UNCLASSIFIED

AD NUMBER

AD336943

CLASSIFICATION CHANGES

TO: unclassified

FROM: secret

LIMITATION CHANGES

TO:
Approved for public release, distribution unlimited

FROM:
Distribution authorized to U.S. Gov’t. agencies only; Foreign Government Information; MAR 1963. Other requests shall be referred to British Embassy, 3100 Massachusetts Avenue, NW, Washington, DC 20008.

AUTHORITY


THIS PAGE IS UNCLASSIFIED
NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

NOTICE:

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 and 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW.
ROYAL AIRCRAFT ESTABLISHMENT
(FARNBOROUGH)

TECHNICAL NOTE No. RAD. 831

THE PRINCIPLES OF ARTIFICIAL GLINT JAMMING
("CROSS EYE")

by

P. E. Redmill, B.Sc., Eng.

MARCH, 1963
ROYAL AIRCRAFT ESTABLISHMENT
(FARNBOROUGH)

THE PRINCIPLES OF ARTIFICIAL GLINT JAMMING
("CROSS EYE")

by

P.E. Rodmill, B.Sc., Eng.

SUMMARY

Two suitably phased coherent jamming sources, of approximately equal amplitudes, located on the extremities of a target, can introduce considerable angular errors or glint into radars tracking that target. A theoretical method of analysis is presented whereby the effect on any tracker of two sources of any amplitude and phase relationship can be computed.

The tactical effect, on various weapon guidance systems, of an artificial glint jammer installed as a self protection device in a single target aircraft, is discussed. It is shown that, in certain circumstances, a useful effect could be obtained with phase tolerances that correspond to attainable engineering limits.

A practicable artificial glint jammer circuit, using a single R.F. tube and a ferrite phasing device, is proposed.
# LIST OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>MATHEMATICAL ANALYSIS</td>
<td>5</td>
</tr>
<tr>
<td>2.1</td>
<td>Response of a tracker to two jamming sources with small angular separation</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>Response of a tracker to two jamming sources with finite angular separation</td>
<td>7</td>
</tr>
<tr>
<td>2.3</td>
<td>Response of a tracker to two jamming sources with varying phase relationship</td>
<td>10</td>
</tr>
<tr>
<td>2.4</td>
<td>Effect of target skin echo</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>TACTICAL CONSIDERATIONS</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>DESIGN OF AN ARTIFICIAL GLINT JAMMER</td>
<td>15</td>
</tr>
<tr>
<td>4.1</td>
<td>Artificial glint jamming circuits</td>
<td>15</td>
</tr>
<tr>
<td>4.2</td>
<td>R.F. power requirements</td>
<td>17</td>
</tr>
<tr>
<td>4.3</td>
<td>Engineering considerations of the double amplifier circuit</td>
<td>17</td>
</tr>
<tr>
<td>4.4</td>
<td>Engineering considerations of the single tube circuit</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>CONCLUSIONS</td>
<td>19</td>
</tr>
<tr>
<td>6</td>
<td>LIST OF SYMBOLS</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>LIST OF REFERENCES</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>APPENDICES 1-3</td>
<td>23</td>
</tr>
<tr>
<td>9</td>
<td>ILLUSTRATIONS - Figs.1-5</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>DETACHABLE ABSTRACT CARDS</td>
<td>28</td>
</tr>
</tbody>
</table>

### LIST OF APPENDICES

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R.F. field set up by two coherent isotropic jamming sources</td>
<td>23 and 24</td>
</tr>
<tr>
<td>2</td>
<td>The condition that contours of equal bias on the tracker response polar charts are circles</td>
<td>25 and 26</td>
</tr>
<tr>
<td>3</td>
<td>Effect of circuit imperfections on the response of a sum and difference monopulse tracker to artificial glint jamming</td>
<td>27 and 28</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

Artificial glint jammer circuits 1
Double amplifier circuit 1a
Single tube circuit 1b
Dual source geometry and radiation pattern 2
Tracker response polar charts 3
Long range case (ψ/ρ = 0) 3a
Medium range case (ψ/ρ = 0.1) 3b
Short range case (ψ/ρ = 0.4) 3c
Medium range case with ζ = 30° (ψ/ρ = 0.1) 3d
Sum and difference monopulse tracker circuit 4
Vector diagram of signals near tracker centre 5
1 INTRODUCTION

Monopulse tracking radars presents a very considerable problem to the electronic countermeasure designer. The ideal monopulse tracker will track accurately on any single R.F. source irrespective of the modulation on that source.

A system consisting of two coherent jamming signal sources of equal amplitude, located on extremities (such as wing tips) of a target aircraft and phased so as to arrive at the tracking radar receiver aerial \( \pi \) radians out of phase, has often been proposed as a universal countermeasure to tracking radars. In particular a double amplifier circuit (shown in Fig.1a) has been proposed as a means of achieving the correct source amplitude and phase relationship for a single dish tracker, irrespective of movements of the target aircraft.

With exactly equal amplitude sources, errors of up to slightly more than half the tracker beamwidth can be introduced into any active radar tracker. However, as previous authors have shown, this requires very large amplifier gains, exactly equal saturated outputs and extremely tight phase tolerances, particularly at long ranges. If the phase tolerances are exceeded the device becomes a high power beacon. The scheme has never progressed beyond the theoretical stage due to the enormous engineering problems.

In this Note methods of theoretical analysis are presented which demonstrate the universal nature of the countermeasure action, and enable the effect on a tracker of two jamming sources with any amplitude and phase relationships to be computed. It is shown that an artificial glint jammer can be considered as identical in its effect to a single jamming source at a position displaced from the target centre.

The displacement \( K \) of the apparent single jamming source is given in terms of the semi-spacing of the real jamming sources; that is, related to the physical size of the target aircraft and not primarily to the tracking radar parameters.

It is shown that if \( K \) is small, but not negligible, the jamming sources need not be equal in amplitude. In fact, it is advantageous that they should be slightly unequal, as considerable phase tolerances are then permissible before \( K \) falls to a very small and useless value, and the jamming power requirements are then reduced.

A beam riding missile, for example, will fly towards the apparent target; hence a reasonably small value of \( K \) will result in an appreciable miss, and reduction of lethality.

The design of a tactically useful artificial glint jammer, with reasonable power requirements and attainable specification limits, can thus be contemplated. A new artificial glint jammer circuit is proposed, which comprises a single R.F. tube (which could be at least part of an existing E.C.M. installation) and a ferrite phasing unit. This scheme appears to have several advantages over the earlier double amplifier scheme, notably that it is feasible from an engineering point of view, and minimises the risk of the jammer acting as a beacon.
particularly as will be seen in Section 4.1 to multiple C.W. trackers. However, it does not have the same capability as the double amplifier circuit to bias a number of pulsed trackers simultaneously.

2 MATHEMATICAL ANALYSIS

2.1 Response of a tracker to two jamming sources with small angular separation (long range case)

Two coherent jamming sources set up the well known lobed interference pattern. Associated with this is a wavefront (contour in space joining all points with equal R.F. phase) which has half wavelength steps in it coinciding with the nulls in the amplitude pattern.

The radiation pattern from two closely spaced sources, and in particular the form of the wavefront, is derived in greater detail in Appendix 1. If the source amplitudes are exactly equal, the minima in the interference pattern are infinitely sharp, the amplitude dropping to zero at these minima. The wavefront steps are then infinitely sharp and can be considered as steps in either direction, while elsewhere in the field the wavefront forms circular arcs centred on the point C, (Fig.2) which is central between the two sources. If the sources are made unequal the amplitude minima and wavefront steps become less sharp, until in the limit when one source is removed completely the amplitude is uniform throughout the field and the wavefront forms a circle round the remaining source. In the general case of unequal source amplitudes, it is convenient and valid to regard the wavefront as essentially a circle centred on the larger source, but with regular distortions caused by the smaller source, as shown in Fig.2.

The long range case is defined, in this investigation, as where the angular separation \( \theta \) of the two jamming sources is small compared with the tracker beam width \( \rho \). The lobe size of the jammer interference pattern is then very large compared with the diameter of the tracker receiver aerial, and it follows that over the diameter of the tracker receiver aerial the wavefront (as defined above) is flat to a very close approximation. The amplitude of the jamming signal, however, is not in general equal at all points over the aerial aperture, particularly if \( \delta \) (jamming source phase relationship) is nearly equal to \( \pi \).

A tracker with "perfect angle sensing" may be defined as one which aligns itself to point in the direction normal to the wavefront, irrespective of any amplitude variation over its aerial aperture. One cause of imperfect sensing and its effect on tracker response will be considered in Appendix 3. In this section only trackers with perfect angle sensing will be considered.

The effect of artificial glint jamming may be considered as equivalent to an apparent single jamming source of amplitude \( |E_s| \) located at a point \( S \) separated from the centre of the sources \( C \) by an angle \( \theta \) or \( K\frac{\pi}{2} \). \( K \) is a dimensionless quantity expressing the position of \( S \) in terms of the source semi-spacing. For a tracker with perfect sensing at long ranges where \( \theta/\rho \) is small the equivalence is exact. \( |E_s| \) and \( K \) can then be derived from the jamming signal amplitude and the tilt of the wavefront at the tracker receiver aerial.
\[ |E_s| \text{ is thus equal to } |E_x| \text{ at } 0. \]

Then from equation (A.6) in Appendix 1 where \( v \) is zero:

\[ |E_s| = \sqrt{|E_a|^2 + |E_b|^2 + 2 |E_a| |E_b| \cos \delta}. \tag{1} \]

As shown in Appendix 1, any source displayed from A is characterised by a progressive change of R.F. phase along OX at a rate proportional to the size of the angle of displacement measured at O. For example, the displacement of source B from A causes a progressive change of relative phase along OX at a rate \( \frac{dv}{da} \) proportional to \( \sin \psi \), as shown in equation (A.4). Conversely, the displacement of the apparent single jamming source S can be determined by the rate of change of the resultant phase \( \phi \) along OX as measured at O; that is to say that

\[ \frac{d\phi}{da} \text{ is proportional to } \sin \left( \left(1 - K \right) \frac{\psi}{2} \right). \]

Therefore

\[ \frac{d\phi}{dv} = \frac{\frac{d\phi}{da}}{\frac{dv}{da}} = \frac{\sin \left( \left(1 - K \right) \frac{\psi}{2} \right)}{\sin \psi}; \]

\[ = \frac{1 - K}{2}, \]

as \( \psi \) is extremely small.

From equation (A.7) in Appendix 1

\[ \phi = \tan^{-1} \left( \frac{|E_b| \sin (\delta + \nu)}{|E_a| + |E_b| \cos (\delta + \nu)} \right); \]

whence, by differentiation and equating \( v \) to zero,

\[ \frac{d\phi}{dv} \text{ at } 0 = \frac{|E_a| |E_b| \cos \delta + |E_b|^2}{\left( |E_a|^2 + |E_b|^2 + 2 |E_a| |E_b| \cos \delta \right)}, \]

and

\[ K = \frac{|E_a|^2 - |E_b|^2}{\left( |E_a|^2 + |E_b|^2 + 2 |E_a| |E_b| \cos \delta \right)} \tag{2} \]

- 6 -

SECRET
or
\[ K = \frac{|E_a|^2 - |E_b|^2}{|E_s|^2}. \]

The latter form of equation (2) shows that large values of bias \( K \) occur simultaneously with low values of \(|E_s|\).

Equations (1) and (2) have been used to construct the tracker response polar chart in Fig.3a.

In the polar chart, the radius represents the source amplitude ratio \(|E_b|/|E_a|\), and the angular co-ordinate gives the source phase relationship \( \delta \). Loci are plotted of all the values of \(|E_b|/|E_a|\) and \( \delta \) for which \( K \) has certain values, and other loci for which \(|E_s|/|E_a|\) has certain values. As an example when \(|E_b|/|E_a| = 0.6 \) and \( \delta = 150^\circ \) then \( K = 2 \) and \(|E_s|/|E_a| = -5 \text{ dBa} \) or \( 0.32 \).

2.2 Response of a tracker to two jamming sources with finite angular separation (effect of tracker aerial beamwidth)

If the angular separation of the two jamming sources is a finite fraction of the tracker beamwidth \( \rho \), the lobe size of the jammer interference pattern is not so many times larger than the diameter of the tracker receiver aerial. In these circumstances the jamming wavefront is not flat over the tracker aerial aperture, nor is the jamming signal amplitude equal at all points. Fundamentally any tracker with perfect sensing (as defined in Section 2.1) must align itself on a weighted average of the wavefront over its aperture. The weighting function depends on the detailed form of the aerial system and tracker circuitry. The response of all trackers with perfect sensing must be broadly similar, but different in detail. A single exact mathematical solution for the response of all trackers is not possible.

However, it is possible to compute the response of any specific tracker from a knowledge of its circuitry and the polar diagram of its aerial system, if these are known. It should be emphasized that the polar diagrams in both amplitude and phase must be known (in most other applications, of course, only amplitude is important). This approach is fundamentally equivalent to, and as equally valid as, computing the weighted average of the slope of the wavefront, and is mathematically much simpler. Moreover it becomes possible to take into account the circuitry. It is a useful cross-check that the response as computed from the polar diagrams of a tracker with perfect sensing at long ranges is identical to that computed from the form of the wavefront in Section 2.1 and displayed on the polar chart in Fig.3a.

Monopulse trackers, as far as tracking in one plane is concerned, function by comparing the signals received at two aerials and deriving therefrom a servo correction signal. Tracking in both planes involves a multiplication of aerials but is fundamentally no different. Conical scan and sequential lobing trackers can be considered, quite validly in this context, as special cases of amplitude comparison monopulse systems. In Appendix 2 it is shown that, if the condition for zero servo correction signal in a tracker can be described by equations of the form of (A.8) and (A.9), then the contours of equal bias on the polar chart...
(Figs. 3a-3d) must be circles. This criterion is fulfilled by nearly all trackers with either perfect or imperfect sensing. With imperfect sensing the polar chart becomes asymmetrical as described in Appendix 3.

Only three points are necessary to define a circle, and only two if the centre of the circle is known to lie on the \( \delta = \pi \) axis. Therefore, to plot contours of equal bias on the polar chart, it is only necessary to obtain and compute an exact mathematical solution for selected restricted cases. For example, it is generally possible to compute tracker bias where \( \delta = \pi \) and where \( |E_a| = |E_b| \). The complete solution for the bias, when both \( \delta \neq \pi \) and \( |E_a| \neq |E_b| \), can then be derived.

Once the bias has been computed, the apparent jammer signal strength \( |E_s| \) can be calculated directly from the vector sum \( V_s \) of the signals in the tracker aerial system.

As an example of this method of solution, the response to artificial glint jamming of a sum and difference type of monopulse tracker will now be calculated. The block diagram of a typical sum and difference monopulse tracker is shown in Fig. 4. In this example the servo correction signal \( \eta \) will be defined by

\[
\eta = \text{real part of } \frac{V_d}{V_s} \quad (3)
\]

For a single source \( E_o \) located at \( C \) and for all values of \( \theta \) the sum signal

\[
V_s = s E_o \exp \left( \frac{-1 + \frac{3}{2}}{\rho^2} \times \theta^2 \right);
\]

and the difference signal

\[
V_d = d E_o \theta \exp \left( \frac{-1 + \frac{3}{2}}{\rho^2} \times \theta^2 \right);
\]

where \( d \) and \( s \) are arbitrary I.F. gain constants under the control of the tracker designer. In this example the sensing will be defined as perfect; that is to say \( d/s \) is real. (In the more general case discussed in Appendix 3, \( d/s \) is complex.) The above definition may not necessarily represent an actual tracker. It does however represent what is probably a tracker designer's ideal in that the sensing is perfect as defined in Section 2.1, there are no side lobes in the aerial polar diagram, and the servo correction signal \( \eta \) is directly proportional to the angular error \( \theta \). For the tracker defined as above the response to artificial glint jamming is derived as follows:-

For signals \( E_a \) and \( E_b \) originating at \( A \) and \( B \) respectively:

\[
V_s = s E_a \exp \left( \frac{-1 + \frac{3}{2}}{\rho^2} \left( \theta - \frac{\pi}{2} \right)^2 \right) + s E_b \exp \left( \frac{-1 + \frac{3}{2}}{\rho^2} \left( \theta + \frac{\pi}{2} \right)^2 \right);
\]
and,

\[ V_d = d E_a \left( \theta - \frac{\psi}{2} \right) \exp \left( -\frac{1.38}{\rho^2} \left( \theta - \frac{\psi}{2} \right)^2 \right) + d E_b \left( \theta + \frac{\psi}{2} \right) \exp \left( -\frac{1.38}{\rho^2} \left( \theta + \frac{\psi}{2} \right)^2 \right) \]

whence

\[ \frac{V_d}{V_s} = \frac{d}{s} \left\{ \theta - \frac{\psi}{2} \right\} \left[ \frac{E_a \exp \left( \frac{1.38}{\rho^2} \theta \psi \right) - E_b \exp \left( -\frac{1.38}{\rho^2} \theta \psi \right)}{E_a \exp \left( \frac{1.38}{\rho^2} \theta \psi \right) + E_b \exp \left( -\frac{1.38}{\rho^2} \theta \psi \right)} \right] \]

\[ = \frac{d}{s} \frac{\psi}{2} \left[ K - \tanh \left( \frac{0.69 K \frac{\psi}{2}}{\rho^2} + \frac{m}{2} + j \frac{5}{2} \right) \right] ; \]

where, as before,

\[ K = \theta/\psi + \frac{E_a}{E_b} = \exp (m-j\delta) . \]

Then:

\[ \eta = \text{real part of } \frac{d}{s} \frac{\psi}{2} \left[ K - \tanh \left( \frac{0.69 K \frac{\psi}{2}}{\rho^2} + \frac{m}{2} - j \frac{5}{2} \right) \right] ; \]

which must be zero from equation (3). i.e.

\[ \text{real part of } \left[ K - \tanh \left( \frac{0.69 K \frac{\psi}{2}}{\rho^2} + \frac{m}{2} - j \frac{5}{2} \right) \right] = 0 \cdot \tag{4} \]

This equation has been evaluated numerically, using tables of functions of complex variables, to obtain the contours of equal bias K in the polar charts in Figs.3b and 3c. These contours are, as proved in Appendix 2, circles.

A few observations on the solution of equation (4) are relevant. It can be shown that over a small area of the polar chart close to the point \(|E_b|/|E_a| = 1\) and \(\delta = \pi\), there are three solutions for K. That is to say, there are three angular positions at which the tracker servo correction signal is zero. The same fact has been noted by previous authors in calculations based on other polar diagrams. The central point is unstable; that is to say, if the tracker were displaced slightly from this point the servo correction signal would be such as to drive it towards one or other of the two remaining stable points. Of these two stable points the one nearest zero is the one at which the tracker will receive the larger jamming signal \(V_s\) and the one to which it will tend to look if artificial glint jamming is suddenly applied. It is this point which is plotted in Figs.3b-3d. It can be seen that this principle value of \(K\) never exceeds about 1.2 \((\rho/\psi)\). The other stable solution for \(K\) may be larger and depends very much on the tracker circuitry and...
polar diagrams. It always corresponds to a very much lower value of $|E_a|$ and very tight phase tolerances, and is hence of no more than theoretical interest.

The magnitude of the apparent jamming signal strength $|E_a|$ is directly related to the magnitude of the sum signal, $|V_s|$, in the tracker when locked onto the stable point. Hence,

$$|E_a| = \frac{1}{s} |V_s|$$

$$= \text{Magnitude of } \left[ E_a \exp \left\{ \frac{-1.38}{\rho^2} \left( \theta - \frac{\psi}{2} \right)^2 \right\} + E_b \exp \left\{ \frac{-1.38}{\rho^2} \left( \theta + \frac{\psi}{2} \right)^2 \right\} \right] \cdot (5)$$

$|E_s|/|E_a|$ is also plotted in Figs.3b and 3c.

2.3 Response of a tracker to two jamming sources with varying phase relationship ($|E_b|/|E_a|$ being constant)

The effect of variation of $\delta$ is to sweep the jammer interference pattern, discussed in Section 2.1, past the tracker. A large error signal, and a simultaneous dip in the apparent jammer signal strength $|E_a|$, is introduced every time an interference pattern minimum passes the tracker receiver aerial. That is, whenever $\delta$ approximated to $\pi$ radians. This represents artificially produced angular noise.

The response of a tracker when $\delta$ is varying must thus depend very largely on the rate of variation of $\delta$ compared with the tracker servo system time constant and A.G.C. time constant. A.G.C. (automatic gain control) is essential in any practical tracker to ensure that the servo correction signal is a function only of angular information, and is independent of mean signal level. This is essential for optimum stable servo operation at all ranges and all normal sizes of target. A.G.C. time constants may be virtually zero for a "true" monopulse system which measures the ratio of aerial input signals\(^\dagger\) instantaneously, but may have larger values, up to slightly less than the servo time constant, for sequential lobing trackers, conical scan trackers, sum and difference type monopulse trackers (in some cases), and C.W. trackers.

If $\delta$ varies at a rate slow enough for the tracker servo system to respond to the angular error signals, large excursions of tracker bias $K$ are produced whenever $\delta$ approximately equals $\pi$ radians. The apparent jammer signal strength $|E_s|$ is approximately proportional to $1/\sqrt{K}$. Equation (2) in Section 2.1 shows this proportionality to be exact at long ranges for a tracker with perfect sensing. The successive excursions of $K$ represent a mean bias $\bar{K}$ combined with angular jitter. Numerical analysis based on Figs.3a-3d show that the R.M.S. value of $K$ can be given approximately by the empirical formula $\sqrt{K}(K_{\text{max}})$, where $K_{\text{max}}$ is the peak bias (produced when $\delta$ equals $\pi$ radians). The R.M.S. jitter, i.e., the fluctuation of $K$ about its mean value, is therefore given approximately by $\sqrt{K}(K_{\text{max}} - \bar{K})$. 

- 10 -

SECRET
Figs. 3a-3d show that \( \bar{R} \) is very roughly inversely proportional to the angular range over which \( \delta \) is varied. If \( \delta \) is varied over a full cycle of 2\( \pi \) radians, \( \bar{R} \) is equal to very slightly less than unity. This is consistent with the concept that the tracker aligns itself on the wavefront averaged over its receiver aerial aperture. The wavefront is shown in Appendix 1 to be essentially a circle centred on the larger source, but with regular deformation caused by the smaller source. Variation of \( \delta \) causes the wavefront pattern to sweep past the aerial aperture. The tracker thus aligns itself on each section of the wavefront in turn. Its mean angle of alignment must nearly coincide with the mean wavefront, which is a circle centred on the larger jamming source.

A low level of angular jitter could thus be generated by sweeping \( \delta \) continuously at a slow rate, accompanied by a modulation of \( |E_b|/|E_a| \) to randomise the form and direction of the excursions of bias \( K \). This technique would present fewer technical problems than maintaining \( \delta \) constant at \( \pi \) radians. The more nearly equal \( |E_a| \) and \( |E_b| \) were maintained, the larger would be the R.M.S. level of the jitter. However, a limit would be set when the dips in \( |E_a| \) fell below the skin echo level, or were pronounced enough to form the basis of a counter countermeasure such as blanking off the tracker servo input signals during periods of low total received signal.

With faster rates of variation of \( \delta \), the tracker, due to its limited speed of servo response, cannot follow the excursions of \( K \), but tracks on a mean value of bias \( \bar{R} \) such that its mean servo correction signal is zero. The excursions of \( K \) could introduce considerable noise into the angular tracking circuits, which may degrade the performance in certain trackers, although this is not a general effect.

If \( \delta \) varies over an angular range of 2\( \pi \) radians at a rate faster than the servo time constant but slower than the A.G.C. time constant of the tracker, \( \bar{R} \) is equal to approximately 1. This means the tracker tracks the largest source. For example \( \bar{R} \) may be evaluated for the sum and difference type of monopulse tracker considered in Section 2.2. As shown in that section, at any instant the servo correction signal is given by:

\[
\frac{\partial}{\partial \delta} \left[ \bar{R} - \tanh \left( 0.69 \frac{\bar{R}}{\rho^2} + \frac{\pi}{2} - j \frac{\delta}{2} \right) \right].
\]

The time average must be zero and as phase is assumed here to vary uniformly with time the phase average also must be zero and hence:

\[
2\pi \bar{R} - \int_0^{2\pi} \tanh \left( 0.69 \frac{\bar{R}}{\rho^2} + \frac{\pi}{2} - j \frac{\delta}{2} \right) d\delta = 0.
\]

The definite integral

\[
\int_0^{2\pi} \tanh \left( 0.69 \frac{\bar{R}}{\rho^2} + \frac{\pi}{2} - j \frac{\delta}{2} \right) d\delta = 11.
\]
is, when evaluated, equal to either $+2\pi$ or $-2\pi$ depending on whether the term $(0.69 \frac{\psi}{2} + \frac{\psi}{2})$ is positive or negative in value respectively. Hence $\bar{x}$ equals either $+1$ or $-1$. That is to say, the tracker must track on one or other of the two sources and select the larger. This is exactly true of some other trackers and approximately true of all trackers.

If $\delta$ varies at a faster rate than the A.G.C. time constant of the tracker, the mean bias $K$ (Ref.15) somewhat less than 1. This is because $|E_s|$ is not constant as $\delta$ varies and is greater when $K$ is small. Again, an exact analysis cannot be made for all trackers. However, to a first approximation any tracker tracks on the "power centre" of the two sources given by:

$$K = \frac{|E_a|^2 - |E_b|^2}{|E_a|^2 + |E_b|^2}$$

### 2.4 Effect of target skin echo

Any practical jammer must radiate sufficient power to over-ride, or pre-dominate in its effect over, the natural target skin echo. The skin echo return of any moving target is very complex, with more or less random fluctuations in amplitude and phase (fading), and fluctuations in apparent position (natural glint or angular noise). A complete mathematical analysis of the effect of the combination of artificial glint jamming and natural skin echo on a tracker is extremely complicated. However, by making two simplifying assumptions an approximate solution can be derived.

Artificial glint jamming is assumed to be identical in its effect to a single jamming source of amplitude $|E_s|$, and position $S$ displaced by angle $\frac{\psi}{2}$ from the target centre $C$. $|E_s|$ and $K$ are the apparent jammer signal strength and the linear bias as derived in sections 2.1 and 2.2. This approximation is least valid when $|E_a|$ is almost exactly equal to $|E_b|$, and $\delta$ nearly equal to $\pi$. There are then two stable solutions for $K$ with positive and negative values (see Section 2.2). However, as will be seen in Section 3, the condition of unequal source amplitudes is of more interest from a practical point of view.

The natural skin echo is assumed to be free of positional fluctuations or natural glint, and to be located at position $C$. It is also assumed to have completely random phase, and a Rayleighian amplitude probability distribution as defined in the List of Symbols.

Making the above assumptions, the problem reduces to an analysis of the effect of two sources, of which the amplitude and phase of one are varying randomly. The tracker response then depends on the rate of amplitude variation of the combined skin echo and jamming signal return, compared with the A.G.C. and servo time constants of the tracker.
If the A.G.C. time constant is short so that it can follow the amplitude changes of the combined signal (jamming plus skin echo), the tracker tries at any instant to look on to whichever of the two sources, real or apparent, is the stronger. The mean bias $K_o$ is then equal to the bias without skin echo $K$ multiplied by the fractional time during which $|E_s|$ exceeds $|E_o|$.

That is:

$$K_o = K \int_{0}^{\infty} 2c \exp(-c^2) dc \cdot$$

$$= K \left[1 - \exp\left(\frac{|E_s|}{|E_o|}\right)^2\right].$$

(6)

If the tracker A.G.C. time constant is too long to be able to follow the amplitude changes in the combined return signal, the tracker tracks approximately on the "power centre" of the apparent single jamming source and the skin echo as shown in Section 2.3.

Then:

$$K_o = K \frac{|E_s|^2}{|E_s|^2 + |E_o|^2}. \quad (7)$$

Equations (6) and (7) when evaluated can be seen to differ only slightly. The tracker A.G.C. time constant thus makes very little difference to the response of a tracker to continuous artificial glint jamming when $\delta$ is constant. If the jamming can be regarded, as above, as equivalent to a displaced single jamming source (that is for unequal real jamming source amplitudes), there is no sharp jamming threshold.

There could also be some angular jitter produced if the servo time constant were short enough to follow the apparent positional changes or angular noise of the combined return signal. In practice most of the frequency components of this angular noise would be too high to enter the servo system, and the jitter would be of small amplitude. However this angular noise might of course affect the circuitry of certain trackers in ways peculiar to those trackers (see Section 2.3).

3  TACTICAL CONSIDERATIONS

The mathematical analysis shows that artificial glint jamming can very considerably degrade the tracking accuracy of a radar tracker by producing either a constant mean angular bias, or angular jitter (varying bias), or both effects simultaneously.

To achieve a mean bias, large in proportion to the source separation, the jamming source phase relationship $\delta$ at the centre of the tracker receiver aerial must be maintained at nearly exactly $\pi$ radians. As far as it is known, the only feasible method of maintaining $\delta$ at $\pi$ radians, independent of aircraft
motion, is to use the tracker transmitted signals, as received at each jamming source aerial, as a phase reference. This is proposed in the "double amplifier" and "single tube" circuits that will be described later. This means, of course, that the jammer interference pattern is fixed in orientation relative to the tracker transmitting aerial.

A high level of angular jitter may be generated deliberately by either: a slow modulation of \( \delta \) over a limited range around \( \pi \) radians, or a slow modulation of the source amplitude ratio \( \left| \frac{E_b}{E_a} \right| \) around a mean value of unity, or modulation of both \( \delta \) and \( \left| \frac{E_b}{E_a} \right| \) simultaneously.

Low level angular jitter can be generated without maintaining \( \delta \) constant, by ensuring that its rate of variation is slow enough for the tracker servo to be able to follow the successive excursions of \( K \). The form and direction of these excursions may be randomised by a slow modulation of \( \left| \frac{E_b}{E_a} \right| \).

Missile guidance systems which depend for accuracy on an active radar tracker are very susceptible to artificial glint jamming. A bias introduced into the guidance tracker of a beam-riding, or a command guidance, missile system would result in an equal miss distance. High explosive warheads have a very limited lethal radius, outside which the probability of target damage decreases rapidly. A miss distance of only a few times the source separation, would result in a very considerable reduction of missile lethality.

Using source aerials on aircraft wing tips, a mean linear bias \( i_c \) even as low as 2 would appear to be nearly as useful tactically as half a radar beamwidth. This is very important, as the mathematical analysis has shown that, by using jamming sources of slightly unequal amplitudes, moderate values of bias \( i_c \) can be obtained at all ranges with reasonably wide tolerances on the source phase relationship \( \delta \), and with moderate source power requirements.

Moderate level angular jitter can also cause appreciable miss distances, due to lags in the missile steering servo response. Also, the missile range may be reduced somewhat by the increased aerodynamic drag, as the missile tries to follow the varying bias or angular jitter.

Fully active homing missiles are very susceptible to artificial glint jamming. These normally use proportional navigation techniques, by which the missile rate of turn is equalized to the rate of change of sight angle multiplied by a "navigation constant". A constant bias can thus only affect the final homing phase of the missile flight, and then only slightly. The exact response depends on the distance at which the homing head can resolve the jamming sources separately, and the ability of the missile servo system to make rapid corrections. However, angular jitter may be very effective, as the missile is then affected at all stages in its flight, and this effect is enhanced by the "navigation constant" above. Unpublished calculations by P.B. Kemshall (Royal Aircraft Establishment, Weapons Department) show that even simple switching of full jamming power between sources at a rate slow enough for the missile steering servo to respond to it, could cause R.M.S. miss distances, in a homing missile, of the order of 0·7 times the source separation. As shown in Section 2.3, angular jitter set up by the artificial glint principle is of considerably greater amplitude, and the R.M.S. miss distances would be proportionally greater.
Artificial glint jamming can, in general, generate a constant angular bias only in active trackers. With suitable adjustment of the phase difference between the two jamming sources, such a bias can be generated in trackers with either a common transmit and receive aerial ("single dish"), or transmit and receive aerials separated by a small fixed distance \( \sigma \) ("twin dish"). In Section 4.1 the implications of this need for correct phase adjustment in the two cases, is discussed. Usually of course pulsed trackers have single dishes and C.I.W. trackers have twin dishes.

Artificial glint jamming may not be expected in general to have much effect on a semi-active missile guidance system. Angular bias or jitter or both could of course be generated in the target illumination radar tracker, but not sufficient to destroy its function of following and illuminating the target. The missile itself would normally be flying through the lobes of the jammer interference pattern at a high rate (several tens of lobes per second). The angular noise received by its homing head would, in general, be of too high a frequency to cause angular jitter. There is no known method of orientating the jammer interference pattern, relative to a passive or semi-active tracking head (short, of course, of using an active radar in conjunction with the jammer). Probably the best dual source countermeasure to passive or semi-active systems is simple switching of full jamming power between sources at a slow rate.

4 DESIGN OF AN ARTIFICIAL GLINT JAMMER

4.1 Artificial glint jammer circuits

The ideal artificial glint jammer would be capable of producing a bias in any active radar tracker. To achieve this, the source amplitude ratio as seen by the tracker must be correct, and the phase relationship at the centre of the tracker receiver aerial must be maintained at \( \phi = \pi \) radians, irrespective of the relative motions of the target and the tracker. In other words, a minimum in the jammer interference pattern must always be located on the centre of the tracker receiver aerial.

A circuit employing two broad-band travelling wave tube (T.W.T.) amplifiers, and considered as the basic "cross-eye" circuit in previous literature, is shown in Fig.1a. The principle is as follows. Tracker transmitted signals received in aerial \( B \), are amplified in T.W.T. amplifier \( A \), and re-radiated from aerial \( A \). Signals received in aerial \( A \), are amplified in T.W.T. amplifier \( B \), delayed by a constant phase delay of \( \phi \), and re-radiated from aerial \( B \). Ferrite circulators, or an equivalent system, have been proposed in order to isolate the input of each T.W.T. amplifier from the output of the other, so that each receives an input consisting only of the tracker transmitted signal. Given such infinite isolation, and T.W.T. amplifiers and aerial feeders with exactly equal phase shifts, the two amplified signals must arrive back at the centre of the tracker transmitter aerial, with a phase relationship of \( \phi \). To bias any "single dish" tracker \( \phi \) must be pre-set to \( \pi \) radians, so as to locate a minimum of the jammer interference pattern on the centre of the tracker dish. At the centre of the receiver dish of a "twin dish" tracker, \( \delta \) would not be equal to \( \phi \), but \( \phi + \frac{2\pi}{\lambda} \sigma \phi \). To produce a bias, \( \phi \) must be equalised to \( \pi - \frac{2\pi}{\lambda} \sigma \). \( \sigma \) is a function of the tracker parameters and the tactical geometry, and must be found for each individual twin dish tracker, either by accurate intelligence, or a
process of phase searching; that is, continuously varying $\phi$ until the tracker is observed, using an auxiliary listening receiver device, to react.

A double amplifier jammer can thus be set, in principle, to bias either all "single dish" trackers, or one selected "twin dish" tracker. If set up to bias one "twin dish" tracker, it would present a large beacon to all other radars. Alternatively, or if $\sigma \psi$ cannot be found, $\phi$ can be swept slowly so as to generate intermittent bias or low level angular noise in all trackers.

The enormous difficulty of constructing T.W.T. amplifiers with identical phase shifts has led to consideration of possible single tube circuits; that is, methods of splitting the output from a single T.W.T. to feed two aerials in the correct phase relationship.

Proposals\textsuperscript{6,11} have been made for generating intermittent bias, or low level angular noise, by feeding two source aerials from a single tube and inserting a slowly varying phase shift in one aerial lead. Unfortunately (for the countermeasure designer) target motion would also produce high rates of change of $\delta$. With a typical aircraft target, $\delta$ might vary at several tens of radians per second. Most of the angular noise frequency components would lie outside the tracker servo bandwidth; i.e. the excursions of bias would be too rapid for the tracker to follow, and the device would servo mainly as a beacon.

Thought was therefore given to the possibility of using the tracker transmitted signals, as received at the source aerials, as a phase reference to control the source phasing quickly and automatically, and thus to maintain $\delta$ constant independent of target motion and generate a tracker bias. The circuit in Fig.1b is proposed. Phase is measured intermittently during "look-through" periods while the jamming signal is suppressed. Signals from the tracker received at aerials A and B pass via feeders, and a variable reciprocal phase shifter $\varphi$, to arms A and B of a $3$ dB hybrid junction. The relative amplitudes of the signals appearing in Arms C and D depend on the phase relationship of the signals entering Arms A and B. A minimum or phase sensing receiver, as shown in Fig.1b, is used to servo-control $\phi$ so that a minimum signal occurs in arm C. This receiver could be similar in form to the monopulse receiver in Fig.4, where arms C and D correspond to the difference and sum channels respectively.

A signal from the tracker is also picked up on a third aerial. This signal is amplified and injected, via a ferrite circulator, into arm C as the jamming signal.

If the microwave network is completely reciprocal, one of the minima of the interference pattern in space, set up by the jammer, must lie on the centre of the tracker transmitter aerial. A single dish tracker would thus be biased.

To bias a "twin dish" tracker, it is necessary to retransmit jamming with a phase relationship differing by $2\pi/\lambda \sigma \psi$ radians from the phase relationship of the received tracker signals (where $\sigma \psi$, as in the double-amplifier circuit, must be found from intelligence sources or by phase searching). One method of altering the phase relationship, is to insert a non-reciprocal ferrite phase shifter with a differential phase shift of $2\pi/\lambda \sigma \psi$ in one of the aerial feeds, as shown by the dotted component in Fig.1b.
A single tube circuit jammer would select the desired tracker on a frequency, p.r.f., or a similar basis. It could only bias one tracker ("single dish" or "twin dish") at any one time, but would generate intermittent bias or low level jitter in all other trackers, within its operating frequency bandwidth.

4.2 R.F. power requirements

As an example of the tolerance and power requirements, an artificial glint jammer to give a mean linear bias $K_0$ of at least 5, at all except short ranges, should have: a source amplitude ratio $|E_b|/|E_a|$ of between 0·8 and 0·9 (that is between 1 and 2 dBs), a phase tolerance on $\phi$ of ±0·2 radians, and a ratio of individual source output power to skin echo power of about 50 (17 dBs).

Assuming jammer aerial gains of 30 (15 dBs), this means T.W.T. amplifier gains of 40 dBs each in the double amplifier circuit, or 43 dBs (or slightly more, allowing for reduction of mean power due to look through) in the single tube circuit. To protect a 10 square metre target, a maximum power output per source of 2 watts mean when deployed against a C.W. tracker giving an illuminating power density of 0·1 watts mean per square metre, and 200 watts peak when deployed against a pulse tracker giving an illuminating power density of 10 watts peak per square metre, would be needed.

4.3 Engineering considerations of the double amplifier circuit

The design of the double amplifier circuit involves two very difficult problems: firstly the construction of two broad-band T.W.T. amplifiers with equal phase shifts and a fixed ratio of output amplitudes for all input levels, and secondly the achievement of a very high isolation between the output of one amplifier and the input of the other. Further problems are the equalisation of phase delays in the aerial feeders, and the matching of source aerial polar diagrams.

One approach to the problem is to design and pre-set the amplifiers to be as similar as possible and then provide a system of monitoring the outputs and phase shifts, while in operation, deriving "error" signals to correct for the differences by a feedback or servo system. Experiments show that, while T.W.T. gains and saturated power output levels can be controlled by variation of electron beam current, this produces large changes of phase shift of the order of 0·7 radians per dB change of gain. This is a fundamental effect due to space charge depression of electron beam potential. It was found preferable to adjust the T.W.T.'s for a constant ratio of saturated output powers by pre-setting the beam current, and to apply the feedback control to servo controlled mechanically variable attenuators in the T.W.T. inputs. T.W.T. phase shifts may be readily controlled by variation of helix voltage (electron beam potential) without greatly affecting the gain. Relative T.W.T. phase shifts might be monitored in one of three ways: use of a test signal of approximately the same level and amplitude as the radar signal, use of the radar signal itself as a test signal, or by phase searching (see Section 4.1).

It is desirable that the input signal to each amplifier, due to leakage from the output of the other amplifier, should not deviate by more than 1 dB in amplitude and 0·1 radians in phase from the "true" input signal from the tracker.
This necessitates an isolation of at least 20 dBs more than the individual amplifier gain. That is, isolation of the order of 60 dBs or more. This degree of isolation is not remotely possible with ferrite devices over anything more than extremely narrow bandwidths. Separate transmitting and receiving aerials for each amplifier (that is four aerials in all) must be considered. A limit is set on the permissible aerial spacings by the effect of intermittent deformations of the target vehicle on the relative radiation path lengths. For example, if the jamming sources were on the tips of wings which each flap by 5° peak to peak, the maximum allowable spacing of aerials, if δ is not to vary by more than 0.1 radians due to this wing flat, is one wavelength.

The only known possible general solution to the problem of achieving high isolation is to use widely spaced transmitting and receiving aerials at each source, to give sufficient isolation, and then to correct for the effects of aircraft deformation (wing flap) by applying automatic correction on the T.W.T. helix voltages, using the following system of phase monitoring. During short intermittent monitoring periods the output of each T.W.T. amplifier is isolated from its appropriate source transmitting aerial and fed into a dummy load. The phase relationship of the tracker signals entering the two source transmitting aerials is measured, and compared with the phase relationship of the outputs of the two amplifiers. Correction voltages are fed to the T.W.T. helixes to make the two phase relationships identical. If the monitoring is carried out at frequent intervals, corrections can be applied, in theory, to allow for the effects of target deformation, unequal aerial feeders, and unequal phase shifts in the amplifiers. While this scheme is possible in principle, it obviously involves great electronic complexity and, if deployed against pulsed trackers, T.W.T. amplifiers whose relative phase shifts remain equal during pulse modulation.

4.4 Engineering considerations of a single tube circuit

The feasibility of a single tube circuit depends on two factors: the availability of a suitable electronically controllable phase shifting component for \( \phi \), and whether the phase sensing look-through can be carried out sufficiently frequently to enable the phasing correction \( \phi \) to be applied with sufficient precision.

Various types of nominally reciprocal magnetically controlled ferrite phase shifters exist, of which the so called "Reggia Spencer" type\(^{16}\) shows particular promise in that it gives large phase shifts for small controlling fields. Measurements on an experimental component of this type show that the difference between the phase shifts in each direction is less than 0.01 radians for all values of controlling field. The phase shift is constant within 0.01 radians for all R.F. power levels up to at least 250 watts. A certain amount of development is probably needed to extend the usable R.F. bandwidth.

When deployed against C.W. trackers, phase sensing look-through can be carried out as frequently as is desirable. Against pulsed trackers, however, the highest practicable look-through frequency is very alternate pulse, while to maximise the proportion of effective jamming return pulses to the tracker a lower look-through frequency is desirable. The lower limit to the permissible rate is set by the rates of aircraft motion (roll, yaw, wing flap, etc.).
Periodic look-through, of course, interrupts the jamming and must result in a loss of effectiveness to some extent, as compared with continuous jamming. Against C.W. trackers, and other trackers where the A.G.C. time constant is longer than the look-through repetition rate, the result is a reduction in the apparent mean jamming power. Against pulsed trackers with short A.G.C. time constants the result is a reduction in the mean bias $K_0$, proportional to the fractional time spent in "look-through".

5 CONCLUSIONS

This investigation indicates that an artificial glint jammer is a feasible proposition, and that in many situations it could perform a useful role as a self protection device in an aircraft.

It would have the effect of seriously degrading the accuracy and performance of guided missile systems relying for guidance on an active tracker, i.e. fully active systems, beam riding and command guidance systems. Systems using an active tracker for mid-course guidance and with a separate terminal homing system, might be degraded in performance but to a lesser extent. Complete loss of angular lock in an active tracker would not, in general, be expected; nor could artificial glint jamming be expected to have much effect on a passive or semi-active radar homing missile system.

The theoretical treatment shows that with sources of slightly unequal amplitude, the phase tolerance and repeater gain requirements are achievable in a single tube circuit.

To obtain full effectiveness against "twin dish" (C.W.) trackers, and trackers with grossly imperfect sensing, some form of listening receiver device would be required to indicate when the tracker is biased in angle.

Artificial glint jamming might contribute usefully to the self protection of a tactical strike aircraft. In this environment the proposed "single tube" circuit has attractive features in that it would be integrated with other electronic countermeasure facilities, without severe penalties of space and weight.

---

LIST OF SYMBOLS (See also Fig.2)

- $E_a$ = electrical field vector set up at the tracker centre $O$ by the larger source $A$
- $E_0$ = electrical field vector $O$ due to smaller source $B$
- $E_x$ = resultant electric field vector at $X$ set up by both jamming sources $A$ and $B$
- $|E_a|, |E_b|, |E_x|$ = magnitudes of $E_a, E_b, E_x$ respectively
- $\delta$ = phase lead of $E_b$ relative to $E_a$ at $O$ (jamming source phase relationship)
- $\delta + \nu$ = jamming source phase relationship at $X$
LIST OF SYMBOLS (Cont'd)

\( s \) = phase of lead \( E_x \) relative to \( E_a \)

\( \theta \) = angular tracker bias caused by artificial glint jamming

\( \psi \) = angular separation of jamming sources A and B measured at tracker centre O

\( \rho \) = tracker beamwidth between "half power" points

\( K = 20/\psi \) = linear bias caused by jamming

\( \overline{K} \) = mean value of \( K \) when \( \delta \) is varying

\( K_{\text{max}} \) = maximum value of \( K \) when \( \delta \) is varying

\( K_0 \) = mean value of \( K \) when skin echo is present

\( \lambda \) = wavelength

\( r \) = tracker to target range

\( m \) = \( \log_e \left( \frac{|E_a|}{|E_b|} \right) \)

\( |E_s| \) = apparent jammer signal strength measured at the tracker

\( V_1, V_2 \) = resultant signals set up by jamming in the aerial feeds of a monopulse tracker

\( V_s \) = sum signal in monopulse tracker

\( V_d \) = difference signal in monopulse tracker

\( |V_s| \) and \( |V_d| \) = amplitudes of \( V_s \) and \( V_d \) respectively

\( \zeta \) = difference of phase shifts in the sum and difference channels of monopulse tracker

\( \sigma \) = dish separation of "twin dish" tracker

\( |E_o| \) = E.M.S. skin echo amplitude

\( \sigma |E_o| \) = instantaneous skin echo amplitude

\( 2\sigma \exp(-\sigma^2) \) = defines Rayleigh amplitude probability distribution of \( \sigma \)

\( \eta \) = tracker servo correction signal
<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Title, etc.</th>
</tr>
</thead>
</table>
**LIST OF REFERENCES (Cont'd)**

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Title, etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>Rhodes, D.R.</td>
<td>Introduction to monopulse. McGraw Hill (Publisher)</td>
</tr>
<tr>
<td>14</td>
<td>Phillips, E.G.</td>
<td>Functions of a complex variable. Oliver and Boyd (Publisher) (Page 40)</td>
</tr>
<tr>
<td>15</td>
<td>Kennelly</td>
<td>Tables of complex hyperbolic and circular functions. (Page 74)</td>
</tr>
<tr>
<td>16</td>
<td>Reggia, F.</td>
<td>A new technique in ferrite phase shifting for beam scanning microwave aerials.</td>
</tr>
</tbody>
</table>

**ATTACHED:-**

- Appendices 1-3
- Drgs. Nos. Rad/P.5535-5542
- Detachable Abstract Cards
APPENDIX 1

R.F. FIELD SET UP BY TWO COHERENT ISOTROPIC RADIATING SOURCES

The geometry of two coherent isotropic R.F. sources A and B, and a
tracker with receiver aerial centre at O, is shown in Fig.2. Polar co-ordinates
with origin A are used. OX is the circular arc centre A and radius r.

At any point \(X(r, \alpha)\) on OX, the electric field vector produced by source A
has equal amplitude \(|E_A|\), and equal phase. As phase is a purely relative
quantity, this will hereafter be used as the reference and defined as zero.

The electric field vector produced by source B at \(X\) is given by:

\[
|E_b| \exp\{i(\delta + \nu)\} \left( \frac{BO}{BX} \right).
\]

From a simple consideration of radiation path lengths,

\[
\nu = \frac{2\pi}{\lambda} (BO - BX).
\]

By trigonometry:

\[
BO^2 = r^2 + AB^2 - 2r AB \cos \beta,
\]

\[
BX^2 = r^2 + AB^2 - 2r AB \cos (\beta - \alpha),
\]

and

\[
BO^2 - BX^2 = 2r AB (\cos (\beta - \alpha) - \cos \beta);
\]

whence

\[
BO - BX = \frac{2r AB (\sin \beta \sin \alpha + \cos \beta (\cos \alpha - 1))}{BO + BX}.
\]

If \(\alpha\) is small the term: \(\cos \beta (\cos \alpha - 1)\), can be neglected and the term
\(BO + BX\) in the denominator is extremely closely equal to \(2BO\).

Then

\[
BO - BX = \frac{2r AB \sin \beta \sin \alpha}{2BO}.
\]

By trigonometry:

\[
\frac{\sin \beta}{BO} = \frac{\sin \psi}{AB},
\]
whence
\[
B_0 - B_X = r \sin \psi \sin \alpha,
\]
\[
\nu = \frac{2\pi}{\lambda} r \sin \psi \sin \alpha,
\]
\[
\approx \frac{2\pi}{\lambda} \psi \alpha,
\]  
(A.3)

if \(\psi\) and \(\alpha\) are small; and
\[
\frac{d\nu}{d\alpha} = \frac{2\pi}{\lambda} \sin \psi \cos \alpha,
\]
\[
\approx \frac{2\pi}{\lambda} \psi.\]  
(A.4)

A displaced source \(B\) is thus characterised by setting up a variation of phase along \(OX\), at a rate proportional to the sine of the angle of displacement.

The resultant electric field vector at \(X\) is given by:
\[
|E_x| \exp (j\epsilon) = |E_a| + |E_b| \exp \left\{ j(\delta + \nu) \right\} \frac{\langle B_0 \rangle}{\langle B_X \rangle},
\]  
(A.5)

neglecting the inverse law term \((B_0/B_X)\).

By plotting the vector diagram of \(E_a, E_b\) and \(E_x\) as in Fig. 5 it may be seen that:
\[
|E_x| = \sqrt{|E_a|^2 + |E_b|^2 + 2 |E_a| |E_b| \cos (\delta + \nu)};
\]  
(A.6)

and
\[
\tan \epsilon = \frac{|E_b| \sin (\delta + \nu)}{|E_a| + |E_b| \cos (\delta + \nu)}.
\]  
(A.7)

From these relations the curves in Fig. 2 of \(|E_x|\) and \(\epsilon\) can be plotted.

The periodic variation of \(\epsilon\) is equivalent to a periodic displacement of the wavefront (contour of constant phase) from the arc \(OX\).
APPENDIX 2

THE CONDITION THAT CONTOURS OF EQUAL BIAS ON THE TRACKER RESPONSE POLAR CHARTS ARE CIRCLES

Monopulse trackers, as far as tracking in each of the two angular dimensions are concerned, function by comparing incoming signals $V_1$ and $V_2$ in two aerial feeds, and deriving therefrom a correction signal for the servo system. It is essential for correct tracker operation, i.e. optimum servo performance irrespective of size of target, that the servo correction signal is a function only of the ratio $V_2/V_1$ (Ref.13). This ratio is in general a complex quantity. It is the function of the receiver circuitry to extract a real quantity to be the servo correction signal.

In most, if not all, monopulse trackers the condition of zero servo correction signal (tracker lock) may be expressed as an equation:

$$\text{Real part of } (F) = 0; \quad (A.8)$$

where

$$F = \left\{ \frac{a + b \frac{V_2}{V_1}}{d + g \frac{V_2}{V_1}} \right\}; \quad (A.9)$$

$a, b, c, g$ being complex constants. This condition certainly holds for a pure amplitude comparison monopulse ($a = -b = d = g$), for pure phase comparison monopulse ($ja = jb = d = g$), and for the generalised sum and difference monopulse discussed in Appendix 3 ($a \exp (-j\zeta) = -b \exp (-j\zeta) = d = g$).

As far as is known by the author, any monopulse tracker fulfilling the criterion that the servo correction signal is a function only of the ratio $V_2/V_1$, also satisfies the conditions in $(A.8)$ and $(A.9)$ (Ref.13).

If the condition expressed in equations $(A.8)$ and $(A.9)$ is satisfied the following proof is valid.

Let $P_1(\theta)$ and $P_2(\theta)$ be the complex functions specifying the response of the two tracker aerials.

Then

$$V_1 = E_a P_1\left(\theta - \frac{\pi}{2}\right) + E_b P_1\left(\theta + \frac{\pi}{2}\right),$$

$$V_2 = E_a P_2\left(\theta - \frac{\pi}{2}\right) + E_b P_2\left(\theta + \frac{\pi}{2}\right),$$

and

$$\frac{V_2}{V_1} = \left\{ \frac{P_2\left(\theta - \frac{\pi}{2}\right) + \frac{E_b}{E_a} P_2\left(\theta + \frac{\pi}{2}\right)}{P_1\left(\theta - \frac{\pi}{2}\right) + \frac{E_b}{E_a} P_1\left(\theta + \frac{\pi}{2}\right)} \right\}. \quad (A.10)$$

SECRET
For a constant bias $\theta$: $P_2\left(\theta + \frac{\pi}{2}\right)$, $P_2\left(\theta - \frac{\pi}{2}\right)$, $P_1\left(0 + \frac{\pi}{2}\right)$, and $P_1\left(0 - \frac{\pi}{2}\right)$, are complex constants. Equation (A.10) can then be regarded as a Mobius or bilinear transformation\textsuperscript{13,14}, in the theory of complex variables, transforming points on the plane of one complex variable $(\frac{E_b}{E_a})$ to the plane of a new complex variable $(\frac{V_2}{V_1})$. Similarly, equation (A.9) is also a bilinear transformation, transforming from the complex plane of $(\frac{V_2}{V_1})$ to the complex plane of $F$.

Properties of bilinear transformations are that they are reversible (i.e., the inverse of a bilinear transformation is also a bilinear transformation), and that points lying on a circular or straight line (circle of infinite radius) locus on one complex plane are transformed to points lying on a circular or straight line locus on the new complex plane.

Equation (A.8) represents a straight line on the plane of the complex variable $F$. Points on this locus must therefore lie on circular or straight line loci on the complex planes of the variables $(\frac{V_2}{V_1})$ and $(\frac{E_b}{E_a})$. The tracker response polar chart is essentially a plot of points on the $(\frac{E_b}{E_a})$ plane. For any specified bias $\theta$ the condition of zero servo correction signal must be represented by a circle on the polar chart.
APPENDIX 3

EFFECT OF CIRCUIT IMPERFECTIONS ON THE RESPONSE OF A SUM AND DIFFERENCE MONOPULSE TRACKER TO ARTIFICIAL GLINT JAMMING (IMPERFECT SENSING)

The basic sum and difference type monopulse tracker using an amplitude comparison (squinted beams) aerial system is shown in Fig. 4. If a phase comparison (interferometer) aerial system is used, the circuitry is identical except that a fixed π/2 radians phase shift is inserted in the difference channel. The twin diode second detector gives outputs proportional to the amplitudes of the vector sum and the vector difference of its sum and difference inputs. The difference of the two diode output signals becomes the servo correction signal η. If correct A.C.C. is employed in the I.F. amplifiers, the servo correction signal η is given approximately by equation (3).

\[ \eta = \text{real part of} \left( \frac{V_d}{V_s} \right) \]

and is zero when V_d and V_s are in quadrature or when |V_d| equals zero.

If the I.F. amplifiers have equal phase shifts, V_d and V_s will be in quadrature if the aerial system is aligned on the mean wavefront over its aperture, irrespective of the R.F. amplitude distribution in the aperture. That is to say the sensing will be perfect as defined in Section 2.1.

If the I.F. amplifiers have a difference of phase shifts given by ζ radians, the sensing is imperfect. When tracking on normal single targets, imperfect sensing is no disadvantage to a tracker as the wavefront of the normal return signal has a uniform amplitude distribution, and a sharp zero |V_d| is produced when the tracker is on target. Imperfect sensing thus does not mean a tracker asymmetry. To a first order of magnitude ζ does not affect tracker accuracy. For example in the A.I.23 monopulse tracker a tolerance on ζ of ±50° is quoted by the manufacturers.

The effect of ζ on the response of a tracker to artificial glint jamming is quite appreciable. At medium and short ranges, as discussed in Section 2.2, the response of a tracker depends in detail on its aerial polar diagram, but for all trackers with equal values of θ/ρ the response is similar. Similarly the effect of ζ is basically similar, but not absolutely identical, for all trackers. To demonstrate the effect, the response will be calculated for the sum and difference of tracker defined in Section 2.2, but with the difference that d/s is no longer real, but has a phase angle or argument ζ. This means that equation (4) must be rewritten as:

\[ \text{real part of} \left( \exp (jζ) \right) \left[ K - \tanh \left( \frac{0.69K}{\rho} \right)^2 + \frac{m - j\delta}{2} \right] = 0 \quad (A.11) \]

This equation has been evaluated numerically, as in Section 2.2, and the results plotted in the polar chart in Fig. 3d for the value of θ/ρ of 0.1. The contours of equal bias K are circles, as the criterion of Appendix 2 is still satisfied.
At long ranges where $\psi/\rho$ is very small equation (A.11) simplifies to:

$$\text{real part of } \left( \exp j\zeta \right) \left[ K - \frac{E_a - E_b}{E_a + E_b} \right] = 0 \quad (A.12)$$

This is a general equation for the response at long ranges of all sum and difference monopulse trackers with imperfect sensing caused by a difference in phase shifts of $\zeta$ in the sum and difference channels. It can be verified as a cross-check, that if $\zeta = 0$ equation (A.12) reduces to equation (1) of Section 2.1.
FIG. 1. ARTIFICIAL GLINT JAMMER CIRCUITS.

(a) DOUBLE AMPLIFIER CIRCUIT.

(b) SINGLE TUBE CIRCUIT.
FIG. 2. DUAL SOURCE GEOMETRY AND RADIATION PATTERN.
CONTOURS OF CONSTANT $K$ SHOWN SOLID
CONTOURS OF CONSTANT $|E_s|/|E_0|$ SHOWN DOTTED.

(a) LONG RANGE CASE $\frac{\psi}{\rho} = 0$

FIG. 3(c)

3. TRACKER RESPONSE POLAR CHART.
FIG. 3. TRACKER RESPONSE POL
CONTOURS OF CONSTANT K SHOWN SOLID
CONTOURS OF CONSTANT $|E_0|$ SHOWN DOTTED

(b) MEDIUM RANGE CASE $\frac{\psi}{\rho} = 1$

KER RESPONSE POLAR CHART.
FIG. 3. TRACKER RESPONSE POLAR CHART.

CONTOURS OF CONSTANT \( K \) SHOWN SOLID
CONTOURS OF CONSTANT \( \frac{|E_E|}{|E_0|} \) SHOWN DOTTED

(c) SHORT RANGE CASE \( \frac{\psi}{\rho} = .4 \)
CONTOURS OF CONSTANT K SHOWN SOLID

CONTOURS OF CONSTANT $|\frac{E_1}{E_0}|$ SHOWN DOTTED

(d) MEDIUM RANGE CASE WITH $\zeta = 30^\circ$ AND $\frac{\hat{y}}{\hat{p}} = 0.1$ (IMPERFECT SENSING AS DISCUSSED IN APPENDIX 3)

KER RESPONSE POLAR CHART.
FIG. 4. SUM AND DIFFERENCE MONOPULSE TRACKER CIRCUIT.
FIG. 5. VECTOR DIAGRAM OF SIGNALS NEAR TRACKER CENTRE.
These abstract cards are inserted in reports and technical notes for the convenience of librarians and others who need to maintain an information index. Detached cards are subject to the same security regulations as the parent document, and a record of their location should be made on the inside of the back cover of the parent document.
A practicable artificial glint jammer circuit, using a single R.F. tube and a ferrite phasing device, is proposed.
This document is now available at the National Archives, Kew, Surrey, United Kingdom.

DTIC has checked the National Archives Catalogue website (http://www.nationalarchives.gov.uk) and found the document is available and releasable to the public.

Access to UK public records is governed by statute, namely the Public Records Act, 1958, and the Public Records Act, 1967. The document has been released under the 30 year rule. (The vast majority of records selected for permanent preservation are made available to the public when they are 30 years old. This is commonly referred to as the 30 year rule and was established by the Public Records Act of 1967).

This document may be treated as UNLIMITED.