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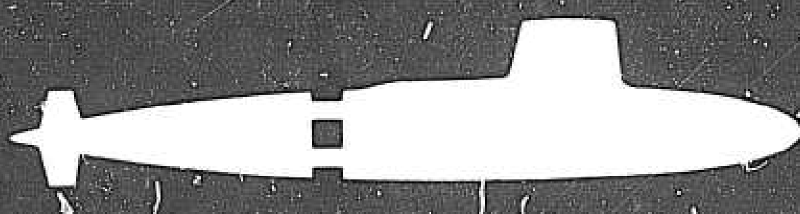

Submarine Integrated Control

OFFICE OF
NAVAL
RESEARCH

GENERAL DYNAMICS CORPORATION
ELECTRIC BOAT DIVISION
GROTON, CONNECTICUT

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PROCESSING OF DATA FROM SONAR SYSTEMS (U)

VOLUME IV

by

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ABSTRACT

This report deals with the following topics:

1) Detection in the presence of interference. Earlier studies of analogue detection in an interference dominated environment were reported in Volume III. Here the emphasis is on detectors using digital techniques. Costs of clipping and sampling are examined for optimal detectors as well as for more readily realizable suboptimal instrumentations. Some attention is given to the problem of recovering part of the clipping loss through multilevel quantization.

2) Adaptation to certain noise field properties. The report examines detectors which adjust the detection threshold in accordance with various measured noise parameters. An instrumentation using measured noise power in setting the threshold possesses non-parametric properties that make the false alarm rate asymptotically independent of the noise amplitude distribution. A second (digital) instrumentation achieves reduced dependence on noise spectral properties by adjusting the threshold in accordance with the count of zeros at each hydrophone.

3) Active Sonar. One possible detector for active sonar signals consists of a pair of widely separated receivers whose outputs are cross-correlated. The performance of such a receiver is analyzed for noise environments dominated by reverberation or by ambient noise. The effect of linear frequency modulation in the transmitted signal is examined.

4) Vertical directionality of ambient noise. Numerical computations are performed to estimate the directionality of ambient noise at shallow depths for various realistic velocity profiles. The basic noise model is that proposed by Talham.

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TABLE OF CONTENTS

* These reports are unclassified and are being issued as Supplement 1 to Volume IV

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FOREWORD

This report is the fourth in a series describing work performed by Yale University under subcontract to Electric Boat division of General Dynamics Corporation. The report covers the period 1 July 1965 to 1 July 1966. An unclassified supplement to this volume has been bound as a separate document, Report No. U417-67-084. Electric Boat is prime contractor of the SUBIC (Submarine Integrated Control) Program under Office of Naval Research contract NOnr 2512(00). LCDR. E.W. Lull, USN, is Project Officer for ONR; J.W. Herring is Project Manager for Electric Boat division under the direction of Dr. A.J. van Woerkom.

I. Introduction

The following is a summary of work performed under contract 8050-31-55001 between Yale University and the Electric Boat division during the period 1 July 1965 to 1 July 1966. More detailed discussions of the results as well as their derivations are contained in a series of six progress reports that are appended. Four of these represent continuations of investigations on passive detection reported in earlier volumes of this series, one initiates a new effort in the direction of active sonar signal processing (continued in subsequent reports) while the final one is concerned with a special topic in noise propagation.

One of the central themes in Volume III of this series was detection in a noise field dominated by a plane wave interference. The performance of the optimal (likelihood-ratio) detector was compared with that of a conventional detector and found to be substantially better in a strong interference environment. Several suboptimal analogue procedures (null steering) were proposed and analyzed. In the present volume this effort is continued with the examination of procedures suitable for digital implementation. The simple introduction of sampling and hard limiting into the previously analyzed suboptimal instrumentations was found to lead to degradations in performance which grow with interference to ambient noise ratio. A study was therefore undertaken, a) to separate the loss into a sampling component and a clipping component, b) to determine what fraction of the loss was independent of the particular implementation (hence a basic clipping loss), and c) to estimate the improvement attainable through use of n rather than 2 level quantization.

In most analytical treatments of the passive detection problem the statistical properties of signal and noise are assumed to be known a priori.

This is clearly unrealistic in most practical sonar problems. As one attempts to relax the assumption one is lead to instrumentations which measure relevant statistical parameters and adapt the detector to the measurements. Depending on one's point of view, the resulting system might be, at one extreme, a configuration adapting to the complete space time structure of the noise field or, at the other extreme, a system adapting only to one or two parameters deemed particularly important by the designer. The null steering procedures mentioned earlier may be said to fall into the second category, the single unknown parameter in this case being the interference bearing. Another parameter almost certainly unknown a priori is the total noise power or signal-to-noise ratio. Since it critically affects the detection threshold, measurement and adaptation suggests itself. Initial studies concerning this problem were contained in Volume III. Two reports in the present volume extend this effort by developing procedures insensitive not only to noise power level, but also noise amplitude distribution and, to some extent, noise spectral properties.

The initial effort in the active sonar field deals with a detector cross-correlating the outputs of two spatially separated receivers in an environment that may be dominated by either ambient noise or reverberation. One of the primary aims of this study was the development of an analytical framework for the discussion of reverberation that could be carried over to future investigations of active signal design and processing.

II. Detection in the Presence of Interference

Two reports in this volume deal with detection in a noise environment dominated by a strong plane wave interference. Both assume that the output of each hydrophone is converted into ligital (generally binary) form prior to further processing. Report No. 26 examines a digital version of the null steering procedure analyzed in Volume III (Report No. 21). The hydrophone outputs are sampled and hard-limited, digitally delayed for alignment with the interference, subtracted pairwise for interference elimination, after which beamforming on the target and detection is accomplished in conventional manner. The sampling procedure degrades performance by two mechanisms.

1) If the sampling period corresponds to the minimum delay increment available, then the sampling rate fixes the number of beams that can be formed. Unless the interference falls precisely on the axis of one of these beams it cannot be eliminated perfectly by the null steering mechanism.

2) Even if the interference lies precisely on a beam axis (and could therefore be eliminated perfectly by an analogue nulling procedure), hard limiting spreads the spectrum over an infinite frequency interval (even if the input spectrum is bandlimited). Therefore any finite sampling rate leads to a certain amount of spectral foldover with attendant loss of information.

The first effect is easily studied by considering a sampled analogue version of the null steering detector. Signal interference and ambient noise are assumed prewhitened to a bandwidth w_o rad/sec so that the Nyquist rate is w_o/π samples/sec. The residual interference power is a maximum when the interference is located midway between the two most closely adjacent beams. In this worst case the sampling rate f_s (samples/sec) required to hold the residual interference to a power no larger than that of the ambient noise is given approximately by

$$f_s = 0.9 \sqrt{\frac{I}{N}} \times \text{(Nyquist rate)} = 0.9 \sqrt{\frac{I}{N}} \times \frac{w_o}{\pi} \quad \text{for} \quad I/N \geq 1 \qquad (1)$$

I/N is the interference to noise ratio. Thus, for an interference to noise ratio of 100 (20 db), one must sample at about 9 times Nyquist rate.

Having isolated the beam steering problem, one can now deal with the clipping loss and the additional sampling loss due to mechanism 2) by constraining the interference to lie on the beam axis. In order to separate the effects of clipping and sampling as much as possible (always keeping in mind that there would be no type 2) sampling loss in the absence of clipping) it is convenient to consider two cases:

a) Sampling at the Nyquist rate of the prewhitened signal, noise and inference processes, so that the samples are statistically independent.

b) Sampling at a rate faster than the above by a sufficient amount so that the output signal-to-noise ratio (considered as a function of sampling rate) closely approaches its asymptotic value.

In case a) one finds that the clipping and sampling loss (ratio of output signal-to-noise ratio without clipping to output signal-to-noise ratio with clipping) is given roughly by

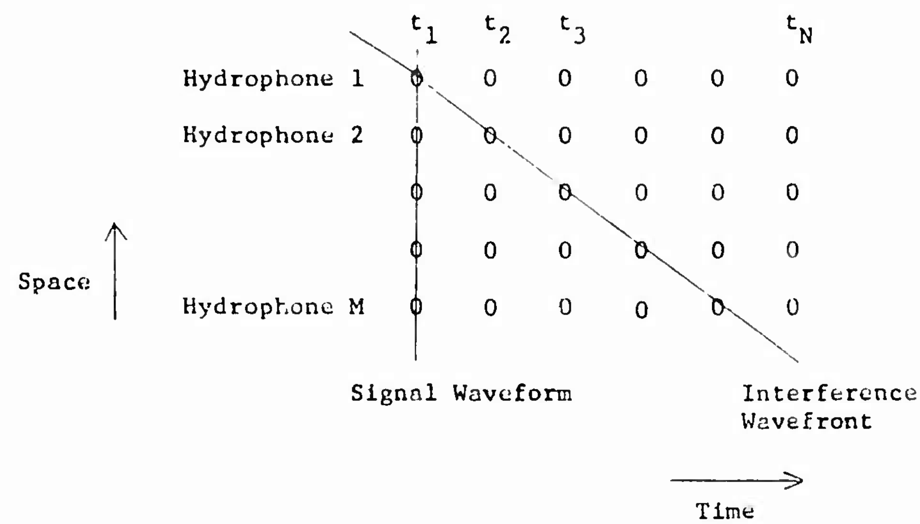
$$\text{clipping and sampling loss (in db)} = 10 \log_{10}\left(1.4 \sqrt{\frac{I}{N}}\right) \qquad \text{for } I/N>1 \quad (2)$$

The above result is essentially independent of the number of hydrophones once the array size exceeds a relatively small minimum. The improvement of the null steering detector over the equivalent (clipped and Nyquist rate sampled) conventional detector is quite substantial for strong interferences. For a 40 element linear array at I/N = 100 (20 db) the improvement amounts to 16.6 db of output signal-to-noise ratio. This figure increases by roughly 6 db for every 10 db increment in interference to noise ratio, but is relatively insensitive to changes in the number of hydrophones.

It is most convenient to express the results of part b) (fast sampling) in terms of those of part a) (Nyquist rate sampling) and an improvement factor I. The latter rises linearly with sampling rate until it approaches an asymptotic value near the sampling rate given by Eq. (1). Thus sampling above that rate brings little benefit. An estimate of the true cost of clipping can be obtained by subtracting from Eq. (2) the asymptotic value of I (expressed in db). The result (for reasonably large arrays) is a loss of about 2 db for I/N = 1 and an increase of roughly 2 db for every 10 db increase in interference to noise ratio. Similar computations for a set of non-white spectra indicate that these results do not depend critically on spectral shape.

Report No. 26 evaluated the cost of clipping for a particular instrumentation (null steering). It therefore leaves open the question whether the very appreciable losses encountered with high interference to noise ratios are peculiar to this particular system or whether they are a fundamental property of all clipped instrumentations. To shed light on this question, Report No. 28 considers a likelihood ratio detector operating on clipped hydrophone outputs. The results indicate that, for large interference to noise ratios, the optimum system is a clipped analogue of Anderson's null steering scheme (beam-form on interference, subtract average value of this interference estimate from each hydrophone, then beam form on the target). The performance difference (large I/N) between this system and the null steering scheme of Report No. 26 is minor (less than 1 db). Thus the clipping cost determined in Report No. 26 appears to be basic and not subject to reduction by improved processing procedures.

An interesting physical insight into the reasons for the clipping loss can be gained from the following argument. Consider a linear array mechanically steering on target so that the signal wave-front reaches all hydrophones simultaneously. The situation is illustrated in the diagram.



The vertical dimension corresponds to spatial displacement along the array, the horizontal to time displacement (successive samples) of the output of one hydrophone. In the absence of interference the noise is independent from hydrophone to hydrophone. The signal wavefront causes one polarity to be preferred along each vertical line and it is this preponderance, however slight, which allows detection when a sufficient number of samples (vertical lines) is examined. In the presence of a strong interference the polarities of samples along an interference wavefront are all the same unless the interference time function is near zero at the particular instant represented by that wavefront. A set of samples whose polarity is completely determined by the interference contributes no information concerning the presence or absence of the signal. Detection must then be accomplished on the basis of the residual samples, those corresponding to sufficiently low instantaneous values of interference so that their polarity is affected by signal and ambient noise. It is a simple matter to calculate the probability that all samples along an interference wavefront have the same polarity. For large interference to noise ratios this probability differs from unity by a factor of the order of $\sqrt{N/I}$. Hence only about $\sqrt{N/I}$ of the total samples are useful in signal detection. More quantitatively, one finds from this line of reasoning that the clipping cost for slow sampling is asymptotically $10 \log_{10} \frac{\pi}{2} \sqrt{\frac{I}{N}}$ [compare with Eq. (2)].

Report No. 28 also examines the improvement attainable through use of multilevel quantization in place of hard limiting. Approximate computations indicate that, for an interference to noise ratio of 20 db, the use of 4 quantization levels reduces the clipping loss from 12 db to 9 db for slow sampling and from 6 db to 4½ db for fast sampling. More than 8 levels are required to reduce the fast sampling loss to 3 db. One must note also that the adjustment of multilevel quantizers becomes more critically dependent on the ambient noise power as the number of levels increases.

III. Adaptation to Certain Noise Field Properties

The problem of detection in a noise environment of unknown power level was first considered in Volume III (Report No. 18). An instrumentation was there proposed which measured the average noise power over the observation interval and used the result to set the detector threshold.

In a Gaussian noise environment and with arrays of more than a very small number of hydrophones the resulting system performed almost as well as one having full a priori knowledge of the noise power. Report No. 23 of the present series examines the performance of the same adaptive threshold detector in more general noise environments. One interesting conclusion is that the false alarm rate becomes independent of the noise probability density as the observation time increases. In this very important sense, therefore, the detector is asymptotically nonparametric. The report also examines the dependence of detection probability (fixed false alarm rate) on the noise amplitude distribution. With Gaussian noise there is a degradation of performance equivalent to the loss of about ½ hydrophone (relative to the performance of a conventional detector operating in a noise environment of known power). On the other hand, it is not difficult to construct examples of non-Gaussian noise in which the detection probability (fixed false alarm rate) exceeds that of a conventional detector. This appears to be true especially with noise of an impulsive character.

Report No. 28 discusses an adaptive threshold for a clipped conventional detector (PCA detector). Clipping eliminates the dependence on noise power, but there remains a dependence on noise spectral properties. In particular, the fact that changes in noise bandwidth affect the dependence between adjacent samples suggests that improved performance might be obtained by adjusting the detector threshold in accordance with the noise bandwidth. An approximate measure of this bandwidth can be obtained from a count of zeros at the output of each hydrophone. The report considers the performance of a detector which ties its threshold to the zero count. The functional dependence of the threshold on the zero count is based on a nominal spectrum and the performance of the detector is then investigated for a wide variety of different noise spectra. The procedure is by no means perfect, but the false alarm rate turns out to be far less dependent on spectral properties than that of a fixed threshold detector.

IV. Active Sonar

Report No. 27 analyzes an active sonar detector using cross-correlation between the signals received at two spatially separated receivers. The transmitted signal consists of a single pulse of linearly frequency modulated carrier. The pulse duration is large compared with the carrier period. Transmitter, target, receiver and all scattering centers are assumed to be stationary, so that no Doppler shifts occur. The received signal is assumed to be a delayed, but otherwise perfect, replica of the transmitted signal and a similar assumption is made concerning the returns from various scattering centers which make up the reverberation. This description of the sonar return is obviously far too idealized for many purposes. It ignores most of the important transformations experienced by the signal in a realistic propagation channel. However, it has the merit of not obscuring the effects of the reverberation model whose development and study is one of the primary purposes of the report.

The reverberation is attributed to the effect of a large number of omnidirectional point scatterers, Poisson distributed throughout the illuminated volume (volume reverberation) or near the illuminated surface (surface reverberation). The Poisson assumption can clearly not account for such physically observed phenomena as a coherent component in the specular direction of a surface reflected wave or the mode structure induced in the scattered energy by a more or less periodic wind-driven surface. However, the model appears not unreasonable for volume reverberation and may adequately represent some of the features of surface reverberation, at least if the back-scattered component is of primary interest.

The report deals at some length with the effect of signal bandwidth. A linearly frequency modulated signal of wide bandwidth (large frequency deviation) imposes a rather narrow range gate on the reverberation. Its width is the correlation distance in the water of the transmitted signal. Only scatterers separated in range from each other or from the target by less than this quantity contribute to the reverberation noise at the output of the cross-correlator. A wideband signal also produces angular discrimination against false targets. This effect becomes significant when the spacing between receivers is large compared with the signal correlation distance.

When many weak scatterers but no strong scatterers (false targets) are illuminated, the model leads to a Gaussian reverberation process. The detector output signal-to-noise ratio is calculated for both reverberation limited and ambient noise limited environments. Under the stated assumptions (no Doppler shift) the output signal-to-noise ratio in a reverberation limited environment is generally maximized by use of the widest possible transmitted bandwidth. In an ambient noise limited environment, on the other hand, narrow-band signals (no frequency modulation) and narrow-band receivers produce the best output signal-to-noise ratio. The output signal-to-noise ratio varies as the square of the input signal-to-noise for low input signal-to-noise ratios, but as the first power of the input signal-to-noise ratio for high input signal-to-noise ratios. The latter behavior is basically that of a coherent detector whereas the former corresponds more nearly to incoherent detection. Even at very high input signal-to-noise ratios the correlation detector is still inferior to the true coherent detector (replica correlator) by 3 db of output signal-to-noise ratio. This is, of course, due to the availability in the replica correlator of a noise free replica, whereas each of the channels of the cross-correlator is noisy.

V. Vertical Directionality of Ambient Noise

Report No. 25 uses an ambient noise model introduced by Talham (JASA 36, 1541, 1964) to examine the vertical directionality of the ambient noise in a number of practically interesting situations. The model attributes ambient noise to a series of statistically independent noise sources uniformly distributed over the ocean surface. Talham's calculated noise distributions for hydrophones located near the bottom showed good agreement with measured results, at least for low sea states. Report No. 25 is concerned primarily with the noise field near the surface (100 - 500 ft. depth). Realistic velocity profiles for several locations and seasons were considered. The results indicate that very substantial differences may exist between the noise intensities in the vertical and horizontal direction (differentials in excess of 20 db were indicated in a number of cases). The computed directionality patterns fall into two classes. If the observation point is in a region of positive velocity gradient the pattern exhibits a peak near the horizontal. If the observation point lies in a region of negative velocity gradient there is a sharp null near the horizontal. Except for this anomaly (which is confined to the immediate vicinity of the horizontal) there are no major qualitative differences between the various calculated patterns. Aside from the major seasonal changes in velocity profiles the results do not appear to be critically dependent on the particular profile used.

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SUBOPTIMAL TECHNIQUES EMPLOYING HARD-LIMITING FOR DETECTING PASSIVE SONAR TARGETS IN THE PRESENCE OF INTERFERENCE

by

Morton Kanefsky

Progress Report No. 26

General Dynamics Electric Boat division

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DEPARTMENT OF ENGINEERING

AND APPLIED SCIENCE

YALE UNIVERSITY

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Summary

This report deals with a suboptimal instrumentation for the detection of a weak target in the presence of interference from a much stronger target as well as ambient noise. The instrumentation considered is identical to the nulling detector proposed in Report No. 21 with the hydrophone inputs being modified by sampling and hard-limiting.

It is shown that this implementation improves detectability considerably, relative to the standard clipped power detector PCA, for point source interference targets. The cost of hard-limiting is approximately 2 db for an interference to ambient noise power ratio $\frac{I}{N}$ of unity and increases almost linearly with $\frac{I}{N}$ at a rate of 2 db per decade. Thus for very strong interfering "point sources" ($\frac{I}{N} \sim 30$ db) the cost of clipping becomes significant. For this $\frac{I}{N}$, the improvement in nulling relative to the PCA is approximately 29 db for a 40 hydrophone array. In addition it is demonstrated that the assumption that such a strong interfering target can be regarded as a point source is in fact not unreasonable.

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I. Introduction

Report No. 21 proposes a nulling detector that approximates the likelihood-ratio detector for the passive detection of a weak sonar target in the presence of ambient noise and interference from a much stronger second target. This report considers the detectability cost of sampling and hard-limiting the hydrophone inputs prior to the nulling instrumentation. Intuition and previous results indicate that this cost is essentially that inherent in hard-limiting. However, should this cost be severe, it will have to be verified that the likelihood-ratio test for clipped inputs is not significantly better. Such verification is extremely difficult and will be omitted for the present.

The nulling detector assumes a linear array consisting of M equally spaced hydrophones. The hydrophone outputs are delayed to align the interference components and are then subtracted pairwise to eliminate the interference. The resulting differences are delayed once more in such a manner as to align their signal components. The outputs of the second set of delays are summed, squared and filtered in the conventional manner. In the modified detector, the hydrophone outputs are sampled and hard-limited prior to any processing.

II. Analog Nulling Detector

The detector proposed in Report No. 21, which will be called the Analog Nulling Detector (D_{an}), calculates the following test statistic

$$S_{an}(\bar{x}) = \sum_{k=1}^{Tf_s} \left| \sum_{i=1}^{M-r} x_i(t+k\sigma) \right|^2 , \tag{1}$$

where T is the decision time, f_s is the sampling rate, σ is the sample interval ($\sigma = 1/f_s$), and M is the number of hydrophones in the array.

The processes $x_i(t)$ are given by

$$x_i(t+k\sigma) = \frac{1}{2} y_i(k) - \frac{1}{2} y_{i,r}(k) , \tag{2}$$

where $y_i(k)$ represents the output of the i^{th} hydrophone delayed first by $i\alpha$, where α represents the delay per hydrophone needed to align the array with the interference, and then delayed again by $i\lambda$, where λ represents the delay per hydrophone needed to align the new array with the signal. The process $y_{i,r}(k)$ represents the output of the $(i+r)^{th}$ hydrophone delayed first by $(i+r)\alpha$ and then again by $i\lambda$. Hence

$$y_i(k) = n_i(t + k\sigma + i\lambda + i\alpha) + i(t + k\sigma + i\lambda) + s(t + k\sigma) ,$$

and

$$y_{i,r}(k) = n_{i+r}(t + k\sigma + i\lambda + (i + r)\alpha) + i(t + k\sigma + i\lambda) + s(t + k\sigma - r\lambda) , \tag{3}$$

where $n(t)$ represents the ambient noise, $i(t)$ represents the interference, and $s(t)$ represents the signal. If either the interference or signal were broadside, the second delay would be

$$\lambda = \frac{d}{c} \sin \theta , \tag{4}$$

where d is the spacings between hydrophones, c is the speed of sound in the ocean, and θ is the angle between the target and interference. The processes $n_j(t)$ where $j = 1, \ldots,$; $i(t)$; and $s(t)$ are assumed to be statistically independent, normal processes with normalized correlation functions $\rho_n()$, $\rho_i()$, and $\rho_s()$ and whose powers are N, I, and S respectively.

Let us first develope an expression for the autocorrelation function $\overline{x_i(t)\, x_j(t+k\sigma)}$ defined as $R_{i,j}(k)$. It is seen that

$$R_{i,j}(k) = \frac{1}{4}\left\{\overline{y_i(o)y_j(k)} - \overline{y_i(o)y_{j,r}(k)} - \overline{y_{i,r}(o)y_j(k)} + \overline{y_{i,r}(o)y_{j,r}(k)}\right\}, \quad (5)$$

where

$$\overline{y_{i,m}(o)y_{j,n}(k)} = N\delta(i+m-j-n)\,\rho_n\left[k\sigma+(j-i)\lambda+(j+n-i-m)a\right]+I\rho_i\left[k\sigma+(j-i)\lambda\right]+S\rho_s\left[k\sigma+(m-n)\lambda\right] \quad (6)$$

We can distinguish three cases of interest.

Case 1 $(i = j)$

$$R_{i,i}(k) = \frac{1}{2}\left\{N\rho_n(k\sigma) + S\left[\rho_s(k\sigma) - \frac{1}{2}\rho_s(k\sigma+r\lambda) - \frac{1}{2}\rho_s(k\sigma-r\lambda)\right]\right\}. \quad (7)$$

Case 2 $(i \pm r = j)$

$$R_{i,i\pm r}(k) = \frac{1}{2}\left\{S\left[\rho_s(k\sigma) - \frac{1}{2}\rho_s(k\sigma+r\lambda) - \frac{1}{2}\rho_s(k\sigma-r\lambda)\right] - \frac{1}{2}N\rho_n(k\sigma \pm r\lambda)\right\}, \quad (8)$$

Case 3 $(j = i \pm p;\ p \neq r)$

$$R_{i,i\pm p}(k) = \frac{1}{2}S\left[\rho_s(k\sigma) - \frac{1}{2}\rho_s(k\sigma+r\lambda) - \frac{1}{2}\rho_s(k\sigma-r\lambda)\right]. \quad (9)$$

It is convenient to define $R'_{i,i\pm r}(k) \equiv R_{i,i\pm r}(k) - R_{i,i\pm p}(k)$, where

$$R'_{i,i\pm r}(k) = -\frac{1}{4}N\,\rho_n(k\sigma \pm r\lambda) \quad (10)$$

We are now in a position to evaluate the signal-to-noise ratio $\overline{SNR}$ of the nulling detector for weak input signals. This ratio is defined by

$$\overline{SNR} = \frac{\Delta}{D}, \quad (11)$$

where

$$\Delta = \lim_{\frac{S}{N}\to 0}\left[E_K\{S(\bar{x})\} - E_H\{S(\bar{x})\}\right], \quad (12)$$

and

$$D^2 = \mathrm{Var}_H\{S(\bar{x})\}. \quad (13)$$

The subscripts H and K refer to the hypothesis of signal absent and the alternative of signal present. It is easily seen that

$$\frac{E\{S(\bar{x})\}}{Tf_s} = \sum^{M}\sum^{-r}_{j}\overline{x_i(t)x_j(t)} = (M-r)R_{i,i}(o)+2\sum_{p=1}^{M-r}(M-r-p)R_{i,i\pm p}(o)+2(M-2r)R'_{i,i\pm r}(o). \quad (14)$$

Hence substituting the results of Equations 7, 9, and 10 into Equation 14 and then into Equation 12 yields

$$A_{an} = Tf_s S \frac{(M-r)^2}{2} \left[1 - \rho_s(r\lambda) \right] . \tag{15}$$

The variance of $S_{an}(\overline{x})$ is given by

$$Var\left[S_{an}(\overline{x})\right] = \sum_{k}^{Tf_s} \sum_{q} \left[\overline{\beta(k)^2\beta(q)^2} - \overline{\beta(k)^2}\,\overline{\beta(q)^2} \right] , \tag{16}$$

where

$$\beta(k) = \sum_{i=1}^{M-r} x_i(t+k\sigma) .$$

Since $\beta(k)$ is normal, it follows that,

$$Var\left[S(\overline{x})\right] = 2 \sum_{k}^{Tf_s} \sum_{q} \overline{\beta(k)\beta(q)}^2$$

$$= 2Tf_s \left[\overline{\beta^2(k)}^2 + \sum_{p=1}^{Tf_s} \left(1 - \frac{p}{Tf_s}\right) \overline{\beta(o)\beta(p)}^2 + \sum_{p=1}^{Tf_s} \left(1 - \frac{p}{Tf_s}\right) \overline{\beta(p)\beta(o)}^2 \right] , \tag{17}$$

where

$$\overline{\beta(o)\beta(\kappa)} = \sum_{i}^{M-r} \sum_{j} \overline{x_i(t)x_j(t+k\sigma)} = (M-r)R_{i,i}(\kappa) + 2\sum_{p=1}^{M-r} (M-r-p)R_{i,i+p}(\kappa)$$

$$+ 2(M-2r)R'_{i,i+r}(\kappa) . \tag{18}$$

Once more combining Equations 7, 9, 10, 13, 17, and 18 results in,

$$D^2_{an} = 2Tf_s\left(\frac{M-r}{2}\right)^2 S^2 \left\{ \left[1 - \left(\frac{M-2r}{M-r}\right)\rho_n(r\lambda)\right]^2 + 2\sum_{k=1}^{Tf_s} \left(1 - \frac{k}{Tf_s}\right) \left[\rho_n(kT) \right.\right.$$

$$\left.\left. - \frac{1}{2}\left(\frac{M-2r}{M-r}\right) \left[\rho_n(k\sigma + r\lambda) + \rho_n(k\sigma - r\lambda)\right] \right]^2 \right\} . \tag{19}$$

From Equations 11, 15, and 19 one finally obtains,

$$\overline{SNR}_{an} = \sqrt{\frac{Tf_s}{2}}\,\frac{S}{N}\,\frac{(n-r)\left[1-\rho_s(r\lambda)\right]}{\sqrt{\left[1-\left(\frac{n-2r}{n-r}\right)\rho_n(r\lambda)\right]^2+2\sum_{k=1}^{Tf_s}\left(1-\frac{k}{Tf_s}\right)\left[\rho_n(k\sigma)-\frac{1}{2}\left(\frac{n-2r}{n-r}\right)\left\{\rho_n(k\sigma+r\lambda)+\rho_n(k\sigma-r\lambda)\right\}\right]^2}} \tag{20}$$

Throughout this report we will make the assumption that $r\lambda$ is sufficiently large so that $\rho_n(r\lambda) \cong \rho_s(r\lambda) \doteq 0$. This is not the same as requiring that the target and interference are far removed in bearing. For this later condition the assumption is satisfied for $r = 1$, and we can subtract adjacent hydrophones. When the interference is relatively close to the target, the assumption can be satisfied by making r large enough. For large arrays this can be accomplished at a fairly small cost for all bearing angles except those in the immediate vicinity of zero. For example if the inputs are pre-whitened and processed out to 5 KC, the hydrophone spacings are 2 feet, and $r = 4$; the assumption is identically satisfied for a relative bearing angle of $3^\circ 35'$ in the vicinity of broadside.

The primary motivation for the previous assumption is that it will greatly simplify the analysis, particularly when we consider hard-limited data. The justification can be found in the discussion of Report No. 21 about the bearing response pattern. It was pointed out that while the $\overline{SNR}$ of Equation 20 is insensitive to the relative bearing, the bearing response pattern looks rather bad for $r = 1$ when the interference is relatively close to the target. It was further noted that the bearing response can be greatly improved by increasing r. In practice the bearing response is sufficiently important as to require the assumption that $\rho_n(r\lambda) \cong \rho_s(r\lambda) \cong 0$ for all bearing angles above some small value. For this implementation, Equation 20 can be closely approximated by,

$$\overline{SNR}_{an} \cong \sqrt{\frac{Tf_s}{2}}\,\frac{S}{N}\,\frac{M - r}{\left\{1 + 2\sum_{k=1}^{Tf_s}\left(1 - \frac{k}{Tf_s}\right)\left[\rho_n^{\,2}(k\sigma) + \frac{1}{2}\left(\frac{M-2r}{M - r}\right)^2 \rho_n^{\,2}(k\sigma - r\lambda)\right]\right\}^{\frac{1}{2}}} \qquad (21)$$

Let us now consider the special case where the inputs are pre-whitened to w_o radians/sec and sampled at the Nyquist rate ($f_s = w_o/\pi$). Equation 21 becomes

$$\overline{SNR}_{an} = \sqrt{\frac{Tw_o}{2\pi}}\,\frac{S}{N}\,\frac{M - r}{\left[1 + \frac{1}{2}\left(\frac{M-2r}{M - r}\right)^2\left(1 - \frac{r\lambda}{T}\right)\right]^{\frac{1}{2}}} ,$$

and for large decision times

$$\overline{SNR}_{an} \cong \sqrt{\frac{Tw_o}{2\pi}}\,\frac{S}{N}\,\frac{M - r}{\sqrt{1 + \frac{1}{2}\left(\frac{M-2r}{M - r}\right)^2}} , \qquad (22)$$

Replacing r by 1 results in Equation 20 of Report No. 21.

As a second case, let us rewrite Equation 21 for large decision times in the following way

$$\overline{SNR}_{an} \cong \frac{\sqrt{T}\,\frac{S}{N}\,(M - r)}{\left\{2\sigma + 4\sum_{k=1}^{Tfs}\left[\rho_n^{\,2}(k\sigma)\sigma + \frac{1}{2}\left(\frac{M-2r}{M-r}\right)^2 \rho_n^{\,2}(k\sigma - r\lambda)\sigma\right]\right\}^{\frac{1}{2}}} .$$

For extremely rapid sampling rates (essentially continuous operation) this approaches

$$\overline{SNR}_{an} \cong \sqrt{\frac{T}{4\int_0^{\infty}\rho_n^{\,2}(\mu)\,d\mu}}\;\frac{S}{N}\,\frac{M - r}{\sqrt{1 + \left(\frac{M-2r}{M - r}\right)^2}} . \qquad (23)$$

It will become apparent that even for pre-whitened inputs one should sample significantly faster than the Nyquist rate for large interference to ambient noise power ratios.

III. Clipped Nulling Detector

The modified nulling detector will be called the Clipped Nulling Detector (D_{cn}). The first modification of the previous analysis occurs at Equation 6. Since the processes y(k) are limited to the values 1 or 0 depending on their sign, Equation 6 becomes

$$\overline{y_{i,m}(o)y_{j,n}(k)} = \frac{2}{\pi}\sin^{-1}\left[\frac{N(i+m-j-n)\rho_n\left[k\sigma+(j-i)\lambda+(j+n-i-m)\sigma\right]+I\rho_i\left[k\sigma+(j-i)\lambda\right]+S\rho_s\left[k\sigma+(m-n)\right]}{N+I+S}\right] \tag{24}$$

It follows that for the three cases of interest

Case 1 (i = j)

$$R_{i,i}(k) = \frac{1}{\pi}\left\{\sin^{-1}\left[\frac{N\rho_n(k\sigma)+I\rho_i(k\sigma)+S\rho_s(k\sigma)}{N+I+S}\right]-\frac{1}{2}\sin^{-1}\left[\frac{I\rho_i(k\sigma)+S\rho_s(k\sigma-r\lambda)}{N+I+S}\right]\right.$$
$$\left.-\frac{1}{2}\sin^{-1}\left[\frac{I\rho_i(k\sigma)+S\rho_s(k\sigma+r\lambda)}{N+I+S}\right]\right\} \quad . \tag{25}$$

Case 2 ($j = i \pm p$; $p \neq r$)

$$R_{i,i\pm p}(k) = \frac{1}{\pi}\left\{\sin^{-1}\left[\frac{I\rho_i(k\sigma\pm p\lambda)+S\rho_s(k\sigma)}{N+I+S}\right]-\frac{1}{2}\sin^{-1}\left[\frac{I\rho_i(k\sigma\pm p\lambda)+S\rho_s(k\sigma\mp r\lambda)}{N+I+S}\right]\right.$$
$$\left.-\frac{1}{2}\sin^{-1}\left[\frac{I\rho_i(k\sigma\pm p\lambda)+S\rho_s(k\sigma\pm r\lambda)}{N+I+S}\right]\right\} \quad . \tag{26}$$

Case 3 ($j = i \pm r$); $R'_{i,i\pm r}(k) = R_{i,i\pm r}(k) - R_{i,i\pm p}(k)$

$$R'_{i,i\pm r}(k) = \frac{1}{2\pi}\left\{\sin^{-1}\left[\frac{I\rho_i(k\sigma\pm r\lambda)+S\rho_s(k\sigma\pm r\lambda)}{N+I+S}\right]-\sin^{-1}\left[\frac{N\rho_n(k\sigma\pm r\lambda)+I\rho_i(k\sigma\pm r\lambda)+S\rho_s(k\sigma\pm r\lambda)}{N+I+S}\right]\right\} \tag{27}$$

Substituting these three Equations into Equations 12 and 14 yields

$$\Delta_{cn} = Tf_s \frac{M-r}{\pi}\left[\left\{\sin^{-1}\left[\frac{I}{N+I}\right]-\sin^{-1}\left[\frac{I+S\rho_s(r\lambda)}{N+I+S}\right]\right\}+\left(\frac{M-2r}{M-r}\right)\left\{\sin^{-1}\left[\frac{N\rho_n(r\lambda)+I\rho_i(r\lambda)}{N+I}\right]\right.\right.$$

$$-\sin^{-1}\left[\frac{N\rho_n(r\lambda)+I\rho_i(r\lambda)+S\rho_s(r\lambda)}{N+I+S}\right]+\sin^{-1}\left[\frac{I\rho_i(r\lambda)+S\rho_s(r\lambda)}{N+I+S}\right]$$

$$\left.\left.-\sin^{-1}\left[\frac{I\rho_i(r\lambda)}{N+I}\right]\right\}+2\sum_{k=1}^{M-r}\left(1-\frac{k}{M-r}\right)\left\{\sin^{-1}\left[\frac{I\rho_i(k\lambda)+S}{N+I+S}\right]-\sin^{-1}\left[\frac{I\rho_i(k\lambda)+S\rho_s(r\lambda)}{N+I+S}\right]\right\}\right] \tag{28}$$

Making the assumption that $p_n(r\lambda) = \rho_s(r\lambda) = \rho_i(r\lambda) = 0$, simplifies the previous equation to

$$\Delta_{cn} \cong Tf_s \frac{M-r}{\pi}\left[\left\{\sin^{-1}\left[\frac{I}{N+I}\right]-\sin^{-1}\left[\frac{I}{N+I+S}\right]\right\}+2\sum_{k=1}^{M-r}\left(1-\frac{k}{M-r}\right)\left\{\sin^{-1}\left[\frac{I\rho_i(k\lambda)+S}{N+I+S}\right]-\sin^{-1}\left[\frac{I\rho_i(k\lambda)}{N+I+S}\right]\right\}\right] \tag{29}$$

It is readily shown, via the Taylor expansion, that

$$\sin^{-1}\left(\frac{A+bs}{N+I+S}\right) = \sin^{-1}\left(\frac{A}{N+I}\right) + \frac{S}{N+I}\,\frac{b-\frac{A}{N+I}}{\sqrt{1-\left(\frac{A}{N+I}\right)^2}} + o\left(\frac{S}{N+I}\right)^2 .$$

Hence Equation 29 can be further simplified to

$$\Delta_{cn} \cong Tf_s \frac{M-r}{\pi}\,\frac{S}{N+I}\left\{\frac{I/N+I}{\sqrt{1-(I/N+I)^2}} + 2\sum_{k=1}^{M-r}\left(1-\frac{k}{M-r}\right)\frac{1}{\sqrt{1-\left(\frac{I}{N+I}\right)^2\rho_i^2(k\lambda)}}\right\}. \tag{30}$$

This can be more conveniently approximated by

$$\Delta_{cn} \cong Tf_s \frac{M-r}{\pi}\,\frac{S}{N+I}\left\{M-r-1+\frac{I/N+I}{\sqrt{1-(I/N+I)^2}} + \left(\frac{I}{N+I}\right)^2\sum_{k=1}^{M-r}\left(1-\frac{k}{M-r}\right)\frac{\rho_i^2(k\lambda)}{\sqrt{1-\left(\frac{I}{N+I}\right)^2\rho_i^2(k\lambda)}}\right\}. \tag{31}$$

We now wish to calculate the variance of the test statistic $S_{cn}(\bar{x})$. Unfortunately the nonlinear properties of hard-limiting make this calculation for the general case impossible to perform exactly. However by suitable approximations we can obtain a reasonable estimate of this variance. Let us first consider a very special case.

Pre-Whitened Inputs and Slow Sampling Rates

The assumption of pre-whitened inputs out to the processing frequency is certainly reasonable if one is concerned with the inherent cost of a particular implementation. Let us further assume that the inputs are sampled at the Nyquist rate, that is all of the samples in the output of a single hydrophone are statistically independent. Since the sampling rate determines the minimum delay it quantizes the permissable bearing angles to which the array can be aligned. Thus point sources can be only crudely nulled out even in the absence of hard-limiting. It will be seen that for interference to ambient noise ratios significantly greater than unity, this "sampling cost" severely degrades the performance of the detectors. It is useful, however, to consider this restrictive case first. Finally let us assume that the array can be aimed directly at the interference. This assumption not only removes the "sampling cost" but greatly simplifies the analysis.

Under the hypothesis (S = 0), given the assumptions, Equations 25, 26, and 27 can be rewritten as

$$R_{i,i}(k) = \begin{cases} \frac{1}{2}\left[1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right] & ; \quad k = 0 \\ 0 & ; \quad \text{otherwise,} \end{cases}$$

$$R_{i,i\pm p}(k) = 0, \tag{32}$$

$$R'_{i,i\pm r}(k) = \begin{cases} -\frac{1}{4}\left[1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right] ; & \text{if } k = \pm rm \\ 0 & ; \quad \text{otherwise,} \end{cases}$$

where $\lambda = m\sigma$. Since $\lambda \leq \frac{d}{c}$ it follows that $m \leq \frac{d}{c\sigma}$. The variance of S_{cn} under the hypothesis is

$$\mathrm{Var}_H(S_{cn}) = \sum_k^{Tf_s}\sum_q \sum_i^{(M-r)}\sum_j\sum_r\sum_s \left[\overline{x_i(t+k\sigma)x_j(t+k\sigma)x_r(t+q\sigma)x_s(t+q\sigma)} - \overline{x_i(t)x_j(t)}\ \overline{x_r(t)x_s(t)}\right] . \tag{33}$$

Consider first the term t_1, obtained by setting $k = q$.

$$t_1 = Tf_s \sum_i^{(M-r)}\sum_j\sum_r\sum_s \left[\overline{x_i(t)x_j(t)x_r(t)x_s(t)} - \overline{x_i(t)x_j(t)}\ \overline{x_r(t)x_s(t)}\right] .$$

From the fact that $x_i(t)$ is either 1, 0, or -1 and from Equation 32 we see that $x_i^2(t) = 1$ with probability $p \equiv R_{i,i}(o) = \frac{1}{2}\left[1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right]$ and zero otherwise. It follows that $x_i^4(t) = 1$ with probability p. Hence,

$$\begin{aligned}\frac{t_1}{Tf_s} &= (M-r)(p-p^2) + 2\sum_{i \neq s}\sum \overline{x_i^2(t)}\ \overline{x_s^2(t)} \\ &= \left[2(M-r)^2 - 3(M-r)\right]p^2 + (M-r)p.\end{aligned} \tag{34}$$

The term t_2 obtained by setting $k \neq q$ and ignoring the special case where $k - q = \pm rm$ can easily be shown to be identically zero. For this special case, let us consider $q = k + rm$ and by symmetry

$$t_3 = 2(Tf_s - rm)\sum_i^{(M-r)}\sum_j\sum_r\sum_s\left[\overline{x_i(t)x_j(t)x_r(t+rm\sigma)x_s(t+rm\sigma)} - \overline{x_i(t)x_j(t)}\ \overline{x_r(t)x_s(t)}\right] .$$

This term has a nonzero value for $r = i - r$, $s = j - r$, and $i \neq j$ (s and r can be reversed) as well as for the case $i = j$ and $r = s = i - r$. Hence

$$\begin{aligned} t_3 &= 4(Tf_s - rm)\sum_{i \neq j}\sum \overline{x_i(t)x_{i-r}(t+rm\sigma)}\ \overline{x_j(t)x_{j-r}(t+rm\sigma)} \\ &+ 2(Tf_s - rm)\sum_i\left[\overline{x_i^2(t)x_{i-r}^2(t+rm\sigma)} - \overline{x_i^2(t)x_{i-r}^2(t)}\right] .\end{aligned}$$

Recognizing that $R_{i,i-r}(rm) = -\frac{1}{2}p$, we obtain

$$t_3 = (Tf_s - rm)\left[(M-2r)^2 - (M-2r)\right]p^2 + (2Tf_s - rm)(M-2r)\mu \ , \tag{35}$$

where

$$\mu = \overline{x_i^{\,2}(t)\,x_{i-r}^{\,2}(t+rm\sigma)} - \overline{x_i^{\,2}(t)}\ \overline{x_{i-r}^{\,2}(t)} \quad .$$

Combining Equations 34 and 35 we obtain

$$D_{cn}^{\,2} = 2Tf_s(M-r)^2 p\left\{p\left[1 - \frac{3}{2}\,\frac{1}{(M-r)} + \frac{1}{2}\left(1 - \frac{rm}{Tf_s}\right)\left(\frac{M-2r}{M-r}\right)^2\right] + \frac{1}{2}\left(\frac{1}{M-r}\right)\left[1 + \left(1 - \frac{rm}{Tf_s}\right)\left(\frac{M-2r}{M-r}\right)(2\mu p^2)\right]\right\} \tag{36}$$

Although we cannot evaluate μ, we do recognize that $2\mu^2 - p^2$ is quite small (of the order μ^2). In fact if $x(t)$ were normal (not true) the term $2\mu^2 - p^2$ would be identically zero. Finally since $\frac{1}{2(M-r)}$ is normally much less than p (except for very large $\frac{I}{N}$ in which case $2\mu - p^2$ is bound to be very much less than p), the error in assuming that $2\mu - p^2 = 0$ is quite negligible. Therefore to a very good approximation

$$D_{cn}^{\,2} = 2Tf_s(M-r)^2 p^2\left\{1 + \frac{1}{2}\left(1 - \frac{r\lambda}{T}\right)\left(\frac{M-2r}{M-r}\right)^2 + \frac{1-3p}{2(M-r)p}\right\} \quad . \tag{37}$$

Combining Equations 11, 31, and 37 we obtain that for the pre-whitened case with slow sampling rates

$$\overline{SNR}_{cn} = \frac{2}{\pi}\sqrt{\frac{Tw_o}{2\pi}}\,\frac{S}{N}\,\frac{1}{\left(1+\frac{I}{N}\right)\left(1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)}\;\frac{M-r-1+\dfrac{I/N+I}{\sqrt{1-(I/N+I)^2}}}{\sqrt{1 + \frac{1}{2}\left(\frac{M-2r}{M-r}\right)^2\left(1 - \frac{r\lambda}{T}\right) + \frac{1-3p}{2(M-r)p}}} \quad . \tag{38}$$

where $p = \frac{1}{2}\left(1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)$. The cost of clipping for these slow sampling rates is obtained by comparing Equations 38 and 22. If the cost (C) is defined as the decrease in $\overline{SNR}$,

$$C = \frac{2}{\pi}\,\frac{1}{\left(1+\frac{I}{N}\right)\left(1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)}\;\frac{M-r+\dfrac{I/N+I}{\sqrt{1-(I/N+I)^2}}}{M-r}\sqrt{\frac{1 + \frac{1}{2}\left(\frac{M-2r}{M-r}\right)^2}{1 + \frac{1}{2}\left(\frac{M-2r}{M-r}\right)^2 + \frac{1-3p}{2(M-r)p}}} \quad . \tag{39}$$

For large arrays, Equation 39 can be approximated by

$$C \cong \frac{2}{\pi} \frac{1}{(1+\frac{I}{N})(1-\frac{2}{\pi}\sin^{-1}\frac{I}{N+I})} \quad . \tag{40}$$

If we now assume that $\frac{I}{N}$ is large and consider only the first two terms of the expansion

$$\sin^{-1}(1-z) = \frac{\pi}{2} - \sqrt{2z}\left[1 + \sum_{k=1}^{\infty} \frac{1\cdot 3\cdot 5\ (2k-1)}{2^{2k}(2k+1)\ k!}\ z^k\right]$$

we obtain

$$C \approx \frac{1}{\sqrt{2(1+\frac{I}{N})}} \quad . \tag{41}$$

It is observed that the cost of clipping increases with $\frac{I}{N}$ and gets quite large for large values of $\frac{I}{N}$. While this cost will get cut roughly in half with more sensible sampling rates, it will remain significant. The following table is an evaluation of Equation 39 for a variety of values of $\frac{I}{N}$ and M under the assumption that r = 1. (Actually the assumptions inherent in Equation 39 make this choice the only logical one, since the sampling rate prohibits the detector from searching for a target at any bearing angle other than those for which $\rho_n(\lambda) \doteq 0$. Equations 38 and 39 are of course based on the assumption that any target present is precisely at one of these bearing angles. These restrictions will soon be relaxed.)

M \ I/N	0	1	10	100	1000
10	1.9	2.9	6.15	10.2	13.9
40	2.0	3.15	6.6	11.1	15.5
100	2.0	3.2	6.7	11.4	16.1
∞	2.0	3.2	6.7	11.5	16.5

Table 1 Cost of Clipping in db for Slow Sampling Rates

While we are considering slow sampling rates, let us calculate the the improvement attained by nulling relative to the standard clipped power detector, the polarity coincidence array (D_{pca}).

IV. Improvement due to Nulling at Slow Sampling Rates

The PCA calculates the statistic

$$S_{pca}(\bar{x}) = \sum_{k=1}^{Tf_s} \left[\sum_{i=1}^{M} x_i(t+k\sigma) \right]^2 , \tag{42}$$

where

$$x_i(t) = \text{sgn}\left[y_i(t)\right] ,$$

and

$$y_i(t) = n_i(t+i\alpha) + i(t+i\lambda) + s(t) ,$$

and where all the symbols are as before. Hence it follows that

$$\overline{x_i(t)x_j(t+k\sigma)} = \frac{2}{\pi} \sin^{-1}\left[\frac{N\delta_{ij}\rho_n(k\sigma) + I\rho_i\left[k\sigma+(j-i)\lambda\right] + S\rho_s(k\sigma)}{N + I + S} \right] . \tag{43}$$

It is readily seen that

$$\frac{E(S_{pca})}{Tf_s} = \sum_i^M \sum_j \overline{x_i(t)x_j(t)} = M + \frac{4}{\pi}\sum_{k=1}^{M} (M-k) \sin^{-1}\left[\frac{I\rho_i(k\lambda) + S}{N + I + S}\right] . \tag{44}$$

Hence from Equation 12

$$\Delta_{pca} = \lim_{S/N\to 0} Tf_s \frac{4}{\pi}\sum_{k=1}^{M} (M-k)\left\{\sin^{-1}\left[\frac{I\rho_i(k\lambda) + S}{N + I + S}\right] - \sin^{-1}\left[\frac{I\rho_i(k\lambda)}{N + I}\right]\right\}. \tag{45}$$

It follows that for pre-whitened inputs and Nyquist rate sampling that

$$\Delta_{pca} = Tf_s \frac{2}{\pi} \frac{S}{N+I} (M^2 - M) . \tag{46}$$

The variance of S_{pca} under the hypothesis ($S = 0$) and with the slow sampling assumption is

$$\mathrm{Var}_H\left[S_{pca}(\bar{x})\right] = \sum_k^{Tf_s}\sum_q\left[\sum_i\sum_j\sum_r^M\sum_s \overline{x_i(t+k\sigma)x_j(t+k\sigma)x_r(t+q\sigma)x_s(t+q\sigma)} - M^2\right], \quad (47)$$

where

$$\overline{x_i(t)x_j(t)} = \begin{cases} 1 \,;\, i \neq j \\ 0 \,;\, i \neq j \end{cases}$$

and

$$\overline{x_i(t)x_j(t \pm p\sigma)} = \begin{cases} \frac{2}{\pi}\sin^{-1}\frac{I}{N+I} & ; \quad j = i \pm p/m \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Since $x_i^2(t) = 1$ for all i, Equation 47 can be rewritten as

$$D_{pca}^2 = \sum_k^{Tf_s}\sum_q \sum_{i\neq j}\sum\sum_{r\neq s}^{M}\sum \overline{x_i(t+k\sigma)x_r(t+q\sigma)x_j(t+k\sigma)x_s(t+q\sigma)} \quad . \quad (48)$$

$$= 2Tf_s\sum_{i\neq j}\sum \overline{x_i^2(t)}\;\overline{x_j^2(t)} + 4\sum_{p=1}(Tf_s-pm)\sum_{i\neq j}\sum \overline{x_i(t)x_{i-p}(t+pm\sigma)}\;\overline{x_j(t)x_{j-p}(t+pm\sigma)}$$

$$= 2Tf_s\left[(M^2-M)\,1 + 2\left(\frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)^2\sum_{p=1}^{M}\left(1-\frac{p\lambda}{T}\right)(M-p)(M-p-1)\right] \cdot \quad (49)$$

Recognizing that $2\sum_{p=1}^{M}(M-p)(M-p-1) = \frac{(2M-4)}{3}(M^2-M)$, Equation 49 becomes for

large decision times

$$D_{pca}^2 = 2Tf_s\left[1 + \bar{M}\left(\frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)^2\right]\left[M^2-M\right] \quad (50)$$

where

$$\bar{M} \cong (2M-4)/3 \quad .$$

Finally from Equations 46 and 50 we obtain

$$\overline{SNR}_{pca} = \frac{2}{\pi}\sqrt{\frac{Tw_o}{2\pi}}\,\frac{S}{N+I}\,\frac{\sqrt{M^2-M}}{\sqrt{1+\bar{M}\left(\frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)^2}} \quad . \quad (51)$$

Comparing this result with Equation 38 we see that the improvement as a consequence of nulling (J) for slow sampling rates is given by

$$J = \frac{\sqrt{1 + \bar{M}\left(\frac{2}{\pi}\sin^{-1} I/N+I\right)^2}}{\sqrt{1 + \frac{1}{2}\left(\frac{M-2r}{M-r}\right)^2 + \frac{1-3p}{2(M-r)p}}} \; \frac{M-r-1+\frac{I/N+I}{\sqrt{1-(I/N+I)^2}}}{\sqrt{M^2-M}} \; \frac{1}{\left(1-\frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)} \quad . \tag{52}$$

This improvement is plotted in db in Table 2 for r = 1.

M \ I/N	0	1	10	100	1000
10	-1.6	1.6	7.8	14.2	20.8
40	-1.0	3.7	10.6	16.6	22.4
100	-.8	5.4	12.6	18.5	23.9

Table 2 Nulling Improvement in db for Slow Sampling Rates

V. Effect of Sampling Rate on Nulling Procedure

As mentioned before the sampling rate quantizes the bearing angle to which the array can be aligned. Let us therefore assume that the array was not aligned exactly with the interfering point source. In the analog nulling detector the difference of a pair of hydrophones is seen to have the power

$$\overline{x^2(t)} = 2N\left\{1 + \frac{I}{N}\left[1 - \rho_j(r\varepsilon)\right]\right\} \quad , \tag{53}$$

where ε represents the error in the delay per hydrophone. If σ is the sampling interval, it is clear that the maximum error in the delay between adjacent hydrophones is $\sigma/2$. Since, in principle the hydrophones need not be all aligned in one direction, the maximum error in the delay for hydrophones further apart is not obvious. We will bypass this difficulty by assuming that the interference and target are sufficiently removed in bearing that r can be set equal to one. Now assuming that the interference spectrum is flat out to the processing frequency, we have that

$$\overline{x^2(t)} \leq 2N\left\{1 + \frac{I}{N}\left[1 - \frac{\sin w_o\sigma/2}{w_o\sigma/2}\right]\right\} \quad . \tag{54}$$

Previously we have assumed sampling at the Nyquist rate ($\sigma = \frac{\pi}{w_o}$). Hence the power of $x(t)$ can be as much as $(1 + .363x\ I/N)$ times greater than the idealized case. Therefore, for interference to ambient noise ratios much greater than unity, the interference will still be dominant.

A more reasonable sampling rate is one for which the "sampling cost" is at most 3 db (when $\epsilon = \sigma/2$). Actually this sampling cost is not only a function of the increased power, but the residual interference introduces a nonzero autocorrelation between the hydrophones as seen for example in Equation 43. This autocorrelation "costs" an additional amount by increasing the test statistic variance (see Equation 50, for example). In fact an increase in the power of $x(t)$ of 50 per cent will "cost" in the order of 3 db. For the sake of simplicity we will assume that a reasonable sampling rate is one for which the power in the modified channels $x(t)$ is at most 50 per cent larger than the idealized case. It follows that a minimum reasonable sampling rate f_{sm} is given by

$$1 - \frac{\sin w_o/2f_{sm}}{w_o/2f_{sm}} = \frac{1}{2}\frac{N}{I} \quad . \tag{55}$$

For large I/N ratios this equation can be approximated by

$$\left(\frac{w_o}{2f_{sm}}\right)^2 \cong 3\frac{N}{I} \quad ,$$

or

$$\frac{2\pi f_{sm}}{w_o} \cong \pi\sqrt{\frac{1}{3}\frac{N}{I}} \quad . \tag{56}$$

These results are tabulated below

$\frac{I}{N}$	1	10	100	1000
$\frac{2\pi f_{sm}}{w_o}$	1.7	5.7	18	57

Table 3 "Minimum" Sampling Rates for Nulling Procedure

It follows that for an interference power 20 db greater than ambient noise, a sampling rate of the order nine times the Nyquist rate is called for.

Let us now consider the sampling cost when hard-limiting is introduced. The equivalent to Equation 54 for D_{cn} is

$$\overline{x^2(t)} \le 2\left\{1 - \frac{2}{\pi}\sin^{-1}\left[\frac{I}{N+I}\,\frac{\sin w_o\sigma/2}{w_o\sigma/2}\right]\right\}. \qquad (57)$$

This can be closely approximated by

$$\overline{x^2(t)} \le 2\left\{1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I} + \frac{2}{\pi}\left(1 - \frac{\sin w_o\sigma/2}{w_o\sigma/2}\right)\frac{I}{N}\sqrt{\frac{N}{N+2I}}\right\}.$$

If we sample at the rate determined by Equation 55, this becomes

$$\overline{x^2(t)} \le 2\left\{1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I} + \frac{1}{\pi}\sqrt{\frac{N}{N+2I}}\right\}.$$

Using the expansion for $\sin^{-1}(1-\varepsilon)$, we see that for large I/N,

$$\overline{x^2(t)} \le 2\left\{\frac{4}{\pi}\sqrt{\frac{N}{2N+2I}} + \frac{1}{\pi}\sqrt{\frac{N}{N+2I}}\right\} . \qquad (58)$$

Thus the maximum "sampling cost" of the clipped nulling detector when sampling at the rate f_{sm} is approximately 1/2 that of the analog nulling detector. The implication of this result will be clear later.

VI. Cost of Hard-Limiting with Rapid Sampling Rates

The problem is to evaluate the variance of the test statistic $S_{cn}(\bar{x})$ under the hypothesis $S = 0$. As mentioned before, an approximation is necessary to overcome the analytic difficulties of hard-limiting. The test statistic $S_{cn}(\bar{x})$ calculates the sum of the squares of $\beta(k)$ where

$$\beta(k) = \sum_{i=1}^{M-r} x_i(t+k\sigma) \quad , \qquad (59)$$

and where $x_i(t)$ is the difference of a pair of clipped hydrophone outputs. Under the hypothesis $S = 0$, the autocorrelation function of $x_i(t)$ with

$x_j(t+k\sigma)$ is determined by

$$R_{i,i}(k) = \frac{1}{\pi}\left\{\sin^{-1}\frac{N\rho_n(k\sigma) + I\rho_i(k\sigma)}{N + I} - \sin^{-1}\left[\frac{I\rho_i(k\sigma)}{N + I}\right]\right\} , \qquad (60)$$

$$R_{i,i\pm p}(k) = 0 \quad ,$$

$$R'_{i,i\pm r}(k) = -\frac{1}{2\pi}\left\{\sin^{-1}\left[\frac{N\rho_n(k\sigma\pm r\lambda) + I\rho_i(k\sigma\pm r\lambda)}{N + I}\right] - \sin^{-1}\left[\frac{I\rho_i(k\sigma\pm r\lambda)}{N + I}\right]\right\} ,$$

where $R'_{i,i\pm r}(k) = R_{i,i\pm r}(k) - R_{i,i\pm p}(k)$. It is clear that the process $x_i(t)$ has an amplitude density function given by Figure 1, where the amplitude of

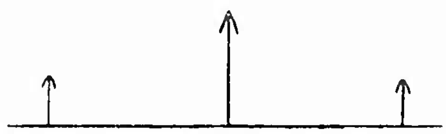


Figure 1 Amplitude Density of $x_i(t)$

the side delta functions is 1/2 $R_{i,i}(0)$. It seems reasonable to assume therefore that for large arrays ($M-r >> 1$), $\beta(k)$ is approximately normally distributed with zero mean. We will see that for <u>slow sampling rates</u>, the results obtained with this assumption are similar to that of Equation 37 except that the last term is missing. While this last term of Equation 37 does indeed vanish for large arrays, it can be seen, by putting in numbers, that for very large I/N ratios (30 db) and reasonable sized arrays ($M = 40$), that it is <u>not</u> negligible. This can be traced to the fact that the amplitude of the side delta functions of Figure 1 gets extremely small for large I/N ratios. We will ignore this problem for the moment since it will soon turn out that this difficulty can be patched up.

The autocorrelation of $\beta(k_1)$ with $\beta(k_1\pm k)$ is determined with the help of Equation 60 to be

$$\overline{\beta(o)\,\beta(\pm k)} = \sum_{i}^{M-r}\sum_{j} x_i(t)x_j(t\pm k\sigma)$$

$$= \frac{M-r}{\pi}\left\{\sin^{-1}\left[\frac{N\rho_n(k\sigma) + I\rho_i(k\sigma)}{N + I}\right] - \sin^{-1}\left[\frac{I}{N+I}\,\rho_i(k\sigma)\right]\right\}$$

$$+ \frac{M-2r}{2\pi}\left\{\sin^{-1}\left[\frac{I\rho_i(k\sigma-r\lambda)}{N + I}\right] - \sin^{-1}\left[\frac{N\rho_n(k\sigma-r\lambda) + I\rho_i(k\sigma-r\lambda)}{N + I}\right]\right\} . \qquad (61)$$

Since $\rho_{i,n}(r\lambda)$ is assumed to be negligible,

$$\overline{\beta^2(k)} = \frac{M-r}{2}\left\{ - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right\} . \qquad (62)$$

We are attempting to determine the variance of $S_{cn}(\overline{x})$ under the hypothesis, or

$$\mathrm{Var}_H\, S_{cn}(\overline{x}) = D_{cn}^{\;2} = \sum_{k_1}^{Tf_s}\sum_{k_2}\left[\overline{\beta^2(k_1)\beta^2(k_2)} - \overline{\beta^2 k}^{\,2}\right]. \qquad (63)$$

The Central Limit Theorem enables us to assume that

$$\overline{\beta^2(k_1)\beta^2(k_2)} \cong \overline{\beta^2(k_1)}\,\overline{\beta^2(k_2)} + 2\,\overline{\beta(k_1)\beta(k_2)}^{\,2} . \qquad (64)$$

Hence

$$D_{cn}^{\;2} \cong 2\sum_{k_1}^{Tf_s}\sum_{k_2}\overline{\beta(k_1)\beta(k_2)}^{\,2}$$

$$= 2Tf_s\,\overline{\beta^2(k)}^{\,2} + 2\sum_{k_1\neq k_2}^{Tf_s}\sum\overline{\beta(k_1)\beta(k_2)}^{\,2}$$

$$= 2Tf_s\,\overline{\beta^2(k)}^{\,2} + 4\sum_{k=1}^{Tf_s}(Tf_s - k)\,\overline{\beta(o)\beta(k)}^{\,2}$$

or

$$D_{cn}^{\;2} \cong 2Tf_s\left[\overline{\beta^2(k)}^{\,2} + \sum_{k=1}^{Tf_s}\left(1 - \frac{k}{Tf_s}\right)\overline{\beta(o)\beta(k)}^{\,2}\right] . \qquad (65)$$

Keeping in mind that $r\lambda/\sigma$ is constrained to be an integer we obtain that

$$\frac{D_{cn}^{\;2}}{2Tf_s} \cong \frac{(M-r)^2}{4}\left[1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right]^2 + 2\left(1 - \frac{r\lambda}{T}\right)\frac{(M-2r)^2}{16}\left[1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right]^2$$

$$+2\sum_{\substack{k=1\\ k\neq \frac{r\lambda}{\sigma}}}^{Tf_s}\left(1-\frac{k}{Tf_s}\right)\left\{\frac{M-r}{\pi}\left[\sin^{-1}\left(\frac{N\rho_n(k\sigma)+I\rho_i(k\sigma)}{N+I}\right)-\sin^{-1}\left(\frac{I}{N+I}\rho_i(k\sigma)\right)\right]\right.$$

$$\left.+\frac{M-2r}{2\pi}\left[\sin^{-1}\left(\frac{I\rho_i(k\sigma-r\lambda)}{N+I}\right)-\sin^{-1}\left(\frac{N\rho_n(k\sigma-t\lambda)+I\rho_i(k\sigma-r\lambda)}{N+I}\right)\right]\right\}^2. \tag{66}$$

If we now make the slightly stronger assumption that $\rho(\frac{r\lambda}{2})$ is quite small, the cross term in the last square can be ignored and Equation 66 becomes

$$\frac{D_{cn}^2}{2Tf_s}\approx\frac{(M-r)^2}{\pi^2}\left\{\left(\frac{\pi}{2}-\sin^{-1}\frac{I}{N+I}\right)^2\left[1+\frac{1}{2}\left(1-\frac{r\lambda}{T}\right)\left(\frac{M-2r}{M-r}\right)^2\right]\right.$$

$$+2\sum_{k=1}^{Tf_s}\left(1-\frac{k}{Tf_s}\right)\left(\sin^{-1}\left[\frac{N\rho_n(k\sigma)+I\rho_i(k\sigma)}{N+I}\right]-\sin^{-1}\left[\frac{I}{N+I}\rho_i(k\sigma)\right]\right)^2$$

$$\left.+\left(\frac{M-2r}{M-r}\right)^2\sum_{k=1}^{Tf_s-\frac{r\lambda}{\sigma}}\left(1-\frac{k+\frac{r\lambda}{\sigma}}{Tf_s}\right)\left(\sin^{-1}\left[\frac{N\rho_n(k\sigma)+I\rho_i(k\sigma)}{N+I}\right]-\sin^{-1}\left[\frac{I}{N+I}\rho_i(k\sigma)\right]\right)^2\right\}. \tag{67}$$

For large decision times, this becomes

$$D_{cn}^2=2Tf_s\frac{(M-r)^2}{4}\left(1-\frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)^2\left[1+\frac{1}{2}\left(1-\frac{r\lambda}{T}\right)\left(\frac{M-2r}{M-r}\right)^2\right]\left[1+2\sum_{k=1}^{\infty}\left(1-\frac{k}{Tf_s}\right)R_{cn}(k\sigma)\right] \tag{68}$$

where

$$R_{cn}^{1/2}(k\sigma)=\left\{\sin^{-1}\left[\frac{N\rho_n(k\sigma)+I\rho_i(k\sigma)}{N+I}\right]-\sin^{-1}\left[\frac{I}{N+I}\rho_i(k\sigma)\right]\right\}\Big/\left\{\frac{\pi}{2}-\sin^{-1}\frac{I}{N+I}\right\} \tag{69}$$

Inserting Equations 69 and 31 into Equation 11 results in

$$\frac{\quad}{D_{cn}}=\frac{\sqrt{2Tf_s}}{\pi}\,\frac{S}{N}\,\frac{M-r-1+\dfrac{I/N+I}{\sqrt{1-(I/N+I)^2}}+\left(\dfrac{I}{N+I}\right)^2\displaystyle\sum_{k=1}^{M-r}\left(1-\frac{k}{M-r}\right)\frac{\rho_i^2(k\lambda)}{\sqrt{1-(I/N+I)^2\rho_i^2(k\lambda)}}}{\left(1+\frac{I}{N}\right)\left(1-\frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right)\sqrt{1+\frac{1}{2}\left(\frac{M-2r}{M-r}\right)^2\left(1-\frac{r\lambda}{T}\right)}\sqrt{1+2\displaystyle\sum_{k=1}^{\infty}\left(1-\frac{k}{Tf_s}\right)R_{cn}(k\sigma)}}. \tag{70}$$

This last result differs from that obtained under the assumption of independent samples (Equation 38) in only two ways. First there is the presence of the additive term in the numerator and multiplicative term in the denominator as a result of the sample dependency. The additional term in the numerator vanishes for interference sources that are far removed in bearing from the target. In fact it can be shown by putting in numbers that this term is always negligible, and hence it will be ignored. The second difference is that there is a term missing in the denominator as a consequence of the assumption that the test statistic is normal. This term, which we mentioned previously, can modify the SNR by as much as 1 db for $I/N = 10^3$ depending on whether or not it gets multiplied by the last term in the denominator of Equation 70. Since the cost of clipping for this high I/N ratio will turn out to be significantly larger than 1 db, this possible error source is not particularly significant. In fact, if we define the signal-to-noise ratio for uncorrelated samples (Equation 38) as $\overline{SNR}'_{cn}$, then we will take as a possibly pessimistic bound

$$\overline{SNR}_{cn} \cong \overline{SNR}'_{cn} \frac{1}{\sqrt{1 + 2\sum_{k=1}^{\infty}\left(1 - \frac{k}{Tf_s}\right)R_{cn}(k\sigma)}} \quad . \tag{71}$$

Correspondingly, an examination of Equation 21 leads us to conclude that for the analog nulling detector D_{an}, we can write

$$\overline{SNR}_{an} = \overline{SNR}'_{an} \frac{1}{\sqrt{1 + 2\sum_{k=1}^{\infty}\left(1 - \frac{k}{Tf_s}\right)\rho_n^{\,2}(k\sigma)}} \tag{72}$$

It follows that the cost of clipping is equal to that given in Table 1 for slow sampling rates modified by the improvement ratio (I) which is

$$I = \left[\frac{1 + 2\sum_{k=1}^{\infty} (1 - \frac{k}{Tf_s}) \rho_n^{\,2}(k\sigma)}{1 + 2\sum_{k=1}^{\infty} (1 - \frac{k}{Tf_s}) R_{cn}(k\sigma)} \right]^{\frac{1}{2}} , \qquad (73)$$

where R_{cn} is given in Equation 69. The inherent cost of clipping is obtained by considering infinite sampling rates or

$$I^2 = \frac{\int_0^{\infty} (1 - M/T)\, \rho_n^{\,2}(M) dM}{\int_0^{\infty} (1 - M/T)\, R_{cn}(M) dM} . \qquad (74)$$

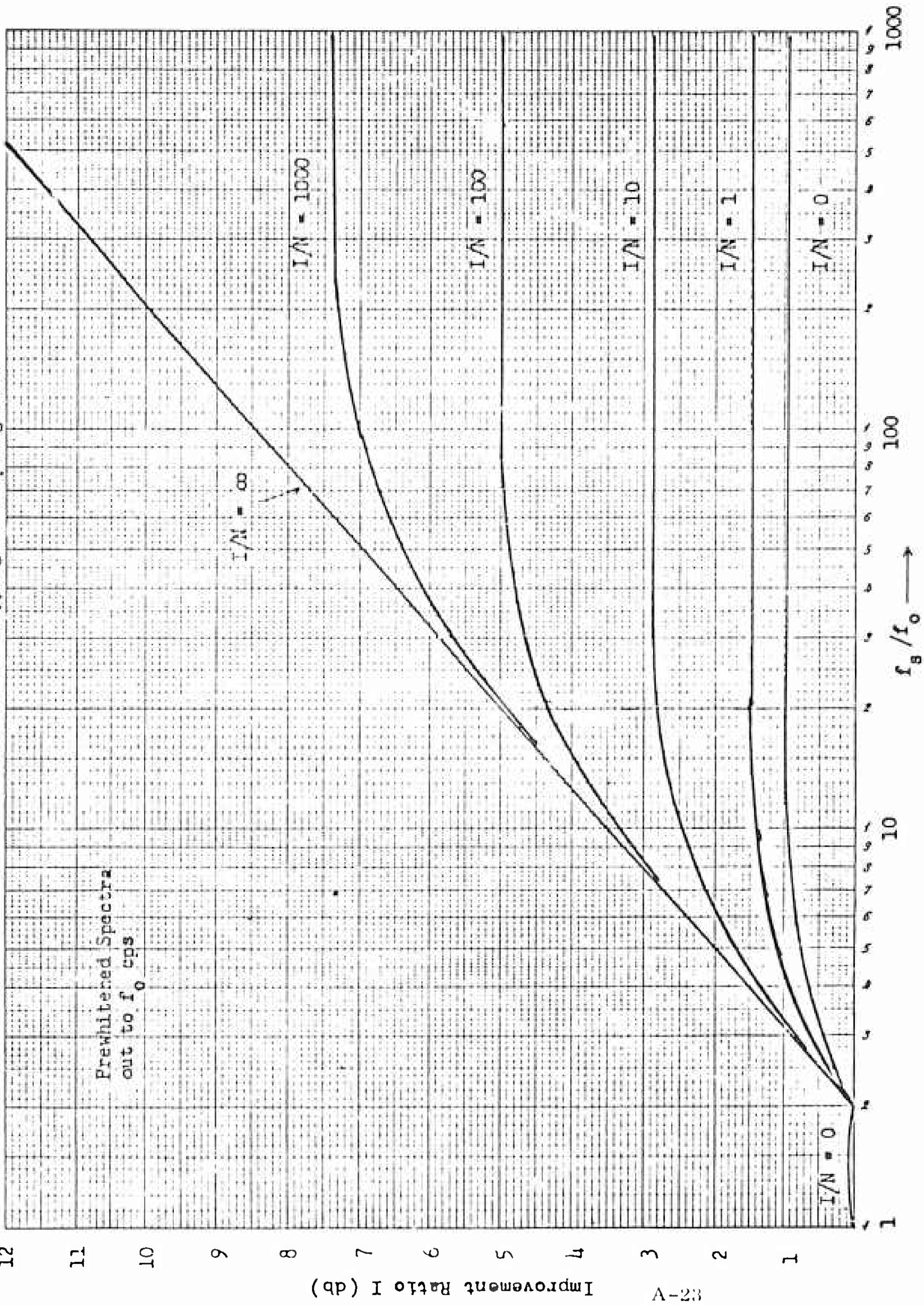
We can readily determine from Equation 69 that

$$R_{cn}(k\sigma) \geqq \rho_n^{\,2}(k\sigma)\, \frac{2}{\pi}\; \frac{1}{(1+\frac{I}{N})(I - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I})} , \qquad (75)$$

if it is assumed that $\rho_i(k\sigma) = \rho_n(k\sigma)$. The equality is satisfied for $\rho_i(k\sigma) = 0$ but is very nearly satisfied for small values of $\rho_n(k\sigma)$. The inequality is a consequence of the fact that hard-limiting always costs something regardless of the spectral shape of the inputs. However the fact that both sides of Equation 75 are nearly equal for an appreciable range of values of $\rho_n(k\sigma)$ is indicative of the fact that a significant portion of the cost of clipping for slow sampling rates is a consequence of the sampling rate rather than the hard-limiting.

Assuming large decision times $(1 - \frac{k}{Tf_s}) \approx 1$), and a noise spectra that is flat out to $f_o = w_o/2\pi$ cps and zero elsewhere, Equation 73 is plotted in Figure 2 as a function of sampling rate with I/N as the parameter. The limiting case (Equation 74) is apparent from this figure and is tabulated on the following page.

Figure 2 Improvement in Cost of Clipping vs Sampling Rate



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I/N	0	1	10	100	1000
I(db)	1	1.5	2.9	5.0	7.4

Table 4

Note also the "improvement" of the PCA relative to the analog devices corresponds to the 1 db given in Table 4 for I/N = 0. Thus this table also represents the improvement of D_{cn} relative to the PCA detector if all the values are reduced by 1 db. As an example let us assume an array of 40 hydrophones. Table 5 gives the net improvement of D_{cn} relative to the PCA as well as the net cost of clipping relative to D_{an}.

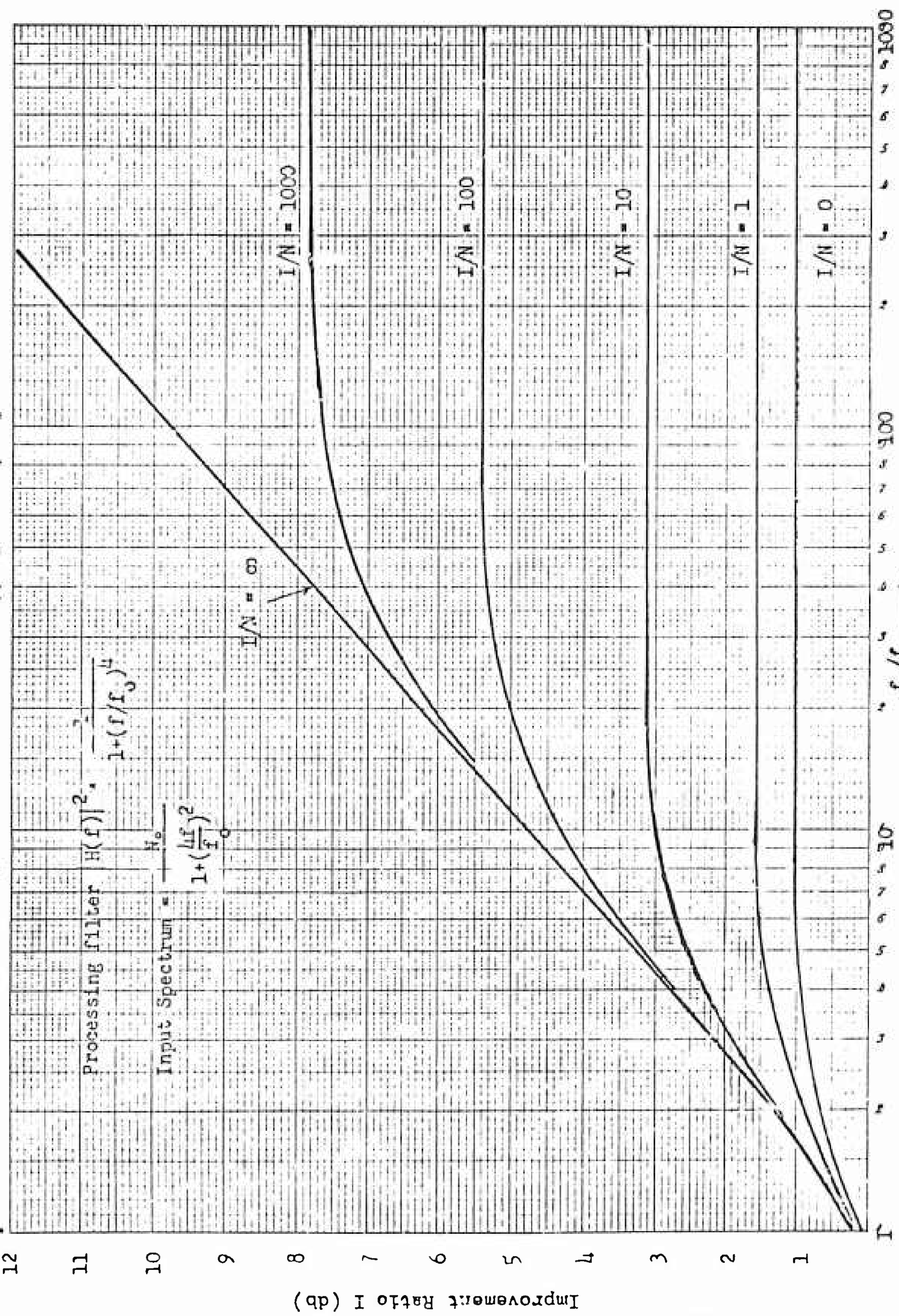
I/N	0	1	10	100	1000
J(db)	-1	4.2	12.5	20.6	28.8
C(db)	1	1.7	3.7	6.2	8.2

Table 5 (M = 40)

Equation 73 is also plotted in Figure 3 for a noise spectrum that is not pre-whitened but falls off at a rate of 6 db/octave beyond $f_o/4$ and is cut-off sharply but not ideally at f_o cps. While the inherent cost of clipping is most appropriately determined for pre-whitened spectra, Figure 3 indicates that the results of Table 5 are relatively insensitive to the spectral shape.

Finally we are concerned with the cost of clipping when sampling at the rather arbitrarily determined "minimum reasonable" sampling rate f_{sm}. For $I/N \leqslant 1$, Nyquist rate sampling is adequate and there is no improvement for "rapid sampling". For large values of I/N, the improvement is approximately 1 db less than that given in Table 4. These results, obtained from Figure 2, are based on the assumption that the array can be aimed directly at the interference. When the interference is midway between permissable bearing angles, the sampling cost is of the order 3 db for D_{an} and $1\frac{1}{2}$ db for D_{cn}. Hence for

Figure 3 Improvement in Cost of Clipping vs Sampling Rate



this later case, the total cost of sampling and clipping is, if anything, less than given in Table 5. Also for other than ideally flattened spectral shapes, the improvement in the cost of clipping for sampling at the f_{sm} rate is greater than that obtained from Figure 2. For example the spectral shape that determines Figure 3 gives an improvement factor at the f_{sm} sampling rate that is almost identical to the results of Table 4, even when the array is aimed directly at the interference. A reasonable rule of thumb for the cost of clipping for moderately large arrays and for sampling rates $f_s \geq f_{sm}$ might be

> The cost of clipping is approximately 2 db for an interference to ambient noise power ratio I/N of unity and increases almost linearly with I/N at a rate of 2 db per decade.

VII. On the Point Source Assumption and Hydrophone Tolerances

The sampling rate discussion of Section V. gives us a convenient frame of reference with which to decide whether or not a source can be regarded as a point source. Let us assume that the detector is sampled at the rates given in Table 3 which are just adequate to null out a point source with a given I/N ratio. The resulting bearing quantization determines that maximum length that a target, at a particular range and with a given I/N ratio, can have and still be regarded as a point source. Thus, if the length of the target is no greater than the product of the range with the minimum change in bearing angle, then there is no ambiguity as to its direction and it is for all practical purposes a point source. Another way of looking at this is to realize that such a target can in principle be nulled out at a cost that is comparable to the average cost of sampling at the f_{sm} rate when nulling out a perfect point source.

If the interference is broadside, then $\sigma = 1/f_s = d/c \sin\Theta$. Hence $\Delta\Theta \cong c/f_{sn}d$, and therefore

$$q \leq \frac{c}{f_o}\frac{r}{d}\frac{f_o}{f_{sm}} , \qquad (76)$$

where q is the effective length of the target and f_s/f_o is given in Table 3 as a function of I/N. For endfire interference Equation 76 becomes

$$q \leq \left[\frac{1}{2}\frac{c}{f_o}\frac{r^2}{d}\frac{f_o}{f_{sm}}\right]^{\frac{1}{2}} . \qquad (77)$$

Let us assume that the inputs are pre-whitened out to 5 kc ($\frac{c}{f_o} = 1$) and a hydrophone spacing of 2 feet. The maximum effective target length is given in Table 6 as a function of range and I/N ratio.

	r \ I/N	1	10	100	1000
Broadside	2000'	555.'	175.5'	55.5'	17.6'
	1000'	277.5'	87.8'	27.8'	8.8'
Endfire	2000'	745.'	419.'	235.5'	132.5'
	1000'	372.5'	209.5'	118.'	66.'

Table 6

Thus if the target dimensions are the effective propellor dimensions, it can come as close as 1000' and supply an I/N ratio of 30 db and still be regarded as a point source. In fact for a 20 db I/N ratio an entire submarine at 1000 yards can be regarded as a point source. We conclude that the point source assumption for interfering shipping is not unreasonable.

Finally let us consider the allowable tolerance on the hydrophone spacings. In order to null out an endfire interference the maximum delay per hydrophone of d/c seconds is required. If the actual hydrophone spacings is Δd feet different from the assumed spacings, then the required delay would be in error by $\Delta d/c$ seconds. If we now require that this error be no greater than $1/2f_{sm}$

seconds, the nulling will still be effective even for endfire interference. Thus a reasonably tight bound on the hydrophone spacings might be

$$\Delta d \leq 6 \frac{c}{f_o} \frac{f_o}{f_{sm}} \text{ inches.} \tag{78}$$

Hence if we assume that the inputs are processed out to 5 kc, the tolerances in the hydrophone spacings needed to null out an endfire interference whose power is I/N that of the ambient noise is given by

I/N	1	10	100	1000
Δd	3"	1"	.3"	.1"

Table 7

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OPTIMAL TECHNIQUES EMPLOYING QUANTIZED INPUTS FOR DETECTING PASSIVE SONAR TARGETS IN THE PRESENCE OF INTERFERENCE

By

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General Dynamics Electric Boat division

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Summary

The likelihood ratio test for detecting a weak signal in the presence of a strong point source interference has been evaluated, given that the inputs have been reduced by hard-limiting. An implementation which very closely approximates this test and is in fact asymptotically optimum as the interference to ambient noise power ratio increases is presented. This procedure aims the array in the direction of the interference and subtracts the average value of the inputs from each of the inputs before proceeding with the standard DIMUS processing. As this procedure is asymptotically optimum for analog inputs as well, it is conjectured to be optimum for arbitrary quantization. It is shown that the degradation due to hard-limiting indicated in report 26 is indeed real and that a relatively large number of quantization levels are needed to significantly reduce this cost for large interference-to-noise-ratios.

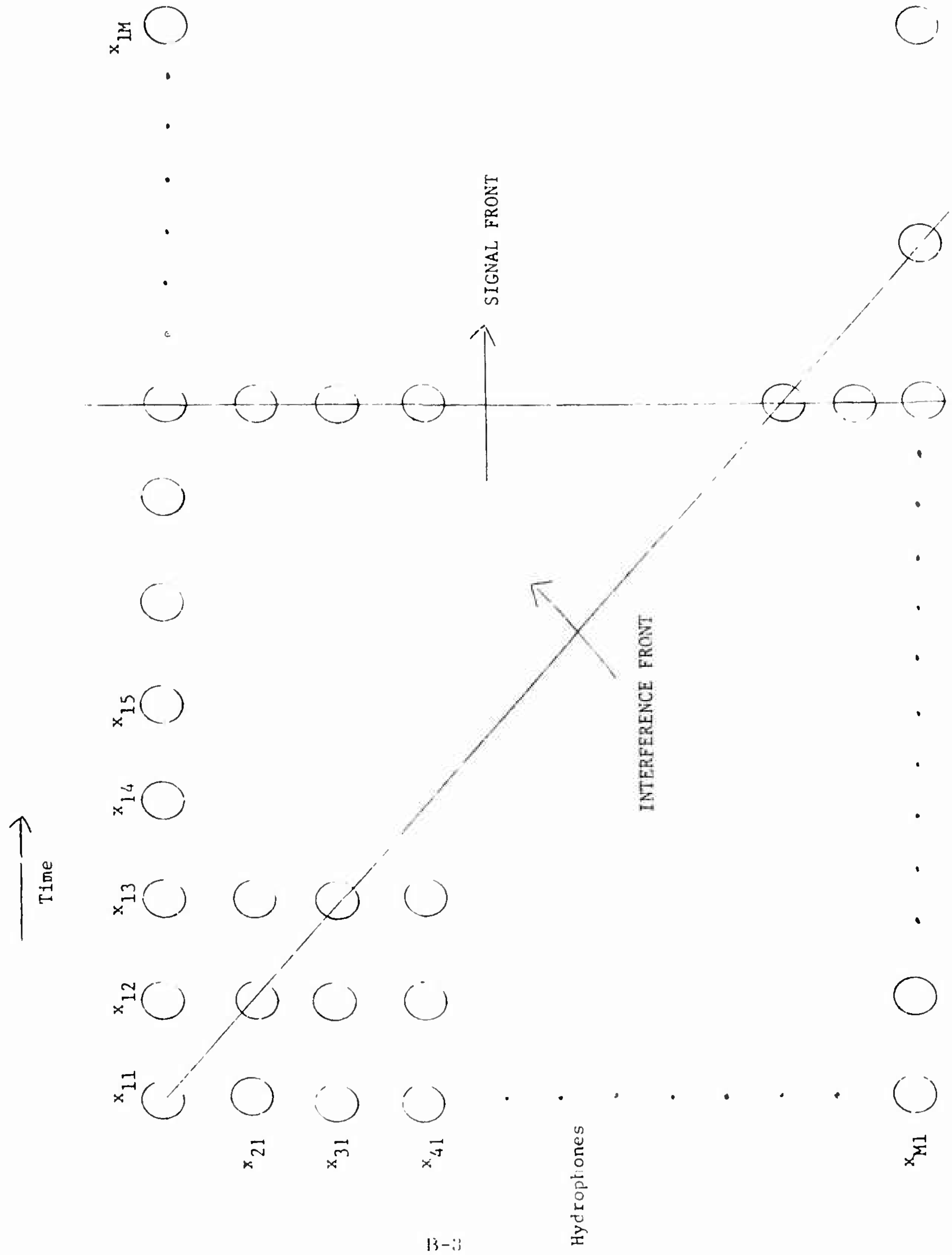
I. Introduction

This report deals with the detection of a sonar target in the presence of a strong point source interference with the constraint that the inputs are reduced by arbitrary quantization. It will be assumed throughout that the hydrophone inputs in the absence of the signal and interference targets are statistically independent. This problem was considered without the data reduction in reports 17 and 21. The likelihood ratio test was evaluated and it was shown that the interference was effectively removed. Furthermore, a suboptimum implementation called the nulling detector was proposed which closely approximated the likelihood ratio test. The nulling implementation was further analyzed in report 26 under the assumption that the inputs were hard-limited. While this instrumentation represents a vast improvement over the standard polarity coincidence array, the results indicated that the degradation in detectability due to hard-limiting is not negligible for strong interfering targets. In order to determine whether or not this degradation is inherent in the problem, one must determine the performance of the likelihood ratio test for the clipped data. If the degradation is indeed significant, it will be useful to estimate the improvement in detectability for multilevel quantization. This result should, in addition to indicating the advisability of going to multilevel quantization, indicate the accuracy required for the detector employing analog data.

There is no known approach for the general solution of the likelihood ratio for quantized inputs. The only known procedure is to assume that the inputs are sampled slowly enough so that the samples are independent (Nyquist-rate sampling for gaussian inputs). Faster sampling rates are required however to achieve a low cost of clipping and to permit reasonable accuracy in bearing measurements. In fact, it was indicated in report 26 that unless one sampled rapidly enough the major source of degradation in the nulling detector is due to the sampling rate rather than the quantization. Once the optimum detector for independent samples has been found, however, it can be analyzed for rapid sampling rates. There are a number of reasons why this procedure is not thought to be restrictive. In the first place there is no a priori reason to assume that the form of the optimum detector would be a function of the sampling rate. For example in the case of analog data, the only difference is the presence of pre-whitening filters at the inputs. Secondly even if the optimum detector did depend on the sampling rate, one would need precise knowledge of the ambient noise power spectrum in order to implement it. The complexity of such a system would rule it out even if such knowledge were available and it generally is not. We will therefore find the likelihood ratio for independent, hard-limited samples of the hydrophone inputs. It will be assumed that the array can be aimed either directly at the interference or the target even though the sampling rate required by this assumption, as discussed in report 26, is inconsistent with the independent sample assumption. Thus the detector we are seeking is to be optimum for independent samples, but it will in fact be operated with dependent samples.

II. The Likelihood Ratio for Weak Signals

A graphical representation of all of the available data is shown in Fig. 1 where it is assumed that the array is aimed directly at the target. The assumption that the array could be aimed at the interference is indicated by the fact that the interference wave front intersects M data points simultaneously. It follows from the assumptions that the noise inputs are all statistically independent and the samples in any channel are independent that the correlation between samples along any vertical line (fixed instant of time) is due solely to the common additive signal. Given that the data is



x_{1M}
SIGNAL FRONT
INTERFERENCE FRONT
Time
x_{11}
x_{12}
x_{13}
x_{14}
x_{15}
x_{21}
x_{31}
x_{41}
x_{M1}
M Hydrophones

Figure 1

hard-limited, the information available as to the presence of the signal is contained in the number of samples along a vertical axis that have the same sign where all of the samples are suitably "weighted" by the number of samples that have the same sign along the diagonal axis which contains the common additive interference. We will assume that the number of samples N is sufficiently large $\left[N >> M(M-1)\right]$ so that the "corners" of Fig. 1 where the diagonal lines have fewer than M elements can be ignored. The optimum implementation will involve aiming first at the interference and then after some processing, aiming at the target. Hence, the data array will really be in the form of a rhombus with the corners missing.

Let us denote the probability that a sample, with an additive signal of value ξ, is positive and x_1 of the M samples along the common interference diagonal are positive by $P^+(x_1,\xi)$. Hence

$$P^+(x_1,\xi) = \int_{-\infty}^{\infty} \binom{M-1}{x_1-1} \left[1-F_{n+S}(-u)\right]^{x_1-1} \left[F_{n+s}(-u)\right]^{M-x_1} \left[1-F_n(-u-\xi)\right] f_I(u)\, du \quad , \tag{1}$$

where F( ) is the gaussian cumulative distribution function and $f_I(\ )$ is the density function of the interference. Similarly $P^-(x_1,\xi)$ is the probability that the sample is negative and x_1 of the samples along the interference diagonal are positive.

$$P^-(x_1,\xi) = \int_{-\infty}^{\infty} \binom{M-1}{x_1} \left[1-F_{n+S}(-u)\right]^{x_1} \left[F_{n+s}(-u)\right]^{M-x_1-1} F_n(-u-\xi)\, f_I(u)\, du \quad . \tag{2}$$

All that matters is the number of samples along a vertical axis that are positive, and the ordering of them is irrelevant. The probability that r samples are positive with an arbitrary distribution along the interference diagonals is given by

$$P_K(r,\underline{x}) = \binom{M}{r} \int_{-\infty}^{\infty} \prod_{i=1}^{r} P^+(x_i,\xi) \prod_{j=1}^{M-r} P^-(x_j,\xi)\; f_s(\xi)\, d\xi \quad , \tag{3}$$

where the subscript K refers to the alternative of signal present. Under the hypothesis of no signal

$$P_H(r,\underline{x}) = \binom{M}{r} \prod_{i=1}^{r} P^+(x_i,0) \prod_{j=1}^{M-r} P^-(x_j,0) \quad . \tag{4}$$

The likelihood ratio $L(r,\underline{x})$ is given by

$$L(r,x) = \frac{P_K(r,\underline{x})}{P_H(r,\underline{x})} \quad , \tag{5}$$

and depends on the signal-to-noise ratio S/N among other things.

The standard procedure is to consider a locally optimum detector that is optimum for vanishingly small signal-to-noise ratios. Hence we in fact implement the first two terms only of the expansion

$$L(r,\underline{x},S/N) = L(r,\underline{x},0) + \frac{\partial L}{\partial (S/N)}\bigg|_{S/N=0} \cdot S/N + \ldots , \tag{6}$$

We will denote these first two terms by $\overline{L}(r,\underline{x})$. This likelihood ratio will be evaluated by expanding the probabilities $P^+(x_i,\xi)$ and $P^-(x_i,\xi)$ and retaining only those terms of order (S/N) or ξ^2 and then repeating this procedure for $P_K(r\ \underline{x})$. It may be that when we are done, some of the terms of $\overline{L}(r,\underline{x})$ will not represent cross-correlation information and hence are useful only when the noise power is a priori knowledge. Ignoring such terms would in fact represent no real loss of information.

Expanding and then collecting all of the terms in Eq. (1) is straightforward. Retaining only the pertinent terms, one obtains:

$$P^+(x_i,\xi) \longrightarrow P(x_i,0) + P_A^+(x_i)\ S/N + P_B^{\ +}(x_i)\xi + P_C^{\ +}(x_i)\xi^2 \quad , \tag{7}$$

where

$$P(x_i,0) = \frac{x_i}{M}\binom{M}{x_i} \int_{-\infty}^{\infty} \left[1-F_n(-u)\right]^{x_i}\left[F_n(-u)\right]^{M-x_i} f_I(u)\, du \quad , \tag{8}$$

$$P_A^+(x_i) = \frac{x_i}{M}\binom{M}{x_i} \frac{M-x_i}{2} \int_{-\infty}^{\infty} \left[1-F_n(-u)\right]^{x_i} f_n(-u)\; u\; f_I(u)\, du \quad ,$$

$$- \frac{x_i}{M}\binom{M}{x_i} \frac{x_i-1}{2} \int_{-\infty}^{\infty} \left[1-F_n(-u)\right]\left[F_n(-u)\right]^{M-x_i} f_N(-u)\; u\; f_I(u)\, du \quad , \tag{9}$$

$$P_B^+(x_i) = \frac{x_i}{M}\binom{M}{x_i} \int_{-\infty}^{\infty} \left[1-F_n(-u)\right]^{x_i-1}\left[F_n(-u)\right]^{M-x_i} f_n(-u)\; f_I(u)\, du \quad , \tag{10}$$

and

$$P_C^+(x_i) = \frac{x_i}{M}\binom{M}{x_i} \int_{-\infty}^{\infty} \left[1-F_n(-u)\right]^{x_i-1}\left[F_n(-u)\right]^{M-x_i} \frac{u}{2N}\; f_n(-u)\; f_I(u)\, du \;. \tag{11}$$

Similarly:

$$P^-(x_i,\xi) \rightarrow \frac{M-x_i}{x_i}\left[P(x_i,0) + P_A^-(x_i)S/N - P_B^-(x_i)\xi - P_C^-(x_i)\xi^2\right] \quad , \tag{12}$$

where

$$P_A^-(x_i) = \frac{x_i}{M}\binom{M}{x_i}\left(\frac{M-x_i-1}{2}\right) \int_{-\infty}^{\infty} \left[1-F_n(-u)\right]^{x_i} F_n(-u)\; f_n(-u)\; u\; f_I(u)\, du$$

$$- \frac{x_i}{M}\binom{M}{x_i} \frac{x_i}{2} \int_{-\infty}^{\infty} \left[F_n(-u)\right]^{M-x_i} f_n(-u)\; u\; f_I(u)\, du \quad , \tag{13}$$

$$P_B^-(x_i) = \frac{x_i}{M}\binom{M}{x_i} \int_{-\infty}^{\infty} \left[1-F_n(-u)\right]^{x_i}\left[F_n(-u)\right]^{M-x_i-1} f_n(-u)\; f_I(u)\, du \quad , \tag{14}$$

and

$$P_C^-(x_i) = \frac{x_i}{M}\binom{M}{x_i}\int_{-\infty}^{\infty}\left[1-F_n(-u)\right]^{x_i}\left[F_n(-u)\right]^{M-x_i-1}\frac{M}{2N}\,f_n(-u)\,f_I(u)\,du\ . \tag{15}$$

If one now substitutes Eq. (7) into the first product term of Eq. (3) and then retains only the pertinent terms, one obtains

$$\prod_{i=1}^{r} P^+(x_i,\xi) \to \prod_{i=1}^{r} P(x_i,0) + \sum_{i=1}^{r} P_A^+(x_i)\prod_{j\neq i}^{r} P(x_j,0)\ S/N$$
$$+\sum_{i=1}^{r} P_B^+(x_i)\prod_{j\neq i}^{r} P(x_j,0)\xi + \sum_{i=1}^{r} P_C^+(x_i)\prod_{j\neq i}^{r} P(x_j,0)\,\xi^2$$
$$+\frac{1}{2}\sum_{i\neq j}^{r}\sum^{r} P_B^+(x_i)\,P_B^+(x_j)\prod_{k\neq j}^{r} P(x_k,0)\xi^2\ , \tag{16}$$

or

$$\prod_{i=1}^{r} P^+(x_i,\xi) \to \prod_{i=1}^{r} P(x_i,0)\left\{1 + S/N\sum_{i=1}^{r}\frac{P_A^+(x_i)}{P(x_i,0)} + \xi\sum_{i=1}^{r}\frac{P_B^+(x_i)}{P(x_i,0)}\right.$$
$$\left.+\xi^2\sum_{i=1}^{r}\frac{P_C^+(x_i)}{P(x_i,0)} + \frac{\xi^2}{2}\sum_{i\neq j}^{r}\sum^{r}\frac{P_B^+(x_i)\ P_B^+(x_j)}{P(x_i,0)\ P(x_j,0)}\right\}\ . \tag{17}$$

Similarly

$$\prod_{j=1}^{M-r} P^-(x_j,\xi) = \prod_{j=1}^{M-r}\frac{M-x_j}{x_j}P(x_j,0)\left\{1 + S/N\sum_{j=1}^{M-r}\frac{P_A^-(x_j)}{P(x_j,0)} - \xi\sum_{j=1}^{M-r}\frac{P_B^-(x_j)}{P(x_j,0)}\right.$$
$$\left.-\xi^2\sum_{j=1}^{M-r}\frac{P_C^-(x_j)}{P(x_j,0)} + \frac{\xi^2}{2}\sum_{i\neq j}\sum^{M-r}\frac{P_B^-(x_i)\ P_B^-(x_j)}{P(x_i,0)\ P(x_j,0)}\right\}\ . \tag{18}$$

It follows from Eq. (4) as well as Eqs. (17) and (18) that

$$P_H(r,\underline{x}) = \binom{M}{r}\prod_{i=1}^{r} P(x_i,0)\prod_{j=1}^{M-r}\frac{M-x_j}{x_j}P(x_j,0)\ . \tag{19}$$

Combining Eqs. (3,5,17,18, and 19) one obtains for the likelihood ratio

$$\bar{L}(r,x) = 1 + S/N\left\{\sum_{i=1}^{r}\frac{P_A^+(x_i)}{P(x_i,0)} + \sum_{i=1}^{M-r}\frac{P_A^-(x_i)}{P(x_i,0)} - N\sum_{i=1}^{r}\sum_{j=1}^{M-r}\frac{P_B^+(x_i)}{P(x_i,0)}\frac{P_B^-(x_i)}{P(x_j,0)}\right.$$

$$+ N\sum_{i=1}^{r}\frac{P_C^+(x_i)}{P(x_i,0)} - N\sum_{i=1}^{M-r}\frac{P_C^-(x_i)}{P(x_i,0)} + \frac{N}{2}\sum^{r}\sum^{r}_{i\neq j}\frac{P_B^+(x_i)}{P(x_i,0)}\frac{P_B^+(x_j)}{P(x_j,0)}$$

$$\left. + \frac{N}{2}\sum^{M-r}\sum^{M-r}_{i\neq j}\frac{P_B^-(x_i)}{P(x_i,0)}\frac{P_B^-(x_j)}{P(x_j,0)}\right\} \qquad (20)$$

It is now conjectured that the terms involving $P_A^+(x_i)$, $P_A^-(x_i)$, $P_C^+(x_i)$, and $P_C^-(x_i)$ either vanish or cancel themselves out depending on the value of the interference-to-noise ratio (I/N). It is not, however, necessary to prove this conjecture, since we can argue that these terms should be ejected in the event that they do not cancel themselves out. One notes, by tracing their origins, that these relatively few terms (for large arrays) do not represent cross-correlation information and hence can only be useful when the noise power is a priori knowledge. They are completely analogous to the M terms $\left[\text{of a total of } M^2\right]$ of the standard array detector that measure the power of the hydrophone inputs rather than the correlation between them. When the noise power is not known, these terms can only add to the variance of the test statistic and hence they should be omitted (see reports 18,22, and 23). Since one indeed never has a priori knowledge of the noise power, the useful likelihood ratio is given by:

$$\bar{L}(r,\underline{x}) = 1 + \frac{S}{2}\left\{\sum^{r}\sum^{r}_{i\neq j}P^+(x_i)P^+(x_j) + \sum^{M-r}\sum^{M-r}_{i\neq j}P^-(x_i)P^-(x_j) - 2\sum_{i}^{r}\sum_{j}^{M-r}P^+(x_i)P^-(x_j)\right\} \qquad (21)$$

where

$$P^{+}(x_i) \triangleq \frac{P_B^{+}(x_i)}{P(x_i,0)} = \frac{\displaystyle\int_{-\infty}^{\infty} \frac{f_n(-u)}{\left[1-F_n(-u)\right]} \left[1-F_n(-u)\right]^{x_i} \left[F_n(-u)\right]^{M-x_i} f_I(u)\, du}{\displaystyle\int_{-\infty}^{\infty} \left[1-F_n(-u)\right]^{x_i} \left[F_n(-u)\right]^{M-x_i} f_I(u)\, du}, \quad (22)$$

and

$$P^{-}(x_i) \triangleq \frac{P_B^{-}(x_i)}{P(x_i,0)} = \frac{\displaystyle\int_{-\infty}^{\infty} \frac{f_n(-u)}{F_n(-u)} \left[1-F_n(-u)\right]^{x_i} \left[F_n(-u)\right]^{M-x_i} f_I(u)\, du}{\displaystyle\int_{-\infty}^{\infty} \left[1 - F_n(-u)\right]^{x_i} \left[F_n(-u)\right]^{M-x_i} f_I(u)\, du} \quad . \quad (23)$$

Let us now evaluate the likelihood ratio for the case of no interference ($I \rightarrow 0$). Substituting for $f_I(u)$ in Eqs. (22 and 23) the delta function $\delta(u)$, one obtains by inspection that

$$P^{+}(x_i) = P^{-}(x_i) = 2\, f_n(-0) = \sqrt{\frac{2}{\pi N}} \quad . \quad (24)$$

It follows from Eq. (21) that

$$\begin{aligned} \overline{L}(r,\underline{x}) &\xrightarrow{I \rightarrow 0} \left\{1 + \frac{1}{\pi}\frac{S}{N}\left[(r^2-r) + (M-r)^2 - (M-r) - 2r(M-r)\right]\right\} \\ &= \left\{1 + \frac{1}{\pi}\frac{S}{N}\left[(2r-M)^2 - M\right]\right\} \quad . \end{aligned} \quad (25)$$

This is exactly the result obtained in report 6 for the standard detection problem, and it therefore acts as a partial check on Eq. (21). It is known that the DIMUS array implements the likelihood ratio of Eq. (25) See report 22 for example .

III. An Implementation of the Likelihood Ratio

It is informative to interpret the meaning of the weights" given by Eqs. (22 and 23). In the case of no interference, we note that

$$P^{+} = \text{Prob}\left\{ n \lesssim 0/n > 0 \right\},$$

and

$$P^-(x_i) = \text{Prob}\left\{n = 0/n < 0\right\} \tag{26}$$

These weights are obviously pertinent for the weak signal case, since they are the probability that a vanishingly small signal determines the sign of the samples. When interference is present, we recognize that

$$\frac{\left[1 - F_n(-u)\right]^{x_i}\left[F_n(-u)\right]^{M-x_i} f_I(u)}{\left[1 - F_n(-u)\right]^{x_i}\left[F_n(-u)\right]^{M-x_i} f_I(u)du} = \text{Prob}\left\{i(t) = u/x_i\right\} . \tag{27}$$

It follows that

$$P^+(x_i) = \text{Prob}\left\{n \lesssim -i(t)/n > -i(t), x_i\right\} ,$$

$$P^-(x_i) = \text{Prob}\left\{n \approx -i(t)/n < -i(t), x_i\right\} . \tag{28}$$

Once again we have the probability that a vanishingly small signal determines the sign of the samples. Note that if we used a standard DIMUS system that did not use the information contained in the vector $\underline{x}$, the weights would be $\sqrt{2/\pi(N+I)}$ which are not the optimum weights given by Eqs. (22 and 23). It is intuitively clear from the form of the likelihood ratio [Eq. (21)], however, that the optimum implementation is a DIMUS system that is preceeded by some type of pre-processing which is based on the number of positive samples (x_i) along the common interference fronts. Insight is gained into the type of pre-processing by considering some special cases.

It is easily shown from Eqs. (22 and 23) that when $x_i = M$, $P^+(x_i) \longrightarrow 0$ as $I/N \rightarrow \infty$ and when $x_i = 0$, $P^-(x_i) \longrightarrow 0$ as $I/N \rightarrow \infty$. It follows that when all of the samples along an interference front have the same sign, they contribute very little information about the presence of the signal and for large interference sources, they should be ejected. It can be numerically demonstrated that the optimum "weights" become negligibly small when the interference-to-noise ratio I/N is large enough so that the probability of the events $x_i = M$ and $x_i = 0$ become significant. Thus, it costs very little to eject such samples regardless of the interference-to-noise ratio. Recall that in the analog case (Report 17), the asymptotically optimum ($I/N \to \infty$) implementation very nearly approximates the optimum for large arrays [introduces a cost of one hydrophone for $I/N = 0$] and does not require a priori or estimated knowledge of the interference-to-noise ratio. It will be seen that this is also the case when the inputs are hard-limited. It can also be numerically demonstrated that for large values of I/N, the probability of the event $x_i = M$ or 0 is 1-t, where t is of the order $\sqrt{N/I}$. The removal of such a large percentage of the samples constitutes the significant degradation due to hard-limiting that was observed in Report 26.

It is also informative to consider the average weight along a common inteference front. This average weight $W(x_i)$ is calculated from the formula

$$W(x_i) = \frac{1}{M}\left[x_i P^+(x_i) - (M - x_i)P^-(x_i)\right] \quad . \tag{29}$$

It turns out that $W(x_i) \to 0$ as $I/N \to \infty$ for all values of x_i. This is difficult to show rigorously, but it can easily be shown to be approximately true by considering a suitable approximation to $P^+(x_i)$ and $P^-(x_i)$. This result has already been demonstrated for $x_i = M$ or 0. For other values of x_i, it can be shown that in the limit as $I/N \to \infty$, the probability of the event $\left[i(t) = u/x_i\right]$ as given by Eq. (27) is an even function about its maximum which occurs for that value of u such that

$$u_{max} = +\sqrt{N}\,\Phi^{-1}\left(\frac{x_i}{M}\right) \quad . \tag{30}$$

The functions $\frac{f_n(-u)}{\left[1-F_n(-u)\right]}$ and $\frac{f_n(-u)}{F_n(-u)}$ are very nearly linear functions and hence $P^+(x_i)$ and $P^-(x_i)$ can be approximated by these functions evaluated at

$u = u_{max}$ (see Eqs. 22 and 23). Thus

$$P^{+}(x_i) \approx \sqrt{\frac{2}{\pi N}}\ \frac{M}{2x_i}\ \exp\left\{-\frac{1}{2}\left[\Phi\left(\frac{x_i}{M}\right)\right]^2\right\},$$

and

$$P^{-}(x_i) \approx \sqrt{\frac{2}{\pi N}}\ \frac{M}{2(M-x_i)}\exp\left\{-\frac{1}{2}\left[\Phi^{-1}\left(\frac{x_i}{M}\right)\right]^2\right\} \tag{31}$$

For these approximate weights, Eq. (29) is identically zero. The exact weight differ in the same direction so that $W(x_i)$ is still identically zero. It is interesting to note that u_{max} is the maximum likelihood estimator of the interference and also that $P^{+}(x_i)$ and $P^{-}(x_i)$ for $x_i = M/2$ are what they should be for $I/N = 0$.

The previous results suggests an implementation [suggested by Anderson for the case of analog inputs] which consists of steering the array at the interference and then subtracting the average value of the samples from each of the samples. This implementation would weight the samples in such a way that the average weight would be indentically zero and in the event $x_i = M$ or 0 the weight would be zero. If x_i samples are positive, the average value would be $1/M\left[x_i-(M-x_i)\right] = 2x_i/M - 1$. It follows that the modified samples would have the values $2(1-x_i/M)$ for positive samples and $-2x_i/M$ for negative samples. This procedure therefore introduces <u>multiplicative</u> weights $2(1-x_i/M)$ and $2x_i/M$. Regardless of the value of I/N, the expected value of x_i is equal to $M/2$ (provided only that $\bar{t} = 0$) and hence the expected value of the weights is equal to unity. This modified DIMUS system is analyzed in Appendix A where it is shown that for weak input signals, the output signal-to-noise ratio becomes

$$\overline{SNR} = \frac{2}{\pi}\sqrt{\frac{TW_o}{2\pi}}\ \frac{S}{N+I}\ \frac{\sqrt{(M^2-M)}\ (1-1/M)}{\left[1-\frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right]\sqrt{1-\frac{4}{3M}-\frac{1}{3M^2}}} \tag{32}$$

Note that for large arrays, the cost of this implementation is quite small (about 4/3 hydrophone) for vanishingly small interference-to-noise ratios even though this implementation is conjectured to be optimum only asymptotically as $I/N \rightarrow \infty$. For large values of I/N we can replace $\sin^{-1}(I/N+I)$ by $\pi/2 - \sqrt{2N/N+I}$ (see report 26). Hence Eq. (32) becomes

$$\overline{SNR} \xrightarrow[I/N \rightarrow \infty]{} \sqrt{\frac{TW_o}{2\pi}} \; \frac{S}{N} \sqrt{M^2-M} \sqrt{\frac{N}{2(N+I)}} \; \frac{(1-1/M)}{\sqrt{1-\frac{4}{3M}-\frac{1}{3M^2}}} \; . \tag{33}$$

A comparison of this result with report 26 shows that the nulling implementation approximates this asymptotically optimum implementation in the identical fashion as in the analog case [an additional cost of 0.88 db]. As a result there is no point in analyzing this detector for rapid sampling rates since the analysis for the nulling detector carried out in report 26 should suffice.

Let us now consider the discrepancy between the weights of the proposed scheme and the optimum weights of Eqs. (22 and 23). Let us replace the weights of the proposed scheme [$2(1-x_1/M)$ and $2(x_1/M)$] by the weights $Kf(x_1)2(1-x_1/M)$ and $Kf(x_1)2(1-x_1/M)$. These modified weights would also satisfy the special cases considered. The constant K would of course not effect the performance of the detector. It is clear from Eq. (31) that these modified weights would approximate the optimum weights of Eqs. (22 and 23) provided

$$f(x_1) = \frac{1}{4\frac{x_1}{M}\left(1-\frac{x_1}{M}\right)} \exp\left\{-\frac{1}{2}\left[\Phi^{-1}\left(\frac{x_1}{M}\right)\right]^2\right\} . \tag{34}$$

This function is however, <u>very nearly</u> equal to unity for all values of x_1/M and is plotted in Fig. 2. In fact the difference between $f(x_1)$ and unity is a result <u>only</u> of the fact the Eq. (31) gives approximate weights rather that the precise weights. <u>The proposed weights are proportional to the optimum</u>

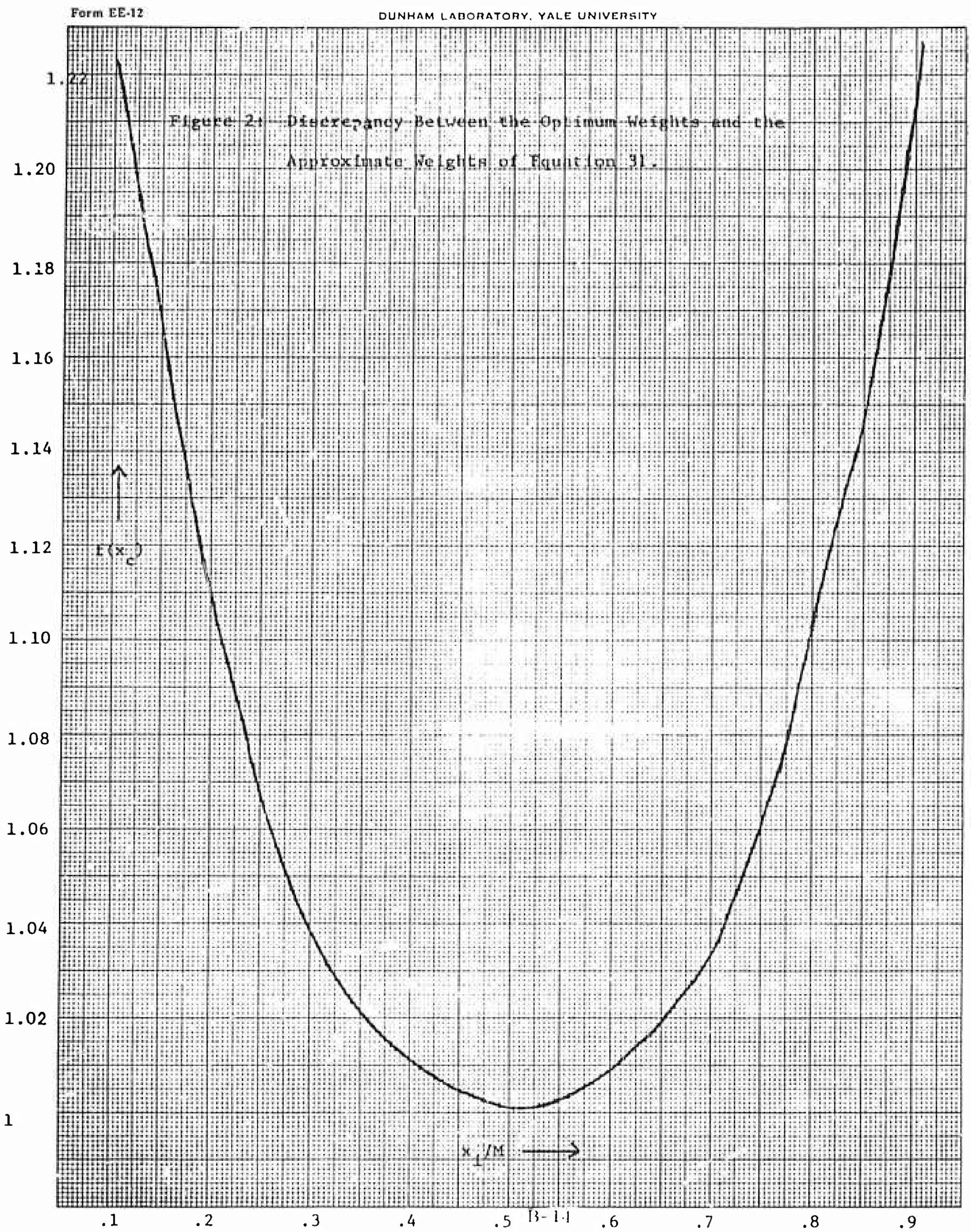


Figure 2: Discrepancy Between the Optimum Weights and the Approximate Weights of Equation 31.

weights of Eqs. (22 and 23). It follows that the modified DIMUS system represents an asymptotically optimum implementation ($I/N \to \infty$) of the likelihood ratio.

IV. On the Improvement in Detectability for Multilevel Quantization

The asymptotically optimum implementation is the same for analog data [Anderson's proposal] as well as for hard-limited data. It is therefore reasonable to assume that this implementation is optimum for an arbitrary degree of quantization. While an analysis of the general case is quite difficult, a figure of merit which gives an estimate of the improvement in detectability for multilevel quantization is rather easily obtained. This figure of merit seems to be accurate enough to give reasonable quantitative results.

For binary quantization, the samples fall into two classes those which have the same sign as the most common sign along the interference front and those which are "switches". The samples are then weighted according to which class they belong to and the best estimate of the interference. Assuming that the interference signal has a zero mean, the probability of a "switch" is identically equal to the probability that the sample has the opposite sign of the interference. Hence the probability of a switch (P_s) is given by

$$P_s = \mathrm{Prob}\left[n+s+i>0/i<0\right] = 2P\left[n+s>-i,i<0\right] = 2\int_0^{\infty}\left[1-F_{n+s}(u)\right] f_I(u)\, du\ . \qquad (35)$$

Assuming all the processes are gaussian, this probability becomes asymptotically for large interference to ambient noise power ratios;

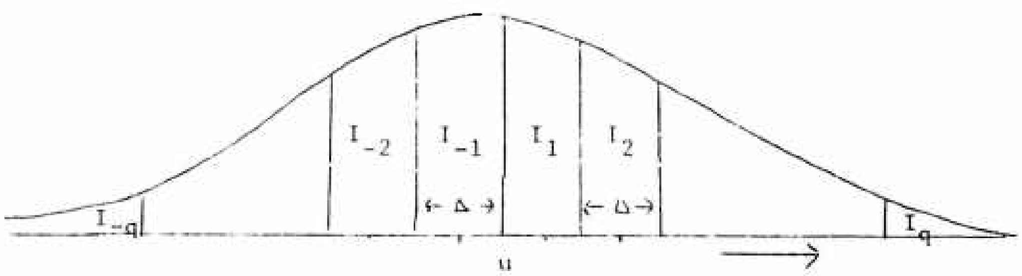
$$P_s \xrightarrow{I/N \to \infty} \sqrt{\frac{2}{\pi}} \int_0^{\infty}\left[1-\Phi\left(\sqrt{\frac{I}{N+S}}\,u\right)\right] du = \frac{1}{\pi}\sqrt{\frac{N+S}{I}} \xrightarrow{S/N \to 0} \frac{1}{\pi}\sqrt{\frac{N}{I}}\ . \qquad (36)$$

The optimum pre-processing effectively reduces the number of samples along any interference front to twice the number of switches. That is the weights of the switches vary from 1 to 2 as the weights of the non-switches vary from 1 to 0. One can postulate a "cost" in terms of the average "effective reduction in sample size". This "cost" becomes asymptotically

$$C \to \frac{2\sqrt{2}}{\pi}\sqrt{\frac{N}{2I}} \quad . \tag{37}$$

This cost is essentially equivalent (to within ½ db) of the actual asymptotic cost in detectability obtained from Eq. (33) for large arrays.

This figure of merit can be generalized for an arbitrary degree of quantization. The asymptotic "cost" converges nicely to unity as the number of quantization levels increase and it is conjectured that the results can be used quantitatively as well as qualitatively. Let us assume that the inputs are now quantized into 2 q equal intervals as in the figure below. Further assume that the interference lies in one of the closed intervals I_k where $(k-1)\Delta < I_K < k\Delta$ and that the samples are assigned values corresponding to the center of the intervals.



The samples fall into three categories: those which "switch" to the left, the switches to the right, and the non-switches. The nulling procedure will modify the weights of the switches to -1 and 1 and remove the non-switches. Hence for such interference fronts, the optimum processing "effectively" reduces the number of samples to the switches. However, for those interference fronts which correspond to the two open intervals, the pre-processing effectively reduces the number of samples to twice the switches.

It is shown in Appendix B that the probability that a sample switches from a closed interval is given by

$$P_{sc}q \rightarrow \frac{1}{\pi}\sqrt{\frac{N}{I}}\left[1 - \sqrt{\pi}\int_{\frac{m}{2}}^{\infty} \text{erfc } x dx\right]\sum_{k=1}^{q-1}\left\{\exp\left[-\frac{1}{2}(k-1)^2 m \frac{N}{I}\right] + \exp\left[-\frac{1}{2}k^2 m^2 \frac{N}{I}\right]\right\}, \quad (38)$$

where the quantization width Δ is given in terms of the number of standard deviations of the ambient noise ($\Delta = m\sqrt{N}$). Furthermore the probability of a switch from the open intervals becomes

$$P_{so}q \rightarrow \frac{1}{\pi}\sqrt{\frac{N}{I}} \exp\left[-\frac{1}{2}(q-1)^2 \frac{N}{I}\right] \quad . \quad (39)$$

Finally, as we have argued, the "cost" figure of merit is given by c ~ $P_{sc}q + 2P_{so}q$. Utilizing the facts that

$$\sqrt{\pi}\int_{\frac{m}{2}}^{\infty} \text{erfc } x dx \xrightarrow[m \to 0]{} 1 - \sqrt{\frac{\pi}{2}}\, m$$

and

$$\sum_{k=0}^{q} \varepsilon_k \exp\left[-\frac{1}{2}k^2 m^2 \frac{N}{I}\right] \xrightarrow{q \to \infty} \frac{2}{m}\sqrt{\frac{I}{N}}\int_0^{\infty} \exp\left[-\frac{1}{2}\xi^2\right] d\xi = \sqrt{\frac{2\pi}{m}}\sqrt{\frac{I}{N}} \quad . \quad (40)$$

in conjunction with Eqs. (38 and 39) it is apparent that the cost figure does indeed converge to unity as the quantization gets finer.

A plot of estimated improvement in detectability for multilevel quantization and slow sampling rates is given in Fig. 3 for an interference to ambient noise ratio of 20 db. It is observed that for a moderate number of levels (≤ 8), the improvement is not very sensitive to the width of the quantization intervals, but this sensitivity increases as the number of levels increases. Quantizing into four levels instead of two reduces the cost from 12 db to 9 db. Recall that this cost is for the case of artificially slow

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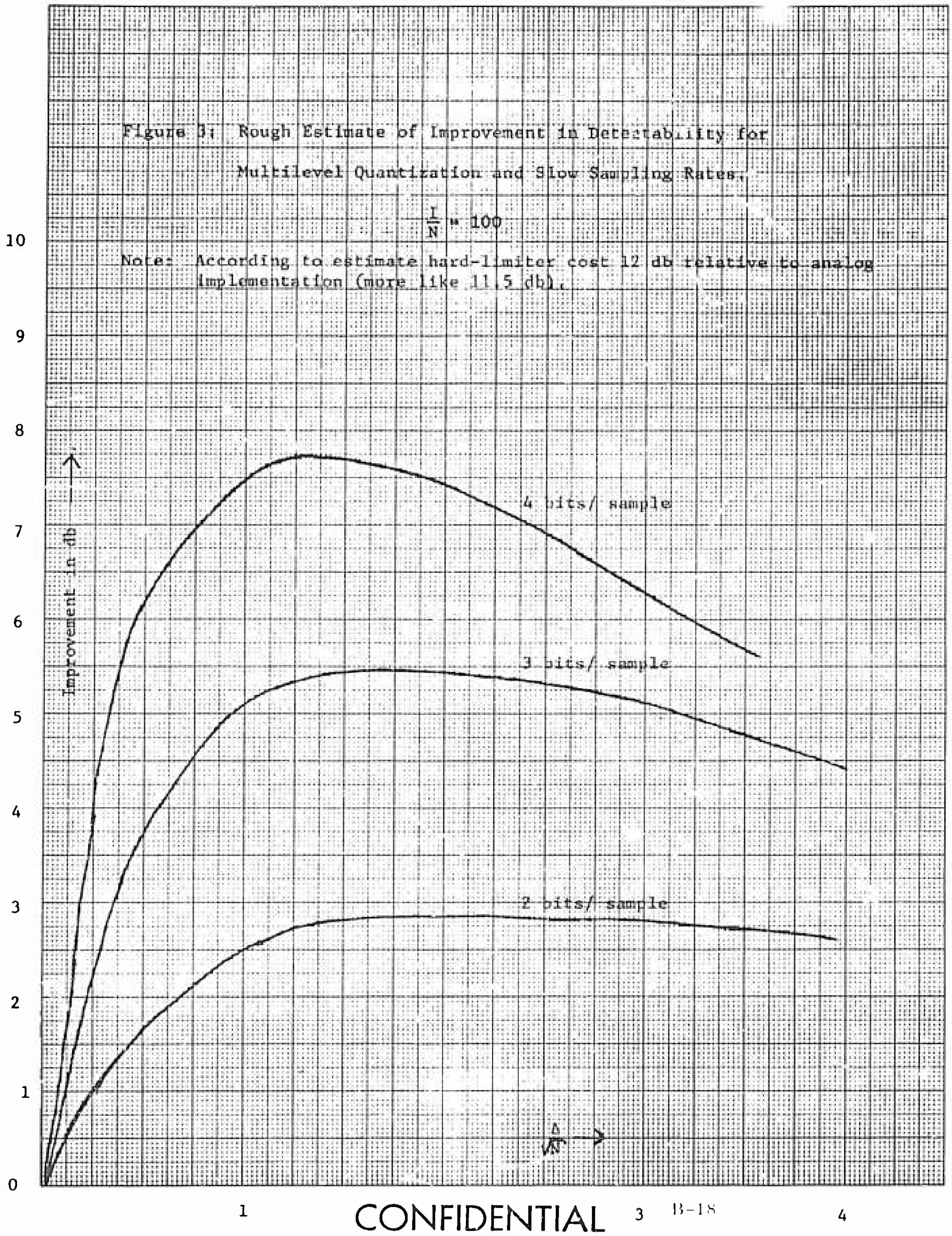
Figure 3: Rough Estimate of Improvement in Detectability for Multilevel Quantization and Slow Sampling Rates.

$\frac{I}{N} = 100$

Note: According to estimate hard-limiter cost 12 db relative to analog implementation (more like 11.5 db).



sampling rates. Reasonable sampling rates reduce the cost, in db, very nearly in half. Hence doubling the no. of quantization levels reduces the cost from approximately 6 db to approximately $4\frac{1}{2}$ db. More than eight levels are needed to reduce the cost to 3 db. While these results are only rough estimates of the actual cost, it is clear that a relatively large no. of quantization levels are needed to reduce the inherent cost of hard-limiting to some minimal value for large interference-to-noise ratios (20 db). However, the accuracy required for analog processing is not very restrictive. It is only necessary to know the value of the samples to within one standard deviation of the ambient noise. The only equipment problem is the rather stringent linearity requirement.

Appendix A

The proposed scheme calculates the following test statistic·

$$S = \sum_{k=1}^{Tf_s} \sum_{i \neq j}^{M} \sum^{M} \left[x_i(k) - y_i\right] \left[x_j(k) - y_j\right] \tag{A-1}$$

where

$$x_i(k) = \text{sgn}\left[u_i(t+k\tau)\right] ,$$

and

$$y_i = \frac{1}{M} \sum_{r=1}^{M} x_r(t + \left[k + m(r-i)\right]\tau) ;$$

where $u_i(t)$ is the input to the i^{th} hydrophone at time t, f_s is the sampling rate, and T is the decision time. The quantity y_i is the average of the hard-limited samples along the interference front associated with $u_i(t+k\tau)$. It has been assumed that the array has been aimed in the direction of the signal by introducing a delay of $m\tau$ seconds per hydrophone where τ is the sampling interval. In the absence of a signal (the hypothesis H) the inputs to the M hydrophones are all uncorrelated and are comprised of the sum of two statistically independent processes n(t) and i(t) with powers N and I respectively. Under the alternative (K), there is a common additive signal s(t) with variance S.

The expected value of the test statistic is given by

$$E(S) = Tf_s(M^2-M)\left\{\overline{x_i(k)x_j(k)} - \frac{1}{M}\sum_{r=1}^{M} \overline{x_j(k)x_r\left[k+m(r-i)\right]} - \frac{1}{M}\sum_{r=1}^{M} \overline{x_j(k)x_r\left[k+m(r-j)\right]}\right.$$

$$\left. + \frac{1}{M^2}\sum_{r}^{M}\sum_{s}^{M} \overline{x_r\left[k+m(r-i)\right]x_s\left[k+m(s-j)\right]}\right\} , \tag{A-2}$$

where i and j are constrained to be unequal. Recognizing that only those samples at the same instant of time are correlated and by the amount $2/\pi \sin^{-1} S/N+I+S$, this mean becomes,

$$E(S) = Tf_s(M^2-M)\ \frac{2}{\pi}\ \sin^{-1}\frac{S}{N+I+S}\ \left(1 - \frac{2}{M} + \frac{1}{M^2}\right) \quad . \tag{A-3}$$

It follows that the shift in the mean of the test statistic becomes asymptotically as the signal-to-noise ratio S/N gets small

$$E_K(S) - E_H(S) \rightarrow \frac{2}{\pi}\ Tf_s\ (M^2-M)\ \left(1 - \frac{1}{M}\right)^2 \frac{S}{N+I} \tag{A-4}$$

Let us now replace the quantity $(x_i(k) - y_i)$ of Eq. (A-1) by the symbol $z_i(k)$. The variance of the test statistic under the hypothesis can be written as

$$\mathrm{Var}_H(S) = \sum_{k=1}^{Tf_s}\sum_{p=1}^{Tf_s}\ \sum_{i\neq j}^{M\ M}\ \sum_{q\neq n}^{M\ M}\ \overline{z_i(k)z_j(k)z_q(p)z_n(p)} \quad . \tag{A-5}$$

The z terms are all pair-wise uncorrelated unless the subscripts differ by the right amount to place them on a common interference front. Therefore

$$\mathrm{Var}_H(S) = 2\sum_{k=1}^{Tf_s}\sum_{i\neq j}^{M,M}\ \overline{z_i^2(k)}\ \overline{z_j^2(k)} + 4\sum_{p=1}^{Tf_s/m}\ (Tf_s - m)\ \sum_{i\neq j}^{M,M}\ \overline{z_i(k)z_{i+p}(k+mp)}\ \overline{z_j(k)z_{j+p}(k+mp)} \tag{A-6}$$

Realizing now that there are only (M-p)(M-p-1) terms that can exist in the second sum, the variance becomes

$$\mathrm{Var}_H(S) = 2Tf_s\ (M^2-M)\ \overline{z_i^2}^2 + 2\sum_{p=1}^{Tf_s/m}\ (Tf_s - pm)(M-p)(M-p-1)\overline{z_i(k)z_{i+p}(k+mp)}^2 \tag{A-7}$$

This can be simplified for large decision times by recognizing that

$$2\sum(M-p)(M-p-1) = (M^2-M)\ \overline{M} \quad . \tag{A-8}$$

where $\overline{M} = (2M-4)/3$. Hence for large decision times

$$\mathrm{Var}_H(S) \simeq 2Tf_s(M^2-M)\ \overline{z_i^2(k)}^2 + \overline{M}\ \overline{z_i(k)\ z_{i+p}(k+mp)}^2 \quad . \tag{A-9}$$

Recognizing that

$$\overline{x_i(k)x_j\left[k+m(j-1)\right]} = \begin{cases} 1 \cdot \text{ if } i=j \\ 2/\pi \sin^{-1} I/N+I \text{; if } j=i+p, \\ 0\text{; otherwise} \end{cases} \tag{A-10}$$

it is straightforward to show that

$$\overline{z_i^{\,2}(k)} = \frac{M-1}{M}\left[1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right] ,$$

and

$$\overline{z_i(k)z_{i+p}(k+mp)} = \frac{1}{M}\left[\frac{2}{\pi}\sin^{-1}\frac{I}{N+I} - 1\right] . \tag{A-11}$$

Combining Eqs. (A-10) and (A-11), it is seen that

$$\mathrm{Var}_H(S) = 2Tf_s(M^2-M)\left[\frac{2}{\pi}\sin^{-1}\frac{I}{N+I} - 1\right]^2\left[1 - \frac{4}{3M} - \frac{1}{3M^2}\right] . \tag{A-12}$$

The output signal-to-noise ratio of the detector for weak signals is given by the ratio of the shift in the mean to the square root of the variance or

$$\mathrm{SNR} \xrightarrow{S/N \to 0} \frac{2}{\pi}\sqrt{\frac{Tf_s}{2}}\,\frac{S}{N+I}\,\frac{\sqrt{M^2-M}\,\left(1-\frac{1}{M}\right)^2}{\left[1 - \frac{2}{\pi}\sin^{-1}\frac{I}{N+I}\right]\sqrt{1 - \frac{4}{3M} - \frac{1}{3M^2}}} . \tag{A-13}$$

<u>Appendix B</u>

Let p_K denote the probability that a sample is contained in a quantization interval I_K given that the interference itself is contained in this interval i.e.

$$p_K = \mathrm{Prob}\left[n + i\varepsilon I_K / i\varepsilon I_K\right] . \tag{B-1}$$

If the interval is closed and of width $\Delta,\left[(k-1)\Delta < I_K < k\Delta\right]$, then

$$p_K = \frac{\int_{(k-1)\Delta}^{k\Delta}\left\{F_n\left[k\Delta-u\right] - F_n\left[(k-1)\Delta-u\right]\right\}f_i(u)\,du}{p(i\varepsilon I_K)} = \frac{\int_0^{\Delta}\left[F_n(x) - F_n(x-\Delta)\right] f_i(k\Delta-x)\,dx}{\int_0^{\Delta} f_i(k\Delta-x)\,dx} \tag{B-2}$$

If $p_{sc}^{(k)}$ is the probability of a 'switch" or the probability that a sample lies outside this interval given $i \epsilon I_K$, then

$$p_{sc}^{(k)} = 1 - p_K = \frac{\int_0^{\Delta}\left[1-F_n(x)\right] f_i(k\Delta - x)dx + \int_0^{\Delta} F_n(x-\Delta)\ f_i(\Delta - x)dx}{p(i\epsilon I_K)} \qquad \text{(B-3)}$$

As the inteference-to-noise power ratio gets large, this probability of a switch becomes

$$p_{sc}(k) \xrightarrow{I/N \rightarrow \infty} \frac{f_i(k\Delta) + f_i\left[(k-1)\Delta\right]}{p(i\epsilon I_K)} \int_0^{\Delta} \left[1 - F_n(x)\right] dx \quad . \qquad \text{(B-4)}$$

Assuming gaussian processes, this can be put into the form

$$p_{sc}(k) \rightarrow \frac{1}{\pi}\sqrt{\frac{N}{I}}\ \frac{\exp\left[-\frac{1}{2}\frac{(k\Delta)^2}{I}\right] + \exp\left[-\frac{1}{2}\frac{\left[(k-1)\Delta\right]^2}{I}\right]}{2p(i\epsilon I_K)}\ \sqrt{\pi} \int_0^{\Delta/\sqrt{2N}} \operatorname{erfc} x dx \qquad \text{(B-5)}$$

The probability of a switch from any of the 2(q-1) closed intervals is given by

$$p_{sc}^{q} = 2 \sum_{k=1}^{(q-1)} p_{sc}(k) p(i\epsilon I_K) \quad . \qquad \text{(B-6)}$$

By normalizing the quantization intervals by the standard deviation of the noise $\left[\Delta = m\sqrt{N}\right]$, p_{sc} becomes

$$p_{sc}^{q} \rightarrow \frac{1}{\pi}\sqrt{\frac{N}{I}}\,\pi \int_0^{m/\sqrt{2}} \operatorname{erfc} x dx \sum_{k=0}^{(q-1)} \left\{\exp\left[-\frac{1}{2} k^2 m^2 \frac{N}{I}\right] + \exp\left[-\frac{1}{2}(k-1)^2 m^2 \frac{N}{I}\right]\right\} \qquad \text{(B-7)}$$

Invoking symmetries, this can be written as

$$p_{sc}^{q} \rightarrow \frac{1}{\pi}\sqrt{\frac{N}{I}} \left[1 - \sqrt{\pi}\int_{m/\sqrt{2}}^{\infty} \operatorname{erfc} x dx\right] \sum_{k=1}^{q-1} \epsilon_k \exp\left[-\frac{1}{2} k^2 m^2 \frac{N}{I}\right] \quad , \qquad \text{(B-8)}$$

where
$$\epsilon_k = \begin{cases} 1; & k=0,\ q-1 \\ 2; & k\neq 0,\ q-1 \end{cases}$$

The probability of a switch given that the interference lies in one of the two open intervals is given by

$$P_{sd}(q) = \frac{\int_{(q-1)}^{\infty} F_n\left[(q-1)\Delta - u\right] f_i(u)\,du}{p(i\epsilon I_q)} = \frac{\int_0^{\infty} \left[1 - F_n(x)\right] f_i\left[x+(q-1)\Delta\right]dx}{p(i\epsilon I_q)} . \qquad \text{(B-9)}$$

This asymptotically approaches

$$P_{so}(q) \rightarrow \frac{f_i\left[(q-1)\Delta\right]}{p(i\epsilon I_q)} \int_0^{\infty} \left[1 - F_n(x)\right] dx \quad , \qquad \text{(B-10)}$$

which becomes for gaussian processes

$$P_{so}(q) \rightarrow \frac{1}{\pi}\sqrt{\frac{N}{I}} \; \frac{\exp - \frac{1}{2}(q-1)^2 \frac{N}{I}}{2p(i\epsilon I_q)} \sqrt{\pi} \int_0^{\infty} \text{erfc } x dx \qquad \text{(B-11)}$$

Recognizing that $\sqrt{\pi} \int_0^{\infty} \text{erfc } x dx = 1$, the probability of switching from either of the open intervals becomes

$$P_{so}{}^{q} \rightarrow \frac{1}{\pi}\sqrt{\frac{N}{I}} \; \exp\left[- \frac{1}{2}(q-1)^2 \frac{N}{I}\right] . \qquad \text{(B-12)}$$

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