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ROYAL AIRCRAFT ESTABLISHMENT
(FARNBOROUGH)

TECHNICAL NOTE No. I.E.E. 32

**SOME NOTES ON TORSION
OSCILLATOR GYROSCOPES**

by

G. H. Hunt, B.Sc., Ph.D.

and

A. E. W. Hobbs, B.Sc.

JUNE, 1963

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SOME NOTES ON TORSION OSCILLATOR GYROSCOPES

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R.A.E. Ref: IEE/3803

SUMMARY

A torsion oscillator can be used as the sensing element of a vibratory gyroscope instead of the more commonly used tuning fork. Some theoretical aspects of torsion oscillators suitable for this application, and practical considerations in their use are both examined.

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1 INTRODUCTION

Of the many types of vibratory gyroscope which are theoretically possible, that which uses the tuning-fork as the vibrating element has been most extensively studied. A number of experimental tuning-fork gyros have been constructed at R.A.E., and the theoretical and experimental properties of these have been described¹⁻⁵.

As the oscillatory sensing element in a vibratory gyroscope, the tuning-fork has a number of disadvantages. These are principally:

- (a) The centres of mass of the two tines move, and being near the ends of the tines their mean position is necessarily sensitive to gravity or acceleration fields.
- (b) The oscillatory motion of the centres of mass of the tines can give rise to error torques which are gravity sensitive.
- (c) The natural frequency of oscillation is gravity sensitive.
- (d) The vibratory motion of the tines can be excited by external vibration of the instrument.
- (e) The natural frequency of oscillation will not, in general, have the same temperature coefficient as that of the torsion oscillator which forms the torque-measuring output system.

These disadvantages should be eliminated if a torsion oscillator is used to replace the tuning-fork, and this Note examines theoretically the ways in which such a torsion oscillator can be used. For the theoretical studies of Sections 2 and 3, the oscillating elements are assumed infinitely rigid and to have their centres of mass exactly coincident with the axis of torsional oscillation, while the torsion stems are assumed to have zero mass. Whilst such assumptions cannot be made to hold in a practical instrument, the conclusions reached from the theory are largely valid in practice.

In Section 4 are described some possible ways in which practical instruments can be designed. It must be emphasized that no attempt is made in this Note to consider the problems of measurement of the torques generated by the oscillator; the methods used for this measurement in tuning-fork gyroscopes have been described elsewhere^{1,3}, and similar methods will almost certainly be used for torsion-oscillator gyroscopes.

2 THE GYROSCOPIC TORQUES PRODUCED BY TORSION OSCILLATORS

Consider a rigid body in which a set of body axes $O1,2,3$ is defined. It is assumed that this body executes small angular oscillations about its $O3$ axis relative to a set of axes Ox,y,z as shown in Fig.1. The angle of oscillation is defined as ϕ , measured about the Oz axis which is coincident with $O3$, and when ϕ is zero then Ox,y,z and $O1,2,3$ are coincident.

The Ox, y, z set of axes may have instantaneous angular velocities $\omega_x, \omega_y, \omega_z$ relative to inertial space measured about the Ox, Oy, Oz axes respectively. The angular velocities of the $O1, 2, 3$ axes measured about those axes are then

$$\left. \begin{aligned} \omega_1 &= \omega_x \cos \phi + \omega_y \sin \phi \\ \omega_2 &= -\omega_x \sin \phi + \omega_y \cos \phi \\ \omega_3 &= \omega_z + \dot{\phi} \end{aligned} \right\} \quad (1)$$

The torques which have to be exerted on the body in order to maintain these motions will be calculated. If these torques can be measured, it is possible knowing the oscillation ϕ to deduce one or more of the angular velocities ω , and a type of gyroscope can be constructed. Ideally Ox, Oy, Oz are the axes of the case of the gyroscope, but in practice the measurement of torque introduces a further angular motion between the body and the case. It will be assumed here that the problems arising from this additional motion can be overcome, and only the simple model of a rigid body having one small oscillatory motion ϕ relative to the case will be considered.

Let the body have moments of inertia I_1, I_2 and I_3 , and products of inertia J_{12}, J_{23} and J_{31} relative to the body axes $O1, 2, 3$.

The angular momenta in the $O1, 2, 3$ axis system are:-

$$\left. \begin{aligned} H_1 &= I_1 \omega_1 - J_{12} \omega_2 - J_{13} \omega_3 \\ H_2 &= I_2 \omega_2 - J_{21} \omega_1 - J_{23} \omega_3 \\ H_3 &= I_3 \omega_3 - J_{31} \omega_1 - J_{32} \omega_2 \end{aligned} \right\} \quad (2)$$

In practice it will be necessary to measure torques in the Ox, y, z case axes, and it is convenient to calculate the angular momenta in these axes. Then

$$\left. \begin{aligned} H_x &= H_1 \cos \phi - H_2 \sin \phi \\ H_y &= H_1 \sin \phi + H_2 \cos \phi \\ H_z &= H_3 \end{aligned} \right\} \quad (3)$$

Substituting for H_1 , H_2 , H_3 , ω_1 , ω_2 and ω_3 , and since $J_{12} = J_{21}$ etc.,

$$H_x = \omega_x \left[I_1 \cos^2 \phi + I_2 \sin^2 \phi + 2 J_{12} \sin \phi \cos \phi \right] \\ + \omega_y \left[\overline{I_1 - I_2} \sin \phi \cos \phi + J_{12} \overline{\sin^2 \phi - \cos^2 \phi} \right] \\ + (\omega_z + \dot{\phi}) [J_{23} \sin \phi - J_{13} \cos \phi]$$

$$H_y = \omega_x \left[\overline{I_1 - I_2} \sin \phi \cos \phi + J_{12} \overline{\sin^2 \phi - \cos^2 \phi} \right] \\ + \omega_y \left[I_1 \sin^2 \phi + I_2 \cos^2 \phi - 2 J_{12} \sin \phi \cos \phi \right] \\ - (\omega_z + \dot{\phi}) [J_{13} \sin \phi + J_{23} \cos \phi]$$

$$H_z = \omega_x [J_{32} \sin \phi - J_{31} \cos \phi] - \omega_y [J_{32} \cos \phi + J_{31} \sin \phi] + I_3 (\omega_z + \dot{\phi}) \\ \dots (4)$$

The torques acting on the body due to these rates of turn are calculated using Euler's equations:

$$\left. \begin{aligned} M_x &= \frac{dH_x}{dt} + \omega_y H_z - \omega_z H_y \\ M_y &= \frac{dH_y}{dt} + \omega_z H_x - \omega_x H_z \\ M_z &= \frac{dH_z}{dt} + \omega_x H_y - \omega_y H_x \end{aligned} \right\} (5)$$

Even with no case rotation rates, it may be seen that H_z contains the large fluctuating term $I_3 \dot{\phi}$, so that the torque M_z about the Oz axis includes the term $I_3 \dot{\phi}$. This term is so large in any practical system compared with the usable terms that it is impossible to use the torque measured about the Oz axis as a measure of input rate. One of the Ox or Oy axes orthogonal to Oz must be used, and since by symmetry the characteristics of the torques measured about either axis must be similar, the Ox axis is chosen for convenience.

Expanding the expression for M_x in terms of the input rates,

$$\begin{aligned}
 M_x = & \left[\frac{I_1}{2} (1 + \cos 2\phi) + \frac{I_2}{2} (1 - \cos 2\phi) + J_{12} \sin 2\phi \right] \frac{d\omega_x}{dt} \\
 & + \left[\frac{I_1}{2} \sin 2\phi - \frac{I_2}{2} \sin 2\phi - J_{12} \cos 2\phi \right] \frac{d\omega_y}{dt} \\
 & + [J_{23} \sin \phi - J_{13} \cos \phi] \left(\frac{d\omega_z}{dt} + \ddot{\phi} \right) \\
 & + [-I_1 \sin 2\phi + I_2 \sin 2\phi + 2 J_{12} \cos 2\phi] \dot{\phi} \omega_x \\
 & + [I_3 + I_1 \cos 2\phi - I_2 \cos 2\phi + 2 J_{12} \sin 2\phi] \dot{\phi} \omega_y \\
 & + [J_{13} \sin \phi + J_{23} \cos \phi] 2 \dot{\phi} \omega_x \\
 & + [J_{13} \sin \phi + J_{23} \cos \phi] \dot{\phi}^2 \\
 & + \left[-\frac{I_1}{2} \sin 2\phi + \frac{I_2}{2} \sin 2\phi + J_{12} \cos 2\phi \right] \omega_x \omega_z \\
 & + \left[I_3 - \frac{I_1}{2} (1 - \cos 2\phi) - \frac{I_2}{2} (1 + \cos 2\phi) + J_{12} \sin 2\phi \right] \omega_y \omega_z \\
 & + [J_{23} \sin \phi - J_{13} \cos \phi] \omega_x \omega_y \\
 & + [J_{23} \cos \phi + J_{13} \sin \phi] (\omega_z^2 - \omega_y^2) . \tag{6}
 \end{aligned}$$

The above expression has made no restrictions on ϕ . If it is now assumed that ϕ is small and oscillatory, and has the form $\phi = \phi_0 \sin nt$, then M_x can be expanded in powers of ϕ_0 . Powers of ϕ_0 higher than the first are small and difficult to use in a practical gyroscope, and it is therefore necessary, in order to work with a torque proportional to one of the input rates, to select a term proportional to ϕ_0 . Such a term is necessarily at the same frequency $n/2\pi$ as ϕ , and the only other terms at this frequency are those due to odd powers of ϕ_0 such as ϕ_0^3 etc. These are negligibly small.

It is therefore realistic to make the approximations $\sin \phi \rightarrow \phi$, $\cos \phi \rightarrow 1$, and ignore powers of ϕ higher than the first. Then equation (6) simplifies to

$$\begin{aligned}
M_x = & \left(2 J_{12} \omega_x + \overline{I_3 + I_1 - I_2} \omega_y + 2 J_{23} \omega_z \right) \phi_0 n \cos nt \\
& + \left(\begin{aligned} & 2 J_{12} \dot{\omega}_x + \overline{I_1 - I_2} \dot{\omega}_y + J_{23} \dot{\omega}_z \\ & + (I_2 - I_1) \omega_x \omega_z + 2 J_{12} \omega_y \omega_z + J_{23} \omega_x \omega_y \\ & + J_{13} (\omega_z^2 - \omega_y^2) \end{aligned} \right) \phi_0 \sin nt \\
& + J_{13} n^2 \phi_0 \sin nt \\
& + I_1 \dot{\omega}_x - J_{12} \dot{\omega}_y - J_{13} \dot{\omega}_z \\
& + J_{12} \omega_x \omega_z + (I_3 - I_2) \omega_y \omega_z - J_{13} \omega_x \omega_y + J_{23} (\omega_z^2 - \omega_y^2) . \quad (7)
\end{aligned}$$

Considering in turn the components of the torque M_x , the first bracket contains terms due to Coriolis acceleration, being the product of angular velocity of the Ox,y,z axis system and the angular velocity of the body within those axes. The second term is due to angular accelerations of the body. The term $J_{13} n^2 \phi_0 \sin nt$ is a function only of the oscillatory motion; in any practical system it is necessary that J_{13} be made very small so that this torque is negligibly small since it can represent steady error, being at the frequency of oscillation $n/2\pi$. The remaining terms are the normal rigid-body torques which are independent of the oscillatory motion ϕ .

The first set of terms is proportional to the steady input angular velocities, and may be regarded as the useful input torque. By choosing different body shapes, any one of the three torques may be used, giving the gyroscope sensitivity to rotation about any of the three Ox,y,z axes.

In practice it is only possible to make a working system having two bodies which are oscillating in antiphase, supported at a nodal point. If this is not so, reaction torques at the base will be so large that although the torque-measuring axis is nominally orthogonal to the axis of oscillation, very large torque errors would result. Consider therefore two bodies A and B which oscillate through angles

$$\left. \begin{aligned} \phi_A &= \phi_{0A} \sin nt \\ \phi_B &= \phi_{0B} \sin nt \end{aligned} \right\} \quad (8)$$

about a common Oz axis. The total reaction torques when the Ox,y,z axes are stationary, are:

$$\begin{aligned}
 M_{x_{A+B}} &= \left(J_{13_A} \phi_{o_A} + J_{13_B} \phi_{o_B} \right) n^2 \sin nt \\
 M_{y_{A+B}} &= \left(J_{23_A} \phi_{o_A} + J_{23_B} \phi_{o_B} \right) n^2 \sin nt \\
 M_{z_{A+B}} &= - \left(I_{3_A} \phi_{o_A} + I_{3_B} \phi_{o_B} \right) n^2 \sin nt \quad . \quad (9)
 \end{aligned}$$

Each of these is ideally zero, and for any practical instrument must be made small. This implies a balance between the A and B bodies such that the point of support is a nodal point of the oscillatory mode.

The useful Coriolis torque for two oscillatory masses is

$$M_{x_{A+B}} = \left[\begin{aligned} & 2 \left(J_{12_A} \phi_{o_A} + J_{12_B} \phi_{o_B} \right) \omega_x \\ & + \left(\overline{I_{3_A} + I_{1_A} - I_{2_A}} \phi_{o_A} + \overline{I_{3_B} + I_{1_B} - I_{2_B}} \phi_{o_B} \right) \omega_y \\ & + 2 \left(J_{23_A} \phi_{o_A} + J_{23_B} \phi_{o_B} \right) \omega_z \end{aligned} \right] n \cos nt \quad (10)$$

and if from equation (9)

$$\begin{aligned}
 J_{23_A} \phi_{o_A} + J_{23_B} \phi_{o_B} &= 0 \\
 I_{3_A} \phi_{o_A} + I_{3_B} \phi_{o_B} &= 0
 \end{aligned}$$

equation (10) reduces to

$$M_{x_{A+B}} = \left[\begin{aligned} & 2 \left(J_{12_A} \phi_{o_A} + J_{12_B} \phi_{o_B} \right) \omega_x \\ & + \left(\overline{I_{1_A} - I_{2_A}} \phi_{o_A} + \overline{I_{1_B} - I_{2_B}} \phi_{o_B} \right) \omega_y \end{aligned} \right] n \cos nt \quad . \quad (11)$$

There are thus only two basic types of instrument which appear to have any practical possibility of success. The first of these uses the first term in equation (11), and by measuring oscillatory torque about the Ox axis measures the angular velocity ω_x about the same axis. The second type uses the second expression and measures angular velocity ω_y about the Oy axis which is orthogonal both to the torque-measuring axis Ox and the oscillation axis Oz. The two types are shown in Figs.2 and 3, and will be considered in more detail.

An ideal instrument of the type shown in Fig.2 has the following properties:

$$\begin{aligned} (I_1 - I_2)_A &= (I_1 - I_2)_B = 0 \\ J_{13}_A &= J_{13}_B = J_{23}_A = J_{23}_B = 0 \\ I_{3A} \phi_{oA} + I_{3B} \phi_{oB} &= 0 \end{aligned} \quad (12)$$

Substituting these values into equation (7), the torque about the Ox axis due to both masses A and B is

$$\begin{aligned} M_{x_{A+B}} &= 2 \left(J_{12}_A \phi_{oA} + J_{12}_B \phi_{oB} \right) \left[\omega_x n \cos nt + \overline{\dot{\omega}_x + \omega_y \omega_z} \sin nt \right] \\ &+ \left(I_{1A} + I_{1B} \right) \dot{\omega}_x - \left(J_{12}_A + J_{12}_B \right) \dot{\omega}_y \\ &+ \left(J_{12}_A + J_{12}_B \right) \omega_x \omega_z + \left(I_{3A} - I_{2A} + I_{3B} - I_{2B} \right) \omega_y \omega_z \end{aligned} \quad (13)$$

Of these torques, the first set are due to the oscillatory motions ϕ_{oA} and ϕ_{oB} , and form the signal torques. The second set are not a function of the oscillations. Provided the gyroscope is protected from input angular velocities ω_x , ω_y , ω_z which have components at frequencies near the oscillation frequency $n/2\pi$, these latter torques should not be important and the torque-measuring transducer will be able to discriminate against them.

The signal torque is very similar to that produced by a tuning-fork gyroscope and which has been examined by Hunt⁴ and others. For a tuning-fork whose moment of inertia about the Ox axis is $(I_o + I_1 \sin nt)$, this torque is

$$\begin{aligned}
 M_x &= \frac{d}{dt} \left[(I_0 + I_1 \sin nt) \omega_x \right] \\
 &= I_0 \dot{\omega}_x + I_1 (\omega_x n \cos nt + \dot{\omega}_x \sin nt) .
 \end{aligned}$$

The additional term in equation (13) proportional to $\omega_y \omega_z \sin nt$ would be found also for a tuning-fork if a modulation of the moment of inertia about the Oy axis were considered. It can be seen from Fig.3 that such a modulation is effectively given by this type of torsion oscillator system.

The principal error torques with this oscillator arise since the conditions of equation (12) are not exactly maintained. In particular, if J_{13_A} and J_{13_B} are not exactly zero, a torque

$$M_{x_{A+B}} = \left(J_{13_A} \phi_{o_A} + J_{13_B} \phi_{o_B} \right) n^2 \sin nt \quad (14)$$

will result. This torque has a phase difference of 90° relative to the steady-state signal torque. It is exactly equivalent to the torques produced by a tuning-fork gyroscope when this has mass asymmetries in the x_s plane; such torques have been examined by Hobbs¹ and by Stratton and Hunt².

The second principal error may arise if the torque-measuring axis is not exactly orthogonal to the axis of oscillation Oz . If this condition exists, then a small component of the M_z torque will be measured. The largest of the M_z torques will be due to the term

$$M_{z_{A+B}} = - \left(I_{3_A} \phi_{o_A} + I_{3_B} \phi_{o_B} \right) n^2 \sin nt \quad (15)$$

which is ideally made zero by the conditions of equation (12). This condition calls for an exact relationship between the magnitudes of oscillation of the two masses A and B, which is a function of the coupling between them. This coupling will be examined in the next section.

The second type of oscillator is shown in Fig.3. This ideally is designed for the following conditions to hold:

$$\begin{aligned}
 J_{12_A} = J_{12_B} = J_{23_A} = J_{23_B} = J_{31_A} = J_{31_B} = 0 \\
 I_{3_A} \phi_{o_A} + I_{3_B} \phi_{o_B} = 0 . \quad (16)
 \end{aligned}$$

Substituting these values into equation (7), the output torque about the Ox axis is

$$M_{x_{A+B}} = \left[\overline{I_{1A} - I_{2A}} \phi_{oA} + \overline{I_{1B} - I_{2B}} \phi_{oB} \right] \left[\omega_y n \cos nt + \overline{\dot{\omega}_y - \omega_x \omega_z} \sin nt \right] \\ + \left(I_{1A} + I_{1B} \right) \dot{\omega}_x + \left(I_{3A} - I_{2A} + I_{3B} - I_{2B} \right) \omega_y \omega_z \quad (17)$$

This equation is very similar to equation (13), with the principal difference that the signal torque is due to rotations about the Oy axis, so that the behaviour is in some ways similar to a conventional rotating-wheel gyroscope. The error torques are identical to those already discussed and will not be examined further.

3 THE STEADY-STATE OSCILLATIONS OF A TORSION OSCILLATOR SYSTEM

A torsion oscillator of the most simple form may be considered as made up of the 3 parts shown in Fig.4, the two masses A and B which will oscillate in antiphase, and the central mass C which is ideally at rest. The masses A and B are joined to C by means of torsion stems, and C is supported by a mount relative to the case of the instrument.

The following parameters are defined:-

Moment of inertia of mass A about torsion stem axis	= I_A
Moment of inertia of mass B about torsion stem axis	= I_B
Moment of inertia of mass C about torsion stem axis	= I_C
Twist of A relative to C about torsion stem axis	= ϕ_A
Twist of B relative to C about torsion stem axis	= ϕ_B
Twist of C relative to instrument case (assumed fixed in inertial space)	= ϕ_C
Elastic stiffness coefficient of left-hand torsion stem	= k_A
Damping stiffness coefficient of left-hand torsion stem	= μ_A
Elastic stiffness coefficient of right-hand torsion stem	= k_B
Damping stiffness coefficient of right-hand torsion stem	= μ_B
Elastic stiffness coefficient of mount of C	= k_C
Damping stiffness coefficient of mount of C	= μ_C

The equations of motion are then derived by considering in turn the torques acting on the three masses A, B and C about the common torsion axis. Products of inertia are assumed zero.

Torques on A:

$$I_A(\ddot{\phi}_A + \ddot{\phi}_C) + \mu_A \dot{\phi}_A + k_A \phi_A = 0 \quad (18)$$

Torques on B:

$$I_B(\ddot{\phi}_B + \ddot{\phi}_C) + \mu_B \dot{\phi}_B + k_C \phi_C = 0 \quad (19)$$

Torques on C:

$$I_C \ddot{\phi}_C + \mu_C \dot{\phi}_C + k_C \phi_C - \mu_A \dot{\phi}_A - k_A \phi_A - \mu_B \dot{\phi}_B - k_B \phi_B = 0 \quad (20)$$

The stiffness of each torsion stem may be characterized by the natural frequency of oscillation of the corresponding mass. Thus we may put

$$k_A = n_A^2 I_A, \quad \mu_A = \frac{n_A I_A}{Q_A} \quad (21)$$

$$k_B = n_B^2 I_B, \quad \mu_B = \frac{n_B I_B}{Q_B} \quad (22)$$

where $n_A/2\pi$ and $n_B/2\pi$ are the natural frequencies of oscillation of A and B separately, Q_A and Q_B the "quality factors" of those oscillations.

Let the steady-state oscillations only be considered, and let these be at a natural frequency $\omega/2\pi$. Then equations (18) to (20) become

$$\phi_C = \left[\left(\frac{n_A^2}{\omega^2} - 1 \right) + \frac{j n_A}{\omega Q_A} \right] \phi_A \quad (23)$$

$$= \left[\left(\frac{n_B^2}{\omega^2} - 1 \right) + \frac{j n_B}{\omega Q_B} \right] \phi_B \quad (24)$$

$$= \left[\frac{\omega^2}{k_C + \mu_C j\omega - I_C \omega^2} \right] \left[\left(\frac{n_A^2}{\omega^2} + \frac{j n_A}{\omega Q_A} \right) I_A \phi_A + \left(\frac{n_B^2}{\omega^2} + \frac{j n_B}{\omega Q_B} \right) I_B \phi_B \right] \dots (25)$$

The three equations (23) to (25) contain four unknowns, ω , ϕ_A , ϕ_B and ϕ_C , and a non-trivial solution can be found for the angles of oscillation. The solution of the equations is simplified by the substitutions

$$S_A = \left(\frac{n_A^2}{\omega^2} - 1 \right) + \frac{j n_A}{\omega Q_A} \quad (26A)$$

$$S_B = \left(\frac{n_B^2}{\omega^2} - 1 \right) + \frac{j n_B}{\omega Q_B} \quad (26B)$$

where S_A and S_B are complex numbers which become small when the frequency of oscillation is nearly equal to the two natural frequencies, and the Q 's are large.

Then by putting

$$K = k_C + \mu_C j\omega - \omega^2(I_A + I_B + I_C) \quad (27)$$

equations (23) and (24) become

$$\phi_C = S_A \phi_A = S_B \phi_B \quad (28)$$

and equation (25) is then

$$\phi_C = \frac{\omega^2}{K + \omega^2(I_A + I_B)} \left[(S_A + 1) I_A \phi_A + (S_B + 1) I_B \phi_B \right]$$

which using equations (28) then transforms to

$$\phi_C = \frac{\omega^2}{K} (I_A \phi_A + I_B \phi_B) \quad (29)$$

One of the most critical properties of the torsion oscillator used as a gyroscopic element is the net torque which it exerts on its mount when it is not rotated in inertial space. Ideally this torque is zero, but as mentioned in the previous section a torque does exist with a practical instrument.

The reaction torque due to the motions of the three masses A, B and C is

$$M = - \left[I_A(\ddot{\phi}_A + \ddot{\phi}_C) + I_B(\ddot{\phi}_B + \ddot{\phi}_C) + I_C \ddot{\phi}_C \right]$$

and from equations (18) to (20) this is

$$M = [k_C + \mu_C j\omega] \phi_C \quad (30)$$

Thus ϕ_C must be found from equations (29) in order to evaluate the net torque. As a measure of the quality of the oscillator it is best found in terms of the useful oscillation of the masses A and B. As no restriction has been placed in this theory on the shapes of the masses, they may be of the type shown in either Fig.2 or Fig.3, where the output signal sensitivity would be proportional to

$$\left(J_{12_A} \phi_{o_A} + J_{12_B} \phi_{o_B} \right) \quad \text{and} \quad \left(\overline{I_{1_A} - I_{2_A}} \phi_{o_A} + \overline{I_{1_B} - I_{2_B}} \phi_{o_B} \right)$$

respectively. In the two cases, the coefficients J_{12_A} and J_{12_B} , and $\overline{I_{1_A} - I_{2_A}}$ and $\overline{I_{1_B} - I_{2_B}}$ are of opposite sign. Thus without making any further restriction on the shapes of the masses A and B, it is most convenient to consider as the useful oscillation the value of

$$(I_A \phi_A - I_B \phi_B)$$

where I_A and I_B are necessarily positive.

The equations (29) are therefore transformed into the alternative form

$$\phi_C = \frac{\omega^2}{K} (I_A \phi_A + I_B \phi_B) \quad (31)$$

$$= \frac{1}{4} \left[\left(\frac{S_A}{I_A} - \frac{S_B}{I_B} \right) (I_A \phi_A - I_B \phi_B) + \left(\frac{S_A}{I_A} + \frac{S_B}{I_B} \right) (I_A \phi_A + I_B \phi_B) \right] \quad (32)$$

and

$$\left(\frac{S_A}{I_A} - \frac{S_B}{I_B} \right) (I_A \phi_A + I_B \phi_B) + \left(\frac{S_A}{I_A} + \frac{S_B}{I_B} \right) (I_A \phi_A - I_B \phi_B) = 0 \quad (33)$$

from which, by eliminating $(I_A \phi_A + I_B \phi_B)$,

$$\phi_C = - \left(\frac{\omega^2}{K} \right) \left[\frac{\frac{S_A}{I_A} + \frac{S_B}{I_B}}{\frac{S_A}{I_A} - \frac{S_B}{I_B}} \right] (I_A \phi_A - I_B \phi_B) \quad (34)$$

and

$$\left(\frac{S_A}{I_A} + \frac{S_B}{I_B}\right) \left[\frac{\omega^2}{K} - \left(\frac{S_A}{I_A} + \frac{S_B}{I_B}\right) \right] + \left(\frac{S_A}{I_A} - \frac{S_B}{I_B}\right)^2 = 0 \quad (35)$$

Equation (34) provides the required relationship between ϕ_C and $(I_A \phi_A - I_B \phi_B)$, provided values of S_A and S_B are found. The latter are contained in equation (35), which does in fact determine the frequency of oscillation $\omega/2\pi$. However an exact solution for ω is not possible and an approximation method will be used.

If the frequency difference between the two sides of the oscillator is small, then the resultant natural frequency $\omega/2\pi$ of the system will be nearly equal to the individual frequencies. If also the damping of each side is small so that Q_A and Q_B are large, then S_A and S_B will both be small. Under these conditions, and if K is not excessively large,

$$\left| \frac{\omega^2}{K} \right| \gg \frac{S_A}{I_A} + \frac{S_B}{I_B}$$

and equation (35) may be approximated to

$$\left(\frac{S_A}{I_A} + \frac{S_B}{I_B}\right) \frac{\omega^2}{K} + \left(\frac{S_A}{I_A} - \frac{S_B}{I_B}\right)^2 = 0 \quad (36)$$

from which equation (34) becomes

$$\phi_C = \frac{1}{4} \left(\frac{S_A}{I_A} - \frac{S_B}{I_B}\right) (I_A \phi_A - I_B \phi_B) \quad (37)$$

It may be noted that this approximation is equivalent to neglecting the last term of equation (32).

If the differences between the natural frequencies of the two halves and the resultant natural frequency are expressed as

$$\begin{aligned} n_A - \omega &= \Delta_A \\ n_B - \omega &= \Delta_B \end{aligned} \quad (38)$$

then provided these differences are small (less than 1%), the values of S_A and S_B are approximately

$$S_A = \frac{2\Delta_A}{\omega} + \frac{j}{Q_A}, \quad S_B = \frac{2\Delta_B}{\omega} + \frac{j}{Q_B} \quad (39)$$

If these values are substituted into equation (35), and powers of Δ and $1/Q$ and their products higher than the first are neglected, then

$$\frac{2\Delta_A}{I_A \omega} + \frac{j}{Q_A I_A} + \frac{2\Delta_B}{I_B \omega} + \frac{j}{Q_B I_B} = 0$$

from which by separating real and imaginary parts,

$$\frac{\Delta_A}{I_A} + \frac{\Delta_B}{I_B} = 0 \quad (40)$$

$$\frac{1}{Q_A I_A} + \frac{1}{Q_B I_B} = 0 \quad (41)$$

The first equation (40) effectively defines the resultant frequency $\omega/2\pi$, and may be reduced to

$$\omega = \frac{n_A I_B + n_B I_A}{I_A + I_B} \quad (42)$$

Equation (41) is the condition for sustained oscillation. It may be noted that either both sides of the oscillator have zero damping, or else the positive damping of one side must be opposed by negative damping or driving of the other.

If the values of S_A and S_B are now substituted into equation (37), we find

$$\begin{aligned} \phi_C &= \frac{1}{4} \left[\frac{2}{\omega} \left(\frac{\Delta_A}{I_A} - \frac{\Delta_B}{I_B} \right) + j \left(\frac{1}{Q_A I_A} - \frac{1}{Q_B I_B} \right) \right] (I_A \phi_A - I_B \phi_B) \\ &= \left[\frac{n_A - n_B}{\omega(I_A + I_B)} + \frac{j}{4} \left(\frac{1}{Q_A I_A} - \frac{1}{Q_B I_B} \right) \right] (I_A \phi_A - I_B \phi_B) \end{aligned} \quad (43)$$

and hence the net residual torque is

$$M = \left[k_C + \mu_C j\omega \right] \left[\frac{n_A - n_B}{\omega(I_A + I_B)} + \frac{j}{4} \left(\frac{1}{Q_A I_A} - \frac{1}{Q_B I_B} \right) \right] (I_A \phi_A - I_B \phi_B)$$

where

$$I_A Q_A = - I_B Q_B \quad (44)$$

Referring now to the axis system of Fig.1 and Section 2, this torque is ideally about the Oz axis, and the torques due to input rates are detected about the Ox axis. In practice due to errors in construction a small amount of this torque may be measured and will give a spurious output. Such a torque is of particular importance if it is in phase with the output torque produced by a true input rate, equivalent to a torque in quadrature with $(I_A \phi_A - I_B \phi_B)$.

From equation (44), this quadrature component is

$$M = j \left[\frac{k_C}{4} \left(\frac{1}{Q_A I_A} - \frac{1}{Q_B I_B} \right) + \mu_C \frac{(n_A - n_B)}{(I_A + I_B)} \right] (I_A \phi_A - I_B \phi_B) \quad (45)$$

The torque is thus produced by a combination of differential damping of the two masses combined with the stiffness of the central mount, and a combination of differential frequency of the two masses combined with the damping of the central mount. Of the two terms it is generally easier to adjust the second to be small. Reduction of the first term depends effectively on accurate balance of driving and damping torques on each of the two masses which is not easily carried out since it is not practicable to experiment on the two masses oscillating separately.

Other torques may be generated about the torque-measuring axis due to the motions of A and B, in particular those described by equation (14) of Section 2 which are due to asymmetries of the oscillating masses in the x, z plane. Any such torques are proportional to the amplitudes of oscillation ϕ_A and ϕ_B , and in general can be expressed in the form

$$M = A(I_A \phi_A - I_B \phi_B) + B(I_A \phi_A + I_B \phi_B)$$

where A and B are real quantities.

From equations (31) and (43) this becomes

$$M = \left[A + B \left(\frac{k_C + \mu_C j\omega - \omega^2 \overline{I_A + I_B + I_C}}{\omega^2} \right) \left(\frac{n_A - n_B}{\omega(I_A + I_B)} + \frac{j}{4} \left(\frac{1}{Q_A I_A} - \frac{1}{Q_B I_B} \right) \right) \right] \times \left[I_A \phi_A - I_B \phi_B \right] \quad \dots (46)$$

In addition, torques may arise due to asymmetries of the central mass C, which in general will oscillate through the small angle ϕ_C ; such torques will be of a form similar to that of equation (45).

The number of possible asymmetries and mechanisms for producing torques is too great to consider in detail any further, but some general conclusions may be deduced from equations (45) and (46). To minimise torques which are in-phase with the torques developed by a rate of turn, damping mechanisms in the oscillator system must be minimised so that $1/Q_A$ and $1/Q_B$ become very small, and external damping of the mounting of the central mass, represented by μ_C , must also be made small. In addition, since one of the largest terms is multiplied by the stiffness of the mount k_C , this should also be minimised.

4 PRACTICAL DESIGNS OF TORSION OSCILLATOR GYROSCOPE

4.1 Single torsion axis

The simplest type of balanced torsion oscillator is shown in Fig.4, and consists of the two oscillating elements each at the end of a torsion stem, the central mass joining the other ends of the stems. It has been demonstrated that there are two arrangements of the oscillating masses which can be used to form a gyroscope; these are shown in Figs.2 and 3. The most simple torsion oscillator gyroscope is then formed by adapting the oscillator of Fig.4 to these arrangements, and the resultant gyroscopes are shown in Figs.5 and 6. In each case the masses oscillate about the Ox torsion axis, and the torques are measured about the Ox axis; no details of the method of measuring these torques will be discussed, but the most likely arrangement is to measure the resulting amplitude of oscillation in a tuned torsion stem.

Three major difficulties are present with this type of oscillator. They are:

(a) The centres of mass of the oscillating elements do not lie in the Ox,y plane containing the output torque axis, and the elements are therefore likely to have large J_{13} products of inertia which will give large error torques.

(b) The masses are at the free ends of the torsion stems, and acceleration or gravity fields may give large bending of these stems which will result in displacements of the centres of mass and corresponding error torques.

(c) In order to minimise error torques it has already been shown in Section 3 that the torsional stiffness of the mounting of the centre mass should be small. With the arrangement shown this also implies that lateral stiffness to Oy acceleration shall be small, and this will again give high sensitivity of the error torques to acceleration fields.

It may be possible to overcome the first of these difficulties by using the arrangement shown in Fig.7 for one of the oscillating masses; such an arrangement could be applied to either of the types shown in Fig.5 or Fig.6. By suitable design the centre of mass of the oscillating element could be made to lie on the Ox axis.

To overcome the second difficulty, it is possible to make the oscillating masses each supported by two torsion stem sections, which gives a considerable increase in lateral stiffness against acceleration. Such an arrangement is shown in Fig.8, and it is seen that the central mass C of Fig.4 has now become of the form of a cradle or gimbal. Because of the different arrangement of torsion stems compared with Fig.4, the theory of Section 2 is no longer directly applicable. It can be shown that the coupling between the two oscillating elements is now much improved compared with the simple oscillator of Fig.4, but the difficulty of the required low torsional stiffness of the mount of the centre mass remains. It is possible to provide the equivalent of such low stiffness by making the outer torsion stems much less stiff than the inner one, but this again introduces problems of lateral stiffness. Again the oscillating masses can be arranged to have the shapes shown in Fig.2 or Fig.3.

A more attractive alternative is shown in Fig.9. In this case there are two oscillating masses of different shape, a central solid mass and an outer mass in the form of a picture frame. There are now two equivalent central masses C at the nodal points of the torsion stems, and these may be joined together in the form of a cradle or gimbal.

Such an arrangement allows the centres of mass of both the oscillating elements A and B to coincide with the Ox axis, and does give good support of these elements since the torsion stems are not effectively "free-ended". One major problem is that the masses are now of completely different shape, and hence the problem of driving and damping these in a symmetrical way to make both $1/Q_A$ and $1/Q_B$ zero becomes much more difficult. In addition there is still the problem of providing effectively small torsional restraint on the centre masses while maintaining good lateral stiffness.

An oscillator of this type formed the basis of two experimental torsion-oscillator gyroscopes developed at R.A.E. These were known as TG1 and TG2, and the oscillator and output torsion system of each are shown in Figs.10 and 11 respectively.

TG1 was fitted with a balanced-torsion output system which was of similar form to the torsion oscillator shown in Fig.8. The two outer ends P and Q were joined together by a massive frame which was then rubber mounted to the instrument case. The torsion counterbalance R had fixed to it two additional masses so that its moment of inertia about the output torsion axis was equal to that of the elements A and B and the central element C which was in the form of a gimbal frame.

Practical tests quickly demonstrated that TG1 was a complete failure. The principal problem was that the frame C was not sufficiently rigid so that considerable torsional flexure about the Ox axis was possible. This completely spoiled the characteristics of the output torsion system, and no realistic testing was possible.

Due to the failure of TG1, a second gyro number TG2 was constructed with a much more rigid frame C, and this is shown in Fig.11. The output torsion system was changed to a balanced, nodally mounted form, basically of the type shown in Fig.9. The weakness of TG1 was found to be completely overcome, and the

output torsion system worked satisfactorily. It was, however, quickly found that the large stiffness and inertia of the frame C effectively prevented adequate coupling between the oscillating elements A and B, and that consequently their relative amplitudes were very much dependant on their damping and driving. The error torques produced by the oscillations were in fact very much larger and more variable than with comparable tuning-fork gyroscopes. This corresponds to large values of the stiffness k_C term of equation (45), and the resultant large variation of M with $(1/Q_A I_A - 1/Q_B I_B)$.

A modification to TG2 was then made to increase the torsional freedom of the torsion-stem mount and reduce the effective masses of the mount and the corresponding moments of inertia I_C . This was done by spark-machining the gimbal frame near the roots of the torsion stem as shown in Fig.12. Such an arrangement allows considerable freedom of the torsion stem while still providing considerable lateral stiffness and also maintaining the torsional stiffness of the gimbal frame C about the Ox axis. Due to the shape of the element of Fig.11, it was not possible to perform this machining by normal cutting or grinding methods.

The immediate result of this modification was successful, and the coupling between the A and B elements much increased. However, no further detailed tests were carried out on this instrument due to deficiencies in the design of the drive of the elements A and B and in the output torsion pickoffs.

4.2 Double torsion axis

Some of the difficulties experienced with the practical design of torsion oscillator discussed in Section 4.1 can be overcome by mounting the two elements A and B on separate axes of torsional oscillation. The two axes are necessarily parallel so that for a perfectly balanced pair oscillating in antiphase, no external torques are generated. The equations of Sections 2 and 3 do in fact still apply to this arrangement.

A number of alternative arrangements are possible, but those shown in Figs.13 and 14 would appear to be the most promising. In each case it will be seen that the Fig.2 type system is used; the Fig.3 system is necessarily asymmetric with two torsion stems, and does not appear to have any possible advantage.

It will be noticed also that in both Fig.13 and Fig.14 the centres of mass of the elements A and B both lie in the Ox,y plane, and that the elements are both supported at both sides on torsion stems. The only likely major problem with such systems will be the difficulty in providing effective coupling between the elements A and B; the centre mass C is necessarily large and stiff, and it will not be easy to give it significant torsional freedom about the Oz axis.

Another problem with this type of oscillator is the difficulty of accurate construction. This is a problem which it shares with the other types discussed, as will be seen also from Figs.10 and 11. Although it is possible that sufficiently accurate forms can be made by hand on a 1-off basis, the problems of machining this type of oscillator in production appear formidable. This thought has led to another concept of torsion oscillators discussed in Section 4.3.

Experiments are currently in progress by Prof. Karolus in Zürich working under M.O.A. contract on an instrument of the type shown in Fig.14.

4.3 Bending-stem oscillators

By comparison with the forms of torsion oscillator already discussed, the tuning-fork is easy to machine and can therefore be made with extreme accuracy. The principal reason for this is that starting with a rectangular block of material of accurate dimensions, virtually all the machining is of surfaces normal to one of the rectangular faces, and the shape can be designed so that the whole of the final machining can be performed using one cutter and not removing the workpiece from the machine. This makes possible the use of automatically-controlled machining.

A form of oscillator having similar constructional properties is shown in Fig.15. It will be seen to consist of the two elements A and B, each mounted on a leaf-spring relative to the central mass C. The feature of this design is that the leaf-spring acts as a hinge, allowing A and B to oscillate in the Ox,y , plane, and that the effective hinge-point which is the centre of any such rotational oscillation is designed to be coincident with the centre of mass of the element. The elements should therefore behave exactly as those of Figs.13 and 14, and therefore as examined theoretically in Section 2. The shape of the element is designed to give the correct position of the centre of mass, and to maximise the product of inertia J_{12} .

Relative to the tuning-fork gyroscope, which it closely resembles, it has the potential advantage that the elements are rigid and not flexible tines, and that since the centres of mass are at the hinge-points it should not be possible to excite oscillations of the elements by lateral vibration. In addition the fact that the centres of mass do not move may provide greater stability of zero signal.

One oscillator of this type, number A7, has been constructed at R.A.E. Preliminary tests show that the coupling between the A and B elements is small due to the large and stiff mass C, and a satisfactory coupling mechanism will have to be devised before construction of a complete gyroscope is undertaken.

5 CONCLUSIONS

A number of alternative forms of oscillator have been described, each of which could be made to operate satisfactorily as an oscillator. Their application to a gyroscope is not easy, principally due to problems of mounting of the central mass, lateral flexure of the torsion stems, and the maintenance of symmetrical oscillation of the two elements. In addition, the structures of the oscillators are in general so complex compared with a tuning-fork that it is unlikely that they can be manufactured to the same high standards of accuracy.

The only two torsion oscillator gyroscopes constructed at R.A.E. were of unsatisfactory design, and it cannot be deduced from their failure that the basic concept of such an instrument is unsound.

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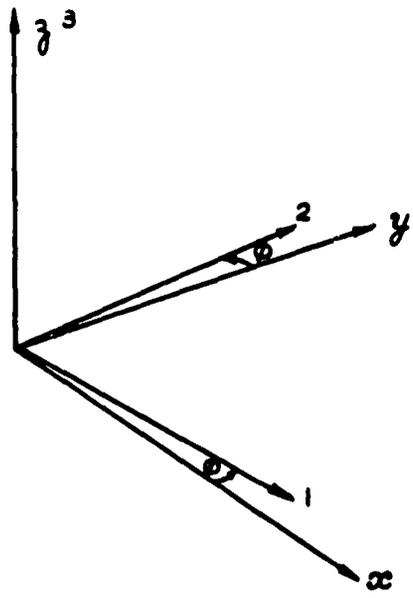


FIG. 1. AXIS SYSTEM.

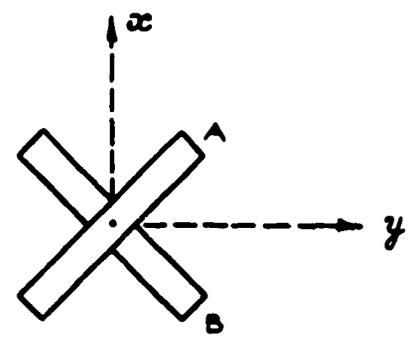


FIG. 2. BASIC TYPE OF TORSION OSCILLATOR.

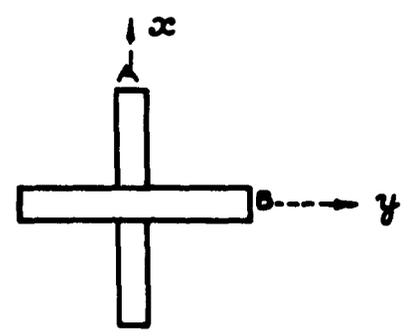


FIG. 3. BASIC TYPE OF TORSION OSCILLATOR.

FIG. 4.

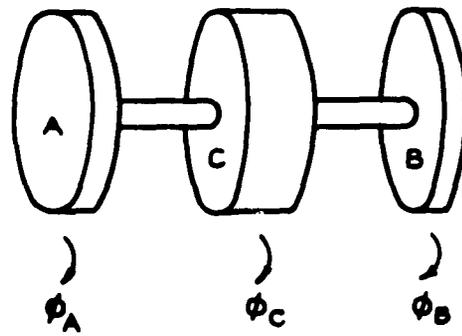


FIG. 4. NODALLY MOUNTED TORSION OSCILLATOR.

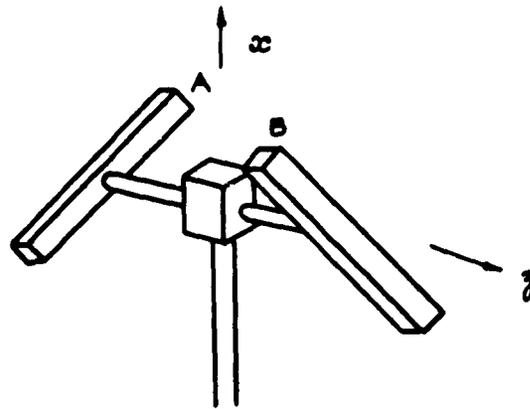


FIG. 5.

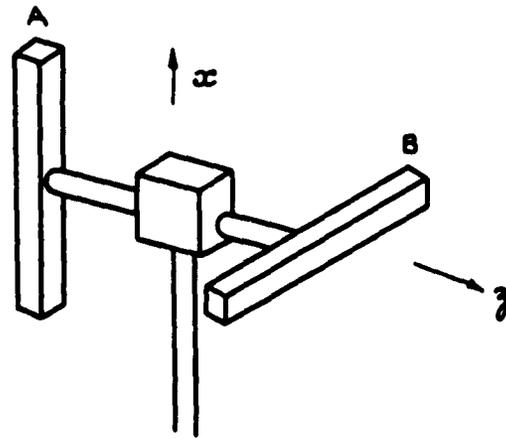


FIG. 6.

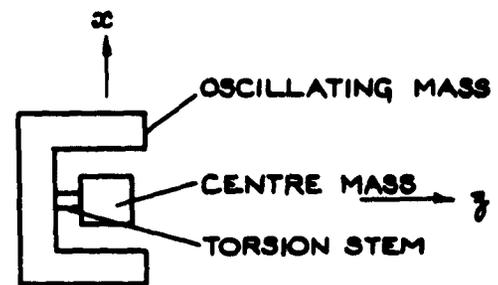


FIG. 7.

FIG. 5-7. FORMS OF BALANCED TORSION OSCILLATOR.

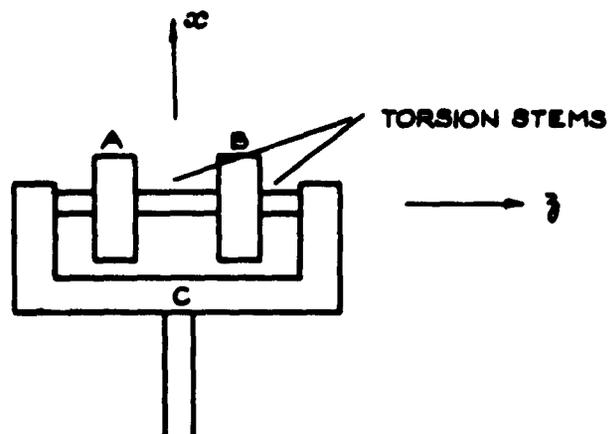


FIG. 8.

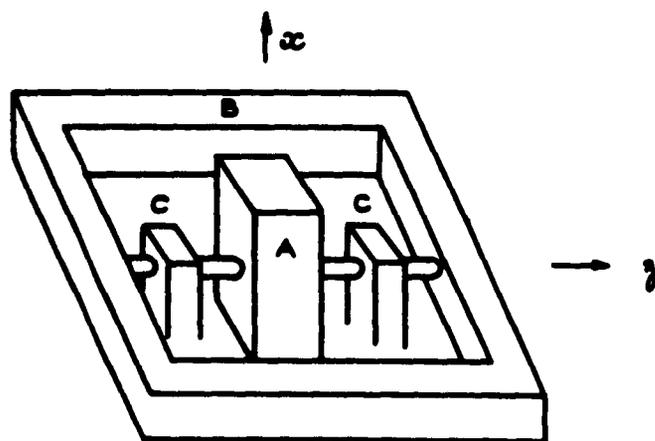


FIG. 9.

FIG. 8 & 9. FORMS OF BALANCED TORSION OSCILLATOR.

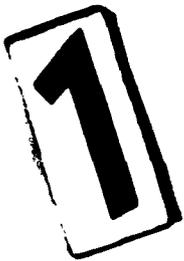
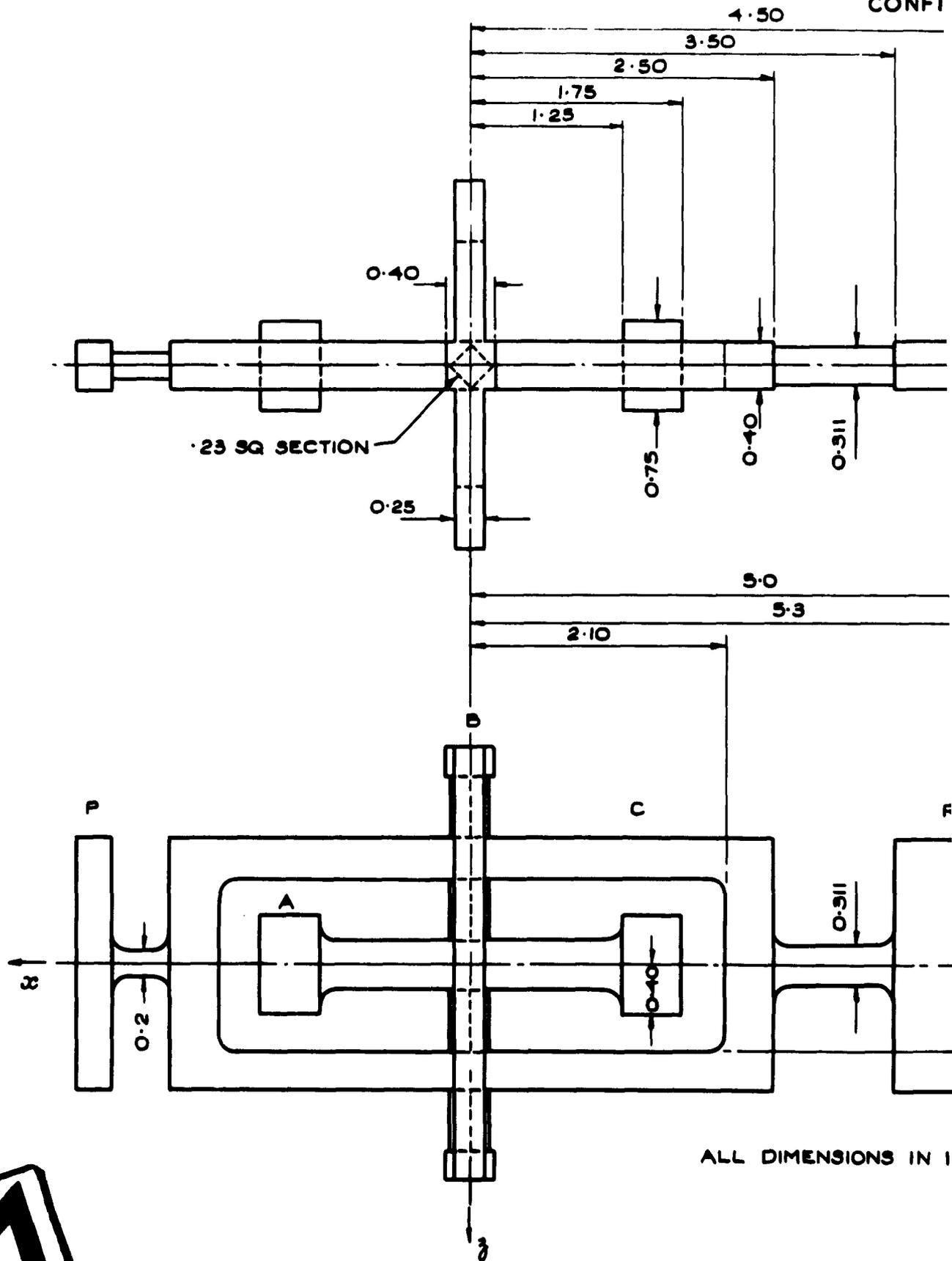
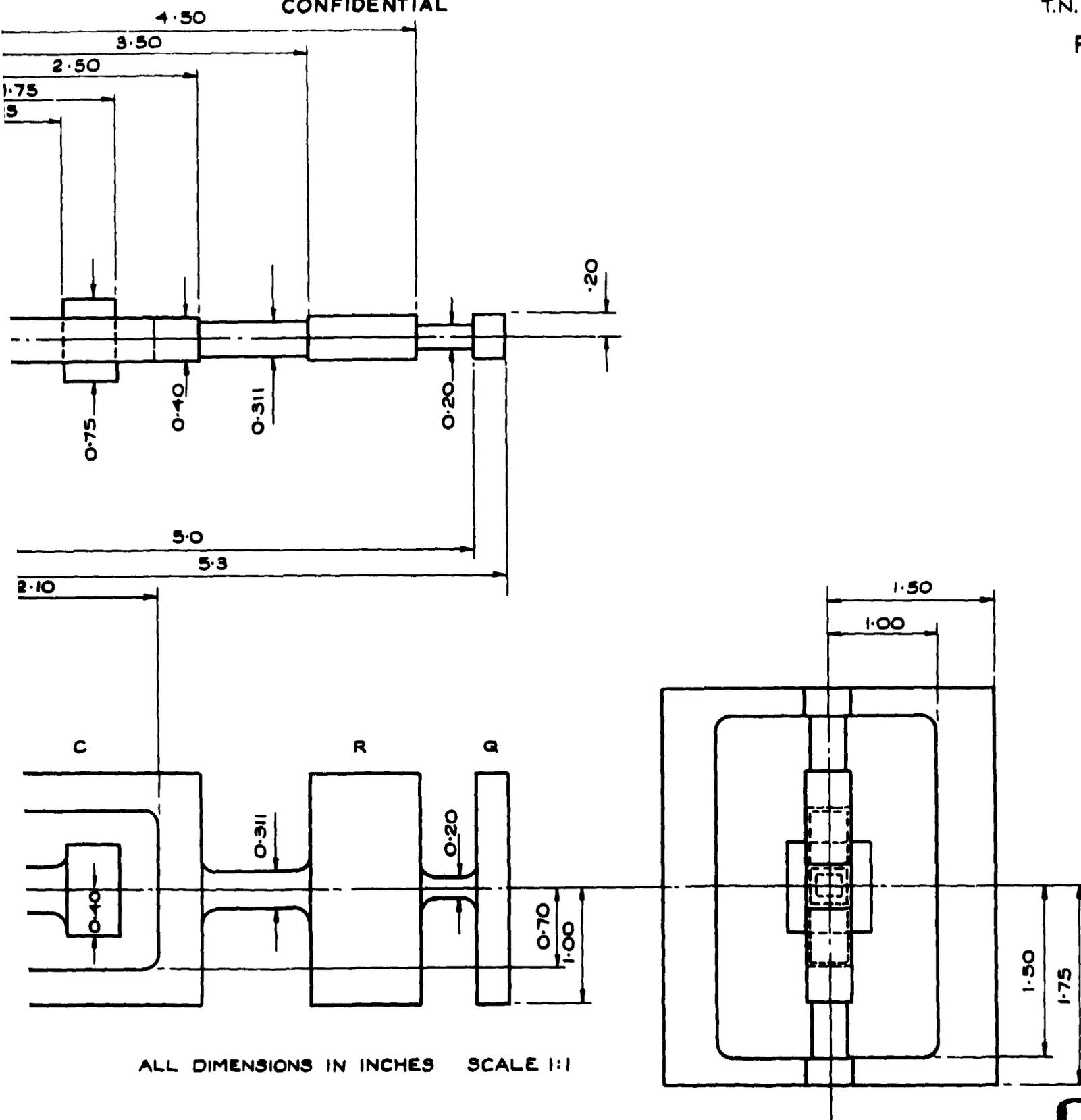


FIG. 10. R.A.E. EXPERIMENT

CONFIDENTIAL

T.N. IEE. 32.

FIG. 10.



ALL DIMENSIONS IN INCHES SCALE 1:1

FIG. 10. R.A.E. EXPERIMENTAL GYRO T.G.I.

2

FIG. II.

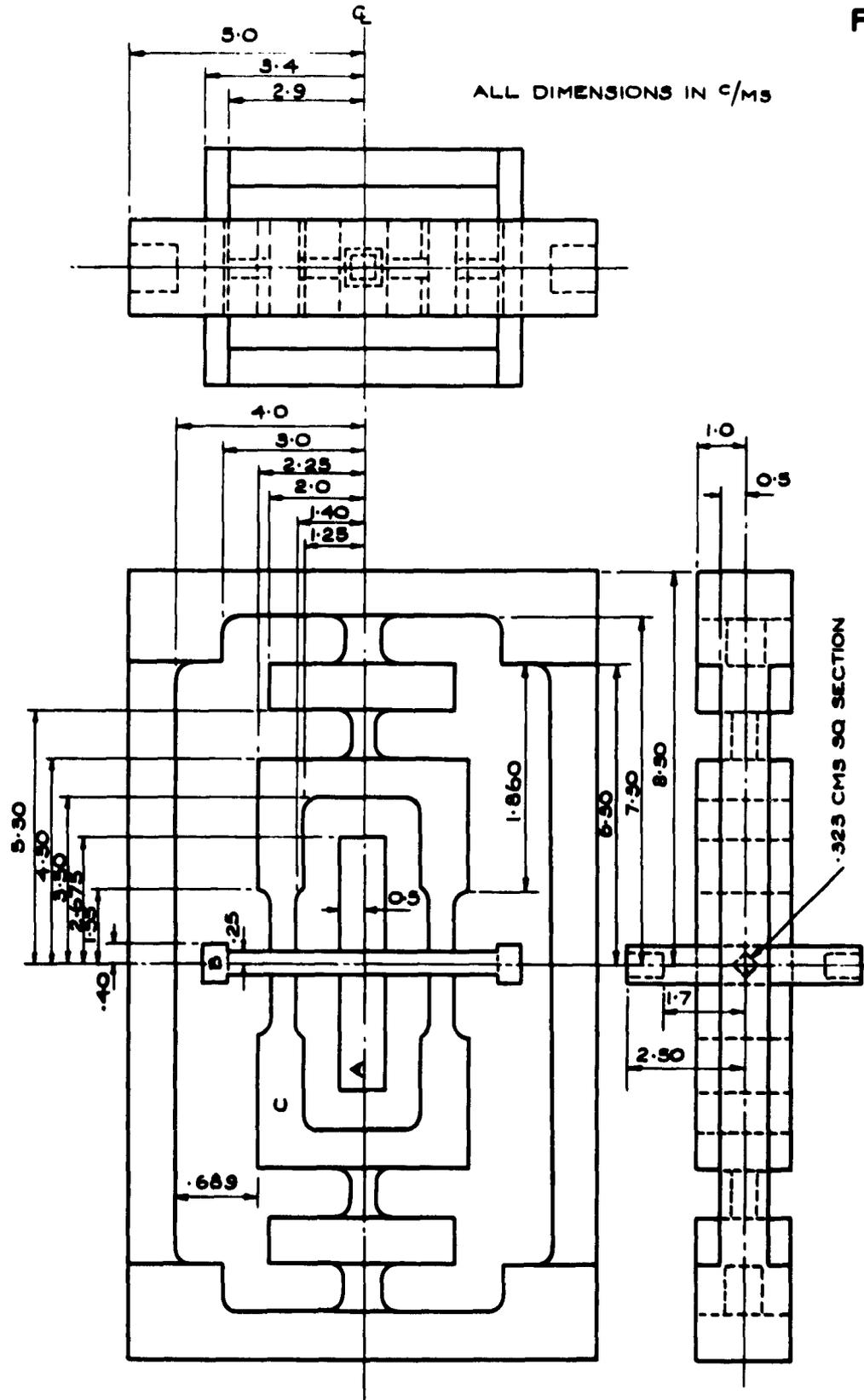
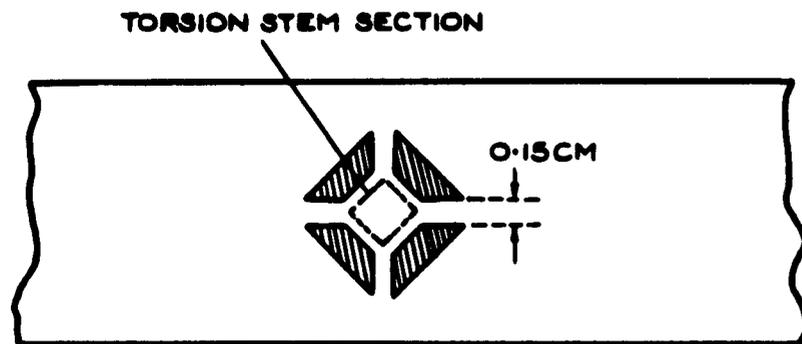


FIG. II. R.A.E. EXPERIMENTAL GYRO T.G.2.



SCALE 2:1

AREAS REMOVED BY SPARK MACHINING SHOWN HATCHED

FIG.12. MODIFICATION OF NODAL MOUNT OF TG2.

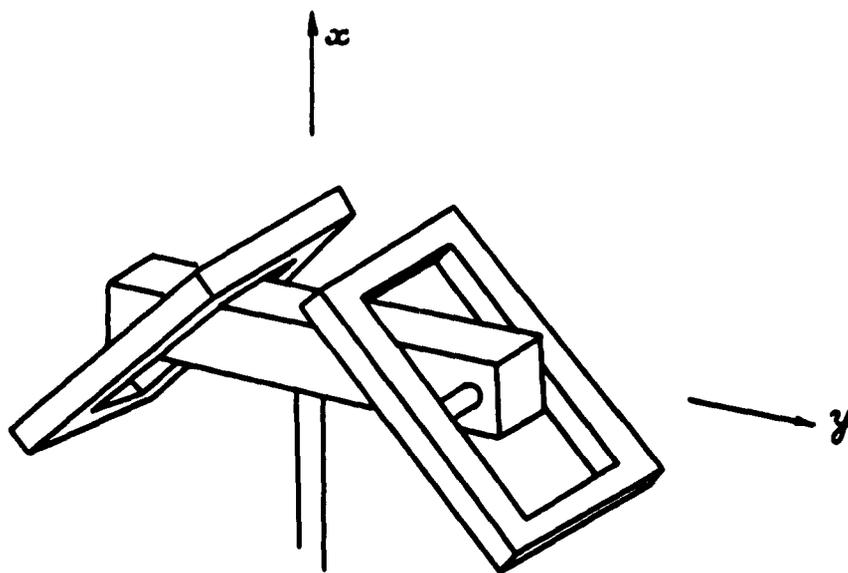


FIG. 13.

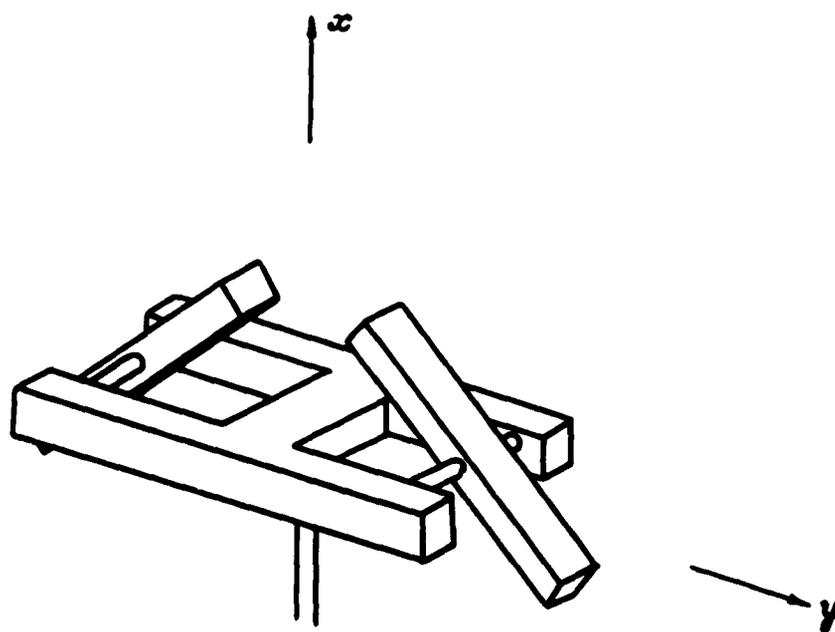


FIG. 14.

FIG.13 & 14. FORMS OF BALANCED TORSION OSCILLATOR.

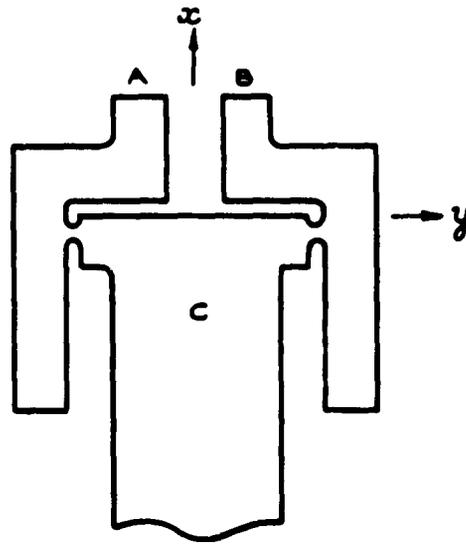


FIG. 15. FORM OF BENDING STEM OSCILLATOR.

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<p align="center">CONFIDENTIAL</p> <p>Technical Note No. IRE 32 Royal Aircraft Establishment</p> <p>SOME NOTES ON TORSION OSCILLATOR GYROSCOPES. Runt, G.H., and Hobbs, A.E.W. June, 1963.</p> <p>A torsion oscillator can be used as the sensing element of a vibratory gyroscope instead of the more commonly used tuning fork. Some theoretical aspects of torsion oscillators suitable for this application, and practical considerations in their use are both examined.</p>	<p align="center">CONFIDENTIAL</p> <p>Technical Note No. IRE 32 Royal Aircraft Establishment</p> <p>SOME NOTES ON TORSION OSCILLATOR GYROSCOPES. Runt, G.H., and Hobbs, A.E.W. June, 1963.</p> <p>A torsion oscillator can be used as the sensing element of a vibratory gyroscope instead of the more commonly used tuning fork. Some theoretical aspects of torsion oscillators suitable for this application, and practical considerations in their use are both examined.</p>	<p align="center">51,385 : 51,803.22</p>
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