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ROYAL AIRCRAFT ESTABLISHMENT

TECHNICAL REPORT No. 65038

TECHNIQUES OF MEASUREMENT

AND AUTOMATIC ANALYSIS

OF DYNAMIC MOTION WITH

APPLICATION TO SOME

RE-ENTRY EXPERIMENTS [U]

by

A. P. Waterfall

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TECHNIQUES OF MEASUREMENT AND AUTOMATIC ANALYSIS OF DYNAMIC MOTION WITH APPLICATION TO SOME RE-ENTRY EXPERIMENTS

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SUMMARY

An automatic technique, utilizing a large high speed computer for the analysis of records obtained from accelerometers, pressure gauges and gyros installed in the re-entry vehicles of the Black Knight rocket, is described. It is shown that this enables a suitable arrangement of only three accelerometers to yield trajectory data, angular motion and the aerodynamic characteristics of the re-entry body. Examples of the analysis of accelerometer and rate gyro data are given and the lines of further development and some applications are indicated.

Departmental Reference: Space 90

CONTENTS

1   INTRODUCTION

Black Knight is a ballistic research test vehicle primarily designed for atmospheric re-entry studies and firings at Woomera have continued since 1958. An important part of the Black Knight programme is the on-board measurement of the dynamic motion and aerodynamic characteristics of the re-entry vehicles flown. These re-entry vehicles are axially symmetric and mainly conical but some are spherical in shape and re-enter the atmosphere at speeds of up to 16000 ft/sec. The current Black Knight programme, Project Dazzle is a re-entry physics programme measuring the radar and optical characteristics of re-entry vehicles. Measurements by ground based instrumentation of either optical or IR radiation or radar returns, due to wake or ionisation effects associated with the re-entry body, are strongly affected by body incidence. In these experiments it is therefore important to correlate the angular motion of the vehicle with the ground measurement, as well as continuing the basic measurement of aerodynamic characteristics. These more exacting requirements plus a higher firing rate made it desirable to devise some form of automatic analysis scheme for the re-entry dynamics data, especially for that obtained from instruments carried by the re-entry vehicle. The techniques of measurement and data analysis that are under development form the substance of this paper.

There are two possible approaches to making aerodynamic measurements in free flight. The first is essentially similar to wind tunnel techniques in which forces and moments are measured directly along with incidence α, so that force and moment coefficients C_A, C_N and C_M may be calculated as functions of α. The alternative approach arrives at values of the aerodynamic derivatives $C_{N\alpha}$, $C_{M\alpha}$, C_{Mq} by studying the dynamic response. This does have limitations in the general case, because the form of the coefficient vs. incidence curve cannot be completely determined. However, adequate results are usually obtained by assuming some polynomial form to the curve. The advantages of the dynamic method is that it needs much less instrumentation than the direct method, and it is much in favour where space is at a premium. It is this type of method with which we are primarily concerned.

Analysis of dynamic response always needs a theoretical dynamic model. The one used for the analysis of the 'Black Knight' data is discussed in Section 2, together with an outline of the theory of re-entry and an appraisal of some of its limitations. This leads to some theoretical considerations, under Section 3, of the kind of instrumentation that may be carried by re-entry vehicles. Accelerometers in particular are considered in detail, because they

are usually the simplest and most compact system. It is shown how some
inherent difficulties may be overcome and how three accelerometers, suitably
deployed, can monitor almost all the data necessary for a complete dynamic
analysis.

Section 3 also gives a brief account of the 'Black Knight' Re-entry
Vehicle instrumentation system. In the most comprehensive system, in which
direct measurements of coefficients are also attempted, rate gyros,
differential pressure systems, accelerometers and ground based ballistic
cameras yield the basic dynamics data. However, the usual practice is to
limit the on-board instrumentation to accelerometers only, because of shortage
of space.

There are two methods of dynamic analysis of re-entry data: tho first
is suitable for a graphical analysis by hand, the second for an automatic
method on a computer. The graphical method, described in detail under
Appendix A is essentially one of progressive simplification of the data.
It starts with an estimate of the trajectory so as to obtain dynamic pressure
as a function of time. The various components of the dynamic motion are
then separated and progressively reduced to a plot of the motion and estimates
of the aerodynamic derivatives.

It may be possible to adopt the graphical method for computer use but
it would be most untidy; it is better to adopt a different approach based
on least squares fitting methods. The basic idea is to begin by making a
guess at the parameters in the dynamic model, which is then used to produce
theoretical numbers for comparison with the actual data. The method of
differential corrections, described in Section 4, is then used to improve
the estimates of the parameters until the best fit to the data, on the least
squares criterion, is obtained. Such a techrique is ideally suited to com-
puter use, and it has the great advantage that the model may be as compre-
hensive as we please, whereas there are practical limits to a graphical
analysis performed by hand.

Automatic methods using a digital computer were developed first for
the analysis of trajectory data from ballistic cameras and accelerometers.
These are briefly described under Section 5.1 and illustrated with actual
results in which a smoothed trajectory is obtained along with estimates of
C_A as a function of height. Such least squares methods have long been used
for analysing the dynamic motion in ballistic ranges to obtain the aerodynamic
derivatives. The dynamic model in this case is fairly simple. The model

described in Section 2 is necessarily more complex to take account of the large
changes in environment during re-entry and a large high-speed computer now
becomes essential if the computing time is to be kept to a reasonable duration.
These programmes, identical in principle but much more complex than the tra-
jectory programmes are described briefly under Section 5.2. Typical results we
also discussed showing how the angular motion is fitted and estimates made of
aerodynamic derivatives.

Operating details of the analysis programmes are given in Appendix C.
The report ends with a discussion of possible future developments and
applications.

2   THE DYNAMIC MODEL

It is convenient, for the purposes of analysis, to consider re-entry
dynamics in two parts, and to take first the motion of a point having given
aerodynamic properties, and then the motion of the body about its C of G as it
descends through the atmosphere. The first loads to a description of the
trajectory of the body in terms of height and velocity as a function of time,
and the latter to the dynamic motion. Strictly speaking the trajectory and
motion are inter-related, but the technique described in this paper is concerned
with re-entry experiments in which the angles of incidence are intended to be
small. In such cases the dynamic motion produces only small perturbations of
the trajectory which can usually be determined quite accurately by making only
very rough allowances for the effect of the lateral motion.

2.1   Equations of motion of a particle over a fixed earth in two dimensions

In the equations of motion, effects arising from the rotation of the
earth are ignored but gravity terms are included although even these are
negligible in comparison with the aerodynamic terms except during the very
early stages of re-entry. The equations of motion then take the following
convenient form for small values of incidence:

$$\dot{V} = (- D - N\bar{\alpha})/m - g \sin \theta \tag{1}$$

$$V\dot{\theta} = (- D\bar{\alpha} + N)/m - g \cos \theta + V^2 \cos \theta/r \tag{2}$$

$$\dot{h} = \dot{r} = V \sin \theta . \tag{3}$$

Where all parameters are defined in Fig.1 and where $N = \frac{1}{2}\rho V^2 s \bar{\alpha} C_{N\alpha}$,
$D = \frac{1}{2}\rho V^2 s C_A$ and $g = g_0 r_0^2/r^2$ and ρ is air density, s the reference area,
C_A axial force coefficient and $C_{N\alpha}$ the normal force derivative. Note that D
rather than A is used for axial force in this paper.

By making certain assumptions about the variation of ρ with height, and ignoring all except the drag term, the above equations may be integrated to obtain expressions for velocity and height as a function of time. However, the advent of very high speed computers makes these rather inflexible trajectory models unnecessary. Much more useful results may be realised by integrating the equations by a numerical method. It is then possible, for example, to determine the variation of C_A with Mach number and altitude rather than the mean ballistic coefficient. This is discussed further in Sections 3 and 4.

2.2 The three-dimensional gyrations of a missile descending through the atmosphere

The basic equations of motion of a spinning body during re-entry have much in common with the stability equations of aircraft and spinning shells, with the extra complication of rapidly varying dynamic pressure and aerodynamic coefficients. Theoretical studies of the dynamics of re-entry have received a great deal of attention of recent years, although many of these have made simplifying assumptions which render them unsuitable for our purpose. The approach given in Ref.1 is, perhaps, the most comprehensive, and has been followed in the authors work. Only relevant results will be quoted and these without proof.

We are concerned with small angular perturbations of a missile about the trajectory. Small perturbations are a necessary assumption of the mathematical solution, but it is not a serious restriction as incidences of up to 40 degrees are allowed. It is assumed that the basic unperturbed trajectory is a straight line. The effect of perturbations is to disturb both the trajectory and the orientation of the missile axis. To specify these angular perturbations we will use an orthogonal reference frame fixed in the body. (N.B. this differs from Ref.1 where the axes are fixed in space.) We take X along the axis of symmetry of the missile and the origin O at the centre of gravity. Then, looking in the direction of motion from behind the C of G, the angular perturbations can be represented as projections on the YZ plane thus:-

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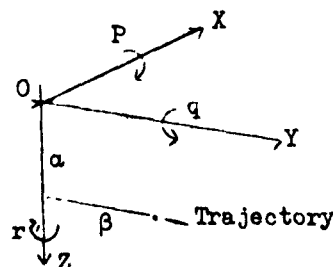
where the missile has perturbations, α (pitch), β (yaw) and angular velocity resolutes P (spin), q (pitch), r (yaw).

The equations of motion in terms of the complex quantities $\overline{\alpha - i\beta}$ are derived in Ref.1. Certain terms are negligible from a practical point of view. On deleting such terms, the following equation remains:-

$$\overline{\ddot{\alpha - i\beta}} + \overline{\dot{\alpha - i\beta}}\left[\tfrac{1}{2}\rho\,Vs\left(\frac{C_{L\alpha}}{m} - \frac{\ell^2 C_{Mq}}{B}\right) + 2iP\left(1 - \frac{A}{2B}\right)\right]$$

$$+ \overline{\alpha - i\beta}\left[-\frac{\tfrac{1}{2}\rho\,V^2 s\ell C_{M\alpha}}{B} - P^2\left(1 - \frac{A}{B}\right)\right.$$

$$\left. + iP\tfrac{1}{2}\rho\,Vs\left(\frac{C_{L\alpha}}{m}\left(1 - \frac{A}{B}\right) - \frac{\ell^2 C_{Mq}}{B}\right)\right]$$

$$= \frac{\tfrac{1}{2}\rho\,V^2 s\,\ell\,C_{M\alpha}}{B}\,\overline{\alpha_e - i\beta_e}\;. \tag{4}$$

This is the body axes counterpart of equation (12) of Ref.1, with the addition of an aerodynamic asymmetry term on the right hand side. It should be noted that $\overline{\alpha_e - i\beta_e}$ is small compared with $\overline{\alpha - i\beta}$ so that terms involving V have been omitted from the right hand side in comparison with V^2 term.

This equation (4) could be used as it stands as the dynamical model for the dynamic motion in the same way as we do for the trajectory. This would be a retrograde step, however, as a great deal of physical insight would be lost - the equation is much too complex, which is not true in the case of the simple trajectory equations. An analytical solution to equation (4) is there-fore required.

The solution may be considered as the sum of two particular integrals, that of motion about trim and that of trim itself. The first is obtained by making $\overline{\alpha_e - i\beta_e}$ zero in free space, $\overline{\alpha - i\beta}$ taking the free space value. The second is obtained by making $\overline{\alpha - i\beta}$ and its derivatives zero in free space and studying the effect of $\overline{\alpha_e - i\beta_e}$ as the body enters the atmosphere. The latter is a complex problem and will be referred to later. The solution offered here is for the case of zero trim or that part of re-entry well away from the resonance region. A quasi-static solution to equation (4) for the motion about trim is derived in Ref.1. In terms of body axes it is as follows:

$$\overline{\alpha - i\beta} \;=\; R \, \exp\left[\, i \int_{o}^{t} (-\,\bar{\omega} + AP/2B - P)\ dt \right] + S \, \exp\left[\, i \int_{o}^{t} (\bar{\omega} + AP/2B - P)\ dt \right].$$

$$\ldots \;(5)$$

where

$$\bar{\omega}^2 \;=\; -\tfrac{1}{2}\,\rho\, V^2 s\, \ell\; C_{M\alpha}/B \;+\; A^2 P^2/4B^2$$

$$R \;=\; R_o \sqrt{\omega_o/\omega}\; \exp\left[-\int_{o}^{t} (a + bAP/2B\bar{\omega})\ \rho\, V\, dt \right]$$

$$S \;=\; S_o \sqrt{\omega_o/\omega}\; \exp\left[-\int_{o}^{t} (a - bAP/2B\bar{\omega})\ \rho\, V\, dt \right]$$

$$a \;=\; \tfrac{s}{4}\left[C_{L\alpha}/m - \ell^2\; C_{Mq}/B \right]$$

$$b \;=\; \tfrac{s}{4}\left[C_{L\alpha}/m + \ell^2\; C_{Mq}/B \right]$$

and $\bar{\omega}_o$, R_o and S_o represent the situation at $t = 0$.

This is the well known form of two arms rotating in opposite directions, the R arm against the spin at a rate $(-\,\bar{\omega} + AP/2B - P)$ and the S arm with the spin at a rate $(\bar{\omega} + AP/2B - P)$. In the absence of spin the arms rotate at equal and opposite rates, and so would draw an ellipse at rate $\bar{\omega}$. The effect of spin is to cause this ellipse to precess at rate $P(1 - A/2B)$, resulting in the familiar flower petal pattern of Fig.4.

The limitations of this model are fully discussed in Ref.1 and it would appear that, for all practical purposes, it is valid for symmetrical missiles with linear aerodynamic properties below 200 000 ft. Above that height it would be seriously in error if the spin were very small, much smaller than would be desirable in practice. The two-arm pattern also remains qualitatively true in the presence of non-linear aerodynamics and the sort of small asymmetries which are the result of manufacturing imperfections. Allowance for some of these effects will be discussed in Section 6. However, experience has shown that this linear symmetric theory provides a very useful dynamical model for a preliminary analysis of re-entry motion.

Before considering methods of measurement it will be helpful to write down the derivatives of $\overline{\alpha - i\beta}$. Define:

$$\gamma_R = \int_0^t (- \bar{\omega} + AP/2B - P)\ dt + \gamma_{R_o} \tag{6}$$

$$\gamma_S = \int_0^t (\bar{\omega} + AP/2B - P)\ dt + \gamma_{S_o} \tag{7}$$

then from equation (5)

$$\overline{\alpha - i\beta} = R\ e^{i\gamma_R} + S\ e^{i\gamma_S} . \tag{8}$$

Now in body axes

$$\overline{q + ir} = - \overline{\alpha - i\beta} - iP\ \overline{\alpha - i\beta} \tag{9}$$

and

$$\overline{q + ir} = - \overline{\ddot{\alpha - i\beta}} - iP\ \overline{\dot{\alpha - i\beta}} \tag{10}$$

substituting we have

$$\overline{q + ir} = - i[- \bar{\omega} + AP/2B]\ R\ e^{i\gamma_R} - i[\bar{\omega} + AP/2B]\ S\ e^{i\gamma_S}$$

$$\overline{\dot{q + ir}} = [- \bar{\omega} + AP/2B][- \bar{\omega} + AP/2B - P]\ R\ e^{i\gamma_R}$$

$$+ [\bar{\omega} + AP/2B][\bar{\omega} + AP/2B - P]\ S\ e^{i\gamma_S}$$

writing out in full, and including constant trim angles α_t and β_t we obtain

$$\left.\begin{array}{l} \alpha = R \cos \gamma_R + S \cos \gamma_S + \alpha_t \\[2ex] \beta = - R \sin \gamma_R - S \sin \gamma_S + \beta_t \end{array}\right\} \tag{11}$$

$$\left.\begin{array}{l} q = R\ [- \bar{\omega} + AP/2B]\ \sin \gamma R + S[\bar{\omega} + AP/2B]\ \sin \gamma s \\[2ex] r = - R\ [- \bar{\omega} + AP/2B]\ \cos \gamma R - S[\bar{\omega} + AP/2B]\ \cos \gamma s \end{array}\right\} \tag{12}$$

$$\dot{q} = R\,[-\bar{\omega} + AP/2B][-\bar{\omega} + AP/2B - P]\,\cos\gamma\,R + S[\bar{\omega} + AP/2B][\bar{\omega} + AP/2B - P]\,\cos\gamma\,s$$

$$\dot{r} = R\,[-\bar{\omega} + AP/2B][-\bar{\omega} + AP/2B - P]\,\sin\gamma R + S[\bar{\omega} + AP/2B][\bar{\omega} + AP/2B - P]\,\sin\gamma s$$

$$\dots\ (13)$$

3   METHODS OF MEASUREMENT

Instrumentation for the monitoring of a particular flight is divided between on board instruments and ground based cameras or radars. The latter are required to monitor position with time and will not be discussed. Trajectory analysis normally consists in combining ground data and airborne accelerometer data to arrive at a best fit to the data. Accelerometers, however, are affected by the oscillations of the body and need correcting before use in a trajectory analysis. It will therefore be appropriate to consider methods of monitoring the dynamic motion with on board instruments before the monitoring of the trajectory.

Three instruments are in common use for the measurement of dynamic behaviour, accelerometers, rate gyros and differential pressure devices. Rate gyros monitor $\overline{q + ir}$ directly and are perhaps the most useful and accurate instrument we can use. Differential pressure measurements give $\overline{\alpha - i\beta}$ assuming dynamic pressure is known. They need pre-flight calibrations and sometimes suffer from slow response. Both the latter are well understood and will not be discussed further. Accelerometers, on the other hand, are used more than any other instrument in ballistic missiles because of their small size and modest power requirements. Unfortunately they are also the least understood, and give the most opportunity of being incorrectly interpreted. They are therefore worthy of careful consideration.

3.1   The use of accelerometers

The usual purpose of accelerometers is to measure the acceleration of the centre of gravity, and for this they must be located at the centre of gravity. The acceleration along the three principal axes $(A_x,\ A_y,\ A_z)$ is given by:

$$A_x = -\tfrac{1}{2}\rho V^2 s\, C_A/m$$

$$A_y = \tfrac{1}{2}\rho V^2 s\, C_{N\alpha}(\beta - \beta_t)/m$$

$$A_z = \tfrac{1}{2}\rho V^2 s\, C_{N\alpha}(\alpha - \alpha_t)/m \quad .$$

A_x is the most used in trajectory determination. A_y and A_z can be used in the same way as differential pressure measurements to determine α and β, or, if these are known to find $C_{N\alpha}$.

As it is frequently impossible to locate accelerometers at or even near the C of G, we shall now consider the acceleration measured at a general point. As before, let the principal axes be CXYZ where O is at the C of G. Let the resolutes of velocity be (u, v, w) and let the resolutes of angular velocity Ω be (P, q, r). We consider a general point L located at (x, y, z).
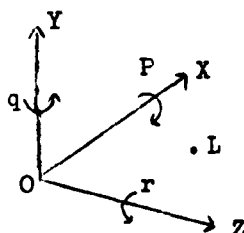
The velocity of L in body axes is given by:

$$V = [u\ v\ w] + \begin{bmatrix} P & q & r \\ x & y & z \end{bmatrix}$$

$$= [u + qz - ry,\ v - Pz + rx,\ w + Py - qx] \quad .$$

The acceleration of L is $\dot{V} + [\Omega v]$ which is

$$= [\dot{u} + \dot{q}z - \dot{r}y,\ \dot{v} - \dot{P}z + \dot{r}x,\ \dot{w} + \dot{P}y - \dot{q}x] +$$

$$\begin{bmatrix} P & q & r \\ u + qz - ry & v - Pz + rx & w + Py - qx \end{bmatrix}$$

$$= [(\dot{u} + qw - rv) - (q^2 + r^2)x + (qP - \dot{r})y + (pr + \dot{q})z,$$

$$(\dot{v} + ru - Pw) + (Pq + \dot{r})x - (P^2 + r^2)y + (qr - \dot{P})z,$$

$$(\dot{w} + Pv - uq) + (Pr - \dot{q})x + (qr + \dot{P})y - (q^2 + P^2)z] \quad .$$

The first terms in each component are actually the C of G acceleration in body axes (Ax, Ay, Az), so that:

Acceleration parallel to X axis

$$a_x = -\tfrac{1}{2}\rho V^2 s\, C_A/m - (q^2 + r^2)\, x + (Pq - \dot{r})\, y + (r + \dot{q})\, z .$$

$$\dots \quad (14)$$

Acceleration parallel to Y axis

$$a_y = \tfrac{1}{2}\rho V^2 s\, C_{N\alpha}(\beta - \beta_t)/m + (Pq + \dot{r})\, x - (P^2 + r^2)\, Y + qrz .$$

$$\dots \quad (15)$$

Acceleration parallel to Z axis

$$a_z = \tfrac{1}{2}\rho V^2 s\, C_{N\alpha}(\alpha - \alpha_t)/m + (Pr - \dot{q})\, x + q\,ry - (P^2 + q^2)\, z$$

$$\dots \quad (16)$$

where P, q, r, α, β, $\dot{q}$, $\dot{r}$ are given by equations (11) to (13) and $\dot{P}$ is assumed to be negligible.

It will readily be appreciated that the general situation is very complicated and that little can be done to simplify the above equations. Fortunately those terms involving qr and q^2 and r^2, which destroy the simple two component form of the C of G accelerations, are often small and may be neglected in analysis by hand. Corrections are readily made for centrifugal effects in automatic analysis techniques when accelerometers have been scattered about a re-entry vehicle in an arbitrary fashion, but are a nuisance particularly in the case of trajectory analysis. However, accelerometers can be made to yield extra useful data if displaced from the C of G with care and thought, especially axial displacement of lateral accelerometers. This special case will now be considered.

3.2   The use of axially displaced lateral accelerometers

Suppose we have an accelerometer measuring acceleration parallel to the Y axis but displaced from the C of G along the X axis. Then

$$a_y = \tfrac{1}{2}\rho V^2 s\, C_{N\alpha}(\beta - \beta_t)/m + (Pq + \dot{r})\, x \qquad (17)$$

substituting for β, P, q and $\dot{r}$ from equations (11), (12) and (13)

$$a_y = [- \tfrac{1}{2} \rho V^2 s \, C_{N\alpha}/m + x(\bar{\omega} - AP/2B)^2] \, R \sin \gamma_R$$

$$+ [- \tfrac{1}{2} \rho V^2 s \, C_{N\alpha}/m + x(\bar{\omega} - AP/2B)^2] \, S \sin \gamma_S \, . \qquad (18)$$

Hence the general effect of an axial displacement is to change the magnitude of the two oscillatory components of the accelerometer reading. A backward displacement (-x) increases, a forward displacement decreases the magnitude of the components.

If we have two accelerometers at axial stations x_1 and x_2, and subtract their outputs, we eliminate terms involving $C_{N\alpha}$ and are left with the centrifugal terms only:

$$\Delta = (x_1 - x_2) \, [(\bar{\omega} - AP/2B)^2 \, R \sin \gamma_R + (\bar{\omega} + AP/2B)^2 \, S \sin \gamma_S] \, . \qquad (19)$$

From a plot of Δ it is possible, in principle, to obtain R and S without even a knowledge of the trajectory. This is the basis of the method suggested by Nelson. In his method, an orthogonal pair of accelerometers is required at each axial station because the data is analysed by the graphical method described in Section 4.1. However, the automatic method of the author can fit a curve to a plot of Δ, or any single record and deduce R and S as well as other parameters. Moreover $C_{N\alpha}$ can be deduced by substitution of an analysis of (19) back into (18). Consequently, with a third accelerometer to measure longitudinal acceleration, a complete dynamic analysis becomes possible. For, C_A is determined from the trajectory, $C_{M\alpha}$ from $\bar{\omega}$, C_{Mq} and $C_{L\alpha}$ from the damping of R and S. In addition $C_{L\alpha}$ is also given by the difference of $C_{N\alpha}$ and C_A. Thus three accelerometers can be deployed so that all the aerodynamic properties of the re-entry vehicle are monitored. The axial displacement $(x_1 - x_2)$ has to be large and the accelerometers need to be very accurate to give superior results to a combination of rate gyros and accelerometers.

It is worth noting that over most of the re-entry $\bar{\omega}$ is very much bigger than AP/2B so that equation (18) may be reduced to the form:

$$a_y = \frac{\tfrac{1}{2} \rho V^2 s \, C_{N\alpha}}{m} \, (\beta - \beta_t)(1 + m\ell \times C_{M\alpha}/B \, C_{N\alpha}) \, . \qquad (20)$$

The term $m\ell \times C_{M\alpha}/B \, C_{N\alpha}$ is a constant (approximately), so that the effect of moving an accelerometer axially from the C of G is to change the magnitude of oscillations by this factor.

3.3   Determination of the trajectory from accelerometers

If a trajectory is to be integrated it is necessary to determine the aerodynamic terms of equations (1) and (2), these are not given directly by acceleration measured in the body. The term $(- D \bar{a} + N)$ of equation (2) is oscillatory and it has been found reasonable to assume that its net effect on $\dot{\theta}$ is zero. The problem is reduced therefore to a determination of the value of $(D + N\bar{a})$ of equation (1).

The major part of this term is measured by the longitudinal accelero-meter but, as will be seen from equation (14) there are complications caused by displacements from the C of G. It may be shown that lateral displacements result in symmetrical oscillatory errors, which may be removed by reading the mean value of acceleration. This leaves the effect of axial displacement only viz:

$$a_x = -\tfrac{1}{2} \rho V^2 s C_A/m - (q^2 + r^2) x .$$

Substituting from equation (10), assuming that $\bar{\omega} \gg AP/2B$, and taking the mean value of a_x we have

$$\bar{a}_x = -\tfrac{1}{2} \rho V^2 s C_A/m - \bar{\omega}^2 (R^2 + S^2) x .$$

Here $\sqrt{R^2 + S^2}$ is the rms of incidence $\bar{a}$ if trim is assumed to be zero. Substituting for $\bar{\omega}$ also we have

$$\bar{a}_x = -(C_A - m\ell \times \bar{a}^2 C_{M\alpha}/B) \tfrac{1}{2} \rho V^2 s/m . \tag{21}$$

Combining with equation (1) we have:

$$\dot{V} = \bar{a}_x - (1 + m\ell \times C_{M\alpha}/B \ C_{N\alpha}) \frac{N\bar{a}}{m} - g \sin \theta . \tag{22}$$

Using the lateral accelerometers an estimate of $\bar{a}$ can be made by estimating the rms value $\bar{a}_y$ which is equal to N/m. Thus:

$$\dot{V} = \bar{a}_x - [1 + m\ell \times C_{M\alpha}/B \ C_{N\alpha}] \ \bar{a}_y^2/\tfrac{1}{2} \rho V^2 s C_{N\alpha} - g \sin \theta \tag{23}$$

and

$$V\dot{\theta} = - g_0 \cos \theta + V^2 \cos \theta/r \tag{24}$$

$$\dot{h} = \dot{r} = V \sin \theta . \tag{25}$$

Given an initial height, velocity and climb angle these equations may be integrated to give a re-entry trajectory, using values of $\bar{a}_x$ and $\bar{a}_y$ from the accelerometer records. It will be shown in Section 5.1 how this may be combined with a least squares fit to ballistic camera observations. Although only approximate allowance has been made for the effect of dynamic motion on the accelerometer and the trajectory, it has been found adequate in practice.

3.4   Black Knight instrumentation system

A considerable quantity of sub-miniature instrumentation can be carried in Black Knight re-entry vehicles. Typical instrumentation payloads, i.e. excluding the outer shell, internal support structure and ballast weights, have been up to 60 lb in weight. Output from transducers can be transmitted to ground receivers via radio telemetry or alternatively recorded on magnetic tape. Provided the recorded tape is stored in an armoured cassette, recovery presents little difficulty.

The latter system which is not troubled by ionisation blackout during re-entry, has been used exclusively for a number of years and a range of magnetic tape recorders has been developed. The most favoured version, the sub-miniature recorder, is fully armoured and provides a total record time of $3\frac{1}{2}$ minutes in 8 tracks on $\frac{1}{2}$ inch tape running at $1^7/8$ inches per sec. The tape recorder is switched on shortly before re-entry commences.

One tape track is used for recording time in the form of pulses at a prf of 2·5 kc/s provided by a crystal controlled source. The main system of recording on other tracks is by frequency modulation. The input to the modulator is ±2·5V D.C., the output is 1·5 kc/s ±600 c/s feeding direct to the recording heads. Each F.M. input is time multiplexed by a 24 way switch running at 3 revs per sec. The principle use of multiplexing on channels recording dynamics information is to permit the insertion of voltage calibrations from a high stability source.

Dynamics measurements in the re-entry vehicles have been of three types, rate gyros, differential pressure gauges and accelerometers. Three rate gyros are usually installed measuring pitch, yaw and roll rate. Power supply is from a rotary invertor and an inductive bridge type of pick-off is used to provide a voltage output proportional to angular velocity.

The acceleration and pressure transducers are also of the inductive bridge type but power requirements are modest so that a typical installation of 7 transducers can be small and compact. Such a system would consist of two

pressure transducers for differential pressure measurements in the pitch and yaw planes; two longitudinal accelerometers, one for acceleration from boosts prior to re-entry and one for deceleration during re-entry; and three lateral accelerometers, one of which is of greater sensitivity to monitor motion during the early part of re-entry. All these transducers are capable of a 1% full scale accuracy under the most severe environment.

In most Black Knight re-entry vehicles, where many other measurements are made, there is insufficient room for a complete dynamics installation and only accelerometers are installed. Such a system is illustrated in Figs.2 and 3. Fig.2 shows the sub-miniature tape recorder and its armoured cassette. Fig.3 shows an instrumentation assembly with the tape recorder visible on the left of the chassis. As we have seen this can be a most useful system although there has often been some difficulty in placing the accelerometers in the optimum positions.

After flight the tape is recovered and replayed on to paper records using an oscillograph or galva recorder. Timing pulses are counted down to give pulses at one millisecond intervals. Synchronisation with ground instrumentation is achieved by simultaneous recording of some event such as a boost ignition. The paper records are read by hand using a semi-automatic method at intervals of 0·01 seconds or less. It is then processed using a 'Pegasus' computer to produce punched paper tape of the parameters as a function of time. A print of this may be plotted for the purposes of graphical analysis or analysed on the larger 'Mercury' computer by the automatic method.

4   METHODS OF ANALYSIS

The automatic method of analysis is essentially a curve fitting technique. Its one drawback is the need to make a rough guess at the parameters of the curve before the automatic processes can proceed. This is because the dynamic model of re-entry behaviour is a highly non-linear function of the parameters to be determined. Whereas the familiar polynomial curve is a linear function of its coefficients and may be fitted by a simpler and more direct approach. To obtain approximate values of the parameters of the dynamic model it is necessary to do the analysis of the first half second or so of re-entry by hand. It will be appropriate, therefore, to briefly consider such an essential preliminary to the automatic technique. At the same time some of the limitations of graphical analysis will be demonstrated.

4.1   Graphical methods

The production of a trajectory by graphical means is fairly obvious.
One proceeds from drawing a curve through a plot of height against time data
to obtain estimates of velocity from the slope and thus estimates of dynamic
pressure. A summary of the procedure is given in Appendix A.

Graphical analysis of accelerometer or rate gyro data to determine the
motion of the body is also fairly straightforward and has been described in
many papers including those by Nelson[2,3]. An illustration is given in Fig.4
of the basic idea of the method. It shows how arm lengths R and S and fre-
quencies may be extracted from suitable plots of the data. Applying the theory
already given and using the results of trajectory analysis, it is possible to
estimate the aerodynamic derivative $C_{M\alpha}$.

This is as far as is necessary to go if numbers for the priming of the
automatic method are required. All that is needed are estimates of $C_{M\alpha}$,
p, R_o, S_o and the phase angles γ_{R_o} and γ_{S_o} at the initial time t_o. If the
whole analysis is to be conducted by hand then the process has to be repeated
for convenient lengths of the whole record of re-entry, perhaps lasting a
total of 10 seconds or more. It is then possible to obtain plots of R and S
against time and to make estimates of C_{Mq} and $C_{L\alpha}$ by studying the decay of
R and S with time. Details of the analysis of dynamic motion by hand are also
given in Appendix A.

There would, of course, be no need for an elaborate automatic technique
if the graphical method were as easy as it sounds. In practice data does not
plot with the beautiful precision of the smoothed data of Fig.4 and corrections
for the effects of accelerometer displacements, for instance, are difficult and
tedious to make. Consequently errors tend to be large and unknown so that it
is often impossible to be sure whether irregularities in re-entry behaviour are
real or apparent. On the other hand, an automatic method is not only able to
remove the tedium of analysis but make more elaborate corrections for known
errors and treat random errors in a proper statistical manner.

4.2   The automatic method

The general theory of the automatic method is given in Appendix B. It
will be helpful here to illustrate the main results by taking the simple one
dimensional problem of fitting a trajectory to observation of height x_i at
times t_i. We will suppose that heights can be calculated from the function

$$\pi_{1o} = \pi(x_1, x_2, t_i) \tag{26}$$

and it is desired to determine x_1 and x_2, which could be initial velocity and height, so as to minimise the function

$$U = \Sigma\, w_i(\pi_i - \pi_{io})^2 \tag{27}$$

which is the sum of the squares of the weighted residuals.

Because π is a non-linear function of x_1 and x_2 we employ the method of differential corrections which starts with approximate values x_k' for x_k and obtains x_k by an iterative procedure. That is, we start with

$$\pi_{ic}' = \pi(x_k', t_i) \tag{28}$$

and

$$x_k' = x_k + \delta x_k \quad (k = 1, 2) \quad . \tag{29}$$

The technique is to expand equation (28) by Taylor's series in terms of the partial derivatives

$$f_{ik} = \frac{\partial \pi(x_k, t_i)}{\partial x_k} \quad .$$

It is shown in Appendix B that δx_k is given by solving the equation:

$$\Psi\, E = C\,W\,D \tag{30}$$

where

$$E = \begin{pmatrix} \delta x_1 \\ \delta x_2 \end{pmatrix} , \quad C = \begin{pmatrix} f_{11} \cdots f_{n1} \\ f_{12} \cdots f_{n2} \end{pmatrix} , \quad W = \begin{pmatrix} w_1, 0, 0 \ . \ . \\ . \quad . \ \ . \ . \ . \\ 0 \quad 0 \ . \ . \ w_n \end{pmatrix}$$

$$D = \begin{pmatrix} \pi_{1c} - \pi_1 \\ . \quad . \quad . \\ \pi_{nc} - \pi_n \end{pmatrix} \quad \text{and} \quad \Psi = C\,W\,C^*$$

where asterisk $*$ denotes the transpose. In long-hand (30) becomes:

$$\delta x_1 \,\Sigma\, w_i f_{11} f_{11} + \delta x_2 \,\Sigma\, w_i f_{11} f_{12} = \Sigma\, w_i f_{11}(\pi_{1o} - \pi_1) \tag{31}$$

$$\delta x_1 \,\Sigma\, w_i f_{11} f_{12} + \delta x_2 \,\Sigma\, w_i f_{12} f_{12} = \Sigma\, w_i f_{12}(\pi_{1o} - \pi_1) \tag{32}$$

these are known as the Normal Equations and are solved by a pivotal condensation scheme to obtain δx_k. The non-linear nature of the problem makes it necessary to repeat the process using the new values of x_k until δx_k is negligible.

The accuracy of the observations is estimated from:

$$\sigma^2 = \sigma_o^2 - \frac{E^* \, C \, W \, D}{n-p}$$

where σ_o^2 is the variance of the uncorrected weighted observations.

At the 95% probability level, the probable error in the parameters x_k is given by

$$\Delta X = 2 \sigma^2 \Psi_k^{-1}$$

where Ψ_k^{-1} is the kth diagonal element of the matrix Ψ^{-1}.

The partial derivatives are computed by a simple perturbation technique. In this case three trajectories would be calculated instead of only one, the first for x_1, x_2, the second for $(x_1 + \xi_1)$, x_2, and the third for x_1, $(x_2 + \xi_2)$. The partial derivatives are then given by:

$$f_{11} = \frac{\pi(x_1, \, x_2, \, t_i) - \pi(x_1 + \xi_1, \, x_2, \, t_i)}{\xi_1}$$

and similarly for f_{12}.

The purpose of the weight w_i is to give the most weight to the most accurate observations. They are therefore defined as numbers inversely proportional to the square of the probable errors of the observations, i.e. the reciprocal of the variances of the individual observations

$$w_i = 1/\delta_i^2 \; .$$

In practice it is found convenient to read in $(1/\delta_i)$ to the computer, rather than the actual weight.

When data is weighted in this way, the estimate of σ, the weighted standard deviation of the residuals, should be little more than unity. Values of 2 or 3 for σ would indicate that δ_i are unduly optimistic or that there are defects in the dynamic model. For most data δ_i are not available in which case all the observations are given unit weight.

This then, very briefly, is the theory of the automatic method. Although a simple case has been taken the theory is completely general and has been programmed for a computer for any number of parameters, dimensions and observations. Into this general programme the subroutine, for the calculation of the dynamic model appropriate to the observations, is inserted. A block schematic diagram of the complete programme is shown in Fig.5 and details are given in Appendix C. The main features are as follows:

(a) Data from the complete re-entry period is split into blocks of a length depending on how long parameters may be considered constant. It is only necessary to estimate initial conditions for the first block, the programme predicts for the remainder.

(b) A facility is incorporated for rejecting data points which are obviously wrong as a result of misreading; this is on the basis of the residual being greater than 4 times the rms of the whole. Only data accepted is counted and its variance σ_o^2 calculated. The programme fails if data accepted is less than p, the number of parameters. This usually arises if the solution diverges.

(c) The fitting procedure is applied several times to each block until the values of δx_k are less than a preset accuracy level. The programme then goes on to the next block after printing out extra information such as the fitted trajectory or motion and the residuals.

5   AUTOMATIC ANALYSIS RESULTS

5.1   Trajectory analysis

Results to be quoted were obtained by a relatively simple one dimensional fit to height data, using the model discussed in Section 2. These particular programmes form only an appendix to a complete system for providing smoothed trajectories from launch to impact in three dimensions for the whole vehicle. However, re-entry test vehicles usually follow near vertical trajectories, so that one dimensional fits to height data, with an allowance for the departure of the trajectory from the vertical, have been found adequate for re-entry dynamics studies.

Two programmes are in common use - TRAJECTORY ANALYSIS 1623/1A and 1623/1B - and fit a trajectory to height data derived from ballistic cameras or radar. Both programmes use atmospheric pressures and temperatures measured at the time of flight. Programme 1A uses a drag table of C_A against Mach number based on the best pre-flight estimate. The first parameter is allocated

to ΔC_A, the mean zero error in the drag table. Second and third parameters are initial height h_o and velocity v_o. No attempt is made to allow for induced drag from the dynamic motion of the vehicle. Programme 1B uses measured values of acceleration A_x and A_y in place of the drag table, otherwise it is similar to 1A. The first parameter is now ΔA_x, the zero error in longitudinal acceleration. The dynamic model is given by equation (23) et seq and makes full allowance for induced drag and accelerometer displacement effects. The programme calculates the drag coefficient C_A. Operating details of these programmes will be found in Appendix C. The form of a typical set of data for input to the computer being reproduced in Table 1.

The result from Programme 1B is reproduced in Table 2. It will be noted that the programme first prints the initial parameters as read in, the second column being the desired perturbations ξ_k. Subsequently the new values of the parameters are printed after each iteration but the second column now lists the best estimates of the standard deviations ΔX of the parameters. Also before each new parameter list the best estimate of the standard deviation of the weighted residuals is printed, followed by the number of degrees of freedom $(n-p)$ where n is number of observations accepted.

It will be noted that one iteration sufficed to obtain the best values of the parameters, the second made no further improvement. In the printed trajectory, the unweighted residuals are printed on the right hand side. The initial fall in C_A with decrease in M is of particular interest and has been ascribed to a viscous effect. Only part of the fitted trajectory is reproduced for reasons of brevity.

5.2 Dynamic analysis

Four programmes have been developed at present for the successful analysis of flight data, one each for differential pressures and rate gyro analysis and two for accelerometers, as follows:

Dynamic analysis

> 1623/1 - For differential pressures -
>   9 parameters
>
> 1623/2 - For accelerometers - 9 parameters
>
> 1623/3 - For accelerometers - 10 parameters
>
> 1623/4 - For rate gyros   - 9 parameters

The extra parameter in the case of No.3 is $C_{N\alpha}$, the other 9 are $C_{M\alpha}$, C_{Mq}, P, R_o, S_o γ_{R_o}, γ_{S_o} and two zero errors E_x, E_y. At present programmes 1 to 3

cannot distinguish between trim and zero errors but later versions, it is
hoped, will do so.

All programmes use the same subsidiary data as Trajectory Analysis 1A
except that the drag table is derived from 1B results, as are the other
trajectory parameters. The trajectory is needed to give values for density
and dynamic pressure and is obtained by integration of equations (22), (24)
and (25). Allowance is made for induced drag so that the calculated trajec-
tory is identical to that deduced by trajectory analysis. It should be noted
that only C_A and $C_{N\alpha}$ are read into the machine, $C_{L\alpha}$ is calculated from the
difference $(C_{L\alpha} = C_{N\alpha} - C_A)$. Thus, except in the case of Programme No.3,
$C_{L\alpha}$ is fixed and damping can only be altered by varying C_{Mq}.

All programmes may be used to fit either a single instrument or an
orthogonal pair. It is thus possible to do a complete analysis from one
instrument only. However, Programme 3 is intended to analyse two accelero-
meters at different axial stations for the determination of $C_{N\alpha}$, rather than
rely on differential damping of R and S to give $C_{L\alpha}$ and thus $C_{N\alpha}$. Operating
details are given in detail in Appendix C.

Up to date, the automatic method has been used to analyse short lengths
of record or blocks of data. The complete record is covered by a series of
blocks. Some examples of the analysis of a single block are shown in
Tables 3 and 4 and Figs.6-8. Table 3 is the data tape and Table 4 the result
of fitting data from a pair of accelerometers with Programme No.2. The result
is plotted in Fig.6. Actual measurements are plotted for comparison with the
smoothed curve of acceleration. The motion plotted in Fig.4 to illustrate
graphical analysis was obtained from the same accelerometer analysis of Fig.6
and is thus the required end product of the analysis, it is, of course,
dependent on the assumed value for $C_{N\alpha}$. It will be noted that the pattern of
Fig.4 and Fig.6 is the result of R and S being almost equal, that of Fig.7
and Fig.8 is for a case when $2R \simeq S$. Programme 2 was again used to analyse
a pair of accelerometers to give Fig.7. Programme 4 was used to analyse a
rate gyro record from the same re-entry but at an earlier time to give Fig.8.
Some of the parameters obtained from this analysis are given along side those
from an accelerometer analysis for the same part of the re-entry. It will be
noted that values of the parameters agree very well but the rate gyro is
claimed to be much more accurate. This is probably because a gyro is not at
the mercy of so many side effects as an accelerometer.

The poor accuracy of C_{Mq} in the figures quoted in Fig.8, especially in the case of the accelerometer, is mainly because the record is for an altitude of 160 000 ft when the effect of C_{Mq} is negligible. Accuracy is greatly improved at lower altitudes and is also helped by analysing a longer period of record. Even so, C_{Mq} is a difficult parameter to measure accurately and it is doubtful if an accuracy of better than ±0·5 can be achieved.

It has been seen that trajectory analysis converges very rapidly. Dynamic analysis is much slower, in spite of good guesses at the initial parameters, because of the highly non-linear form of the dynamic model. This is illustrated by the plot of parameter values against iteration number in Fig.9 for an accelerometer analysis. The standard deviation of the residuals and the number of degrees of freedom are also plotted, and it will be seen that initially half the observations were not used. This probably accounts for some of the erratic variations in parameter values as more observations were accepted. It has been found in practice that up to six iterations are necessary to fit the motion at the beginning of re-entry when the initial estimates are the result of rough graphical analysis. At present a complete re-entry record is split into 10 blocks of about 40-50 readings and covering 20000 ft of altitude. Having made an initial guess at the first block the remaining nine are handled automatically in about one hour of computing. The time is the same whether one instrument or an orthogonal pair are being analysed.

6   FUTURE DEVELOPMENTS

This paper has been primarily concerned with the basic approach to the analysis of re-entry data by automatic means. At present this scheme uses the simplest dynamic model which can reasonably be assumed - the case of linear aerodynamics and a symmetric missile. It has been found, however, that it works surprisingly well and is adequate for a primary analysis of re-entry data.

The analysis of several firings on this basis has shown that there are at least two defects in the model which must be corrected before analysis of those firings can be satisfactorily completed. The most common defect is the non-linear behaviour of $C_{M\alpha}$. It is this which makes it necessary to analyse over unduly short time intervals. To analyse over longer periods assuming a constant $C_{M\alpha}$ is impossible because $C_{M\alpha}$ governs the aerodynamic frequency $\bar{\omega}$ and small errors soon build up into unacceptably large phase errors. As a result of taking data in short groups it is impossible to obtain a smoothed curve of

incidence through re-entry and accurate estimates of C_{Mq} and $C_{L\alpha}$. It is thus essential to permit $C_{M\alpha}$ to vary in some way.

Results have shown that incidence has the dominant influence on $C_{M\alpha}$ and that a function of the form

$$C_{M\alpha} = c_o + c_1 \bar{\alpha}^2 \tag{36}$$

is a good approximation to the truth. It is proposed to extend the dynamic model in this way, at the same time putting a restriction on spin rate P. This is necessary because non-linearities in $C_{M\alpha}$ also manifest themselves as apparent spin. Spin will be set at the best known free space value if nothing better is available. This modification should enable the length of record that may be analysed to be greatly extended.

It will be noted that $C_{M\alpha}$ will be made a function of the mean incidence $\bar{\alpha}$ or $\sqrt{R^2 + S^2}$, however, it it is really a function of $\overline{\alpha - i\beta}$ and not only affects pitching frequency but the waveform of the oscillations. This ought to be taken into account. Theoretical work on this problem by Rasmussen[4] and others has been successfully applied to ballistic range experiments. Nelson[3] has extended the non-linear theory to a varying environment on an intuitive basis. All these workers use graphical methods of analysis and it is not clear how to apply these methods in an automatic scheme. Moreover it is doubtful whether re-entry data is of sufficient quality to justify a detailed non-linear analysis scheme.

A further limitation of the dynamic model is the assumption of aero-dynamic symmetry. On at least one round this defect has proved serious and has made it impossible to fit data over long periods up to resonance, because of the shift in phase and amplitude of trim. After resonance the trim is in phase with the S arm and gives no further trouble. Equation (4) in Section 2 includes the effect of asymmetry and it was mentioned that the solution is the sum of two particular integrals. That for the motion about trim forms the present dynamic model, it remains to add the particular integral for the behaviour of trim. An analytical solution for this has been obtained by G.S. Green at the R.A.E. for the case of no damping and lift, it is a series arising from a modified form of Bessel's Function. The series converges rapidly well away from resonance and is useful for determining the general effect of asymmetry. In the region of resonance it can go to hundreds of terms and is quite unsuitable for our purpose. Some other approach is desired.

It has been decided that the simplest way to determine the particular integral for the motion of trim is to integrate equation (4) numerically with $\overline{\alpha - i\beta}$ zero at infinity. The motion about trim would still use the analytical solution. This is not as bad as it sounds because $\overline{\alpha_e - i\beta_e}$ is a scale factor and the only important variables are $C_{M\alpha}$ and perhaps C_{Mq}. It would involve adding 12 differential equations to the mathematics of the programme and should cause no undue difficulties.

From experience with the automatic scheme using the linear symmetric model it is felt that the above two modifications will enable a complete re-entry to be analysed in a single run on the computer. This will not necessarily speed the analysis up but will enable accurate mean values for $C_{N\alpha}$, $C_{L\alpha}$ and C_{Mq} to be obtained as well as a continuous history of incidence with time. It is hoped to try it out in the first four months of 1965.

7   CONCLUSIONS

The automatic method of analysis has proved to be a considerable advance on the graphical methods formerly used for the analysis of re-entry data. Not only has the primary objective, of developing a quick and efficient method of treating large quantities of re-entry data, been achieved, but it has enabled a more sophisticated instrumentation scheme using accelorometers to be recommended. This was an unexpected bonus and means that a very simple arrangement of accelerometers can monitor, not only the motion, but all the aerodynamic characteristics of the re-entry vehicle. This principle might be applied with profit in ballistic range experiments.

Computer methods also enable more complex mathematical models of motion to be used than is normally practicable in hand analysis, such as the treatment of non-linear aerodynamics and asymmetric effects. In addition experimental errors are treated in a consistent manner which is important when results of several firings are to be compared. These considerations are quite general and do not only apply to re-entry work. It is probable that such methods can be applied to other experiments in the aero-space field. Re-entry experiments cannot be the only case where simpler instrumentation can be gained at the expense of more elegant analysis methods.

Appendix A

GRAPHICAL METHODS OF ANALYSIS

The term graphical methods includes all the many techniques for the analysis of flight data which do not require a digital computer; although it is difficult to draw a definite dividing line. The ideal automatic method should not need the intervention of a human operator but all methods of the curve fitting type require a guess at the initial parameters and it is here that graphical methods must be employed. Graphical methods therefore form an essential preliminary to an automatic analysis and have not been included for historical interest. These techniques vary greatly with the data available and the whim of the analyst and are extensively described in the literature. Only a brief outline will be given in this paper.

Trajectory analysis

The steps in the process would normally include the following:

(a) The synchronisation of ground and airborne time which can be tricky when an airborne tape recorder is in use. The usual method is to record a flash both on the ground and on board at some convenient time.

(b) The determination of velocity V and height H at a time shortly before re-entry (say 250 000 ft altitude). This will be provided by analysis of vacuum trajectory data such as from radar and is a subject in itself.

(c) The integration of the longitudinal acceleration record, starting with the initial V and H, to obtain velocity and height as functions of time.

(d) The calculation of $\frac{1}{2} \rho V^2$ using measured pressures and temperatures of the atmosphere at the time of firing.

(e) A cross check and adjustment (if necessary) of the trajectory with spot heights from Ballistic Camera Data.

(f) The estimation of the drag coefficient.

Dynamic analysis

The analysis would include some or all of the following steps although not necessarily in the order given. The components of the record from a particular instrument are referred to as R' and S', and could be actual incidence or angular velocity or some other variable.

(a) For the early part of re-entry when pitching rate $\bar{\omega}$ is low and of the same order as spin rate P, extract the frequency and amplitude of R' and S' by study of a record from a single instrument. Fig.4 demonstrates in diagrammatic form how this can be done. By application of equation (6) $\bar{\omega}$ and P may be extracted and plotted against time, along with R' and S'. This type of approach for just the first few re-entry oscillations is all that is required to prime the automatic system.

(b) As an alternative to the above, especially when $\bar{\omega}$ is large, plot the pitch readings against yaw readings to obtain a graph similar to Fig.4. The precise form of this will depend on the relative sizes of $\bar{\omega}$ and P, and R' and S'^2. Measure the time and angle between adjacent peaks as shown, perhaps taking several peaks to obtain a mean value for a chosen time. Note that angles must be measured from the trim centre.

Then

$$\bar{\omega} = \pi/(t_2 - t_1) \quad \text{and} \quad P(1 - A/2B) = (\pi - \Delta\theta)/(t_2 - t_1)$$

where t_2 and t_1 corresponds to time between peaks.

Measure the amplitude of maxima and minima from the trim centre as shown. The maximum equals $(R' + S')$ and the minimum is $(R' - S')$ so that R' and S' may be obtained as before.

(c) A plot of R' and S' against time is obtained for each type of instrument or different pairs of accelerometers by repeated application of (a) or (b) above at a number of different times. From $\bar{\omega}$, P and $\frac{1}{2}\rho V^2$ from the trajectory, R' and S' can be processed to obtain the incidence components R and S, the pitching moment derivative $C_{M\alpha}$ and perhaps the normal force derivative $C_{N\alpha}$. This is achieved by application of the appropriate equations in Sections 2 and 3.

(d) The final stage in the analysis is the study of the variation of R and S with time to derive the stability derivatives $C_{L\alpha}$ and C_{mq}. First it is necessary to remove the effect of the term $\sqrt{\omega_0}/\bar{\omega}$ - often the dominant term for much of the re-entry. Hence compute and plot against time:

$$\bar{R} = \sqrt{\omega_0}/\bar{\omega}\, R \quad \text{and} \quad \bar{S} = \sqrt{\omega_0}/\bar{\omega}\, S .$$

The slopes of the resulting corrected curves $d\bar{R}/dt$ and $d\bar{S}/dt$ are then measured. Then it follows from equation 5

$$a + \frac{b}{\omega} \cdot \frac{AP}{2B} = -\frac{d\bar{\bar{R}}}{dt} \quad V \rho \bar{R}$$

$$a - \frac{b}{\omega} \cdot \frac{AP}{2B} = -\frac{d\bar{\bar{S}}}{dt} \quad V \rho \bar{S} \quad .$$

Since a and b are functions of $C_{L\alpha}$ and C_{Mq} it is theoretically possible to solve these equations to obtain the derivatives. Accuracy is usually too poor to make this possible but $C_{L\alpha}$ can be obtained separately from knowledge of $C_{N\alpha}$ and C_A for

$$C_{L\alpha} = C_{N\alpha} - C_A \quad .$$

Hence a mean value for C_{Nq} may be determined.

Appendix B

GENERAL THEORY OF AUTOMATIC METHOD OF ANALYSIS

The differential correction procedure

In general, a dynamic observation will be a vector such as position, velocity, or acceleration, and is best defined by the components in some cartesian axes system. In practice only one or two particular components may be measured, but the following theory assumes three dimensions although it is equally valid for any number.

Let α_i, β_i, γ_i be observations of the vector at a time t_i, and let n such observations be made at different times. If there are P undetermined parameters X_k (k = 1, 2, ... p) in the dynamic model, and if there are no errors in the observations or the model then the observations would be related to the parameters by 3n equations of the form

$$\alpha_i = \alpha(X_1, X_2, \ldots X_P, t_i)$$

$$\beta_i = \beta(X_1, X_2, \ldots X_P, t_i)$$

$$\gamma_i = \gamma(X_1, X_2, \ldots X_P, t_i)$$

the functions α, β, γ depending on the dynamic model. When $3n > P$, the parameters may be obtained by solving the equations redundantly, using a least squares procedure to determine the best values.

Let α_{1o}, β_{1o}, γ_{1o} be computed values close to the observed values based on estimates of the parameters x_k, such that

$$\alpha_{ic} = \alpha(x_1, x_2, \ldots x_P, t_i)$$

$$\beta_{ic} = \beta(x_1, x_2, \ldots x_P, t_i)$$

$$\gamma_{ic} = \gamma(x_1, x_2, \ldots x_P, t_i)$$

and let the residuals be of the form:

$$R_{\alpha i} = \alpha_i - \alpha_{ic} \text{ etc } .$$

Then the procedure adopted is to determine the values of x_k so as to minimise the quantity

$$U = \sum_{i=0}^{n} w_{\alpha i} R_{\alpha i}^2 + \sum_{i=0}^{n} w_{\beta i} R_{\beta i}^2 + \sum_{i=0}^{n} w_{\gamma i} R_{\gamma i}^2$$

where $w_{\alpha i}$, $w_{\beta i}$ and $w_{\gamma i}$ are the weights of the observations. Suppose x_k' are approximations to x_k so that

$$x_k' = x_k + \delta x_k$$

$$\alpha_{ic}' = \alpha(x_k' , t_i) , \quad \text{etc.} \tag{B1}$$

Linear equations may be obtained by expanding equation (B1) as a Taylor series, thus

$$\alpha_{ic}' = \alpha_{ic} + \sum_{k=1}^{P} f_{ki} \, \delta x_k + 0((\delta x_k)^2 , t_i)$$

$$\beta_{ic}' = \beta_{ic} + \sum_{k=1}^{P} g_{ki} \, \delta x_k + 0((\delta x_k)^2 , t_i)$$

$$\gamma_{ic}' = \gamma_{ic} + \sum_{k=1}^{P} h_{ki} \, \delta x_k + 0((\delta x_k)^2 , t_i)$$

where

$$f_{ki} = \frac{\partial \alpha(x_k, t_i)}{\partial x_k} , \quad g_{ki} = \frac{\partial \beta(x_k, t_i)}{\partial x_k} , \quad h_{ki} = \frac{\partial \gamma(x_k, t_i)}{\partial x_k} .$$

The correction procedure is based on the assumption that $(\delta x_k)^2$ is negligible. In practice this is not generally true, and the minimisation of U is obtained by repeated application of the procedure.

Let

$$R_{\alpha i}' = \alpha_i - \alpha_{ic}' , \quad \text{etc}$$

so that

$$R'_{\alpha i} = R_{\alpha i} - \sum_{k=1}^{P} f_{ki}\, \delta x_k$$

$$R'_{\beta i} = R_{\beta i} - \sum_{k=1}^{P} g_{ki}\, \delta x_k \qquad\qquad \text{(B2)}$$

$$R'_{\gamma i} = R_{\gamma i} - \sum_{k=1}^{P} h_{ki}\, \delta x_k$$

and

$$U' = \sum_{i=1}^{n} \left(w_{\alpha i}\, R'^{2}_{\alpha i} + w_{\beta i}\, R'^{2}_{\beta i} + w_{\gamma i}\, R'^{2}_{\gamma i} \right). \qquad\qquad \text{(B3)}$$

It is required to determine δx_k so as to minimise U', that is to find values of δx_k for which $\partial U'/\partial(\delta x_k)$ is zero. For this purpose it is helpful to pose the problem in matrix form.

Let

$$C = \begin{pmatrix} f_{11}, & f_{12}, \cdots, & g_{11}, g_{12}, \cdots, & h_{11}, h_{12}, \cdots \\ f_{21}, & f_{22}, \cdots, & g_{21}, g_{22}, \cdots, & h_{21}, h_{22}, \cdots \\ \cdot, & \cdot, \cdots, & \cdot, \cdot, \cdots, & \cdot, \cdot, \cdots \\ f_{k2}, & f_{k2}, \cdots, & g_{k1}, g_{k2}, \cdots, & h_{k1}, h_{k2}, \cdots \end{pmatrix}$$

$$E = \begin{pmatrix} \delta x_1 \\ \delta x_k \end{pmatrix}, \qquad\qquad D = \begin{pmatrix} R_{\alpha i} \\ R_{\beta i} \\ R_{\gamma i} \end{pmatrix}$$

and W a diagonal matrix with the weights along the diagonal in the order $(w_{\alpha i}, w_{\alpha 2}, \cdots w_{\beta 1}, w_{\beta 2}, \cdots w_{\gamma 1}, w_{\gamma 2})$ all other terms being zero. So that equations (B2) and (B3) may be written:

$$D' = D - C^* E \qquad\qquad \text{(B4)}$$

$$U' = D'^* W D' \qquad\qquad \text{(B5)}$$

where the asterisk $*$ denotes the transpose.

From equations (B4) and (B5)

$$D'^* \, W \, D' \; = \; (D^* - E^* \, C) \, W(D - C^* \, E) \tag{B6}$$

therefore

$$\frac{\partial U'}{\partial(\delta x_k)} \; = \; \frac{\partial(D'^* \, W \, D')}{\partial(\delta x_k)}$$

$$= \; - \, 2 \, C \, W \, D \, + \, 2 \, C \, W \, C^* \, E \; = \; O$$

that is

$$C \, W \, C^* \, E \; = \; C \, W \, D \tag{B7}$$

which is the matrix form of what are commonly called the normal equations.

Defining

$$\Psi \; = \; C \, W \, C^*$$

$$\Psi E \; = \; C \, W \, D$$

and

$$E \; = \; \Psi^{-1} \, C \, W \, D$$

which is the desired solution and gives the increment to be added to the parameters. The procedure is repeated using the new parameters as starting values, until E is negligible when $(\delta x_k)^2$ must be negligible, and the best estimate of the parameters obtained.

Estimation of the accuracy of the observations

To estimate the accuracy of the observations it is necessary to determine their covariance matrix. Let us suppose that D is redefined as a vector of residuals based on the true values of the parameters. That is a typical element in D is $[a_i - a(X_k, \, t_i)]$ rather than $[a_i - a(x_k, \, t_i)]$. Define $\bar{D}$ as the mean value of D. Then the weighted covariance matrix of D is

$$\text{Cov}(D) \; = \; \text{Expectation} \; (D - \bar{D}) \, W(D^* - \bar{D}^*) \quad . \tag{B8}$$

To determine Cov(D) certain assumptions have to be made: that the dynamical model exactly fits the true observations and that the errors are random, uncorrelated and normally distributed. In which oase the covariance matrix reduces to:

$$\text{Cov}(D) \; = \; \text{Expectation} \; (D^* W \, D) \; = \; \sigma^2 \tag{B9}$$

a best estimate of σ^2 is obtained from the minimum value of U', given by substituting (B7) into (B6).

Thus

$$D'^* W D' = D^* W D - D^* W C^* E - E^* C W D + E^* C W C^* E$$

$$= D^* W D - D^* W C^* E - E^* Y E + E^* Y E$$

$$= D^* W D - E^* C W D .$$

Best estimate of

$$\sigma^2 = \frac{U'}{3n - P} = \frac{(D'^* W D')}{3n - P}$$

so

$$\sigma^2 = \frac{D^* W D}{3n - P} - \frac{E^* C W D}{3n - P}$$

or

$$\sigma^2 = \sigma_0^2 - \frac{E^* C W D}{3n - P} \qquad (B10)$$

where σ_0^2 is the variance of the uncorrected weighted observations.

Estimation of the accuracy of the dynamic parameters

An estimation of the accuracy of the parameters determined by the correction procedure is most important, otherwise there will be a danger of drawing unjustified conclusions from the results.

Let $\bar{E}$ be defined as a vector of deviations from the true values of the parameters X_k. That is a typical element of $\bar{E}$ is $(X_k - x_k)$. Then the covariance matrix of E is given by taking the expectation of the product $(E - \bar{E})(E^* - \bar{E}^*)$.

Now

$$E - \bar{E} = Y^{-1} C W (D - \bar{D})$$

so that

$$\text{Exptn } (E - \bar{E})(E^* - \bar{E}^*) = \text{Exptn } Y^{-1} C W (D - \bar{D})(D - \bar{D})^* W^* C^* Y^{-1*}$$

from equation (B9),

$$\text{Exptn } W(D - \bar{D})(D - \bar{D})^* = \sigma^2 .$$

Hence

$$\text{Cov}(E) = \sigma^2 Y^{-1} C W^* C^* Y^{-1*}$$

noting that Y and W are symmetric matrices so that they are unchanged by transposition, we have

$$\text{Cov}(E) = \sigma^2 \Psi^{-1} . \tag{B11}$$

Provided the parameters are uncorrelated all the terms in $\text{Cov}(E)$ will vanish except for the diagonals. These diagonal terms, therefore give the variance of the individual parameters, since the variances of δX_k must be the variance of X_k.

Let the variance of X_k be S_k^2 and Ψ_k^{-1} be the kth diagonal element of Ψ^{-1}, then

$$S_k^2 = \sigma^2 \Psi_k^{-1} . \tag{B12}$$

The best estimate of the standard deviation of X_k will be S_k, although a correction may be necessary for a small number of observations (usually $(n-P) < 20$). It is usual to quote accuracy at the 95% probability level, in which case the probable error in X_k is given by

$$\Delta X = 2 S_k = 2 \sigma^2 \Psi_k^{-1} . \tag{B13}$$

It should be emphasised that this estimate is only valid when $(\delta x_k)^2$ is negligible, a situation which is usually reached after several repetitions of the correction procedure. The non diagonal terms in Ψ^{-1} must also be negligible.

Appendix C

OPERATING DETAILS OF THE AUTOMATIC ANALYSIS PROGRAMMES

The automatic scheme described in Appendix B is completely general and could be applied to all manner of different problems. It has been developed to meet the particular needs of re-entry data analysis and advantage has therefore been taken of features peculiar to these problems. As far as possible a standard procedure has been adopted for both trajectory and dynamic analysis programmes, because it considerably eases the task of writing new programmes and modifying others. The programmes have been written for exclusive use on a Mercury Computer. For this reason much of the programme is in machine code mixed with Autocode, with the aim of achieving the maximum computing speed. This would limit the use of the programmes on other computers. Details are given first of general features of the data input and operating procedure, followed by details of specific programmes for trajectory and dynamic analysis.

The format of data input

A structural format has been used for all programmes. The data is grouped into five tables of numbers; a table of atmosphere properties; a table of drag properties of the body or of measured accelerations to enable the trajectory to be calculated, a list of variable parameters, a list of constant parameters, and finally the experimental data to be analysed. The data is headed by a title and is punched on five hole paper tape.

(a) Atmosphere table

This contains measurements of temperature and pressure made at the time of flight, the units being $^{\circ}$C and millibars. Each row of the table consists of:

Height (ft)   Pressure (mbs)   Temperature ($^{\circ}$C)

the maximum number of rows is 28 and the table is terminated by the warning character (*), an asterisk. The programme uses a log-linear interpolation to obtain density and Mach number as a function of height.

(b) Drag table

For trajectory analysis drag coefficient is read in as a function of Mach number (in the form: Mach number, C_A) but for dynamic analysis height is a more useful variable (viz: height, C_A). When trajectory analysis makes use of in flight measurements of acceleration this table takes the form:

$$\text{Time (sec)} \quad \bar{a}_x \ (g) \quad \bar{a}_y \ (g)$$

the maximum number of rows is 32 and the table is terminated by an asterisk (*) warning character.

(c)   Variable parameter list

In the present programmes up to 12 parameters may be variables. The initial estimates of the parameters x_k' are read as a table along with the desired perturbations ξ_k and is headed by the initial time, thus:

$$
\begin{array}{ll}
t_o & \\
x_1 & \xi_1 \\
x_2 & \xi_2 \\
\cdots\cdots & \\
x_P & \xi_P \ *
\end{array}
$$

The number of parameters P is not set, instead the table is terminated by a warning character *. If desired, parameters may be omitted from the bottom of the list when they are assumed to be zero by the programme unless also set in the list of constants.

(d)   Constant parameter list

This is a list of all constants needed by the dynamic model and of certain numbers which control the programme. The actual list depends on the particular programme.

(e)   Experimental data table

This table is headed by an integer q which is one less than the number of dimensions of the experimental data. The table is terminated by an asterisk *. If desired this table may be split into blocks, headed by q and terminated by *. The programme then fits each block in turn and uses the value of the parameters obtained for a block as the initial estimates for the subsequent block. In the general three dimensional case each row would be written thus:

$$t_i \quad \alpha_i \quad \beta_i \quad \gamma_i \quad w_i$$

That is the time followed by the components of the observation followed by the weight. The value of q in this case would be 2. The trajectory programmes are one dimensional so that q is zero and is not read in. The dynamic analysis

programmes are either one or two dimensional so that if q is 1 the programme looks for α_1 and β_1, but if zero only for α_1. In these programmes w_1 is not read in at present because weights are assumed to be unity.

Operating details

The programme tape is first read into the computer using the Mercury Autocode Input Scheme or the Binary Input. The machine then calls and reads the data tape. The operation of the programme is entirely automatic and con-trolled by numbers included on the data tape. Certain automatic controls, however, can be over-ridden using the hand-switches as will be explained below. For normal operation only hand-switch 4 is set to suppress optional printing, all others are cleared.

(a)   Accuracy

The accuracy to which the data is fitted depends on the number of itera-tions made and the speed of convergence. Clearly when all the changes in the parameters δx_k are all zero the limit of accuracy has been reached but this would take an infinite time. Accordingly it is arranged that when the δx_k are less than a preset level, the fitting process is terminated and the fitted motion printed out (see Fig.5). This accuracy level is set to k times ξ_k where ξ_k are the perturbations being made in the parameters. These perturbations are arbitrary provided they are small enough and so are given this additional use.

After each iteration the new values of the parameters are printed out along with the variance of the errors and the probable error in the parameters. The user is thus able to see how fast the solution is converging. If desired he may terminate the fitting process at any time by setting hand-switch 1. The last fit to the data is then printed and the programme calls in the next block of data.

(b)   Rejection of data

Some means of rejecting data that is obviously wrong is necessary. As mentioned in Section 6 this is done by rejecting data when the error is greater than 4 times the standard deviation of all errors. On the first fit, however, this level ε_R is set on the data tape and it is advisable to make it as large as possible, say about ten times the expected errors otherwise data is rejected unnecessarily.

(c)   Interpolation and extrapolation of data

The programme is fitting observations at discrete time intervals but
it is often useful to have the fitted motion at more frequent intervals.
This is obtained by use of the third control parameter on the data tape,
the maximum step length h. The programme will print out the fitted motion
at this interval as well as at the times of observations so that interpolation
is easy. If h is greater than the interval between two observations no
intermediate value is printed.

To extrapolate before the first observation, it is only necessary to
set the initial time to the appropriate value. The motion is then printed
at the interval h up to the time of the first observations. To extrapolate
beyond the last observation, the easiest way is to add a block of one
observation only, $(q\ t_e\ *)$ will do. The programme will then extend the
fitted motion to time t_e. It will not attempt to fit a block of less than
P observations, it will merely print the fitted motion using the latest
values of parameters.

(d)   Use of the programmes for prediction

The facility described above enables the programmes to be used for pre-
flight predictions. All that is necessary is to replace the experimental
data by $(0\ t_e\ 0\ *)$. The motion between t_o and t_e will then be printed at
the interval h.

Trajectory analysis programmes

Trajectory analysis 1623/1A. This programme uses a table of Mach
number against C_A based on the best pre-flight estimates. The parameter
lists are as follows:

$$t_o$$
$$\Delta C_A \quad \xi_1$$
$$V_o \quad \xi_2 \quad (ft/sec)$$
$$h_o \quad \xi_3 \quad * \quad (ft)$$

V_o (ft/sec), h_o (ft), θ_o (deg), s (sq ft), m (pounds), Initial Rejection
error ε (ft), Accuracy factor k, Max step length h (sec).

The data fitted consists of time and height, the weights are the reciprocals
of the probable errors of each ballistic camera height.

Trajectory analysis 1623/1B. This programme uses measured values of acceleration $\bar{a}_x$ and $\bar{a}_y$ in place of the drag table as already described. The parameter lists are as follows.

$$t_o$$

$\Delta\bar{a}_x$	ξ_1	(ft/sec^2)
V_o	ξ_1	(ft/sec)
h_o	ξ_3 *	(ft)

V_o (ft/sec), h_o (ft), θ_o (deg), s (sq ft), m (pounds), $m \ell \times C_{m\alpha}/B$, $C_{N\alpha}$, ε (ft), k, h (sec).

Experimental data, which is height from ballistic cameras, is the same as programme 1A. Table 1 is an annotated copy of data as read into the computer and Table 2 the resulting fitted trajectory.

Dynamic analysis programmes

Four programmes are available at present, one each for differential pressure gauges and rate gyro analysis and two for accelerometer analysis, as follows.

Dynamic analysis 1623/1 – For differential pressure gauges	– 9 parameters
1623/2 – For accelerometers	– 9 parameters
1623/3 – For accelerometers	– 10 parameters
1623/4 – For rate gyros	– 9 parameters

All programmes determine the parameters $C_{M\alpha}$, C_{Mq}, P, R_o, S_o, γ_{R_o}, γ_{S_o} and two zero errors for programme 4 only, and two trim angles in the case of programmes 1-3. At present the latter cannot distinguish between zero errors and trim. In the case of 1623/3 the extra parameter is $C_{N\alpha}$ and intended to be used when two accelerometers with a large mutual axial displacement are being analysed. The parameter lists are arranged in the same way for all four programmes and are as follows:-

$$t_o$$

$C_{M\alpha}$	ξ_1	
C_{Mq}	ξ_2	
P	ξ_3	(Radians per sec)
$C_{N\alpha}$	ξ_4	Included only in Prog. 1623/3
R_o	ξ_5	(Rad)
S_o	ξ_6	(Rad)

$$\begin{array}{lll}
\gamma_{R_o} & \xi_7 & \text{(Rad)} \\
\gamma_{S_o} & \xi_8 & \text{(Rad)} \\
\alpha_t & \xi_9 & \text{(Rad)} \\
\beta_t & \xi_{10} & \text{(Rad)} \\
\ast
\end{array} \qquad \text{or} \begin{bmatrix} \theta_x \text{ rad/sec} \\ \theta_y \text{ rad/sec} \end{bmatrix} \text{for Prog. 1623/4}$$

A (slugs ft sq), B (slugs ft sq), ℓ (ft), s (sq ft), m (pounds), $C_{N\alpha}$,
V_o (ft/sec), h_o (ft), θ_o (deg)
ε (initial rejection error), k (accuracy factor) h (sec, max step length)

$$\left. \begin{array}{lll}
x_1/32\cdot2 & y_1/32\cdot2 & z_1/32\cdot2 \\
x_2/32\cdot2 & y_2/32\cdot2 & z_2/32\cdot2
\end{array} \right\} \begin{array}{l} \text{Accelerometer coordinates included only in} \\ \text{Progs. 1623/2, 1623/3} \end{array}$$

$C_{\Delta P}$ For Prog. 1623/1 only. The differential pressure for unit dynamic
   pressure at one radian incidence in pounds per square foot.

Experimental data follows the standard form except that weights are
omitted, either one instrument or an ortnogonal pair may be analysed. An
illustration of the use of Programme 1623/2 for the analysis of actual
flight data is given in Tables 3 and 4. Table 3 is an annotated version of
the data tape and Table 4 the result.

SYMBOLS

A	roll M of I. Also for acceleration
B	pitch M of I
C	matrix of partial derivatives
C_A	axial force coefficient
$C_{L\alpha}$	lift force derivative
$C_{M\alpha}$	pitching moment derivative
$C_{N\alpha}$	normal force derivative
C_{Mq}	damping in pitch derivative
D	axial force. Also matrix of residuals
E	matrix of errors
N	normal force
R	component of incidence. Also for residual
S	component of incidence. Also standard deviation
U	sum of squares
V	velocity
W	matrix of weights
X_k	true parameter value
a	constant, also acceleration
b	constant
f	partial derivative
g	partial derivative or gravity
ℓ	reference length
m	mass
n	number of observations
P,p	spin, number of parameters
q,r	pitch and yaw rate
s	reference area
t	time
w_i	weight of observation
x_k	parameter
π_i	observation
$\mathbb{Y}$	matrix
α,β	incidence or observation
θ	climb angle
ξ_i	perturbation
ρ	air density
ω	aerodynamic frequency

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TABLE 1
TRAJECTORY ANALYSIS 1623/1B DATA

ATMOSPHERE

H	PRESS	TEMP
0	996.1	23
6000	807.1	24
8000	750.5	11
16750	540.0	-10.5
24000	401.6	-24
30000	310.0	-41
35000	245.5	-52
41000	182.5	-63
43000	167.5	-64
44000	159.3	-62
52000	106.9	-68.5
58000	78.9	-69.5
65000	55.7	-63
70000	43.7	-55
75000	34.5	-53
80000	27.4	-48.5
86000	20.8	-47
91000	16.7	-47

ACCELERATION TABLE

T(SECS)	AX	AY
825.745	-7.7	5
826.215	-12.9	7
826.825	-17.6	10
827.295	-24.0	11
827.765	-33.1	12
828.265	-44.0	11
828.735	-52.0	10
829.765	-65.5	5
830.255	-67.3	2
830.765	-65.9	0
831.265	-62.1	0
831.765	-57.4	0
832.265	-52.6	2
832.765	-47.5	0
833.265	-41.8	0
833.765	-37.1	0
834.265	-31.9	0
834.765	-27.5	0
835.265	-23.6	0

VARIABLE PARAMETERS
826.1
-30   5.0
15450   5.0
86500   25

CONSTANT PARAMETERS
15450   86500   88.4   1.285   121   0.6   1.8
5000   2.0   0.5

Table 1(cont'd)

TABLE I (CONT)

BALLISTIC CAMERA DATA

T(SECS)	H(FT)	SQRT W
826.5	80467	0.048
826.6	78959	0.020
826.7	77473	0.032
826.8	75970	0.036
826.9	74447	0.033
827.0	72947	0.032
827.1	71455	0.028
827.2	69970	0.026
827.3	68485	0.024
827.4	67006	0.021
827.5	65550	0.026
827.6	64089	0.020
827.7	62648	0.037
827.8	61239	0.016
827.9	59800	0.019
828.0	58385	0.020
828.1	56983	0.018
828.2	55603	0.019
828.3	54230	0.020
828.4	52868	0.023
828.5	51515	0.024
828.6	50188	0.024
828.7	48869	0.023
828.8	47590	0.022
828.9	46309	0.024
829.0	45046	0.026
829.1	43801	0.029
829.2	42584	0.024
829.3	41373	0.024
829.4	40227	0.018
829.5	39088	0.012
829.6	37968	0.009
829.7	36866	0.008
829.8	35778	0.008
829.9	34699	0.008
830.0	33644	0.008
830.1	32601	0.009
830.2	31566	0.011
830.3	30551	0.016
830.4	29553	0.026
830.5	28604	0.030
830.6	27682	0.028
830.7	26778	0.028
830.8	25894	0.028
830.9	25031	0.030
831.0	24206	0.013
831.1	23391	0.014
831.2	22603	0.014
831.3	21813	0.014
831.4	21054	0.013
831.5	20297	0.013
831.6	19589	0.011
835.2	4300	1.0

TABLE II
TRAJECTORY ANALYSIS 1623/1B RESULT.

```
 -30      5.0
15450     5.0
86500    25.0

0.46977   49
 -62      2.9
15466     8.1
86651     8.6
```

0.46975	49	SIGMA	N-P
-62	2.9	AX ERROR (FT/SEC.SQ)	
15466	8.1	Vo (FT/SEC)	
86651	8.6	Ho (FEET)	

TIME	H	V	AX (G)	DYN P	M NO	CA	RES
826.1	86651	15466	-12.5	7233	15.6	0.154	
826.5	80503	15280	-16.0	9409	15.5	0.153	-36
826.6	78978	15228	-16.8	10070	15.5	0.151	-19
826.7	77459	15172	-17.6	10789	15.5	0.147	14
826.8	75945	15114	-18.3	11551	15.5	0.144	25
826.9	74437	15054	-19.5	12351	15.4	0.144	10
827.0	72936	14989	-20.9	13181	15.4	0.145	11
827.1	71441	14919	-22.3	14054	15.3	0.145	14
827.2	69953	14845	-23.6	14975	15.3	0.144	17
827.3	68473	14767	-25.0	16096	15.3	0.142	18
827.4	67001	14683	-27.0	17281	15.3	0.143	5
827.5	65538	14593	-28.9	18530	15.3	0.143	12
827.6	64084	14497	-30.8	19806	15.2	0.143	5
827.7	62640	14395	-32.8	21117	15.2	0.143	8
827.8	61206	14286	-34.8	22480	15.1	0.143	33
827.9	59784	14171	-37.0	23893	15.0	0.143	16
828.0	58373	14048	-39.1	25350	15.0	0.143	18
828.1	56975	13919	-41.3	26723	14.8	0.143	8
828.2	55591	13782	-43.5	28072	14.7	0.143	12
828.3	54220	13639	-45.5	29432	14.5	0.143	10
828.4	52864	13489	-47.2	30803	14.3	0.142	4
828.5	51524	13334	-48.9	32118	14.2	0.141	-9
828.6	50199	13174	-50.6	33317	14.0	0.141	-11
828.7	48890	13008	-52.3	34496	13.7	0.141	-21
828.8	47598	12837	-53.8	35649	13.5	0.140	-8
828.9	46324	12662	-55.1	36775	13.3	0.139	-15
829.0	45067	12483	-56.4	37866	13.1	0.138	-21
829.1	43828	12299	-57.7	39006	12.9	0.137	-27
829.2	42608	12111	-59.0	40377	12.7	0.136	-24
829.3	41407	11919	-60.3	41055	12.5	0.136	-34
829.4	40226	11722	-61.6	41667	12.3	0.137	1
829.5	39064	11522	-63.0	42307	12.0	0.138	24

Table 2 (cont'd)   C 280

TABLE 11 (CONT)

829.6	37922	11317	−64.3	42665	11.7	0.140	46
829.7	36801	11108	−65.6	43034	11.4	0.142	65
829.8	35702	10895	−66.6	43310	11.2	0.143	76
829.9	34623	10680	−66.9	43432	10.9	0.143	76
830.0	33567	10464	−67.3	43345	10.6	0.144	77
830.1	32532	10246	−67.7	43182	10.4	0.146	69
830.2	31518	10028	−68.0	42939	10.1	0.147	48
830.3	30527	9808	−68.1	42618	9.8	0.148	24
830.4	29557	9590	−67.8	42116	9.5	0.150	−4
830.5	28610	9372	−67.6	41425	9.3	0.151	−6
830.6	27684	9154	−67.3	40687	9.0	0.154	−2
830.7	26779	8938	−67.0	39902	8.8	0.156	−1
830.8	25897	8723	−66.6	39073	8.5	0.158	−3
830.9	25035	8510	−65.8	38212	8.2	0.160	−4
831.0	24195	8299	−65.0	37324	8.0	0.162	11
831.1	23376	8091	−64.3	36447	7.8	0.164	15
831.2	22578	7885	−63.5	35552	7.6	0.166	25
831.3	21800	7682	−62.7	34633	7.3	0.168	13
831.4	21042	7481	−61.8	33694	7.1	0.170	12
831.5	20304	7284	−60.8	32740	6.9	0.172	−7
831.6	19585	7090	−59.9	31774	6.7	0.175	4
−62	0.0						
7090	0.0						
19585	0.0						
831.6	19585	7090	−59.9	31774	6.7	0.175	
832.1	16277	6164	−55.1	26779	5.8	0.191	
832.6	13411	5316	−50.1	21610	4.9	0.214	
833.1	10949	4552	−44.6	17005	4.2	0.242	
833.6	8846	3876	−39.6	13100	3.5	0.278	
834.1	7061	3279	−34.6	9738	2.9	0.325	
834.6	5555	2761	−29.9	7117	2.4	0.383	
835.1	4289	2314	−25.8	5228	2.0	0.448	
835.2	4063	2232	−25.0	4904	2.0	0.463	

TABLE III   C 281   Table 3

RE ENTRY DYNAMIC ANALYSIS 1623/2 DATA

ATMOSPHERE TABLE
```
  H     PRESS  TEMP
87500   19.8   -49.0
95500   13.8   -45.4
105500   8.74  -44.7  *
```

DRAG TABLE
```
  H      CA
45000   0.162
180000  0.155  *
```

VARIABLE PARAMETERS
```
787.680
-0.30   0.01
-2.0    1.0
23.0    0.1
0.06    0.001
0.05    0.001
-0.6    0.03
3.7     0.03
*       0.001
*       0.001  *
```

CONSTANT PARAMETERS
```
0.510   2.265   1.333   1.4   114.5   1.85
14225.7  96394.4   84.0   3.0   3.0   0.05
```

ACCELEROMETER COORDS(FT/G)
```
-0.00985  0.00560  -0.00168
0.00200   0.00543  -0.00218
```

ACCELEROMETER DATA
```
  TIME        Y ACC   Z ACC
1
 787.700       8.14   -5.24
 787.720       4.64   -9.45
 787.740      -1.79   -8.31
 787.760      -4.22   -1.51
 787.780       0.50    5.04
 787.800       7.16    6.04
 787.820       8.38    0.76
 787.840       1.53   -2.93
 787.860      -6.81   -0.78
 787.880     -10.07    5.90
 787.890      -8.20    9.05
 787.900      -5.96   10.38
 787.960      -9.83   -4.44
 787.970     -13.32   -4.18
 788.000     -13.75    2.20
 788.010     -10.37    3.30
 788.020      -6.85    2.44
 788.040      -0.29   -4.51
 788.060      -3.56  -12.08
 788.080      -9.70  -11.62
 788.100      -9.80   -4.51
 788.120       0.71    0.73
 788.140       8.58   -1.48
 788.160       8.45   -7.17
 788.180       1.09   -6.40  *
```

Table 4

TABLE IV

RE ENTRY DYNAMICS ANALYSIS 1623/2 RESULT.

```
-9.30000      0.0100
-2.00000      1.0000
25.00000      0.1000
 0.06000      0.0010
 0.05000      0.0010
-0.60000      0.0300
 3.70000      0.0300
 0.00000      0.0010
 0.00000      0.0010

 0.32880      17
-0.33027      0.0042
-8.11967      1.4500
23.42858      0.1864
 0.07555      0.0019
 0.06678      0.0028
-0.43314      0.0472
 3.58381      0.0555
 0.00510      0.0020
 0.02624      0.0018

 0.55789      41
-0.32710      0.0021
-1.37170      0.6773
23.12657      0.1108
 0.06919      0.0020
 0.05790      0.0020
-0.48595      0.0356
 3.53643      0.0440
 0.00307      0.0012
 0.02165      0.0011
```

```
 0.54972      41
-0.32883      0.0018      CMA
-1.88662      0.5813      CMQ
23.15920      0.0967      P (RAD/SEC)
 0.06975      0.0017      Ro (RAD)
 0.05896      0.0017      So (RAD)
-0.46695      0.0344      GAMMA Ro (RAD)
 3.52026      0.0435      GAMMA So (RAD)
 0.00349      0.0012      ALPHA TRIM (RAD)
 0.02152      0.0011      BETA TRIM (RAD)
```

T(SECS)	V(FT/SEC)		H(FT)	OMEGA(RAD/SEC)			
R(DEG)	S(DEG)	ALPHA	BETA	Y ACC	Z ACC	RES Y	RES Z
787.680	14225.70		96394.40	32.96			
4.00	3.58	0.43	3.05	1.95	-0.07		
787.700	14220.82		96111.46	33.16			
3.96	3.35	-2.58	5.93	7.16	-4.79	0.983	-0.449

TABLE IV (CONT)

```
787.720   14215.89   95828.67        33.37
3.93    3.31   -5.50    4.53    4.86   -9.47   -0.222    0.017
787.740   14210.90   95545.93        33.57
3.89    3.28   -4.67    0.94   -1.48   -8.28   -0.314   -0.028
787.760   14205.84   95263.39        33.78
3.85    3.25   -0.28   -0.64   -4.30   -1.26    0.076   -0.248
787.780   14200.72   94980.90        33.99
3.81    3.22    3.81    1.67   -0.08    5.39    0.579   -0.354
787.800   14195.54   94698.46        34.21
3.78    3.19    4.09    5.34    6.80    5.86    0.358    0.178
787.820   14190.29   94416.17        34.42
3.74    3.15    1.14    6.18    8.47    0.94   -0.092   -0.182
787.840   14184.97   94133.94        34.64
3.71    3.12   -1.08    2.84    2.13   -2.86   -0.596   -0.072
787.860   14179.59   93851.87        34.85
3.67    3.09    0.45   -1.65   -6.55   -0.29   -0.263   -0.488
787.880   14174.15   93569.85        35.07
3.63    3.06    4.40   -3.15   -9.53    6.61   -0.535   -0.712
787.890   14171.40   93428.95        35.18
3.61    3.04    5.95   -2.36   -8.03    9.37   -0.169   -0.315
787.900   14168.63   93287.97        35.29
3.60    3.03    6.54   -0.93   -5.25   10.46   -0.715   -0.080
787.950   14154.54   92583.76        35.84
3.51    2.95   -1.55   -1.03   -5.69   -3.92
787.960   14151.67   92443.06        35.95
3.49    2.93   -2.18   -3.18  -10.15   -5.12    0.318    0.632
787.970   14148.78   92302.27        36.06
3.47    2.92   -1.77   -5.08  -14.13   -4.42    0.813    0.237
788.000   14140.01   91880.25        36.40
3.42    2.87    2.07   -5.07  -14.32    2.69    0.573   -0.488
788.010   14137.05   91739.64        36.51
3.40    2.85    2.42   -3.19  -10.41    3.36    0.042   -0.058
788.020   14134.07   91599.05        36.62
3.38    2.84    1.73   -1.11   -5.98    2.06   -0.870    0.382
788.040   14128.06   91318.00        36.85
3.35    2.81   -2.05    1.43   -0.44   -5.17    0.148    0.665
788.060   14121.97   91037.01        37.08
3.31    2.76   -5.55    0.22   -3.00  -12.07   -0.563   -0.007
788.080   14115.81   90756.20        37.31
3.27    2.74   -5.17   -2.43   -8.84  -11.48   -0.863   -0.143
788.100   14109.57   90475.51        37.54
3.24    2.71   -1.61   -2.41   -8.81   -4.48   -0.992   -0.028
788.120   14103.26   90194.89        37.77
3.20    2.68    0.96    1.22   -0.60    0.63    1.306    0.104
788.140   14096.87   89914.45        38.00
3.17    2.65   -0.09    4.93    8.09   -1.57    0.491    0.090
788.160   14090.40   89634.09        38.23
3.13    2.62   -2.71    4.94    8.28   -7.06    0.165   -0.105
788.180   14083.85   89353.91        38.47
3.10    2.59   -2.77    1.69    0.71   -7.32    0.382    0.917
```

FIG. I DEFINITION OF NOTATION & FORCES
ON MISSILE

Fig.2
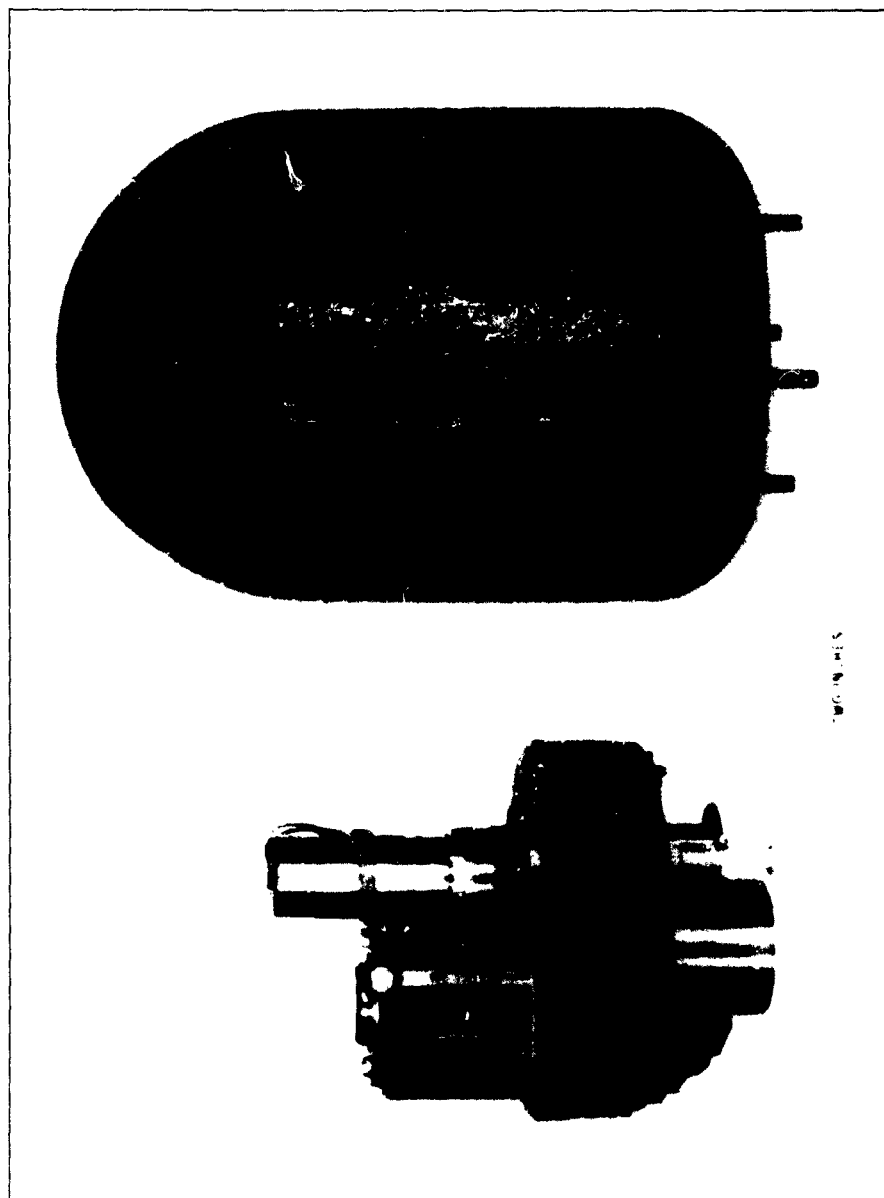
Fig.2 The sub-miniature tape recorder

Fig.3

Fig.3 A Black Knight instrumentation assembly

Fig.4

SPA/P/1426
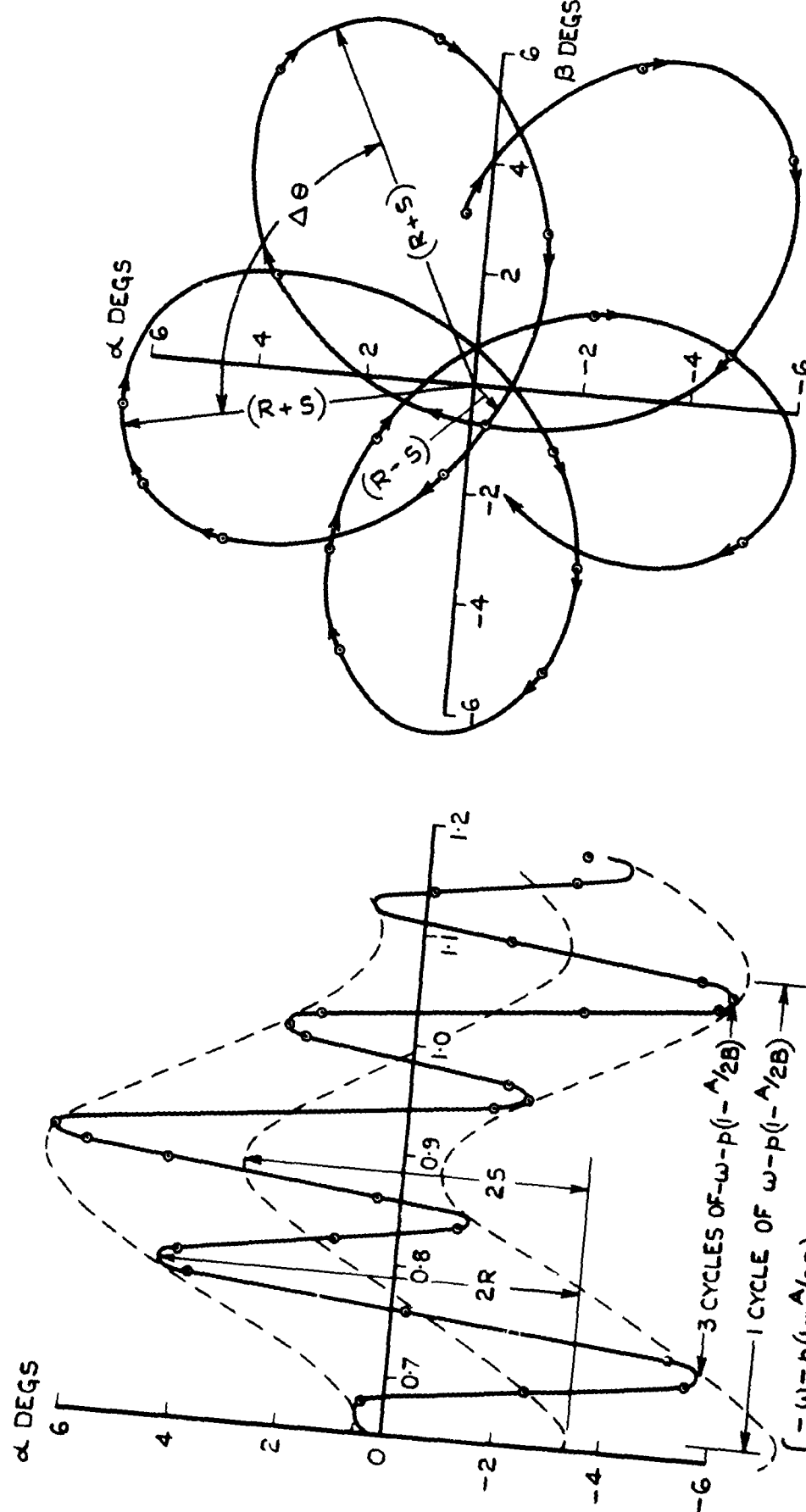
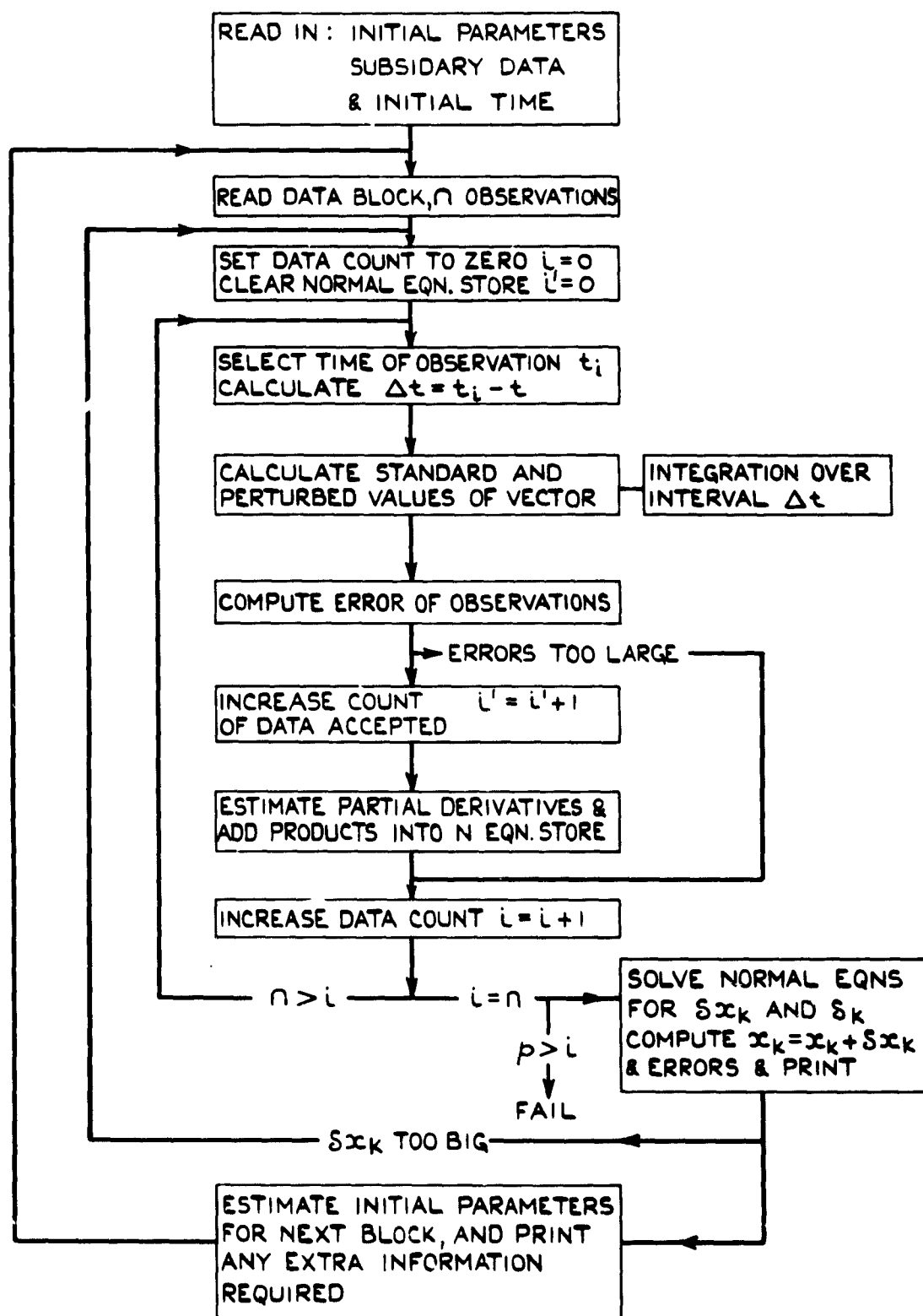FIG.4 GRAPHICAL METHOD FOR EXTRACTION OF AMPLITUDE & FREQUENCY
INFORMATION FROM RE-ENTRY RECORDS

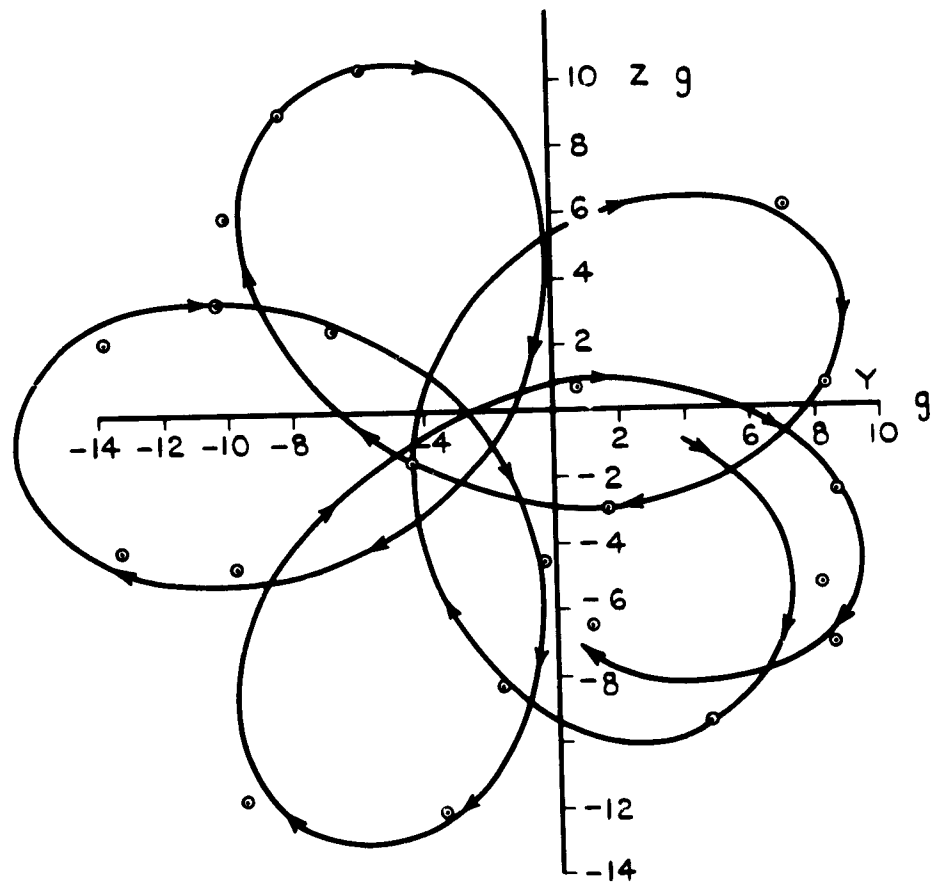FIG. 5 FLOW DIAGRAM OF AUTOMATIC
ANALYSIS PROGRAMME

Fig.6

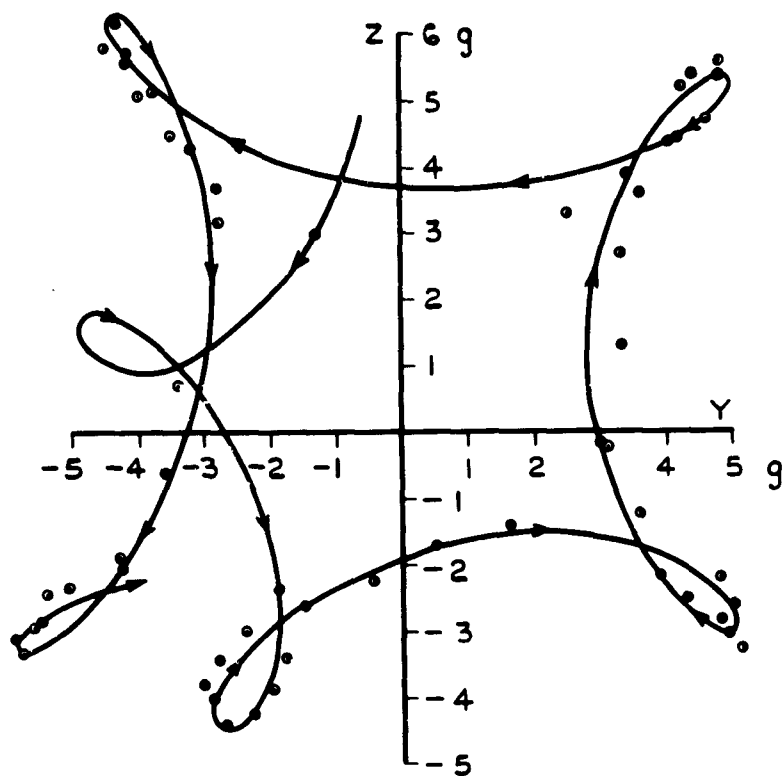FIG. 6 FITTED CURVE TO DATA FROM PAIR
OF ACCELEROMETERS (R≃S)

FIG. 7 FITTED CURVE TO DATA FROM PAIR
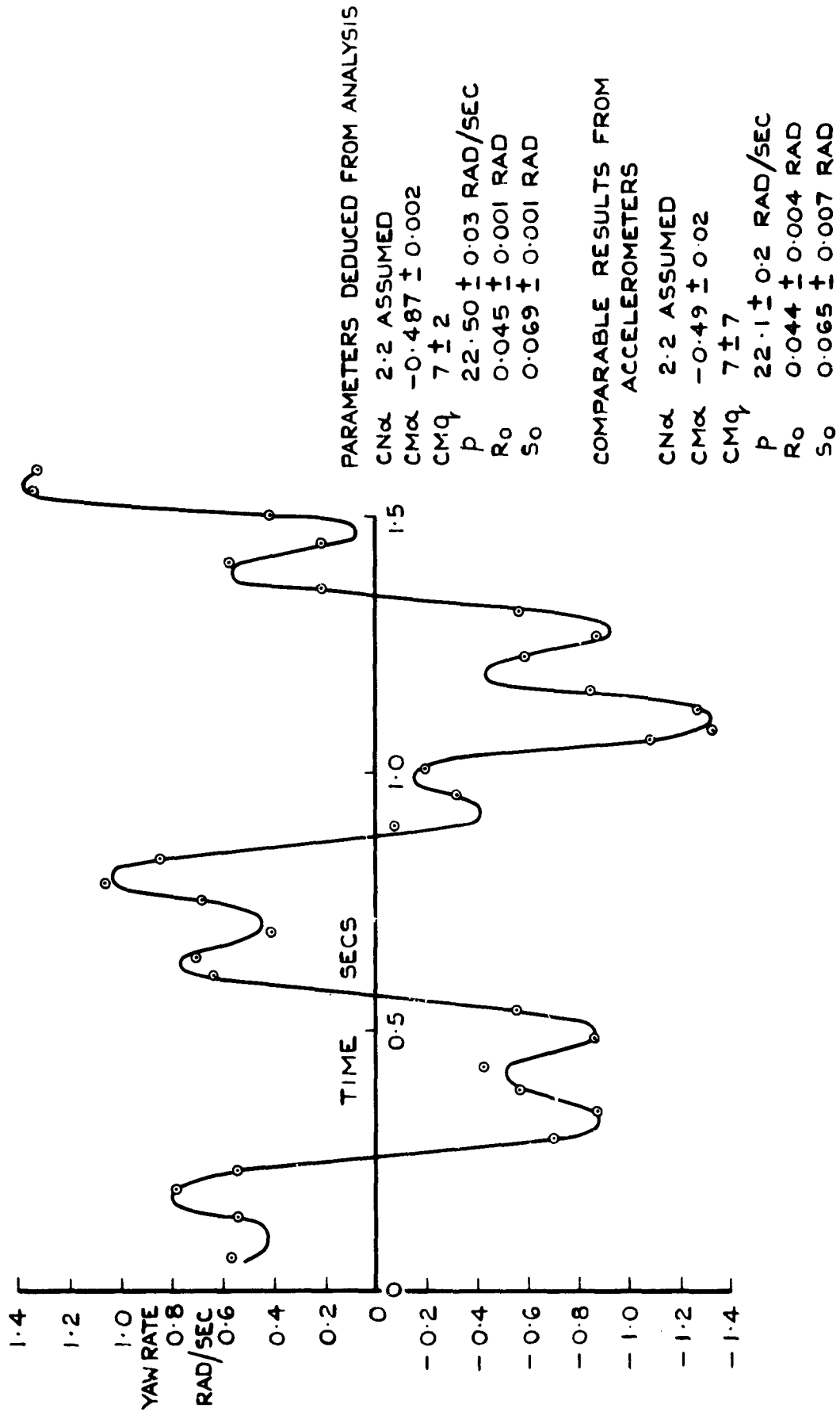OF ACCELEROMETERS (2R≃S)

Fig.8 SPA / P / 1430
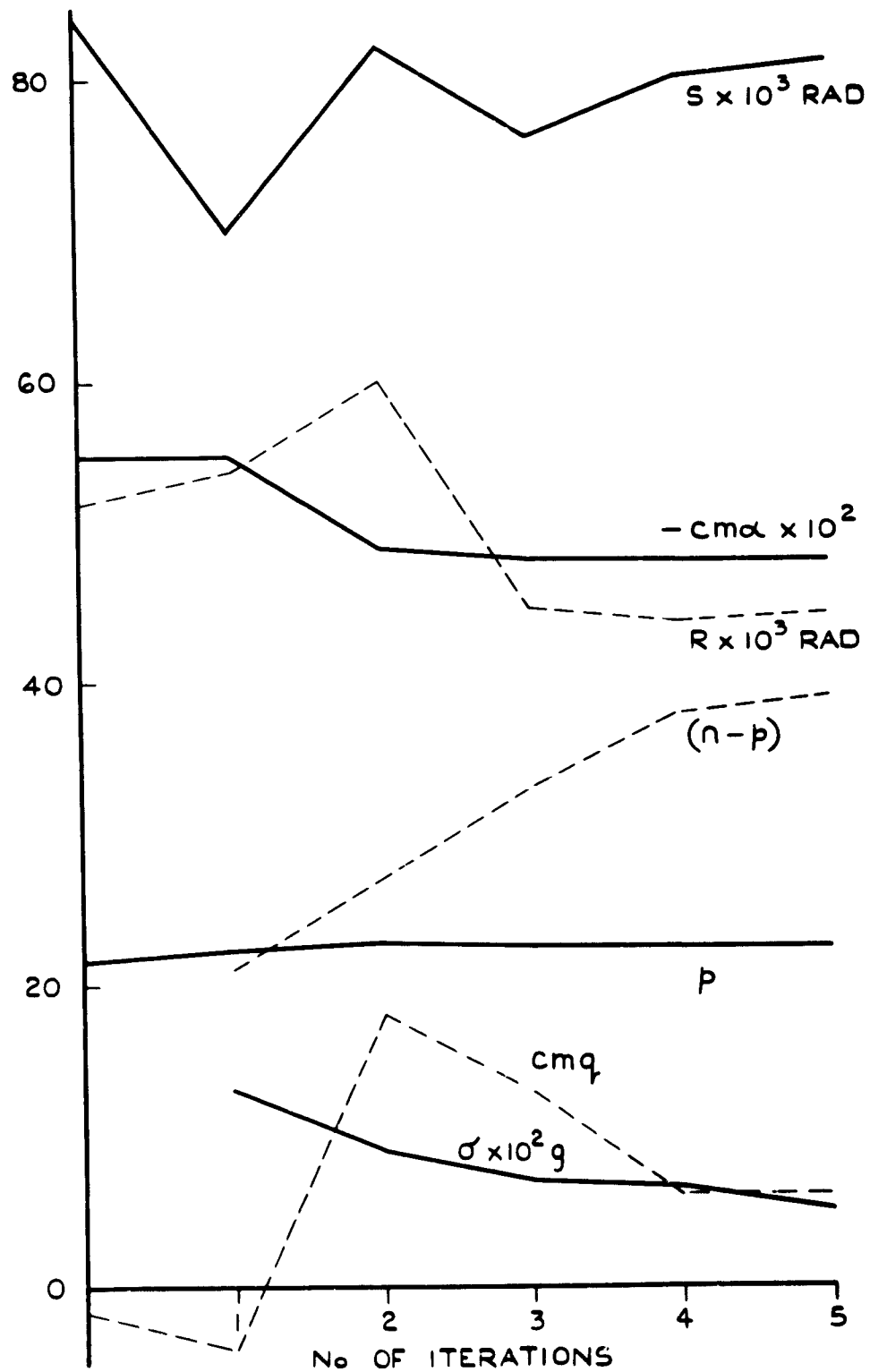

FIG. 8 FITTED CURVE TO SINGLE RATE GYRO RECORD

FIG. 9 VARIATION OF PARAMETER VALUE WITH
ITERATION No FOR AN ACCELEROMETER ANALYSIS

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629.19.097.2 :
533.665 :
533.6.013.6 :
629.19.077.2 :
629.19.077.4

Waterfall, A. P.

TECHNIQUES OF MEASUREMENT AND AUTOMATIC ANALYSIS OF
DYNAMIC MOTION WITH APPLICATION TO SOME RE-ENTRY
EXPERIMENTS

Royal Aircraft Establishment Technical Report 65038   February 1965

An automatic technique, utilizing a large high speed computer for the analysis of records obtained from accelerometers, pressure gauges and gyros installed in the re-entry vehicles of the Black Knight rocket, is described. It is shown that this enables a suitable arrangement of only three accelerometers to yield trajectory data, angular motion and the aerodynamic characteristics of the re-entry body. Examples of the analysis of accelerometer and rate gyro data are given and the lines of further development and some applications are indicated.

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