made for this arrangement on the grounds that metals are more familiar to the reader, and that electrovalency is a simpler concept than covalency at this stage, but the treatment is not without precedent. In differentiating the two classes of elements, the conventional distinction by physical properties is, of course, mentioned, but the new approach is exemplified by emphasis on the ability to ionise by electron loss or gain respectively—a route which leads

immediately and easily into the electronic concept of valency. The student is first introduced to the Periodic Classification in Part I, so that he is not unfamiliar with this when the elements are discussed individually and their positions in the Classification defined. Although the metals and non-metals are discussed as such, opportunities are taken to introduce other topics as appropriate. Thus, it is encouraging to find a lucid account of hydrogen bonding and some of its implications in the section devoted to water.

Throughout the book the experimental and preparative aspect is kept to the fore by the description of illustrative experiments suited to the school laboratory, and two useful appendices on volumetric analysis and the molecular theory of gases round the text off neatly.

The preparation of logical and useful indexes is always a laborious and time-consuming task, and that in the present book leaves a certain amount to be desired. Thus, picking at random, "Texas" was obscure until it was seen to be cited as a source of sulphur, while "Cheddar" was found to relate to calcium carbonate and not cheese.

All-in-all, this text is a refreshing departure from the conventional approach of its many predecessors, and should form a useful introduction to advanced level chemistry.

**E. N. Dodd**

**Telemetry Systems.** By L. E. Foster. Pp. x + 308. New York; London; John Wiley & Sons Inc. 1965. Price 96s.

The development of Telemetry as a separate branch of communications science has closely followed the growth of the guided weapon and space industries. Readers will therefore not be greatly disturbed to find that this book whilst often referring to other possible uses, derives most of the descriptive examples from the use of telemetry in space and missile systems. Indeed the author describes himself as "Manager of the ACE-S/C Engineering Apollo Support Department" and ex manager "Telemetry and Command Systems Engineering Section," of the General Electric Company (of America). Telemetry in this context is therefore a radio communication system which makes use of the most advanced methods of radio transmission, reception, and multi-channel modulation techniques. With such a large field to cover in a limited number of pages selection has to take place, and as the book's title indicates the author has chosen to describe the system aspect of telemetry.

The book is divided into seven chapters covering: an introduction to telemetry; design considerations; fundamental engineering measurements; telemetry data reduction; television space communication; communication for manned space craft; and range instrumentation. This breakdown is a fair indication of the scope of the book, although the chapter headings do not altogether describe the contents of the chapters. Thus the introductory chapter is much more a description and comparison of coding methods whilst the chapter labelled "Design of Telemetry Systems" in reality works through the r.f. performance capabilities and describes modulation systems and their errors. In a similar way the chapter "Fundamental Engineering Measurements" aims at describing the optimum methods of measurements for telemetry systems. However, it is largely devoted to descriptions of transducers, although ways in which telemetry techniques can be used to measure such factors as engine wear are touched on. The later chapters describe some of the uses of telemetry and the

reduction of the data obtained to a usable form together with its integration with other range instrumentation.

Block diagrams are used widely throughout the text to illustrate the systems described and wherever applicable equations are stated usually with numerically worked examples applying to the particular system, although no attempt is made to derive or justify the equations. It is unfortunate that although there is an extensive bibliography, no notation references are made in the text to it, and about 20% of the bibliography is made up of either company reports or internal reports which presumably are not accessible to the general reader.

A great deal of effort has been expended to include up-to-date descriptions of the various parts of the latest space telemetry equipment and range equipment. Only one section is devoted to telemetry standards and this refers particularly to modulation and multiplexing techniques. Only passing reference is made to the standards of range equipment and no attempt is made to codify in table form. This makes it difficult for the reader to appreciate the overall constraints which are put on any telemetry system by the various aspects of transducers, modulation, transmission, reception, data reduction and correlation with other records, which add up to provide the information content and accuracy of the telemetry system.

Production and quality of the book is good and all block diagrams and illustrations are clear, although it is difficult to justify the inclusion of some of the half tone illustrations purely on their information content. Only occasionally does the author use jargon invented by the American space industry and it can usually be ignored.

As a description of the telemetry systems used in America this book is very informative and can be recommended, especially to any reader who wishes to get a grounding in telemetry without needing to delve too deeply into the theory of the techniques. There are some signs that further editing and re-arrangement would be an advantage but this is no doubt because of the need to produce an up-to-date book. The price asked is likely to prevent its wide sale on this side of the Atlantic but it would be a useful addition to the shelves of a technical library.

R. D. Rayner

**Automatic Control of Aircraft and Missiles.** By J. H. Blakelock. Pp. x + 348. New York, London, Sydney; John Wiley and Sons Ltd. 1965. Price 120s.

In June 1914 one of the first automatic controls for an aircraft was successfully demonstrated by Lawrence Sperry. He was seen flying close to the ground, standing in the cockpit with his hands over his head, while his mechanic walked up and down on the wing. Since then the autopilot has become an integral part of any aircraft, and the rapid development of aircraft has ensured that the controls designer has been continually faced with new problems. Nowadays flights of aircraft (and satellites) can be made "hands off" from start to finish. The author claims that this is the first book devoted exclusively to the automatic control of aircraft and missiles.

The book begins with the standard derivation of the aircraft equations of motion (using NACA derivative notation), the equations are linearised, and, following the usual assumptions, separated into longitudinal and lateral modes. The remainder of Chapter 1 considers the longitudinal equations in more detail. A number of longitudinal autopilots are analysed in Chapter 2, ranging from the simple displacement type to the integrated control system including glide slope and flare control. Analysis throughout the book is by root locus techniques.

Suitable transfer functions for autopilots for a four engine jet transport are given, except for those cases where the problems are not applicable to an aircraft of this type. Constructional details of the autopilots are not discussed, although there is a brief section on the use of gyros.

Chapter 3 considers the lateral equations of motion in more detail, and lateral autopilots are analysed in Chapter 4. In the analysis of longitudinal and lateral autopilots the results of analogue computer solutions are presented, illustrating the performance of the autopilots under discussion. The evolution of the high performance aircraft has resulted in the problem of inertial cross-coupling (not applicable to the jet transport). A control to cope with this problem is analysed in Chapter 5 and applied to an aircraft which exhibits this phenomenon.

The problems of designing controls for all operating conditions of the high performance aircraft has led to the development of self-adaptive controls. Three such controls are presented in Chapter 6, one of which (MH-90) is a forerunner of the system fitted in the X-15. Security classification undoubtedly precludes description of the most up-to-date control systems, and for this reason also the chapter on missile control systems is fairly brief. Systems for the Vanguard missile are analysed.

The aeroelasticity of aircraft and missiles can have serious effects; a method for determining the transfer function of a flexible missile is given in Chapter 8. Also included in this chapter is a discussion on propellant sloshing. The use of statistical design principles is introduced in the final chapter. Appendices follow dealing briefly with vectors, gyros, basic servo theory, fundamental aerodynamics and matrices. Finally longitudinal and lateral aerodynamics data are given for the F 94 A in a variety of conditions.

This is a book for the specialist, of great value to those embarking on a career in the design of autopilots, and a useful work of reference for the professional designer. It is a well written and beautifully produced book, as indeed it should be at this price; a price which probably precludes purchase by those not directly concerned although they would find it extremely interesting.

J. B. Spencer

**Bituminous Materials: Asphalts, Tars and Pitches,** Volume I. Edited by A. J. Højberg. Pp. xiii + 432. John Wiley & Sons, London and New York. 1964. Price 132s.

Bituminous materials are primary engineering materials which function largely because they are highly water resistant and thermoplastic. They have excellent adhesive and cohesive properties and being low in cost can be used in relatively thick films to give excellent protection against corrosive influences, such as sea water, and the weather. In spite of the availability of more sophisticated protective coatings, bituminous materials are still widely used as marine coatings the world over.

This book is the first of three on bituminous materials and concerns mainly fundamentals, modifiers and appropriate tests. It consists of nine chapters each written by different authors, and ranges from an introductory chapter on nomenclature and terms to specialised subjects such as the irradiation of bituminous materials and microbial action on bituminous materials. This type of approach to the subject inevitably leads to variations in style and some duplication of subject matter. Omissions are also evident and it is a pity that in such an authoritative work no reference is made to the use of cathodic protection in

conjunction with bituminous coatings for the protection of buried metals and immersed structures. Again, while a chapter has been devoted to rubber modifiers, there is no mention of modifications of tars and pitches with epoxy resins, which are widely used on Naval ships and submerged structures.

These criticisms apart, the book is well written in an authoritative style, each chapter being well supported with suitable diagrams, tables, photographs and bibliographies. The result is an excellent book on asphalts, tars and pitches, which should be useful to those in many parts of the industry as well as those wishing to gain a better understanding of the broad principles of the utilisation of bituminous materials.

**J. C. Kingcome**

**Theoretical Hydromechanics.** By N. E. Kochin, I. A. Kibel and N. V. Roze. Translated by D. Boyanovitch. Pp. v + 557. London: New York; John Wiley and Sons Ltd. 1964. Price 150s.

This is a translation of the fifth edition of *Theoretical Hydromechanics*, Volume 1, published in Moscow in 1955, which is clearly a standard Russian textbook on the classical theory of the mechanics of an incompressible inviscid fluid. Discussion of viscous fluids was deferred to a second volume.

It can be said at once that the text is both a comprehensive and a carefully detailed presentation of classical hydrodynamics. Arguments are sometimes long-winded with proofs including all the algebraic steps. For instance Thomson's Minimum Energy Theorem takes two pages whereas Milne-Thomson using vector notation takes less than half a page in his textbook. Vector notation is indeed offered from the first page but, as with other authors, once detailed proofs are begun full sets of equations in three-dimensional coordinates are generally resorted to. At least, given all the steps of a mathematical argument, the reader is spared the often misleading "It clearly follows . . ."

The content is very much what would be expected. The opening chapters cover fluid kinematics and the equations of motion. Quite unexpectedly there is then interpolated a 30-page chapter on hydrostatics. Returning to fluid motion comes a chapter on the simplest cases of integration of the equations of motion followed by a study of flows involving vortices. This leads up to a major chapter on two-dimensional flow problems including such topics as Joukowski profiles, thin aerofoils, cascades and discontinuous flow. The juxtaposition here of sections on aerofoils and unsteady motion could lead to misunderstanding; the very restrictive assumption is made of constant circulation round a plane contour in unsteady motion but this is indeed neither necessary nor appropriate in unsteady aerofoil theory. There is a shorter discussion on three-dimensional flow exemplified by the general motion of an ellipsoid; wing theory is not touched on. Finally, the longest chapter is on wave motion including theories of the tides and an introduction to ship waves. Personal predilection of the authors is shown perhaps in the space devoted to Karman vortex streets and planing of flat plates while in contrast flow normal to a flat plate with a cavity behind it is only discussed for the re-entrant jet case.

The table of contents is of little use to anyone new to the subject. For example the 130 pages on two-dimensional motion is given a chapter heading only with no further breakdown; some subdivision would be a help to understanding the logical development of the subject.

The five-page index is quite comprehensive but this serves a different purpose and is not an alternative to a contents list. An author list is also given but the present reviewer found this of limited value. The list is far from complete and it is not clear how an author qualifies for inclusion. Even Euler is omitted although his name is often quoted in the text. Again the reader will often not know whom a Russian writer will quote for, say, some well-known law. The equation of state for a perfect gas  $pV = RT$  is referred to on page 58 as Clapeyron's equation and on page 170 as the law of Gay-Lussac and Boyle-Marriotte. The differing attribution arises presumably from the fact that the respective chapters are the work of different authors.

The translation is very readable; in only one or two places might the editor (J. R. M. Radok) have clarified a point. The scalar product of two vectors is referred to as a "vector product," something quite different. By the word "union" (of forces) on pages 62 and 65 is meant the word "sum." "Piezometric head" on page 112 could surely have been written "static pressure." A student new to the subject could only but be puzzled by the words "recalling the definition of vorticity . . ." on page 3 when there has been no previous mention of it. Of the misprints noticed only a few might cause more than a moment's hesitation. The first quote, on page 66, of Bernoulli's equation has  $\rho$  in place of  $p$  ( $\rho$  is taken as unity). On pages 198-200, several quotations have terms involving  $(z-z_0)$  where these should read  $1/(z-z_0)$ .

For the benefit of the student a number of exercises are included, sometimes with solution. It is a pity however, that none are given on the major chapters on two- and three-dimensional flow due to the motion of a body.

The above are criticisms only of detail and the book is indeed a significant addition to the text-books available on classical hydrodynamics. With its own presentation it offers a useful complement to any standard text the English reader might already possess and it could well be used in place of the latter.

**D. L. Ryall**

**Guide to Radio Technique.** Vol. I. Fundamentals, Valves, Semi-conductors. By E. Julander. Pp. x + 238. Phillips Technical Library: London; Cleaver-Hume Press Ltd. (Macmillan and Co. Ltd.) 1965. Price 37s. 6d.

Volume II of the *Guide to Radio Technique* will deal with "Passive Elements and Fundamental Circuits" and Volume III with "Technique of A.M. Radio Reception, Transmitters, Measuring Instruments and Servicing."

This first volume is divided into six chapters of which chapters 1, 4 and 5 deal with the subjects mentioned in the title, Fundamentals of electricity, Valves and semi-conductors in that order; these three chapters form more than two thirds of the book, the chapter on valves being considerably longer than either of the other two.

Chapters 2 and 3, both very short, explain the complex numeral system and define the transmission units used in telecommunications and acoustics. Chapter 6, a mere 30 pages, is entitled "Generation, Propagation and Modulation of Electromagnetic Waves," and some idea of the elementary nature of the book can be gained from this magnificent compression.

This type of book has been written many times and the author himself is perhaps best qualified to assess how well it meets the needs of the readers for whom it is intended. Mr. Julander has lectured for many years at Engineering and Technical Colleges and doubtless knows the needs well; he has written the book in the hope

that it will be used by students at these colleges as well as commercial colleges and other teaching institutions. Of the urgent need for a fairly elementary textbook of this type there is little doubt; with the increasing use of electronic tools in all branches of industry, including management, and the ever increasing number of technicians required in the radio, television and other entertainments industries, the gulf between the specialist-technician and the rest of the population must be narrowed.

There is also a need to keep books of this type continually up-to-date and, although for example Mr. Julander would probably like now to change the balance of the valve and semi-conductor chapters, he has succeeded in discarding many of the trees which have in the past made the wood so difficult to see.

The material on multi-grid and multi-system valves is doubtless aimed at the technicians, mentioned in the preface, dealing with radio and television servicing; one might now feel that even these technicians could profit more from an introduction to silicon transistors and to uses of the NPN transistor. However no author can be blamed for the inability of textbooks to keep pace with the rapid development of techniques; a criticism which the author of this book may have to accept is his under-estimation, to use his own words, "for reasons which we need not go into here," of the importance of understanding the complex current flow within a PNP transistor, particularly when the convention he adopts results in a confusing multiplicity of minus signs.

The original Swedish version of this book was printed in Stockholm in 1959 and the Phillips Technical Library contains editions in German, Dutch, French and Spanish. This English edition, printed in the Netherlands in 1955, has been translated from the German by E. Grubba with no apparent errors of sense and an agreeable style which is often the result of a multi-lingual background.

The stated purpose of the book, which also gives some idea of the language style, is "to provide an introduction of electronics and radio technique which is easy to understand and does not involve great mathematical difficulties." The style and presentation certainly makes reading and understanding easy; sentences are rigidly restricted to an absolute minimum length, so that the text becomes a series of economically explicit statements easily assimilated; reference indexing and layout of text and diagrams could hardly be improved; the use of 214 explanatory diagrams and illustrations must be most welcome to readers looking for a fairly elementary textbook and a 14 page appendix provides some useful tables and nomograms for radio technicians.

Many authors have endeavoured to cover the same ground without introducing mathematical difficulties and the level at which difficulties occur is obviously a matter of personal opinion; Mr. Julander's opinion can best be conveyed by the fact that he uses an 18 page chapter to explain the complex numeral system before using it, a better appreciation of difficulties, one feels, than many of his predecessors.

R. L. Short

**Electronics Data Handbook.** By M. Clifford. Pp. 158. New York; Gernsback Library Inc.; London; Iliffe Books Ltd. 1965. Price 16s.

The publishers claim that this book "with its mass of information and its comprehensive index, will be invaluable as a work of constant reference for students and technicians alike." The index is certainly comprehensive but the information must disappoint any student or

technician looking for a useful addition to the forbidding amount of literature already available.

The purpose of the book is to provide in one volume the formulae most frequently required, with explanatory text to clarify the use or derivation of the equations. The intention is good, but the need has long been recognised and frequently met; a book of this type must supplement or improve on the material already in existence and it is difficult to see how this book succeeds in this respect; many publishers, including the Dorset House stable from which this book came, are annually involved in meeting the exact requirements which this book purports to do, and they throw in a diary, to boot, at a considerably lower price.

The trade diaries, which contain all of the material in this book, and often more, also have the considerable advantage of periodic revision; it is significant that a 1960, six shilling diary contains more useful transistor data than this *Electronics Data Handbook*.

All the commonly used formulae and equations for D.C. and A.C. circuits are covered up to Kirchoff's Laws and basic  $\tau$ ,  $\pi$  and M-derived filters. The valve is adequately covered with simple equivalent circuits but the transistor is sadly neglected without even an approximate equivalent circuit. The 28 pages of tables and data are the same miscellaneous collection that most of the trade diaries provide, ranging from the conversion of "Liters to Gals", through the "Fundamentals of Boolean Algebra" and back to rods, hogsheads and pecks!

A good diary every two or three years would, one feels, provide much better value for money.

R. L. Short

**Navies of the Second World War. German Submarines.** (2 Vols.) By H. T. Lenton. Pp. Vol. 1 126, Vol. 2 128. London: Macdonald & Co. (Publishers) Ltd. 1965. Price 15s. each volume.

Each of these pocket size volumes begins with a brief introductory survey of German Submarine development. Vol. 1 has nine pages dealing with the period 1934-39, and Vol. 2 has eight pages on the period 1939-45. The layout of each volume is exactly similar; following the introduction the submarines are tabulated in numerical order, against each entry is given the type number, name of builder, date of launch and fate. More detail about each type of submarine is given on pages interspersed throughout the table. This detail takes the form of line drawings (plan, side elevation, and two or three cross sections) and lists the displacement, dimensions, machinery, bunkers, radius of action, armament, complement and notes of any variations. There are also a number of photographs of submarines.

These volumes are two of a series dealing with navies of the second world war, and it is certainly of interest to have these details of German submarines collected together. The naval historian will probably find them of great value. For the more general reader there should be a key to the line drawings, indicating the various compartments. For some reason this has only been provided in one case, for the type XVII B in the second volume. Reproduction of the photographs is not as good as it could be, indeed some of the photographs could well have been omitted, particularly those of boats under attack where little can be seen of the submarines themselves. At fifteen shillings each these books are unlikely to find their way into many pockets, and as works of reference it is surprising to find them of such small dimensions. Nevertheless, for the interested there are details of each submarine (projected ones, as well as those built), and it is fascinating to study the developments that were considered by the Germans.

J. B. Spencer

**Ergonomics: Man in his Working Environment.** By K. F. H. Murrell. Pp. xix + 496. London; Chapman and Hall Ltd. 1965. Price 63s.

No doubt most readers of this Journal will have met the term 'ergonomics', but some may wonder just how wide-ranging are the studies which ergonomists actually pursue of 'man in his working environment.' In military circles the subject tends to hide under the less precise labels of 'personnel research' and 'human engineering.' On the other hand, in industry it has too often been regarded as an extension of 'work study,' or even as a kind of sophisticated 'study of seating.'

Now that ergonomics has begun to achieve the status of an academic discipline, the time is right for an authoritative exposition of just what the subject is all about. After all, there is now a Chair in Ergonomics at one of our new technological universities, and the subject is taught both at postgraduate and undergraduate levels in a number of other universities and technological institutions. Ergonomists (in name, rather than just by conversion from other disciplines) will soon be in the market as applied scientists.

An authoritative exposition is indeed what the author has set out to produce. His achievement is all the more remarkable considering the formidable range of empirical data he has encompassed without a simple and consistent theoretical framework into which everything readily fits. The book is a pioneering venture in this respect, being the first text published in English which explicitly tackles the whole scope of ergonomic topics. No-one could be better equipped to write it than Mr. Murrell. He was one of the principal founders of the first Ergonomics Research Society in 1949. (At the time, he was a principal scientific officer in the R.N.S.S. and Head of the Naval Motion Study Unit which he was transforming into one of the first ergonomics groups). During the last fifteen years, he has accumulated a unique background of industrial and academic research into ergonomic problems; a background which is informed and guided by constant reference to practical problems.

The Author presents Ergonomics both as an interdisciplinary science and a technology. The science is contributed from experimental psychology, climatic and work physiology, functional anatomy, and anthropometry. Its application, tempered by the practical experience of design and production engineers, is to problems of working in 'man-machine' environments. The environments in which Mr. Murrell is most interested are industrial, so it is not surprising that most of his examples and his explicit recommendations are related to Industry. Yet a high proportion of the ergonomic data themselves are, of necessity, drawn from studies which had a mainly military (often naval) objective. Most of the human problems of work with which the book is concerned have their close counterparts in the design of military equipments and the conditions under which servicemen work.

There are two parts to the book. The first, on the "Elements of Ergonomic Practice," is really a student's guide to the underlying psychological, physiological, and anatomical, basis of human occupational behaviour. There is a lot of ground to cover in the 125 pages allowed here. Among the fundamental mechanisms described are the skeletal and muscular system, metabolic 'regulators,' sensory- and neuro-physiology, and perception.

These early chapters are inevitably open to the criticism that nothing new is presented for the specialist. Nevertheless for readers who are not ergonomists by circumstance or profession, the outline in Part I is extremely useful to have; even though it is incomplete and sketchy in parts.

The least satisfactory discussion is of human perception. If, as the author says on page 97, "Perception in its widest sense can be taken to cover all those processes which lie between the input and output of an organism," then it is scarcely adequate to tackle the fundamentals of it in five pages. There are, of course, references to perceptual processes (not in the widest sense) elsewhere in the book. Indeed, one is left with a conviction that in the long run it would have been more helpful (even to student's for whom the Author has been particularly mindful throughout) to introduce all the 'theory' and quantitative data of Part I wherever they appeared most relevant in later chapters.

At the end of Part I, however, is a novel and welcome feature in a British text: a chapter on "Man as a System Component." This section will appeal particularly to systems engineers. It deals with man as a 'detector' and a 'data-processing' component. Much of the chapter is given over to quite the best available summary of the stimulating and creative work on the system concepts (somewhat quaintly called 'quickening' and 'unburdening') which originated in work at the U.S. Naval Research Laboratory by F. Taylor and H. Birmingham.

Part II, on "Practical Ergonomics," occupies the remaining three-quarters of the book. Here the Author displays his skill as an interpreter and collator of research, as well as a substantial contributor to it. Nearly all he has to say is relevant, much of it is important, and his manner of saying it is fresh and unambiguous. For some R.N.S.S. readers who can recollect acting as 'guinea-pigs' for Mr. Murrell during a series of experiments in 1950-51 on dial-reading, there will be the added satisfaction of seeing the results of their labour featured prominently in the section on instrumental displays. (Page 167 f.).

Fourteen chapters comprise Part II, arranged under three main headings. Under Design Factors are chapters on layout of equipment, seating, instrumental displays, controls, and compatibility among various critical features of control and display design. The section on Environmental Factors surveys temperature, noise, lighting and the visual environment, and vibration. Organisational Factors are represented by methods of investigating and of organising work itself, inspection, shift work, and age. By and large, these chapters afford an expected and comprehensive spread of subject matters, though the inclusion of aging (and, perhaps, the exclusion of training in occupational skills) is a reasonable choice of the Author's rather than a distinctively-ergonomic topic.

Mr. Murrell has interesting and useful things to say about all these topics. That he has little to say on some matters, like design for ease of maintenance (one page), and perhaps a disproportionate amount on some others (e.g. 36 pages on design of quantitative instruments: dials) is a consequence of the evident unevenness in the balance of research and development in ergonomics as a whole. In some areas, such as control design, there is evidently a wide array of empirical data without very much in the way of confirmatory experimentation or theoretical understanding. Much of the information presented in the chapter on design characteristics of controls is necessarily based still on *ad hoc* studies from World War II. For naval engineering designers it may be disappointing to find no mention of electronic displays, and a brief account only (two pages) of auditory displays. There is, of course, considerably more ergonomic data available on these matters; but the Author has presumably rated them of relatively little importance to his industrial readers. One of the most intriguing chapters is on Organisation of Work, containing much that is both novel and controversial on topics like 'fatigue' (a term which the

Author wishes, commendably, to abolish), pacing, and monitoring behaviour.

The Author is to be congratulated on the breadth and up-to-date nature of his references, of which there are nearly 500. They do ample justice to British work, including some of the neglected early studies of the pre-war Industrial Health Research Board. There are plenty of informative sketches and diagrams, together with some relevant photographs. The checklists of ergonomic data (Pages 141 and 417), and the glossary of terms used in connection with instrumental displays (Page 452), are useful features; and the subject index is adequate for the purpose of finding one's way to the principal contents. On the debit side, it must be noted that there are a number of proof-reading or printing errors scattered throughout the text, not all of them obvious to a reader fresh to ergonomics.

D. Wallis

**The Aircraft of the World.** By W. Green and G. Pollinger. Pp. 360. London; Macdonald and Co. (Publishers) Ltd. 1965. Price 95s.

It is ten years since "The Aircraft of the World" last appeared and since then it has increased in size from 211 pages to 360 pages and in price from 35s. to 95s. It is still the only book, in Britain at least, of its scope, and well worth its place on the bookshelf of anyone interested in aviation.

Some readers may disagree with the word "aircraft" in the title as this term includes all types of flying machines, both heavier than air and lighter than air, and thus embraces gliders and airships, which the book does not. With this proviso, the authors have done a good job in collecting details and illustrations of over fifteen hundred different types of aircraft, with three-view silhouettes of nearly eight hundred of them. The introduc-

tion claims that this volume includes details of *all* aircraft types that are still actually flying; certainly any omissions are not immediately obvious.

This edition is laid out differently from the two previous editions, and is in alphabetical order of countries of origin and then makers. Following the general section there are two specialised sections, one of Rotorcraft and one of Veteran and Replica aircraft. The Rotorcraft section lists 112 types, of which 11 are Russian, 12 are British and 57 American. The Veteran and Replica section is particularly interesting with 47 types described; these vary from the Avro 504K, a Bristol Boxkite replica and a Bleriot XI to the Spitfire, Hurricane and Swordfish.

For each type there is a short history and description, followed by details of Power Plant, Performance, Weights and Dimensions. With a book of this scope some errors are almost bound to occur. A printer's error probably accounts for the Beagle Airedale having a wing area of 109 sq. ft. instead of its 190 sq. ft. and there are some minor errors in the performance figures or dimensions of, for example, the Vanguard 950, the VC 10, the BAC 1-11, the Trident 1E, the Super Caravelle and the Dassault Mystere 20. There is, too, some inconsistency in the use of maker's names in the United Kingdom section: if the Vulcan and Comet must now be regarded as Hawker Siddeley products then surely the Lightning and the VC 10 should be claimed by BAC. However, the excellent index lists each type under its name as well as the maker, and enables any aircraft to be easily found.

Although the errors found were rather more numerous than one would like to find in a book of this class, they were not sufficiently serious to detract from its value. All that need be said of the presentation and paper is that both are up to Macdonald's usual high standard.

"The Aircraft of the World" can be thoroughly recommended.

P. D. Hocking



Notes

and

News

#### Admiralty Experiment Works

Recent visitors to the establishment have included the Mechanics Committee of the Aeronautical Research Council who toured the laboratories on the 21st June. On the 28th June, Dr. F. H. Todd, Scientific Adviser to the Commanding Officer and Director of the David Taylor Model Basin, accompanied by Mr. McCandless were entertained at the establishment, whilst on the 28th

July a visit was made by Dr. Bruce Johnson, the Associate Professor of Engineering at the U.S. Naval Academy, Annapolis. Mr. C. E. Sherwin (D.N.C.), Mr. Perry (D.D.N.C./X) and Mr. Moss (Superintendent, N.C.R.E.) visited the establishment on the 5th August when they held discussions with senior officers.

Mr. P. Lover (Chief Constructor) has recently visited Aden on a technical mission and Mr. G. Cox has been invited to take the chair at the Symposium on Integrated Ship Control Systems, which is to be held at the Marine Engineering Laboratory, Minneapolis, U.S.A.

Mr. S. G. Lankester has been elected Chairman of the local branch of the Institute of Strain Measurement.

The Institute of Electrical Engineer has awarded a prize to Mr. J. W. Randall.

#### Admiralty Materials Laboratory

Vice Admiral Sir Raymond Hawkins, C.B., Chief of Naval Supplies and Transport and Vice Controller, accompanied by his Naval Assistant, Captain A. K. Dodds, R.N., visited the laboratory on Wednesday, 17th August, when they were introduced to key members of the staff and were shown selected highlights of the research programme.

Mr. N. L. Parr left A.M.L. on the 1st September to take up his new appointment as Director Materials Research (Naval). He had been head of the Metallurgy

Division since 1954 and all staff at A.M.L. wish him every success in his new appointment.

A paper entitled "Artefacts and Trivial Effects in the Measurement of Delayed Fluorescence," by C. A. Parker and Thelma A. Joyce, has been published in the Journal of the Chemical Society, 1966 T. 821. In collaboration

with H. C. Borresen of the Institute of Clinical Biochemistry, University of Oslo, Dr. C. A. Parker has also published a note on "Some Precautions Required in the Calibration of Fluorescent Spectrometers in the Ultraviolet Region" in Analytical Chemistry, Vol. 38 (1966) 1073.



### "Tenovus" Cancer Fund Pedal Kart Race

Every year the Apprentices' Association of S.R.D.E., Christchurch, in conjunction with the "Tenovus" Cancer Fund hold a Pedal Kart Race, and this year the A.M.L. Apprentices were easy winners of the race, which was held on Saturday, 6th July. They competed against teams from local industry, public utilities and other R. & D. establishments in the area. The A.M.L. team shown in the photograph are left to right: Messrs: A. Mogg, P. Grant, A. Fairclough (in driver's seat), D. Taylor, D. Runyard, S. Beer, I. Taylor and A. Savage. Mr. Grant and Mr. Fairclough have recently completed their apprenticeship at A.M.L., being the first A.M.L. Apprentices to do so and are now graded as Laboratory Mechanics.

## NOTICE TO READERS

The Editor extends to all readers of the JOURNAL a cordial invitation to contribute articles of R.N.S.S., Naval or general scientific and technical interest.

Authors are requested particularly to note that as the JOURNAL is in the Restricted category the printing of material within its pages does not constitute open publication and does not prejudice the subsequent use of the material in the journal of a learned Society or Institution, or in the scientific and technical press.

Manuscript should be typewritten if possible, in double spacing and on one side of the paper only. It should include, in addition to the title, the name of the author together with initials, degrees and Department, Establishment or Ship. Pseudonyms are acceptable only in special circumstances. A convenient length is between 3,000 and 5,000 words, but other lengths are acceptable and no contribution would be debarred on this ground alone.

Illustrations are in most cases a desirable addition. Photographs should be of good quality, glossy, unmounted, and preferably between two and three times the size of the required final picture. Graphs and line drawings should be made on a similar large scale, with bold lines to permit reduction in block making.

Views and opinions expressed in the JOURNAL are not necessarily endorsed either by the R.N.S.S. or by the Editor.

All communications should be addressed to:—

H. L. R. Hinkley, Editor,

Journal of the Royal Naval Scientific Service,

Ministry of Defence,

Block 'B,' Station Square House,

St. Mary Cray, Orpington, Kent.

*Telephone: Orpington 32111 Ext 258*

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